DYNAMIC MODEL

Assumptions:
- The quadrotor is modeled as a rigid body.
- 1st order motor model.
- Sensor noises are included.
- Wind disturbances are applied both as vertical force and torques.

Rotor Thrust Mapping

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -l & 0 & l \\ l & 0 & -l & 0 \\ c & -c & c & -c \end{bmatrix}, \quad b = \begin{bmatrix} T_{\text{total}} \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix}$$

$$T_{\text{cmd}} = A^{-1} b$$

$$\dot{T} = \frac{T_{\text{cmd}} - T_{\text{act}}}{\tau_{\text{motor}}}$$

$$T_{\text{act}} = T_{\text{act}} + \dot{T} \times dt$$

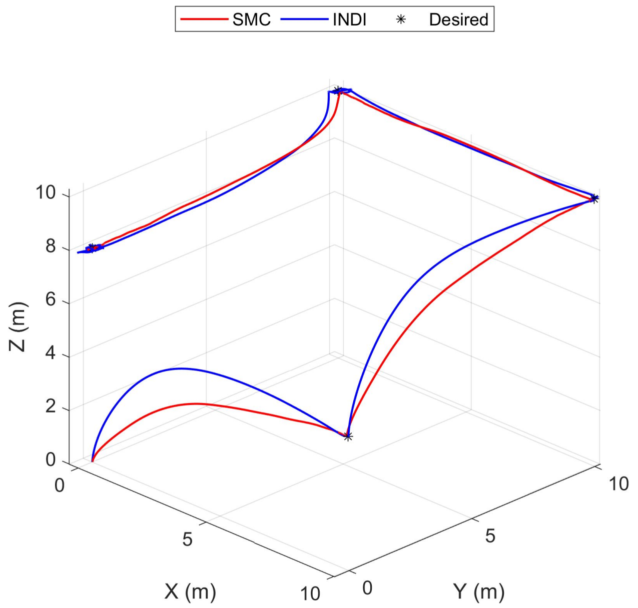
τ : *motor time constant*

Total Force and Moments

$$F_z = \sum T_{\text{act}} + F_{z,\text{wind}}$$
$$\tau_\phi^{\text{act}} = l \times (-T_{\text{act}}(2) + T_{\text{act}}(4)) + \tau_{\phi,\text{wind}}$$
$$\tau_\theta^{\text{act}} = l \times (T_{\text{act}}(1) - T_{\text{act}}(3)) + \tau_{\theta,\text{wind}}$$
$$\tau_\psi^{\text{act}} = c \times (T_{\text{act}}(1) - T_{\text{act}}(2) + T_{\text{act}}(3) - T_{\text{act}}(4)) + \tau_{\psi,\text{wind}}$$

Accelerations

$$\dot{p} = \frac{\tau_\phi}{I_{xx}} \qquad a_x = \frac{-\sin\theta \cdot F_z + F_{x,wind}}{m}$$
$$\dot{q} = \frac{\tau_\theta}{I_{yy}} \qquad a_y = \frac{\sin\phi \cdot F_z + F_{y,wind}}{m}$$
$$\dot{r} = \frac{\tau_\psi}{I_{zz}} \qquad a_z = \frac{F_z}{m} - g$$



SLIDING MODE CONTROL

Position Control

$$s_x = \lambda_x e_x - \dot{x}_{\text{meas}}$$
$$\theta_{\text{des}} = -K_x \cdot \text{sat}(s_x, 0.2)$$

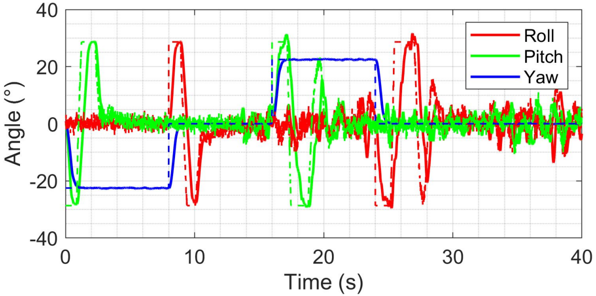
$$s_y = \lambda_y e_y - \dot{y}_{\text{meas}}$$
$$\phi_{\text{des}} = K_y \cdot \text{sat}(s_y, 0.2)$$

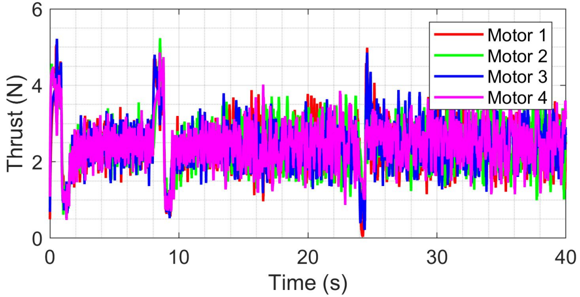
Angle Control

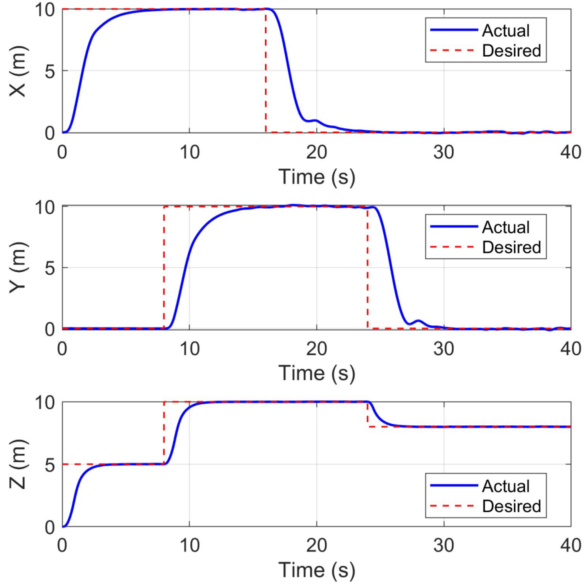
$$e_\Theta = \Theta_{\text{des}} - \Theta_{\text{meas}}$$
$$\dot{e}_\Theta = -\omega_{\text{meas}}$$
$$s_\Theta = \lambda_\Theta e_\Theta + \dot{e}_\Theta$$
$$\tau = I_{xx}(\lambda_\Theta \dot{e}_\Theta + K_\Theta \text{sat}(s_\Theta, 0.1))$$

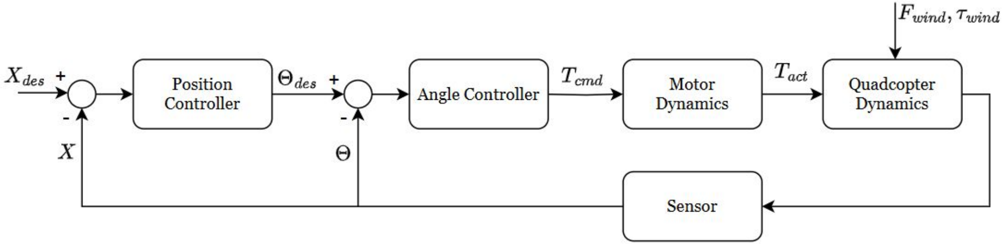
Altitude Control

$$e_z = z_{\text{des}} - z_{\text{meas}}$$
$$s_z = \lambda_z e_z - \dot{z}_{\text{meas}}$$
$$T_{\text{total}} = m\,(g + K_z \cdot \text{sat}(s_z, 0.2))$$









INCREMENTAL NONLINEAR DYNAMIC INVERSION - 1

Position Control (PD)

$$\theta_{des} = -(K_{px}e_x + K_{dx}(-\dot{x}))$$
$$\phi_{des} = K_{py}e_y + K_{dy}(-\dot{y})$$

Angle Control

$$p_{\text{des}} = \text{Kp}_{\phi}(\phi_{\text{des}} - \phi_{\text{meas}})$$
$$q_{\text{des}} = \text{Kp}_{\theta}(\theta_{\text{des}} - \theta_{\text{meas}})$$
$$r_{\text{des}} = \text{Kp}_{\psi}(\psi_{\text{des}} - \psi_{\text{meas}})$$

Desired Angular Accelerations

$$\dot{p}_{\text{des}} = K_p e_p + K_i \int e_p \, dt + K_d \dot{e}_p$$
$$\dot{q}_{\text{des}} = K_p e_q + K_i \int e_q \, dt + K_d \dot{e}_q$$
$$\dot{r}_{\text{des}} = K_p e_r + K_i \int e_r \, dt + K_d \dot{e}_r$$

Calculate Control Increments

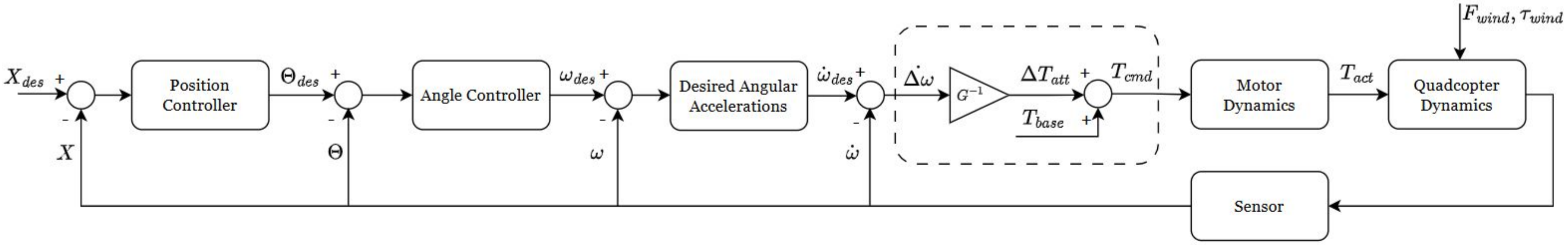
$$\Delta\dot{p} = \dot{p}_{\text{des}} - \dot{p}$$
$$\Delta\dot{q} = \dot{q}_{\text{des}} - \dot{q}$$
$$\Delta\dot{r} = \dot{r}_{\text{des}} - \dot{r}$$

Calculate Thrust Increments

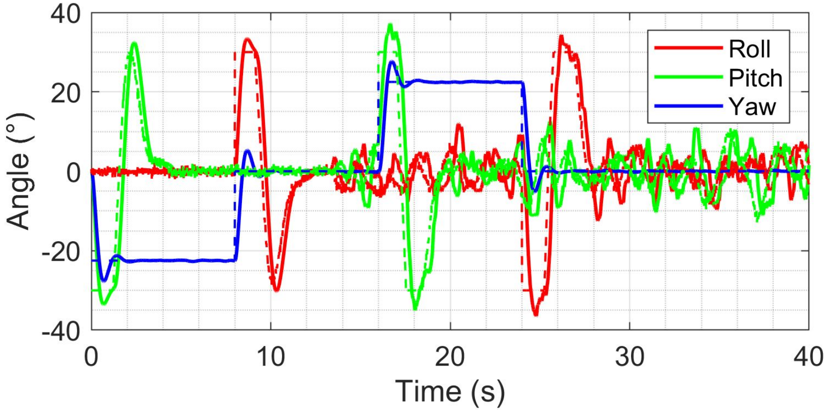
$$\Delta T_{\text{att}} = \begin{bmatrix} 0 & -\frac{l}{I_{xx}} & 0 & \frac{l}{I_{xx}} \\ \frac{l}{I_{yy}} & 0 & -\frac{l}{I_{yy}} & 0 \\ \frac{c}{I_{zz}} & -\frac{c}{I_{zz}} & \frac{c}{I_{zz}} & -\frac{c}{I_{zz}} \end{bmatrix}^{-1} \begin{bmatrix} \Delta\dot{p} \\ \Delta\dot{q} \\ \Delta\dot{r} \end{bmatrix}$$

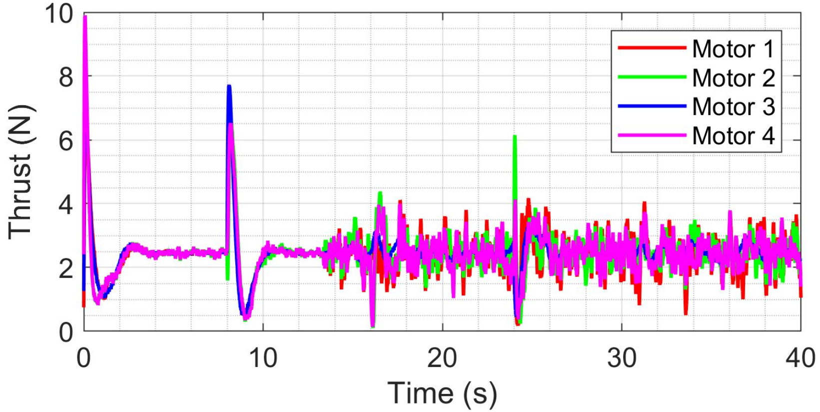
Combine Control Law

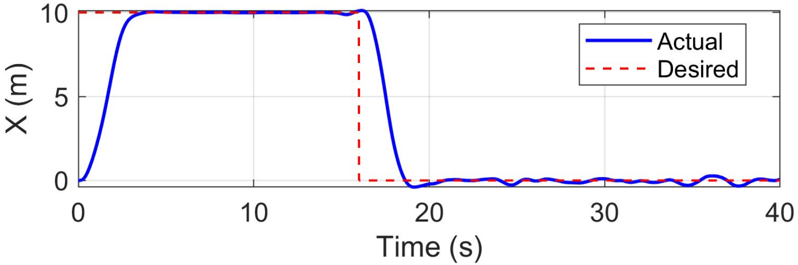
$$T_{\text{base}} = \frac{T_{\text{total}}}{4} \times [1; \quad 1; \quad 1; \quad 1]$$
$$T_{\text{cmd}} = T_{\text{base}} + \Delta T_{\text{att}}$$

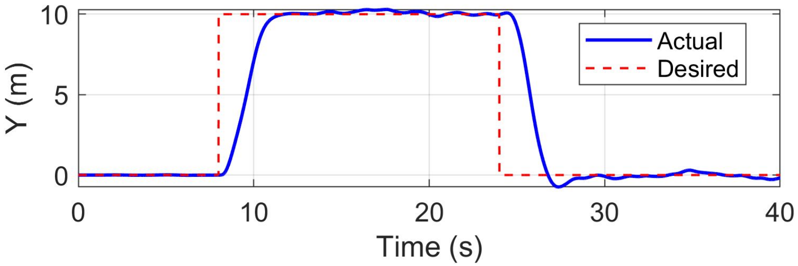


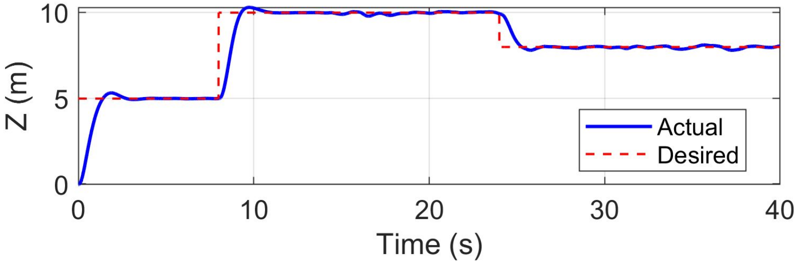
INCREMENTAL NONLINEAR DYNAMIC INVERSION - 2





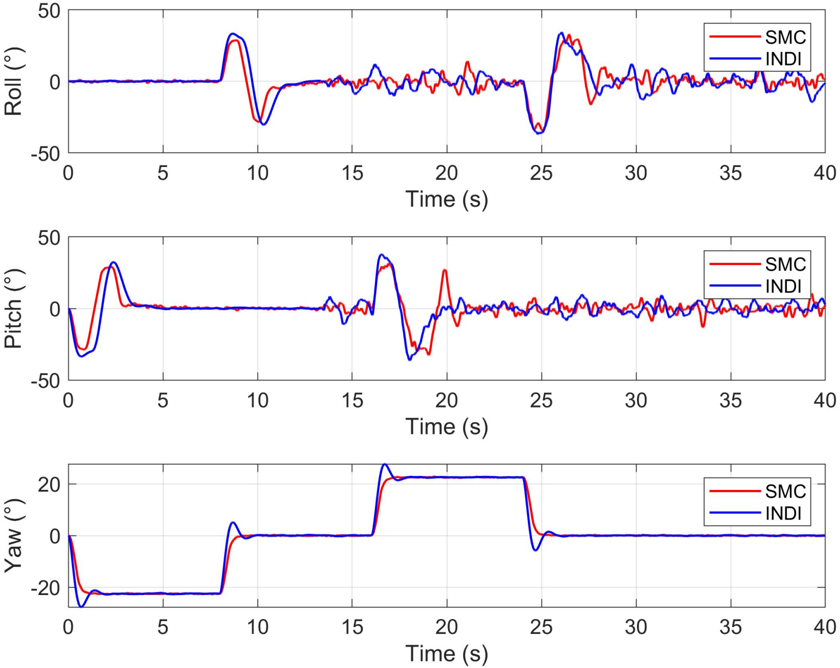






CONCLUSION

In this study, position control is added in addition to the previous study. The angle commands required by the INDI controller are generated by a PD controller. According to the scenario, after the second waypoint, the wind effect is taken into account. With the wind effect, larger oscillations are observed in the output of the INDI. However, by optimizing the control coefficients more precisely, these oscillations can be reduced and the system performance can be further improved. On the other hand, SMC generally performs very well with strong stability characteristics; however, high frequency control signals can be difficult or impossible to physically implement, especially in actuators, and can lead to performance losses of system components in the long term.



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