



Modelling Software-based Systems Lecture 4 System Engineering using Refinement-based Methodology

Master Informatique

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General Summary

Refinement of models

- 2 Summary on Event-B
- 3 The Access Control

4 Conclusion

Current Summary

- 1 Refinement of models
- 2 Summary on Event-B
- 3 The Access Control
- **4** Conclusion

Summing up...

- Refinement relates Event-B models
- Problem for starting a refinement-based development
- Problem for finding the best abstract model
- Problem for discharging unproved proof obligations generated for each refinement step
- The Access Control Problem

Current Summary

- Refinement of models
- 2 Summary on Event-B
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- **4** Conclusion

Simple Form of an Event

- An event of the simple form is denoted by :

```
 < event.name > \widehat{=} \\ WHEN \\ < condition > \\ THEN \\ < action > \\ END
```

where

- $< event_name >$ is an identifier
- < condition > is the firing condition of the event
- < action > is a generalized substitution (parallel "assignment")

Non-deterministic Form of an Event

- An event of the non-deterministic form is denoted by :

```
 < event\_name > \stackrel{\frown}{=} \\ \text{ANY} < variable > \text{WHERE} \\ < condition > \\ \text{THEN} \\ < action > \\ \text{END}
```

where

- $< event_name >$ is an identifier
- < variable > is a (list of) variable(s)
- < condition > is the firing condition of the event
- < action > is a generalized substitution (parallel "assignment")

Shape of a Generalized Substitution

```
A generalized substitution can be
```

```
- Simple assignment : x := E
- Generalized assignment : x : P(x, x')
- Set assignment : x :\in S
T
- Parallel composition : \cdots
```

Invariant Preservation Verification (0)

INVARIANT ∧ GUARD ⇒

ACTION establishes INVARIANT

Invariant Preservation Verification (1)

- Given an event of the simple form :

```
\begin{array}{c} \text{EVENT e} \; \cong \\ \text{WHEN} \\ G(x) \\ \text{THEN} \\ x := E(x) \\ \text{END} \end{array}
```

$$| I(x) \wedge G(x) \implies I(E(x))$$

Invariant Preservation Verification (2)

- Given an event of the simple form :

```
EVENT e \widehat{=}
WHEN
G(x)
THEN
x:|P(x,x')
END
```

$$I(x) \wedge G(x) \wedge P(x,x') \implies I(x')$$

Invariant Preservation Verification (3)

- Given an event of the simple form :

```
\begin{array}{c} \text{EVENT e} \; \triangleq \\ \text{WHEN} \\ G(x) \\ \text{THEN} \\ x :\in S(x) \\ \text{END} \end{array}
```

$$I(x) \wedge G(x) \wedge x' \in S(x) \implies I(x')$$

Invariant Preservation Verification (4)

- Given an event of the non-deterministic form :

```
EVENT e \stackrel{\triangle}{=}
ANY v WHERE
G(x, v)
THEN
x := E(x, v)
END
```

$$I(x) \wedge G(x,v) \implies I(E(x,v))$$

Refinement Technique (1)

- Abstract models works with variables \boldsymbol{x} , and concrete one with \boldsymbol{y}
- A gluing invariant J(x,y) links both sets of vrbls
- Each abstract event is refined by concrete one (see below)

Refinement Technique (2)

- Some new events may appear : they refine "skip"
- Concrete events must not block more often than the abstract ones
- The set of new event alone must always block eventually

Correct Refinement Verification (1)

- Given an abstract and a corresponding concrete event

```
EVENT ae \cong WHEN G(x) THEN x := E(x) END
```

```
EVENT ce \stackrel{	o}{=} WHEN H(y) THEN y := F(y) END
```

and invariants I(x) and J(x,y), the statement to prove is :

$$I(x) \wedge J(x,y) \wedge H(y) \implies G(x) \wedge J(E(x),F(y))$$

Correct Refinement Verification (2)

- Given an abstract and a corresponding concrete event

```
EVENT ae \widehat{=}
ANY v WHERE
G(x, v)
THEN
x := E(x, v)
END
```

EVENT ce
$$\triangleq$$
ANY w WHERE
 $H(y, w)$
THEN
 $y := F(y, w)$
END

$$\begin{array}{cccc} I(x) & \wedge & J(x,y) & \wedge & H(y,w) \\ \Longrightarrow & \\ \exists v \cdot (G(x,v) & \wedge & J(E(x,v),F(y,w))) \end{array}$$

Correct Refinement Verification (3)

- Given a NEW event

```
\begin{array}{c} \text{EVENT ce} & \cong \\ \text{WHEN} \\ H(y) \\ \text{THEN} \\ y := F(y) \\ \text{END} \end{array}
```

and invariants I(x) and J(x,y), the statement to prove is :

$$I(x) \wedge J(x,y) \wedge H(y) \implies J(x,F(y))$$

Current Summary

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A Case Study by J.-R. Abrial

- To control accesses into locations.
- People are assigned certain authorizations
- Each person is given a magnetic card
- Doors are "one way" turnstyles
- Each turnstyle is equipped with :
 - a card reader
 - two lights (one green, the other red)



Access Protocol (after introducing card in reader)

```
    If access permitted { - green light turned on - turnstyle unblocked for 30 sec
    Passing, or 30 sec elapsed { - green light turned off - turnstyle blocked again
    If access refused { - red light turned on for 2 sec - turnstyle stays blocked
```

Goal of System Study

- Sharing between Control and Equipment
- For this : constructing a closed model
- Defining the physical environment
- Possible generalization of problem
- Studying safety questions
- Studying synchronisation questions
- Studying marginal behaviour

Basic System Properties

- P1 : The model concerns people and locations
- P2 : A person is authorized to be in some locations
- P3 : A person can only be in one location at a time
- D1 : Outside is a location where everybody can be
- P4: A person is always in some location
- P5 : A person is always authorized to be in his location

Example

Sets

$$\begin{array}{lll} \text{persons} & = & \{ \, \text{p1}, \, \text{p2}, \, \text{p3} \, \} \\ \text{locations} & = & \{ \, \text{l1}, \, \text{l2}, \, \text{l3}, \, \text{l4} \, \} \end{array}$$

Authorizations

p1	12, 14
p2	I1, I3, I4
р3	12, 13, 14

Correct scenario

p1	14		p1	12		p1	12		p1	14		p1	14
p2	14	\rightarrow	p2	14	\rightarrow	p2	l1	\rightarrow	p2	l1	\rightarrow	p2	1
р3	14		р3	14		рЗ	14		рЗ	14		рЗ	13

Model (1)

Basic sets: persons P and locations B (prop. P1) Constant: authorizations A (prop. P2) A is a binary relation between P and B

 $A \in P \leftrightarrow B$

Model (2)

Constant : outside is a location where everybody is authorized to be (decision D1)

$$outside \in B$$

$$P \times \{outside\} \subseteq A$$

Model (3)

Variable: situations c (prop. P3 and P4) c is a total function between P and B A total function is a special case of a binary relation

$$c \in P \to B$$

Invariant : situations compatible with auth. (prop. P5) The function ${\rm C}$ is included in the relation ${\rm A}$

$$\mathbf{C}\subseteq\mathbf{A}$$

A magic event which can be observed

```
- GUARD :  \begin{cases} \text{- Given some person } p \text{ and location } l \\ \text{- } p \text{ is authorized to be in } l: p, l \in \mathbf{A} \\ \text{- } p \text{ is not currently in } l: \mathbf{C}(p) \neq l \end{cases}
```

- ACTION : - p jumps into l

```
\begin{array}{l} \text{EVENT observation1} & \widehat{=} \\ \text{ANY } p, l \text{ WHERE} \\ p \in P & \wedge \\ l \in B & \wedge \\ p \mapsto l \in A & \wedge \\ \text{c}(p) \neq l \\ \text{THEN} \\ \text{c}(p) := l \\ \text{END} \end{array}
```

Relation overriding

Given two relations a and bOverriding a by b yields a new relation $a \lessdot b$

$$a \mathrel{\vartriangleleft} b \quad \widehat{=} \quad (\mathsf{dom}\,(b) \mathrel{\vartriangleleft} a) \ \cup \ b$$

Abbreviation

$$f(x) := y \quad \widehat{=} \quad f := f \mathrel{\vartriangleleft} \{x \mapsto y\}$$

Invariant Preservation Proof

$$\begin{split} \mathbf{C} &\subseteq \mathbf{A} \quad \wedge \\ p \in \mathbf{P} \quad \wedge \\ l \in \mathbf{B} \quad \wedge \\ p \mapsto l \in \mathbf{A} \\ &\Longrightarrow \\ (\{p\} \lhd \mathbf{C}) \cup \{p \mapsto l\} \subseteq \mathbf{A} \end{split}$$

First Refinement : Introducing Geometry

P6: The geometry define how locations communicate

P7: A location does not communicate with itself

P8: Persons move between communicating locations

Refined Model

Constant: communication STRUCTURE (prop. P6 and P7) STRUCTURE is a binary relation between B The intersection of STRUCTURE with the identity relation on B is empty

STRUCTURE
$$\in B \leftrightarrow B$$

STRUCTURE
$$\cap$$
 id(B) = \emptyset

Correct Refinement Verification (reminder)

Concrete events do not block more often than abstract ones

$$I(x) \wedge J(x,y) \wedge$$
 disjunction of abstract guards \Longrightarrow disjunction of concrete guards

New events block eventually (decreasing the same quantity V(y))

$$I(x) \wedge J(x,y) \wedge H(y) \wedge V(y) = n \implies V(F(y)) < n$$

Refined Event

Event (prop. P8) The guard is strengthened The current location of p and the new location l must communicate

```
 \begin{array}{ll} \text{EVENT observation1} & \widehat{=} \\ \text{ANY } p, l \text{ WHERE} \\ p \in P & \wedge \\ l \in B & \wedge \\ p \mapsto l \in A & \wedge \\ \text{C}(p) \neq l \\ \text{THEN} \\ \text{C}(p) := l \\ \text{END} \end{array}
```

Proofs

Invariant preservation : Success Guard strengthening : Success

$$\exists (p,l) \cdot (p \mapsto l \in A \land C(p) \mapsto l \in STRUCTURE) \Rightarrow \\ \exists (p,l) \cdot (p \mapsto l \in A \land C(p) \neq l)$$

Deadlockfreeness: Failure

$$\exists (p, l) \cdot (p \mapsto l \in A \land C(p) \neq l)$$

$$\Rightarrow$$

$$\exists (p, l) \cdot (p \mapsto l \in A \land C(p) \mapsto l \in STRUCTURE)$$

Safety Problem

P9: No person must remain blocked in a location.

Solution

P10 : Any person authorized to be in a location must also be authorized to go in another location which communicates with the first one.

$$A \subseteq A ; STRUCTURE^{-1}$$

$$p \mapsto l \in A \implies \exists m \cdot (p \mapsto m \in A \land l \mapsto m \in STRUCTURE)$$

Example

p1	12	p2	4
p1	14	р3	12
p2	11	рЗ	13
p2	I3	р3	14

11	13	
l1	14	
13	12	
14	l1	
14	12	
14	13	

11	14
12	13
12	14
13	11
13	14
14	1

p1	1	p
p1	I3	p
p1	14	p
p2	11	p

A; STRUCT

Δ

STRUCTURE STRUCTURE⁻¹

- Opening a door between I2 and I4
- Authorizing p2 to go to l2



Solution

p1	12	p2	14
p1	14	рЗ	12
p2	11	рЗ	13
p2	12	рЗ	14
p2	13		

Α

l1	13	
11	14	
12	14	
13	12	
14	l1	
14	12	
14	13	

11	14
12	13
12	14
13	l1
13	14
14	11
14	12

p1	1	p2
p1	12	p2
p1	l3	р3
p1	14	р3
p2	1	р3
p2	12	р3

STRUCTURE S

STRUCTURE⁻¹

A; STRUCTURE

Decision

 $\mathsf{D2}$: The system that we are going to construct does not guarantee that people can move "outside".

A better solution (1)

Constante : exit is a function, included in com, with no cycle

$$exit \in B - \{outside\} \to B$$

$$exit \subseteq com$$

$$\forall s \cdot (s \subseteq B \implies (s \subseteq exit^{-1}[s] \implies s = \emptyset))$$

$$\forall x \cdot (x \in s \implies \exists y \cdot (y \in s \land (x, y) \in exit))$$

$$\Longrightarrow$$

$$s = \emptyset$$

exit is a tree spanning the graph represented by com

A better solution (2)

P10': Every person authorized to be in a location (which is not "outside") must also be authorized to be in another location communicating with the former and leading towards the exit.

$$A \Rightarrow \{outside\} \subseteq A ; exit^{-1}$$

$$p \mapsto l \in A \land$$

$$l \neq outside$$

$$\Rightarrow$$

$$p \mapsto exit(l) \in A$$

For the experts

Show that no cycle implies the possibility to prove property by induction and vice-versa

$$\forall s \cdot (s \subseteq B \land s \subseteq exit^{-1}[s] \implies s = \emptyset)$$

$$\Leftrightarrow \forall t \cdot (t \subseteq B \land outside \in t \land exit^{-1}[t] \subseteq t \implies t = B)$$

$$t \subseteq \mathbf{B}$$

$$outside \in t$$

$$\forall (x, y) \cdot ((x \mapsto y) \in exit \land y \in t \implies x \in t)$$

$$\Longrightarrow$$

$$t = \mathbf{B}$$

Second Refinement : Introducing Doors

- P11 : Locations communicate via one-way doors.
- P12: A person get through a door only if accepted.
- P13: A door is acceptable by at most one person at a time.
- P14: A person is accepted for at most one door only.
- P15 : A person is accepted if at the origin of the door.
- P16 : A person is accepted if authorized at destination.

Extending the Model (1)

Set : the set ${\tt DOORS}$ of doors Constants : The origin ${\tt ORG}$ and destination ${\tt DST}$ of a door (prop. P11)

$$\begin{split} & \text{ORG} \in \text{doors} \to B \\ & \text{DST} \in \text{doors} \to B \\ & \text{structure} = (\text{ORG}^{-1} \;; \text{DST}) \end{split}$$

Extending the Model (2)

Variable: the rel. DAP between persons and doors (prop. P12 to P16)

$$\begin{array}{l} \text{DAP} \in P \rightarrowtail \text{DOORS} \\ (\text{DAP} \ ; \text{ORG}) \subseteq C \\ (\text{DAP} \ ; \text{DST}) \subseteq A \end{array}$$

Second Refinement : More Properties

P17: Green light of a door is lit when access is accepted. P18: When a person has got through, the door blocks. P19: After 30 seconds, the door blocks automatically. P20: Red light is lit for 2 sec.when access is refused. P21: Red and green lights are not lit simultaneously.

Extending the Model (3)

Definition: GREEN is exactly the range of DAP (prop. P17 to P19)

GREEN
$$\hat{=}$$
 ran (DAP)

Extending the Model (4)

Variable: The set red of red doors (prop. P20)

$$red \subseteq doors$$

Invariant: GREEN and red are incompatible (prop. P21)

GREEN
$$\cap red = \emptyset$$

Condition for Admission

- $\mathsf{P22}:\mathsf{Person}\ p$ is accepted through door d if
 - p is situated within the origin of d
 - p is authorized to move to the dest. of d
 - p is not engaged with another door

```
\begin{array}{l} \mathsf{admitted}\ (p,d) \ \ \widehat{=} \\ \mathrm{ORG}(d) = \mathrm{C}(p) \ \ \land \\ p \mapsto \mathrm{DST}(d) \in \mathrm{A} \ \ \land \\ p \not \in \mathsf{dom}\ (dap) \end{array}
```

A New Event (1)

```
EVENT accept \stackrel{\frown}{=}
ANY p, d WHERE
p \in P \land d \in \text{DOORS} \land d \notin \text{GREEN} \cup \textit{red} \land \text{admitted}(p, d)
THEN
\text{DAP}(p) := d
END
```

A New Event (2)

```
\begin{array}{l} \text{EVENT refuse} & \cong \\ \text{ANY } p, d \text{ WHERE} \\ p \in P \land \\ d \in \text{DOORS} \land \\ d \notin \text{GREEN} \cup \textcolor{red}{red} \land \\ \neg \text{ admitted} \ (p, d) \\ \text{THEN} \\ \textcolor{blue}{red} := \textcolor{blue}{red} \cup \{d\} \\ \text{END} \end{array}
```

Refining Event OBSERVATION2

```
\begin{array}{l} \text{EVENT observation2} \\ \text{ANY } p, l \text{ WHERE} \\ p \in P \\ l \in B \\ p, l \in A \\ c(p) \mapsto l \in \text{STRUCTURE} \\ \text{THEN} \\ c(p) := l \\ \text{END} \end{array}
```

```
EVENT observation3 \widehat{=}
REFINES observation2
ANY d WHERE
d \in GREEN
THEN
C(DAP^{-1}(d)) := DST(d)
DAP := DAP \triangleright \{d\}
END
```

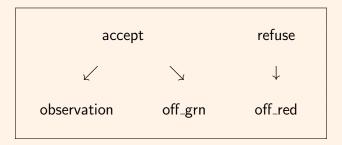
New Event (3)

Turning lights off

```
\begin{array}{l} \text{EVENT off.grn} \ \ \cong \\ \text{ANY } d \text{ WHERE} \\ d \in \text{GREEN} \\ \text{THEN} \\ \text{DAP} := \text{DAP} \bowtie \{d\} \\ \text{END} \end{array}
```

```
\begin{array}{l} \text{EVENT off\_red} \;\; \widehat{=} \\ \text{ANY } d \, \text{WHERE} \\ d \in red \\ \text{THEN} \\ red := red - \{d\} \\ \text{END} \end{array}
```

Synchronization



Proofs

- Event observation is a correct refinement : OK
- Other events refine skip : OK
- Event observation does not deadlock more: OK
- New events do not take control indefinitely : FAILURE

Permanent Obstruction of Card Readers

DANGER

- People without the required authorizations try indefinitely to enter some locations.
- Other people with the authorization always change mind at the last moment.

SOLUTIONS

- Make such practice impossible???
- Card Readers can "swallow" a card

Final Decision

- D3 : The system we are going to construct will not prevent people from blocking doors indefinitely :
 - either by trying indefinitely to enter places into which they are not authorized to enter,
 - or by indefinitely abandoning "on the way" their intention to enter the places in which they are in fact authorized to enter.

Third Refinement : Introducing Card Readers

A decision

D4 : Each card reader is supposed to stay blocked between :

- the sending of a card to the system
- the reception of an acknowledgement.

Third Refinement: Model Extension

The set BLR of blocked Card Readers The set mCard of messages sent by Card Readers The set mAckn of acknowledgment messages

$$BLR \subseteq DOORS$$

 $mCard \in DOORS \leftrightarrow P$
 $mAckn \subseteq DOORS$

Third Refinement: Invariant

dom(mCard), GREEN, red, mAckn partition BLR

 $\operatorname{dom}(mCard) \cup \operatorname{GREEN} \cup \operatorname{red} \cup \operatorname{mAckn} = \operatorname{BLR}$ $\operatorname{dom}(mCard) \cap (\operatorname{GREEN} \cup \operatorname{red} \cup \operatorname{mAckn}) = \varnothing$ $\operatorname{mAckn} \cap (\operatorname{GREEN} \cup \operatorname{red}) = \varnothing$

Events (1)

```
\begin{array}{l} \text{EVENT CARD} & \stackrel{\triangle}{=} \\ \text{ANY } p, d \\ \text{WHERE} \\ p \in P \\ d \in \text{DOORS} - BLR \\ \text{THEN} \\ BLR := BLR \cup \{d\} \\ mCard := mCard \cup \{d \mapsto p\} \\ \text{END} \end{array}
```

Events (2)

```
\begin{array}{l} \text{EVENT accept3} & \widehat{=} \\ \text{ANY } p, d \\ \text{WHERE} \\ p \in P \\ d \in \text{DOORS} \\ d \notin \text{GREEN} \cup \textit{red} \\ \text{admitted} (p, d) \\ \text{THEN} \\ \text{DAP}(p) := d \\ \text{END} \end{array}
```

```
\begin{array}{l} \text{EVENT accept4} \;\; \widehat{=} \\ \text{REFINES} \;\; \text{accept3} \\ \text{ANY} \; p, d \\ \text{WHERE} \\ d \mapsto p \in mCard \\ \text{admitted} \; (p, d) \\ \text{THEN} \\ \text{DAP}(p) \; := \; d \\ mCard \; := \; mCard - \{d \mapsto p\} \\ \text{END} \end{array}
```

Events (3)

```
\begin{array}{l} \text{EVENT refuse4} & \widehat{=} \\ \text{REFINES} & \text{refuse3} \\ \text{ANY } p, d \\ \text{WHERE} & d \mapsto p \in mCard \\ \neg \text{ admitted } (p, d) \\ \text{THEN} & red := red \cup \{d\} \\ mCard := mCard - \{d \mapsto p\} \\ \text{END} & \end{array}
```

Events (4)

```
EVENT observation4 \widehat{=}
REFINES observation3
ANY d
WHERE
d \in \text{GREEN}
THEN
C(\text{DAP}^{-1}(d)) := \text{DST}(d)
\text{DAP} := \text{DAP} \triangleright \{d\}
mAckn := mAckn \cup \{d\}
END
```

Events (5)

```
\begin{array}{l} \text{EVENT off\_grn} & \cong \\ \text{ANY } d \text{ WHERE} \\ d \in \text{GREEN} \\ \text{THEN} \\ \text{DAP} := \text{DAP} \bowtie \{d\} \\ mAckn := mAckn \cup \{d\} \\ \text{END} \end{array}
```

```
\begin{array}{l} \text{EVENT off.red} & \cong \\ \text{ANY } d \text{ WHERE} \\ d \in red \\ \text{THEN} \\ red := red - \{d\} \\ mAckn := mAckn \cup \{d\} \\ \text{END} \end{array}
```

Events (6)

```
\begin{array}{l} \text{EVENT ACKN} & \widehat{=} \\ \text{ANY } d \text{ WHERE} \\ d \in mAckn \\ \text{THEN} \\ BLR := BLR - \{d\} \\ mAckn := mAckn - \{d\} \\ \text{END} \end{array}
```

Synchronization



Fourth Refinement : Physical Doors and Lights

Decisions

D5 : When a door has been cleared, it blocks itself automatically

without any intervention from the control system.

D6: Each door incorporates a local clock for

- the extinction of the green light after 30 sec.
- the extinction of the red light after 2 sec.

Extending the Model : the Green Chain (1)

The set mAccept of acceptance messages (to doors) The set GRN of physical green doors The set mPass of passing messages (from doors) The set $mOff_grn$ of messages (from doors)

$$mAccept \subseteq doors$$
 $GRN \subseteq doors$ $mPass \subseteq doors$ $mOff_grn \subseteq doors$

Extending the Model: the Green Chain (2)

mAccept, GRN, mPass, mOff_grn partition grn

$$mAccept \cup GRN \cup mPass \cup mOff_grn = grn$$

 $mAccept \cap (GRN \cup mPass \cup mOff_grn) = \varnothing$
 $GRN \cap (mPass \cup mOff_grn) = \varnothing$
 $mPass \cap mOff_grn = \varnothing$

Extending the Model: the Red Chain (1)

The set mRefuse of messages (to doors) The set RED of phyical red doors The set $mOff_red$ of messages (from doors)

 $mRefuse \subseteq doors$ $RED \subseteq doors$ $mOff_red \subseteq doors$

Extending the Model: the Red Chain (2)

 $mRefuse, RED, mOff_red$ partition red

$$mRefuse \cup RED \cup mOff_red = red$$
 $mRefuse \cap (RED \cup mOff_red) = \varnothing$
 $RED \cap mOff_red = \varnothing$

Events (1)

```
\begin{array}{l} \text{EVENT accept} & \widehat{=} \\ & \text{ANY } p, d \text{ WHERE} \\ & d, p \in mCard \ \land \\ & \text{admitted } (p, d) \\ & \text{THEN} \\ & \text{DAP}(p) := d \\ & mCard := mCard - \{d \mapsto p\} \\ & mAccept := mAccept \cup \{d\} \\ & \text{END} \end{array}
```

Events (2)

```
\begin{array}{l} \text{EVENT ACCEPT} & \widehat{=} \\ \text{ANY } d \text{ WHERE} \\ d \in mAccept \\ \text{THEN} \\ GRN := GRN \cup \{d\} \\ mAccept := mAccept - \{d\} \\ \text{END} \end{array}
```

Events (3)

```
\begin{array}{l} \text{EVENT PASS} & \cong \\ \text{ANY } d \text{ WHERE} \\ d \in GRN \\ \text{THEN} \\ GRN := GRN - \{d\} \\ mPass := mPass \cup \{d\} \\ \text{END} \end{array}
```

Events (4)

```
 \begin{array}{ll} \text{EVENT observation5} & \cong \\ \text{REFINES observation4} & \text{ANY } d \text{ WHERE} \\ d \in mPass \\ \hline \text{THEN} \\ & \text{C}(\text{DAP}^{-1}(d)) := \text{DST}(d) \\ & \text{DAP} := \text{DAP} \Rightarrow \{d\} \\ & mAckn := mAckn \ \cup \ \{d\} \\ & mPass := mPass - \{d\} \\ \hline \text{END} \\ \end{array}
```

Events (5)

```
\begin{array}{l} \text{EVENT OFF\_GRN} & \cong \\ \text{ANY } d \text{ WHERE} \\ d \in GRN \\ \text{THEN} \\ GRN := GRN - \{d\} \\ mOff\_grn := mOff\_grn \cup \{d\} \\ \text{END} \end{array}
```

Events (6)

```
\begin{array}{l} \text{EVENT off\_grn} & \cong \\ & \text{ANY } d \text{ WHERE} \\ & d \in mOff\_grn \\ & \text{THEN} \\ & \text{DAP} := \text{DAP} \rhd \{d\} \\ & mAckn := mAckn \cup \{d\} \\ & mOff\_grn := mOff\_grn - \{d\} \\ & \text{END} \end{array}
```

Events (7)

```
\begin{array}{l} \text{EVENT refuse} & \widehat{=} \\ \text{ANY } p, d \text{ WHERE} \\ d, p \in mCard \quad \land \\ \neg \text{ admitted} \left(p, d\right) \\ \text{THEN} \\ red := red \cup \left\{d\right\} \\ mCard := mCard - \left\{d \mapsto p\right\} \\ mRefuse := mRefuse \cup \left\{d\right\} \\ \text{END} \end{array}
```

Events (8)

```
EVENT REFUSE \triangleq
ANY d WHERE
d \in mRefuse
THEN
RED := RED \cup \{d\}
mRefuse := mRefuse - \{d\}
END
```

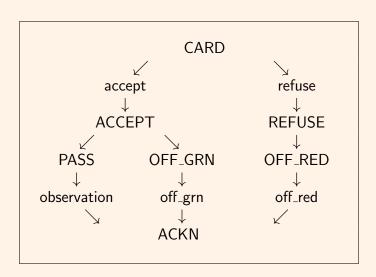
Events (9)

```
 \begin{array}{ll} \text{EVENT OFF\_RED} & \cong \\ & \text{ANY } d \text{ WHERE} \\ & d \in RED \\ & \text{THEN} \\ & RED := RED - \{d\} \\ & \textit{mOff\_red} := \textit{mOff\_red} \cup \{d\} \\ & \text{END} \end{array}
```

Events (10)

```
\begin{array}{l} \text{EVENT off\_red} & \cong \\ \text{ANY } d \text{ WHERE} \\ d \in mOff\_red \\ \text{THEN} \\ red := red - \{d\} \\ mAckn := mAckn \cup \{d\} \\ mOff\_red := mOff\_red - \{d\} \\ \text{END} \end{array}
```

Synchronization



Communications

Hardware		Network		Software
CARD	\rightarrow	mCard	\rightarrow	accept (1) refuse (2)
ACCEPT	←	mAccept	\leftarrow	(1)
PASS	\rightarrow	mPass	\rightarrow	observation (3)
OFF_GRN	\rightarrow	$mOff_grn$	\rightarrow	off_grn (3)
REFUSE	←	mRefuse	\leftarrow	(2)
OFF_RED	\rightarrow	$mOff_red$	\rightarrow	off_red (3)
ACKN	←	mAckn	\leftarrow	(3)

Decomposition (1)

Software Data

 $aut \in P \leftrightarrow B$ $ORG \in doors \rightarrow B$ $DST \in doors \rightarrow B$ $A \subseteq A; DST^{-1}; ORG$ $C \in P \rightarrow B$

Decomposition (2)

Network data

$$mCard \in doors \Rightarrow P$$
 $mAckn \subseteq doors$
 $mAccept \subseteq doors$
 $mPass \subseteq doors$
 $mOff_grn \subseteq doors$
 $mRefuse \subseteq doors$
 $mOff_red \subseteq doors$

Decomposition (3)

"Physical" Data

 $BLR \subseteq DOORS$

 $GRN \subseteq DOORS$

 $RED \subseteq DOORS$

Software Operations

EVENT test_soft(p, d)

 ${
m EVENT}$ accept_soft(p,d)

 ${
m EVENT}$ refuse_soft(d)

 $EVENT pass_soft(d)$

 $EVENT off_grn_soft(d)$

EVENT off_red_soft(d)

Physical Operations

 $(p,d) \longleftarrow \mathsf{CARD}_\mathsf{-}\mathsf{HARD}$

 $\mathsf{ACCEPT_HARD}(d)$

 $\mathsf{REFUSE_HARD}(d)$

 $d \longleftarrow \mathsf{PASS_HARD}$

 $d \longleftarrow \mathsf{OFF_GRN_HARD}$

 $d \longleftarrow \mathsf{OFF_RED_HARD}$

 $ACKN_HARD(d)$

Network Software Operations

 $(p,d) \longleftarrow \mathsf{read_card}$ $write_accept(d)$ $write_refuse(d)$ $d \leftarrow \mathsf{read_pass}$ $d \leftarrow \mathsf{read_off_grn}$ $d \leftarrow \text{read_off_red}$ $write_ackn(d)$

Network Physical Operations

 $SEND_CARD(p, d)$

 $\mathsf{d} \longleftarrow \mathsf{RCV}_\mathsf{ACCEPT}$

 $SEND_PASS(d)$

 $SEND_OFF_GRN(d)$

 $SEND_OFF_RED(d)$

 $d \leftarrow RCV_ACKN$

```
 \begin{array}{l} \text{EVENT CARD} & \widehat{=} \\ \text{VAR } p, d \text{ IN} \\ (p, d) \longleftarrow \text{READ\_HARD}; \\ \text{SEND\_CARD}(p, d) \\ \text{END} \end{array}
```

 $\begin{array}{l} \text{EVENT ACCEPT} & \widehat{=} \\ \text{VAR } d \text{ IN} \\ d \longleftarrow \text{RCV_ACCEPT}; \\ \text{ACCEPT_HARD}(d) \\ \text{END} \end{array}$

 $\begin{array}{ll} \text{EVENT REFUSE} & \widehat{=} \\ \text{VAR } d \text{ IN} \\ \text{d} & \longleftarrow \text{RCV_REFUSE}; \\ \text{REFUSE_HARD}(q) \\ \text{END} \end{array}$

 $\begin{array}{l} \text{EVENT PASS} & \widehat{=} \\ \text{VAR } d \text{ IN} \\ d \longleftarrow \text{PASS_HARD}; \\ \text{SEND_PASS}(d) \\ \text{END} \end{array}$

 $\begin{array}{l} \text{EVENT OFF_GRN} & \widehat{=} \\ \text{VAR } d \text{ IN} \\ d \longleftarrow \text{OFF_GRN_HARD}; \\ \text{SEND_OFF_GRN}(d) \\ \text{END} \end{array}$

 $\begin{array}{l} \text{EVENT OFF_RED} & \widehat{=} \\ \text{VAR } d \text{ IN} \\ d \longleftarrow \text{OFF_RED_HARD}; \\ \text{SEND_OFF_RED}(d) \\ \text{END} \end{array}$

$$\begin{split} & \text{EVENT observation } \widehat{=} \\ & \text{VAR } d \text{ IN} \\ & d \leftarrow \text{read_pass;} \\ & \text{EVENT pass_soft}(d); \\ & \text{write_ackn}(d) \\ & \text{END} \end{split}$$

 $\begin{array}{l} \text{EVENT off_grn} & \widehat{=} \\ \textbf{VAR} \ d \ \textbf{IN} \\ d \leftarrow \text{read_off_grn}; \\ \text{EVENT off_grn_soft}(d); \\ \text{write_ackn}(d) \\ \text{END} \end{array}$

 $\begin{array}{l} {\rm EVENT~off.red} & \widehat{=} \\ {\rm VAR}~d~{\rm IN} \\ d & \longleftarrow {\rm read.off.red}; \\ {\rm EVENT~off.red.soft}(d); \\ {\rm write.ackn}(d) \\ {\rm END} \end{array}$

 $\begin{array}{ll} \text{EVENT ACKN} & \widehat{=} \\ \text{VAR } d \text{ IN} \\ \text{d} & \longleftarrow \text{RCV_ACKN}; \\ \text{ACKN_HARD}(d) \\ \text{END} \end{array}$

Conclusion

22 Properties et 6 "System" Decisions - One Problem Generalization

- Access between locations
- One Negative Choice :
- Possible Card Readers Obstructions
- Three Physical Decisions
- Automatic Blocking of Doors
- Automatic Blocking of Card Readers
- Setting up of Clocks on Doors
- The overall development required 183 proofs
- 147 automatic (80%)
- 36 interactive

Current Summary

- Refinement of models
- 2 Summary on Event-B
- 3 The Access Control
- **4** Conclusion

Conclusion

- Identify an abstract model
- Identify constants and states
- Identify components
- Plan the refinement
- Start as long as the model is not well defined!