CENG 483 - Intro. to Computer Vision

Clustering, segmentation & k-means clustering

Adapted from slides by Juan Carlos Niebles, and Ranjay Krishna

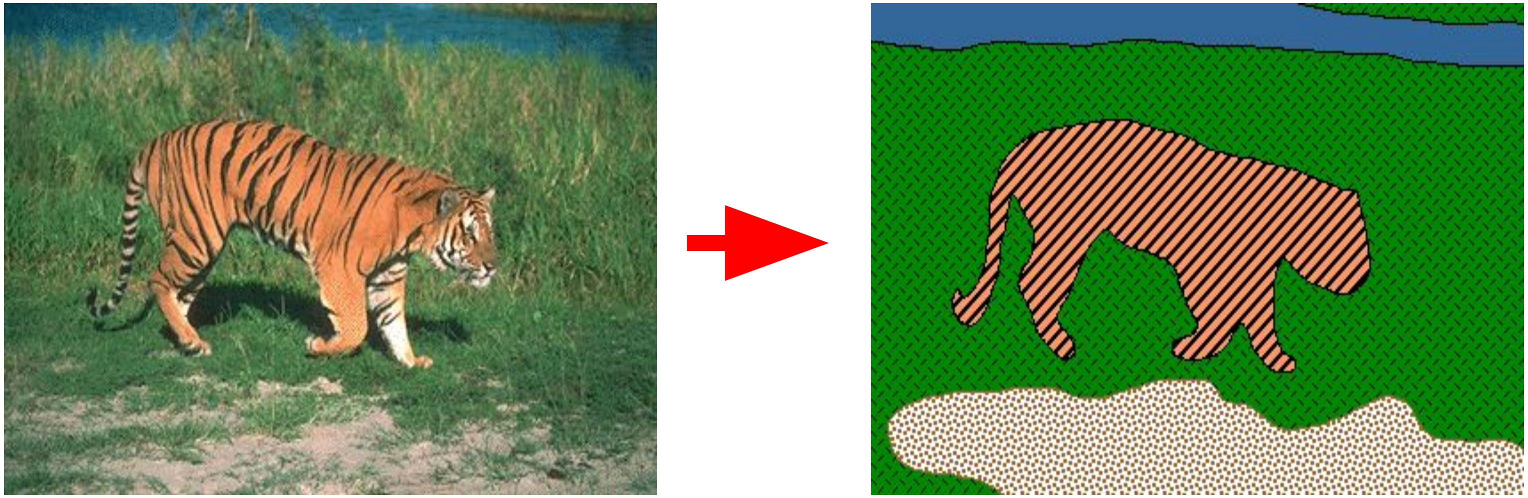
What we will learn today

- **Introduction to segmentation and clustering**
- Gestalt theory for perceptual grouping
- K-means clustering

Adapted from slides by Juan Carlos Niebles, and Ranjay Krishna

Image Segmentation (unsupervised)

- A very commonly studied **grouping problem**
- Goal: identify groups of pixels that go together



Slide credit: Steve Seitz, Kristen Grauman

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Segmentation vs Semantic Segmentation

Typically:

- **Image (unsupervised) segmentation**: unsupervised, grouping based
  - This is what we mean throughout this lecture
- **Semantic segmentation**: supervised, category-level

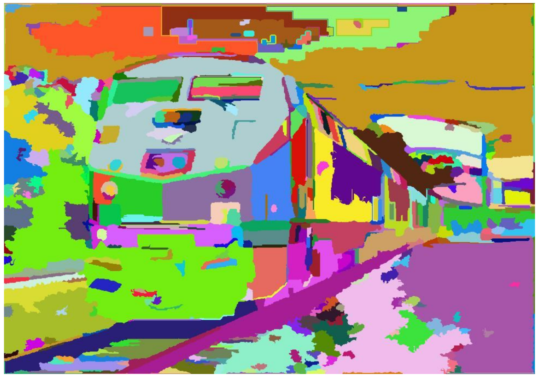
The Goals of Segmentation

- **Example application #1** Separate image into coherent "objects"

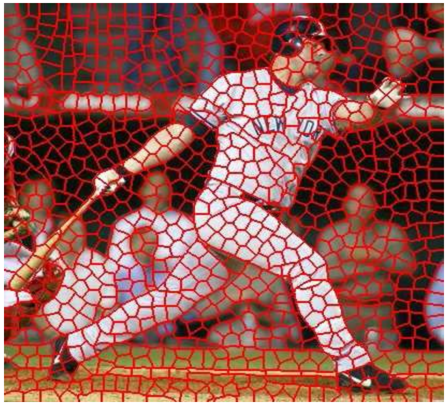
https://arxiv.org/pdf/2102.06191.pdf

The Goals of Segmentation

- **Example application #2** Group similar pixels for efficiency of further processing (superpixels, etc)



[Felzenszwalb and Huttenlocher 2004]



[Hoiem et al. 2005, Mori 2005]

[Shi and Malik 2001]

X. Ren and J. Malik. [Learning a classification model for segmentation.](#) ICCV 2003.

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The Goals of Segmentation

- **Example application #3** Better feature support

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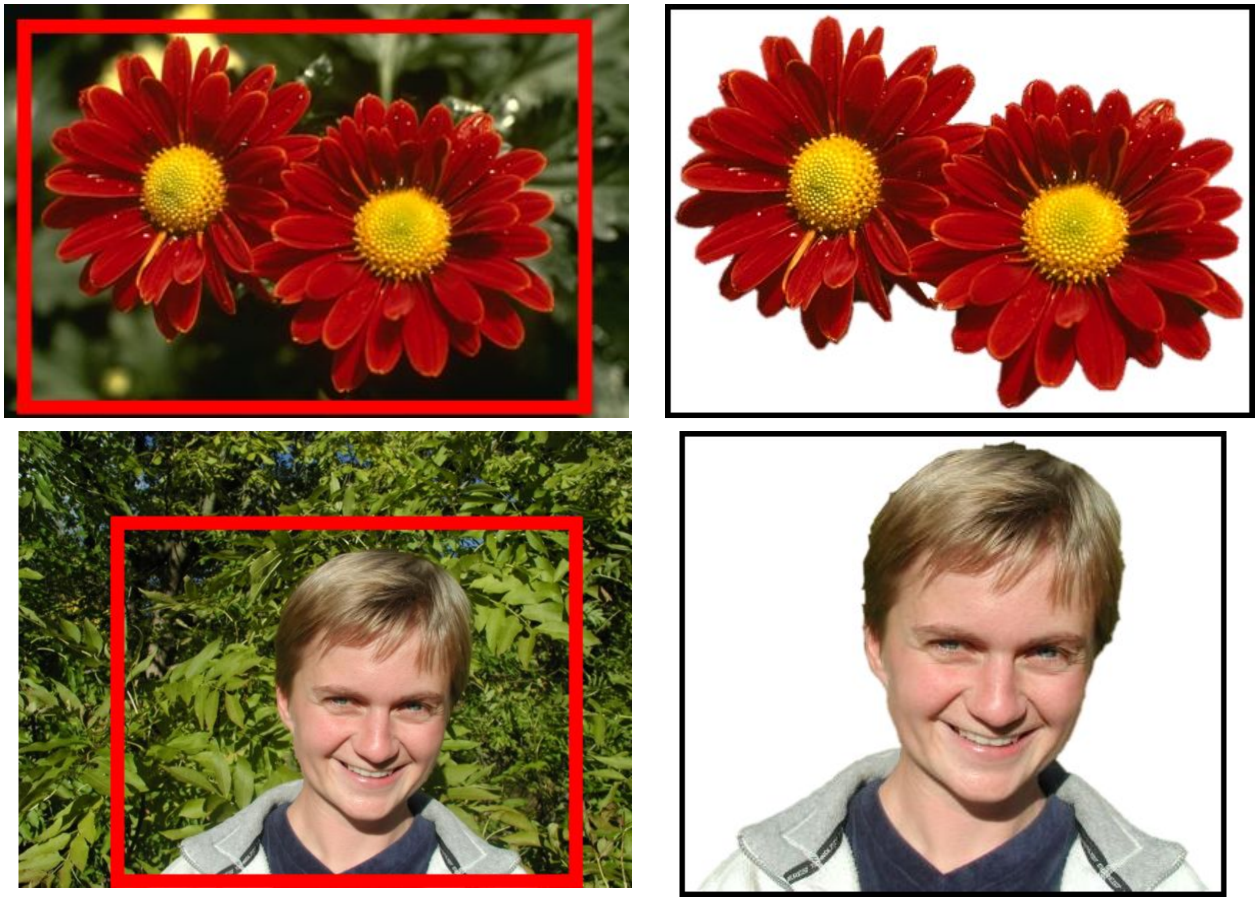
The Goals of Segmentation

- **Example application #4** Image editing



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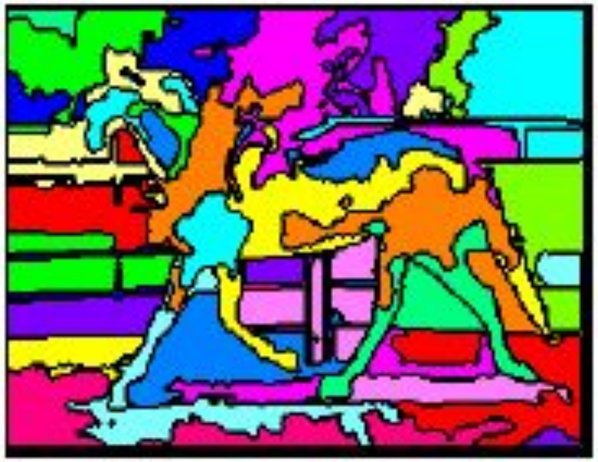
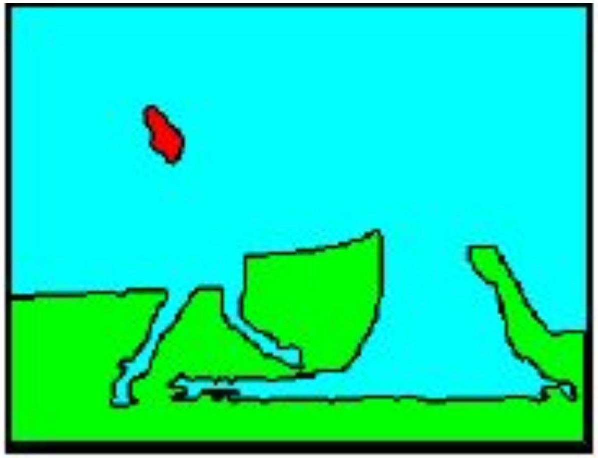
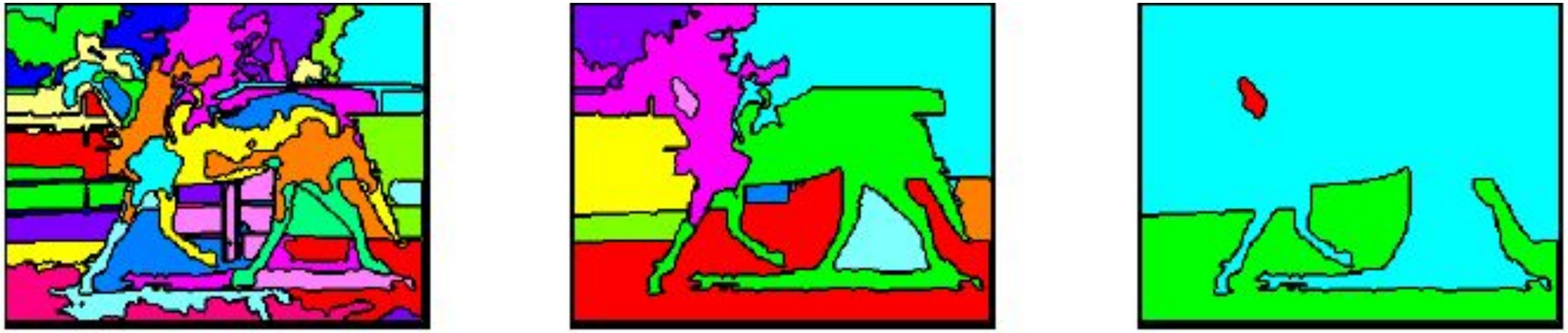
Types of segmentations



Oversegmentation



Undersegmentation



Multiple Segmentations

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One way to think about "segmentation" is Clustering

Clustering: group together similar data points and represent them with a single token

Key Challenges:

1) What makes two points/images/patches similar?

2) How do we compute an overall grouping from pairwise similarities?

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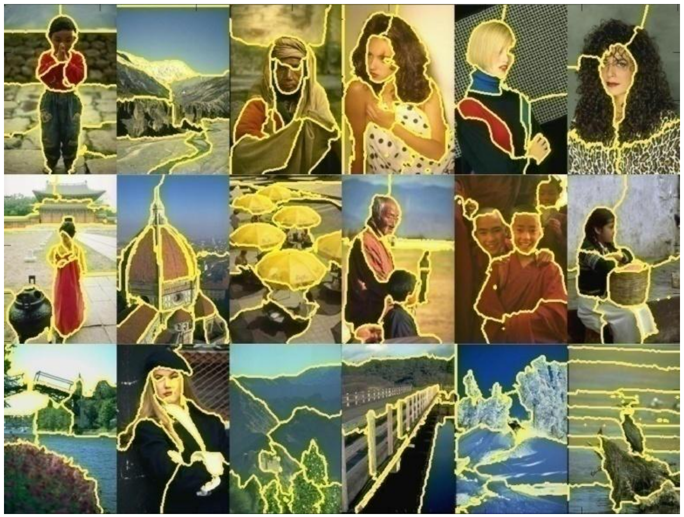
There are many other grouping tasks in Vision



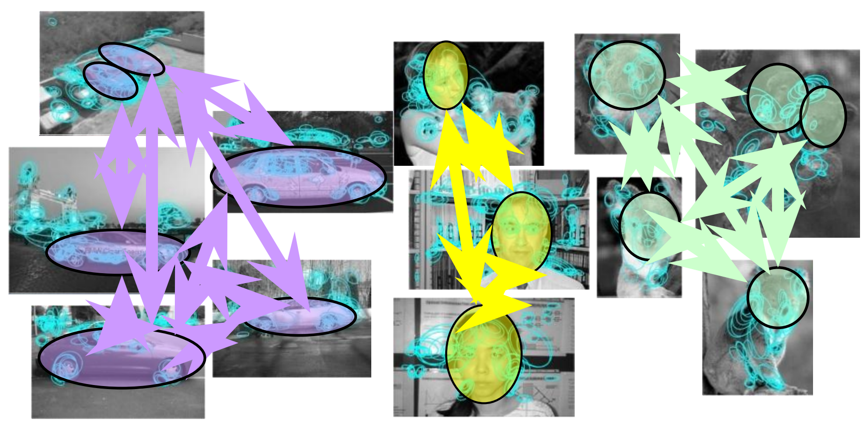
Determining image regions

Grouping video frames into shots

Figure-ground

What things should be grouped?

What cues indicate groups?



Object-level grouping

Original version by: Kristen Grauman

Similarity

Slide credit: Kristen Grauman

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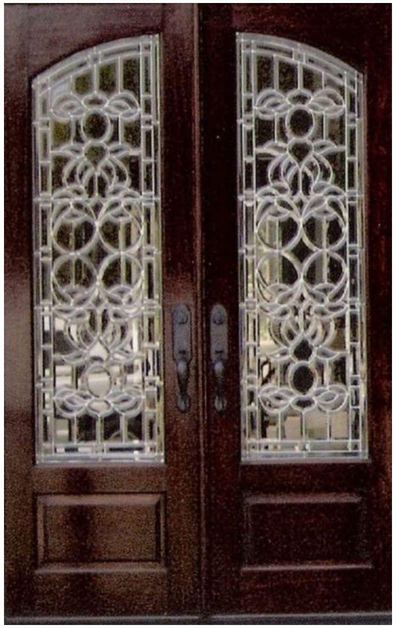
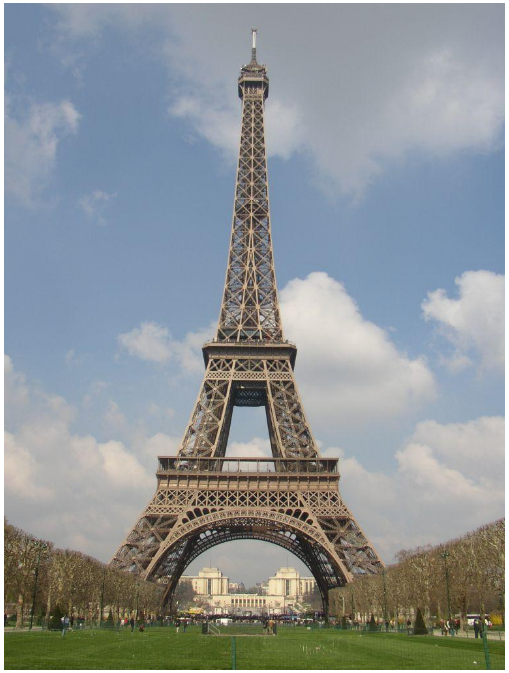
Symmetry





Slide credit: Kristen Grauman

Common Fate

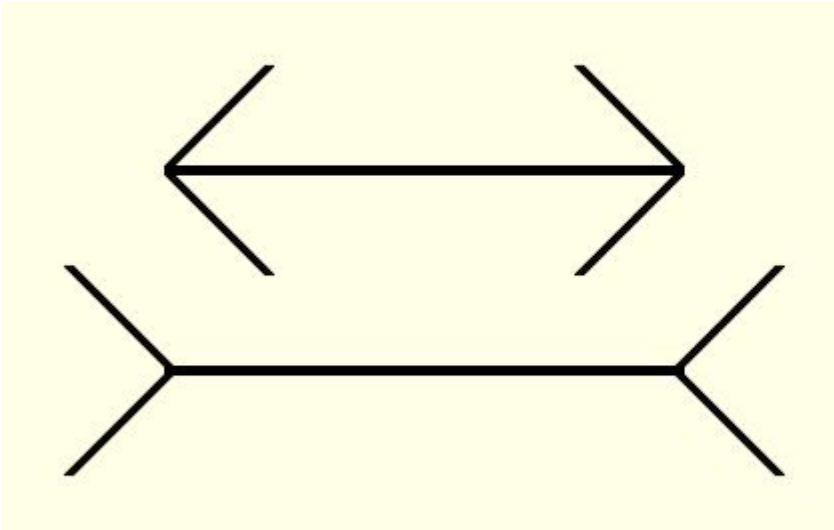
Image credit: Arthus-Bertrand (via F. Durand)

Slide credit: Kristen Grauman

Proximity

Slide credit: Kristen Grauman

Muller-Lyer Illusion



- What makes the bottom line look longer than the top line? (Answer is not fully clear)

See for potential explanations, e.g. misleading 3D cues:
https://en.wikipedia.org/wiki/M%C3%BCller-Lyer_illusion

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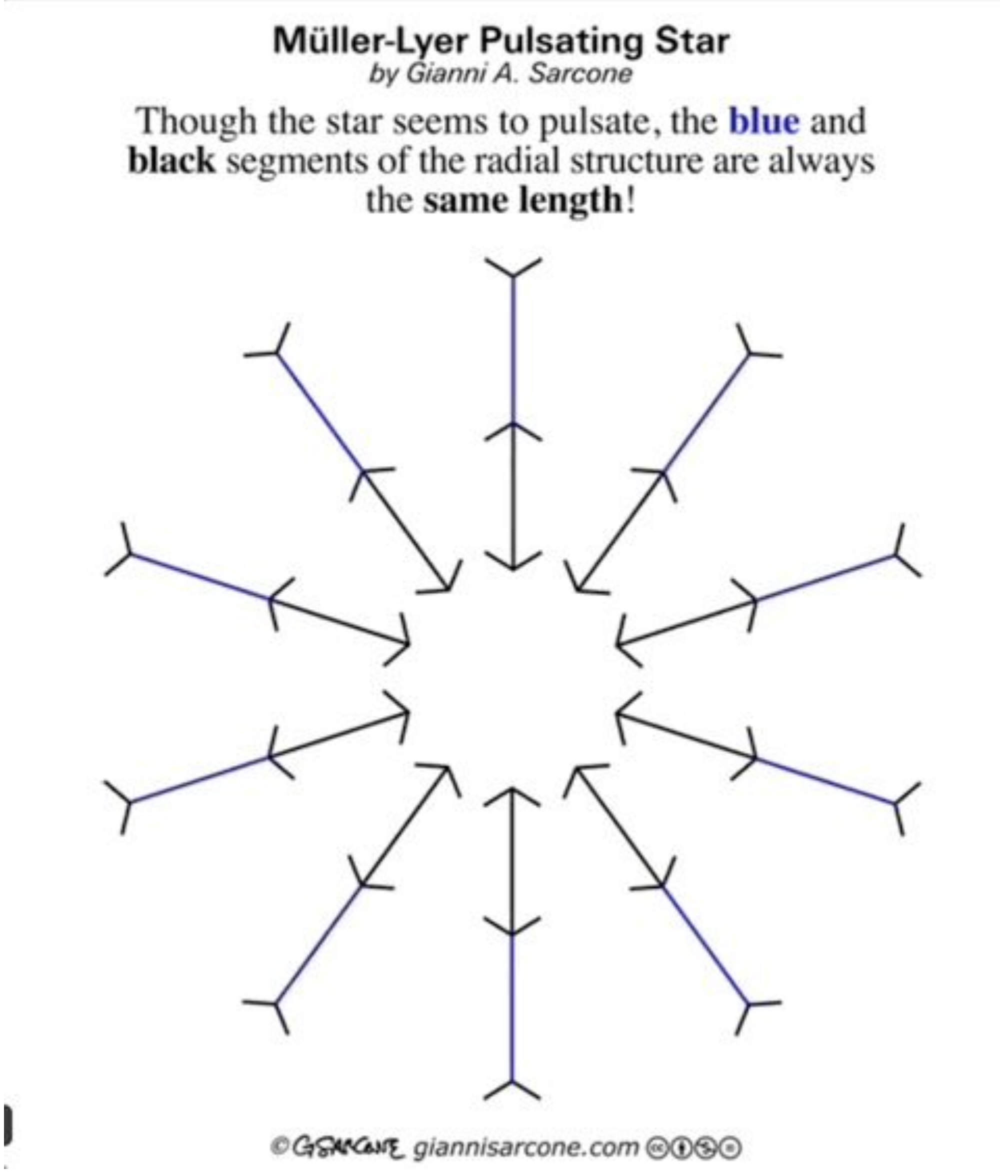

https://twitter.com/social_brains/status/972325994391486464

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What we will learn today

- Introduction to segmentation and clustering
- **Gestalt theory for perceptual grouping**
- K-means clustering

The Gestalt School

- *A school of psychology that emerged in the early twentieth century in Austria and Germany as a theory of perception.*

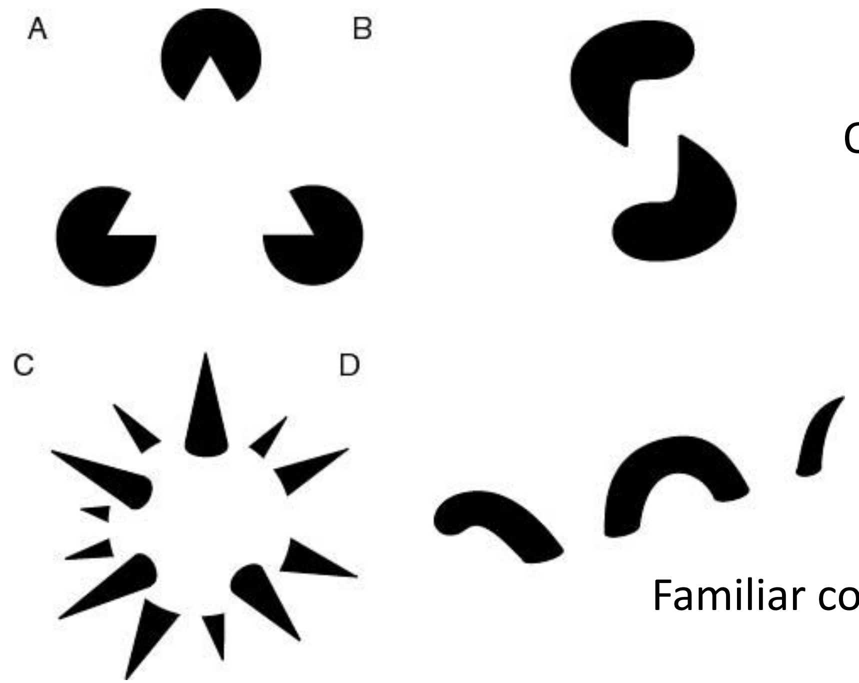
- gestaltism = configurationism

http://en.wikipedia.org/wiki/Gestalt_psychology

The Gestalt School

- Grouping is key to visual perception
- Elements in a collection can have properties that result from **relationships**
  - "The whole is greater than the sum of its parts"

Illusory/subjective contours



Occlusion

Familiar configuration

http://en.wikipedia.org/wiki/Gestalt_psychology

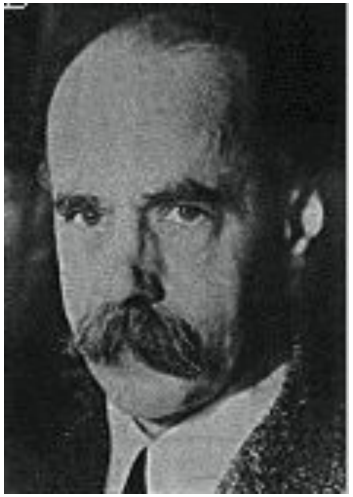
Slide credit: Svetlana Lazebnik

Gestalt Theory

- Gestalt: whole or group
  - Whole is greater than sum of its parts
  - Relationships among parts can yield new properties/features
- Psychologists identified series of factors that predispose set of elements to be grouped (by human visual system)

"I stand at the window and see a house, trees, sky. Theoretically I might say there were 327 brightnesses and nuances of colour. Do I have "327"? No. I have sky, house, and trees."

Max Wertheimer
(1880-1943)



Untersuchungen zur Lehre von der Gestalt,
***Psychologische Forschung*, Vol. 4, pp. 301-350, 1923**
http://psy.ed.asu.edu/~classics/Wertheimer/Forms/forms.htm

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Gestalt Factors

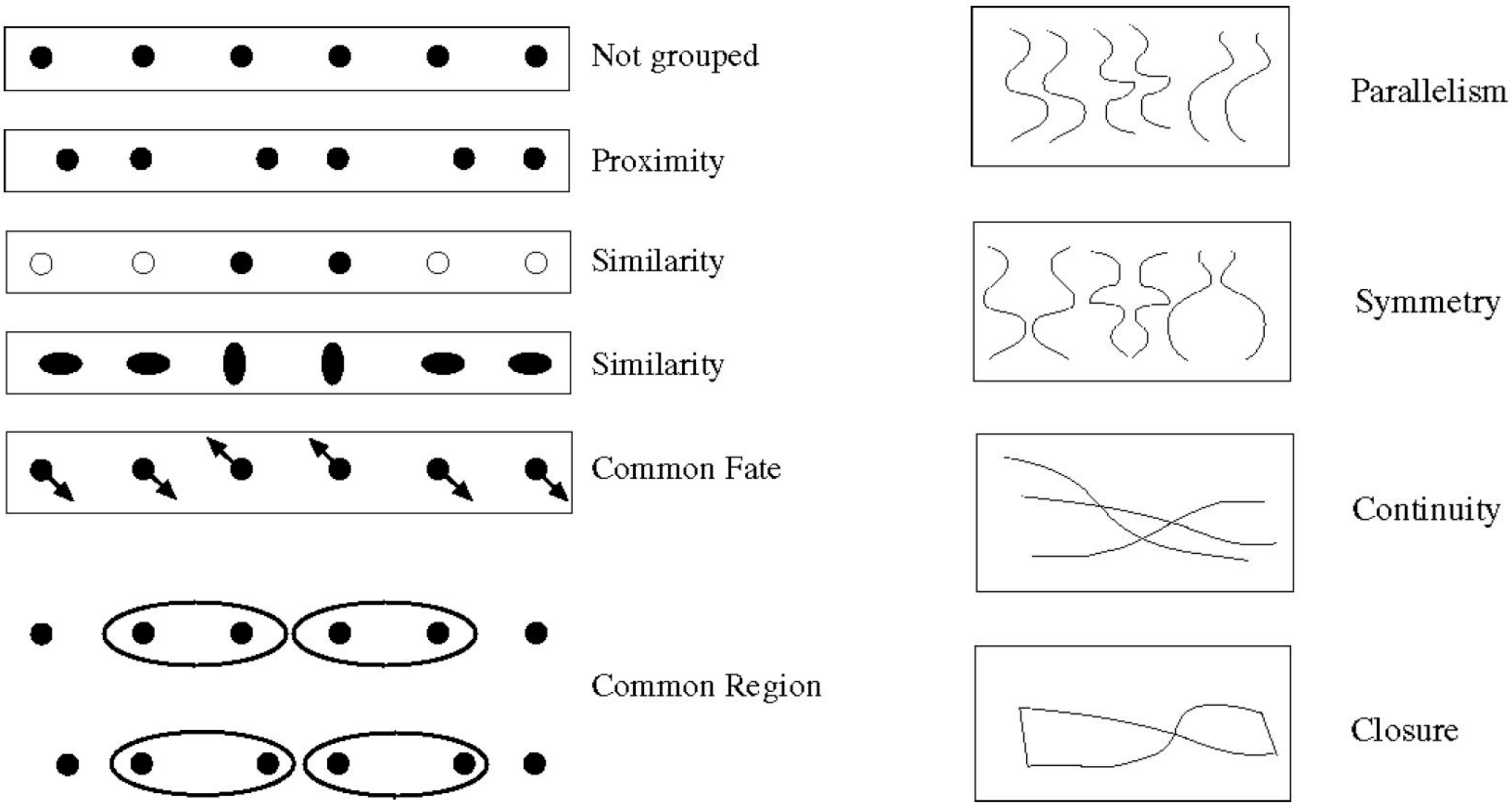
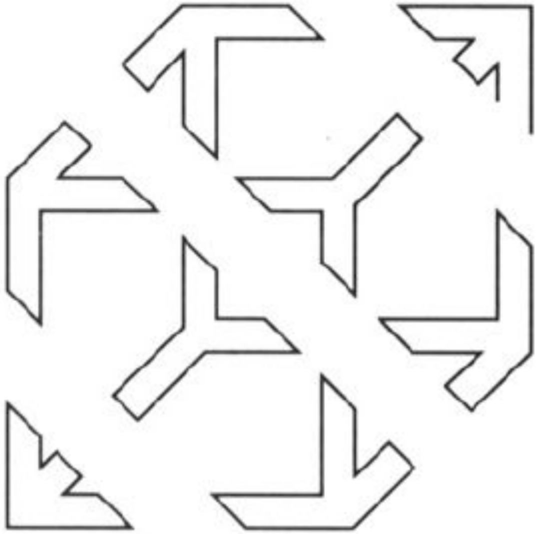


Image source: Forsyth & Ponce

- These factors make intuitive sense, but are very difficult to translate into algorithms.

Continuity through Occlusion Cues



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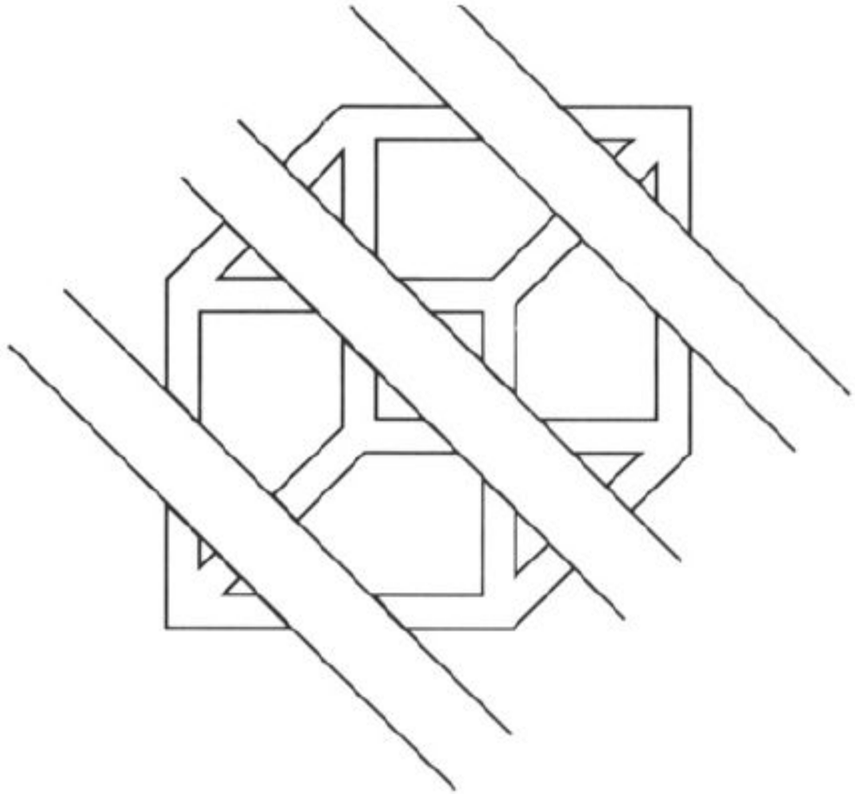
Continuity through Occlusion Cues



Continuity, explanation by occlusion

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Continuity through Occlusion Cues

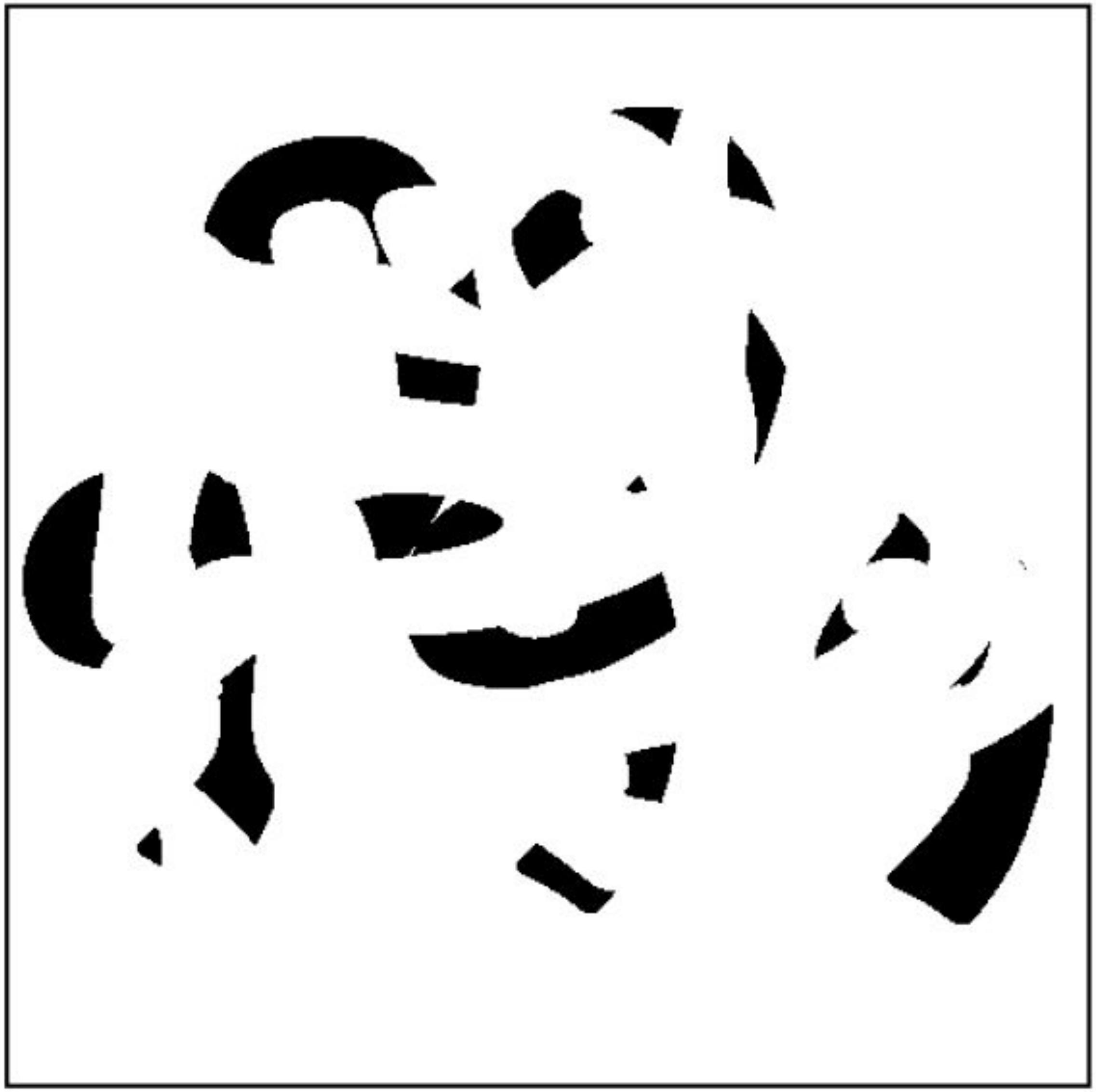


Image source: Forsyth & Ponce

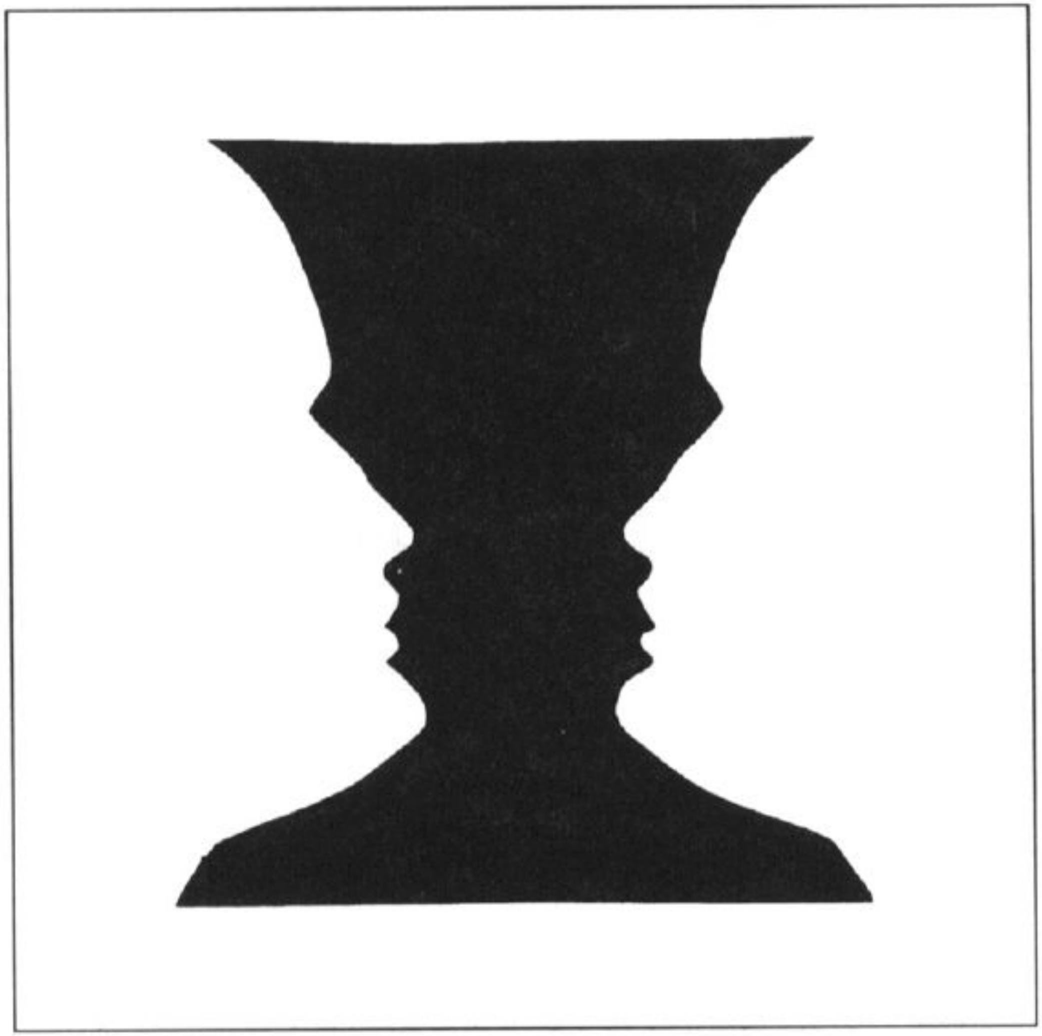
Adapted from slides by Ju

Continuity through Occlusion Cues

Image source: Forsyth & Ponce

Adapted from slides by

Figure-Ground Discrimination



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The Ultimate Gestalt?

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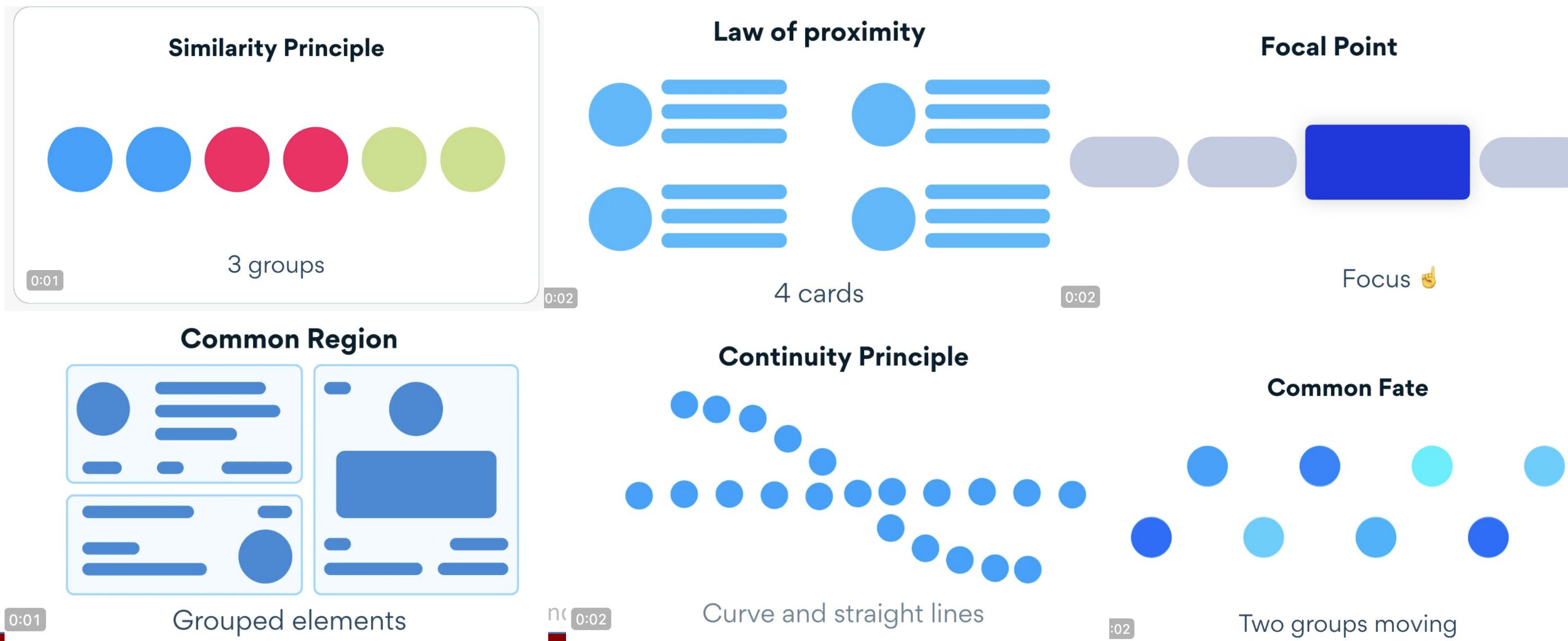
Side-topic: Gestalt Principles in Presentations

- https://twitter.com/pablostanley/status/974303621092225024



What we will learn today

- Introduction to segmentation and clustering
- Gestalt theory for perceptual grouping
- **K-means clustering**

Why do we cluster?

- **Summarizing data**
  - Look at large amounts of data
  - Patch-based compression or denoising
  - Represent a large continuous vector with the cluster number

- **Counting**
  - Histograms of texture, color, SIFT vectors

- **Segmentation**
  - Separate the image into different regions

- **Prediction**
  - Images in the same cluster may have the same labels

Slide: Derek Hoiem

General ideas

- Tokens
  - whatever we need to group (pixels, points, surface elements, etc.)
- Bottom up clustering
  - tokens belong together because they are locally coherent
- Top down clustering
  - tokens belong together because they lie on the same visual entity (object, scene…)
- Model based clustering
  - Based on fitting a model to the data set

How do we cluster?

Examples

- Agglomerative clustering (we'll skip)
  - Start with each point as its own cluster and iteratively merge the closest clusters
- K-means (we'll cover)
  - Iteratively re-assign points to the nearest cluster center
- Mean-shift clustering (we'll skip)
  - Estimate modes of pdf

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Clustering's technical elements

Clustering is an unsupervised learning method. Given items $x_1, \ldots, x_n \in \mathbb{R}^D$, the goal is to group them into clusters.

Dependending on the algorithm we need to make assumptions / take inputs, such as:
- a pairwise distance/similarity function
- the desired number of clusters
- thresholds on intra/inter cluster distance limits
- a data distribution model (eg. mixture of gaussian)

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Defining Similarity / Distance Measures

Some algorithms, such as agglomerative clustering, explicitly use a similarity or distance function.

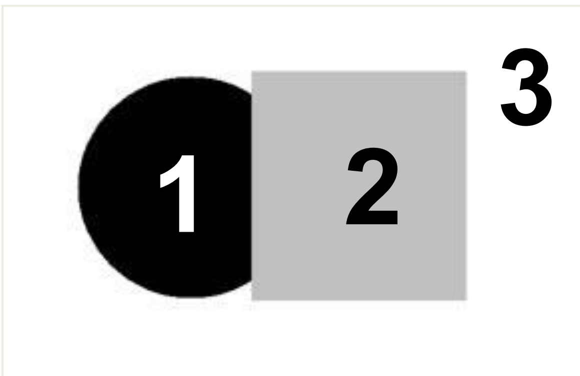
Some others, eg. k-means, implicitly assume a distance metric through the distribution assumption.

Example similarity & distance functions:

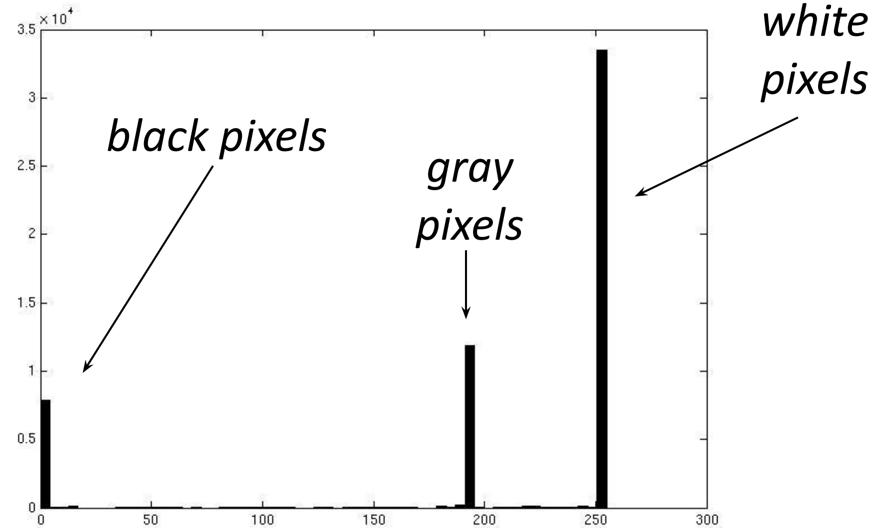
$$\text{cosine similarity} = S_C(A, B) := \cos(\theta) = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|}$$

$$\text{cosine distance} = D_C(A, B) := 1 - S_C(A, B)$$

Image Segmentation: Toy Example



input image



intensity

- These intensities define the three groups.
- We could label every pixel in the image according to which of these primary intensities it is.
  - i.e., segment the image based on the intensity feature.
- What if the image isn't quite so simple?

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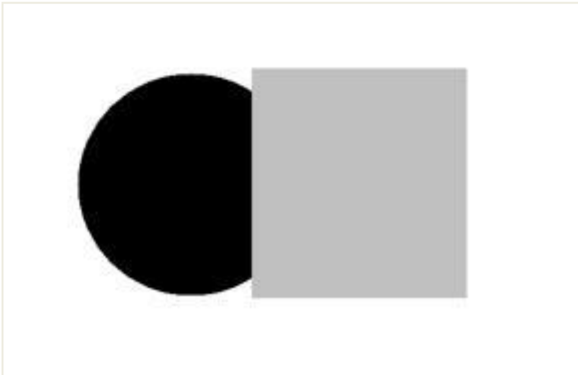
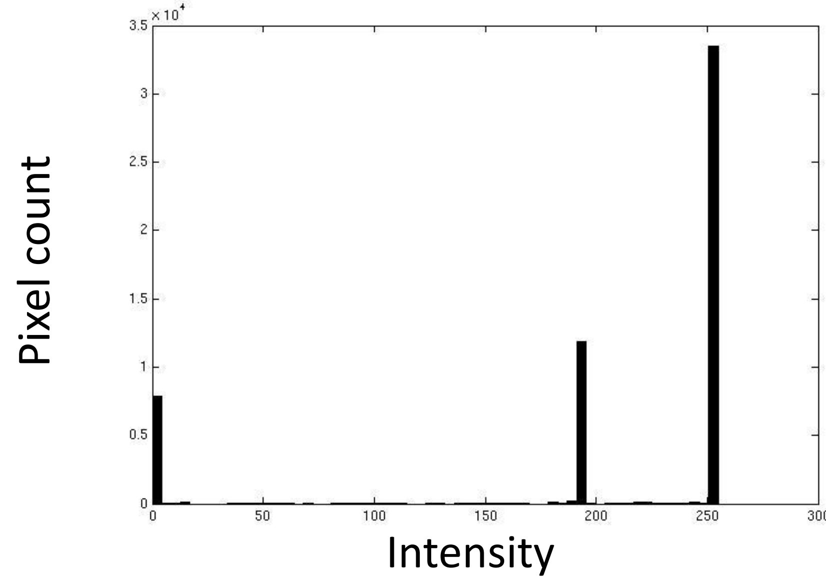
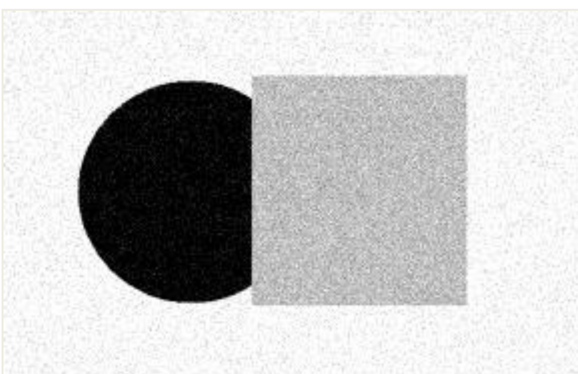
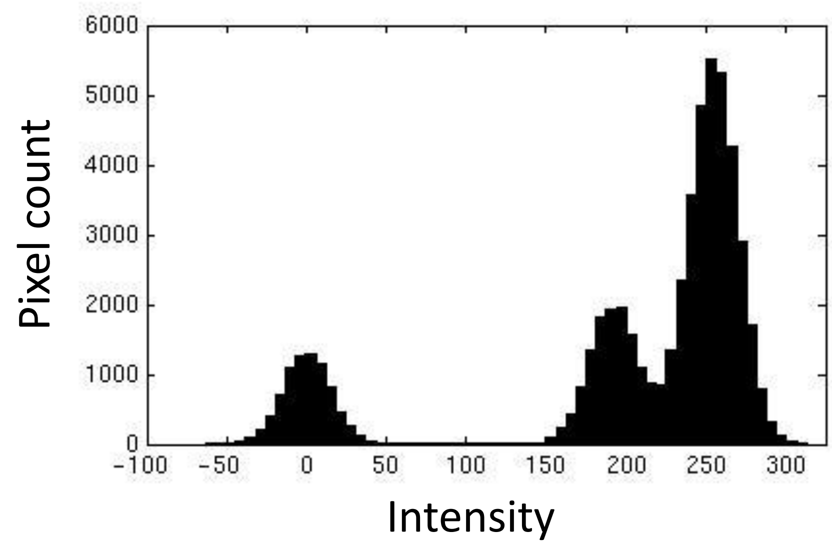


Input image





Input image



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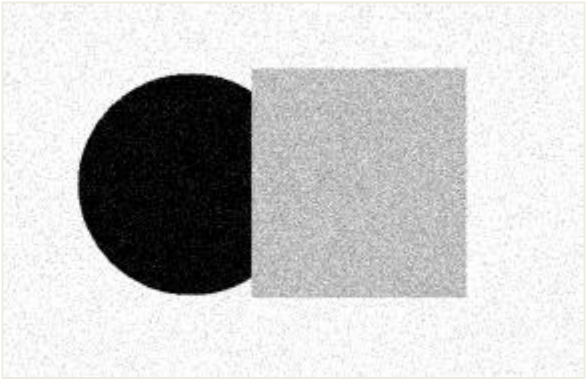
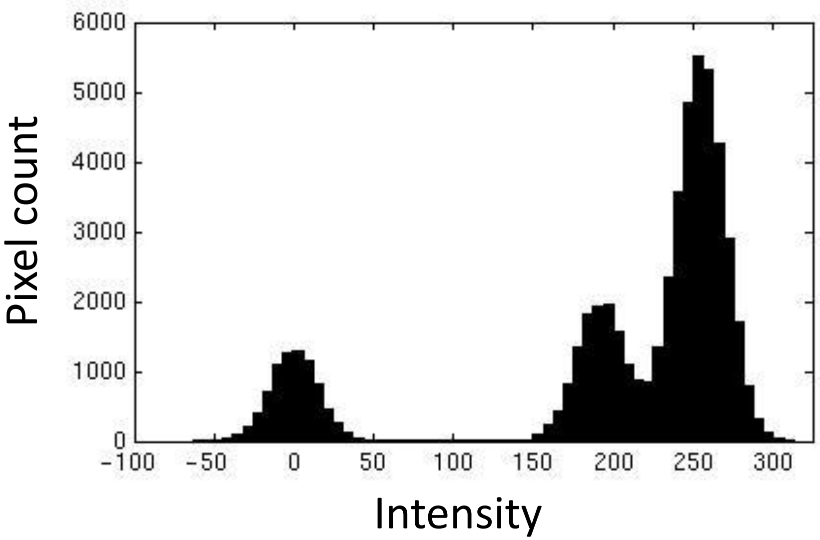


Input image



- Now how to determine the three main intensities that define our groups?
- We need to cluster.

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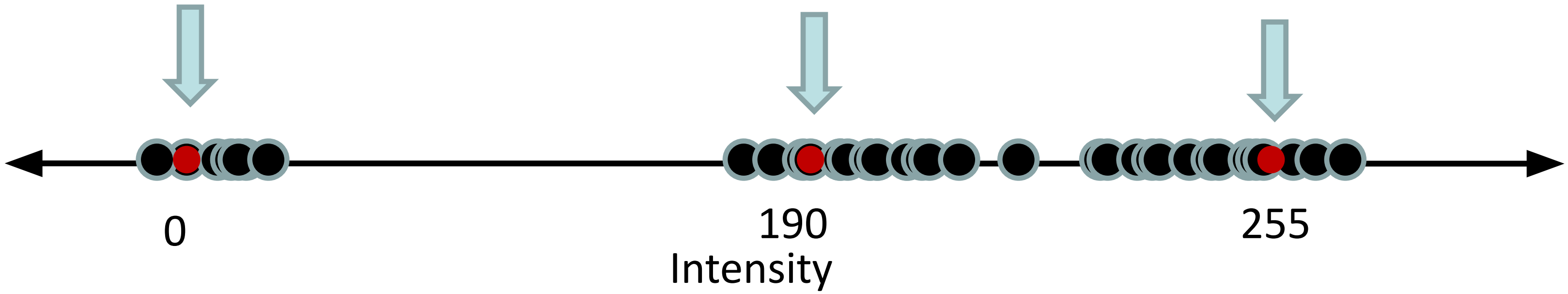

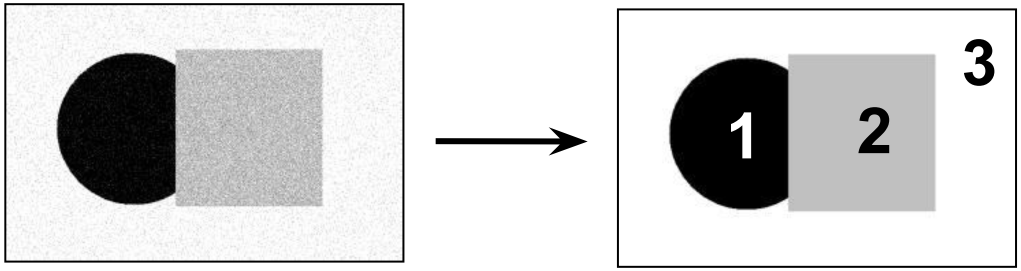

- **General idea**: choose three "centers" as the representative intensities, and label every pixel according to which of these centers it is nearest to.
- **Best cluster centers** are defined as those that minimize Sum of Square Distance (SSD) between all points and their nearest cluster center c_i:

$$SSD = \sum_{cluster\,i} \sum_{x \in cluster\,i} (x - c_i)^2$$

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Clustering for Summarization

Goal: cluster to minimize variance in data given clusters

– Preserve information

Cluster center Data

$$c^*,\ \delta^* = \underset{c,\,\delta}{\arg\min} \frac{1}{N} \sum_{j}^{N} \sum_{i}^{K} \delta_{ij} \left(c_i - x_j\right)^2$$

Whether x_j is assigned to c_i

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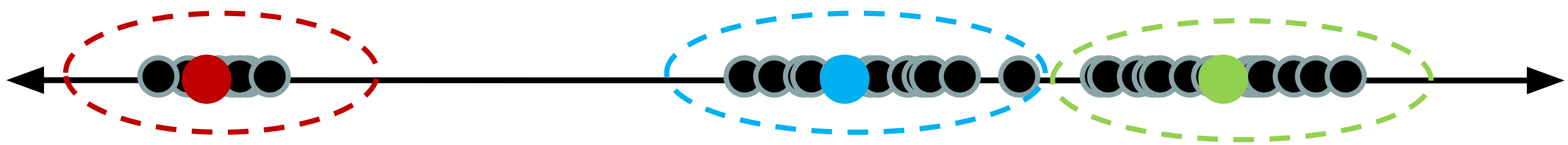
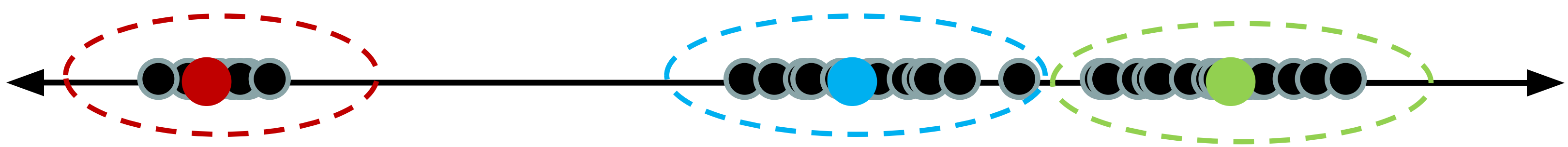
Clustering - but how?

- With this objective, it is a “chicken and egg” problem:
  - If we knew the *cluster centers*, we could allocate points to groups by assigning each to its closest center.



  - If we knew the *group memberships*, we could get the centers by computing the mean per group.



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K-means clustering

1. Initialize ($t = 0$): cluster centers $c_1, \ldots, c_K$

2. Compute δ^t: assign each point to the closest center
   – δ^t denotes the set of assignment for each x_j to cluster c_i at iteration *t*

$$\delta^t = \underset{\delta}{\operatorname{argmin}} \frac{1}{N} \sum_{j}^{N} \sum_{i}^{K} \delta_{ij}^{t-1} \left(c_i^{t-1} - x_j \right)^2$$

3. Computer c^t: update cluster centers as the mean of the points

$$c^t = \underset{c}{\operatorname{argmin}} \frac{1}{N} \sum_{j}^{N} \sum_{i}^{K} \delta_{ij}^{t} \left(c_i^{t-1} - x_j \right)^2$$

4. Update $t = t + 1$, Repeat Step 2-3 till stopped

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Slide: Derek Hoiem

K-means clustering

1. Initialize ($t = 0$): cluster centers $c_1, \ldots, c_K$
   - Commonly used: random initialization

2. Compute δ^t: assign each point to the closest center

3. Computer c^t: update cluster centers as the mean of the points

$$c^t = \underset{c}{\operatorname{argmin}} \frac{1}{N} \sum_j^N \sum_i^K \delta_{ij}^t \left(c_i^{t-1} - x_j \right)^2$$

4. Update $t = t + 1$, Repeat Step 2-3 till stopped

   - c^t (or assignments) doesn't change anymore
     $\Rightarrow$ Convergence, stop.

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K-means clustering

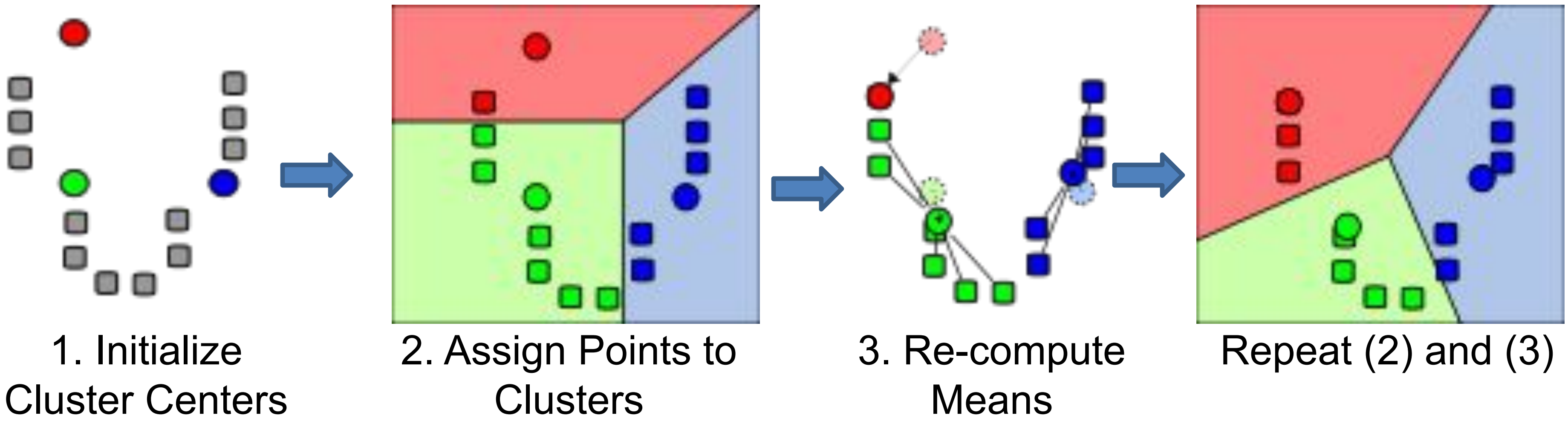

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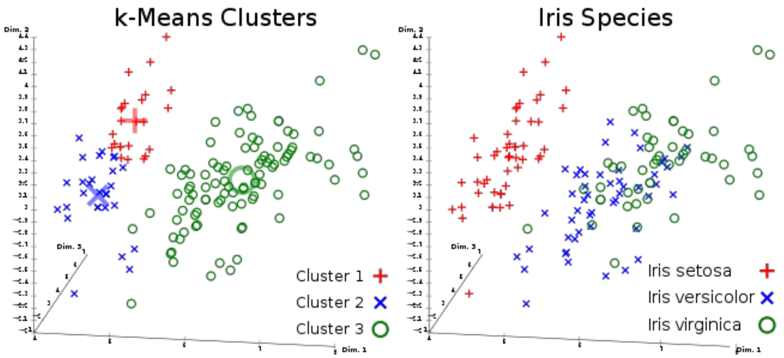
Illustration Source: wikipedia

K-means clustering

- Converges to a *local minimum* solution
  - Initialize multiple runs

- Better fit for spherical data



- Need to pick K (# of clusters)

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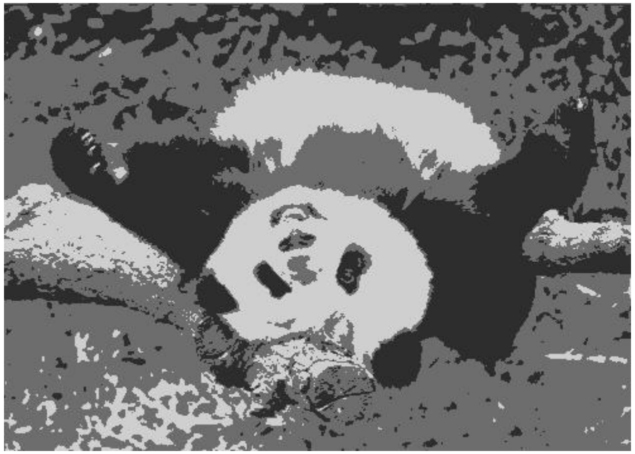
Segmentation as Clustering

Original image

2 clusters



3 clusters

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Feature Space

- Depending on what we choose as the *feature space (=representation)*, we can group pixels in different ways.

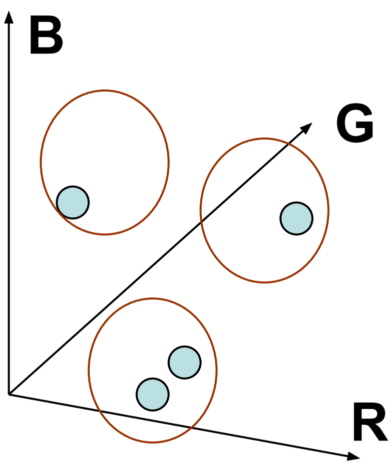
- Grouping pixels based on intensity similarity

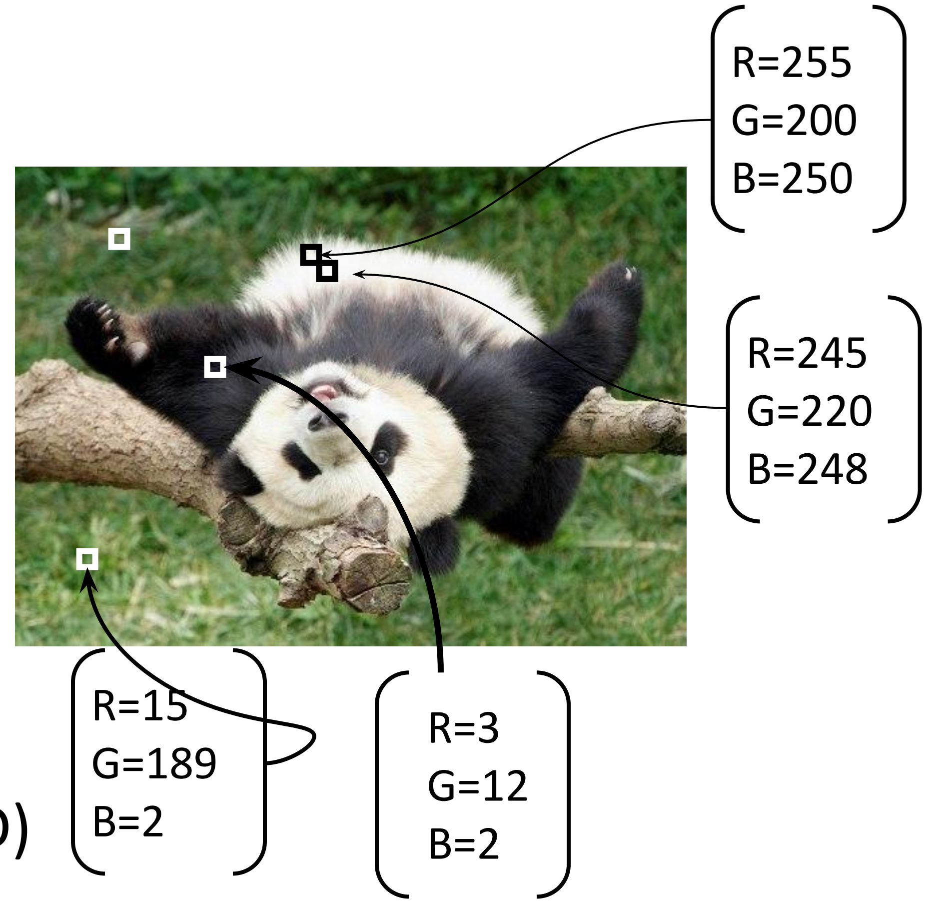
- Feature space: intensity value (1D)

Slide credit: Kristen Grauman

Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.
- Grouping pixels based on color similarity





- Feature space: color value (3D)

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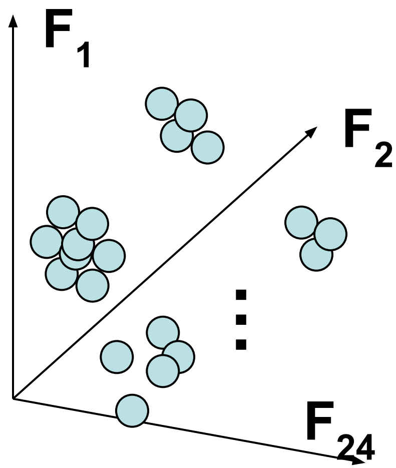
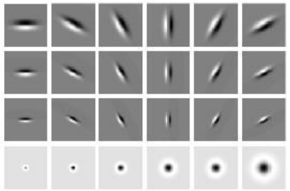
ebles, and Ranjay Krishna

Feature Space

- Depending on what we choose as the *feature space*, we can group pixels in different ways.

- Grouping pixels based on texture similarity





Filter bank of 24 filters

- Feature space: filter bank responses (e.g., 24D)

Slide credit: Kristen Grauman

Features greatly affect K-Means

Image | Intensity-based clusters | Color-based clusters

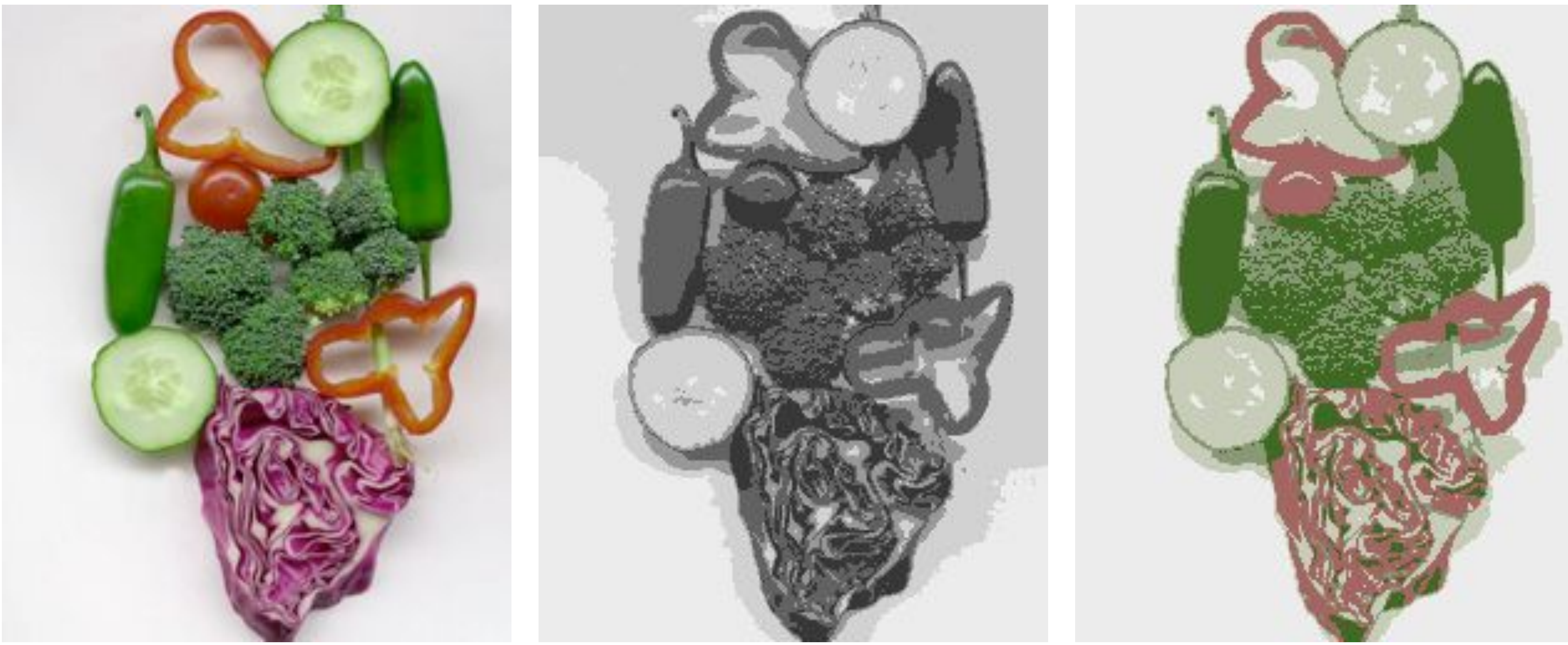


Image source: Forsyth & Ponce

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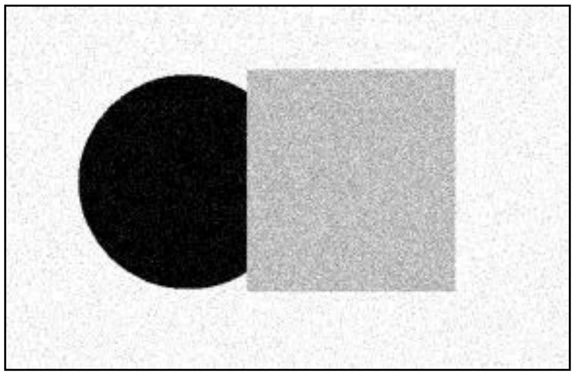
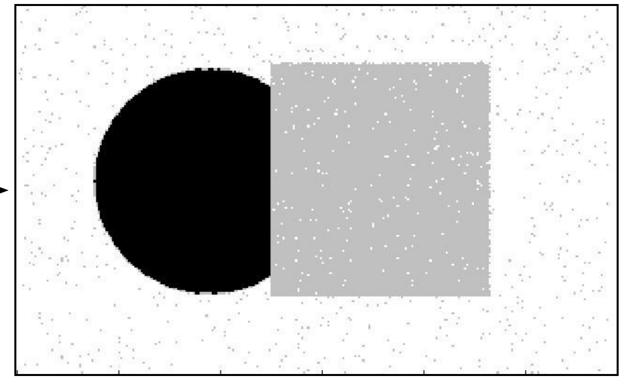
Smoothing Out Cluster Assignments

- Assigning a cluster label per pixel may yield outliers:

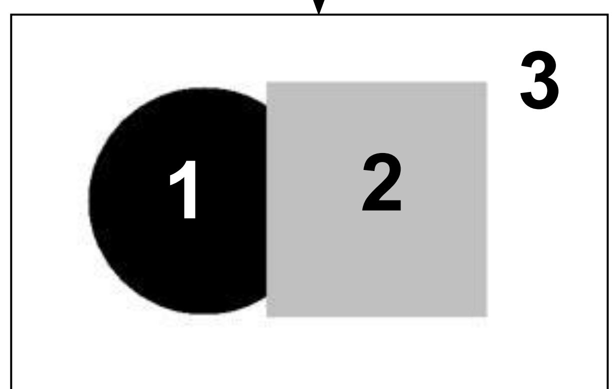


Original



Labeled by cluster center's intensity

?



- How can we ensure they are spatially smooth?

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…ebles, and Ranjay Krishna

Segmentation as Clustering

- Depending on what we choose as the *feature space*, we can group pixels in different ways.

- Grouping pixels based on *intensity+position* similarity

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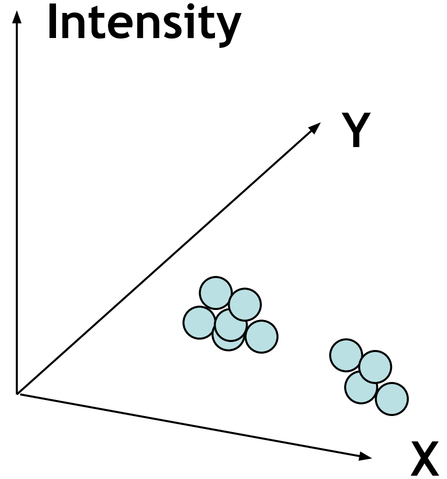
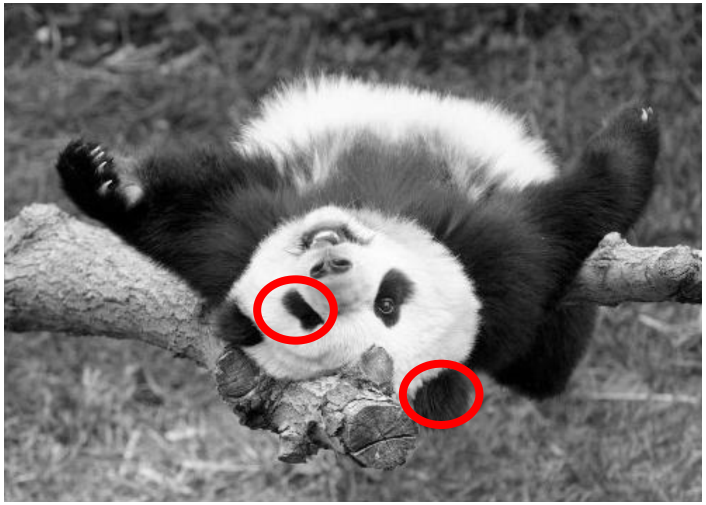




⇒ Way to encode both *similarity* and *proximity.*

Slide credit: Kristen Grauman

Adapted from slides by Juan Carlos Niebles, and Ranjay Krishna

How to evaluate clusters?

- Generative
  - How well are points reconstructed from the clusters?
- Discriminative
  - How well do the clusters correspond to labels?
    - Can we correctly classify which pixels belong to the panda?
  - Note: unsupervised clustering does not aim to be discriminative as we don’t have the labels.

Adapted from slides by Juan Carlos Niebles, and Ranjay Krishna

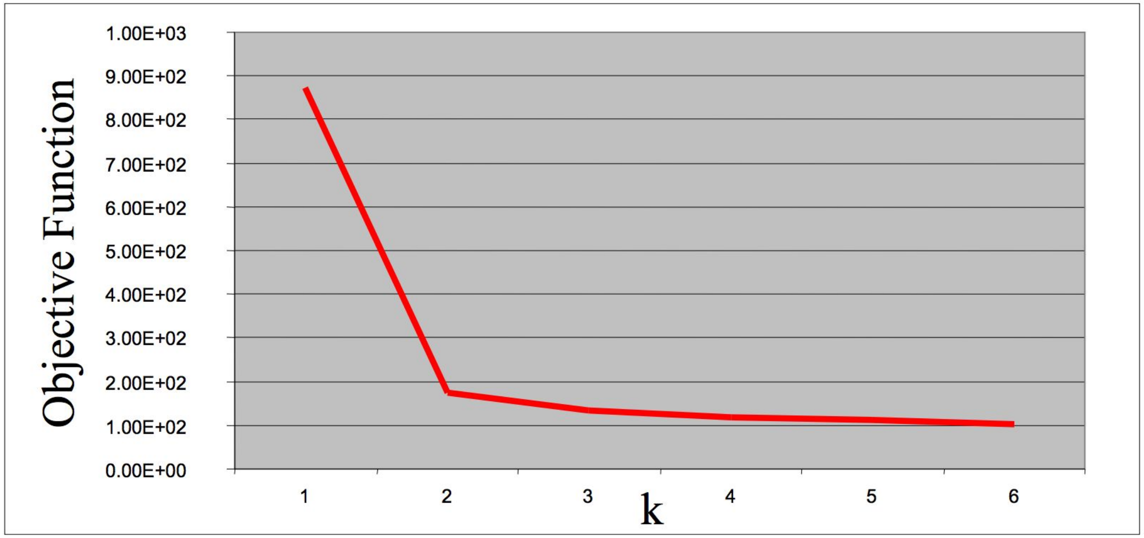
Slide: Derek Hoiem

How to choose the number of clusters?

Try different numbers of clusters in a validation set and look at performance.

We can plot the objective function values for k equals 1 to 6…

The abrupt change at k = 2, is highly suggestive of two clusters in the data. This technique for determining the number of clusters is known as “knee finding” or “elbow finding”.



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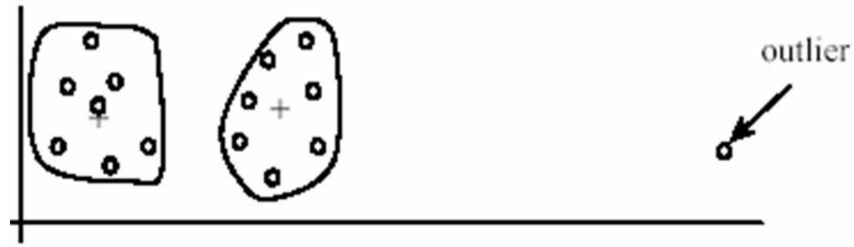
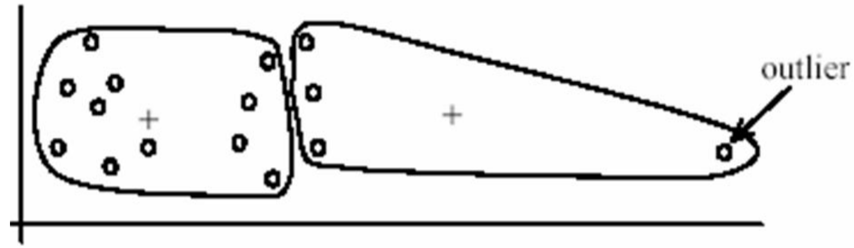
Slide: Derek Hoiem

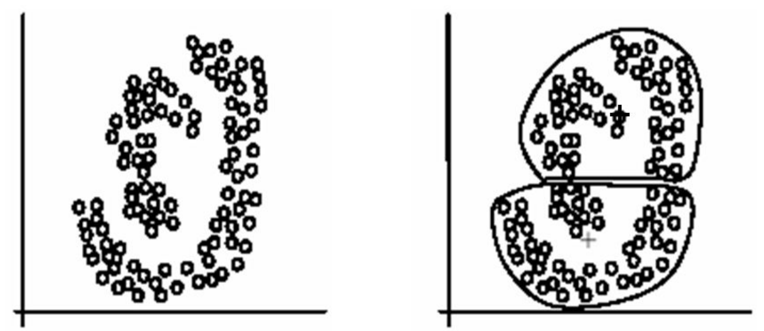
K-Means pros and cons

- Pros
  - Finds cluster centers that minimize conditional variance (good representation of data)
  - Simple and fast, Easy to implement
- Cons
  - Need to choose K
  - Sensitive to outliers
  - Prone to local minima
  - All clusters have the same parameters (e.g., distance measure is non-adaptive)
  - *Can be slow: each iteration is O(KNd) for N d-dimensional points
- Usage
  - Unsupervised clustering
  - Rarely used for pixel segmentation



(B): Ideal clusters





(A): Two natural clusters (B): *k*-means clusters