Dependable Model-Based Hybrid Systems Design via Event-B

Lab 2

Jan 12, 2021

Exercise 1.1. Laplace Transform of Ramp Function: $f(t) = \begin{cases} 0 & t < 0 \\ At & t \geq 0 \end{cases}$, (hint: Recall that integration by parts: $\int_a^b u\,dv = uv|_a^b - \int_a^b v\,du$)

Exercise 1.2. Laplace Transform of *sin* Function: $f(t) = \begin{cases} 0 & t < 0 \\ Asin(\omega t) & t \geq 0 \end{cases}$, (hint: Recall that the Euler formulation: $sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$)

Exercise 1.3. Laplace Transform of *cos* Function: $f(t) = \begin{cases} 0 & t < 0 \\ Acos(\omega t) & t \geq 0 \end{cases}$

Exercise 2.1. Find the time-domain function corresponding to each of the following Laplace transforms using partial fraction:

- $\frac{s^2+2s+3}{(s+1)^3}$
- $\frac{s-1}{s^2+2s+2}$
- $\frac{2}{s(s+2)}$
- $\frac{10}{s(s+1)(s+10)}$
- $\frac{3s+2}{s^2+4s+20}$

Exercise 2.2. Recall the Laplace transform of spring-mass damper system given in the lecture:

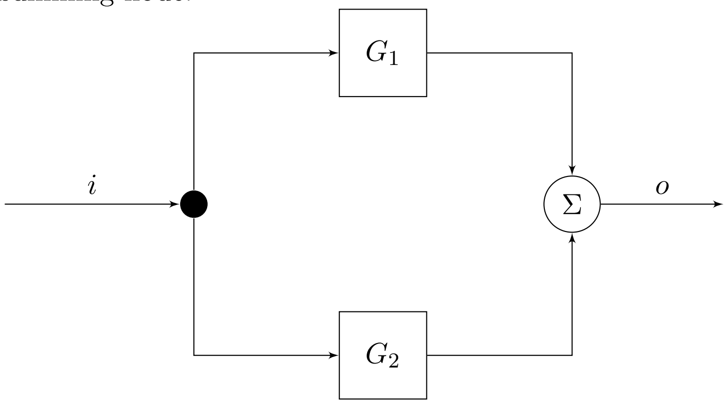
$$X(s) = \frac{\frac{1}{ms} + sx(0) + \dot{x}(0) + \frac{b}{m}x(0)}{s^2 + \frac{b}{m}s + \frac{k}{m}}$$

Find its time-domain correspondence under $x(0) = -2$, $\dot{x}(0) = 3$, and $m = 1, k = 2, b = 3$.

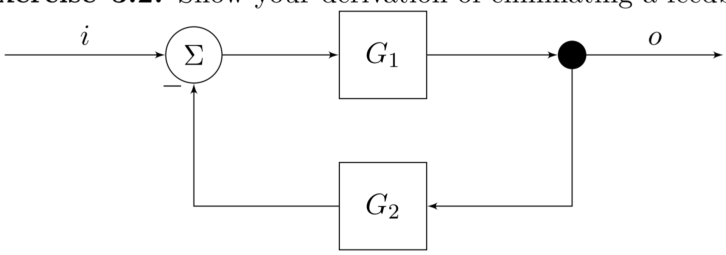
Exercise 2.3. Solve the following ODEs using Laplace method:

- $\ddot{y} + \dot{y} + 3y = 0; y(0) = 1, \dot{y}(0) = 2$
- $\ddot{y} + 2\dot{y} = e^t; y(0) = 1, \dot{y}(0) = 2$

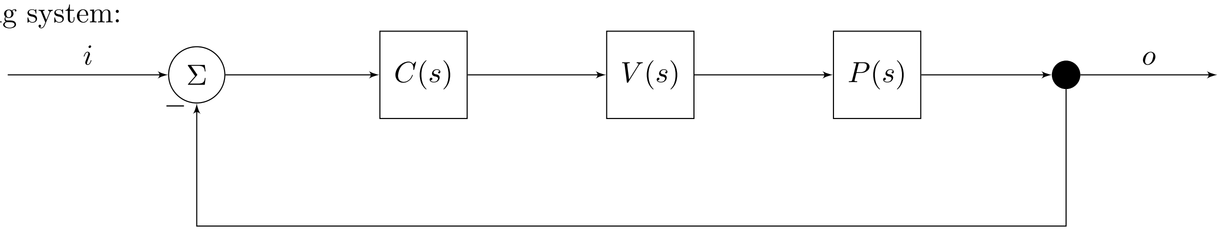
Exercise 3.1. Recall from block diagram algebra, show your derivation of eliminating a summing node:



Exercise 3.2. Show your derivation of eliminating a feedback loop:



Exercise 3.3. Using block diagram algebra to reduce the design of a car position tracking system:



Exercise 4.1. Consider the Exercise 3.3., the component V is under differential equation of $\dot{v} = a$, the component P is under differential equation of $\dot{p} = v$,

a. Compute Laplace transform of $V(s)$ and $P(s)$, assuming zero initial conditions.

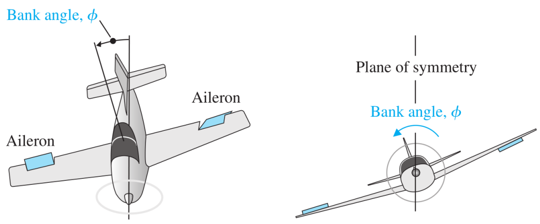
b. Assume $C(s) = K_p + K_d s$, where $K_p = 2, K_d = 0.14$, numerically compute the close loop function $H(s)$ using block diagram algebra.

c. Reduce $H(s)$ to a 2nd-order system.

d. Estimate the DC-gain, overshoot, peak time, rise time, settling time of the reduced 2nd-order system subjected to a step input of amplitude 1.

e. Is the car position never exceed the input reference? justify your answer.

Exercise 4.2. An aileron forms part of the trailing edge of each wing of a fixed-wing aircraft. By actuating the aileron up or down, the bank angle[1] of the aircraft can be modified. The goal of the design is to develop a controller that gradually actuates the aileron to maintain the bank angle at a desired constant ϕ_d. We define a requirement for the design such that the bank angle does not overshoot (for a 2nd-order system, this means $\zeta > 1$).



After simplifying the system design, we can obtain a standard 2nd-order closed-loop transfer function:

$$\hat{H}(s) = \frac{11.29K_p}{s^2 + \sqrt{1.92 - 2.91K_p}s + 11.29K_p}$$

Therefore, excite the system with a step input $U(s)$ that commands the desired bank angle, we have $\hat{Y}(s) = \hat{H}(s)U(s)$ that represents the output bank angle of the approximated system.

The closed-loop transfer function $\hat{H}(s)$ is parameterized by a symbolic gain K_p.

a. Select the range for K_p, such that the system is stable.

b. Is there a K_p we can choose such that the system does not overshoot?

c. Are all K_p chosen will cause the system with no overshoot? justify your answer.

[1] Bank angle is the angle between the wings and the horizon.