

Modelling Software-based Systems

Lecture 2

Proof Obligation Generation

Master Informatique

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General Summary

- 1 Overview of machines, contexts and proof obligations
- 2 Proof Obligations for Contexts and Machines

PO thm/THM (context)

PO thm/THM (machine)

PO evt/inv/INV

PO evt/act/FIS

PO evt/NAT

PO NAT

PO evt/VAR (arithmetic)

PO evt/VAR (set-theoretic)

- 3 Proof Obligations for Refinement

PO evt/grd/GRD

PO evt/act/SIM

PO evt/NAT

PO NAT

PO evt/VAR (arithmetic)

PO evt/VAR (set-theoretic)

PO evt/x/WFIS

- ① Overview of machines, contexts and proof obligations
- ② Proof Obligations for Contexts and Machines
- ③ Proof Obligations for Refinement

Analysis of the Event-B Models



MACHINE

m

REFINES

am

SEES

c

VARIABLES

u

INVARIANTS

$I(s, c, u)$

THEOREMS

$Q(s, c, u)$

VARIANT

$exp(s, c, u)$

EVENTS

INITIALIZATION

...

e

...

END

MACHINE

m

REFINES

am

SEES

c

VARIABLES

u

INVARIANTS

$I(s, c, u)$

THEOREMS

$Q(s, c, u)$

VARIANT

$exp(s, c, u)$

EVENTS

INITIALIZATION

...

e

...

END

- $\Gamma(m)$: environment for the machine m defined by the context c and it provides a list of seen axioms $Ax(s, c)$ and a list of seen theorems $Th(s, c)$ for the sets s and constants c .
- $\Gamma(m) \vdash \forall u. \text{INIT}(s, c, u) \Rightarrow I(s, c, u)$
- For each event e in E :
 $\Gamma(m) \vdash \forall u, u'. I(s, c, u) \wedge BA(e)(u, u') \Rightarrow I(u')$
- For each event e in E :
 $\Gamma(m) \vdash \forall u. I(s, c, u) \wedge GRD(e)(s, c, u) \Rightarrow \exists u'. BA(e)(u, u')$
- $\Gamma(m) \vdash \forall u. I(s, c, u) \Rightarrow Q(s, c, u)$
- Generated proof obligations are derived from those conditions.

Three kinds of events

Events are divided into three kinds of events :

- An event is **ordinary** and, when it is observed, it modifies variables according to a guard and an action.
- An event is **anticipated** and, when it is observed, it means that something is observed but later in the further refinement.
- An event is **convergent** and, when it is observed, it decreases a variant which is member of naturals or is a set..

Checking the well formation of Event-B expressions

- Event-B expressions are contexts, machines, properties, equations, set-theoretical expressions ...
- e is an Event-B expression and $\text{wd}(e)$ is a logical property expressing the well definition of e .
- $\text{wd}(1 = 2) \triangleq \text{wd}(1) \wedge \text{wd}(2)$
- $\text{wd}(a/b) \triangleq b \neq 0 \wedge \text{wd}(a) \wedge \text{wd}(b)$
- $\text{wd}(f(g)) \triangleq g \in \text{dom}(f) \wedge f \in A \leftrightarrow B$

- ① Overview of machines, contexts and proof obligations
- ② Proof Obligations for Contexts and Machines
- ③ Proof Obligations for Refinement

PO thm/THM (context)

CONTEXTS

c
EXTENDS

ac
SETS

s
CONSTANTS

c
AXIOMS

$Ax(s, c)$
THEOREMS

$th_1 : P_1(s, c)$

...

$th_n : P_n(s, c)$

$th : P(s, c)$

...

END

s *seen sets*

c *seen constants*

$Ax(s, c)$ *seen axioms*

$Th(s, c)$ *previous proved theorems*

$Th(s, c) = \{P_i(s, c) | i \ 1..n\}$

$P(s, c)$ *property over s and c*

PO th/THM

$Ax(s, c), Th(s, c) \vdash P(s, c)$

PO thm/THM (machine)

MACHINE

m

...
VARIABLES

u

INVARIANTS

$I(s, c, u)$

THEOREMS

$Q(s, c, u)$

$th : P(s, c, u)$

...
END

s

seen sets

c

seen constants

u

variables

$Ax(s, c)$

seen axioms

$Th(s, c)$

seen theorems

$I(s, c, u)$

invariants

$Q(s, c, u)$

theorems

$P(s, c, u)$

property over s, c and u

PO th/THM

$Ax(s, c), Th(s, c), I(s, c, u) \vdash P(s, c, u)$

PO evt/inv/INV

```
EVENT evt
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
```

$BA(\text{evt}) \hat{=}$

$\exists x. \left(\begin{array}{l} \wedge G(x, s, c, u) \\ \wedge BAP(x, s, c, u, u') \end{array} \right)$

$GRD(\text{evt}) \hat{=} G(x, s, c, u)$

$ACT(\text{evt}) \hat{=} BAP(x, s, c, u, u')$

s

c

u

$Ax(s, c)$

$Th(s, c)$

$I(s, c, u)$

$Q(s, c, u)$

evt

x

$G(x, s, c, u)$

$BAP(x, s, c, u, u')$

$\text{inv} : \text{inv}(s, c, u')$

seen sets

seen constants

variables

seen axioms

seen theorems

invariants

theorems

event name

event parameter

event guard

event before-after predicate

specific modified invariant

PO evt/inv/INV

$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), BAP(x, s, c, u, u') \vdash \text{inv}(s, c, u')$

EVENT evt
ANY x **WHERE**
 $G(x, s, c, u)$
THEN
 $u : |BAP(x, s, c, u, u')$
END

$BA(\text{evt}) \hat{=}$

$\left(\begin{array}{l} \wedge G(x, s, c, u) \\ \wedge BAP(x, s, c, u, u') \end{array} \right)$

$GRD(\text{evt}) \hat{=} G(x, s, c, u)$

$ACT(\text{evt}) \hat{=}$

$BAP(x, s, c, u, u')$

s

c

u

$Ax(s, c)$

$Th(s, c)$

$I(s, c, u)$

$Q(s, c, u)$

evt

x

$G(x, s, c, u)$

$BAP(x, s, c, u, u')$

seen sets

seen constants

variables

seen axioms

seen theorems

invariants

theorems

event name

event parameter

event guard

event before-after predicate

PO evt/act/FIS

$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), \vdash \exists u'. BAP(x, s, c, u, u')$

```

EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
...
VARIANT
  exp(s, c, u)
    
```

s

c

u

$Ax(s, c)$

$Th(s, c)$

$I(s, c, u)$

$Q(s, c, u), R(s, c, u, v)$

evt, ce

x

$G(x, s, c, u)$

$BAP(x, s, c, u, u')$

$exp(s, c, u)$

seen sets

seen constants

abstract variables

seen axioms

seen theorems

abstract invariants

abstract and concrete theorems

event name

event parameters

abstract event guard

event before-after predicate

arithmetic expression

PO evt/NAT $Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u) \vdash exp(s, c, u) \in \mathbb{N}$

```

EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
...
VARIANT
  exp(s, c, u)
    
```

s
 c
 u, v
 $Ax(s, c)$
 $Th(s, c)$
 $I(s, c, u)$
 $Q(s, c, u), R(s, c, u, v)$
 evt, ce
 x
 $G(x, s, c, u)$
 $BAP(x, s, c, u, u')$
 $setexp(s, c, u)$

seen sets
seen constants
abstract variables
seen axioms
seen theorems
abstract invariants
abstract and concrete theorems
event name
event parameters
abstract event guard
event before-after predicate
set expression

PO evt/NAT

$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u) \vdash finite(setexp(s, c, u))$

```

EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
...
VARIANT
  exp(s, c, u)
    
```

s
 c
 u, v
 $Ax(s, c)$
 $Th(s, c)$
 $I(s, c, u)$
 evt
 x
 $G(x, s, c, u)$
 $BAP(x, s, c, u, u')$
 $exp(s, c, u)$

seen sets
seen constants
abstract and concrete variables
seen axioms
seen theorems
abstract invariants
event name
event parameters
abstract event guard
event before-after predicate
arithmetic expression

PO evt/VAR

$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), BAP(x, s, c, u, u') \vdash$
 $exp(s, c, u') < exp(s, c, u)$


```
EVENT ae
  ANY x WHERE
     $G(x, s, c, u)$ 
  THEN
     $u : |BAP(x, s, c, u, u')$ 
  END
...
VARIANT
   $setexp(s, c, u)$ 
```

s

c

u, v

$Ax(s, c)$

$Th(s, c)$

$I(s, c, u)$

evt

x

$G(x, s, c, u)$

$BAP(x, s, c, u, u')$

$setexp(s, c, u)$

seen sets

seen constants

abstract and concrete variables

seen axioms

seen theorems

abstract invariants

event name

event parameters

abstract event guard

event before-after predicate

set-theoretic expression

PO evt/VAR

$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), BAP(x, s, c, u, u') \vdash$
 $setexp(s, c, u') \subset setexp(s, c, u)$

- ① Overview of machines, contexts and proof obligations
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- ③ Proof Obligations for Refinement

PO evt/grd/GRD

```
EVENT ae
  ANY x WHERE
     $G(x, s, c, u)$ 
  THEN
     $u : |ABAP(x, s, c, u, u')$ 
  END
```

```
EVENT ce
  REFINES
    ae
  ANY y WHERE
     $H(y, s, c, v)$ 
  WITH
     $x : W(x, y, s, c, v)$ 
  THEN
     $v : |CBAP(y, s, c, v, v')$ 
  END
```

s
 c
 u, v
 $Ax(s, c)$
 $Th(s, c)$
 $I(s, c, u)$
 $J(s, c, u, v)$
 $Q(s, c, u), R(s, c, u, v)$
 ae, ce
 x, y
 $G(x, s, c, u)$
 $H(y, s, c, v)$
 $ABAP(x, s, c, u, u')$
 $CBAP(x, s, c, u, u')$
 $W(x, y, s, c, v)$

seen sets

seen constants

abstract and concrete variables

seen axioms

seen theorems

abstract invariants

concrete invariants

abstract and concrete theorems

abstract and concrete event names

event parameters

abstract event guard

concrete event guard

abstract event before-after predicate

concrete event before-after predicate

witness predicate

PO evt/grd/GRD

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), W(x, y, s, c, v), H(y, s, c, v), \vdash$
 $G(x, s, c, u, u')$

PO evt/act/SIM

```

EVENT ae
  ANY x WHERE
     $G(x, s, c, u)$ 
  THEN
     $u : |ABAP(x, s, c, u, u')$ 
  END
    
```

```

EVENT ce
  REFINES
    ae
  ANY y WHERE
     $H(y, s, c, v)$ 
  WITH
     $x : WP(x, y, s, c, v)$ 
     $u' : WV(y, u', s, c, v)$ 
  THEN
     $v : |CBAP(y, s, c, v, v')$ 
  END
    
```

s
 c
 u, v
 $Ax(s, c)$
 $Th(s, c)$
 $I(s, c, u)$
 $J(s, c, u, v)$
 $Q(s, c, u), R(s, c, u, v)$
 ae, ce
 x, y
 $G(x, s, c, u)$
 $H(y, s, c, v)$
 $ABAP(x, s, c, u, u')$
 $CBAP(x, s, c, u, u')$
 $WP(x, y, s, c, v)$
 $WV(y, u', s, c, v)$

seen sets

seen constants

abstract and concrete variables

seen axioms

seen theorems

abstract invariants

concrete invariants

abstract and concrete theorems

abstract and concrete event names

event parameters

abstract event guard

concrete event guard

abstract event before-after predicate

concrete event before-after predicate

witness parameter predicate

witness variable predicate

PO evt/act/SIM

$$\left(\begin{array}{l} Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v) \\ WP(x, y, s, c, v), WV(y, u', s, c, v) \\ H(y, s, c, v), CBAP(y, s, c, v, v') \end{array} \right) \vdash ABAP(x, s, c, u, u')$$

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
...
VARIANT
  exp(s, c, u)
```

s
 c
 u, v
 $Ax(s, c)$
 $Th(s, c)$
 $I(s, c, u)$
 $J(s, c, u, v)$
 $Q(s, c, u), R(s, c, u, v)$
evt, ce
 x
 $G(x, s, c, u)$
 $BAP(x, s, c, u, u')$
 $exp(s, c, u)$

seen sets
seen constants
abstract and concrete variables
seen axioms
seen theorems
abstract invariants
concrete invariants
abstract and concrete theorems
event name
event parameters
abstract event guard
event before-after predicate
arithmetic expression

PO evt/NAT

$$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u) \vdash exp(s, c, u) \in \mathbb{N}$$

```

EVENT ae
  ANY x WHERE
     $G(x, s, c, u)$ 
  THEN
     $u : |BAP(x, s, c, u, u')$ 
  END
...
VARIANT
   $exp(s, c, u)$ 

```

s
 c
 u, v
 $Ax(s, c)$
 $Th(s, c)$
 $I(s, c, u)$
 $J(s, c, u, v)$
 $Q(s, c, u), R(s, c, u, v)$
 evt, ce
 x
 $G(x, s, c, u)$
 $BAP(x, s, c, u, u')$
 $setexp(s, c, u)$

seen sets
seen constants
abstract and concrete variables
seen axioms
seen theorems
abstract invariants
concrete invariants
abstract and concrete theorems
event name
event parameters
abstract event guard
event before-after predicate
set expression

PO evt/NAT $Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u) \vdash$
 $finite(setexp(s, c, u))$

```

EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
...
VARIANT
  exp(s, c, u)
    
```

s

c

u, v

$Ax(s, c)$

$Th(s, c)$

$I(s, c, u)$

$J(s, c, u, v)$

$Q(s, c, u), R(s, c, u, v)$

evt, ce

x

$G(x, s, c, u)$

$BAP(x, s, c, u, u')$

$exp(s, c, u)$

seen sets

seen constants

abstract and concrete variables

seen axioms

seen theorems

abstract invariants

concrete invariants

abstract and concrete theorems

event name

event parameters

abstract event guard

event before-after predicate

arithmetic expression

PO evt/VAR

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u), BAP(x, s, c, u, u') \vdash$
 $exp(s, c, u') < exp(s, c, u)$

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
...
VARIANT
  setexp(s, c, u)
```

s
 c
 u, v
 $Ax(s, c)$
 $Th(s, c)$
 $I(s, c, u)$
 $J(s, c, u, v)$
 $Q(s, c, u), R(s, c, u, v)$
evt, ce
 x
 $G(x, s, c, u)$
 $BAP(x, s, c, u, u')$
 $setexp(s, c, u)$

seen sets
seen constants
abstract and concrete variables
seen axioms
seen theorems
abstract invariants
concrete invariants
abstract and concrete theorems
event name
event parameters
abstract event guard
event before-after predicate
set-theoretic expression

PO evt/VAR

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u), BAP(x, s, c, u, u') \vdash$
 $setexp(s, c, u') \subset setexp(s, c, u)$


```

EVENT ae
  ANY x WHERE
     $G(x, s, c, u)$ 
  THEN
     $u : |ABAP(x, s, c, u, u')$ 
  END
    
```

```

EVENT ce
  REFINES
    ae
  ANY y WHERE
     $H(y, s, c, v)$ 
  WITH
     $x : WP(x, y, s, c, v)$ 
     $u' : WV(y, u', s, c, v)$ 
  THEN
     $v : |CBAP(y, s, c, v, v')$ 
  END
    
```

s
 c
 u, v
 $Ax(s, c)$
 $Th(s, c)$
 $I(s, c, u)$
 $J(s, c, u, v)$
 $Q(s, c, u), R(s, c, u, v)$
 ae, ce
 x, y
 $G(x, s, c, u)$
 $H(y, s, c, v)$
 $ABAP(x, s, c, u, u')$
 $CBAP(x, s, c, u, u')$
 $WP(x, y, s, c, v)$
 $WV(y, u', s, c, v)$

seen sets

seen constants

abstract and concrete variables

seen axioms

seen theorems

abstract invariants

concrete invariants

abstract and concrete theorems

abstract and concrete event names

event parameters

abstract event guard

concrete event guard

abstract event before-after predicate

concrete event before-after predicate

witness parameter predicate

witness variable predicate

PO evt/x/WFIS

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), H(y, s, c, v) \vdash$
 $\exists x. WP(x, y, s, c, v)$