



# Cours MALG & MOVEX

## Vérification d'une annotation

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$$\begin{array}{l} \ell_1 : x = 3 \ \land \ y = z + x \ \land z = 2 \cdot x \\ y := z + x \\ \ell_2 : x = 3 \ \land \ y = x + 6 \end{array}$$

#### On définit un contrat comme suit :

variables x, y, z 
$$\begin{array}{l} \text{requires } x0=3 \wedge y0=z0+x0 \wedge z0=2.x0 \\ \text{ensures } x_f=3 \wedge y_f=x_f+6 \\ \\ \begin{bmatrix} \text{begin} \\ \ell_1: x=3 \ \wedge \ y=z+x \ \wedge z=2\cdot x \\ y:=z+x \\ \ell_2: x=3 \ \wedge \ y=x+6 \\ \text{end} \\ \end{array}$$

## On pose les assertions suivantes à partir de l'annotation :

- $ightharpoonup pre(x_0, y_0, z_0) \stackrel{def}{=} x0 = 3 \land y0 = z0 + x0 \land z0 = 2.x0$
- $ightharpoonup prepost(x_0, y_0, z_0, x, y, z) \stackrel{def}{=} x = 3 \land y = x + 6$
- $ightharpoonup Q_1(x_0, y_0, z_0, x, y, z) \stackrel{def}{=} x = 3 \land y = z + x \land z = 2 \cdot x$
- $Q_2(x_0, y_0, z_0, x, y, z) \stackrel{def}{=} x = 3 \land y = x + 6$

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#### On établit les trois cpnditions pour valider le contrat :

- ightharpoonup (init)  $pre(x_0, y_0, z_0) \land (x, y, z) = (x_0, y_0, z_0) \Rightarrow Q_1(x_0, y_0, z_0, x, y, z)$
- ► (concl)  $pre(v_0) \land Q_2(x_0, y_0, z_0, x, y, z) \Rightarrow prepost(x_0, y_0, z_0, x, y, z)$
- (induct)  $pre(x_0, y_0, z_0) \wedge Q_1(x_0, y_0, z_0, x, y, z) \wedge TRUE \wedge (x', y', z') = (x, z+x, z) \Rightarrow Q_2(x_0, y_0, z_0, x', y', z')$

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  - x0 = 3, y0 = z0+x0, z0 = 2.x0, x = 3, y = z+x, z = 0
  - $2 \cdot x, TRUE, (x', y', z') = (x, z + x, z) \vdash x' = 3$

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  - x = 3 est une hypothèse à gauche. Le séquent est valide.

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  - x0 = 3, y0 = z0+x0,  $z0 = (x, z+x, z) \vdash z \cdot x+x = x+c$
  - $2 \cdot x \cdot 0 = 3, y \cdot 0 = 20 + x \cdot 0, z \cdot 0 = 2 \cdot x \cdot 0, x = 3, y = z + x, z = 2 \cdot x, TRUE, (x', y', z') = (x, z + x, z) \vdash 2.3 + 3 = 3 + 6$

  - $2 \cdot x, TRUE, (x', y', z') = (x, z + x, z) \vdash 9 = 9$

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    - $2 \cdot x, TRUE, (x', y', z') = (x, z + x, z) \vdash 2.3 + 3 = 3 + 6$
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  - Réflexivité de l'égalité.