Modelling Software-based Systems
Lecture 5 Checking contracts with Event-B

Master Informatique

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General Summary

❶ Programming by contract

❷ Verification

❸ Floyd to Hoare

Current Summary

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Verifying program correctness

A program P *satisfies* a (pre,post) contract :

- P transforms a variable v from initial values v_0 and produces a final value v_f : $v_0 \xrightarrow{\text{P}} v_f$
- v_0 satisfies pre : $\text{pre}(v_0)$ and v_f satisfies post : $\text{post}(v_0, v_f)$
- $\text{pre}(v_0) \land v_0 \xrightarrow{\text{P}} v_f \Rightarrow \text{post}(v_0, v_f)$
- $\mathbb{D}$ est le domaine RTE de V

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```
requires pre(v_0)
ensures post(v_0, v_f)
variables X
        ⌐ begin
        │ 0 : P_0(v_0, v)
        │ instruction_0
        │ . . .
        │ i : P_i(v_0, v)
        │ . . .
        │ instruction_{f-1}
        │ f : P_f(v_0, v)
        └ end
```

- $pre(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$
- $pre(v_0) \wedge P_f(v_0, v) \Rightarrow post(v_0, v)$
- For any pair of labels ℓ, ℓ' such that $\ell \longrightarrow \ell'$, one verifies that, pour any values $v, v' \in \text{MEMORY}$

$$\left(\begin{array}{c} \left(\begin{array}{c} pre(v_0) \wedge P_\ell(v_0, v)) \\ \wedge cond_{\ell,\ell'}(v) \wedge v' = f_{\ell,\ell'}(v) \end{array} \right) \\ \Rightarrow P_{\ell'}(v_0, v') \end{array} \right),$$

Contracts - Verification Conditions

```
contract P
variables v
requires $pre(v_0)$
ensures $post(v_0, v_f)$
        ┌ begin
        │ $0 : P_0(v_0, v)$
        │ $S_0$
        │ . . .
        │ $i : P_i(v_0, v)$
        │ . . .
        │ $S_{f-1}$
        │ $f : P_f(v_0, v)$
        └ end
```

Verification conditions are listed as follows :

```
contract P
variables v
requires pre(v_0)
ensures post(v_0, v_f)
        ┌ begin
        │ 0 : P_0(v, v)
        │ S_0
        │ . . .
        │ i : P_i(v_0, v)
        │ . . .
        │ S_{f-1}
        │ f : P_f(v_0, v)
        └ end
```

- (initialisation)
  $pre(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$

- (finalisation)
  $pre(v_0) \wedge P_f(v_0, v) \Rightarrow post(v_0, v)$

- (induction)
  For each labels pair ℓ, ℓ'
  such that $\ell \longrightarrow \ell'$, one checks that,
  for any value $v, v' \in \text{MEMORY}$
  $$\left(\begin{pmatrix} pre(v_0) \wedge P_\ell(v_0, v)) \\ \wedge cond_{\ell, \ell'}(v) \wedge v' = f_{\ell, \ell'}(v) \end{pmatrix} \Rightarrow P_{\ell'}(v_0, v') \right),$$

Three kinds of verification conditions should be checked and we justify the method in the full version..

From PAP to Rodin . . .

```
MACHINE M
SEES   C0
VARIABLES
   v, pc
INVARIANTS
   typing : v ∈ D
   control : pc ∈ L
   . . .
   atℓ : pc = ℓ ⇒ Pℓ(v0, v)
   . . .
th1 : pre(v0) ∧ v = v0 ⇒ P0(v0, v)
th2 : pre(v0) ∧ Pf(v0, v)
              ⇒ post(v0, v)
. . .
END
. . .
END
```

$$\text{atℓ} : pc = \ell \Rightarrow P_\ell(v0, v)$$

$$th1 : pre(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$$

$$th2 : pre(v_0) \wedge P_f(v_0, v) \Rightarrow post(v_0, v)$$

From PAP to Rodin . . .

```
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              ⇒ post(v0, v)
. . .
END
. . .
END
```

$$\text{atℓ} : pc = \ell \Rightarrow P_\ell(v0, v)$$

$$th1 : pre(v_0) \land v = v_0 \Rightarrow P_0(v_0, v)$$

$$th2 : pre(v_0) \land P_f(v_0, v) \Rightarrow post(v_0, v)$$

```
MACHINE M
EVENTS
INITIALISATION
BEGIN
(pc, v) : | ( pc' = l0 ∧ v' = v0
             ∧pre(v0) )
END
. . .
e(ℓ, ℓ')
   WHEN
      pc = ℓ
      condℓ,ℓ'(v)
   THEN
      pc := ℓ'
      v := fℓ,ℓ'(v)
   END
. . .
END
```

$$(pc, v) : \left| \left(\begin{array}{l} pc' = l0 \land v' = v0 \\ \land pre(v0) \end{array} \right) \right.$$

$$e(\ell, \ell')$$

$$pc = \ell$$
$$cond_{\ell,\ell'}(v)$$
$$pc := \ell'$$
$$v := f_{\ell,\ell'}(v)$$

Technical problems for students

(Induction Principle (I))

A property $S(z0, z)$ is a safety for an annotated program P if, and only if, there exists a property $I(z0, z)$ satisfying :

1. $\forall z0, z \in \mathsf{L} \times \mathsf{D}.init(z0) \wedge z = z0 \Rightarrow I(z0, z)$
2. $\forall z0, z, z' \in \mathsf{L} \times \mathsf{D}.init(z0) \wedge I(z0, z) \wedge (z \xrightarrow[P]{} z') \Rightarrow I(z0, z')$
3. $\forall z0, z \in \mathsf{L} \times \mathsf{D}.init(z0) \wedge I(z0, z) \Rightarrow S(z0, z)$

(Induction Principle (II))

A property $S(\ell0, x0, \ell, x)$ is a safety property for an annotated program P if, and only if, there exists a property $I(\ell0, x0, \ell, x)$ satisfying :

1. $\forall \ell0, \in \mathsf{L}, x0 \in \mathsf{D}.\ell0 \in \mathsf{L0} \wedge pre(x0) \wedge x = x0 \wedge pc = \ell0 \Rightarrow J(\ell0, x0, \ell, x)$
2. $\forall \ell, \ell' \in \mathsf{L}, x, x0 \in \mathsf{D}.\ell0 \in \mathsf{L0} \wedge pre(x0) \wedge J(\ell0, x0, \ell, x) \wedge BA(e(\ell, \ell'),)(\ell, x, \ell', x') \Rightarrow J(\ell0, x0, \ell', x')$
3. $\forall \ell0, \ell \in \mathsf{L}, x0, x \in \mathsf{D}.pre(x0) \wedge \ell0 \in \mathsf{L0} \wedge J(\ell0, x0, \ell, x) \Rightarrow S(\ell0, x0, \ell, x)$

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1. $\forall \ell0, \in \mathsf{L}, x0 \in \mathsf{D}.\ell0 \in \mathsf{L0} \wedge pre(x0) \wedge x = x0 \wedge pc = \ell0 \Rightarrow J(\ell0, x0, \ell, x)$
2. $\forall \ell, \ell' \in \mathsf{L}, x, x0 \in \mathsf{D}.\ell0 \in \mathsf{L0} \wedge pre(x0) \wedge J(\ell0, x0, \ell, x) \wedge BA(e(\ell, \ell'),)(\ell, x, \ell', x') \Rightarrow J(\ell0, x0, \ell', x')$
3. $\forall \ell0, \ell \in \mathsf{L}, x0, x \in \mathsf{D}.pre(x0) \wedge \ell0 \in \mathsf{L0} \wedge J(\ell0, x0, \ell, x) \Rightarrow S(\ell0, x0, \ell, x)$

(Induction Principle (III))

A property $S(x0, \ell, x)$ is a safety for an annotated program P with one entry point if, and only if, there exists a property $I(x0, \ell, x)$ satisfying :

1. $\forall x0 \in \mathsf{D}.pre(x0) \wedge x = x0 \wedge \ell = \ell0 \Rightarrow J(x0, \ell, x)$
2. $\forall \ell, \ell' \in \mathsf{L}, x, x0 \in \mathsf{D}.pre(x0) \wedge J(x0, \ell, x) \wedge BA(e(\ell, \ell'),)(\ell, x, \ell', x') \Rightarrow J(x0, \ell', x')$
3. $\forall \ell \in \mathsf{L}, x0, x \in \mathsf{D}.pre(x0) \wedge J(x0, \ell, x) \Rightarrow S(x0, \ell, x)$

Soundness of the translation

(Soundness of the method)

If the initialisation init, the generalisation gen and the step induction are proved to be correct by the Rodin platform, the property $S(x0, \ell, x)$ is a correct safety property for the program P. In particular, one can handle the partial correctness and the run time error safety properties.

(Soundness of the method)

If the initialisation init, the generalisation gen and the step induction are proved to be correct by the Rodin platform, the property $S(x0, \ell, x)$ is a correct safety property for the program P. In particular, one can handle the partial correctness and the run time error safety properties.

- Contract and verification conditions are translated into Event-B and are discharged by Rodin and its provers.
- Verification conditions are derived from Floyd's method.
- Annotation as assertion

A short example

s

contract SIMPLE
variables x
requires $x_0 \in \mathbb{N}$
ensures $x_f = 0$
begin
$\ell_0 : \{0 \le x \le x_0 \land x_0 \in \mathbb{N}\}$
while $0 < $ x **do**
$\quad \ell_1 : \{0 < x \land x \le x_0 \land x_0 \in \mathbb{N}\}$
$\quad$ x := x − 1;
od
$\ell_2 : \{x = 0\}$end

INVARIANTS
$inv1 : x \in \mathbb{N}$
$inv2 : l \in L$
$inv3 : l = l0 \Rightarrow$
$0 \le x \land x \le x0 \land x0 \in \mathbb{N}$
$inv4 : l = l1 \Rightarrow$
$0 < x \land x \le x0 \land x0 \in \mathbb{N}$
$inv5 : l = l2 \Rightarrow x = 0$
$requires : x0 \in \mathbb{N} \land x = x0$
$\Rightarrow x = x0 \land x0 \in \mathbb{N}$
$ensures : x = 0 \land x = x0$
$\Rightarrow x = 0$

Event $el0l2$
WHEN
$\quad grd1 : l = l0$
$\quad grd2 : \neg(0 < x)$
THEN
$\quad act1 : l := l2$

Event $Init$
THEN
$\quad act1 : x := x0$
$\quad act2 : l := l0$

Event $el0l1$
WHEN
$\quad grd1 : l = l0$
$\quad grd2 : 0 < x$
THEN
$\quad act1 : l := l1$

Event $el1l0$
WHEN
$\quad grd1 : l = l1$
THEN
$\quad act1 : l := l0$
$\quad act2 : x := x − 1$

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Summary

Annotation of programs

$$\ell : \{P_\ell(v)\}$$
$$cond_{\ell,\ell'}(v) \longrightarrow v := f_{\ell,\ell'}(v)$$
$$\ell' : \{P_{\ell'}(v)\}$$

$$\ell_0^1 : \{x = 0\}$$
$$x := x + 1;$$
$$\ell_0^1 : \{x = 1\}$$

$e(\ell, \ell')$
  WHEN
    $c = \ell$
    $cond_{\ell,\ell'}(v)$
  THEN
    $c := \ell'$
    $v := f_{\ell,\ell'}(v)$
  END

- v is the state meory variable or list of memory variables; v includes the local variables and the results variables.

- c is a new variable which is modelling the control flow and its type is LOCATIONS.

- $e(\ell, \ell')$ is simulating the computation flow starting from ℓ and moving to ℓ'; v is updated.

From annotations to invariants

INVARIANTS
    $inv_i : c \in \text{LOCATIONS}$
    $inv_j : v \in Type$
. . .
    $inv_k : c = \ell \Rightarrow P_\ell(v)$
    $inv_m : c = \ell' \Rightarrow P_{\ell'}(v)$
. . .
    $th_n : A(c, v)$

- $Type$ is the type of the variables v and is a set of possible values defined in the context C.

- The annotation is giving us for free the conditions satisfied by v when the control is in ℓ, (resp. in ℓ').

- $A(c, v)$ is a safety property that we are supposed to check and the case of Event-B, it is a theorem.

Partial correctness using Event-B models

For each pair of successive labels ℓ, ℓ', the three statements are equivalent :

- $P_\ell(v) \land cond_{\ell,\ell'}(v) \land v' = f_{\ell,\ell'}(v) \Rightarrow P_{\ell'}(v')$
- $I(c,v) \land c = \ell \land cond_{\ell,\ell'}(v) \land c' = \ell' \land v' = f_{\ell,\ell'}(v) \Rightarrow (c' = \ell' \Rightarrow P_{\ell'}(v'))$
- $I(c,v) \land BA(e(\ell,\ell'))(c,v,c',v') \Rightarrow (c' = \ell' \Rightarrow P_{\ell'}(v'))$

L

et AA an annotated algorithm with precondition **pre**$(AA)(v)$ and postcondition **post**$(AA)(v_0,v)$. Let the context C and the machine M generated from AA using the construction given previously. We assume that ℓ_0 is the first label and ℓ_e is the last label. We add the following safety properties in the machine M :

- $c = \ell_0 \land \mathbf{pre}(AA)(v) \Rightarrow P_{\ell_0}(v)$
- $c = \ell_e \Rightarrow (P_{\ell_e}(v) \Rightarrow \mathbf{post}(AA)(v_0, v)$

If proof obligations are discharged, then the annotated algorithm AA is partially correct with respect to ist pre/post specification.

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- $\forall x_f, x_0.\mathsf{pre}(x_0) \land x_0 \xrightarrow{\mathsf{P}} x_f \Rightarrow \mathsf{post}(x_0, x_f)$

- $\forall x_f, x_0.\mathsf{pre}(x_0) \wedge x_0 \xrightarrow{\mathsf{P}} x_f \Rightarrow \mathsf{post}(x_0, x_f)$
- $\forall x_f, x_0.\mathsf{pre}(x_0) \Rightarrow x_0 \xrightarrow{\mathsf{P}} x_f \Rightarrow \mathsf{post}(x_0, x_f)$

- $\forall x_f, x_0.\mathsf{pre}(x_0) \wedge x_0 \xrightarrow{\mathsf{P}} x_f \Rightarrow \mathsf{post}(x_0, x_f)$
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- $\forall x_0.\mathsf{pre}(x_0) \Rightarrow \forall x_f.x_0 \xrightarrow{\mathsf{P}} x_f \Rightarrow \mathsf{post}(x_0, x_f)$

From Floyd to Hoare

- $\forall x_f, x_0.\mathsf{pre}(x_0) \wedge x_0 \xrightarrow{\mathsf{P}} x_f \Rightarrow \mathsf{post}(x_0, x_f)$

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- $\forall x_0.\mathsf{pre}(x_0) \Rightarrow [P]\mathsf{post}(x_0, x_f)$

- wlp calculus is introduced

From Floyd to Hoare

- $\forall x_f, x_0.\mathsf{pre}(x_0) \land x_0 \xrightarrow{\mathsf{P}} x_f \Rightarrow \mathsf{post}(x_0, x_f)$

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- $[x := e]P(x) = P[x \mapsto e]$

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- $[x := e]P(x) = P[x \mapsto e]$

- $[\mathsf{if}\ b(x)\ \mathsf{then}\ S1\ \mathsf{else}\ S2\ ]P(x) = b(x) \wedge [S1]P(x) \vee \mathsf{not}\ b(x)\ [S2]P(x)$

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- $\forall x_f, x_0.\mathsf{pre}(x_0) \land x_0 \xrightarrow{\ \mathsf{P}\ } x_f \Rightarrow \mathsf{post}(x_0, x_f)$

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- [if $b(x)$ then $S1$ else $S2$]$P(x) = b(x) \land [S1]P(x) \lor$ not $b(x)\ [S2]P(x)$

- Frama-c uses the HOARE logic for defining the verification conditions as R. Leino in DAFNY.

From Floyd to Hoare

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- Questions of termination require the wp calculus . . .