# An Optimization Problem

IE-202

Course Project Phase 1

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# Dry & Happy

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Dry & Happy

### *Description of the System*

Dry & Happy is a company which sells diapers. Our aim by building this model is to minimize Dry & Happy's cost but also satisfying the total demand for diapers. There is one central factory where all diapers are produced and these are distributed into warehouses each period. Each shop satisfies their demand from the warehouse located closest to it. In our model we assume that there is only a fixed cost for the shop to supply diapers from a certain warehouse, and this fixed cost is determined by the distance from each warehouse. The shop can only supply its diapers from one warehouse, and the maximum amount of diapers it can supply is the total inventory in the warehouse. If the demand for diapers exceed the inventory, then the shop must supply the remaining diapers from the central factory where variable costs are placed upon. The variable costs differ for each shop. In addition, we assume that there are no delays or additional costs for the central factory to produce the amount demanded from the warehouses. In accordance with our assumptions and the system, we have constructed the model shown below.

### *Parameters*

$$\begin{aligned}
i &:= \text{shop i} \\
k &:= \text{warehouse k} \\
t &:= \text{time t} \\
C_k &:= \text{inventory cost in warehouse k} \\
B_k &:= \text{capacity of inventory in warehouse k} \\
f_{ik} &:= \text{fixed cost of shop i ordering from warehouse k} \\
V_i &:= \text{variable cost of shop i ordering from the central} \\
d_{it} &:= \text{demand at time t from shop i}
\end{aligned}$$

### *Decision Variables*

$$\begin{aligned}
O_{ikt} &:= \begin{cases} 1, & \text{if shop i orders diapers from warehouse k at time t} \\ 0, & \text{otherwise} \end{cases} \\
X_{ikt} &:= \text{amount that shop i orders from warehouse k at time t} \\
Y_{it} &:= \text{amount of diapers that shop i orders from the central at time t} \\
I_{kt} &:= \text{inventory in warehouse k at time t} \\
M_{kt} &:= \text{amount of diapers warehouse k gets from the central at time t}
\end{aligned}$$

***Model***

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{k=1}^{K} f_{ik}O_{ikt} + \sum_{t=1}^{T}\sum_{i=1}^{N} V_i Y_{it} + \sum_{t=1}^{T}\sum_{k=1}^{K} C_k I_{kt} \\
\text{subject to} \quad & I_{kt} \leqslant B_k && \forall t, \forall k && (1) \\
& \sum_{k=1}^{K} X_{ikt} + Y_{it} = d_{it} && \forall t, \forall i && (2) \\
& \sum_{k=1}^{K} O_{ikt} = 1 && \forall t, \forall i && (3) \\
& I_{kt-1} + M_{kt} = \sum_{i=1}^{N} X_{ikt} + I_{kt} && \forall k, \forall t && (4) \\
& X_{ikt} \leqslant M * O_{ikt} && \forall i, \forall k, \forall t && (5) \\
& I_{k0} = 5 && \forall k && (6) \\
& O_{ikt} \in \{0,1\} && \forall k, \forall t, \forall i && (7) \\
& X_{ikt} \geq 0, \quad Y_{it} \geq 0, \text{integer} && \forall k, \forall t, \forall i && (8) \\
& I_{kt} \geq 0, \quad M_{kt} \geq 0, \text{integer} && \forall k, \forall t && (9)
\end{aligned}
$$

***Explanation of the Constraints***

(1) The inventory of warehouse k may not exceed its capacity.

(2) The amount of diapers supplied from each warehouse and plus the central factory must be equal to the demand in shop i.

(3) Each shop may only use one warehouse to supply its diapers.

(4) The amount of inventory remained from last period plus the amount of diapers supplied from the central factory must be equal to the total diapers supplied to the shops and the inventory left this period.

(5) Binary constraint.

(6) The inventory of each warehouse at time 0 is 5 diapers.

(7) The order variable is binary.

(8) The amount that the shop supplies diapers from the warehouse and the central factory is non-negative and integer.

(9) The warehouses amount of inventory and amount of supplied diapers from the central factory is non-negative and integer.