

EXTENDS *Naturals, Integers, TLC*

CONSTANT *MAXINT*, *x10*, *x20*

```
--algorithm gcdscm{
  variables x1 = x10;  1st integer
            x2 = x20;  2nd integer
            y1;
            y2;
            y3;
            y4;
            z1, z2;
```

```
{
  print ⟨x1, x2⟩;
  y1 := x1;
  y2 := x2;
  y3 := x2;
  y4 := 0;
  while ( y1 ≠ y2 ) {
    while ( y1 > y2 ) {
      y1 := y1 − y2;
      y4 := y4 + y3;
    } ;
    while ( y2 > y1 ) {
      y2 := y2 − y1;
      y3 := y4 + y3;
    } ;
  } ;
  z1 := y1;
  z2 := y3 + y4;
  print ⟨x1, x2, z1, z2⟩
}
```

BEGIN TRANSLATION (*chksum(pcal)* = “ab42efb3” ∧ *chksum(tla)* = “9f011883”)

CONSTANT *defaultInitValue*

VARIABLES *x1*, *x2*, *y1*, *y2*, *y3*, *y4*, *z1*, *z2*, *pc*

vars \triangleq ⟨*x1*, *x2*, *y1*, *y2*, *y3*, *y4*, *z1*, *z2*, *pc*⟩

Init \triangleq Global variables
 ∧ *x1* = *x10*
 ∧ *x2* = *x20*
 ∧ *y1* = *defaultInitValue*

$$\begin{aligned}
& \wedge y2 = \text{defaultInit Value} \\
& \wedge y3 = \text{defaultInit Value} \\
& \wedge y4 = \text{defaultInit Value} \\
& \wedge z1 = \text{defaultInit Value} \\
& \wedge z2 = \text{defaultInit Value} \\
& \wedge pc = \text{"Lbl_1"} \\
\\
\text{Lbl_1} & \triangleq \wedge pc = \text{"Lbl_1"} \\
& \wedge \text{PrintT}(\langle x1, x2 \rangle) \\
& \wedge y1' = x1 \\
& \wedge y2' = x2 \\
& \wedge y3' = x2 \\
& \wedge y4' = 0 \\
& \wedge pc' = \text{"Lbl_2"} \\
& \wedge \text{UNCHANGED } \langle x1, x2, z1, z2 \rangle \\
\\
\text{Lbl_2} & \triangleq \wedge pc = \text{"Lbl_2"} \\
& \wedge \text{IF } y1 \neq y2 \\
& \quad \text{THEN } \wedge pc' = \text{"Lbl_3"} \\
& \quad \wedge \text{UNCHANGED } \langle z1, z2 \rangle \\
& \quad \text{ELSE } \wedge z1' = y1 \\
& \quad \wedge z2' = y3 + y4 \\
& \quad \wedge \text{PrintT}(\langle x1, x2, z1', z2' \rangle) \\
& \quad \wedge pc' = \text{"Done"} \\
& \wedge \text{UNCHANGED } \langle x1, x2, y1, y2, y3, y4 \rangle \\
\\
\text{Lbl_3} & \triangleq \wedge pc = \text{"Lbl_3"} \\
& \wedge \text{IF } y1 > y2 \\
& \quad \text{THEN } \wedge y1' = y1 - y2 \\
& \quad \wedge y4' = y4 + y3 \\
& \quad \wedge pc' = \text{"Lbl_3"} \\
& \quad \text{ELSE } \wedge pc' = \text{"Lbl_4"} \\
& \quad \wedge \text{UNCHANGED } \langle y1, y4 \rangle \\
& \wedge \text{UNCHANGED } \langle x1, x2, y2, y3, z1, z2 \rangle \\
\\
\text{Lbl_4} & \triangleq \wedge pc = \text{"Lbl_4"} \\
& \wedge \text{IF } y2 > y1 \\
& \quad \text{THEN } \wedge y2' = y2 - y1 \\
& \quad \wedge y3' = y4 + y3 \\
& \quad \wedge pc' = \text{"Lbl_4"} \\
& \quad \text{ELSE } \wedge pc' = \text{"Lbl_2"} \\
& \quad \wedge \text{UNCHANGED } \langle y2, y3 \rangle \\
& \wedge \text{UNCHANGED } \langle x1, x2, y1, y4, z1, z2 \rangle
\end{aligned}$$

Allow infinite stuttering to prevent deadlock on termination.

$$\text{Terminating} \triangleq pc = \text{"Done"} \wedge \text{UNCHANGED vars}$$

$$Next \triangleq Lbl_1 \vee Lbl_2 \vee Lbl_3 \vee Lbl_4 \\ \vee Terminating$$

$$Spec \triangleq Init \wedge \Box[Next]_{vars}$$

$$Termination \triangleq \Diamond(pc = \text{"Done"})$$

END TRANSLATION

$$Property \triangleq pc = \text{"Done"} \Rightarrow z1 * z2 = x1 * x2$$

$$Q1 \triangleq pc \neq \text{"Done"}$$

$$Qpc \triangleq pc = \text{"Done"} \Rightarrow z1 = gcd(x1, x2) \wedge z2 = scm(x1, x2)$$

$$D \triangleq 0 \dots MAXINT \cup \{defaultInitValue\}$$

$$Qpc1 \triangleq pc = \text{"Done"} \Rightarrow (x1 \% z1 = 0) \wedge (x2 \% z1 = 0)$$

$$Qpc2 \triangleq pc = \text{"Done"} \Rightarrow (z2 \% x1 = 0) \wedge (z2 \% x2 = 0)$$

$$Qproperty1 \triangleq pc = \text{"Done"} \Rightarrow x1 * x2 = z1 * z2$$

$$Qef \triangleq x1 \in D \wedge x2 \in D \wedge y1 \in D \wedge y3 \in D \wedge z1 \in D \wedge z2 \in D$$

\ * Modification History
 \ * Last modified *Wed Nov 29 16:59:35 CET 2023* by *mery*
 \ * Created *Wed Sep 09 17:02:47 CEST 2015* by *mery*