DYNAMIC MODEL

Assumptions:
- The quadrotor is modeled as a rigid body.
- Only vertical translation is considered for translational motion.
- 1st order motor model.
- Sensor noises are included.
- Wind disturbances are applied both as vertical force and torques.

Rotor Thrust Mapping

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -l & 0 & l \\ l & 0 & -l & 0 \\ c & -c & c & -c \end{bmatrix}, \quad b = \begin{bmatrix} T_{\text{total}} \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix}$$

$$T_{\text{cmd}} = A^{-1} b$$

$$\dot{T} = \frac{T_{\text{cmd}} - T_{\text{act}}}{\tau_{\text{motor}}}$$

$$T_{\text{act}} = T_{\text{act}} + \dot{T} \times dt$$

τ *: motor time constant*

Total Force and Moments

$$F_z = \sum T_{\text{act}} + F_{z,\text{wind}}$$

$$\tau_\phi^{\text{act}} = l \times (-T_{\text{act}}(2) + T_{\text{act}}(4)) + \tau_{\phi,\text{wind}}$$

$$\tau_\theta^{\text{act}} = l \times (T_{\text{act}}(1) - T_{\text{act}}(3)) + \tau_{\theta,\text{wind}}$$

$$\tau_\psi^{\text{act}} = c \times (T_{\text{act}}(1) - T_{\text{act}}(2) + T_{\text{act}}(3) - T_{\text{act}}(4)) + \tau_{\psi,\text{wind}}$$

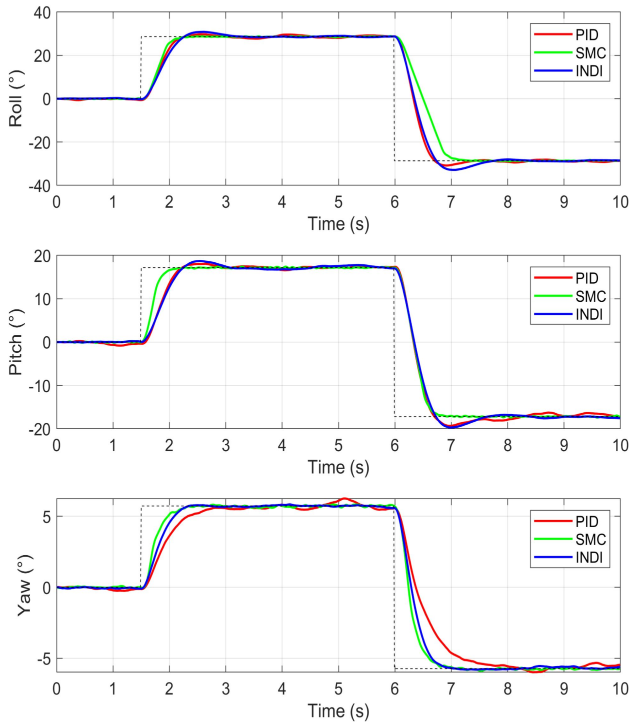
Accelerations

$$a_z = F_z/m - g$$

$$\dot{p} = \tau_\phi^{\text{act}}/I_{xx}$$

$$\dot{q} = \tau_\theta^{\text{act}}/I_{yy}$$

$$\dot{r} = \tau_\psi^{\text{act}}/I_{zz}$$



PID CONTROL

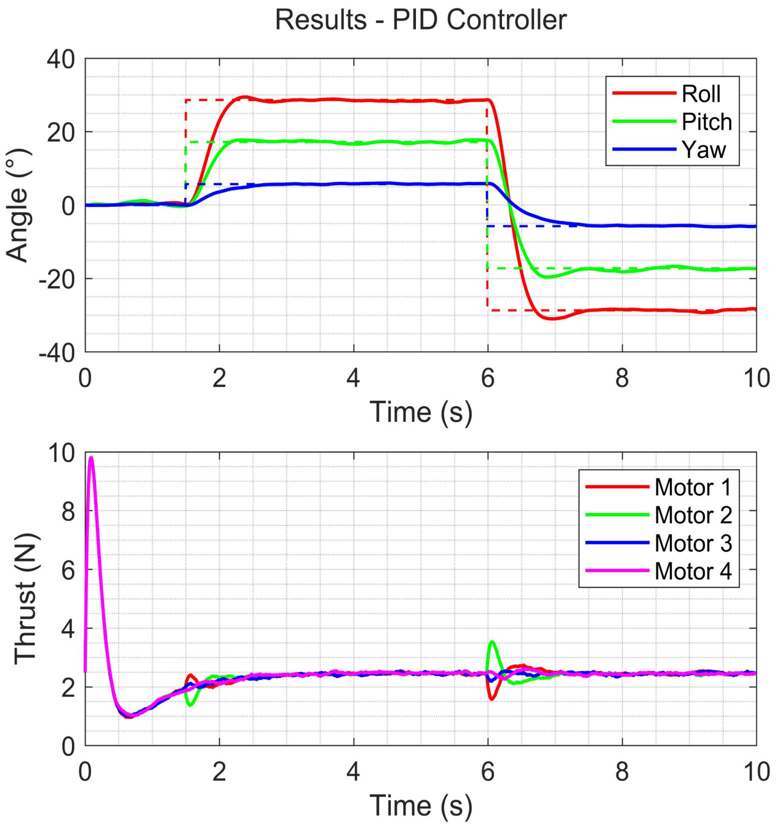
Outer loop for angle control

$$p_{\text{des}} = K_p^{\phi}(\phi_{\text{des}} - \phi_{\text{meas}})$$
$$q_{\text{des}} = K_p^{\theta}(\theta_{\text{des}} - \theta_{\text{meas}})$$
$$r_{\text{des}} = K_p^{\psi}(\psi_{\text{des}} - \psi_{\text{meas}})$$

Inner loop for angular velocity control

$$\tau_{\phi} = I_{xx}\left(K_p^p e_p + K_i^p \int e_p \, dt + K_d^p \dot{e}_p\right)$$
$$\tau_{\theta} = I_{yy}\left(K_p^q e_q + K_i^q \int e_q \, dt + K_d^q \dot{e}_q\right)$$
$$\tau_{\psi} = I_{zz}\left(K_p^r e_r + K_i^r \int e_r \, dt + K_d^r \dot{e}_r\right)$$

$$\begin{aligned} & e_p = p_{\text{des}} - p_{\text{meas}} \\ where,\ & e_q = q_{\text{des}} - q_{\text{meas}} \\ & e_r = r_{\text{des}} - r_{\text{meas}} \end{aligned}$$



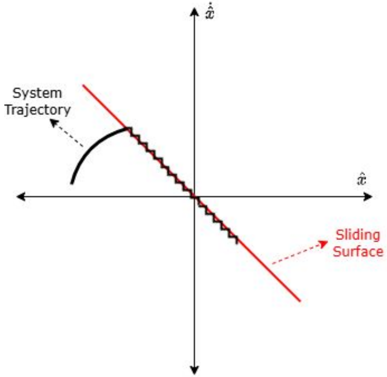
SLIDING MODE CONTROL

Description:
Sliding mode control (SMC) is a robust control method that works by pulling the system dynamics to a predetermined “sliding surface (s)”. When the system remains on this surface, it shows high resistance to uncertainties and disturbances.

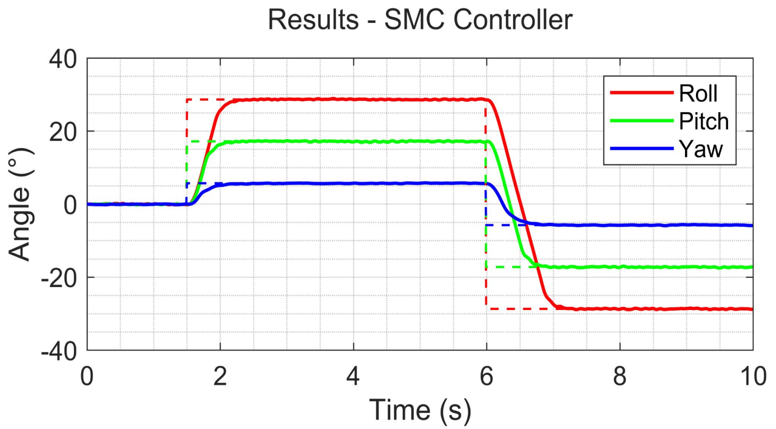
Pros and cons:
Its advantages include its simple structure, fast response and robustness to parameter changes. However, it can lead to some disadvantages in practice due to the generation of high-frequency chattering and the continuous switching of the control signal.

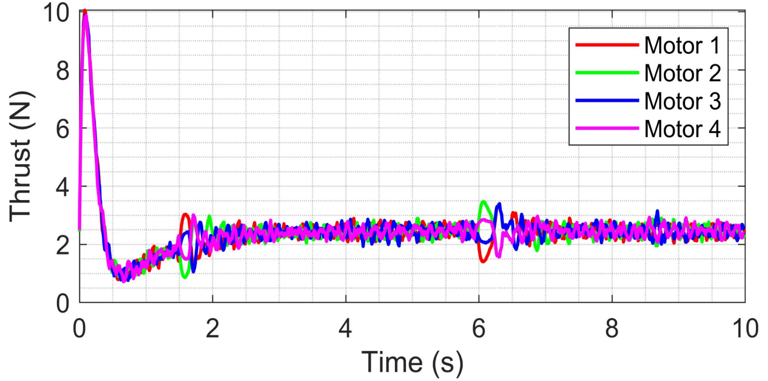
Angle Control

$$e_\phi = \phi_{\text{des}} - \phi_{\text{meas}}$$
$$\dot{e}_\phi = -p_{\text{meas}}$$
$$s_\phi = \lambda_\phi e_\phi + \dot{e}_\phi$$
$$\tau_\phi = I_{xx}(\lambda_\phi \dot{e}_\phi + K_\phi \text{sat}(s_\phi, 0.1))$$



... Same for pitch and yaw angles.





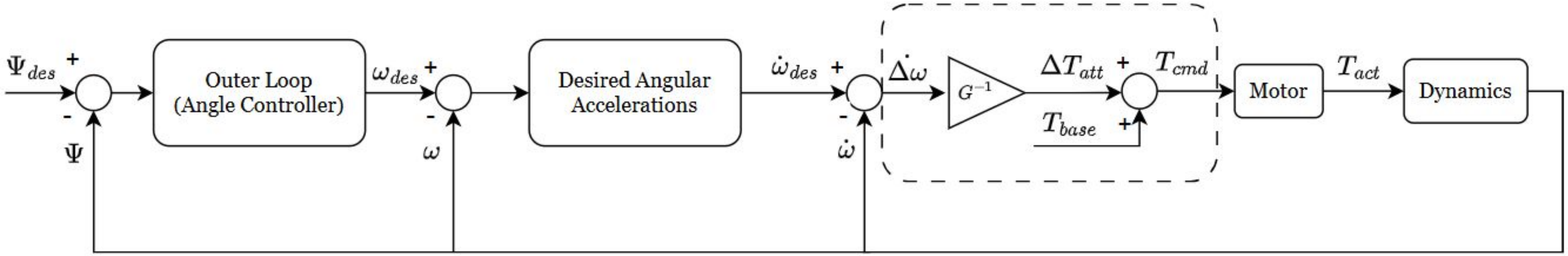
INCREMENTAL NONLINEAR DYNAMIC INVERSION - 1

Description:

Incremental Nonlinear Dynamic Inversion (INDI) is a variation of the traditional Nonlinear Dynamic Inversion (NDI) method and is an advanced control technique used especially in aircrafts.

The basic principle of INDI is to incrementally compute the control action based on measurable quantities such as acceleration without the need for a full dynamic model of the system. This provides high robustness against model uncertainties and external disturbances.

While in the NDI method, the control input is inversely computed over the full model of the system, in INDI only the variation of the dynamics is taken into account, which simplifies the controller design and reduces the problems caused by model errors.



INCREMENTAL NONLINEAR DYNAMIC INVERSION - 2

1. Outer Loop

$$p_{\text{des}} = \mathrm{Kp}_{\phi}(\phi_{\text{des}} - \phi_{\text{meas}})$$
$$q_{\text{des}} = \mathrm{Kp}_{\theta}(\theta_{\text{des}} - \theta_{\text{meas}})$$
$$r_{\text{des}} = \mathrm{Kp}_{\psi}(\psi_{\text{des}} - \psi_{\text{meas}})$$

2. Desired Angular Accelerations

$$\dot{p}_{\text{des}} = K_p e_p + K_i \int e_p \, dt + K_d \dot{e}_p$$
$$\dot{q}_{\text{des}} = K_p e_q + K_i \int e_q \, dt + K_d \dot{e}_q$$
$$\dot{r}_{\text{des}} = K_p e_r + K_i \int e_r \, dt + K_d \dot{e}_r$$

3. Calculate Control Increments

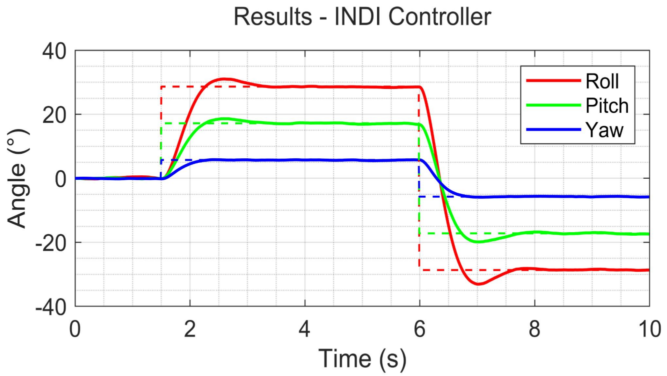
$$\Delta\dot{p} = \dot{p}_{\text{des}} - \dot{p}$$
$$\Delta\dot{q} = \dot{q}_{\text{des}} - \dot{q}$$
$$\Delta\dot{r} = \dot{r}_{\text{des}} - \dot{r}$$

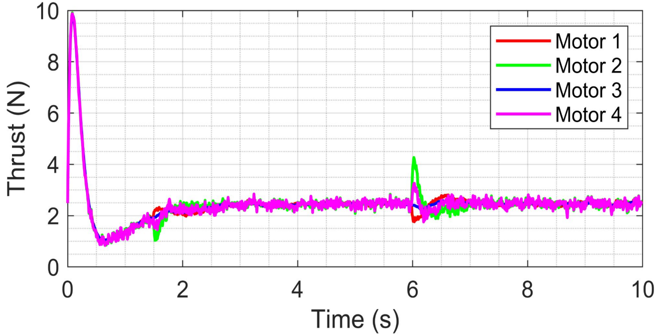
4. Calculate Thrust Increments

$$\Delta T_{\text{att}} = \underbrace{\begin{bmatrix} 0 & -\frac{l}{I_{xx}} & 0 & \frac{l}{I_{xx}} \\ \frac{l}{I_{yy}} & 0 & -\frac{l}{I_{yy}} & 0 \\ \frac{c}{I_{zz}} & -\frac{c}{I_{zz}} & \frac{c}{I_{zz}} & -\frac{c}{I_{zz}} \end{bmatrix}^{-1}}_{G} \begin{bmatrix} \Delta\dot{p} \\ \Delta\dot{q} \\ \Delta\dot{r} \end{bmatrix}$$

5. Combine Control Law

$$T_{\text{base}} = \frac{T_{\text{total}}}{4} \times [1; \quad 1; \quad 1; \quad 1]$$
$$T_{\text{cmd}} = T_{\text{base}} + \Delta T_{\text{att}}$$





Mertcan A. - 2025