```
— MODULE TLAPROOF12 —
EXTENDS Integers, TLC, TLAPS
Constants x0, y0, z0
pre \; \stackrel{\Delta}{=} \; x0 = 10 \wedge z0 = 2*x0 \wedge y0 = z0 + x0
L \triangleq \{\text{"l1"}, \text{"l2"}\}
typeInt(u) \stackrel{\Delta}{=} u \in Int
Assume pre
--algorithm test {
variables x = x0, z = z0, y = y0;
l1: y := z + x;
 BEGIN TRANSLATION (chksum(pcal) = "66c6fc76" \land chksum(tla) = "678ca025")
Variables x, z, y, pc
vars \triangleq \langle x, z, y, pc \rangle
Init \stackrel{\triangle}{=} Global variables
            \wedge x = x0
             \wedge z = z0
             \wedge y = y0
            \land \mathit{pc} = \text{`'l1''}
l1 \stackrel{\triangle}{=} \land pc = "l1"
         \wedge \ y' = z + x
         \land pc' = \text{``Done''}
         \wedge UNCHANGED \langle x, z \rangle
 Allow infinite stuttering to prevent deadlock on termination.
Terminating \stackrel{\triangle}{=} pc = "Done" \land UNCHANGED vars
Next \triangleq l1
                 \vee Terminating
Spec \stackrel{\triangle}{=} Init \wedge \Box [Next]_{vars}
Termination \triangleq \Diamond(pc = \text{``Done''})
```

END TRANSLATION

## ASSUME pre

$$\begin{array}{l} \mathit{MAX} \triangleq 32767 \quad \text{16 bits} \\ D \triangleq -32768 \ldots 32767 \\ x \leq 32760 \\ \\ \mathit{DD}(X) \triangleq (X \in D) \\ \mathit{InductiveInvariant} \triangleq \\ & \land \mathit{pc} \in \{\text{"l1"}, \text{"Done"}\} \\ & \land x \in \mathit{Int} \land y \in \mathit{Int} \land z \in \mathit{Int} \\ & \land \mathit{pc} = \text{"l1"} \Rightarrow x = 10 \land z = 2 * x \land y = z + x \\ & \land \mathit{pc} = \text{"Done"} \Rightarrow x = 10 \land y = x + 2 * 10 \\ \\ \mathit{Inv} \triangleq \mathit{InductiveInvariant} \\ \mathit{Safety\_Partial\_Correctness} \triangleq \mathit{pc} = \text{"Done"} \Rightarrow x = 10 \land y = x + 2 * 10 \\ \\ \mathit{Safety\_rte} \triangleq \mathit{DD}(x) \land \mathit{DD}(y) \land \mathit{DD}(z) \\ \\ \mathit{check} \triangleq \mathit{Inv} \land \mathit{Safety\_Partial\_Correctness} \land \mathit{Safety\_rte} \\ \\ \mathit{prop} \triangleq \Box(x = x0) \\ \mathit{thepre} \triangleq \mathit{pre} \\ \\ \end{array}$$

## Assume $Assumption \triangleq pre$

THEOREM  $InitProperty \stackrel{\triangle}{=} Init \Rightarrow InductiveInvariant$  $\langle 1 \rangle$  suffices assume *Init* 

PROVE InductiveInvariant

## OBVIOUS

- $\langle 1 \rangle 1. \ x = x0$ by Assumption Def Init
- $\langle 1 \rangle 2.$  y = y0by Assumption def Init
- $\langle 1 \rangle 3. \ z = z 0$  by Assumption def Init
- $\langle 1 \rangle 4$ . pc = "I1" by Assumption Def Init
- $\langle 1 \rangle$ .QED

BY  $\langle 1 \rangle 1$ ,  $\langle 1 \rangle 2$ ,  $\langle 1 \rangle 3$ ,  $\langle 1 \rangle 4$  DEF InductiveInvariant, typeInt, pre

Theorem  $Init \Rightarrow InductiveInvariant$ 

BY Assumption DEF Init, InductiveInvariant, typeInt, thepre, pre

LEMMA  $truc \triangleq$ 

ASSUME InductiveInvariant, l1

PROVE InductiveInvariant'

BY DEFS InductiveInvariant, l1, typeInt

```
THEOREM NextProperty \triangleq
Assume InductiveInvariant, [Next]_{\langle x \rangle}
PROVE InductiveInvariant'
\langle 1 \rangle SUFFICES ASSUME InductiveInvariant \land [Next]_{\langle x \rangle}
PROVE InductiveInvariant'
OBVIOUS
\langle 1 \rangle 1. \ x' \in 0... x0 \land typeInt(x') \Rightarrow InductiveInvariant'
BY PTL DEF InductiveInvariant
\langle 1 \rangle 2.
ASSUME InductiveInvariant, l1
PROVE InductiveInvariant'
BY Zenon, SMT, PTL DEF InductiveInvariant, l1, typeInt
\langle 1 \rangle.QED
BY \langle 1 \rangle 1, \langle 1 \rangle 2, Zenon, SMT, PTL DEF InductiveInvariant, typeInt, thepre, pre
THEOREM Correctness \stackrel{\triangle}{=} Spec \Rightarrow \Box Inductive Invariant
\langle 1 \rangle 1. Init \Rightarrow InductiveInvariant
  By Assumption Def Init, InductiveInvariant, typeInt, thepre, pre
\langle 1 \rangle 2. InductiveInvariant \wedge [Next]<sub>vars</sub> \Rightarrow InductiveInvariant'
  BY PTL DEF InductiveInvariant, Next, typeInt, thepre, vars,
\langle 1 \rangle.QED BY \langle 1 \rangle 1, \langle 1 \rangle 2, PTL DEF Spec
```