DYNAMIC MODEL

Rotor Thrust Mapping

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -l & 0 & l \\ l & 0 & -l & 0 \\ c & -c & c & -c \end{bmatrix}, \quad b = \begin{bmatrix} T_{\text{total}} \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix}$$

$$T_{\text{cmd}} = A^{-1} b$$

$$\dot{T} = \frac{T_{\text{cmd}} - T_{\text{act}}}{\tau_{\text{motor}}}$$

$$T_{\text{act}} = T_{\text{act}} + \dot{T} \times dt$$

$$\tau : motor\ time\ constant$$

Accelerations

$$a_x = \frac{(-c_\phi s_\theta c_\psi - s_\phi s_\psi) F_z + F_{x,slung} + F_{x,wind}}{m_{total}}$$

$$a_y = \frac{(-c_\phi s_\theta s_\psi + s_\phi c_\psi) F_z + F_{y,slung} + F_{y,wind}}{m_{total}}$$

$$a_z = \frac{c_\phi c_\theta F_z + F_{z,slung} + F_{z,wind}}{m_{total}} - g$$

$$\dot{p} = \frac{\tau_\phi}{I_{xx}} \qquad \dot{q} = \frac{\tau_\theta}{I_{yy}} \qquad \dot{r} = \frac{\tau_\psi}{I_{zz}}$$

Moments

$$\tau_\phi = l\big(-T_2 + T_4\big) + \tau_{\phi,\text{wind}} + \tau_{\phi,\text{slung}}$$

$$\tau_\theta = l\big(T_1 - T_3\big) + \tau_{\theta,\text{wind}} + \tau_{\theta,\text{slung}}$$

$$\tau_\psi = c\big(T_1 - T_2 + T_3 - T_4\big) + \tau_{\psi,\text{wind}}$$

Assumptions:
- The quadrotor is modeled as a rigid body.
- 1st order motor model.
- Sensor noises are included.
- Wind disturbances are applied both as vertical force and torques.

SLUNG LOAD DYNAMICS

Assumptions:

- The cable has constant length and has no mass.
- The load is considered a point mass.
- Movement in the x–z and y–z planes is independent of each other (decoupled).

Parameters:

L : cable length,
d : damping coefficient,
m_L : slung load mass,
g : gravitational acceleration,
a_x, a_y : accelerations of the quadrotor along x and y axes

$$\mathbf{x} = \begin{bmatrix} \alpha & \dot{\alpha} & \beta & \dot{\beta} \end{bmatrix}^T$$

$$\left.\begin{aligned} \ddot{\alpha} &= -\frac{g}{L}\sin\alpha - d\,\dot{\alpha} - \frac{a_x}{L} \\ \ddot{\beta} &= -\frac{g}{L}\sin\beta - d\,\dot{\beta} - \frac{a_y}{L} \end{aligned}\right\} \text{Angular accelerations}$$

Gravitational Effect

Damping Effect

External Acceleration

The cable tension is calculated as:

$$T = m_L \left(g\cos(\alpha)\cos(\beta) + L\left(\dot{\alpha}^2 + \dot{\beta}^2\right)\right)$$

The slung load exerts the following forces on the quadcopter:

$$F_{slung} = \begin{bmatrix} -T_{\text{cable}} \sin\alpha\,\cos\beta \\ -T_{\text{cable}} \sin\beta \\ T_{\text{cable}} \cos\alpha\,\cos\beta \end{bmatrix}$$

Also creates moments about the quadcopter's center of mass:

$$\tau_{\phi,slung} = -h \cdot F_{x,slung}$$

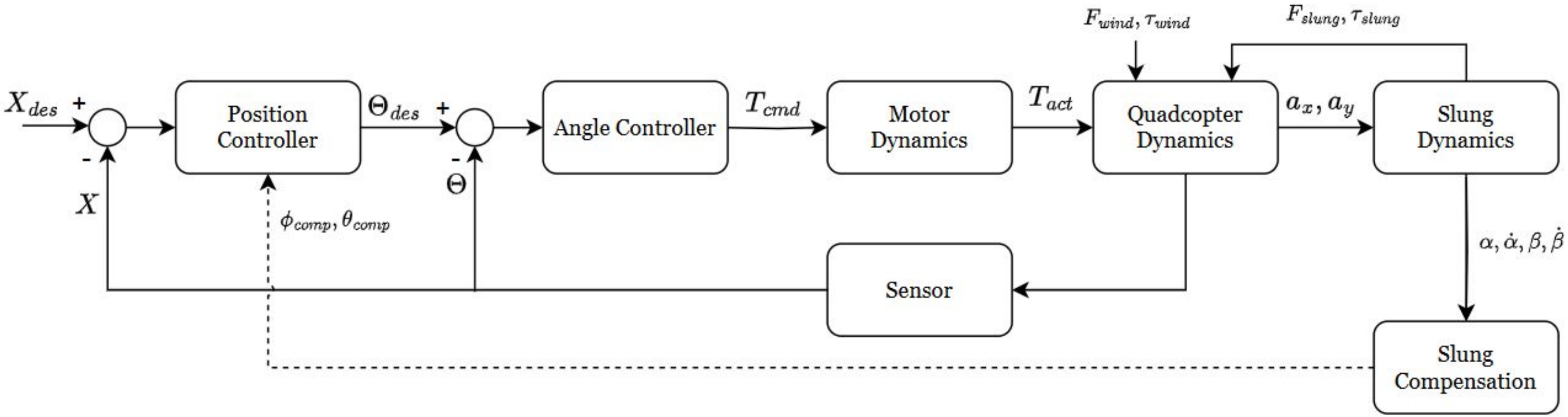
$$\tau_{\theta,slung} = h \cdot F_{y,slung}$$

where h is the vertical distance from the quadcopter's center of mass to the cable attachment point.

SLUNG LOAD COMPENSATION

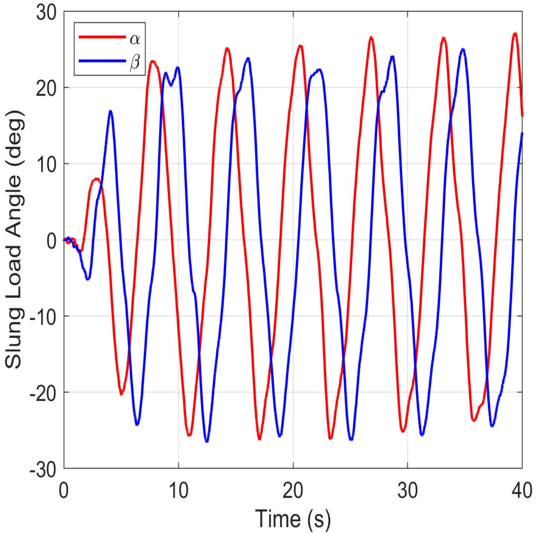
During aggressive maneuvers with heavy loads, high amplitude oscillations were observed, threatening system stability. To dampen these oscillations, a PD control based compensation mechanism was integrated into the position controller. The compensation system is designed under the assumption that the slung angles (α, β) and angular velocities are directly obtained, and it applies the necessary correction signals to the quadcopter's pitch and roll angles using this information. In real applications, state estimation techniques such as Kalman filtering can be utilized due to measurement limitations.

$$a_{x,comp} = -L\left(K_p\alpha + K_d\dot{\alpha}\right) \qquad \theta_{comp} = \arctan\left(\frac{a_{x,comp}}{g}\right)$$
$$a_{y,comp} = -L\left(K_p\beta + K_d\dot{\beta}\right) \qquad \phi_{comp} = -\arctan\left(\frac{a_{y,comp}}{g}\right)$$
$$\Rightarrow \quad \theta_{des} = \theta_{p,des} + \theta_{comp}, \qquad \phi_{des} = \phi_{p,des} + \phi_{comp}$$

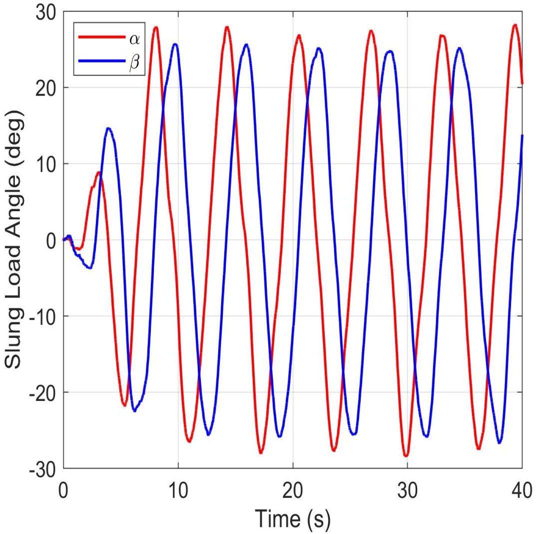


SIMULATION RESULTS

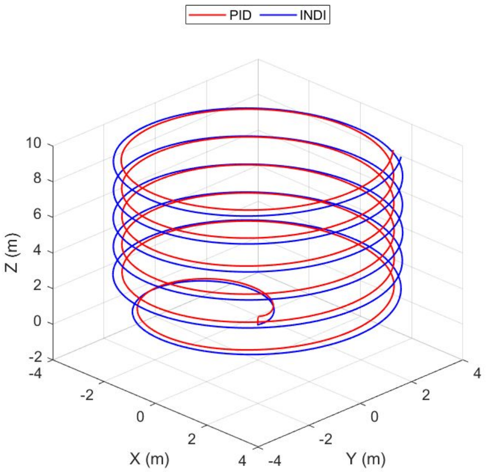
In the first scenario, trajectory tracking was performed with a heavy load, and both the PID and INDI controllers performed command tracking well. Since there were no sudden or aggressive changes in the commands, no high amplitude oscillations were observed in the load. The load oscillated within the range of -25 to 25 degrees on both axes.



PID

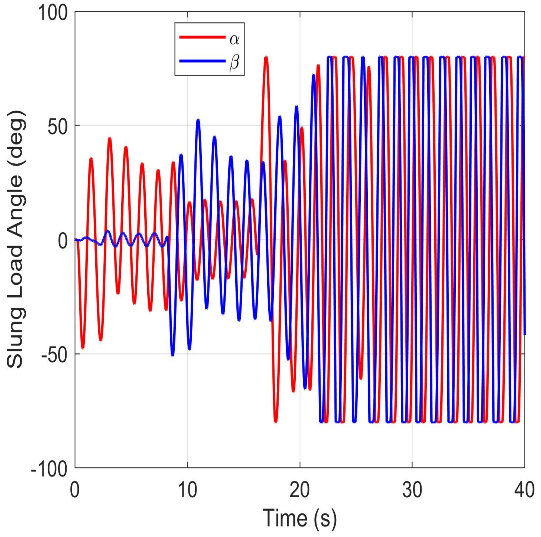


INDI

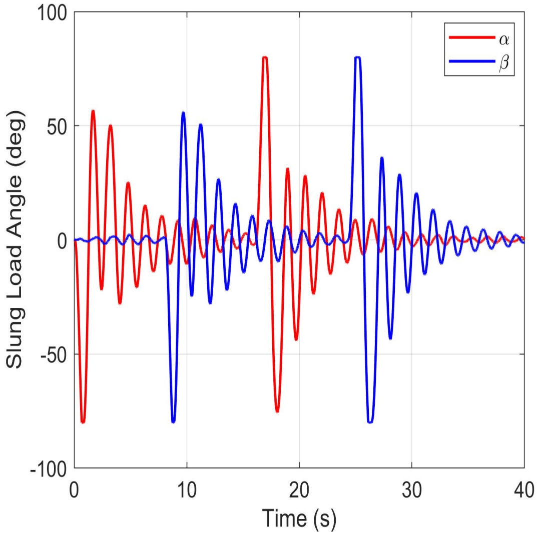


SIMULATION RESULTS

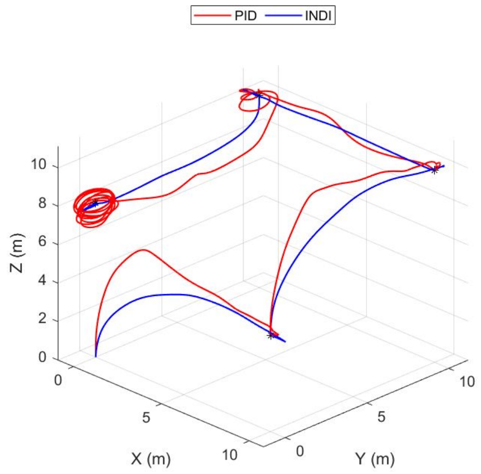
In the second scenario, aggressive commands were applied with a heavy load. While INDI control showed better results against sudden changes, high amplitude oscillations occurred in PID due to the slung effect.



PID

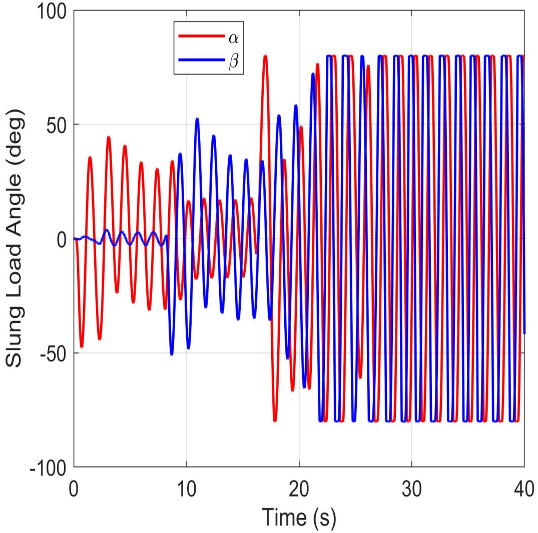


INDI

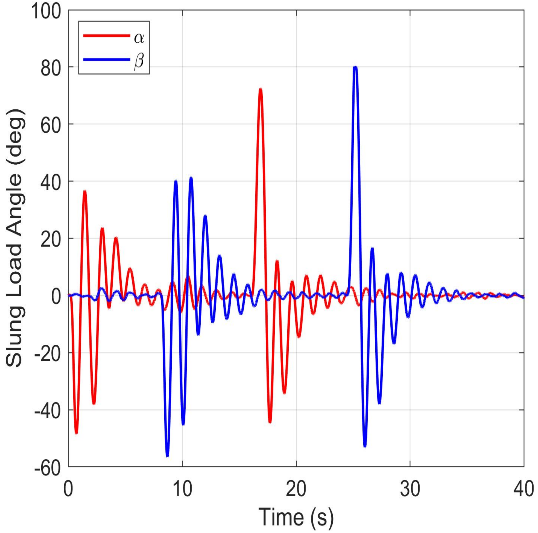


SIMULATION RESULTS

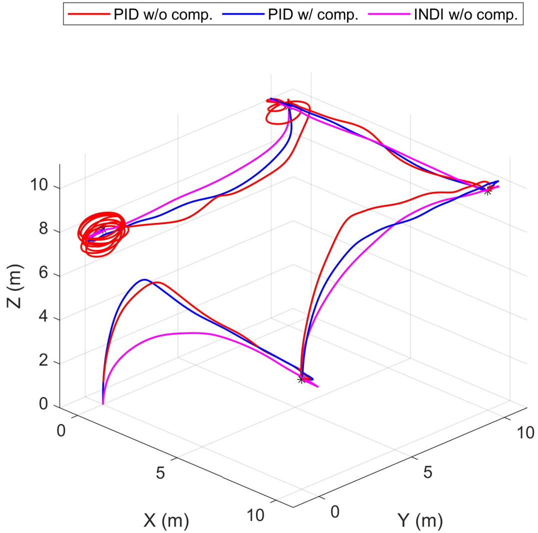
Finally, slung compensation was activated and the high amplitude oscillations that occurred in the previous scenario were damped, and command tracking with PID was successfully performed.



PID without Compensation



PID with Compensation



CONCLUSION

This study investigates the changes in quadcopter dynamics caused by the slung load effect, the impact of these changes on control systems, and the damping of these effects using PD based compensation. By removing the assumption that the α and β dynamics in the current model are independent of each other, coupled slung load dynamics can be developed, thereby enabling more realistic load behaviour to be modelled. Furthermore, by using advanced state estimation techniques such as the Kalman filter or Extended Kalman Filter instead of the assumption that swing angles can be measured directly, more realistic and higher accuracy simulation results can be obtained.

References

- Manalathody, Abhishek & Krishnan, Kishoor & Subramanian, Jisnu & Thangavel, Sakthivel & Pushpangathan, Jinraj & Paranjothy, Hariprasanth. (2025). Nonlinear and Linear PID Controllers-Based Hybrid Flight Control Strategy for a Quadcopter With Slung Load. IEEE Access. PP. 1-1. 10.1109/ACCESS.2025.3551622.

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