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Axioms

1. Non-negativity

For any event A, $p(A) \geq 0$

2. Normalization

The probability of the sample space = 1

$p(S) = 1$

3. Additivity

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

1. Prove from the axioms that if $Y \subseteq Z$, then $p(Y) \leq p(Z)$

$Z = Y \cup (Z \setminus Y)$

using additivity

$P(Z) = P(Y \cup (Z \setminus Y)) = P(Y) + P(Z \setminus Y)$

$P(Y) = P(Y \cap Z) \leq P(Z)$

if $Y \subseteq Z$, $P(Y) \leq P(Z)$

2. additivity

$P(S) = P(S + \phi) = P(S) + P(\phi)$

$\boxed{P(\phi) = 0}$

3. Non-negativity → any event A, $p(A) \geq 0$ normalization → probability of sample space = 1

$P(X|Z) = \dfrac{P(X \cap Z)}{P(Z)}$ → positive

$P(X|Z)$ ↓ positive; $P(Z)$ → positive

$P(X|Z)$ → range $[0,1]$

4. $p(X) = 1 - p(\bar{X})$ → $\bar{X}$ = E-X (E = samplespace)

$E = X \cup \bar{X}$

$\boxed{P(E) = 1}$ → normalization

→ $P(X) + P(\bar{X}) = 1$ → $P(X) = 1 - P(\bar{X})$

5. $$\frac{p(\overset{x}{\text{singing}} \text{ AND } \overset{y}{\text{rainy}})}{p(\text{rainy})} = p(\text{signing} \backslash \text{rainy})$$

$$p(A|B) = \frac{p(A \cap B)}{p(B)}$$

$A \rightarrow$ singing
$B \rightarrow$ rainy

$$P(A \cap B) = p(A|B) * p(B)$$

$$P(\text{singing} \wedge \text{rainy}) = p(\text{singing}|\text{rainy}) * p(\text{rainy})$$

$$\boxed{p(\text{singing} \cap \text{rainy} | \text{rainy})} = \frac{p(\text{singing} \wedge \text{rainy} \wedge \text{rainy})}{p(\text{rainy})} = \frac{p(\text{singing} \wedge \text{rainy})}{p(\text{rainy})} = \boxed{p(\text{singing} | \text{rainy})}$$

6. $$p(x|y) = 1 - p(\bar{x}|y)$$

$$P(x|y) = \frac{P(x \cap y)}{P(y)}$$

$$P(x) = 1 - P(\bar{x})$$

$$\boxed{P(x'|y)} = \frac{P(\bar{x} \wedge y)}{P(y)} = \frac{P(y) - P(x \wedge y)}{P(y)} = \boxed{1 - P(x|y)}$$

$$P(x|y) = 1 - P(\bar{x}|y)$$

7. $$\underbrace{\underbrace{\big(P(x|y).P(y)}_{P(x \cap y)} + \underbrace{P(x|\bar{y}).P(\bar{y})\big)}_{\frac{P(x \wedge \bar{y})}{P(x)}}}_{P(x)} . \underbrace{\frac{P(\bar{z}|x)}{P(\bar{z})}}_{} = \frac{P(x) . P(\bar{z}|x)}{P(\bar{z})} = \frac{P(x \wedge \bar{z})}{P(\bar{z})} = \boxed{P(x|\bar{z})}$$

8. $$P(x|y) = 0$$

$$p(x \wedge y) = 0$$

$$p(x \wedge y \wedge z) = \underbrace{P(x|y \wedge z)}_{0} . P(y \wedge z) = 0$$

$$P(x|y,z) = 0$$

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