# Contents

**CONTEXT** POWER20

**CONSTANTS**

x

u

w

v

C

D

E

**AXIOMS**

axm1: $x \in N$

axm13: $\forall A \cdot (A \subseteq N \land 0 \in A \land (\forall i \cdot i \in N \land i \in A \Rightarrow i + 1 \in A)) \Rightarrow N \subseteq A$

axm2: $u \in N \rightarrow N$

axm3: $u(0) = 0$

axm4: $\forall n \cdot n \in N \Rightarrow u(n + 1) = u(n) + 1$

axm5: $w \in N \rightarrow N$

axm6: $w(0) = 0$

axm7: $\forall n \cdot n \in N \Rightarrow w(n + 1) = w(n) + 2$

axm8: $v \in N \rightarrow N$

axm9: $v(0) = 0$

axm10: $\forall n \cdot n \in N \Rightarrow v(n + 1) = v(n) + w(n) + 1$

axm11: $\forall a, b \cdot a \in Z \land b \in Z \Rightarrow (a + b) * (a + b) = a * a + 2 * a * b + b * b$

axm14: $C \subseteq N \land C = \{k | k \in N \land u(k) = k\}$

axm15: ⟨theorem⟩ $C = N$

axm12: ⟨theorem⟩ $\forall n \cdot n \in N \Rightarrow u(n) = n$

axm16: $D \subseteq N \land D = \{k | k \in N \land w(k) = 2 * k + 1\}$

axm20: $D = N$

axm17: ⟨theorem⟩ $\forall n \cdot n \in N \Rightarrow w(n) = 2 * n + 1$

axm19: $E \subseteq N \land E = \{k | k \in N \land v(k) = k * k\}$

axm21: ⟨theorem⟩ $E = N$

axm18: ⟨theorem⟩ $\forall n \cdot n \in N \Rightarrow v(n) = n * n$

**END**

**MACHINE** POWER21
**SEES** POWER20
**VARIABLES**
r
ok
**INVARIANTS**
inv1: $r \in Z$
inv2: $ok \in BOOL$
inv3: $ok = TRUE \Rightarrow r = x * x$
**EVENTS**
**Initialisation**
**begin**
act1: $r := 0$
act2: $ok := FALSE$
**end**
**Event** computing ⟨ordinary⟩ $\widehat{=}$
**when**
grd1: $ok = FALSE$
**then**
act1: $r := v(x)$
act2: $ok := TRUE$
**end**
**END**

**MACHINE** POWER22
**REFINES** POWER21
**SEES** POWER20
**VARIABLES**

r
vv
ww
uu
k
ok

**INVARIANTS**

inv1: $vv \in N \pfun N$
inv2: $ww \in N \pfun N$
inv3: $uu \in N \pfun N$
inv4: $k \in N$
inv5: $\forall i \cdot i \in dom(uu) \Rightarrow uu(i) = u(i)$
inv6: $\forall i \cdot i \in dom(vv) \Rightarrow vv(i) = v(i)$
inv7: $\forall i \cdot i \in dom(ww) \Rightarrow ww(i) = w(i)$
inv8: $dom(uu) = 0 \upto k \land dom(vv) = dom(uu) \land dom(ww) = dom(vv)$
inv9: $k \leq x$
th1: ⟨theorem⟩ $ok = TRUE \Rightarrow r = x * x$

**EVENTS**
**Initialisation**
**begin**
act1: $r := 0$
act2: $vv := \{0 \mapsto 0\}$
act3: $ww := \{0 \mapsto 0\}$
act4: $k := 0$
act5: $uu := \{0 \mapsto 0\}$
act6: $ok := FALSE$
**end**
**Event** computing ⟨ordinary⟩ $\widehat{=}$
**refines** computing
**when**
grd1: $x \in dom(vv)$
grd2: $ok = FALSE$
**then**
act1: $r := vv(x)$
act2: $ok := TRUE$
**end**
**Event** step ⟨ordinary⟩ $\widehat{=}$
**when**
grd1: $x \notin dom(vv)$
grd2: $ok = FALSE$
**then**
act1: $uu(k+1) := uu(k) + 1$
act2: $vv(k+1) := vv(k) + ww(k) + 1$
act3: $ww(k+1) := ww(k) + 2$
act4: $k := k + 1$
**end**
**END**

**MACHINE** POWER23
**REFINES** POWER22
**SEES** POWER20
**VARIABLES**

r
vv
ww
uu
k
cu
cv
cw
ok

**INVARIANTS**

inv1: $cu \in dom(uu)$
inv2: $cv \in N$
inv3: $cw \in N$
inv4: $cu = uu(k)$
inv5: $cv = vv(k)$
inv6: $cw = ww(k)$
inv7: $cu = k$
inv8: $cw = 2 * cu$
inv9: $cv = cu * cu$
inv10: $cw = 2 * k$
inv11: $cv = k * k$
inv12: ⟨theorem⟩ $ok = TRUE \Rightarrow r = x * x$

**EVENTS**
**Initialisation**
**begin**
act1: $r := 0$
act3: $vv := \{0 \mapsto 0\}$
act4: $ww := \{0 \mapsto 0\}$
act5: $k := 0$
act6: $uu := \{0 \mapsto 0\}$
act7: $cu := 0$
act8: $cv := 0$
act9: $cw := 0$
act10: $ok := FALSE$
**end**
**Event** computing ⟨ordinary⟩ $\widehat{=}$
**refines** computing
**when**
grd1: $k = x$
grd2: $ok = FALSE$
**then**
act1: $r := cv$
act2: $ok := TRUE$
**end**
**Event** step ⟨ordinary⟩ $\widehat{=}$
**refines** step
**when**
grd1: $k < x$
grd2: $ok = FALSE$
**then**

act1: $uu(k+1) := uu(k) + 1$
act2: $vv(k+1) := vv(k) + ww(k) + 1$
act3: $ww(k+1) := ww(k) + 2$
act4: $k := k + 1$
act5: $cu := cu + 1$
act6: $cv := cv + cw + 1$
act7: $cw := cw + 2$

**end**

**END**

**MACHINE** POWER24
**REFINES** POWER23
**SEES** POWER20
**VARIABLES**

r
k
cv
cw
ok

**INVARIANTS**

inv1: ⟨theorem⟩ $cv = k * k$
inv2: ⟨theorem⟩ $cw = 2 * k$
inv3: ⟨theorem⟩ $k \leq x$
inv4: ⟨theorem⟩ $0 \leq k$

**EVENTS**
**Initialisation**
**begin**

act1: $r := 0$
act5: $k := 0$
act8: $cv := 0$
act9: $cw := 0$
act10: $ok := FALSE$

**end**
**Event** computing ⟨ordinary⟩ $\widehat{=}$
**refines** computing
**when**

grd1: $k = x$
grd2: $ok = FALSE$

**then**

act1: $r := cv$
act2: $ok := TRUE$

**end**
**Event** step ⟨ordinary⟩ $\widehat{=}$
**refines** step
**when**

grd1: $k < x$
grd2: $ok = FALSE$

**then**

act4: $k := k + 1$
act6: $cv := cv + cw + 1$
act7: $cw := cw + 2$

**end**
**END**

**MACHINE** POWER25
**REFINES** POWER24
**SEES** POWER20
**VARIABLES**

r
k
cv
cw
ok

**EVENTS**
**Initialisation**
**begin**
act1: $r := 0$
act5: $k := 0$
act8: $cv := 0$
act9: $cw := 0$
act10: $ok := FALSE$
**end**
**Event** computing ⟨ordinary⟩ $\widehat{=}$
**refines** computing
**when**
grd1: $k = x$
grd2: $ok = FALSE$
**then**
act1: $r, k, cv, cw, ok :| (k = x \Rightarrow r' = cv \wedge ok' = TRUE \wedge k' = k \wedge cv' = cv \wedge cw' = cw)$
**end**
**Event** step ⟨ordinary⟩ $\widehat{=}$
**refines** step
**when**
grd1: $k < x$
grd2: $ok = FALSE$
**then**
act4: $r, k, cv, cw, ok :| (k < x \Rightarrow r' = r \wedge ok' = ok \wedge k' = k + 1 \wedge cv' = cv + cw + 1 \wedge cw' = cw + 2)$
**end**
**END**

**MACHINE** POWER27
**REFINES** POWER25
**SEES** POWER20
**VARIABLES**

r
k
cv
cw
ok

**EVENTS**
**Initialisation**
**begin**
act1: $r := 0$
act5: $k := 0$
act8: $cv := 0$
act9: $cw := 0$
act10: $ok := FALSE$
**end**
**Event** computing ⟨ordinary⟩ $\widehat{=}$
**refines** computing
**when**
grd1: $k = x$
grd2: $ok = FALSE$
**then**
act1:
$r, k, cv, cw, ok :| (k = x \Rightarrow r' = cv \wedge ok' = TRUE \wedge k' = k \wedge cv' = cv \wedge cw' = cw)$
$\wedge (k < x \Rightarrow r' = r \wedge ok' = ok \wedge k' = k + 1 \wedge cv' = cv + cw + 1 \wedge cw' = cw + 2)$
**end**
**Event** step ⟨ordinary⟩ $\widehat{=}$
**refines** step
**when**
grd1: $k < x$
grd2: $ok = FALSE$
**then**
act4:
$r, k, cv, cw, ok :| (k = x \Rightarrow r' = cv \wedge ok' = TRUE \wedge k' = k \wedge cv' = cv \wedge cw' = cw)$
$\wedge (k < x \Rightarrow r' = r \wedge ok' = ok \wedge k' = k + 1 \wedge cv' = cv + cw + 1 \wedge cw' = cw + 2)$
**end**
**END**

**MACHINE** POWER28
**REFINES** POWER27
**SEES** POWER20
**VARIABLES**

r
k
cv
cw
ok

**EVENTS**
**Initialisation**
**begin**
act1: $r := 0$
act5: $k := 0$
act8: $cv := 0$
act9: $cw := 0$
act10: $ok := FALSE$
**end**
**Event** final ⟨ordinary⟩ $\widehat{=}$
**refines** computing, step
**when**
grd2: $ok = FALSE$
**then**
act1:
$r, k, cv, cw, ok :| (k = x \Rightarrow r' = cv \wedge ok' = TRUE \wedge k' = k \wedge cv' = cv \wedge cw' = cw)$
$\wedge (k < x \Rightarrow r' = r \wedge ok' = ok \wedge k' = k + 1 \wedge cv' = cv + cw + 1 \wedge cw' = cw + 2)$
**end**
**END**