Discrete Mathematics
LECTURE 4
Functions

**Assistant Professor Gülüzar ÇİT**

Outline

➢Functions
  ➢Function
    ➢one-to-one function
    ➢onto function
➢References

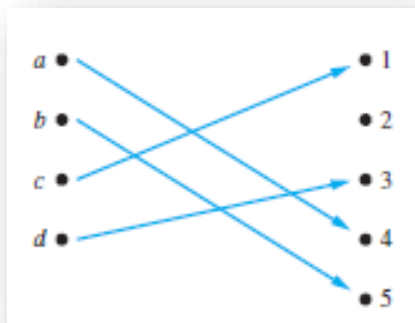# Functions

➢ **Function**

    ➢ a relation that maps each element of set A to only one element of set B

    ➢ denoted by $f : A \rightarrow B$

    ➢ also called *f maps A* to *B*.

    ➢ set A is called the **definition/domain**

    ➢ set B is called the **value/codomain**

    ➢ if *b* is the unique element of *B* assigned by the function *f* to the element *a* of *A*

        ➢ we write f (a) = b

        ➢ we say that b is the **image** of a

        ➢ we say a is a **preimage** of b.

        ➢ the **range**, or **image** of f is the set of all images of elements of A.
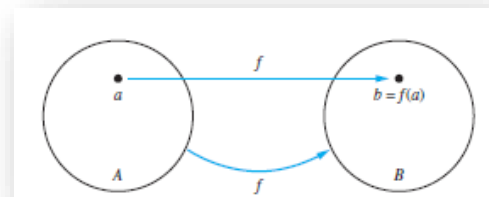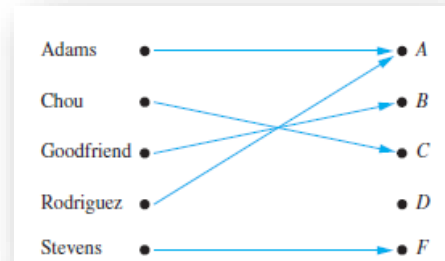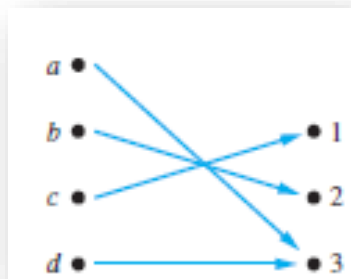
Functions…

➢ **Function…**
  ➢ A function f is said to be **one-to-one** or an **injunction**
    ➢ if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.
    ➢ Such a function is said to be **injective**
    ➢ functions that never assign the same value to two different domain elements.

Functions…

➢**Function…**

    ➢A function f is said to be **onto** or an **surjection**

        ➢if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) =$ b

        ➢such a function is said to be **surjective**
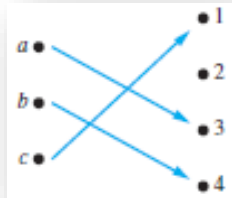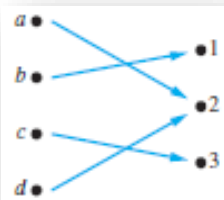
Functions…

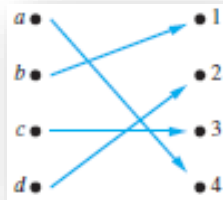➢ **Function…**

➢ Example: Different types of correspondences

one-to-one, not onto	onto, not one-to-one	one-to-one and onto

neither one-to-one nor onto	not a function

SAKARYA ÜNİVERSİTESİ
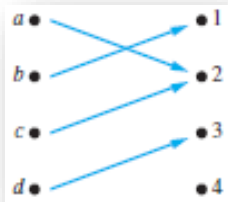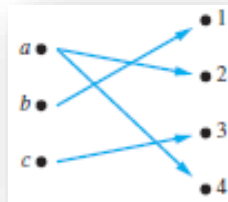
Functions…

> **Function…**
>> Example: Let $A = \{1,2,3\}$, $B = \{a, b, c, d\}$ and $f = \{(1, b), (2, a), (3, d)\}$. Is this function one-to-one?
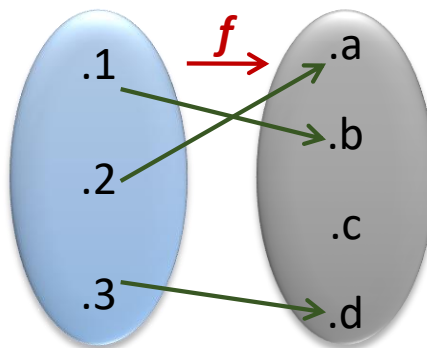
> if f is one-to one, $\forall x_1, x_2 \in A, x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$
>> $x_1 = 1$ and $x_2 = 2 \Rightarrow 1 \neq 2 \Rightarrow f(1) \neq f(2) \Rightarrow b \neq a$
>> $x_1 = 1$ and $x_2 = 3 \Rightarrow 1 \neq 3 \Rightarrow f(1) \neq f(3) \Rightarrow b \neq d$
>> $x_1 = 2$ and $x_2 = 3 \Rightarrow 2 \neq 3 \Rightarrow f(2) \neq f(3) \Rightarrow a \neq d$
> f is one-to-one ✔

Functions…

> **Function…**
>> [Example:](#) Let $A = \{1,2,3\}$, $B = \{a, b, c, d\}$ and $f\{(1, b), (2, a), (3, c), (4, b)\}$. Is this function onto?
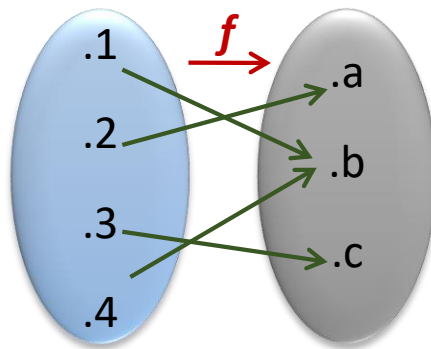
> if f is one-to one, $\forall y(y \in B \Rightarrow \exists x \in A), f(x) = y$
>> $y = a \Rightarrow f(2) = a \Rightarrow 2 \in A$
>> $y = b \Rightarrow f(1) = b \Rightarrow 1 \in A$ and $y = b \Rightarrow f(4) = b \Rightarrow 4 \in A$
>> $y = c \Rightarrow f(3) = c \Rightarrow 3 \in A$

> f is onto ✔

References

➢ K.H. Rosen, Discrete Mathematics and Its Applications, Seventh Edition, Mc Graw Hill, 2012.

➢ R.P. Grimaldi, Discrete and Combinatorial Mathematcics, An Applied Introduction, Fifth Edition, Pearson, 2003.

➢ S.S. Epp, Discrete Mathemtics with Applications, Fouth Edition, 2010.

➢ N. Yurtay, "Ayrık İşlemsel Yapılar" Lecture Notes, Sakarya University.