MODULE *appex4_3*

EXTENDS *Naturals*, *Integers*, *TLC*

CONSTANT *MAXINT*, *u*, *v*

```
--algorithm gcdscm{
  variables x1 = u ;   1st integer
            x2 = v ;    2nd integer
            y1 ;
            y2 ;
            y3 ;
            y4 ;
            z1, z2 ;


{
    print ⟨x1, x2⟩ ;

   y1 := x1 ;
   y2 := x2 ;
   y3 := x2 ;
   y4 := 0 ;
   while ( y1 ≠ y2 ) {

     while  ( y1 > y2 ) {
       y1 := y1 − y2 ;
       y4 := y4 + y3 ;
      } ;
     while  ( y2 > y1 ) {
       y2 := y2 − y1 ;
       y3 := y4 + y3 ;
      } ;

    } ;
   z1 := y1 ;
   z2 := y3 + y4 ;
   print ⟨x1, x2, z1, z2⟩
}

}
```

BEGIN TRANSLATION

CONSTANT *defaultInitValue*

VARIABLES $x1$, $x2$, $y1$, $y2$, $y3$, $y4$, $z1$, $z2$, pc

$vars \triangleq \langle x1, x2, y1, y2, y3, y4, z1, z2, pc \rangle$

$Init \triangleq$ Global variables

$\land\ x1 = u$

$\land\ x2 = v$
$\land\ y1 = defaultInitValue$
$\land\ y2 = defaultInitValue$
$\land\ y3 = defaultInitValue$
$\land\ y4 = defaultInitValue$
$\land\ z1 = defaultInitValue$
$\land\ z2 = defaultInitValue$
$\land\ pc = \text{“Lbl\_1”}$

$Lbl\_1 \triangleq$ $\land\ pc = \text{“Lbl\_1”}$
$\land\ PrintT(\langle x1,\ x2\rangle)$
$\land\ y1' = x1$
$\land\ y2' = x2$
$\land\ y3' = x2$
$\land\ y4' = 0$
$\land\ pc' = \text{“Lbl\_2”}$
$\land$ UNCHANGED $\langle x1,\ x2,\ z1,\ z2\rangle$

$Lbl\_2 \triangleq$ $\land\ pc = \text{“Lbl\_2”}$
$\land$ IF $y1 \neq y2$
THEN $\land\ pc' = \text{“Lbl\_3”}$
$\land$ UNCHANGED $\langle z1,\ z2\rangle$
ELSE $\land\ z1' = y1$
$\land\ z2' = y3 + y4$
$\land\ PrintT(\langle x1,\ x2,\ z1',\ z2'\rangle)$
$\land\ pc' = \text{“Done”}$
$\land$ UNCHANGED $\langle x1,\ x2,\ y1,\ y2,\ y3,\ y4\rangle$

$Lbl\_3 \triangleq$ $\land\ pc = \text{“Lbl\_3”}$
$\land$ IF $y1 > y2$
THEN $\land\ y1' = y1 - y2$
$\land\ y4' = y4 + y3$
$\land\ pc' = \text{“Lbl\_3”}$
ELSE $\land\ pc' = \text{“Lbl\_4”}$
$\land$ UNCHANGED $\langle y1,\ y4\rangle$
$\land$ UNCHANGED $\langle x1,\ x2,\ y2,\ y3,\ z1,\ z2\rangle$

$Lbl\_4 \triangleq$ $\land\ pc = \text{“Lbl\_4”}$
$\land$ IF $y2 > y1$
THEN $\land\ y2' = y2 - y1$
$\land\ y3' = y4 + y3$
$\land\ pc' = \text{“Lbl\_4”}$
ELSE $\land\ pc' = \text{“Lbl\_2”}$
$\land$ UNCHANGED $\langle y2,\ y3\rangle$
$\land$ UNCHANGED $\langle x1,\ x2,\ y1,\ y4,\ z1,\ z2\rangle$

Allow infinite stuttering to prevent deadlock on termination.

$Terminating \triangleq pc = \text{“Done”} \land \text{UNCHANGED } vars$

$Next \triangleq Lbl\_1 \lor Lbl\_2 \lor Lbl\_3 \lor Lbl\_4 \lor Terminating$

$Spec \triangleq Init \land \Box[Next]_{vars}$

$Termination \triangleq \Diamond(pc = \text{“Done”})$

END TRANSLATION

$Q1 \triangleq pc \neq \text{“Done”}$

$Qpc \triangleq pc = \text{“Done”} \Rightarrow z1 = gcd(x1, x2) \land z2 = scm(x1, x2)$

$D \triangleq 0 \,..\, MAXINT \cup \{defaultInitValue\}$

$Qpc1 \triangleq pc = \text{“Done”} \Rightarrow (x1 \% z1 = 0) \land (x2 \% z1 = 0)$

$Qpc2 \triangleq pc = \text{“Done”} \Rightarrow (z2 \% x1 = 0) \land (z2 \% x2 = 0)$

$Qproperty1 \triangleq pc = \text{“Done”} \Rightarrow x1 * x2 = z1 * z2$

$Qef \triangleq x1 \in D \land x2 \in D \land y1 \in D \land y3 \in D \land z1 \in D \land z2 \in D$

\ * Modification History
\ * Last modified *Thu Nov* 19 20:42:02 *CET* 2020 by *mery*
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