Lecture:

Image Formation

Introduction to Computer Vision

Adapted from slides by Juan Carlos Niebles, and Ranjay Krishna

Image formation

- How are objects in the world captured in an image?

Physical parameters of image formation

- Geometric
  - Type of projection
  - Camera pose
- Optical
  - Sensor's lens type
  - focal length, field of view, aperture
- Photometric
  - Type, direction, intensity of light reaching sensor
  - Surfaces' reflectance properties

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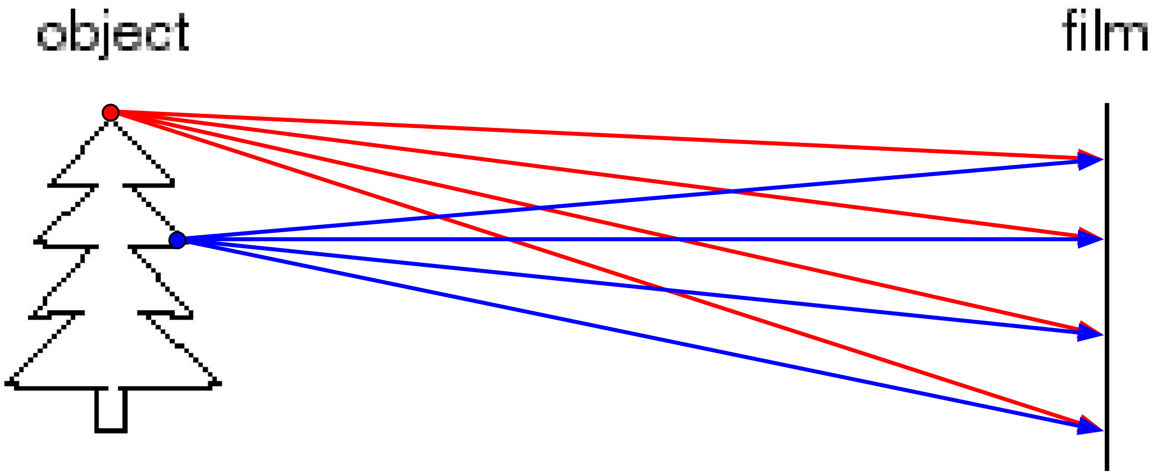
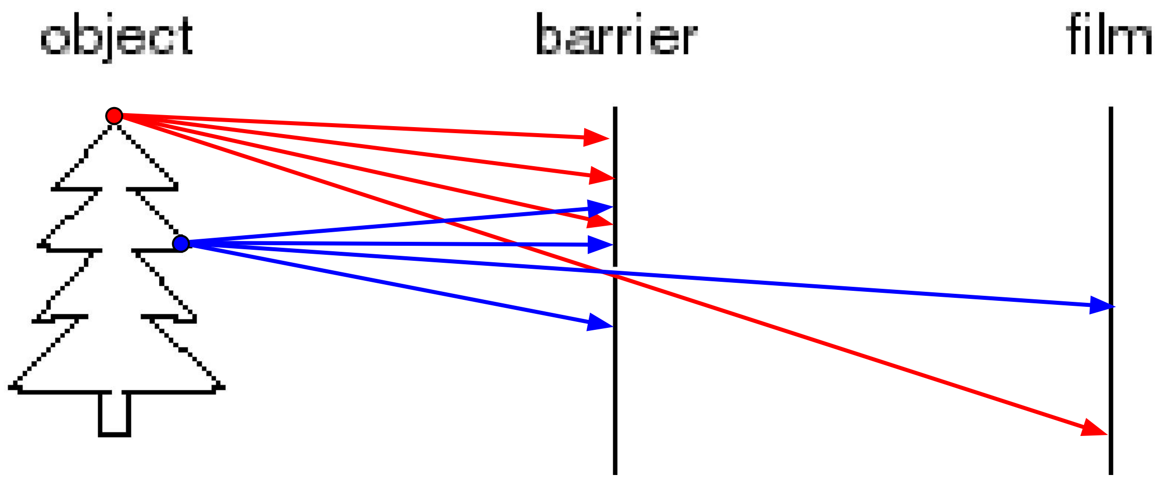
Image formation



- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening is known as the **aperture**
  - How does this transform the image?

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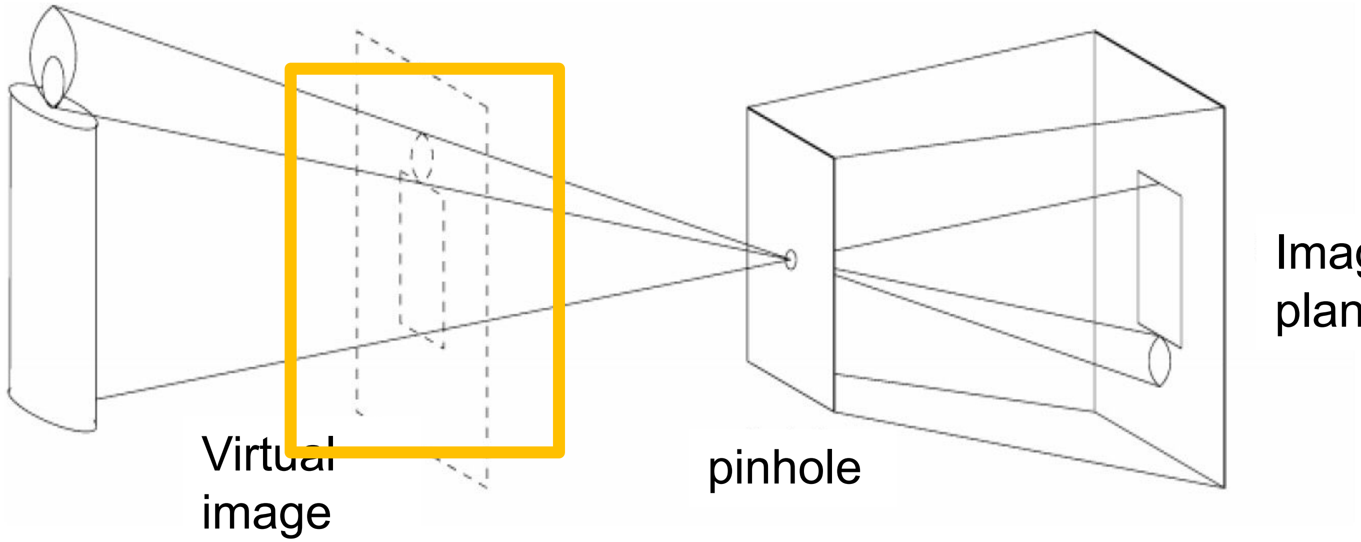
Slide by Steve Seitz

Pinhole camera

- Pinhole camera is a simple model (based on **perspective projection**) to approximate imaging process



If we treat pinhole as a point, only one ray from any given point can enter the camera.

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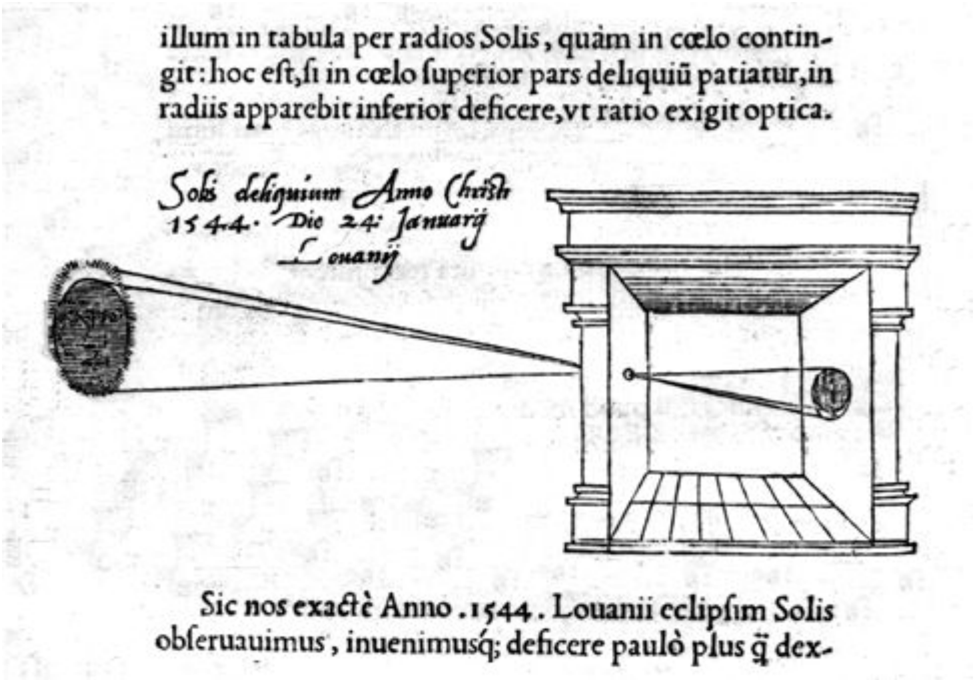
Camera obscura



In Latin, means 'dark room'

"**Reinerus Gemma-Frisius**, observed an eclipse of the sun at Louvain on January 24, 1544, and later he used this illustration of the event in his book De Radio Astronomica et Geometrica, 1545. It is thought to be the first published illustration of a camera obscura..." Hammond, John H., The Camera Obscura, A Chronicle

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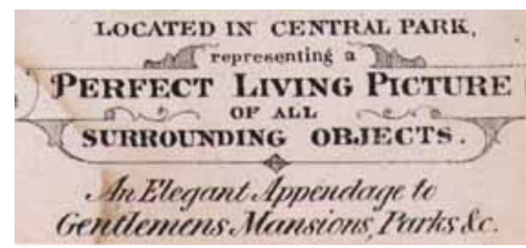
http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html

Camera obscura

Jetty at Margate England, 1898.



Around 1870s

An attraction in the late 19th century

Camera obscura at home

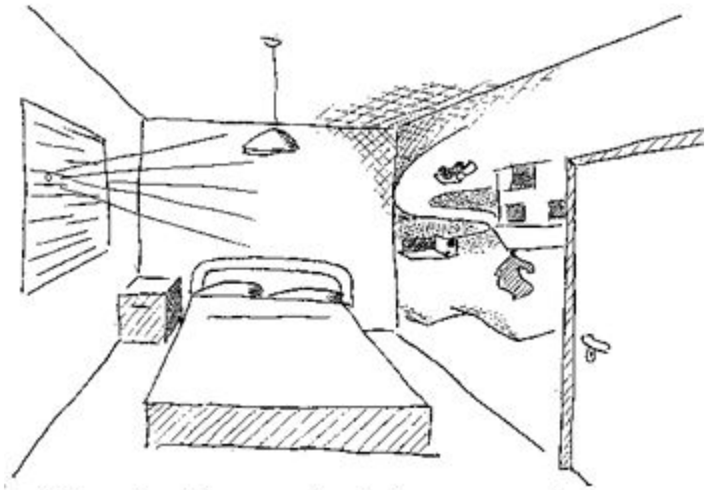


Figure 1 - A lens on the window creates the image of the external world on the opposite wall and you can see it every morning, when you wake up.

https://blackcreek.ca/how-to-make-your-own-camera-obscura/

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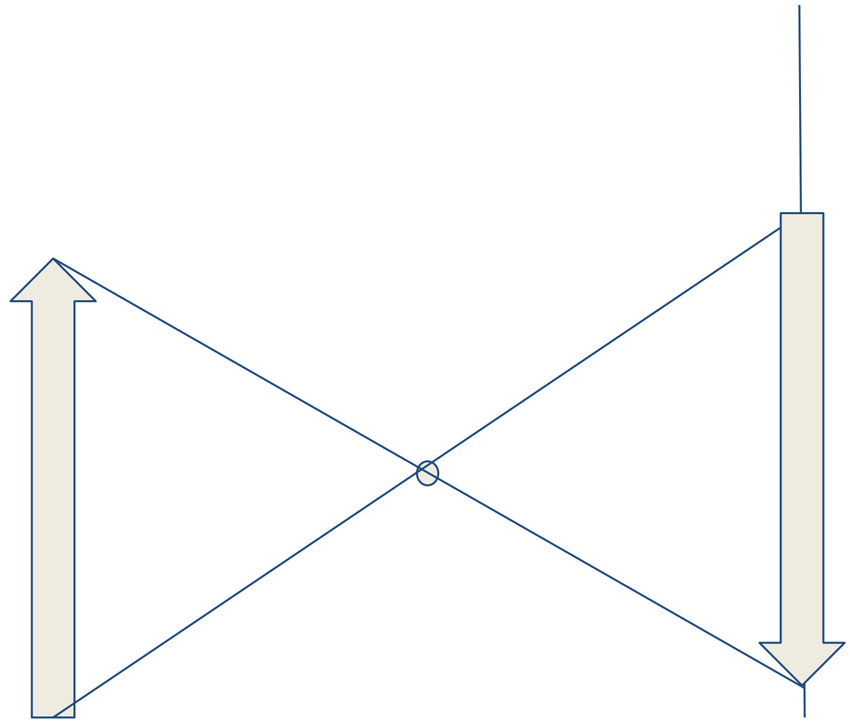
Sketch from http://www.funsci.com/fun3_en/sky/sky.htm

http://blog.makezine.com/archive/2006/02/how_to_room_sized_camera_obscu.html

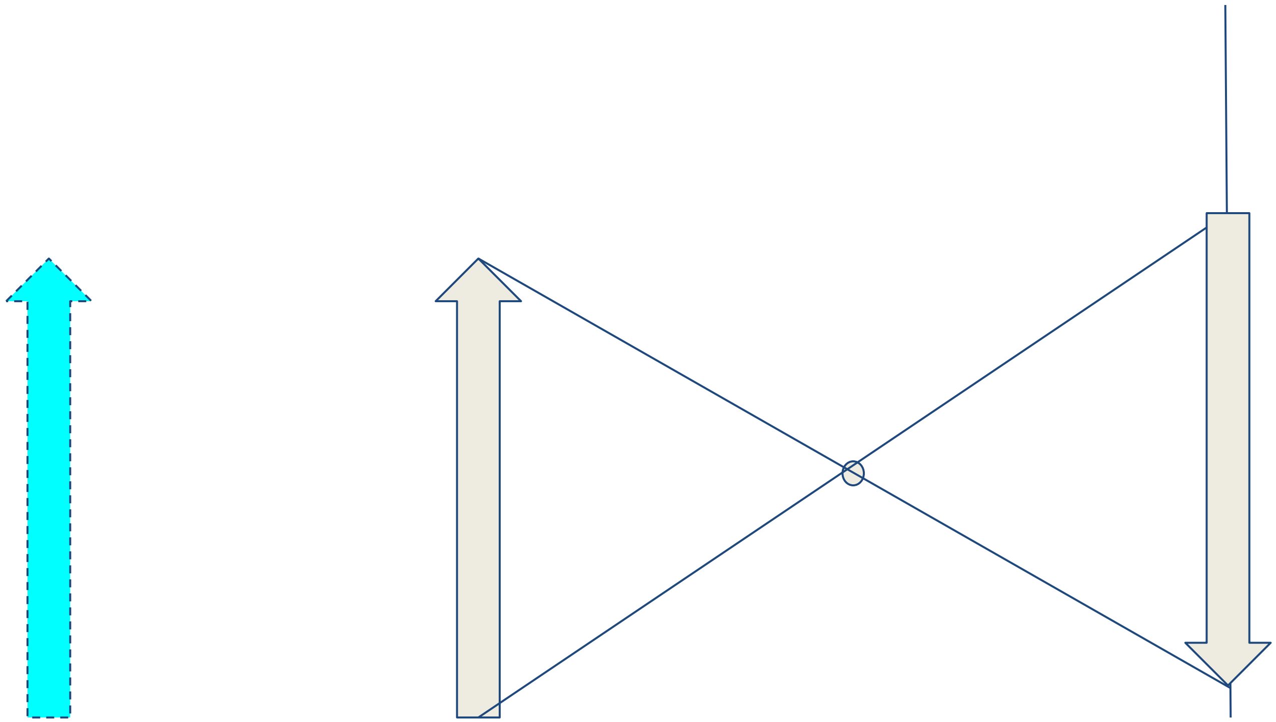
Perspective effects

- Far away objects appear smaller



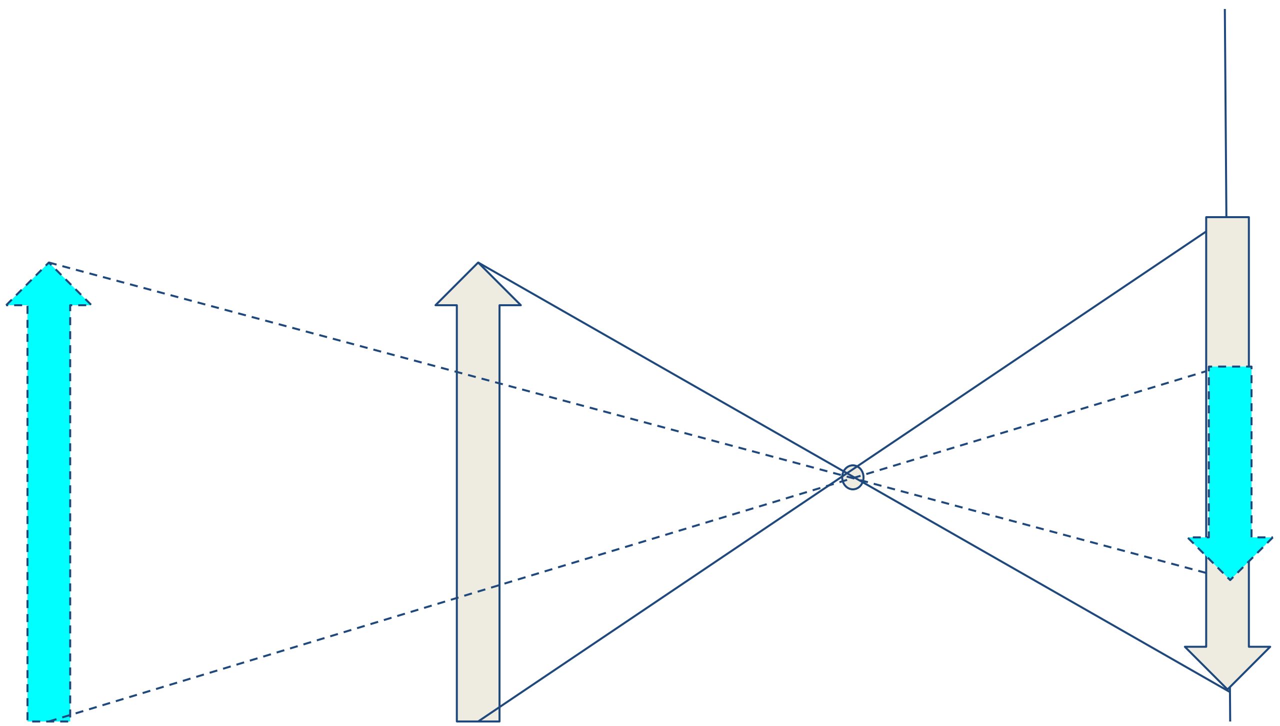
Perspective effects

- Far away objects appear smaller



Perspective effects

- Far away objects appear smaller



Perspective effects

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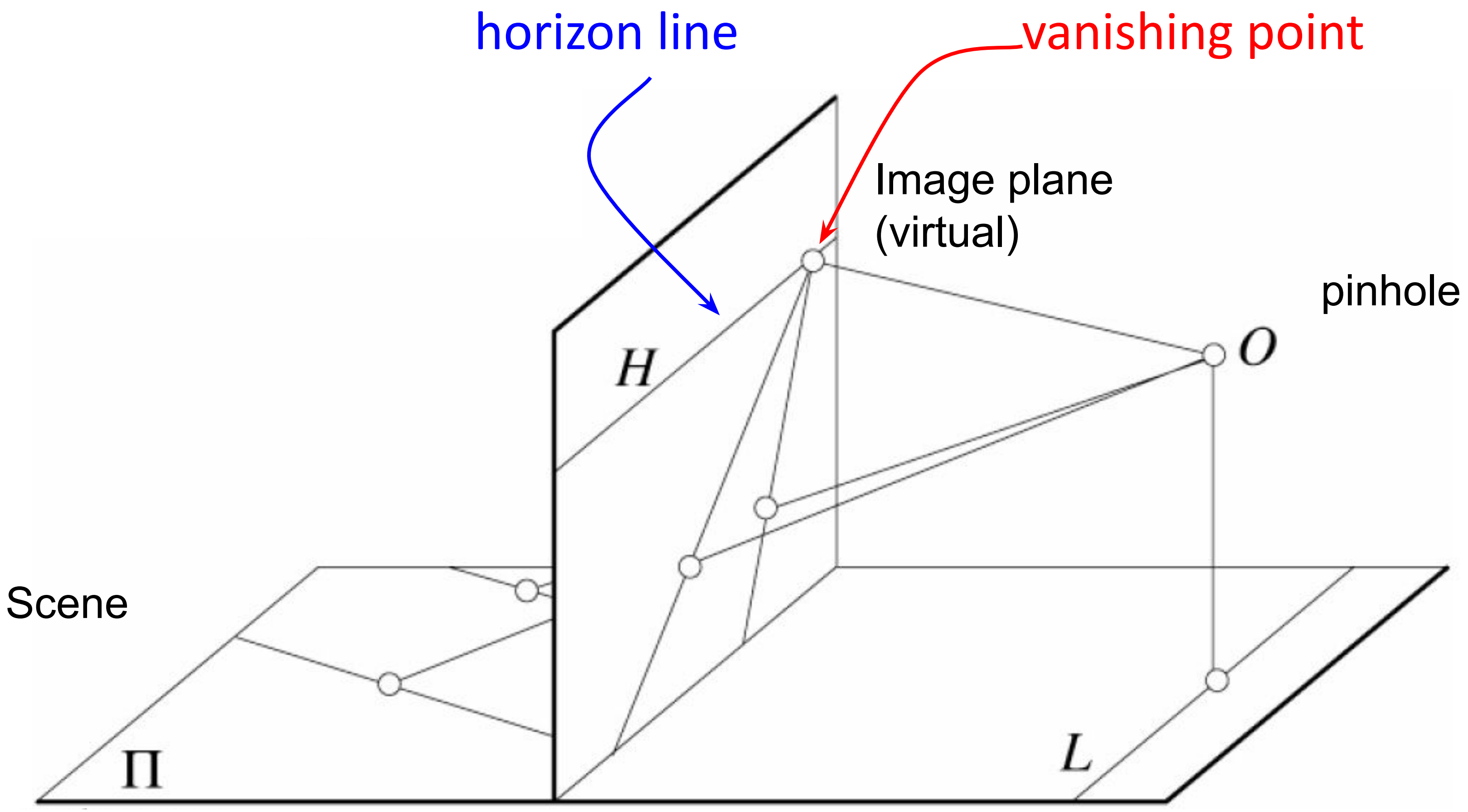
Vanishing points & horizon

- Each set of parallel lines meets at a different point
  - The **vanishing point** for this direction

- Sets of parallel lines on the same plane lead to collinear vanishing points.
  - The line is called the **horizon** for that plane

https://www.cs.umd.edu/class/spring2016/cmsc426/lectures/image-formation-1.pdf

Vanishing points & horizon



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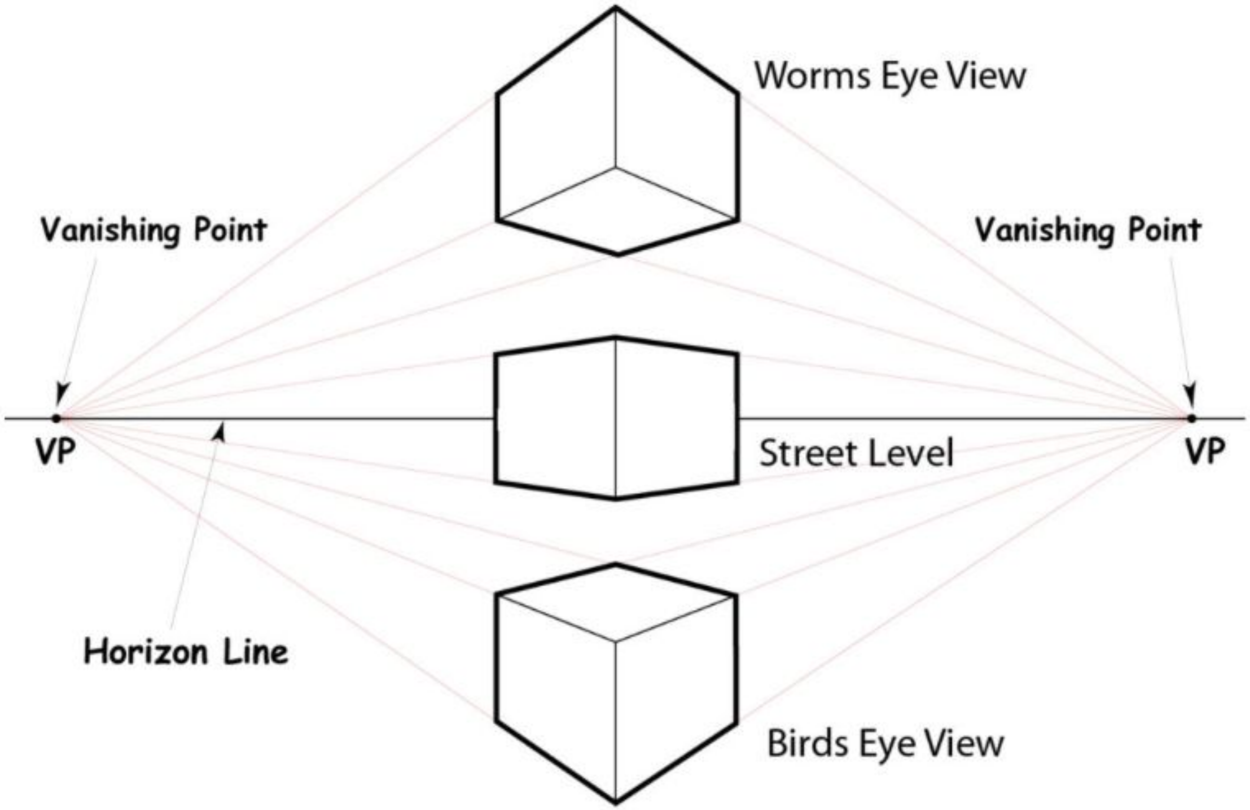
Vanishing points & horizon



https://garybolyer.com/2017/08/09/understanding-drawing-perspective/

Vanishing points & horizon

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Projection properties

- Many-to-one: any points along same ray map to same point in image
- Points $\longrightarrow$ points
- Lines $\longrightarrow$ lines (collinearity preserved)
- Distances and angles are **not** preserved

- Degenerate cases:
  - – Line through focal point projects to a point.
  - – Plane through focal point projects to line
  - – Plane perpendicular to image plane projects to part of the image (e.g. surface/road as in previous image)

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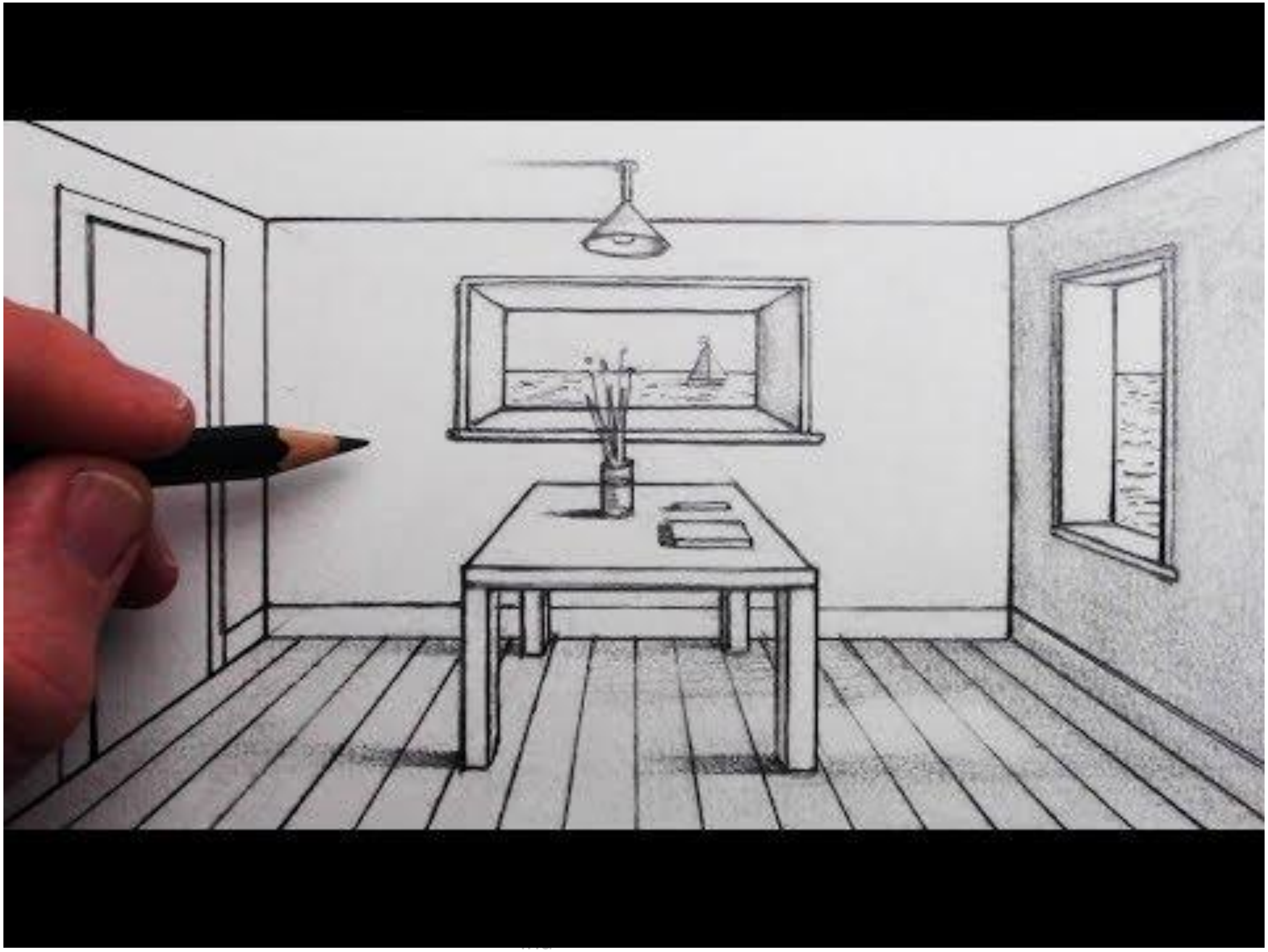
Perspective and art

- Use of correct perspective projection indicated in 1st century B.C. frescoes
- Skill resurfaces in Renaissance: artists develop systematic methods to determine perspective projection (around 1480-1515)

Raphael

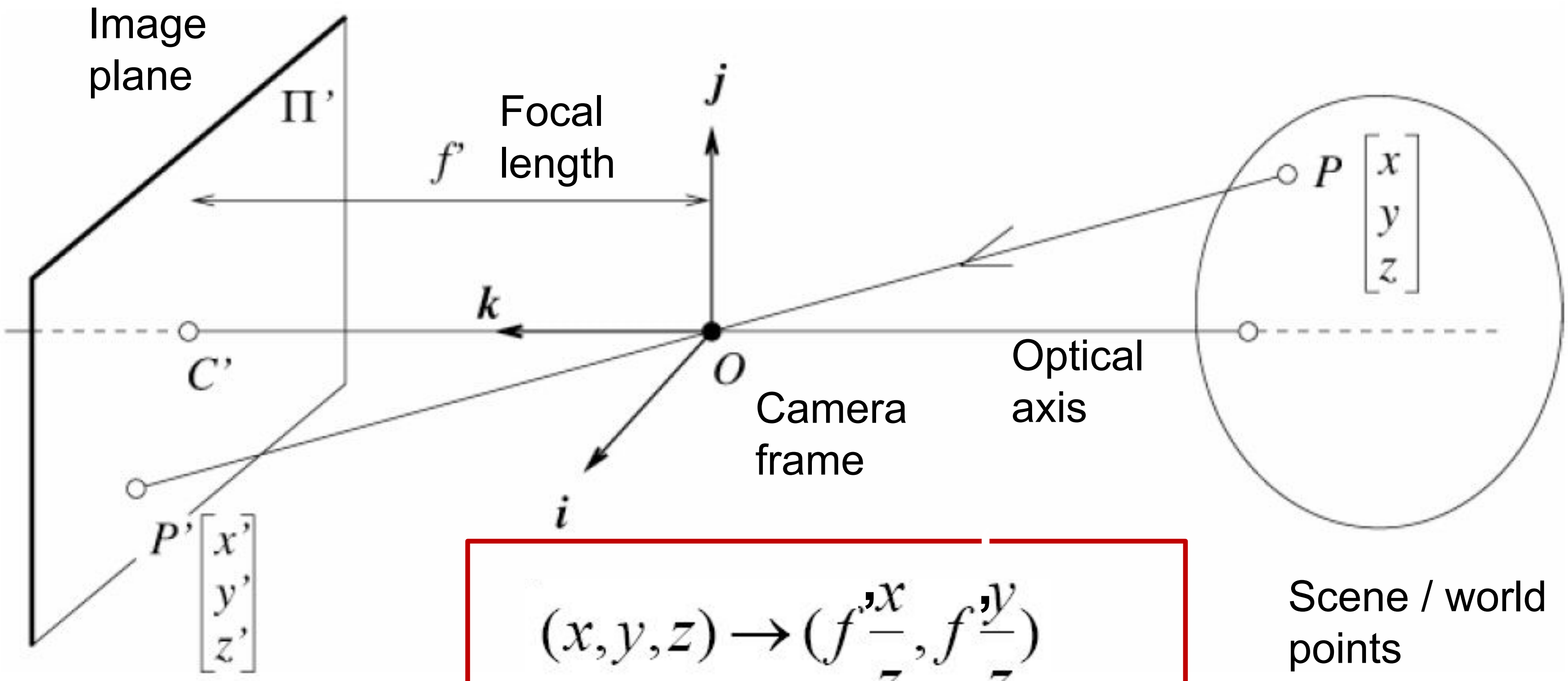
Durer, 1525

Perspective drawing



Perspective projection equations

- 3d world mapped to 2d projection in image plane



$$(x,y,z) \rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$$

Scene point $\longrightarrow$ Image coordinates

Homogeneous coordinates

Is this (scene $\rightarrow$ image point) a linear transformation?

- no—division by z is nonlinear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

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Slide by Steve Seitz

Perspective Projection Matrix

- Projection is a matrix multiplication using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f' & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f' \end{bmatrix} \Rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$$

divide by the third coordinate to convert back to non-homogeneous coordinates

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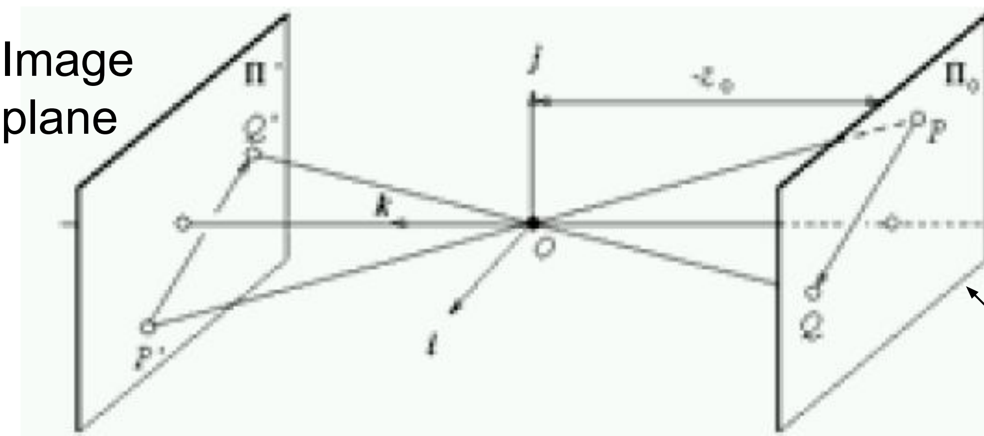
Slide by Steve Seitz

Weak perspective projection

- Approximation: treat magnification as constant
- Assumes scene depth << average distance to camera (ie. z is <u>constant</u> for all points.)

Image plane



World points:

$z = z_0$

$$x' = f\frac{x}{z} \approx \frac{f}{z_0}x$$

$$y' = f\frac{y}{z} \approx \frac{f}{z_0}y$$

Generalized weak perspective: again constant z, but with different scalings in x and y coordinates.

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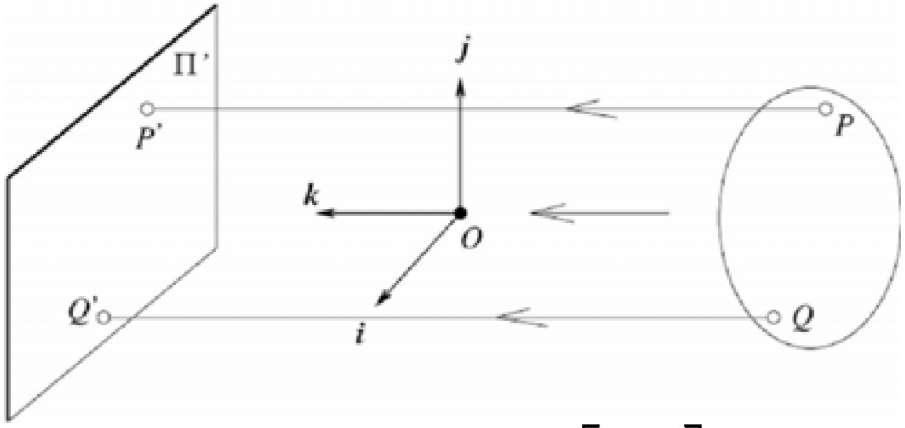
Orthographic projection

- Given camera at **constant** distance from scene
- World points projected along rays parallel to optical access (ie. weak perspective without scaling factor)



$$x' = x$$
$$y' = y$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

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Physical parameters of image formation

- Geometric
  - Type of projection
  - Camera pose
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Pinhole size / aperture

How does the size of the aperture affect the image we'd get?

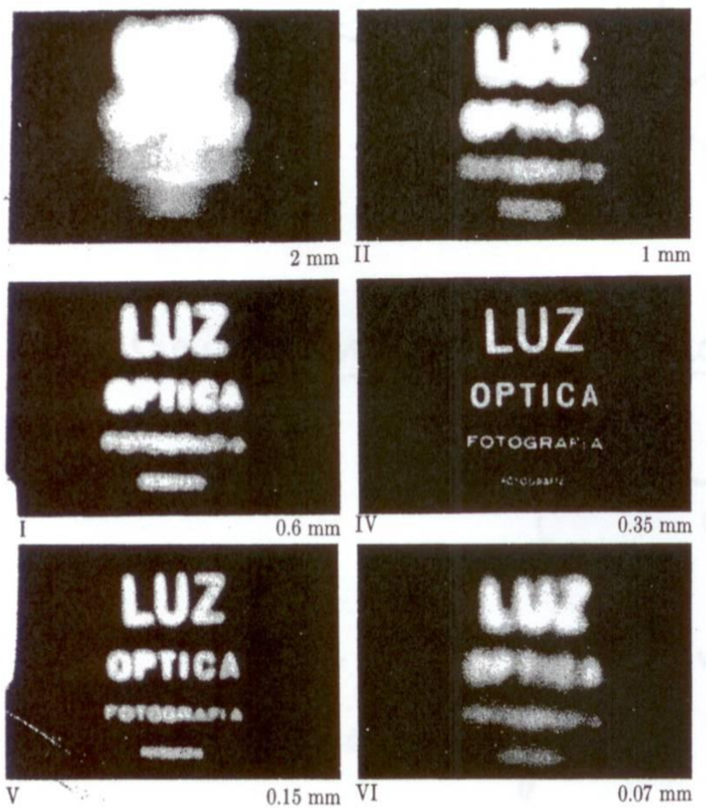
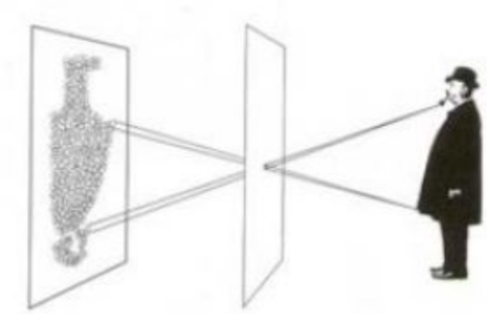


Fig. 5.96 The pinhole camera. Note the variation in image clarity as the hole diameter decreases. [Photos courtesy Dr. N. Joel, UNESCO.]

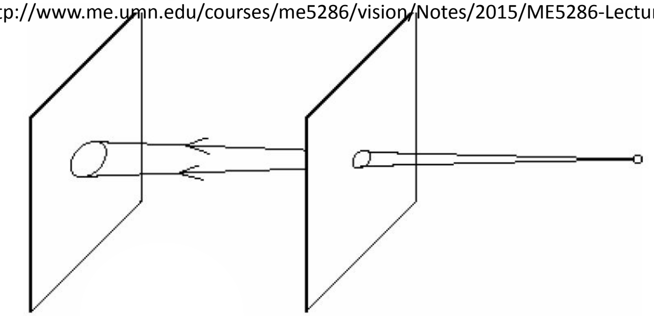
Adapte

Larger

Smaller

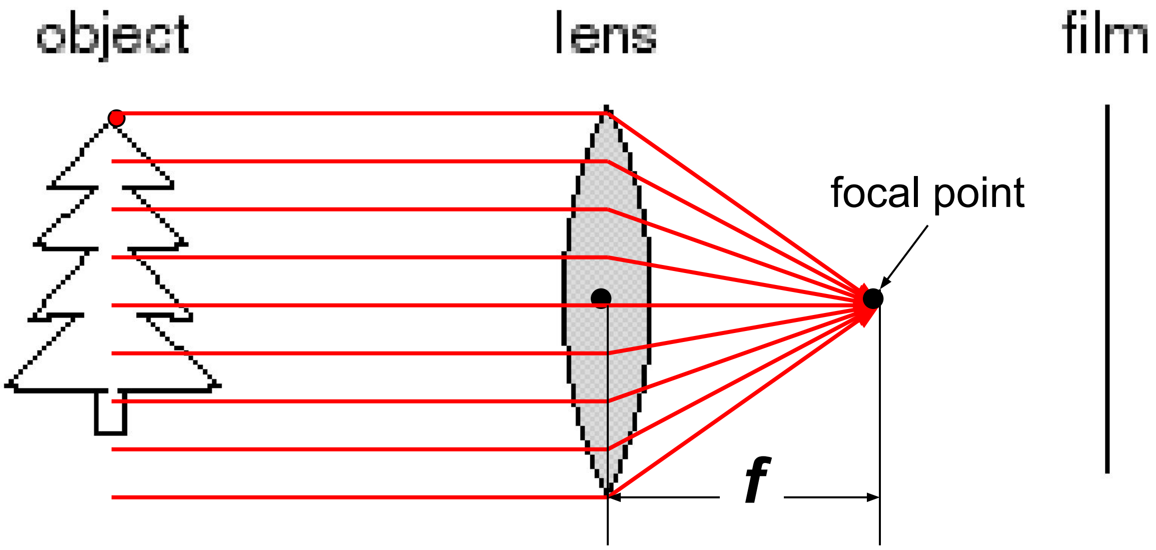


http://www.me.umn.edu/courses/me5286/vision/Notes/2015/ME5286-Lecture2.pdf



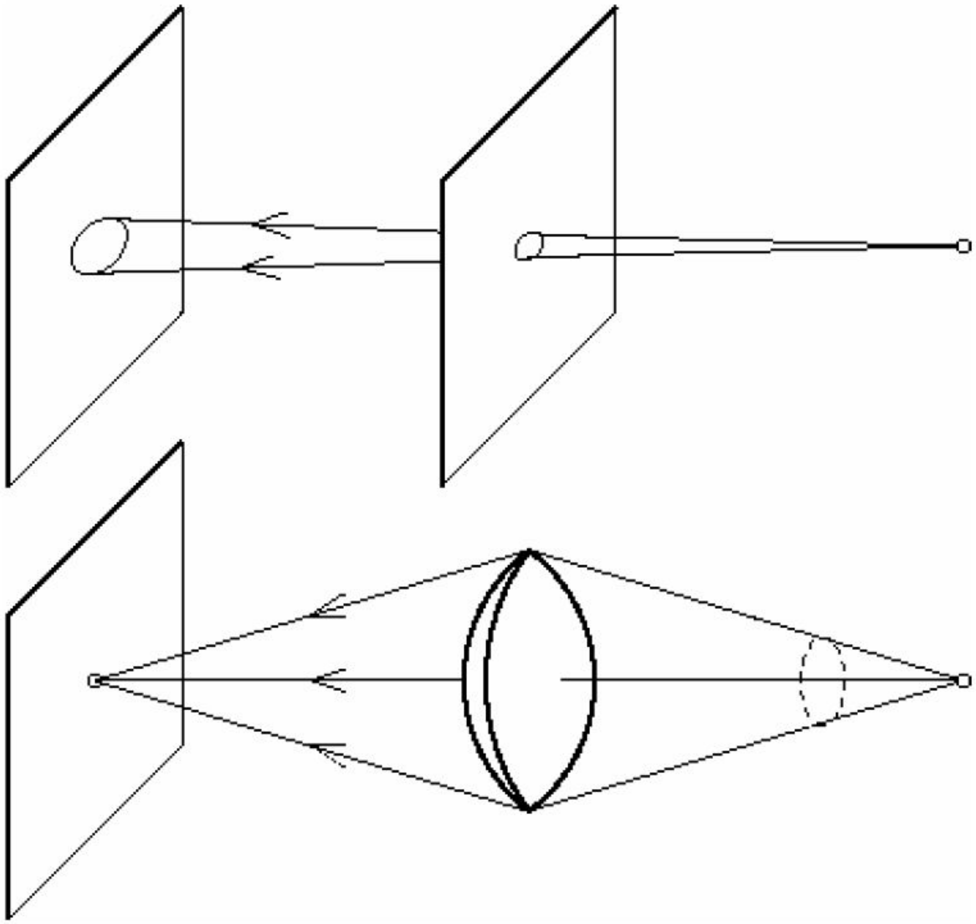
Diffraction: Here, light does not travel in a straight line through the hole, gets scattered in many directions.

Adding a lens



- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the *focal length f*

Pinhole vs. lens



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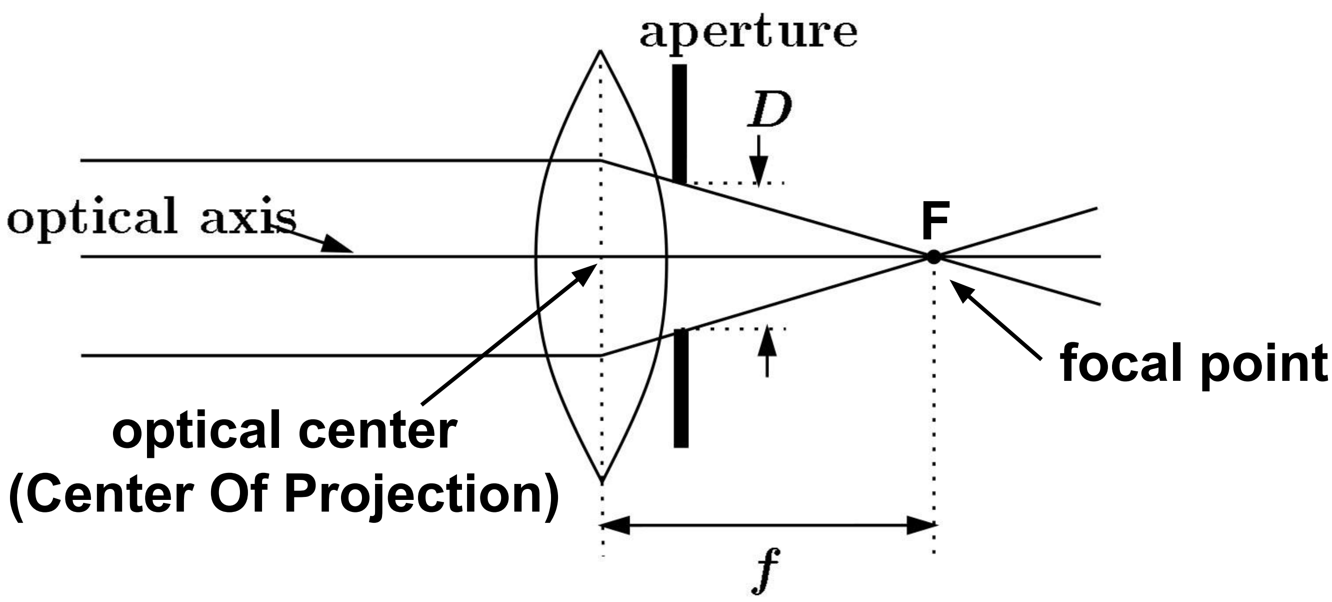
Cameras with lenses



- A lens focuses parallel rays onto a single focal point
- Gather more light, while keeping focus; make pinhole perspective projection practical

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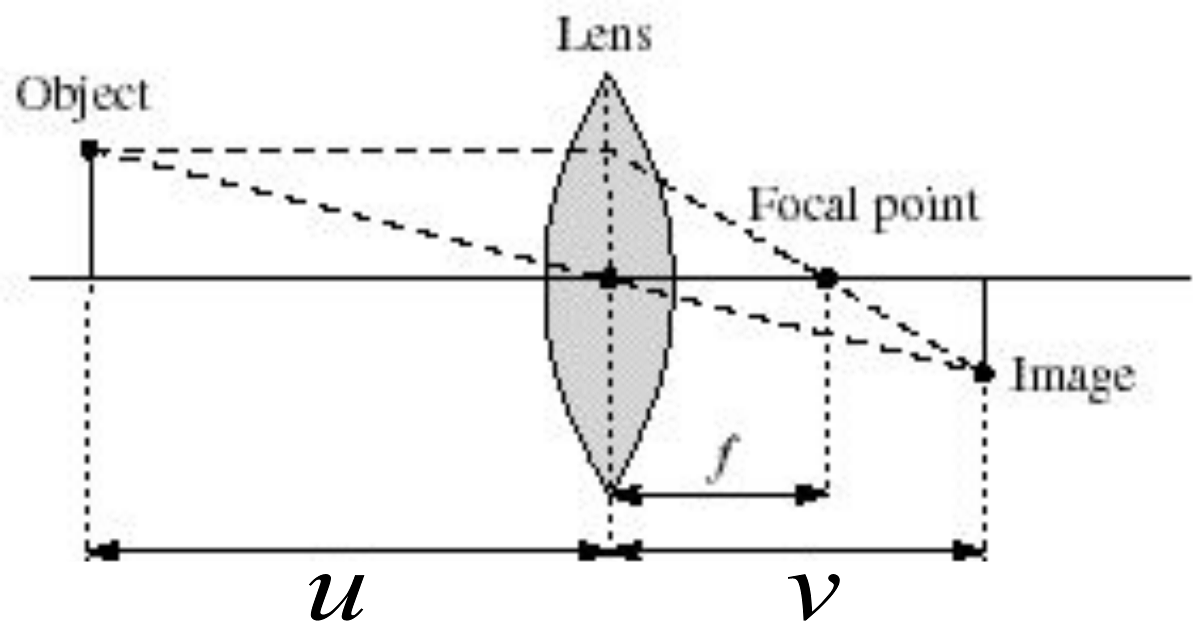
Thin lens equation



- How to relate distance of object from optical center (u) to the distance at which it will be in focus (v), given focal length f?

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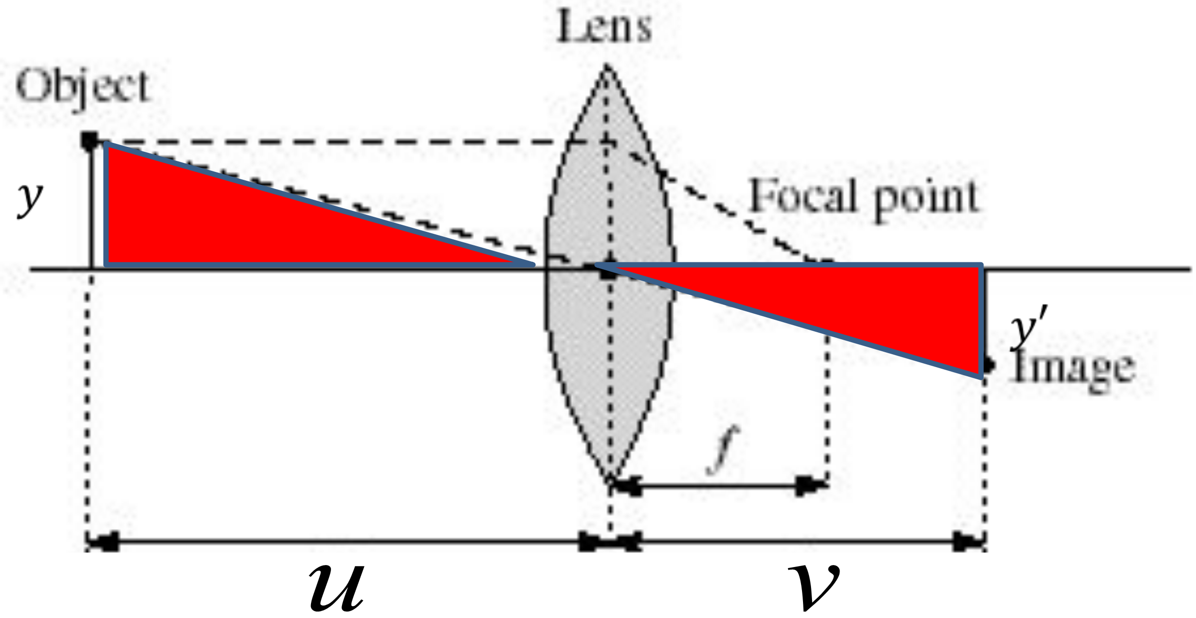
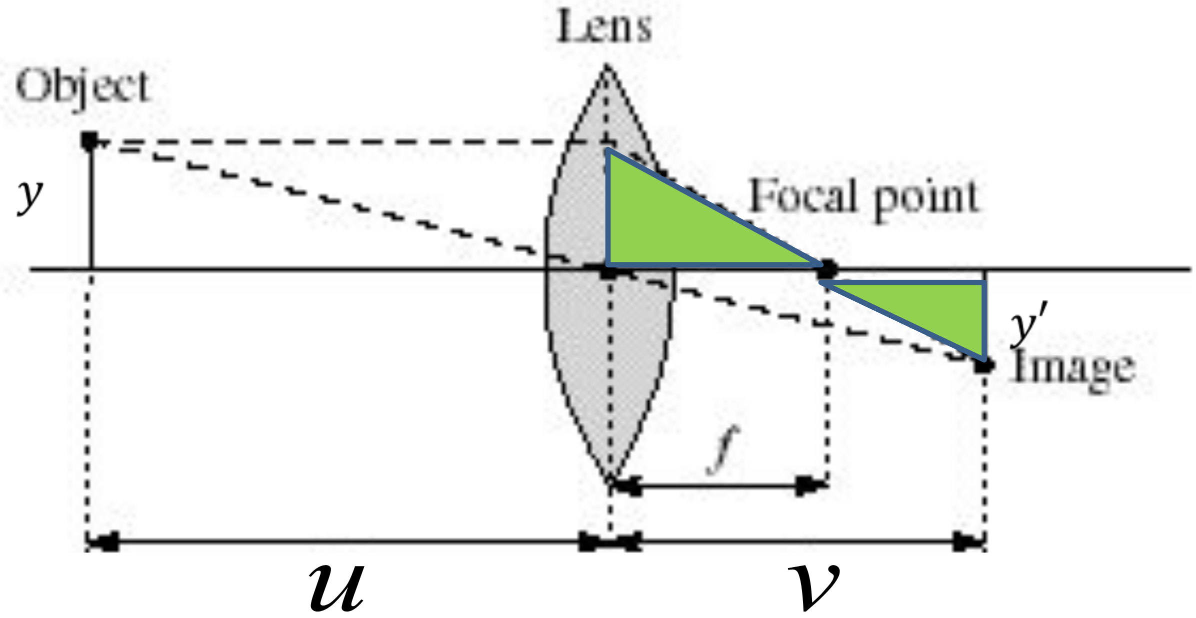
Thin lens equation



$$\frac{y'}{y} = \frac{v}{u}$$

- How to relate distance of object from optical center (u) to the distance at which it will be in focus (v), given focal length f?

Thin lens equation



$$\frac{y'}{y} = \frac{v}{u}$$

$$\frac{y'}{y} = \frac{(v-f)}{f}$$

- How to relate distance of object from optical center (u) to the distance at which it will be in focus (v), given focal length f?

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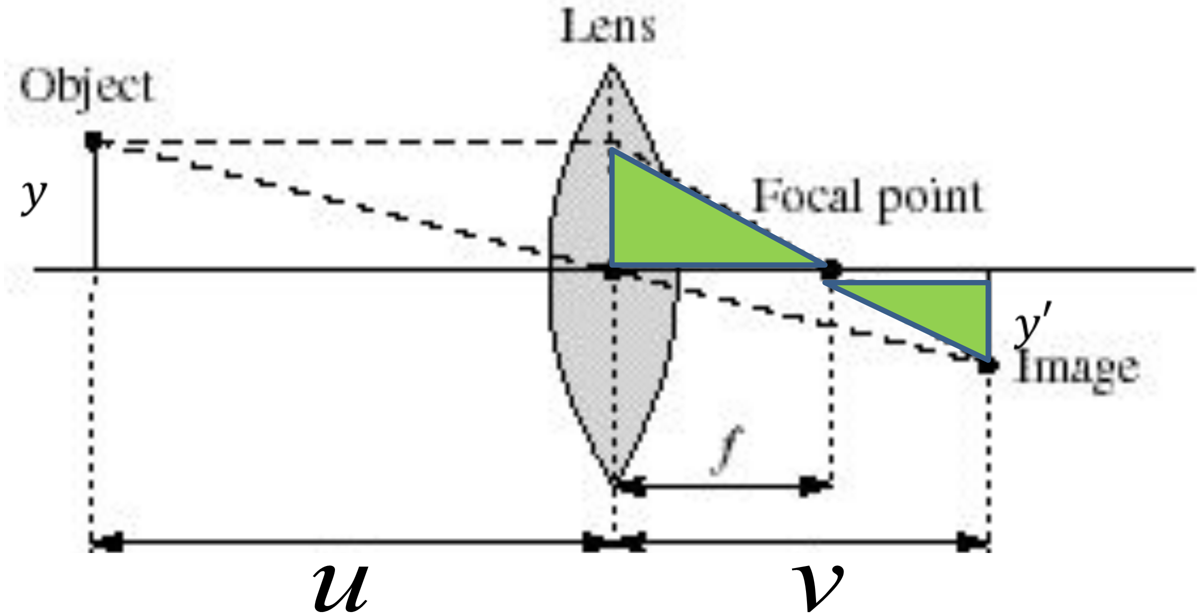
Thin lens equation



$$\frac{y'}{y} = \frac{v}{u}$$

$$\frac{y'}{y} = \frac{(v-f)}{f}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

- Any object point satisfying this equation (approximately) is in focus

Adapted from slides by Juan Carlos Niebles, and Ranjay Krishna

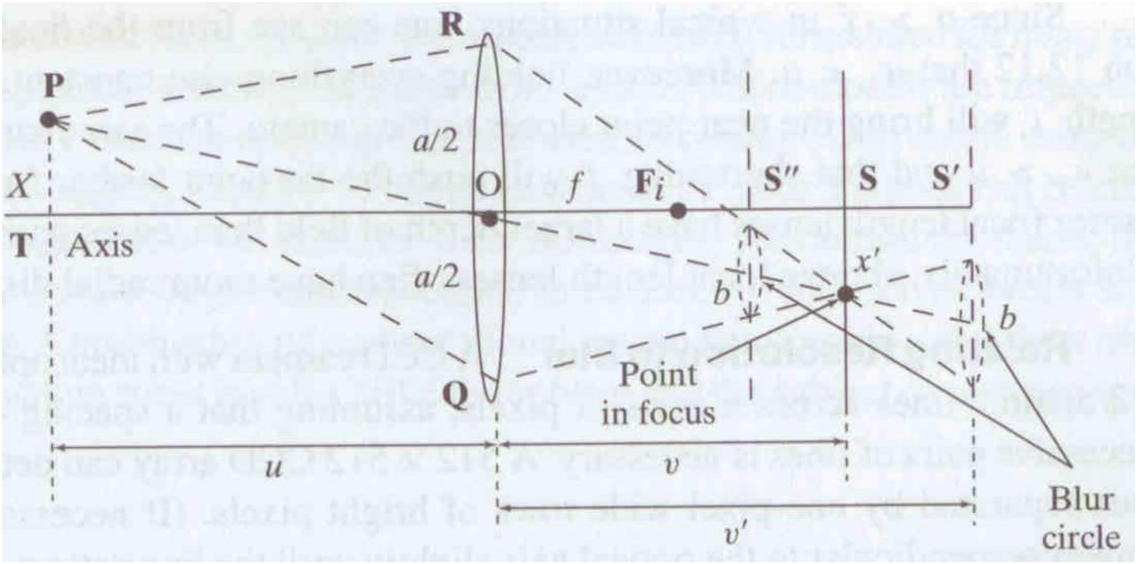
Focus and depth of field

Adapted from slides by Juan Carlos Niebles, and Ranjay Krishna

Image credit: cambridgeincolour.com

Focus and depth of field

- Depth of field: distance between image planes where blur is tolerable



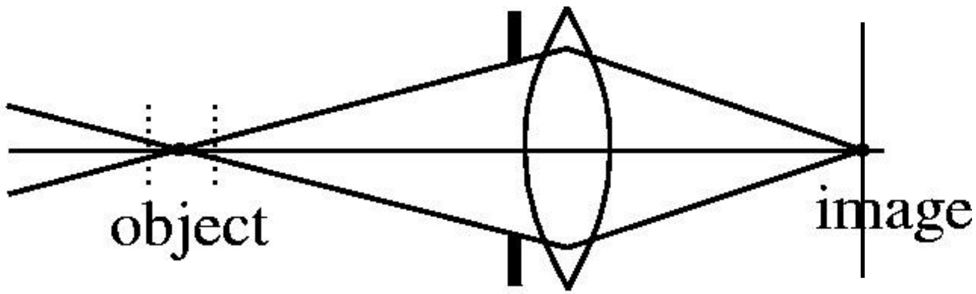
← "circles of confusion" →

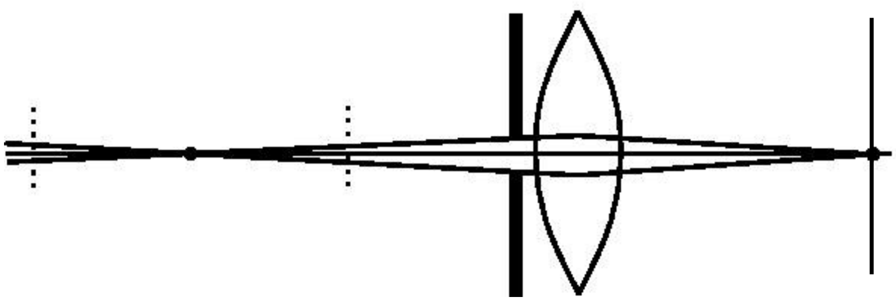
Thin lens: scene points at distinct depths come in focus at different image planes.

(Real camera lens systems have greater depth of field.)

Focus and depth of field

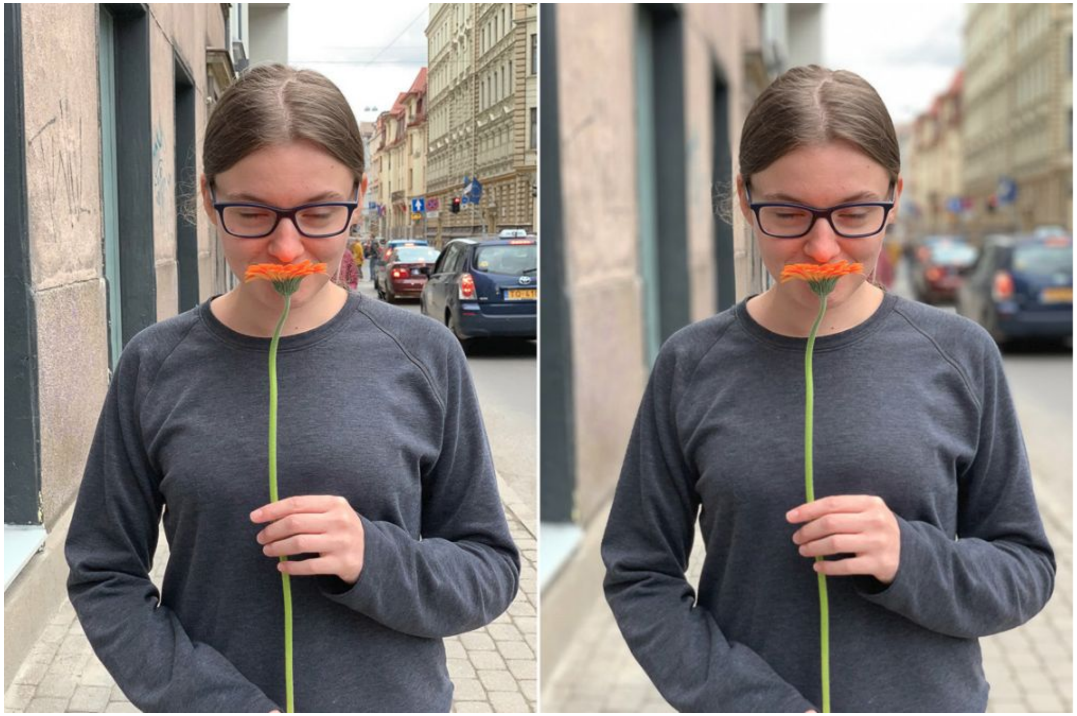
- How does the aperture affect the depth of field?





- A smaller aperture increases the range in which the object is approximately in focus (while decreasing the overall brightness)

Depth of Field - Modern Tricks



https://iphonephotographyschool.com/portrait-mode/

Field of view

- Angular measure of portion of 3d space seen by the camera

28 mm lens, 65.5° × 46.4°

50 mm lens, 39.6° × 27.0°

70 mm lens, 28.9° × 19.5°

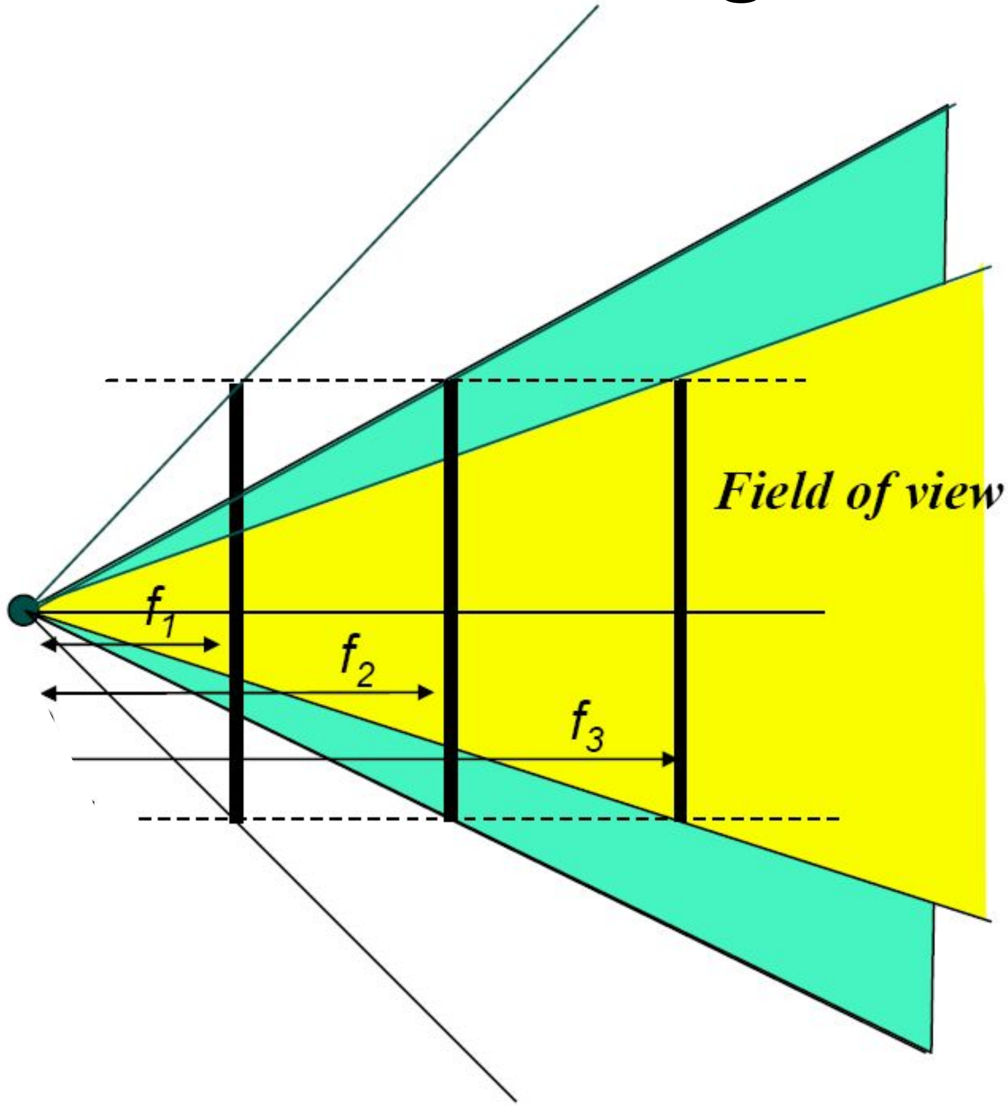
210 mm lens, 9.8° × 6.5°

Adapted from slides by Juan Carlos Niebles, and Ranjay Krishna

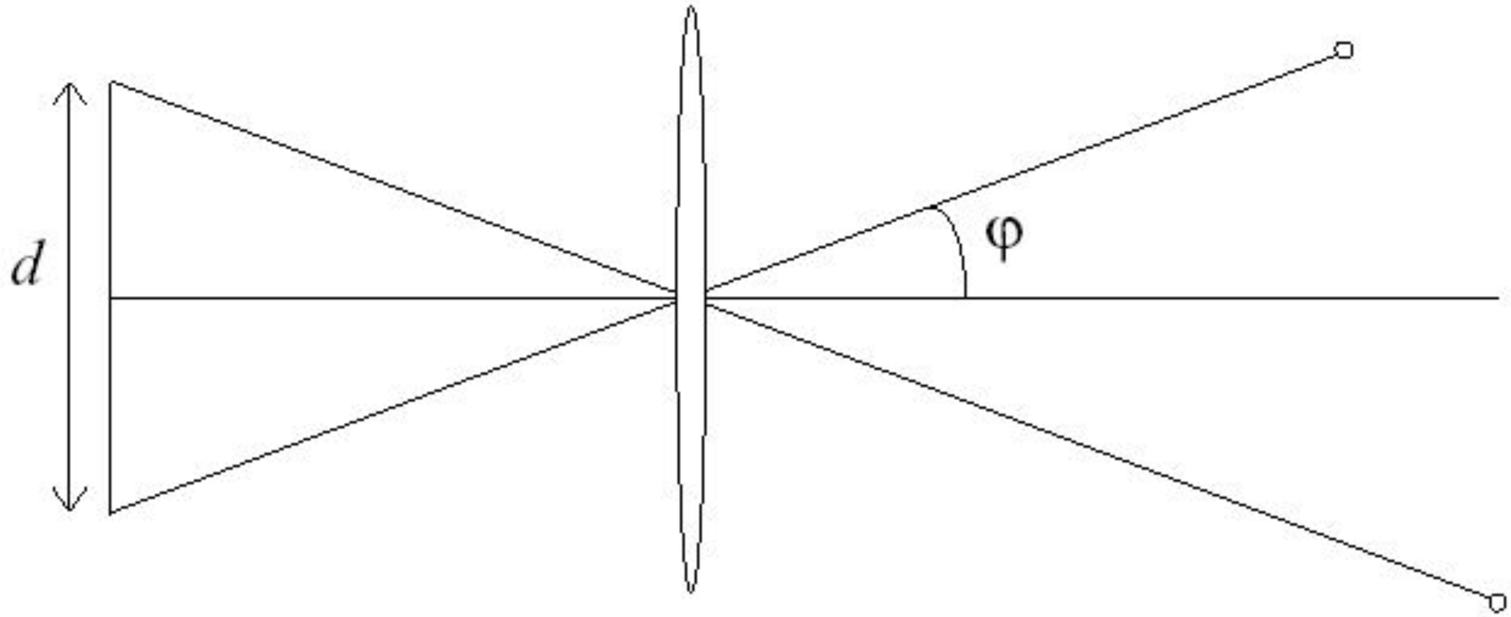
Images from http://en.wikipedia.org/wiki/Angle_of_view

Field of view depends on focal length

- As ***f*** gets smaller, image becomes more *wide angle*
  - more world points project onto the finite image plane
- As ***f*** gets larger, image becomes more *telescopic*
  - smaller part of the world projects onto the finite image plane



Field of view depends on focal length



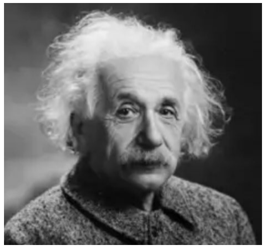
Size of field of view governed by size of the camera retina:

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

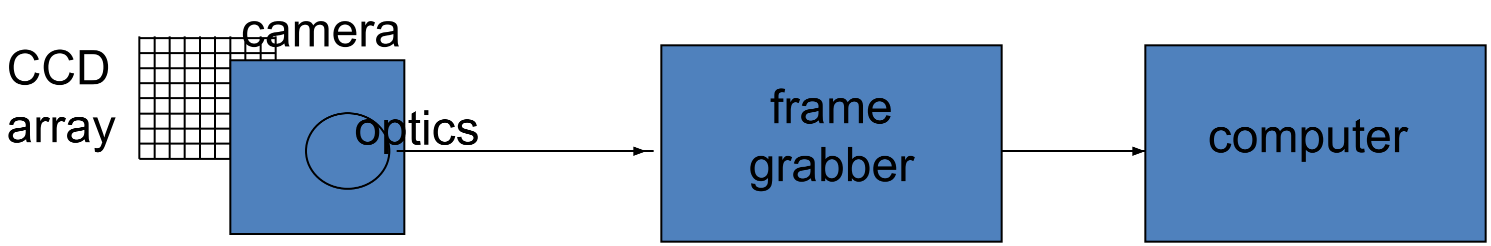
Smaller FOV = larger Focal Length

Slide by A. Efros

Digital cameras



- Film ⟶ sensor array
- Often an array of charge coupled devices
- Each CCD is light sensitive diode that converts photons (light energy) to electrons



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Summary

- Image formation affected by geometry, photometry, and optics.
- Projection equations express how world points mapped to 2d image.
  – Homogenous coordinates allow linear system for projection equations.
- Lenses make pinhole model practical.
- Parameters (focal length, aperture, lens diameter,…) affect image obtained.

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