Analytics for Business Intelligence

Last Name: Bhatnagar First Name: Meru Section Number: 40

Home Work 2 Total marks are 15 which provide 7.5% to the total assessment. Students must implement the homework using R language, cut-and-paste of outputs from text book are not permitted (and easily detectable).

The Steps of this homework should be implemented using RStudio and printouts inserted in this document after each step.

Regression with interaction (3 marks)

Step 1 You need create the working directory and connect Boston data file as you did this in Home Work 1.

OUTPUT:

```
setwd("C:/Users/NICSI/Desktop/ABI/HW 2")
library(MASS)
data(Boston)
```

Step 2 include interaction terms in a linear model using the Im() function. The syntax Istat:black tells R to include an interaction term between Istat and black. The syntax Istat*age simultaneously includes Istat, age, and the interaction term Istatxage as predictors; it is a shorthand for Istat+age+Istat:age.

OUTPUT:

```
Console R Markdown ×
C:/Users/NICSI/Desktop/ABI/HW 2/ A
> summary(lm(medv ~ lstat*age, data = Boston))
call:
lm(formula = medv ~ lstat * age, data = Boston)
Residuals:
    Min
             1Q Median
                              30
                                      Max
-15.806 -4.045 -1.333
                          2.085 27.552
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 36.0885359 1.4698355 24.553 < 2e-16 ***
            -1.3921168 0.1674555
                                    -8.313 8.78e-16
lstat
            -0.0007209 0.0198792 -0.036
                                               0.9711
age
lstat:age 0.0041560 0.0018518
                                    2.244
                                              0.0252
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
Residual standard error: 6.149 on 502 degrees of freedom
Multiple R-squared: 0.5557, Adjusted R-squared: 0.5557, Adjusted R-squared: 0.5557, p-value: < 2.2e-16
                                 Adjusted R-squared: 0.5531
```

Step 3 (0.5 mark) Use command dim to check how many observations and variables are in the Boston file. Find the names of variables in this file.

RCODE & OUTPUT:

```
Console R Markdown ×
C:/Users/NICSI/Desktop/ABI/HW 2/ A
> dim(Boston)
[1] 506 14
> names(Boston)
[1] "crim" "zn"
                             "indus"
                                                                                        "dis"
                                         "chas"
                                                     "nox"
                                                                 "rm"
                                                                                                    "rad"
                                                                             "age"
                                                                                                                "tax"
[11] "ptratio" "black"
                            "lstat"
                                         "medv"
```

Step 4 (0.5 mark) Run the multivariate regression of medv against lstat and age with interaction.

OUTPUT:

```
Console R Markdown *

C:/Users/NICSI/Desktop/ABI/HW 2/ 

> g1<- lm(medv ~ lstat*age, data = Boston)

^
```

Step 5 (0.5 mark) Is interaction term significant or not. Is the answer difference for confidence probability 5 % and 1%? Formulate the appropriate hypotheses and make a conclusion based on the relevant p-values.

OUTPUT:

```
R Markdown ×
Console
C:/Users/NICSI/Desktop/ABI/HW 2/ 🙈
> summary(g1)
call:
lm(formula = medv ~ lstat * age, data = Boston)
Residuals:
           1Q Median
-15.806 -4.045 -1.333
                       2.085 27.552
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 36.0885359 1.4698355 24.553 < 2e-16 ***
           lstat
           -0.0007209 0.0198792 -0.036
                                        0.9711
age
1stat:age
          0.0041560 0.0018518 2.244
                                        0.0252 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '1
Residual standard error: 6.149 on 502 degrees of freedom
Multiple R-squared: 0.5557,
                            Adjusted R-squared: 0.5531
F-statistic: 209.3 on 3 and 502 DF, p-value: < 2.2e-16
```

ANSWER:

P-value of Lstat:Age <- .0252

Lstat:Age: H(95% Confidence Interval): if P-value <.05: Reject Null Hypothesis

Lstat:Age: H(5% Probable Interval): If .05>P-value >.01 : Reject Null Hypothesis

Lstat:Age: H(1% Probable Interval): If P-value < .01 : Reject Null Hypothesis.

Conclusion:

We can see that the last hypothesis is not getting satisfied hence null hypothesis is getting accepted for 1% interval. So the interaction term is significant for 95% and 5% interval but not significant for 1% interval. The interaction term (Istat:age) is highly significant as the p-value of Istat:age is .0252 which is less than 0.05 so we can reject the null hypothesis with 95% confidence.

The answer will be different for confidence probability 5% and 1% because the interaction term has p-value of 0.0252 which is less than 0.05 but higher than 0.01. So for 5% probability confidence the interaction term (lstat:age) will be highly significant as the null hypothesis is rejected while for 1% probability confidence, the interaction term will be insignificant as we cannot reject null hypothesis in this case.

Step 6 (1 Mark) Implement the multiple linear regression of medv against Istat and age without interaction (or just see the results in HW1). Compare these two models with and without interact-ion.

OUTPUT:

Step 7 (0.5 mark) Compare the residual standard errors for these two models.

ANSWER:

From the above calculated summary of the two models, Residual standard error for g1 model (with interactio n) = 6.149 on 502 degrees of freedom.

The residual Standard error of both the models are almost the same so it won't make any significance impact on the model.

Nonlinear transform of predictors (3 marks)

Step 1 (1 mark) The Im() function can also accommodate non-linear transformations of the predictors. For instance, given a predictor X, we can create a predictor X^2 using $I(X^2)$. The function I() is needed since the $^$ has a special meaning I() in a formula; wrapping as we do allows the standard usage in R, which is to raise X to the power P. We now perform a regression of P0 onto P1 stat and P2.

OUTPUT:

```
Console R Markdown ×
                                                                                                                     \neg \Box
C:/Users/NICSI/Desktop/ABI/HW 2/ A
> int1<- (Boston$1stat)^2
> g2<- lm(medv~lstat+int1,data=Boston)
> summary(g2)
lm(formula = medv ~ lstat + int1, data = Boston)
Residuals:
    Min
               1Q Median
                                  3Q
                                          мах
-15.2834 -3.8313 -0.5295 2.3095 25.4148
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 42.862007 0.872084 49.15 <2e-16 ***
lstat -2.332821 0.123803 -18.84 <2e-16 ***
int1 0.043547 0.003745 11.63 <2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 5.524 on 503 degrees of freedom
Multiple R-squared: 0.6407, Adjusted R-squared: 0.6393
F-statistic: 448.5 on 2 and 503 DF, p-value: < 2.2e-16
```

Step 2 (1 mark) Build the single regression of medv (results can be taken from HW1) against lstat. Compare linear and quadratic model using the adjuster R-squared.

OUTPUT:

```
Console R Markdown ×
C:/Users/NICSI/Desktop/ABI/HW 2/ A
> g3<- lm(medv~lstat,data= Boston)
> summary(g3)
call:
lm(formula = medv ~ lstat, data = Boston)
Residuals:
             1Q Median
                              30
   Min
                                      Max
-15.168 -3.990 -1.318 2.034 24.500
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 34.55384  0.56263  61.41  <2e-16 ***
            -0.95005
                        0.03873 -24.53 <2e-16 ***
lstat
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 6.216 on 504 degrees of freedom
Multiple R-squared: 0.5441, Adjusted R-squared: 0. F-statistic: 601.6 on 1 and 504 DF, p-value: < 2.2e-16
                                 Adjusted R-squared: 0.5432
```

INFERENCE:

The adjusted R-squared value increases only if the new term improves the model more than would be expected by chance. It decreases when a predictor improves the model by less than expected by chance.

In the Quadratic model we see the adjusted R-squared value of **0.6393(g2)** is higher than the adjusted R-squared value of **0.5432(g3)** in Linear model. Since the value 0.6393 in quadratic model is closer to 1 and, 0.5432 in linear model is closer to 0, this indicates that the Quadratic model has a better fit hence making it a better model.

Step 3 (1 mark) Use the anova() function to further quantify the extent to which the quadratic fit is superior to the linear fit. The anova() function performs a hypothesis test comparing the two models. The null hypothesis is that the two models fit the data equally well, and the alternative hypothesis is that the full model is superior.

OUTCOME:

```
Console R Markdown * C:/Users/NICSI/Desktop/ABI/HW 2/ > C:/Users/NICSI/Desktop/ABI/HW 2/ > anova(g2,g3)

Analysis of Variance Table

Model 1: medv ~ lstat + int1

Model 2: medv ~ lstat

Res.Df RSS Df Sum of Sq F Pr(>F)

1 503 15347

2 504 19472 -1 -4125.1 135.2 < 2.2e-16 ***

---

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

INFERENCE:

The F-statistic is 135.2 and the associated p-value is almost zero. This shows clear evidence that the model containing the predictors lstat and lstat2(Quadratic model) is far superior to the model that only contains the predictor lstat(Linear Model).

Hence, it can be inferred that quadratic model is a better model then linear model.

Classification: Logistic regression (9 marks)

We will begin by examining some numerical and graphical summaries of the Smarket data, which is part of the ISLR library. This data set consists of percentage returns for the S&P 500 stock index over 1, 250 days, from the beginning of 2001 until the end of 2005. For each date, we have recorded the percentage returns for each of the five previous trading days, Lag1 through Lag5. We have also recorded Volume (the number of shares traded on the previous day, in billions), Today (the percentage return on the date in question) and Direction (whether the market was Up or Down on this date).

Step 1 (0.5 mark) Open library (ISLR) and check the name of variables in Smarket file and provide the summary statistics for all variables.

Output:

```
Console R Markdown ×
C:/Users/NICSI/Desktop/ABI/HW 2/ A
> library(ISLR)
 data(Smarket)
> str(Smarket)
'data.frame':
                1250 obs. of 9 variables:
           : num 2001 2001 2001 2001 2001
 $ Year
            : num    0.381    0.959    1.032    -0.623    0.614    ...
 $ Lag1
            : num -0.192 0.381 0.959 1.032 -0.623 ...
 $ Lag2
 $ Lag3
            : num -2.624 -0.192 0.381 0.959 1.032 ...
                  -1.055 -2.624 -0.192 0.381 0.959 ...
 $ Lag4
            : num
            : num 5.01 -1.055 -2.624 -0.192 0.381 ...
 $ Lag5
           : num 1.19 1.3 1.41 1.28 1.21 .
 $ Volume
            : num 0.959 1.032 -0.623 0.614 0.213 ..
 $ Today
 $ Direction: Factor w/ 2 levels "Down", "Up": 2 2 1 2 2 2 1 2 2 2 ...
> summary(Smarket)
                                                                                   Lag4
                      :-4.922000
                                           :-4.922000
Min.
       :2001
                Min.
                                    Min.
                                                         Min.
                                                                :-4.922000
                                                                             Min.
                                                                                     :-4.922000
1st Ou.:2002
               1st Qu.:-0.639500
                                    1st Qu.:-0.639500
                                                         1st Qu.:-0.640000
                                                                             1st Qu.:-0.640000
                                    Median : 0.039000
                                                         Median : 0.038500
                                                                             Median : 0.038500
Median:2003
                Median: 0.039000
Mean
        :2003
                Mean
                      : 0.003834
                                    Mean
                                           : 0.003919
                                                         Mean
                                                               : 0.001716
                                                                             Mean : 0.001636
 3rd Qu.:2004
                3rd Qu.: 0.596750
                                    3rd Qu.: 0.596750
                                                         3rd Qu.: 0.596750
                                                                              3rd Qu.: 0.596750
Max.
        :2005
               Max.
                      : 5.733000
                                            : 5.733000
                                                         Max.
                                                                : 5.733000
                                                                                    : 5.733000
      Lag5
                        Volume
                                          Today
                                                          Direction
        :-4.92200
                    Min.
                                     Min.
                                            :-4.922000
Min.
                           :0.3561
                                                          Down: 602
1st Qu.:-0.64000
                                     1st Qu.:-0.639500
                    1st Qu.:1.2574
                                                          Up :648
Median : 0.03850
                    Median :1.4229
                                     Median : 0.038500
       : 0.00561
                    Mean
                           :1.4783
                                      Mean
                                            : 0.003138
 3rd Qu.: 0.59700
                    3rd Qu.:1.6417
                                      3rd Qu.: 0.596750
       : 5.73300
                    Max.
                           :3.1525
                                            : 5.733000
Max.
                                     Max.
```

Step 2 (0.5 mark) Use the cor() function produces a matrix that contains all of the pairwise correlations among the predictors in a data set. Use the parameter [,9] because the Direction (#9) variable is qualitative.

OUTPUT:

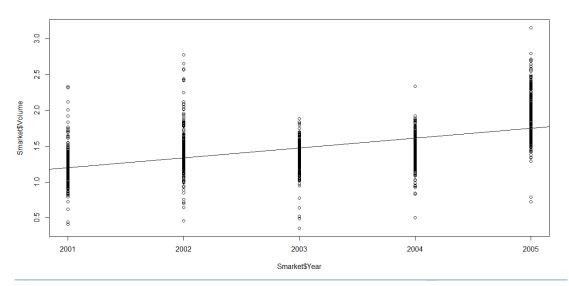
```
Console
       R Markdown ×
C:/Users/NICSI/Desktop/ABI/HW 2/ A
> cor(Smarket[, -9])
             Year
                          Lag1
                                                                                          volume
                                       Lag2
       1.00000000
                  0.029699649
                                0.030596422
                                             0.033194581
                                                           0.035688718
                                                                        0.029787995
                                                                                      0.53900647
                                                                                                  0.030095229
       0.02969965
                  1.000000000 -0.026294328 -0.010803402 -0.002985911 -0.005674606
                                                                                      0.04090991 -0.026155045
Lag1
       0.03059642 -0.026294328 1.000000000 -0.025896670 -0.010853533 -0.003557949 -0.04338321 -0.010250033
Lag2
       0.03319458 -0.010803402 -0.025896670 1.000000000 -0.024051036 -0.018808338 -0.04182369 -0.002447647
Lag3
Lag4
       0.03568872 -0.002985911 -0.010853533 -0.024051036
                                                          1.000000000 -0.027083641 -0.04841425 -0.006899527
       0.02978799 - 0.005674606 - 0.003557949 - 0.018808338 - 0.027083641 1.000000000 - 0.02200231 - 0.034860083
Lag5
Volume 0.53900647 0.040909908 -0.043383215 -0.041823686 -0.048414246 -0.022002315 1.00000000
                                                                                                 0.014591823
       0.03009523 -0.026155045 -0.010250033 -0.002447647 -0.006899527 -0.034860083
                                                                                      0.01459182
                                                                                                  1.000000000
```

Step 3 (1 mark) Explain why volume is correlated with year. Illustrate this graphically.

OUTPUT:

Volume has maximum collinearity with year and the values .539.

```
> library(ggplot2)
> plot(Smarket$Year,Smarket$volume)
> #Reflect a positive relationship between the two variables.
>
> #The trend line of the same:
> abline(lm(Volume~Year,data=Smarket))
```



By both the plots, we can see a gradual rise in the Volume from Year 2001 to 2005. So, we can say that the Volume is increasing over the Years in a linear model.

Step 4 (1 mark) Fit a logistic regression model in order to predict Direction using Lag1 through Lag5 and Volume. The glm() function fits *generalized linear models*, a class of models that includes logistic regression. The syntax of the glm() function is similar to that of lm(), except that we must pass in the argument family=binomial in order to tell R to run a logistic regression rather than some other type of generalized linear model.

RCODE & OUTPUT:

```
> g4<-glm(Direction~Lag1+Lag2+Lag3+Lag4+Lag5+Volume,family=binomial,data=Smarket)
> |
```

Step 5 (1 mark) Analyze the logistic Table in order our ability to predict trading Volume using the information about lags.

Output:

```
Console R Markdown ×
C:/Users/NICSI/Deskton/ABI/HW 2/ 🖒
Restuuat Deviance, 1/20
                                 AIC. 1/42
> summary(g4)
call:
glm(formula = Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 +
    volume, family = binomial, data = Smarket)
Deviance Residuals:
            1Q Median
                            3Q
  Min
                                   Max
-1.446 -1.203
               1.065
                         1.145
                                 1.326
Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.126000
                       0.240736
                                 -0.523
            -0.073074
                        0.050167
                                  -1.457
                                             0.145
Lag1
Lag2
            -0.042301
                        0.050086
                                  -0.845
                                             0.398
Lag3
             0.011085
                        0.049939
                                   0.222
                                             0.824
Lag4
             0.009359
                        0.049974
                                   0.187
                                             0.851
             0.010313
                        0.049511
                                   0.208
Lag5
                                             0.835
volume
             0.135441
                        0.158360
                                   0.855
                                             0.392
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 1731.2 on 1249 degrees of freedom
Residual deviance: 1727.6 on 1243 degrees of freedom
AIC: 1741.6
Number of Fisher Scoring iterations: 3
```

INFERENCE:

Lag1 has the smallest p-value of 0.145 out of all the predictors but it is still large enough. Therefore, the lag variables are not associated with the response Direction.

Also, a negative coefficient indicates a reverse association. So, if the market had a positive return yesterday, then it will mostly have a less positive or negative return today.

Step 6 (0.5 mark) Use the coef() function in order to access just the coefficients for this fitted model. Check the consistency with the previous step.

```
Console R Markdown ×

C:/Users/NICSI/Desktop/ABI/HW 2/ 

> coef (g4)

(Intercept) Lag1 Lag2 Lag3 Lag4 Lag5 Volume

-0.126000257 -0.073073746 -0.042301344 0.011085108 0.009358938 0.010313068 0.135440659

> |
```

INFERENCE:

The results for the coefficients for g4 model are consistent with the previous step since, the values of the coefficients are same.

Step 7 (0.5 mark) Use the summary() function to access particular aspects of the fitted model, such as the p-values for the coefficients.

OUTPUT:

```
Console R Markdown ×
C:/Users/NICSI/Desktop/ABI/HW 2/ 🙈
RESTUUAT DEVTATICE, 1/20
                                AIC. 1/42
> summary(g4)
glm(formula = Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 +
    volume, family = binomial, data = Smarket)
Deviance Residuals:
           1Q Median
                            3Q
                                   Max
-1.446 -1.203 1.065 1.145
                                 1.326
Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.126000
                      0.240736 -0.523
                                            0.601
Lag1
           -0.073074
                        0.050167
                                  -1.457
                                            0.145
           -0.042301
                        0.050086
                                 -0.845
                                            0.398
Lag2
Lag3
             0.011085
                       0.049939
                                   0.222
                                            0.824
Lag4
             0.009359
                      0.049974
                                   0.187
                                            0.851
Lag5
             0.010313
                       0.049511
                                   0.208
                                            0.835
Volume
             0.135441
                       0.158360
                                   0.855
                                            0.392
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 1731.2 on 1249 degrees of freedom
Residual deviance: 1727.6 on 1243 degrees of freedom
AIC: 1741.6
Number of Fisher Scoring iterations: 3
> |
```

Since the p-value for all the lag variables is greater than .05 hence we can say it is safe to remove the lag variable for classification .

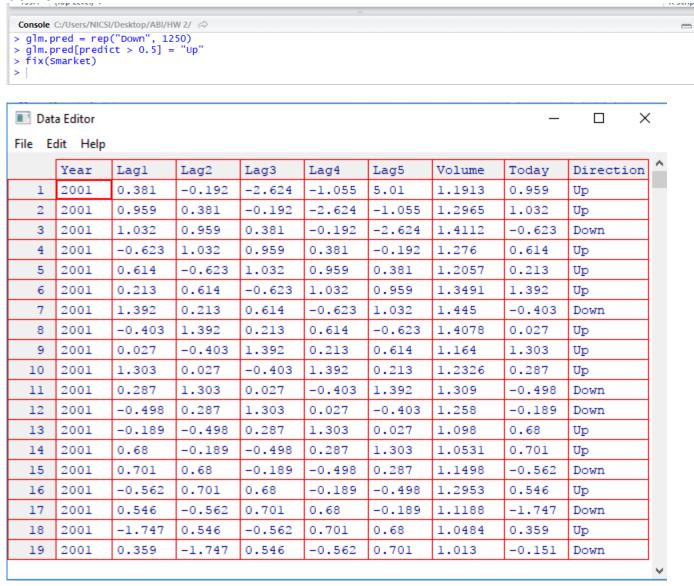
Step 8 (0.5 mark) Use the predict() function, which can be used to predict the probability that the market will go up, given values of the predictors. The type="response" option tells R to output probabilities of the form P(Y=1|X), as opposed to other information such as the logit. If no data set is supplied to the predict() function, then the probabilities are computed for the training data that was used to fit the logistic regression model. Print the first ten probabilities.

OUTPUT:

Step 9 (0.5 mark) Use contrasts() function, which indicates that R has created a dummy variable with a 1 for Up, to check if these values correspond to the probability of the market going up, rather than down.

Step 10 (0.5 mark) Convert these predicted probabilities into class labels, Up or Down, in order to make our prediction more transparent. Check the result with command fix.

OUTPUT:



Step 11 (0.5 mark) Given these predictions, use the table() function to produce a confusion matrix in order to determine how many observations were correctly or incorrectly classified.

OUTPUT:

```
Console C:/Users/NICSI/Desktop/ABI/HW 2/ 

> table(glm.pred, Direction)
    Direction
glm.pred Down Up
    Down 145 141
    Up 457 507

> |
```

Elements on the diagonal of the matrix represent individuals whose default statuses were correctly predicted, while off-diagonal elements represent individuals that were misclassified.

Total correctly classified UP : 507
Total Correctly Classified DOWN: 145

Total Incorrectly Classified UP: 141
Total Incorrectly Classified DOWN: 457

Step 12 (0.5 mark) Calculate the proportion of correct predictions using command mean. Check the results manually using the information from the confusion table.

```
Console C:/Users/NICSI/Desktop/ABI/HW 2/ 

> (507+145)/(1250)

[1] 0.5216

> mean(glm.pred==Smarket$Direction)

[1] 0.5216

> |
```

INFERENCE:

Comparing the mean, manually in the confusion matrix and by computing the mean function, shows us that the diagonal elements of confusion matrix gives correct predictions while the off-diagonal elements show incorrect predictions. Therefore, the model correctly predicted that market will go in Up direction on 507 days and will go in Down direction on 145 days.

The proportion of correct prediction of the movement is 0.5216 or 52.16%.

Step 13 (0.5 mark) The statistics in Step 13 corresponds to the training set equal to the total set of observation. Create the new training set for records from 2001 to 2004. We will then use this vector to create a held out data set of observations from 2005. How many records remains in the test set?

RCODE & OUTPUT:

INFERENCE:

252 observations from 9 variables remain in the test set.

Step 14 (1 mark) Implement command >Direction.2005= Direction [! train]

The object train is a vector of 1, 250 elements, corresponding to the observations in our data set. The elements of the vector that correspond to observations that occurred before 2005 are set to TRUE, whereas

those that correspond to observations in 2005 are set to FALSE. The object train is a *Boolean* vector, since its elements are TRUE and FALSE.

Fit a logistic regression model using only the subset of the observations that correspond to dates before 2005, using the subset argument. Compute the predictions for 2005 and compare them to the actual movements of the market over that time period. Interpret the results.

OUTPUT:

```
Console C:/Users/NICSI/Desktop/ABI/HW 2/
                                                                                                                                             \neg \Box
> Direction. 2005 = Direction[!train]
> # Fit the logistic regression using only the train data
> glm.fit<-glm(Direction~Lag1+Lag2+Lag3+Lag4+Lag5+Volume, data=Smarket, family =binomial, subset= subset)
> # Test the model by fitting the test data set into the fitted model
> glm.probs <- predict(glm.fit, Smarket.2005, type = "response")
> # compute the predictions for 2005 and compare them to the actual
> # movements of the market over that time period.
> glm.pred = rep("Down", nrow(Smarket.2005))
> glm.pred[glm.probs > 0.5] = "Up"
> table(glm.pred, Direction.2005)
         Direction. 2005
glm.pred Down Up
     Down 77 97
     Up
             34 44
> # Test error rate
> mean(glm.pred == Direction.2005)
[1] 0.4801587
> # Training error rate
> mean(glm.pred != Direction.2005)
[1] 0.5198413
```

The training error rate for this model is 52% approximately which is not good for the model. Due to these factors the model would be having significant number of outliers. Hence we refit the model using 2 variables lag1 and lag2..

```
Console C:/Users/NICSI/Desktop/ABI/HW 2/ 

> # Refit the model using only 2 predictors lag1 and lag2

> glm.fit = glm(Direction ~ Lag1 + Lag2, data = Smarket, family = binomial, subset = train)

> glm.probs = predict(glm.fit, Smarket.2005, type = "response")

> glm.pred = rep("Down", nrow(Smarket.2005))

> glm.pred[glm.probs > 0.5] = "Up"

> table(glm.pred, Direction.2005)

    Direction.2005

glm.pred Down Up

    Down 35 35

    Up 76 106

> mean(glm.pred == Direction.2005)

[1] 0.5595238

> |
```

Actual Movement for 2005:

As can be seen the test error rate: (Predicted correctly/ Total observations of the test data set) has been increased to 56 %(roughly) which is not very good.

To predict Direction for new values of Lag1-Lag2 we simply use the predict() function and feed in a data frame of new values. We want to predict Direction on a day when Lag1 and Lag2 equal 1.2 and 1.1, respectively, and on a day when they equal 1.5 and -0.8.