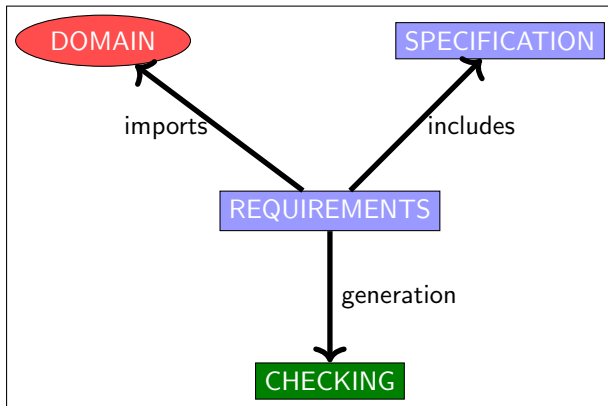


Cours MVSI  
Modélisation et Vérification  
des Systèmes Informatiques

# Modélisation, spécification et vérification (I)

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Telecom Nancy, Université de Lorraine  
(5 septembre 2025 at 8:20 P.M.)

Année universitaire 2025-2026



- ▶ Typing Properties using Typechecker (see for instance functional programming languages as ML, CAML, OCAML, ...)
- ▶ Invariance and safety (*A nothing bad will happen !*) properties for a program  $P$  :
  - Transformation of  $P$  into a relational model  $M$  simulating  $P$
  - Expression of safety properties :
$$\forall s, s' \in \Sigma. (s \in \text{Init}_S \wedge s \xrightarrow{*} s') \Rightarrow (s' \in A).$$
  - Definition of the set of reachable states of  $P$  using  $M$  :
$$\text{REACHABLE}(M) = \text{Init}_S \cup \longrightarrow [\text{REACHABLE}(M)]$$
  - Main property of  $\text{REACHABLE}(M)$  :  $\text{REACHABLE}(M) \subseteq A$
  - Characterization of  $\text{REACHABLE}(M)$  :
$$\text{REACHABLE}(M) = \text{FP}(\text{REACHABLE}(M))$$

- ▶ Proving automatically  $\text{REACHABLE}(M) \subseteq A$  :

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- ▶ Proving automatically  $\text{REACHABLE}(M) \subseteq A$  : changing the domain and solving in another domain as abstract interpretation if making possible
- ▶ Proving automatically  $\text{REACHABLE}(M) \subseteq A$  : approximating semantics of programs



- A problem  $x \in P$  is generally stated by the function  $\chi_{x \in P}$  where  $\chi_{x \in P}(u) = 1$ , if  $P(u)$  is true and  $\chi_{x \in P}(u) = 0$ , if  $P(u)$  is false :
- Problem 1 :  $x \in 0..n$  where  $n \in \mathbb{N}$
  - Problem 1 :  $w \in \mathcal{L}(G)$  where  $G$  is a grammar over the finite set of alphabet symbols  $\Sigma$  and  $\mathcal{L}(G) \subseteq \Sigma^*$ .

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- ▶ Problem of the correctness of a program :
  - Assume that  $\mathcal{F}$  is the set of unary function over natural numbers :  $\mathcal{F} = \mathbb{N} \rightarrow \mathbb{N}$ .
  - $\mathcal{C} \subseteq \mathcal{F}$  : the set of computable (or programmable) functions is  $\mathcal{C}$
  - $f \in \mathcal{C} = \{\Phi_0, \Phi_1, \dots, \Phi_n, \dots\}$  : the set of computable functions is denumerable.
  - The problem  $x \in \text{dom}(\Phi_y)$  is not decidable and it expresses the correctness of programs.



### Implicite versus explicite

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- ▶ Ecrire  $101 = 5$  peut avoir une signification
- ▶ Le code du nombre  $n$  est  $101$  à gauche du symbole  $=$  et le code du nombre  $n$  est sa représentation en base 10 à droite.
- ▶  $n_{10} = 5$  et  $n_2 = 101$
- ▶ Vérification :  $base(2, 10, 101) = 1.2^2 + 0.2 + 1.2^0 = 5_{10}$

- ▶ A train moving at absolute speed  $spd1$
- ▶ A person walking in this train with relative speed  $spd2$ 
  - One may compute the absolute speed of the person
- ▶ Modelling
  - Syntax. Classical expressions
    - ▶ Type  $Speed = Float$
    - ▶  $spd1, spd2 : Speed$
    - ▶  $AbsoluteSpeed = spd1 + spd2$
  - Semantics
    - ▶ If  $spd1 = 25.6$  and  $spd2 = 24.4$  then  $AbsoluteSpeed = 50.0$
    - ▶ If  $spd1 = "val"$  and  $spd2 = 24.4$  then exception raised
  - Pragmatics
    - ▶ What if  $spd1$  is given in *mph* (miles per hour) and  $spd2$  in *km/s* (kilometers per second) ?
    - ▶ What if  $spd1$  is a relative speed ?

- ▶ Un programme  $P$  *produit* des résultats à partir de données en accord avec une sémantique :
  - STATES est l'ensemble de tous les états de  $P$  :  $STATES = X \rightarrow \mathbb{Z}$  où  $X$  désigne les variables de  $P$ .
  - $s_0$  et  $s_f$  deux états de STATES :  $\mathcal{D}(P)(s_0) = s_f$  signifie que  $P$  est exécuté à partir d'un état  $s_0$  et produit un état  $s_f$ .
  - Pour un état  $s$  de  $P$  courant, on notera  $s(X) = x$  pour distinguer la valeur de la variable  $X$  et sa valeur courante en  $s$  :

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$$s_0(X) = x_0, s_f(X) = x_f, s'(X) = x'$$

- $\mathcal{D}(P)(s_0) = s_f$  définit la relation suivante sur l'ensemble des valeurs :

$$x_0 \xrightarrow{P} x_f$$

- Un programme  $P$  *remplit* un contrat (pre,post) :

- $P$  transforme une variable  $x$  à partir d'une valeur initiale  $x_0$  et produisant une valeur finale  $x_f$  :  $x_0 \xrightarrow{P} x_f$
- $x_0$  satisfait pre :  $\text{pre}(x_0)$
- $x_f$  satisfait post :  $\text{post}(x_0, x_f)$
- $\text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$

Un programme  $P$  *remplit* un contrat ( $pre, post$ ) :

- ▶  $P$  transforme une variable  $x$  à partir d'une valeur initiale  $x_0$  et produisant une valeur finale  $x_f$  :  $x_0 \xrightarrow{P} x_f$
- ▶  $x_0$  satisfait  $pre$  :  $pre(x_0)$  and  $x_f$  satisfait  $post$  :  $post(x_0, x_f)$
- ▶  $pre(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow post(x_0, x_f)$

requires  $pre(x_0)$

ensures  $post(x_0, x_f)$

variables  $X$

begin

$0 : P_0(x_0, x)$

instruction<sub>0</sub>

...

$i : P_i(x_0, x)$

...

instruction <sub>$f-1$</sub>

$f : P_f(x_0, x)$

end

▶  $pre(x_0) \wedge x = x_0 \Rightarrow P_0(x_0, x)$

▶  $P_f(x_0, x) \Rightarrow post(x_0, x)$

▶ some conditions for verification related to pairs  $\ell \longrightarrow \ell'$





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- ▶  $\{P\} S \{Q\}$  : *asserted program*

---



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- ▶  $P \Rightarrow WP(S)(Q)$  : *logical formula*



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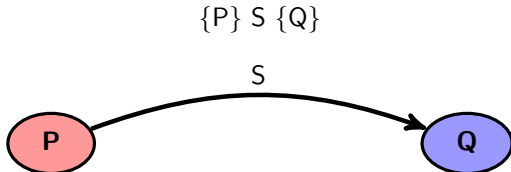
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## Predicate Transformer

$WP(S)(Q)$  is the Weakest-Precondition of  $S$  for  $Q$  and is a predicate transformer but  $WP(S)(.)$  is not a computable function over the set of predicates.



## Method for verifying program properties

correctness and Run Time Errors

A program  $P$  *satisfies* a (pre,post) contract :

- ▶  $P$  transforms a variable  $x$  from initial values  $x_0$  and produces a final value  $x_f : x_0 \xrightarrow{P} x_f$
- ▶  $x_0$  satisfies pre :  $\text{pre}(x_0)$  and  $x_f$  satisfies post :  $\text{post}(x_0, x_f)$
- ▶  $\text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
- ▶  $\mathbb{D}$  est le domaine RTE de  $X$

requires  $\text{pre}(x_0)$

ensures  $\text{post}(x_0, x_f)$

variables  $X$

begin

$0 : P_0(x_0, x)$

instruction<sub>0</sub>

...

$i : P_i(x_0, x)$

...

instruction <sub>$f-1$</sub>

$f : P_f(x_0, x)$

end

▶  $\text{pre}(x_0) \wedge x = x_0 \Rightarrow P_0(x_0, x)$

▶  $\text{pre}(x_0) \wedge P_f(x_0, x) \Rightarrow \text{post}(x_0, x)$

▶ For any pair of labels  $\ell, \ell'$   
such that  $\ell \longrightarrow \ell'$ , one verifies that,  
pour any values  $x, x' \in \text{MEMORY}$   
$$\left( \begin{array}{l} \text{pre}(x_0) \wedge P_\ell(x_0, x) \\ \wedge \text{cond}_{\ell, \ell'}(x) \wedge x' = f_{\ell, \ell'}(x) \end{array} \right) \Rightarrow P_{\ell'}(x_0, x')$$

▶ For any pair of labels  $m, n$   
such taht  $m \longrightarrow n$ , one verifies that,  
 $\forall x, x' \in \text{MEMORY} : \text{pre}(x_0) \wedge$   
 $P_m(x_0, x) \Rightarrow \text{DOM}(m, n)(x)$

\_\_\_\_\_

```
VARIABLES  $X$   
REQUIRES ...  
ENSURES ...  
WHILE  $0 < X$  DO  
     $X := X - 1$ ;  
OD;
```

## Example of an annotation

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**VARIABLES**  $X$   
**REQUIRES** ...  
**ENSURES** ...  
**WHILE**  $0 < X$  **DO**  
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```
CONTRACT  $EX$   
VARIABLES  $X(int)$   
REQUIRES  $x_0 \in \mathbb{N}$   
ENSURES  $x_f = 0$   
   $\ell_0 : \{ x = x_0 \wedge x_0 \in \mathbb{N} \}$   
WHILE  $0 < X$  DO  
   $\ell_1 : \{ 0 < x \leq x_0 \wedge x_0 \in \mathbb{N} \}$   
   $X := X - 1;$   
   $\ell_2 : \{ 0 \leq x \leq x_0 \wedge x_0 \in \mathbb{N} \}$   
OD;  
   $\ell_3 : \{ x = 0 \}$ 
```



### Modelling, specification and verification

### Modelling, specification and verification

- ▶ Set-theoretical notations using the Event-B modelling language
- ▶ Relational modelling of a program or an algorithm
- ▶ Program properties as safety, invariance, pre and post conditions
- ▶ Design by contract
- ▶ Method for proving invariance properties of programs and induction principles as Floyd's method, Hoar logics,
- ▶ Techniques for Model-Checking
- ▶ Tools : the toolset RODIN, the toolset TLAPS, the toolset PAT, the toolset Eiffel Studio, the toolset Frama-c, ...





## Logics

### Logics

- ▶ Propositional Formulae and first order formulae
- ▶ Models
- ▶ Sequents Calculus
- ▶ Proofs and deduction
- ▶ Resolution
- ▶ Tools : the toolset Rodin



## Fixed-Point Theory

# Fixed-Point Theory

- ▶ Complete Partially Ordered Sets (CPO) and Complete Lattices
- ▶ Fixed-Point Theorems : Kleene, Tarski, ...
- ▶ Abstract Interpretation
- ▶ Galois Lattices
- ▶ Static analysis of Programs by abstract interpretation.
- ▶ Semantics of programs



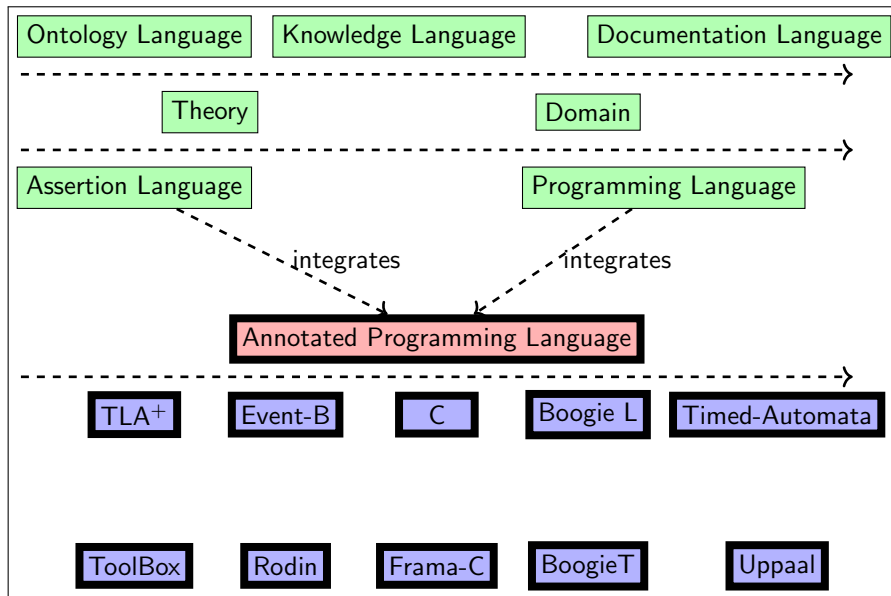
### Computability, Decidability, complexity, Undecidability



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- ▶ Models of computing : Turing Machines, Partially Recursive Functions, URM, ...
- ▶ Church's Thesis
- ▶ Decidability
- ▶ Undecidability
- ▶ Complexity

## Summary of concepts





## Tools

### Tools

- ▶ The TLA<sup>+</sup> ToolBox
- ▶ The RODIN platform
- ▶ Frama-C
- ▶ Boogie and the Visual Studio Suite
- ▶ UPPAAL

- ▶ Documents sur Arche MODÈLES ET ALGORITHMES avec le mot de passe *mery2023*
- ▶ Alternance des cours et des TDs avec des séances sur machines.
- ▶ Intervention de Rosemary Monahan de NUI Maynooth en cours d'année pour un cours et un TD dupliqué pour IL.
- ▶ Deux groupes de TD
- ▶ Machine virtuelle et machines telecom avec les logiciels installés.



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- ▶  $\mathcal{R}$  : pre/post.
- ▶  $\mathcal{D}$  : integers, reals, ...
- ▶  $\mathcal{S}$  : code, procedure, program, ...

A program  $P$  satisfies a (pre,post) contract :

- ▶  $P$  transforms a variable  $v$  from initial values  $v_0$  and produces a final value  $v_f : v_0 \xrightarrow{P} v_f$
- ▶  $v_0$  satisfies pre :  $\text{pre}(v_0)$  and  $v_f$  satisfies post :  $\text{post}(v_0, v_f)$
- ▶  $\text{pre}(v_0) \wedge v_0 \xrightarrow{P} v_f \Rightarrow \text{post}(v_0, v_f)$
- ▶  $\mathbb{D}$  est le domaine RTE de  $V$

requires  $\text{pre}(v_0)$   
 ensures  $\text{post}(v_0, v_f)$   
 variables  $X$

```
begin
  0 :  $P_0(v_0, v)$ 
  instruction0
  ...
  i :  $P_i(v_0, v)$ 
  ...
  instructionf-1
  f :  $P_f(v_0, v)$ 
end
```

- ▶  $\text{pre}(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$
- ▶  $\text{pre}(v_0) \wedge P_f(v_0, v) \Rightarrow \text{post}(v_0, v)$
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
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- ▶ For any pair of labels  $m, n$  such that  $m \longrightarrow n$ , one verifies that,  $\forall v, v' \in \text{MEMORY}$  :
 
$$\text{pre}(v_0) \wedge P_m(v_0, v) \Rightarrow \text{DOM}(m, n)(v)$$



## Example of an annotation

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VARIABLES  $X$   
REQUIRES ...  
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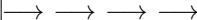
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**ENSURES** ...  
**WHILE**  $0 < X$  **DO**  
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**OD;**

→ → → →

**CONTRACT**  $EX$   
**VARIABLES**  $X(int)$   
**REQUIRES**  $x_0 \in \mathbb{N}$   
**ENSURES**  $x_f = 0$   
     $\ell_0 : \{ x = x_0 \wedge x_0 \in \mathbb{N} \}$   
**WHILE**  $0 < X$  **DO**  
     $\ell_1 : \{ 0 < x \leq x_0 \wedge x_0 \in \mathbb{N} \}$   
     $X := X - 1;$   
     $\ell_2 : \{ 0 \leq x \leq x_0 \wedge x_0 \in \mathbb{N} \}$   
    **OD;**  
     $\ell_3 : \{ x = 0 \}$



An observation of a system  $S$  is based on the following points :

- ▶ a state  $s \in \Sigma$  allows you to observe elements and reports on these elements, such as the number of people in the meeting room or the capacity of the room :  $s(np)$  and  $s(cap)$  are two positive integers.
- ▶ a relationship between two states  $s$  and  $s'$  observes a transformation of the state  $s$  into a state  $s'$  and we will note  $s \xrightarrow{R} s'$  which expresses the observation of a relationship  $R$  :  
 $R = s(np) \in 0..s(cap)-1 \wedge s'(np) = s(np)+1 \wedge s'(cap) = s(cap)$  is an expression of  $R$  observing that one more person has entered the room.
- ▶ a trace  $s_0 \xrightarrow{R_0} s_1 \xrightarrow{R_1} s_2 \xrightarrow{R_2} s_3 \xrightarrow{R} \dots \xrightarrow{R_{i-1}} s_i \xrightarrow{R_i} \dots$  is a trace generated by the different observations  $R_0, \dots R_p, \dots$

- ▶ observing changes of state that correspond either to physical or biological phenomena or to artefactual structures such as a program, a service or a platform.
- ▶ An observation generally leads to the identification of a few possible transformations of the observed state, and the closed-model hypothesis follows naturally.
- ▶ One consequence is that there are visible transformations and invisible transformations.
- ▶ These invisible transformations of the state are expressed by an identity relation called event skip (or stuttering [?]).
- ▶ A modelling produces a closed model with a skip event modelling what is not visible in the observed state.

- ▶ a language of assertions  $\mathcal{L}$  (or a language of formulae) is supposed to be given :  $\mathcal{P}(\Sigma)$  (the set of parts of  $\Sigma$ )
- ▶  $\varphi(s)$  (or  $s \in \hat{\varphi}$ ) means that  $\varphi$  is true in  $s$ .
- ▶ Properties of a system  $S$  which interest us are the state properties expressing that *nothing bad can happen*.
- ▶ Examples : *the number of people in the meeting room is always smaller than the maximum allowed by law* or *the computer variable storing the number of wheel revolutions is sufficient and no overflow will happen*.
- ▶ Safety properties : the partial correctness (PC) of an algorithm  $A$  with respect to its pre/post specifications (PC), the absence of errors at runtime (RTE) ...
- ▶ Properties are expressed in the language  $\mathcal{L}$  whose elements are combined by logical connectors or by instantiations of variable values in the computer sense called flexible.

- ▶ hypothesis : a system  $S$  is modelled by a set of states  $\Sigma$ , and  $\Sigma \stackrel{def}{=} \text{Var} \longrightarrow D$  where  $\text{Var}$  is the variable (or list of variables) of the system  $S$  and  $D$  is the domain of possible values of variables.
- ▶ The interpretation of a formula  $P$  in a state  $s \in \Sigma$  is denoted  $\llbracket P \rrbracket(s)$  or sometimes  $s \in \hat{P}$ .
- ▶ A distinction is made between flexible variable symbols  $x$  and logical variable symbols  $v$ , and constant symbols  $c$  are used.



- ①  $\llbracket \mathbf{x} \rrbracket(s) = s(\mathbf{x}) = x : x$  is the value of the variable  $\mathbf{x}$  in  $s$ .
- ②  $\llbracket \mathbf{x} \rrbracket(s') = s'(\mathbf{x}) = x' : x'$  is the value of the variable  $\mathbf{x}$  in  $s'$ .
- ③  $\llbracket c \rrbracket(s)$  is the value of  $c$  in  $s$ , in other words the value of the constant  $c$  in  $s$ .
- ④  $\llbracket \varphi(x) \wedge \psi(x) \rrbracket(s) = \llbracket \varphi(x) \rrbracket(s)$  et  $\llbracket \psi(x) \rrbracket(s)$  where *and* is the classical interpretation of symbol  $\wedge$  according to the truth table.
- ⑤  $\llbracket \mathbf{x} = 6 \wedge y = \mathbf{x} + 8 \rrbracket(s) \stackrel{def}{=} \llbracket \mathbf{x} \rrbracket(s) = \llbracket 6 \rrbracket(s)$  **and**  $\llbracket y \rrbracket(s) = \llbracket x \rrbracket(s) + \llbracket 8 \rrbracket(s) = (x = 6$  **and**  $y = x + 8$  where  $y$  is a logical variable distinct of  $\mathbf{x}$  and where  $\llbracket \mathbf{x} \rrbracket(s) = s(\mathbf{x}) = x$ .

- ▶  $\llbracket x \rrbracket(s)$  is the value of  $x$  in  $s$  and its value will be distinguished by the font used :  $x$  is the `tt` font of  $\text{\LaTeX}$  and  $x$  is the math font of  $\text{\LaTeX}$ .
- ▶ Using the name of the variable  $x$  as its current value, i.e.  $x$  and  $\llbracket x \rrbracket(s')$  is the value of  $x$  in  $s'$  and will be noted  $x'$ .
- ▶ The transition relation as a relation linking the state of the variables in  $s$  and the state of the variables in  $s'$  using the prime notation as defined by L. Lamport for TLA.
- ▶ Types of variable depending on whether we are talking about the computer variable, its value or whether we are defining constants such as  $np$ , the number of processes, or  $\pi$ , which designates the constant  $\pi$ .
- ▶ a current observation refers to a current state for both endurant and perdurant information data in the sense of the Dines Bjørner.







### axiom of system S

An axiom  $ax(s,c)$  of S is a logical expression describing a constant or constants of S and can be defined as an expression depending on symbols of constants expressing a set-theoretical expression using symbols of sets and symbols of constants already defined.

### Examples of axiom

- ▶  $ax1(fred \in PEOPLE) : fred \text{ is a person from the set } PEOPLE$
- ▶  $ax2(suc \in \mathbb{N} \rightarrow \mathbb{N} \wedge (!i.i \in \mathbb{N} \Rightarrow suc(i) = i+1)) : The \text{ function } suc \text{ is the total function which associates any natural } i \text{ with its successor. successor}$
- ▶  $ax3(\forall A.A \subseteq \mathbb{N} \wedge 0 \in \mathbb{N} \wedge suc[A] \subseteq A \Rightarrow \mathbb{N} \rightarrow \subseteq A) : This \text{ axiom states the induction property for natural numbers. It is an instantiation of the fixed-point theorem.}$
- ▶  $ax4(\forall x.x = 2 \Rightarrow x+2 = 1) : This \text{ axiom poses an obvious problem of consistency and care should be taken not to use this kind of statement as axiom.}$

⊠ Definition(theorem for S)

A property  $P(s, c)$  is a theorem for  $S$ , if  $AX(S, s, c) \vdash P(s, c)$  is a valid sequent.

Let  $s, s'$  be two states of  $S$  ( $s, s' \in \mathcal{Var}(S) \rightarrow \text{VALS}$ ).  $s \xrightarrow{R} s'$  will be written as a relation  $R(x, x')$  where  $x$  and  $x'$  designate values of  $x$  before and after the observation of  $R$ .

Let  $\mathcal{Var}(S)$  be the set of flexible variables of  $S$ . Let  $s$  be the basis sets and  $c$  the constants of  $S$ . An event  $e$  for  $S$  is a relational expression of the form  $R(s, c, x, x')$ , denoted  $BA(e)(s, c, x, x')$

.....

### ⊠ Definition(event-based model of a system)

Let  $\mathcal{Var}(S)$  be the set of flexible variables of  $S$  denoted  $x$ . Let  $s$  be the list of basis sets of the system  $S$ . Let  $c$  be the list of constants of the system  $S$ . Let  $D$  be a domain containing sets  $s$ . An event-based model for a system  $S$  is defined by

$$(AX(s, c), x, \text{VALS}, \text{Init}(x), \{e_0, \dots, e_n\})$$

where

- ▶  $AX(s, c)$  is an axiomatic theory defining the sets, constants and static properties of these elements.
- ▶  $\text{Init}(x)$  defines the possible initial values of  $x$ .
- ▶  $\{e_0, \dots, e_n\}$  is a finite set of events of  $S$  and  $e_0$  is a particular event present in each event-based model defined by  
 $BA(e_0)(x, x') = (x' = x)$ .

The event-based model is denoted

$$EM(s, c, x, \text{VALS}, \text{Init}(x) \{e_0, \dots, e_n\}) = (AX(s, c), x, \text{VALS}, \text{Init}(x), \{e_0, \dots, e_n\}).$$



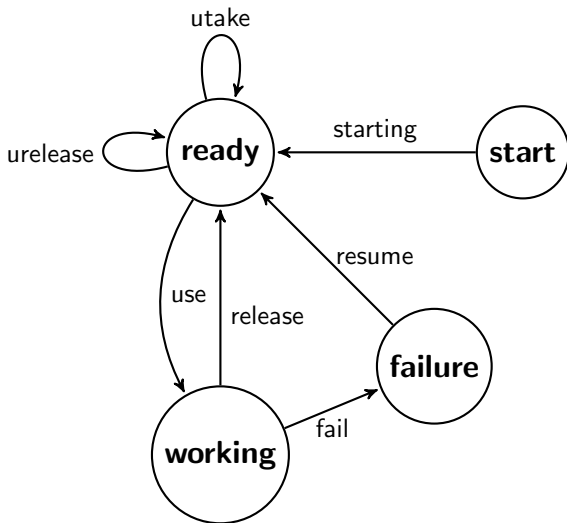
- ▶  $Next(x, x')$  or  
 $Next(s, c, x, x') \stackrel{def}{=} BA(e_0)(s, c, x, x') \vee \dots \vee BA(e_n)(s, c, x, x')$ .
- ▶ the transitive reflexive closure of the relation  $Next^*(s, c, x_0, x) \stackrel{def}{=} \begin{cases} \vee x = x_0 \\ \vee Next(s, c, x_0, x) \\ \vee \exists xi \in VALS. Next^*(s, c, x_0, xi) \wedge Next(s, c, xi, x) \end{cases}$

.....  
☒ Definition(safety property)

A property  $P(x)$  is a safety property for the system  $S$ , if

$$\forall x_0, x \in VALS. Init(x_0) \wedge Next^*(s, c, x_0, x) \Rightarrow P(x).$$

.....



- ▶ there is an implicit control variable  $pc \in \{\mathbf{start}, \mathbf{ready}, \mathbf{working}, \mathbf{failure}\}$  expressing the current visited state.

```
ℓ0[Q := 0];  
ℓ1[R := X];  
IF ℓ5[Y > 0]  
    WHILE ℓ2[R ≥ Y]  
        ℓ3[Q := Q + 1];  
        ℓ4[R := R - Y]  
    ENDWHILE  
ELSE  
    ℓ6[skip]  
ENDIF
```



## A small system as an automaton

$x \leq 5, x := x+1$



$x \leq 5, x := x+1$



► **safety1** :  $0 \leq x \leq 5$

$x \leq 5, x := x+1$



► safety1 :  $0 \leq x \leq 5$  et ...

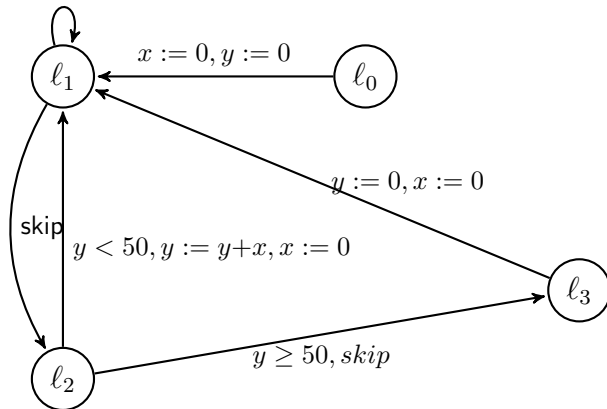
$x \leq 5, x := x+1$



► safety1 :  $0 \leq x \leq 5$  et ... safety2 :  $0 \leq y \leq 56$

## A small system as an automaton

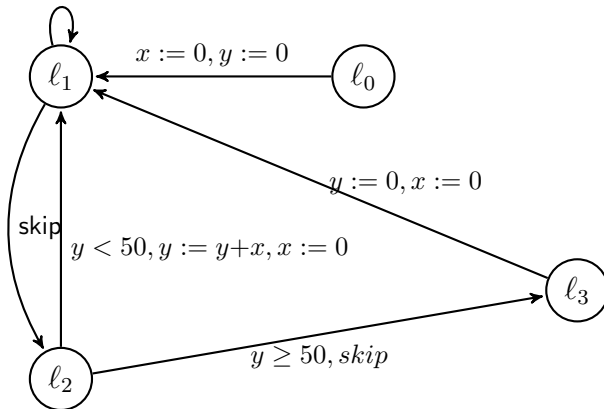
$x \leq 5, x := x+1$





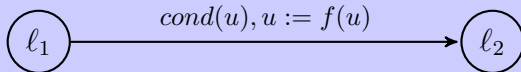
## A small system as an automaton

$x \leq 5, x := x+1$



- ▶  $\text{safety1} : 0 \leq x \leq 5$  et  $\text{safety2} : 0 \leq y \leq 56$
- ▶  $\text{skip} = x := x, y := y$
- ▶  $\text{skip} = \text{TRUE}, x := x, y := y = \text{TRUE}, \text{skip}$

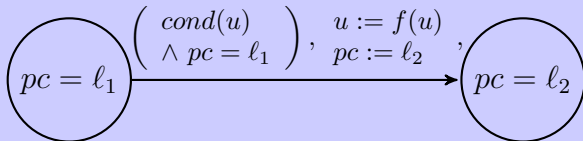
### Transition between two control states



### Transition between two control states



### Transition between two control states



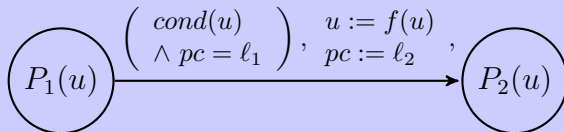
### Transition between two control states



### Transition between two control states



### Transition between two predicates





.....

☒ Definition

Soit  $(Th(s, c), X, VALS, INIT(x), \{r_0, \dots, r_n\})$  un modèle relationnel d'un système  $\mathcal{S}$ . La relation NEXT associée à ce modèle est définie par la disjonction des relations  $r_i$  :

$$NEXT \stackrel{def}{=} r_0 \vee \dots \vee r_n$$

.....

pour une variable  $x$ , nous définissons les valeurs suivantes :

- ▶  $x$  est la valeur courante de la variable X.
- ▶  $x'$  est la valeur suivante de la variable X.
- ▶  $x_0$  ou  $\underline{x}$  sont la valeur initiale de la variable X.
- ▶  $\bar{x}$  ou  $x_f$  est la valeur finale de la variable X, quand cette notion a du sens.

.....

### ⊠ Definition(assertion)

Soit  $(Th(s, c), X, VALS, INIT(x), \{r_0, \dots, r_n\})$  un modèle relationnel  $M$  d'un système  $\mathcal{S}$ . Une propriété  $A$  est une propriété assertionnelle de sûreté pour le système  $\mathcal{S}$ , si

$$\forall x_0, x \in VALS. Init(x_0) \wedge NEXT^*(x_0, x) \Rightarrow A(x).$$

.....

.....

### ⊠ Definition(relation)

Soit  $(Th(s, c), X, VALS, INIT(x), \{r_0, \dots, r_n\})$  un modèle relationnel  $M$  d'un système  $\mathcal{S}$ . Une propriété  $R$  est une propriété relationnelle de sûreté pour le système  $\mathcal{S}$ , si

$$\forall x_0, x \in VALS. Init(x_0) \wedge NEXT^*(x_0, x) \Rightarrow R(x_0, x).$$

.....

- ▶ P. et R. Cousot développent une étude complète des propriétés d'invariance et de sûreté en mettant en évidence correspondances entre les différentes méthodes ou systèmes proposées par Turing, Floyd, Hoare, Wegbreit, Manna ... et reformulent les principes d'induction utilisés pour définir ces méthodes de preuve (voir les deux cubes des 16 principes).





```
ℓ0[Q := 0];  
ℓ1[R := X];  
IF ℓ5[Y > 0]  
    WHILE ℓ2[R ≥ Y]  
        ℓ3[Q := Q + 1];  
        ℓ4[R := R - Y]  
    ENDWHILE  
ELSE  
    ℓ6[skip]  
ENDIF
```







$x \leq 5, x := x+1$



$x \leq 5, x := x+1$



► **safety1** :  $0 \leq x \leq 5$

$x \leq 5, x := x+1$



► **safety1** :  $0 \leq x \leq 5$  et ...

$x \leq 5, x := x+1$



► **safety1** :  $0 \leq x \leq 5$  et ... **safety2** :  $0 \leq y \leq 56$



$x \leq 5, x := x+1$



$x \leq 5, x := x+1$



- ▶  $\text{safety1} : 0 \leq x \leq 5$  et  $\text{safety2} : 0 \leq y \leq 56$
- ▶  $\text{skip} = x := x, y := y$
- ▶  $\text{skip} = \text{TRUE}, x := x, y := y = \text{TRUE}, \text{skip}$

### Transition entre deux états de contrôle



### Transition entre deux états de contrôle



### Transition entre deux états de contrôle



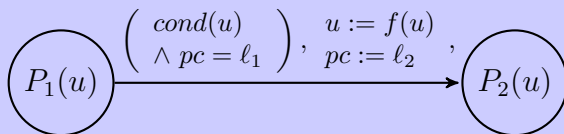
### Transition entre deux états de contrôle



### Transition entre deux états de contrôle



### Transition entre deux prédicats



MODULE *pgcd*

EXTENDS *Naturals, TLC*

CONSTANTS *a, b*

VARIABLES *x, y*

*Init*  $\triangleq x = a \wedge y = b$

*a1*  $\triangleq x > y \wedge x' = x - y \wedge y' = y$

*a2*  $\triangleq x < y \wedge y' = y - x \wedge x' = x$

*over*  $\triangleq x = y \wedge x' = x \wedge y' = y$

*Next*  $\triangleq a_1 \vee a_2 \vee over$

*test*  $\triangleq x \neq y$

```
----- MODULE pgcd -----  
EXTENDS Naturals,TLC  
CONSTANTS a,b  
VARIABLES  x,y  
  
-----  
Init == x=a /\ y=b  
  
-----  
a1 == x > y /\ x'=x-y /\ y'=y  
a2 == x < y /\ y'=y-x /\ x'=x  
over == x=y /\ x'=x /\ y'=y  
  
-----  
Next == a1 \/ a2 \/ over  
  
-----  
test == x # y  
=====
```

MODULE *ex1*

modules de base importables

EXTENDS *Naturals*, *TLC*

un système contrôle l'accès à une salle dont la capacité est de 19 personnes ; écrire  
un modèle de ce système en vérifiant la propriété de sûreté

VARIABLES *np*



### Première tentative

$$\text{entrer} \triangleq np' = np + 1$$
$$sortir \triangleq np' = np - 1$$
$$next \triangleq entrer \vee sortir$$
$$init \triangleq np = 0$$

Seconde tentative

$$\text{entrer}_2 \triangleq np < 19 \wedge np' = np + 1$$
$$\text{next}_2 \triangleq \text{entrer}_2 \vee \text{sortir}$$

---

Troisième tentative

$$\text{sortir}_2 \triangleq np > 0 \wedge np' = np - 1$$

$$\text{next}_3 \triangleq \text{entrer}_2 \vee \text{sortir}_2$$

$$\text{safety}_1 \triangleq np \leq 19$$

$$\text{question}_1 \triangleq np \neq 6$$

---

Soit  $(Th(s, c), x, VALS, INIT(x), \{r_0, \dots, r_n\})$  un modèle relationnel M d'un système  $\mathcal{S}$ . Une propriété  $A$  est une propriété de sûreté pour le système  $\mathcal{S}$ , si

$$\forall x_0, x \in VALS. Init(x_0) \wedge NEXT^*(x_0, x) \Rightarrow A(x).$$

Soit  $(Th(s, c), x, VALS, INIT(x), \{r_0, \dots, r_n\})$  un modèle relationnel M d'un système  $\mathcal{S}$ . Une propriété  $A$  est une propriété de sûreté pour le système  $\mathcal{S}$ , si

$\forall x_0, x \in VALS. Init(x_0) \wedge NEXT^*(x_0, x) \Rightarrow A(x).$

►  $x$  est une variable ou une liste de variable : VARIABLES  $x$

Soit  $(Th(s, c), x, VALS, INIT(x), \{r_0, \dots, r_n\})$  un modèle relationnel  $M$  d'un système  $\mathcal{S}$ . Une propriété  $A$  est une propriété de sûreté pour le système  $\mathcal{S}$ , si

$\forall x_0, x \in VALS. Init(x_0) \wedge NEXT^*(x_0, x) \Rightarrow A(x).$

- ▶  $x$  est une variable ou une liste de variable : `VARIABLES x`
- ▶  $Init(x)$  est une variable ou une liste de variable : `init == Init(x)`

Soit  $(Th(s, c), x, VALS, INIT(x), \{r_0, \dots, r_n\})$  un modèle relationnel  $M$  d'un système  $\mathcal{S}$ . Une propriété  $A$  est une propriété de sûreté pour le système  $\mathcal{S}$ , si

$\forall x_0, x \in VALS. Init(x_0) \wedge NEXT^*(x_0, x) \Rightarrow A(x).$

- ▶  $x$  est une variable ou une liste de variable : `VARIABLES x`
- ▶  $Init(x)$  est une variable ou une liste de variable : `init == Init(x)`
- ▶  $NEXT^*(x_0, x)$  est la définition de la relation définissant ce que fait le système : `Next == a1 \/\ a2 \/\ .... \/\ an`





- ▶ TLA (Temporal Logic of Actions) sert à exprimer des formules en logique temporelle :  $\Box P$  ou *toujours P*
- ▶ TLA<sup>+</sup> est un langage permettant de déclarer des constantes, des variables et des définitions :
  - $\langle \text{def} \rangle == \langle \text{expression} \rangle$  : une définition  $\langle \text{def} \rangle$  est la donnée d'une expression  $\langle \text{expression} \rangle$  qui utilise des éléments définis avant ou dans des modules qui *étendent* ce module.
  - Une variable  $x$  est soit sous la forme  $x$  soit sous la forme  $x'$  :  $x'$  est la valeur de  $x$  après.
  - Un module a un nom et rassemble des définitions et il peut être une extension d'autres modules.
  - $[f \text{ EXCEPT! } [i] = e]$  est la fonction  $f$  où seule la valeur en  $i$  a changé et vaut .
- ▶ Une configuration doit être définie pour évaluer une spécification

- ▶ Limitation des actions :

$$\begin{aligned} \text{nom} &\triangleq \\ &\wedge \text{cond}(v, w) \\ &\wedge v' = e(v, w) \\ &\wedge w' = w \end{aligned}$$

- ▶  $e(v, w)$  doit être codable en Java.
- ▶ Modules standards : Naturals, Integers, TLC ...

- ▶ Téléchargez l'application le site de Microsoft pour votre ordinateur.
- ▶ Ecrivez des modèles et testez les !
- ▶ Limitations par les domaines des variables.



**Permettre un raisonnement symbolique quel que soit l'ensemble des états**