Correct by Construction Algorithms by Refinement

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Summary

Current Summary

Current Summary

Detecting overflows of computations

Listing 1: Function average

```
#include <stdio.h>
#include <limits.h>
int average(int a,int b)
{
  return((a+b)/2);
}

int main()
{
  int x,y;
  x=INT_MAX;y=INT_MAX;
  printf("Average--for-%d-and-%d-is-%d\n",x,y,
         average(x,y));
  return 0;
}
```

Execution

Execution produces a result

```
Average  for 2147483647 and 2147483647 is -1
```

Execution

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```
Average  for 2147483647 and 2147483647 is -1
```

Using frama-c produces a required annotation

```
int average(int a, int b)
{
  int __retres;
  /*@ assert rte: signed_overflow: -2147483648 <= a + b; */
  /*@ assert rte: signed_overflow: a + b <= 2147483647; */
  __retres = (a + b) / 2;
  return __retres;
}
```

Annotated Example 1

Listing 2: Function average.....

```
#include <stdio.h>
#include <limits.h>
/*@  requires  0 <= a;
     requires a <= INT_MAX ;
     requires  0 <= b;
     requires b <= INT_MAX ;
     requires  0 <= a+b;
     requires a+b <= INT_MAX ;
     ensures \result <= INT_MAX;
*/
int average(int a,int b)
{
  return((a+b)/2);
}

int main()
{
  int x,y;
  x=INT_MAX / 2;y=INT_MAX / 2;
  //  printf("Average  for %d and %d is %d\n",x,y,
  //      );
  return average(x,y);
}
```

Current Summary

Nose Gear Velocity

- Estimated ground velocity of the aircraft should be available only if it is within 3 `km/hr` of the true velocity at some moment within past 3 seconds

Characterization of a System (I)

- NG velocity system:
  - **Hardware**:
    - *Electro-mechanical sensor*: detects rotations
    - *Two 16-bit counters*: Rotation counter, Milliseconds counter
    - *Interrupt service routine*: updates rotation counter and stores current time.
  - **Software**:
    - *Real-time operating system*: invokes update function every `500 ms`
    - *16-bit global variable*: for recording rotation counter update time
    - *An update function*: estimates ground velocity of the aircraft.
- Input data available to the system:
  - *time*: in `milliseconds`
  - *distance*: in `inches`
  - *rotation angle*: in `degrees`
- Specified system performs velocity estimations in *imperial* unit system
- Note: expressed functional requirement is in *SI* unit system (km/hr).

Characterization of a System (II) cont.

What are the main properties to consider for formalization?

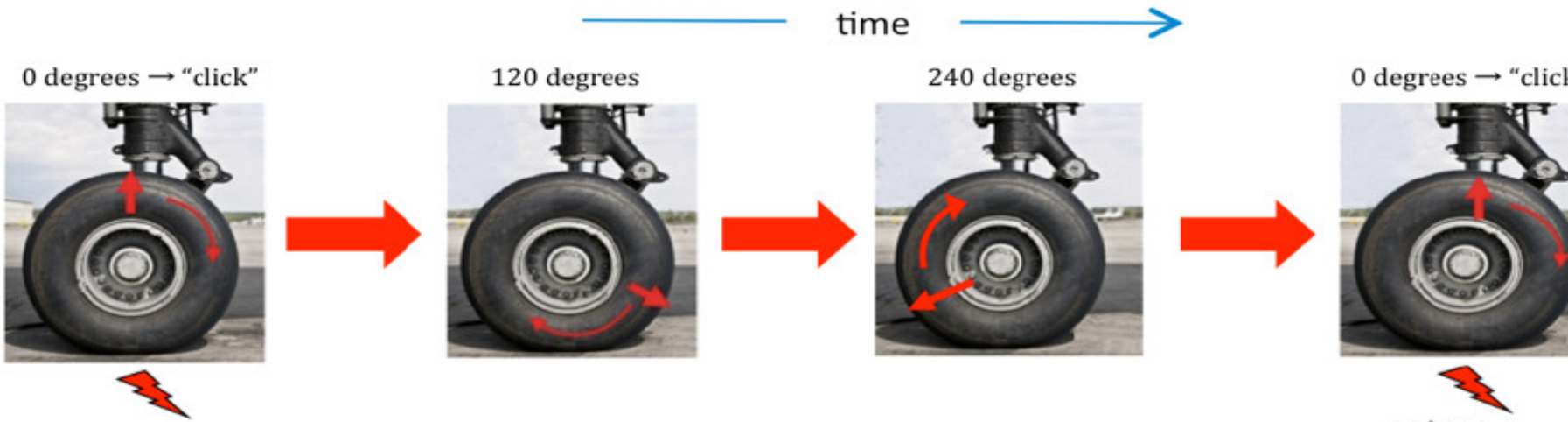
- Two different types of data:
  - counters with modulo semantics
  - non-negative values for time, distance, and velocity
- Two dimensions: *distance* and *time*
- Many units: distance (`inches`, `kilometers`, `miles`), time (`milliseconds`, `hours`), velocity (`kph`, `mph`)
- And interaction among components

How should we model?

- Designer needs to consider units and conversions between them to manipulate the model
- One approach: Model units as *sets*, and conversions as constructed types – *projections*.
- Example:

1 $estimateVelocity \in \texttt{MILES} \times \texttt{HOURS} \longrightarrow \texttt{MPH}$
2 $mphTokph \in \texttt{MPH} \rightarrowtail\!\!\!\!\!\rightarrow \texttt{KPH}$

Sample Velocity Estimation



NGClickTime = 4123 millisecs
NGRotations = 8954

NGClickTime = 4367milli
NGRotations = 8955

WHEEL_DIAMETER = 22 inches
PI = 3.14

12 inches/foot
5280 feet/mile

estimatedGroundVelocity = distance travel/elapsed time
= ((3.14 * 22)/(12*5280))/((4367-4123)/(1000*3600
= 16 mph

Safety Property Run Time Error (RTE)

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Safety Property

- Storing the number of `NGClick` in a n-bit variable VNGClick
- Integers are denoted by the set *Int* and is simply defined by the interval $Int \hat{=} INT\_MIN..INT\_MAX$.
- Safety requirement:
  The value of `VNGClick` is always in the range of implementation Int
  or equivalently $VNGClick \in Int$

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- $Length = \pi * diameter * VNGClick$ (**mathematical property**)
- $Length \leq 6000$ (**domain property**)
- $\pi * diameter * VNGClick \leq 6000$
- $VNGClick \leq 6000/(\pi * diameter)$
  - if n=8, then $2^7 - 1 = 127$ and $6000/(\pi * [22, inch]) = 6000/(\pi * 55, 88) = 6000/(3, 24 * [55, 88, cm]) = 6000/(3, 24 * 0.5588) \approx 3419$ and the condition of safety can not be satisfied in any situation.
  - if n=16, then $2^{15} - 1 = 65535$ and $6000/(\pi * [22, inch]) \approx 3419$ and the condition of safety can be satisfied in any situation since $3419 <= 65535$.

Safety Property Run Time Error (RTE)

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- Storing the number of `NGClick` in a n-bit variable VNGClick
- Integers are denoted by the set *Int* and is simply defined by the interval $Int \hat{=} INT\_MIN..INT\_MAX$.
- Safety requirement:
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$$RTE\_VNGClick : 0 \leq vNGClick \leq INT\_MAX \tag{1}$$

- The current value of VNGClick is always bounded by the two values 0 and INT_MAX.

Current Summary

bug0.c

Listing 3: Bug bug0

```c
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y;
    // Seed the random number generator with the current time
    srand(time(NULL));
    // Generate a random number between 1 and 100
    x = rand() % 100 + 1;
    // Perform some calculations
    y = x / (100 - x);
    printf("Result: %d\n", y);
    return 0;
}
```

Listing 4: Bug bug00

```
// Heisenbug
#include <stdio.h>
#include <stdlib.h>
#include <time.h>



int main() {
  int x, y,i=0;

    for (i = 0; i <= 100000; i++) {
    // Seed the random number generator with the current time
    srand(time(NULL));

    // Generate a random number between 1 and 100
    x = rand() % 100 + 1;
        printf("Result: x= %d\n",x);
    // Perform some calculations
    y = x / (100 - x);

    printf("Result: i=%d  and y=%d\n",i, y);
    }

    return 0;
}
```

Listing 5: Bug bug000

```
// Heisenbug
#include <stdio.h>
#include <stdlib.h>
#include <time.h>



int main() {
  int x, y,i=0;

    for (i = 0; i <= 200000; i++) {
    // Seed the random number generator with the current time
    srand(time(NULL)+i);

    // Generate a random number between 1 and 100
    x = rand() % 100 + 1;
        printf("Result: x= %d\n",x);
    // Perform some calculations
    y = x / (100 - x);

    printf("Result: i=%d  %d\n",i, y);
    }

    return 0;
}
```

Current Summary

Verifying program correctness (Run Time Errors, . . .)

A program P *satisfies* a (pre,post) contract:

- P transforms a variable v from initial values v_0 and produces a final value v_f: $v_0 \xrightarrow{P} v_f$
- v_0 satisfies pre: $\text{pre}(v_0)$ and v_f satisfies post : $\text{post}(v_0, v_f)$
- $\text{pre}(v_0) \wedge v_0 \xrightarrow{P} v_f \Rightarrow \text{post}(v_0, v_f)$
- $\mathbb{D}$ est le domaine RTE de V

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requires $pre(v_0)$
ensures $post(v_0, v_f)$
variables X
begin
$0 : P_0(v_0, v)$
$\texttt{instruction}_0$
...
$i : P_i(v_0, v)$
...
$\texttt{instruction}_{f-1}$
$f : P_f(v_0, v)$
end

- $pre(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$
- $pre(v_0) \wedge P_f(v_0, v) \Rightarrow post(v_0, v)$
- For any pair of labelsℓ, ℓ' such that $\ell \longrightarrow \ell'$, one verifies that, pour any values $v, v' \in \text{MEMORY}$

$$\left(\begin{array}{l} \left(\begin{array}{l} pre(v_0) \wedge P_\ell(v_0, v)) \\ \wedge cond_{\ell,\ell'}(v) \wedge v' = f_{\ell,\ell'}(v) \end{array} \right) \\ \Rightarrow P_{\ell'}(v_0, v') \end{array} \right),$$

- For any pair of labels m, n such taht $m \longrightarrow n$, one verifies that, $\forall v, v' \in \text{MEMORY}$: $pre(v_0) \wedge P_m(v_0, v) \Rightarrow \textbf{DOM}(m, n)(v)$

Example of an annotation

Variables X
Requires $\ldots$
Ensures $\ldots$
While $0 < X$ **DO**
$X := X - 1;$
Od;

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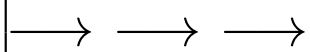

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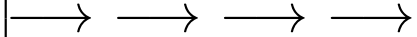

Example of an annotation

Variables X
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Contract EX
Variables $X(int)$
Requires $x_0 \in \mathbb{N}$
Ensures $x_f = 0$
$\ell_0 : \{ x = x_0 \wedge x_0 \in \mathbb{N}\}$
While $0 < X$ **DO**
$\ell_1 : \{0 < x \leq x_0 \wedge x_0 \in \mathbb{N}\}$
$X := X - 1;$
$\ell_2 : \{0 \leq x \leq x_0 \wedge x_0 \in \mathbb{N}\}$
Od;
$\ell_3 : \{x = 0\}$

A Simple C Function

Listing 6: Simple contract

```
/*@ requires \false ;
    @ ensures \false ; */
int f1(int x)
{ if (f1(x) <= 0)
    { return(1);
    }
  else
    { return(0);
    }
}
```

A Simple C Function

Listing 7: Simple contract

```
#include <stdio.h>
#include <math.h>


/*@ requires  \false ;
   @ ensures \false ; */
int f1(int x)
{ if (f1(x) <= 0)
    { return(1);
    }
  else
    { return(0);
    }
}
```

A Simple C Function

Listing 8: Simple contract

```
#include <stdio.h>
#include <math.h>


/*@ requires  \false ;
   @ ensures \false ; */
int f1(int x)
{ if (f1(x) <= 0)
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```

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- HOARE triple:

$$\{\mathbf{prex_0}) \wedge \mathbf{x} = \mathbf{x_0}\}\text{ALG}\{\mathbf{post}(\mathbf{x_0}, \mathbf{x})\}$$

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- Given

  Contract EX
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- Design of the algorithm ALG fulfiling the contract
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Idea?

From Cntract EX, using a step by step approach to find an algorithm ALG satisfying it using refinement and Event-B models.
Applying the CLEANROOM model but with tooml support (Thanks to Dr Wu!)

Current Summary

Short Summary on Event-B (context)

- Context: static properties of Event-B models
  - Sets: user-defined types
  - Constants: static object in development
  - Axioms: presumed properties about sets and constants
  - Theorems: derived properties about sets and constants

SETS

A

CONSTANTS

B, C, f

AXIOMS

$ax1 : B \subseteq A$

$ax2 : C \subseteq A$

$ax3 : g \in B \pfun C$

$ax4 : \forall A.A \subseteq \mathbb{N} \land 0 \in A \land suc[A] \subseteq A \Rightarrow \mathbb{N} \subseteq A$

...

Short Summary on Event-B (discrete)

- Machine: behavioral properties of Event-B models
  - Variables: states
  - Invariants: properties of variables that always need to hold
  - Theorems: derived properties about variables
  - Events: possible state changes

EVENT e
ANY t
WHERE
$G(c, s, t, x)$
THEN
$x :| (P(c, s, t, x, x'))$
END

- c et s are constantes and visible sets by e
- x is a state variable or a list of variabless
- $G(c, s, t, x)$ is the condition for observing e.
- $P(c, s, t, x, x')$ is the assertion for the relation over x and x'.
- $BA(e)(c, s, x, x')$ is the *before-after* relationship for e and is defined by $\exists t.G(c, s, t, x) \ \wedge \ P(c, s, t, x, x')$.

Short Summary on Event-B (refinement)

- Given an abstract and a corresponding concrete event

EVENT ae $\widehat{=}$
any v **where**
$G(x, v)$
then
$x := E(x, v)$
end

EVENT ce $\widehat{=}$
any w **where**
$H(y, w)$
then
$y := F(y, w)$
end

$$I(x) \quad \wedge \quad J(x, y) \quad \wedge \quad H(y, w)$$
$$\Longrightarrow$$
$$\exists v \cdot (G(x, v) \quad \wedge \quad J(E(x, v), F(y, w)))$$

- $BA(ae)(x, x') \ \widehat{=} \ \exists v.G(x, v) \wedge x' = E(x)$
- $BA(ce)(y, y') \ \widehat{=} \ \exists w.H(y, w) \wedge y' = F(y)$

Modelling systems in Event-B

Modelling systems in Event-B

MACHINE
m
SEES
c
VARIABLES
x
INVARIANT
$I(x)$
THEOREMS
$Q(x)$
INITIALISATION
$Init(x)$
EVENTS
$\ldots e$
END

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c defines the static environment for the proofs related to m: sets, constants, axioms, theorems $\Gamma(m)$.

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$\forall e :$
$\Gamma(m) \vdash \forall x, x', u \in Values : \mathrm{I}(x) \wedge R(u, x, x') \Rightarrow \mathrm{I}(x')$

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e
ANY
u
WHERE
$G(x, u)$
THEN
$x :| (R(u, x, x'))$
END

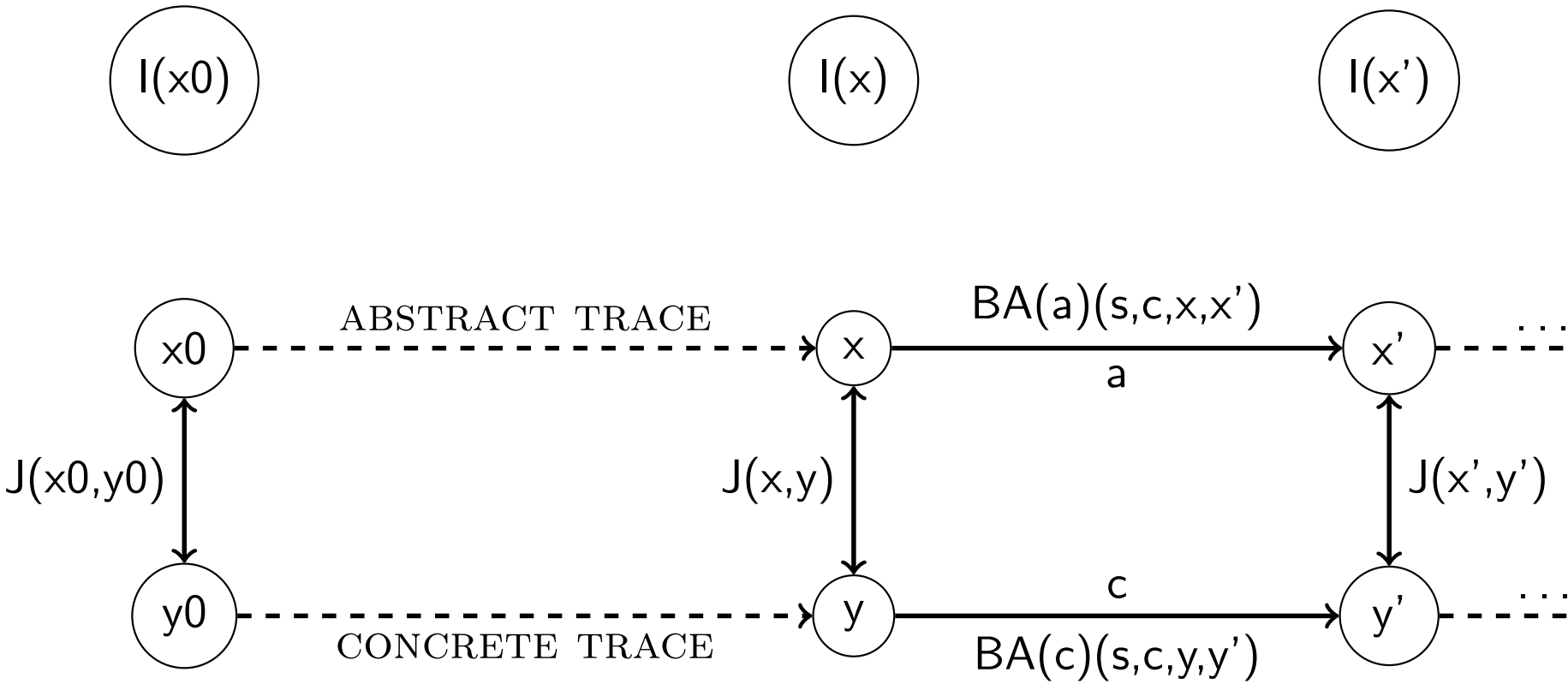
or e is **observed** $x \xrightarrow{e} x'$

Event B Structure and Proofs

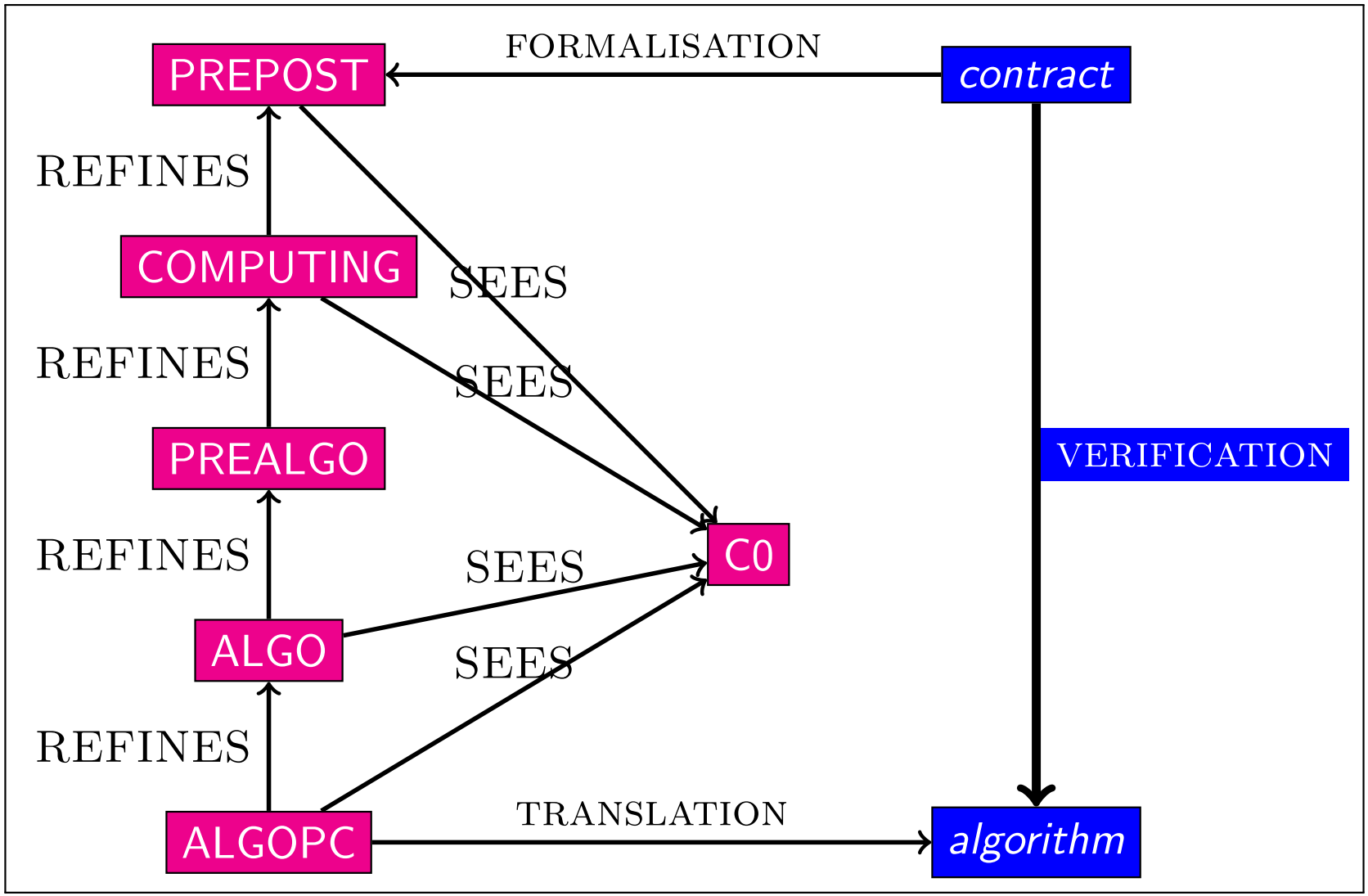
CONTEXT	MACHINE
ctxt_id_2	*machine_id_2*
EXTENDS	**REFINES**
ctxt_id_1	*machine_id_1*
SETS	**SEES**
s	*ctxt_id_2*
CONSTANTS	**VARIABLES**
c	v
AXIOMS	**INVARIANTS**
$A(s,c)$	$I(s,c,v)$
THEOREMS	**THEOREMS**
$T_c(s,c)$	$T_m(s,c,v)$
END	**VARIANT**
	$V(s,c,v)$
	EVENTS
	EVENT e
	any x
	where $G(s,c,v,x)$
	then
	$v :\mid BA(s,c,v,x,v')$
	end
	END

Invariant preservation	$A(s,c) \wedge I(s,c,v)$ $\wedge G(s,c,v,x)$ $\wedge BA(s,c,v,x,v')$ $\Rightarrow I(s,c,v')$
Event feasibility	$A(s,c) \wedge I(s,c,v)$ $\wedge G(s,c,v,x)$ $\Rightarrow \exists v'.BA(s,c,v,x,v')$
Variant modelling progress	$A(s,c) \wedge I(s,c,v)$ $\wedge G(s,c,v,x)$ $\wedge BA(s,c,v,x,v')$ $\Rightarrow V(s,c,v') < V(s,c,v)$
Theorems	$A(s,c) \Rightarrow T_c(s,c)$ $A(s,c) \wedge I(s,c,v)$ $\Rightarrow T_m(s,c,v)$

Refinement between two machines



The Iterative Pattern



Current Summary

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Computing two sums

First n numbers and first odd/even numbers

- First the **pre/post specification** . . .
- Possibility to use a programming language with contracts too

Computing two sums

First n numbers and first odd/even numbers

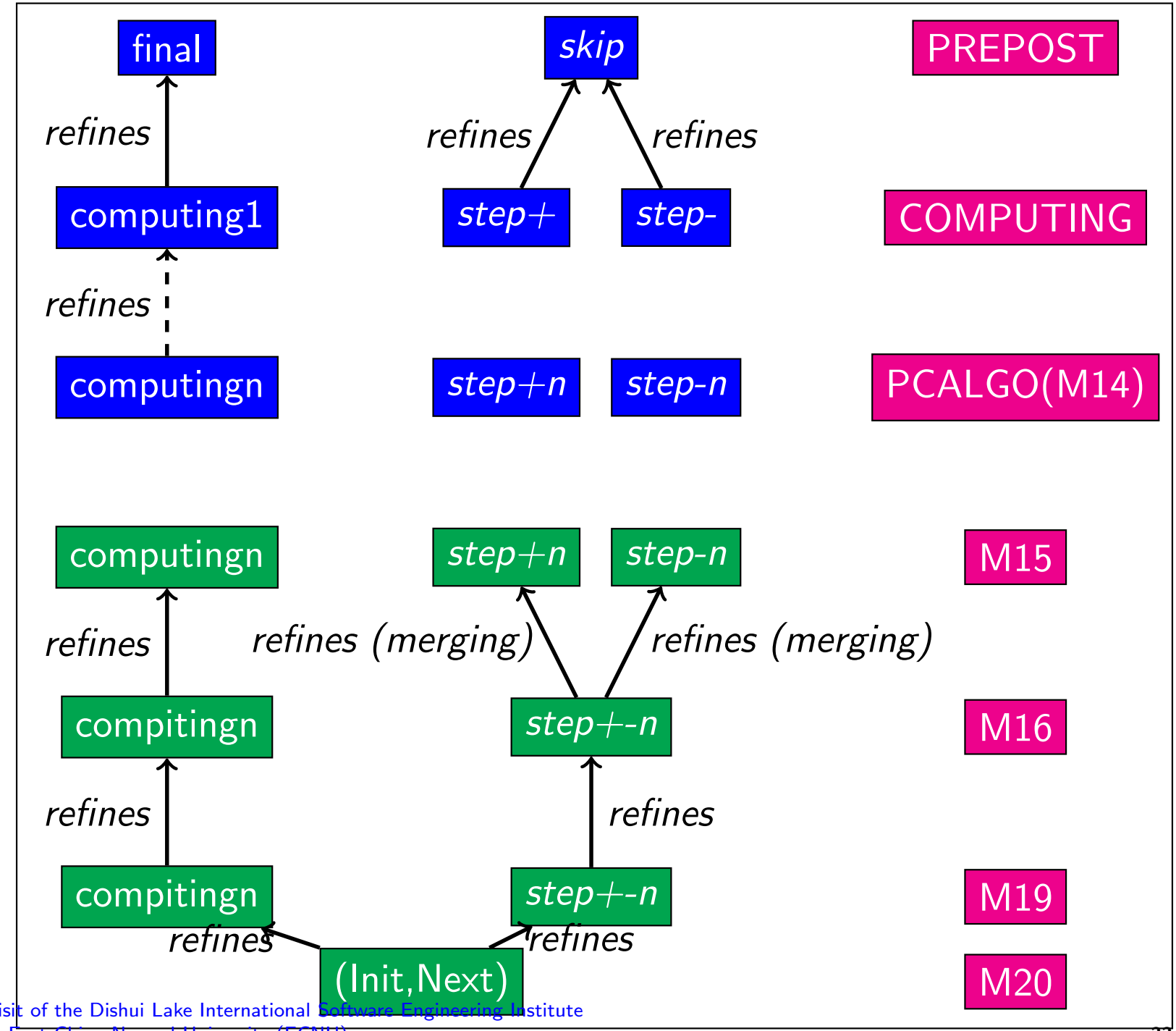
- First the **pre/post specification** ...
- Possibility to use a programming language with contracts too

requires $input_0 \geq 0 \wedge rs_0, re_0 \in \mathbb{Z}$

ensures $\begin{cases} rs_f = s(input_0) \\ re_f = es(input_0) \\ input_f = input_0 \end{cases}$

variables input, re, rs

- Find two sequences s and es computing the sum of first n natural numbers and first even numbers smaller than n
- Prove that:
  - $\forall n.n \in \mathbb{N} \Rightarrow s(n) = \sum_{i=0}^{i=n} i.$
  - $\forall n.n \in \mathbb{N} \Rightarrow es(n) = \sum_{i=0}^{i=n/2} 2 * i.$


final
skip
PREPOST
refines
refines
refines
computing1
step+
step-
COMPUTING
refines
computingn
step+n
step-n
PCALGO(M14)
computingn
step+n
step-n
M15
refines
refines (merging)
refines (merging)
compitingn
step+-n
M16
refines
refines
compitingn
step+-n
M19
refines
refines
(Init,Next)
M20

First n numbers and first odd/even numbers

$$axm1 : n \in \mathbb{N}$$
$$axm2 : s \in \mathbb{N} \to \mathbb{N} \land os \in \mathbb{N} \to \mathbb{N} \land es \in \mathbb{N} \to \mathbb{N}$$
$$axm3 : es(0) = 0 \land os(0) = 0 \land s(0) = 0$$
$$axm4 : \forall i, l \cdot i \in \mathbb{N} \land l \in \mathbb{N} \land i = 2 * l \Rightarrow s(i+1) = s(i) + i + 1 \land es(i+1) =$$
$$axm5 : \forall i, l \cdot i \in \mathbb{N} \land l \in \mathbb{N} \land i = 2 * l + 1 \Rightarrow s(i+1) = s(i) + i + 1 \land es(i+$$
$$axm6 : suc \in \mathbb{N} \to \mathbb{N} \land (\forall i \cdot i \in \mathbb{N} \Rightarrow suc(i) = i + 1)$$
$$axm7 : \forall A \cdot A \subseteq \mathbb{N} \land 0 \in A \land suc[A] \subseteq A \Rightarrow \mathbb{N} \subseteq A$$
$$th1 : \forall i \cdot i \in \mathbb{N} \Rightarrow s(i+1) = s(i) + i + 1$$
$$th2 : \forall u, v \cdot u \in \mathbb{N} \land v \in \mathbb{N} \land 2 * u = v \Rightarrow u = v/2$$
$$th3 : \forall k \cdot k \in \mathbb{N} \Rightarrow 2 * s(k) = k * k + k$$
$$th4 : \forall k \cdot k \in \mathbb{N} \Rightarrow s(k) = (k * k + k)/2$$
$$th5 : \forall k \cdot k \in \mathbb{N} \Rightarrow es(2 * k) = 2 * s(k)$$
$$th6 : \forall k \cdot k \in \mathbb{N} \Rightarrow es(2 * k + 1) = 2 * s(k)$$
$$th7 : \forall k \cdot k \in \mathbb{N} \land k \neq 0 \Rightarrow os(2 * k) = k * k$$
$$th8 : \forall k \cdot k \in \mathbb{N} \Rightarrow os(2 * k + 1) = (k + 1) * (k + 1)$$

Stating the pre/post specification in Event-B

INVARIANTS

$$inv1 : input \in \mathbb{Z}$$
$$inv6 : input = n$$
$$inv2 : rs \in \mathbb{Z} \land re \in \mathbb{Z}$$
$$inv3 : ok \in BOOL$$
$$inv4 : ok = TRUE \Rightarrow rs = s(input) \land re = es(input)$$

END

Stating the pre/post specification in Event-B

then

$act1 : ok := FALSE$

$act2 : rs :\in \mathbb{Z}$

$act3 : re :\in \mathbb{Z}$

$act4 : input := n$

EVENT computing

when

$grd1 : ok = FALSE$

then

$act1 : rs := s(input)$

$act2 : ok := TRUE$

$act3 : re := es(input)$

END

Refining for computing (1)

INVARIANTS

$inv1 : cur \in 0 \,..\, n$

$inv2 : ee \in 0 \,..\, n \pfun \mathbb{N}$

$inv3 : ss \in 0 \,..\, n \pfun \mathbb{N}$

$inv5 : dom(ss) = 0 \,..\, cur \land dom(ee) = dom(ss) \land dom(ss) \subseteq \mathbb{N}$

$inv6 : \forall i \cdot i \in 0 \,..\, cur \Rightarrow ee(i) = es(i) \land ss(i) = s(i)$

Variant

then

$act1 : ok := FALSE$

$act2 : rs :\in \mathbb{Z}$

$act3 : re :\in \mathbb{Z}$

$act4 : input := n$

$act5 : ee := \{0 \mapsto 0\}$

$act6 : ss := \{0 \mapsto 0\}$

$act7 : cur := 0$

END

Refining for computing (2)

EVENT computing11
when
$grd1 : ok = FALSE$
$grd2 : cur = n$

then
$act1 : rs := ss(input)$
$act2 : ok := TRUE$
$act3 : re := ee(input)$

END

Refining for computing (3)

EVENT step11+

Any

$k,$

when

$grd1 : ok = FALSE$

$grd2 : k \in \mathbb{N} \wedge cur = 2 * k + 1$

$grd3 : cur < n$

$grd4 : cur < n$

then

$act1 : ss(cur + 1) := ss(cur) + cur + 1$

$act2 : ee(cur + 1) := ee(cur) + cur + 1$

$act3 : cur := cur + 1$

END

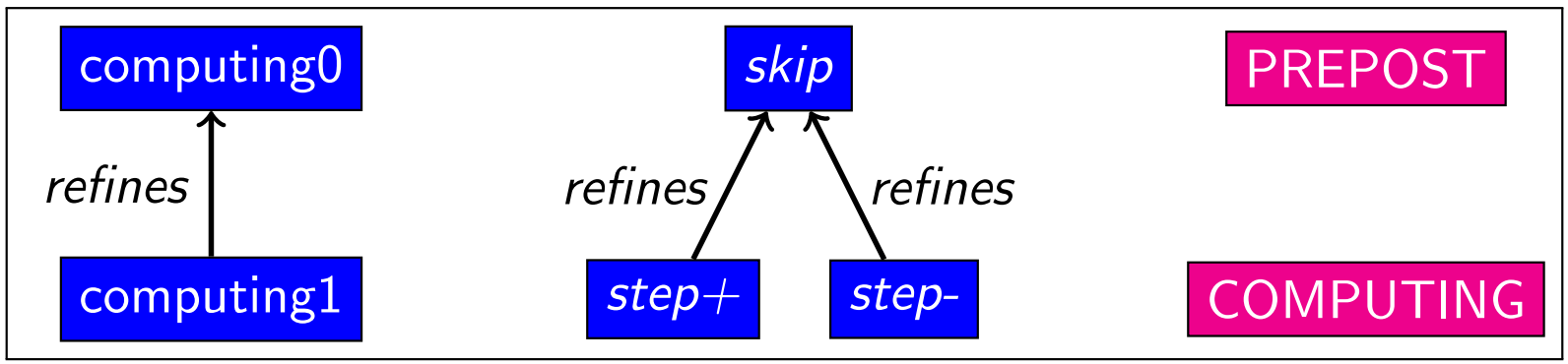
Refining for computing (4)

EVENT step11=
Any
$k,$
when
$grd1 : ok = FALSE$
$grd2 : k \in \mathbb{N} \wedge cur = 2 * k$
$grd3 : cur < n$

then
$act1 : ss(cur + 1) := ss(cur) + cur + 1$
$act2 : ee(cur + 1) := ee(cur)$
$act3 : cur := cur + 1$

END

Diagram of events



Completing the analysis

MACHINE $M1$ **SEES** $C0$

VARIABLES

$input, rs, re, ok,$

INVARIANTS

$inv1 : input \in \mathbb{Z}$

$inv6 : input = n$

$inv2 : rs \in \mathbb{Z} \wedge re \in \mathbb{Z}$

$inv3 : ok \in BOOL$

$inv4 : ok = TRUE \Rightarrow rs = s(input) \wedge re = es(input)$

Completing the analysis

MACHINE $M11$ **SEES** $C0$

VARIABLES

$input, rs, re, ok, ee, ss, cur,$

INVARIANTS

$inv1 : cur \in 0 \,..\, n$

$inv2 : ee \in 0 \,..\, n \pfun \mathbb{N}$

$inv3 : ss \in 0 \,..\, n \pfun \mathbb{N}$

$inv5 : dom(ss) = 0 \,..\, cur \land dom(ee) = dom(ss) \land dom(ss) \subseteq \mathbb{N}$

$inv6 : \forall i \cdot i \in 0 \,..\, cur \Rightarrow ee(i) = es(i) \land ss(i) = s(i)$

Completing the analysis

MACHINE $M12$ **SEES** $C0$

VARIABLES

$input, rs, re, ok, ee, ss, cur, cs, ce,$

INVARIANTS

$inv1 : cs = ss(cur)$

$inv2 : ce = ee(cur)$

Completing the analysis

MACHINE $M13$ **SEES** $C0$

VARIABLES

$input, rs, re, ok, ee, ss, cur, cs, ce,$

INVARIANTS

$inv1 : cs = ss(cur)$

$inv2 : ce = ee(cur)$

$inv4 : ce = es(cur)$

$inv3 : cs = s(cur)$

Completing the analysis

MACHINE $M14$ **SEES** $C0, C1$

VARIABLES

$input, rs, re, cur, cs, ce, l,$

INVARIANTS

$inv1 : cs = s(cur)$

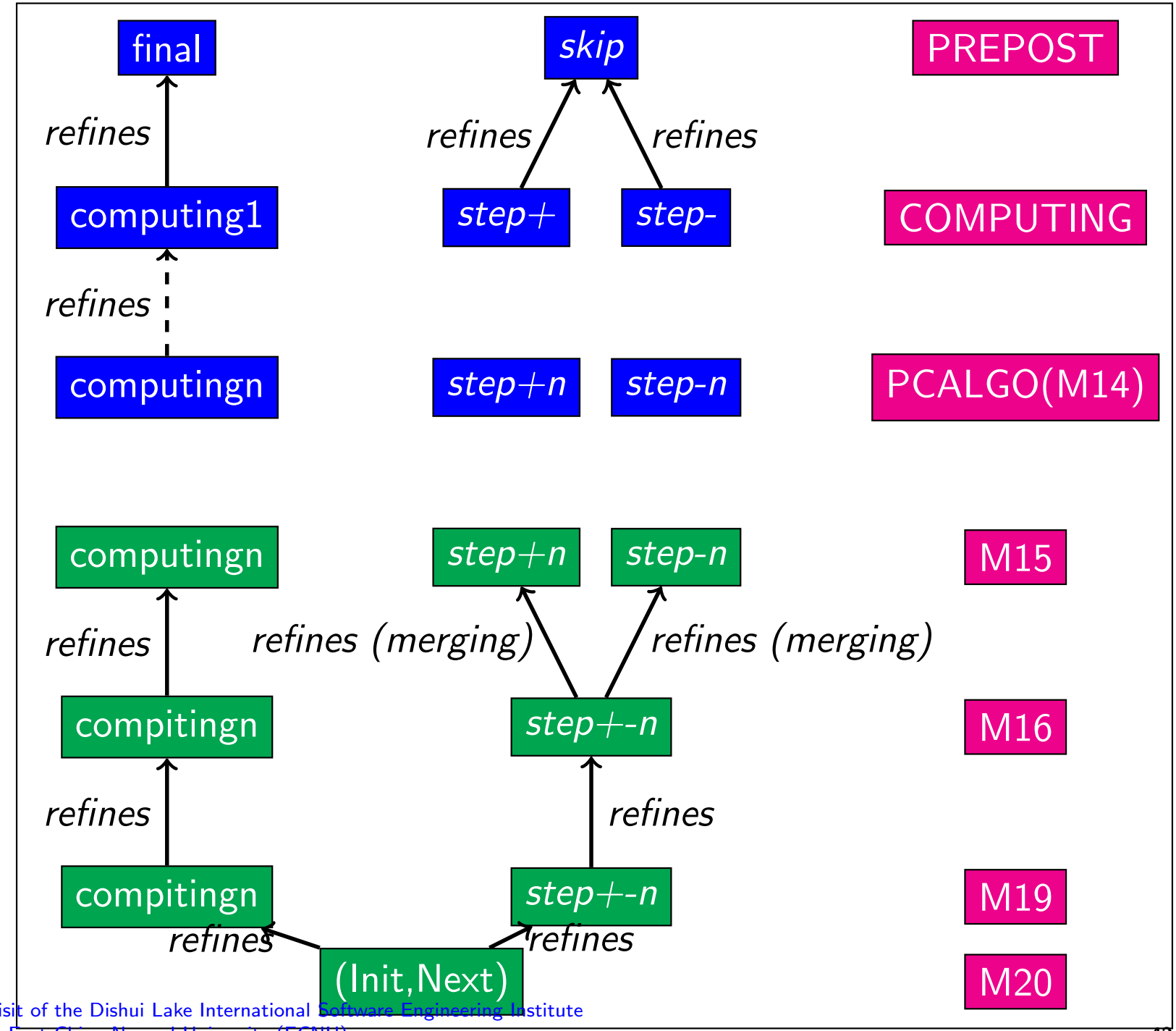
$inv5 : l = start \Leftrightarrow ok = FALSE$

$inv6 : l = end \Leftrightarrow ok = TRUE$

$inv2 : cs = s(cur)$

$inv3 : l \in L$

$inv4 : l = end \Rightarrow re = es(n) \wedge rs = s(n)$


final
skip
PREPOST
refines
refines
refines
computing1
step+
step-
COMPUTING
refines
computingn
step+n
step-n
PCALGO(M14)
computingn
step+n
step-n
M15
refines
refines (merging)
refines (merging)
compitingn
step+-n
M16
refines
refines
compitingn
step+-n
M19
refines
refines
(Init,Next)
M20

Generation of an algorithm from M14

Translation from Evenrts

```
struct sums  codesum(int n)
{int  k,ce,cs; struct sums r;
  r.s=0;r.se=0;k=0;ce=0;cs=0;
  while (k<n)
    {
      if ( k % 2 != 0)
        { ce = ce + k + 1;cs = cs +k+1;        k = k +1;}
      else
        { ce = ce ;cs = cs +k+1;        k = k +1;}
       }
  r.s=cs;r.se=ce;
  return(r);
}
```

Generation of an algorithm from M14

Improving and checking with Frama-c

```
#include "structure.h"
#include "trans-even.h"
struct sums  codesum(int n)
{int  k,ce,cs; struct sums r;
  r.s=0;r.se=0;k=0;ce=0;cs=0;
  /*@ loop invariant k >= 0 && k <= n && mathsum(k) == cs;
    @ loop invariant  ( (k % 2 == 0) ==> (mathse(k) == ce));
    @ loop invariant  ( (k % 2 != 0) ==> (mathse(k) == ce));
    loop assigns k, ce,cs;
    loop variant n-k;
   */
  while (k<n)
    {
      if ( k % 2 != 0)
        { ce = ce + k + 1;cs = cs +k+1;       k = k +1;}
      else
        { ce = ce ;cs = cs +k+1;       k = k +1;}
      }
  r.s=cs;r.se=ce;
  return r;}
```

Generation of an algorithm from M14

Improving and checking with Frama-c

```
#include "structure.h"
#include "trans-even.h"
struct sums  codesum(int n)
{int  k,ce,cs; struct sums r;
  r.s=0;r.se=0;k=0;ce=0;cs=0;
  /*@ loop invariant k >= 0 && k <= n && mathsum(k) == cs;
    @ loop invariant  ( (k % 2 == 0) ==> (mathse(k) == ce));
    @ loop invariant  ( (k % 2 != 0) ==> (mathse(k) == ce));
    loop assigns k, ce,cs;
    loop variant n-k;
   */
  while (k<n)
    {
      if ( k % 2 != 0)
        { ce = ce + k + 1;}
      cs = cs +k+1;        k = k +1;
 cp trabs-e*          }
  r.s=cs;r.se=ce;
  return(r);
}
```

Generation of an algorithm from M14

Using the Plugin EB2Algo

```
rs :∈ ℤ ||
re :∈ ℤ ||
input = n ||
cur = 0 ||
ce = 0 ||
cs = 0

while cur≠n do
    if cur mod 2≠0 then
        cur = cur+1 ||
        cs = cs+cur+1 ||
        ce = ce+cur+1
    else
        cur = cur+1 ||
        cs = cs+cur+1 ||
        ce = ce
```

Current Summary

Merging events

EVENT e1
any
x
where
$G1(s, c, x, u)$
then
$u : |(R(s, c, x, u, u'))$
end

EVENT e2
any
x
where
$G2(s, c, x, u)$
then
$u : |(R(s, c, x, u, u'))$
end

Merging events

EVENT e1
any
x
where
$G1(s, c, x, u)$
then
$u : |(R(s, c, x, u, u'))$
end

EVENT e2
any
x
where
$G2(s, c, x, u)$
then
$u : |(R(s, c, x, u, u'))$
end

EVENT e $refines$ $e1, e2$
any
x
where
$H(s, c, x, u)$
then
$u : |(R(s, c, x, u, u'))$
end

Merging events

EVENT e1
any
x
where
$G1(s, c, x, u)$
then
$u : |(R(s, c, x, u, u'))$
end

EVENT e2
any
x
where
$G2(s, c, x, u)$
then
$u : |(R(s, c, x, u, u'))$
end

EVENT e $refines$ $e1, e2$
any
x
where
$H(s, c, x, u)$
then
$u : |(R(s, c, x, u, u'))$
end

Checking the Proof Obligation:

$$Ax(s, c), I(s, c, u), H(s, c, x, u) \vdash G1(s, c, x, u) \vee G2(s, c, x, u)$$

Synthesis Phase

- Preparing merging by transforming actions.

EVENT e
any
x
where
$G(s, c, x, u)$
then
$u :| (R(s, c, x, u, u'))$
end

EVENT f refines e
any
x
where
$G(s, c, x, u)$
then
$u :| (G(s, c, x, u) \Rightarrow R(s, c, x, u, u'))$
end

- Internalizing the action as a before after relation
- $G(s, c, x, u) \wedge (G(s, c, x, u) \Rightarrow R(s, c, x, u, u')) \Leftrightarrow$
  $G(s, c, x, u) \wedge R(s, c, x, u, u')$

Synthesis Phase

- Preparing merging by transforming actions.

EVENT e
any
x
where
$G(s, c, x, u)$
then
$u :| (R(s, c, x, u, u'))$
end

EVENT f refines e
any
x
where
$G(s, c, x, u)$
then
$u :| (G(s, c, x, u) \Rightarrow R(s, c, x, u, u'))$
end

- $G(s, c, x, u) \wedge (G(s, c, x, u) \Rightarrow R(s, c, x, u, u')) \Leftrightarrow$
  $G(s, c, x, u) \wedge R(s, c, x, u, u')$
- $A(s, c, u) \wedge I(s, c, u) \wedge BA(f)(s, c, u, u') \Rightarrow$
  $I(s, c, u') \wedge BA(e)(s, c, u, u')$

Synthesis Phase

- Preparing merging by transforming actions.

EVENT e
any
x
where
$G(s, c, x, u)$
then
$u :| (R(s, c, x, u, u'))$
end

EVENT f refines e
any
x
where
$G(s, c, x, u)$
then
$u :| (G(s, c, x, u) \Rightarrow R(s, c, x, u, u'))$
end

- $G(s, c, x, u) \wedge (G(s, c, x, u) \Rightarrow R(s, c, x, u, u')) \Leftrightarrow$
  $G(s, c, x, u) \wedge R(s, c, x, u, u')$
- $A(s, c, u) \wedge I(s, c, u) \wedge BA(f)(s, c, u, u') \Rightarrow$
  $I(s, c, u') \wedge BA(e)(s, c, u, u')$
- $A(s, c, u) \wedge I(s, c, u) \wedge G(s, c, x, u) \wedge (G(s, c, x, u) \Rightarrow$
  $R(s, c, x, u, u')) \Rightarrow I(s, c, u') \wedge G(s, c, x, u) \wedge R(s, c, x, u, u')$

Synthesis Phase: review of machines from M15-M20

MACHINE $M15$ **SEES** $C0, C1$
VARIABLES
$input, rs, re, cur, cs, ce, l,$

Synthesis Phase: review of machines from M15-M20

MACHINE $M16$ **SEES** $C0, C1$
VARIABLES
$input, rs, re, cur, cs, ce, l,$

Synthesis Phase: review of machines from M15-M20

MACHINE $M17$ **SEES** $C0, C1$
VARIABLES
$input, rs, re, cur, cs, ce, l,$

Synthesis Phase: review of machines from M15-M20

MACHINE $M18$ **SEES** $C0, C1$
VARIABLES
$input, rs, re, cur, cs, ce, l,$

Synthesis Phase: review of machines from M15-M20

MACHINE $M19$ **SEES** $C0, C1$
VARIABLES
$input, rs, re, cur, cs, ce, l,$

Synthesis Phase: review of machines from M15-M20

MACHINE $M20$ **SEES** $C0, C1$
VARIABLES
$input, rs, re, cur, cs, ce, l,$

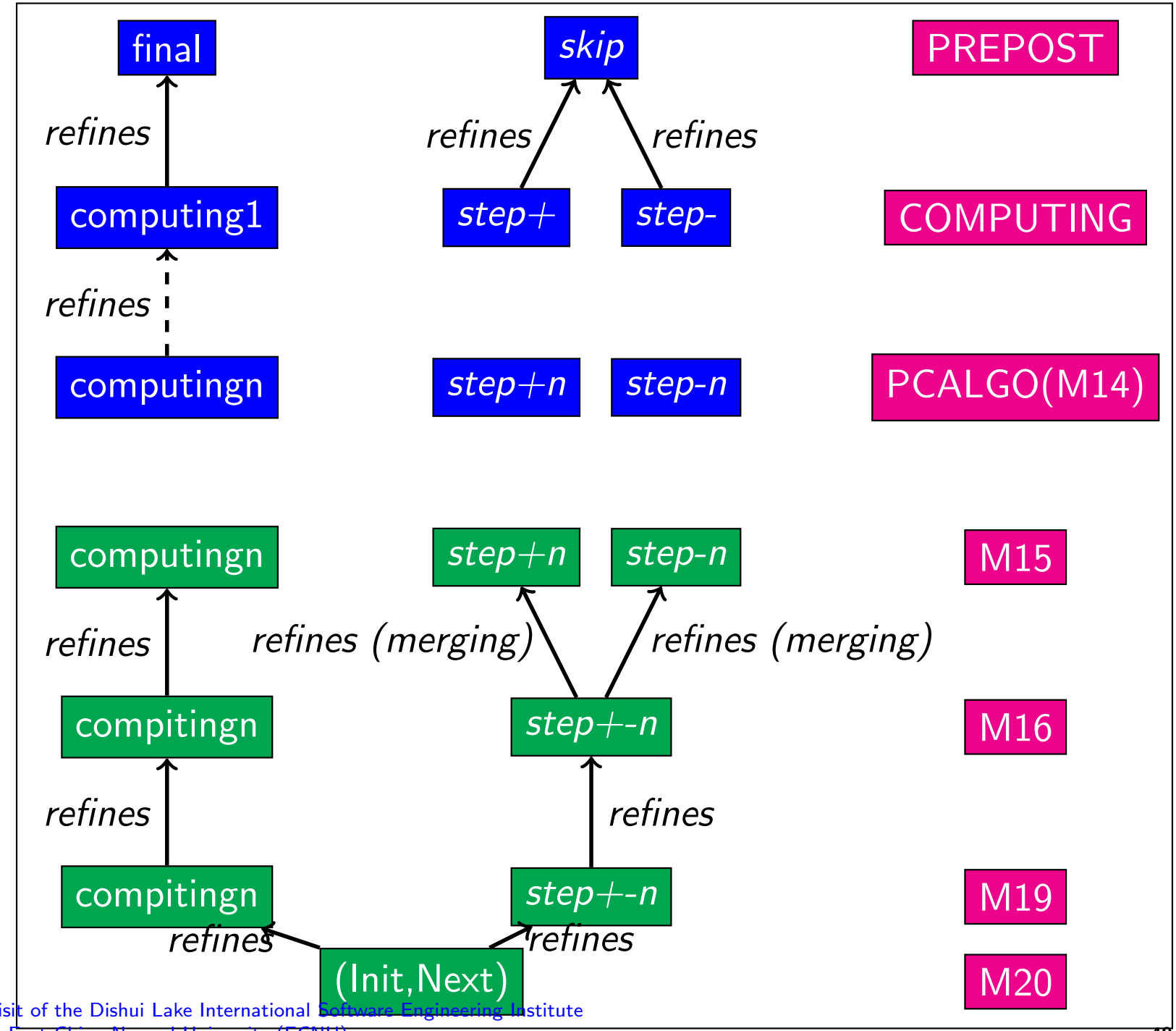
Generating a TLA specification

```
----------------------------
next1 ==
      (l="start" /\   cur < n  /\ ( cur % 2 = 0))
      /\   (cur'=cur+1 /\  ce'=ce /\  l'=l
      /\  cs'=cs+cur+1 /\  re'=re /\  rs'=rs)
o
delsnext2 ==
      (l="start" /\   cur < n  /\ ( cur % 2 # 0))
      /\ (cur'=cur+1 /\  ce'=ce+cur+1 /\  l'=l
      /\  cs'=cs+cur+1 /\  re'=re /\ rs'=rs)

next3 ==
  (l="start" /\  cur=n) /\   (rs'=cs /\  re'=ce
  /\  l'="end" /\  cur'=cur /\ cs'=cs/\ ce'=ce)

Next ==
      (next1 \/ next2 \/ next3)

Init == l="start" /\   cur=0 /\ rs=0 /\ cs=0 /\ re=0 /\ ce =0
=============================================================
```


final
skip
PREPOST
refines
refines
refines
computing1
step+
step-
COMPUTING
refines
computingn
step+n
step-n
PCALGO(M14)
computingn
step+n
step-n
M15
refines
refines (merging)
refines (merging)
compitingn
step+-n
M16
refines
refines
compitingn
step+-n
M19
refines
refines
(Init,Next)
M20

Current Summary

Conclusion

- Paradigm for planning refinements.
- Teaching why and how sequential algorithms are working.
- Relating Event-B to TLA
- Application to controller synthesis: events versus operations.

Conclusion

- Paradigm for planning refinements.
- Teaching why and how sequential algorithms are working.
- Relating Event-B to TLA
- Application to controller synthesis: events versus operations.

Next

- Atlas of *correct-by-construction* sequential algorithms
- Definition of link between events and codes
- Integration of **Cyclone** for validating Event-B models

Case studies

Case studies

- primes
- binary search
- power functions: x^n
- gcd
- fibonacci-like functions


Questions
???