SAKARYA ÜNİVERSİTESİ

Discrete Mathematics
LECTURE 12
Tree

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Outline

- Trees
- References

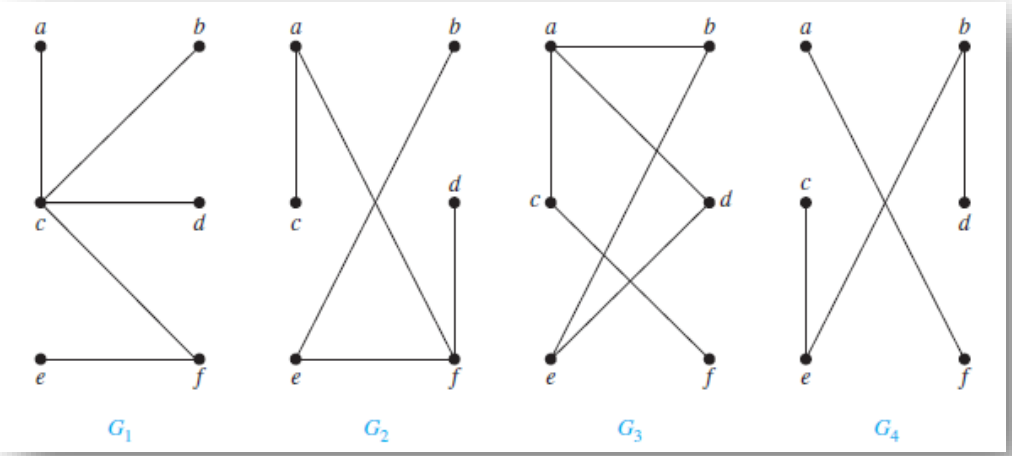
Trees

- **Tree**
  - a connected graph that contains no simple circuits is
  - a particular type of a graph
  - so named because such graphs resemble trees
  - cannot have a simple circuit, so cannot contain multiple edges or loops. therefore any tree must be a simple graph.

- An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

Trees…

➢ **Example:** Which of the graphs shown in the figure below are trees?

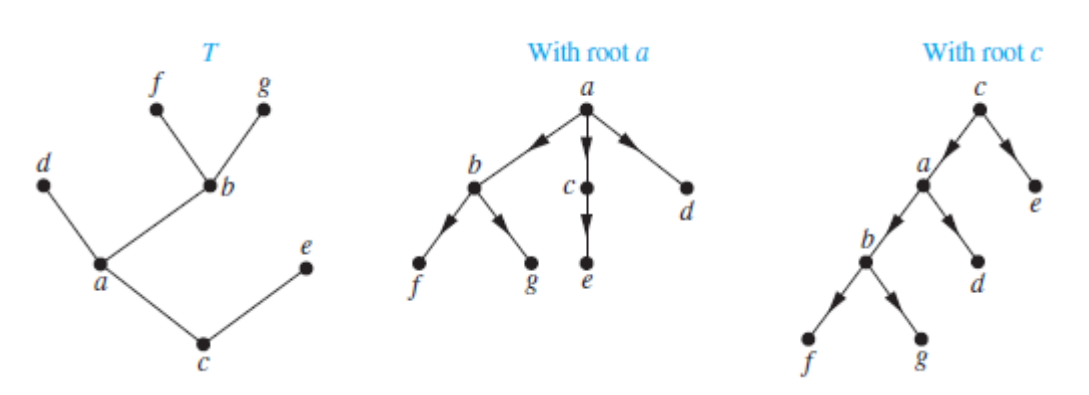


➢ **Solution:**

- G_1 is a tree, because it is a connected graph with no simple circuits.
- G_2 is a tree, because it is a connected graph with no simple circuits.
- G_3 is not a tree, because it includes a loop.
- G_4 is not a tree, because it is not a connected graph ✓

Trees

- **Rooted Tree**
  - a tree in which one vertex has been designated as the root and every edge is directed away from the root.
  - can also be defined recursively
  - we can change an unrooted tree into a rooted tree by choosing any vertex as the root.



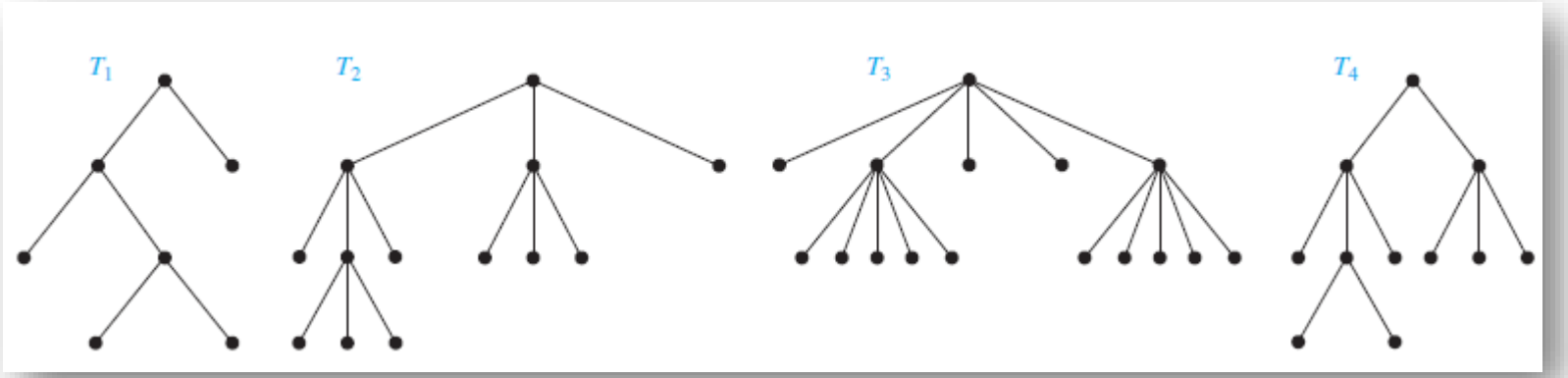
- **A Tree and Rooted Trees Formed by Designating Two Different Roots.**

Trees

- **Rooted Tree…**
  - A rooted tree is called an **m-ary tree** if every internal vertex has no more than m children.
  - The tree is called a **full m-ary tree** if every internal vertex has exactly m children.
  - An m-ary tree with m = 2 is called a **binary tree**.

Trees…

➢ **Example:** Are the rooted trees in the figure below are full *m*-ary trees for some positive integer *m*?



➢ **Solution:**

- T_1 is a full binary tree because each of its internal vertices has two children.
- T_2 is a full 3-ary tree because each of its internal vertices has three children.
- In T_3 each internal vertex has five children, so T_3 is a full 5-ary tree.
- T_4 is not a full *m*-ary tree for any *m* because some of its internal vertices have two children and others have three children. ✓

References

- K.H. Rosen, Discrete Mathematics and Its Applications, Seventh Edition, Mc Graw Hill, 2012.
- R.P. Grimaldi, Discrete and Combinatorial Mathematcics, An Applied Introduction, Fifth Edition, Pearson, 2003.
- S.S. Epp, Discrete Mathematics with Applications, Fouth Edition, 2010.
- N. Yurtay, "Ayrık İşlemsel Yapılar" Lecture Notes, Sakarya University.