Sliding Window Temporal Graph Coloring

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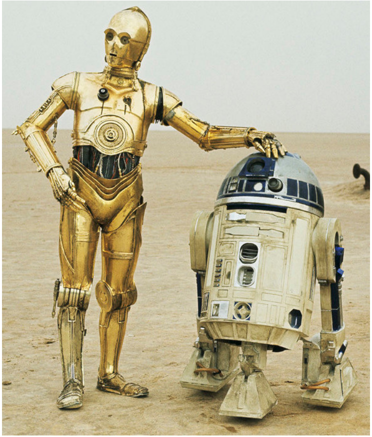
Department of Computer Science
Liverpool University

Main question:

- What is the natural problem that extends graph coloring to the case where the graph changes over time?

A motivating scenario:

- Mobile agents broadcast information



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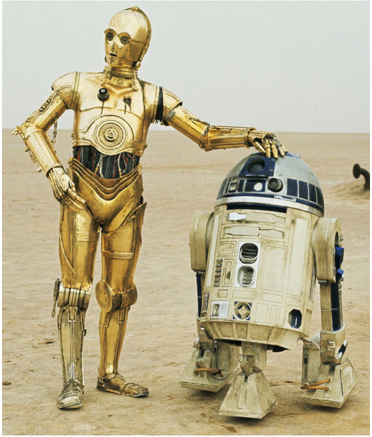

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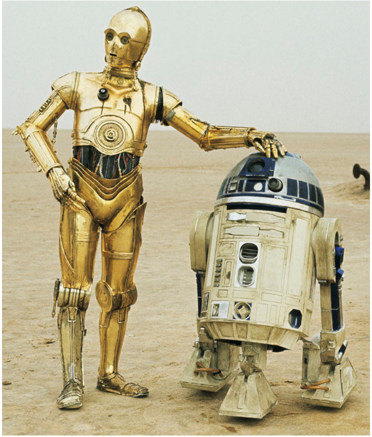

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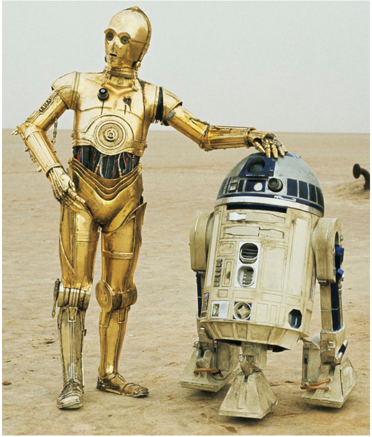

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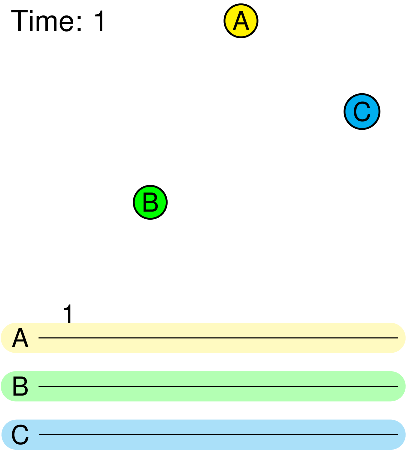

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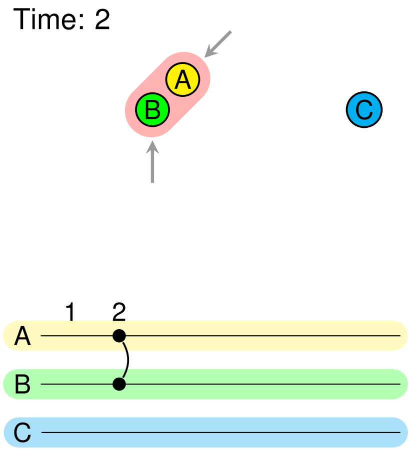

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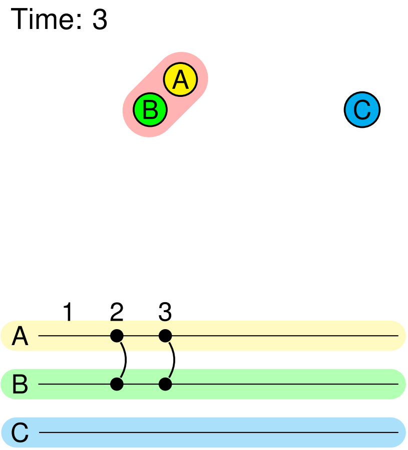

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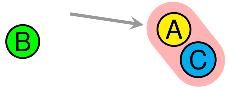

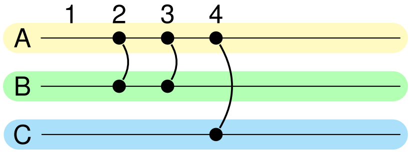

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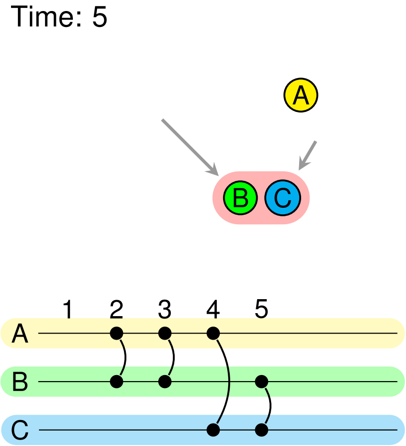

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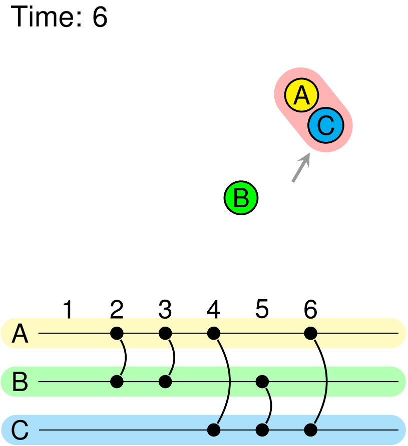

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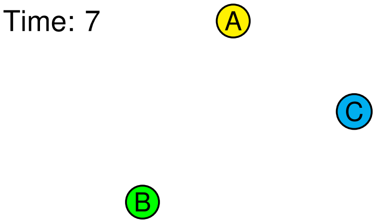

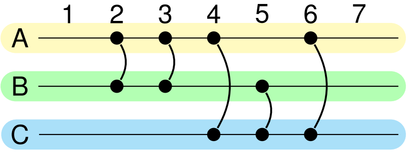

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Modeling:

- Vertices in a temporal graph

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Modeling:

- Vertices in a temporal graph
- Time-edges
- Vertices need to be **differently colored** in order to exchange information
- Each time-edge should be "properly colored" at least once in each Δ-window in which it exists

Temporal Graph

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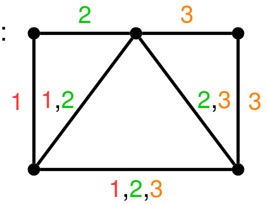

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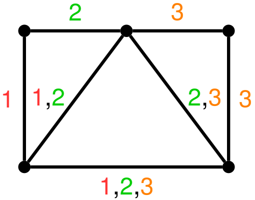
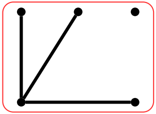


G_1:



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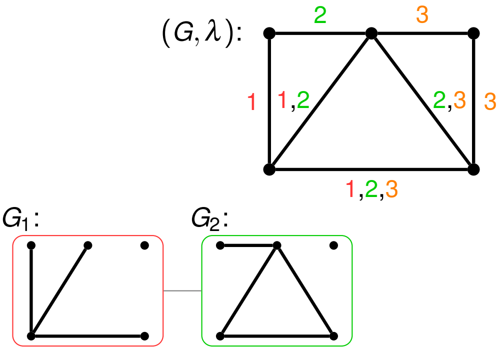

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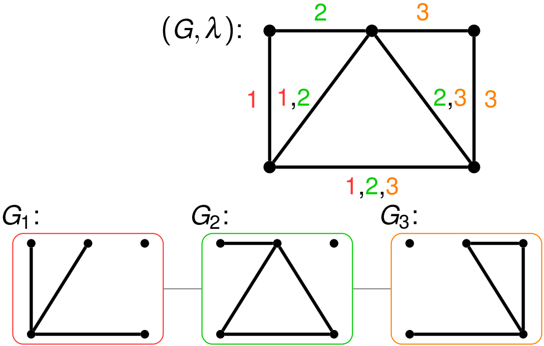

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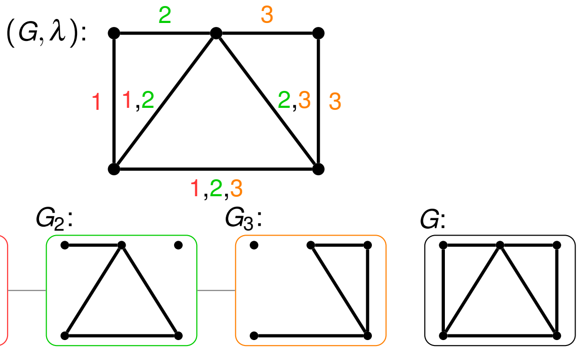

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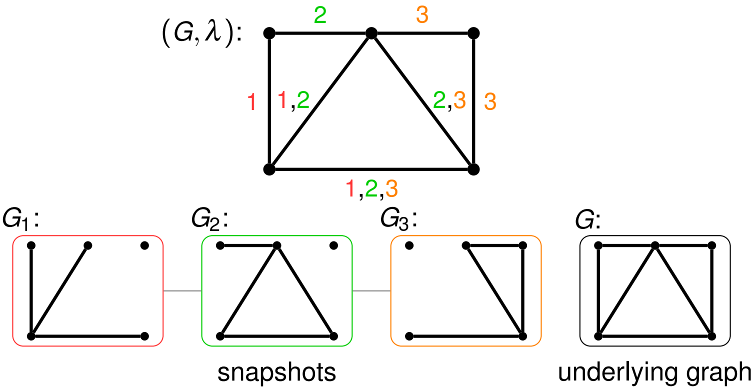

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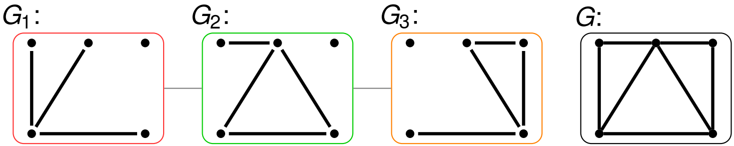

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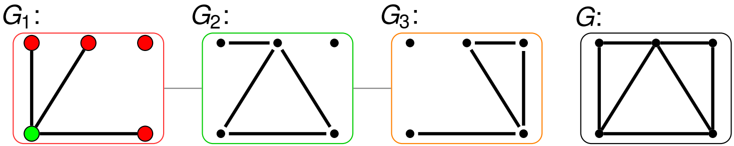

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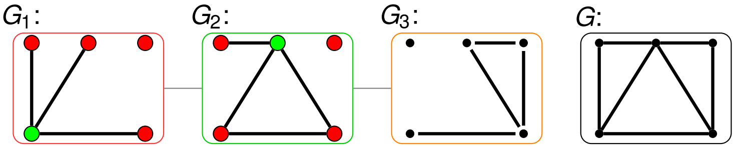

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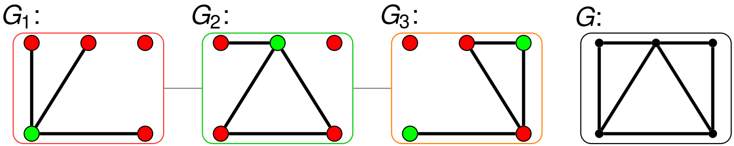

Sliding Window Temporal Coloring (SWTC)

Input: A temporal graph (G, λ), and two integers $k \in \mathbb{N}$ and $\Delta \leq T$.
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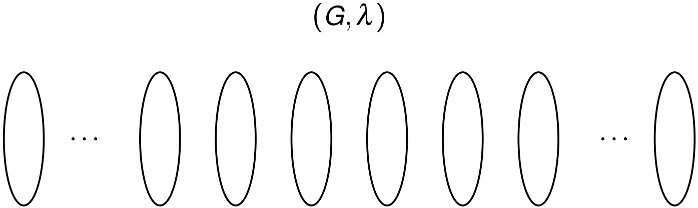

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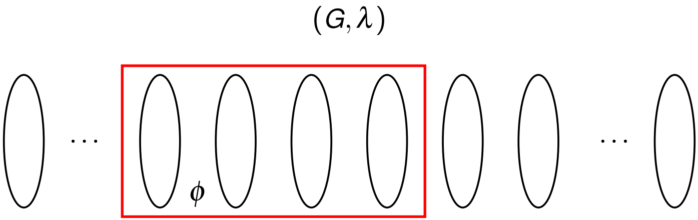

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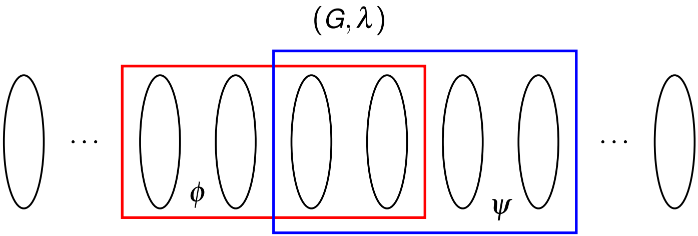

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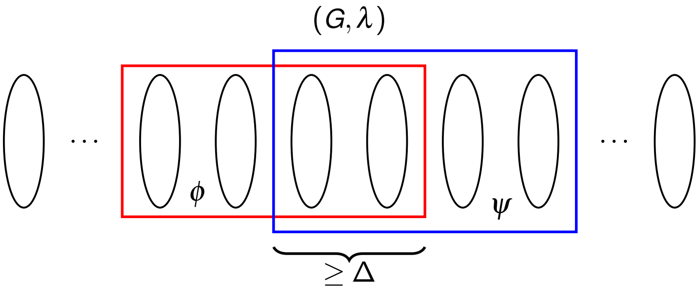

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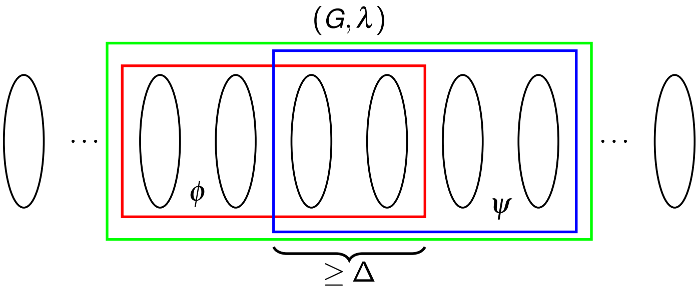

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1. For 2Δ-windows $W_i = [i\Delta + 1, (i+2)\Delta]$ for $i \in \{0, 1, \ldots, T/\Delta - 2\}$, enumerate all partial sliding Δ-window temporal colorings ϕ_{W_i}, where each trivial snapshot is colored in some fixed (but arbitrary) way.

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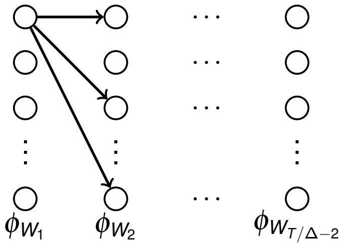
2. Create a DAG with ϕ_{W_i} as vertices and connect ϕ_{W_i} to $\phi_{W_{i+1}}$ if the two colorings agree on the overlapping part.



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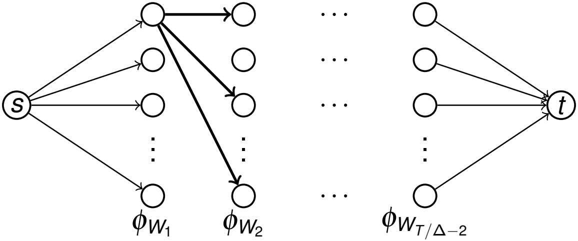

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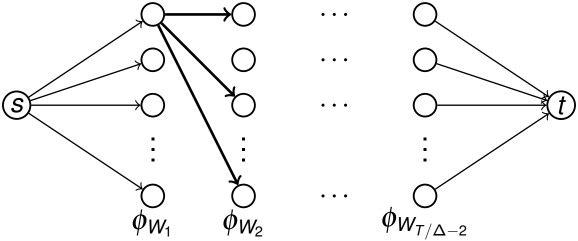
4. If there is a path from s to t, answer YES, otherwise NO.



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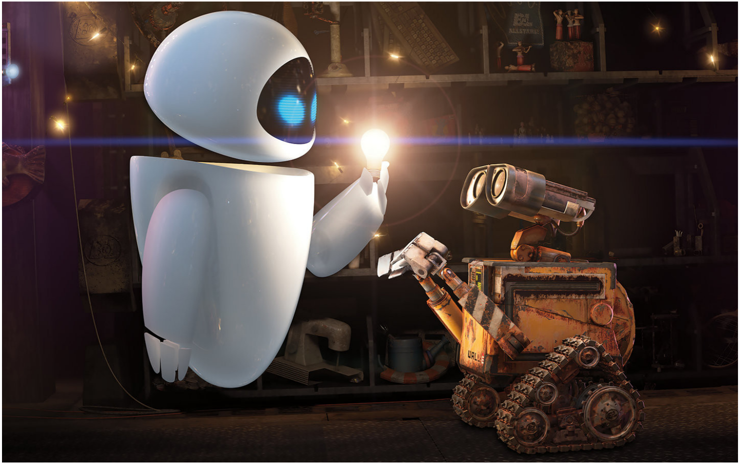
ETH Lower Bound

SWTC does not admit a $2^{o(n) \cdot f(T+k)}$-time algorithm for any computable function f unless ETH fails.



How to exploit few vertices?

Sliding Window Temporal Graph Coloring

Fixed-Parameter Tractability II

Observation

If a snapshot appears n^2 times in a Δ-window, all its edges can be colored properly. *[just properly color every edge of it once]*

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Lemma

If the reduction rule is not applicable, each Δ-window contains at most $2n^2 \cdot 2^{n^2}$ (non-trivial) snapshots.

Sliding Window Temporal Graph Coloring

Fixed-Parameter Tractability III

Recall our first exponential-time algorithm:

Theorem

Sliding Window Temporal Coloring can be solved in $O(k^{4\Delta \cdot n} \cdot T)$ time.

Therefore, since every Δ-window has at most $2n^2 \cdot 2^{n^2}$ snapshots (and since $k \leq n$):

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SWTC is linear-time fixed-parameter tractable (FPT) with respect to n (i.e. in $O(f(n) \cdot T)$ time).

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One of our hardness results:

Theorem

SWTC is NP-hard, even if the vertex cover number of the underlying graph G is at most $2k + 13$ (where $k =$ number of colors).

Thus we cannot hope for an (exact) FPT algorithm with respect to the parameter "vertex cover number of the underlying graph".

Sliding Window Temporal Graph Coloring

Fixed-Parameter Tractability III

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SWTC admits a linear-time FPT-approximation algorithm for parameter "vertex cover number of G" with an additive error one. (Objective: Minimize number of colors.)

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Idea:

- Compute in linear FPT-time a minimum vertex cover of G (the rest is independent set in every slot!)
- Use our exponential algorithm to optimally solve SWTC in the temporal graph induced by the vertex cover vertices
- This is a lower bound on the number of colors needed for (G, λ)
- Color all other vertices in all slots with a fresh color

Reduction from **Monotone Exactly 1-in-3 SAT**.

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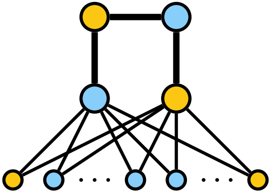
Variable Gadget:



2 colors, sliding window size $\Delta = 2$.

Sliding Window Temporal Graph Coloring

Constant Underlying VC Reduction – Main Ideas II

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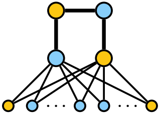
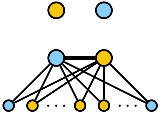
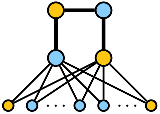


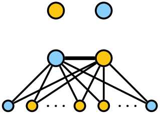
Type 1 snapshot.



Type 2 snapshot.



Type 3 snapshot.



Type 4 snapshot.

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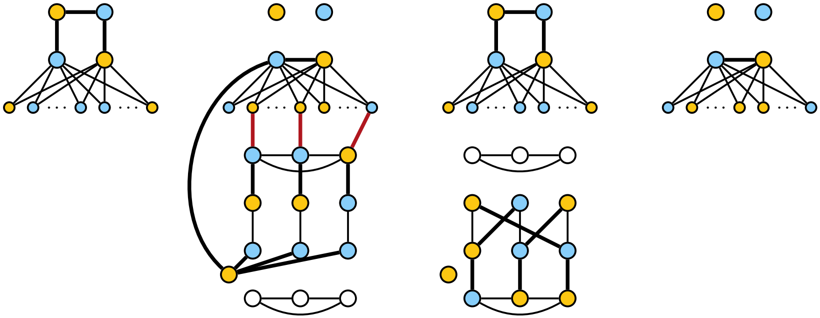

Type 1 snapshot. Type 2 snapshot. Type 3 snapshot. Type 4 snapshot.

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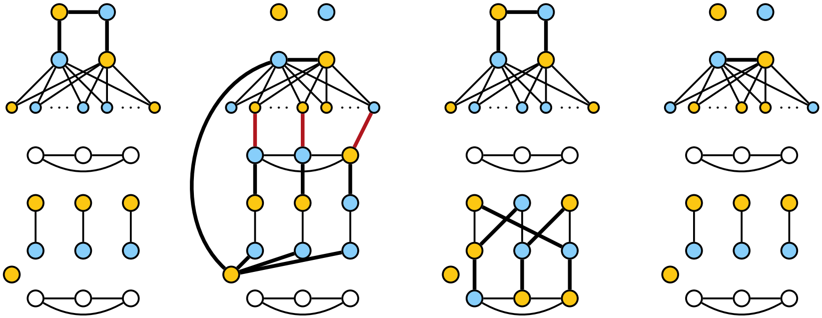

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Outlook

and Future Work

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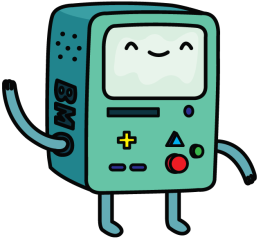

https://arxiv.org/pdf/1811.04753.pdf

Link to arXiv.

Thank you!