Cours MALG & MOVEX

Vérification mécanisée de contrats (II) (The ANSI/ISO C Specification Language (ACSL))

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(4 mars 2025 at 10:10 A.M.)

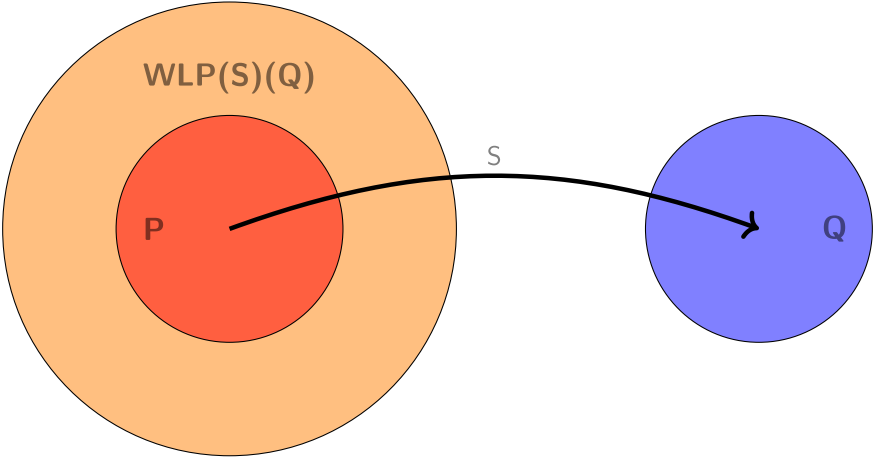
Année universitaire 2024-2025

Plan

Sommaire

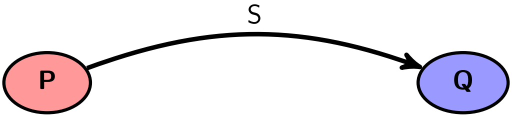
Sommaire

Asserted Program $\{P\}$ S $\{Q\}$



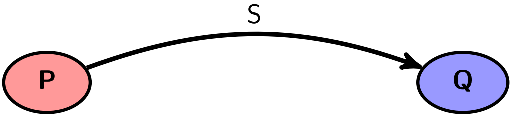
Asserted Program $\{P\}$ S $\{Q\}$

$\{P\}$ S $\{Q\}$



Asserted Program {P} S {Q}

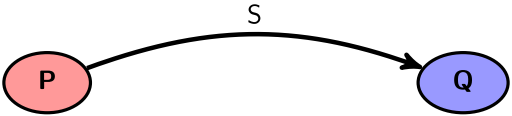
{P} S {Q}



$$P \Rightarrow WLP(S)(Q)$$

Asserted Program {P} S {Q}

{P} S {Q}



$$P \Rightarrow WLP(S)(\mathsf{Q})$$

Computing WLP(S)(Q) ?

Writing a simple contract

variables x

requires $x >= 0 \wedge x <= 10;$

ensures $\left\{ \begin{array}{lcl} x\%2 = 0 & \Rightarrow & 2\cdot\text{result} = x+; \\ x\%2 \neq 0 & \Rightarrow & 2\cdot\text{result} = x-1; \end{array} \right.$

begin
$int\ y;$
$y = x/2;$
$return(y);$
end

- result is the value returned by the command `rezturn(y)`.
- `return(y)` is equivalent to result`:= y`.

(Writing a simple contract.)

Listing 1 – project-divers/annotation.c

```
/*@ requires x <= 0 && x >= 10;
  @ assigns \nothing;
  @ ensures  x % 2 == 0 ==> 2*\result == x;
  @ ensures  x % 2 != 0 ==> 2*\result == x-1;
  @*/
int annotation(int x)
{
  int y;
  y  = x / 2;
  return(y);
  }
```

Writing a simple contract

(Writing a simple contract.)

Listing 2 – project-divers/annotationwp.c

```
/*@ requires 0 <= x  && x <=  10;
  @ assigns \nothing;
  @ ensures  x % 2 == 0 ==> 2*\result == x;
  @ ensures  x % 2 != 0 ==> 2*\result == x-1;
  @*/
int annotation(int x)
{
/*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
/*@ assert   x % 2 != 0 ==> 2* (x / 2) == x-1; */
  int y;
/*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
/*@ assert   x % 2 != 0 ==> 2* (x / 2) == x-1; */
  y  = x / 2;
/*@ assert x % 2 == 0 ==> 2*y == x; */
/*@ assert   x % 2 != 0 ==> 2*y == x-1; */
  return(y);
/*@ assert x % 2 == 0 ==> 2*y == x; */
/*@ assert   x % 2 != 0 ==> 2*y == x-1; */

  }
```

Property to check

$$x \geq 0 \wedge x \leq 10 \Rightarrow \begin{cases} x\%2 \neq 0 & \Rightarrow \ 2\cdot(x/2) = x-1 \\ x\%2 = 0 & \Rightarrow \ 2\cdot(x/2) = x \end{cases}$$

Writing a simple contract

(Checking the precondition.)

Listing 3 – project-divers/annotation0.c

```
g/*@ requires x >= 0 && x <  0;
  @ assigns \nothing;
  @ ensures \result == 0;
  @*/
int annotation(int x)
{
  int y;
  y  =  y / (x-x);
  return(y);
  }
```

Writing a simple contract

(Checking the precondition.)

Listing 4 – project-divers/annotation0wp.c

```
/*@ requires x >= 0 && x <  0;
  @ assigns \nothing;
  @ ensures \result == 0;
  @*/
int annotation(int x)
{
  /*@ assert y / (x-x) == 0; */
  int y;
  /*@ assert y / (x-x) == 0; */
  y  =  y / (x-x);
  /*@ assert y == 0; */
  return(y);
  /*@ assert y == 0; */
  }
```

Property to check

$x \geq 0 \wedge < 0 \Rightarrow y/(x{-}x) = 0$

Transformations of annotated programs (1)

$//@$ assert $P(v0, v)$:
$S1; S2$
$//@$ assert $Q(v0, v)$:

- Applying the property : $wp(S1; S2)(A) = wp(S1)(wp(S2)(A))$
-

$//@$ assert $P(v0, v)$:
$S1;$
$//@$ assert $wp(S2)(Q(v0, v))$:
$S2;$
$//@$ assert $Q(v0, v)$:

$//@$ assert $P(v0, v)$:
$//@$ assert $xp(S1)(wp(S2)(Q(v0, v)))$:
$S1;$
$//@$ assert $wp(S2)(Q(v0, v))$:
$S2;$
$//@$ assert $Q(v0, v)$:

Transformations of annotated programs (2)

```
//@ assert P(v0, v) :
IF B THEN
   S1
ELSE
   S2
FI
//@ assert Q(v0, v) :
```

- Applying the property : $wp(if(B, S1, S2)(A) = b \wedge wp(S1)(A) \vee \neg B \wedge wp(S2)(A)$.
-

```
//@ assert P(v0, v) :
IF B THEN
   S1
ELSE
   S2
FI
//@ assert Q(v0, v) :
```

```
//@ assert P(v0, v) :
IF B THEN
   S1
//@ assert Q(v0, v) :
ELSE
   S2
//@ assert Q(v0, v) :
FI
//@ assert Q(v0, v) :
```

Transformations of annotated programs (2)

//@ assert $P(v0, v)$:
IF B THEN
  $S1$
//@ assert $Q(v0, v)$:
ELSE
  $S2$
//@ assert $Q(v0, v)$:
FI
//@ assert $Q(v0, v)$:

Transformations of annotated programs (2)

$//@$ assert $P(v0,v)$:
IF B THEN
  $S1$
$//@$ assert $Q(v0,v)$:
ELSE
  $S2$
$//@$ assert $Q(v0,v)$:
FI
$//@$ assert $Q(v0,v)$:

$//@$ assert $P(v0,v)$:
IF B THEN
$//@$ assert $B \wedge wp(S2)(Q(v0,v))$:
  $S1$
$//@$ assert $Q(v0,v)$:
ELSE
$//@$ assert $\neg B \wedge wp(S2)(Q(v0,v))$:
  $S2$
$//@$ assert $Q(v0,v)$:
FI

Transformations of annotated programs (2)

//@ assert $P(v0, v)$:
IF B THEN
  $S1$
//@ assert $Q(v0, v)$:
ELSE
  $S2$
//@ assert $Q(v0, v)$:
FI
//@ assert $Q(v0, v)$:

//@ assert $P(v0, v)$:
IF B THEN
//@ assert $b \wedge wp(S1)(Q(v0, v))$:
  $S1$
//@ assert $Q(v0, v)$:
ELSE
//@ assert $\neg b \wedge wp(S2)(Q(v0, v))$:
  $S2$
//@ assert $Q(v0, v)$:
FI
//@ assert $Q(v0, v)$:

//@ assert $P(v0, v)$:
IF B THEN
//@ assert $B \wedge wp(S2)(Q(v0, v))$:
  $S1$
//@ assert $Q(v0, v)$:
ELSE
//@ assert $\neg B \wedge wp(S2)(Q(v0, v))$:
  $S2$
//@ assert $Q(v0, v)$:
FI

Transformations of annotated programs (2)

```
//@ assert P(v0,v) :
IF B THEN
  S1
//@ assert Q(v0,v) :
ELSE
  S2
//@ assert Q(v0,v) :
FI
//@ assert Q(v0,v) :
```

```
//@ assert P(v0,v) :
IF B THEN
//@ assert B ∧ wp(S2)(Q(v0,v)) :
  S1
//@ assert Q(v0,v) :
ELSE
//@ assert ¬B ∧ wp(S2)(Q(v0,v)) :
  S2
//@ assert Q(v0,v) :
FI
```

```
//@ assert P(v0,v) :
IF B THEN
//@ assert b ∧ wp(S1)(Q(v0,v)) :
  S1
//@ assert Q(v0,v) :
ELSE
//@ assert ¬b ∧ wp(S2)(Q(v0,v)) :
  S2
//@ assert Q(v0,v) :
FI
//@ assert Q(v0,v) :
```

- $b \wedge P(v0,v) \Rightarrow b \wedge wp(S1)(Q(v0,v))$
- $\neg b \wedge P(v0,v) \Rightarrow \neg b \wedge wp(S2)(Q(v0,v))$

Transformations of annotated programs (3)

```
//@ assert P(v0, v) :
//@ loop invariant I(v0, v) :
WHILE B THEN
   S
OD
//@ assert Q(v0, v) :
```

- Applying the iteration rule of Hoare Logic :

Transformations of annotated programs (3)

```
//@ assert P(v0,v) :
//@ loop invariant I(v0,v) :
WHILE B THEN
   S
OD
//@ assert Q(v0,v) :
```

- Applying the iteration rule of Hoare Logic :

```
//@ assert P(v0,v) :
//@ loop invariant I(v0,v) :
//@ assert I(v0,v) :
WHILE B THEN
//@ assert b ∧ I(v0,v) :
   S
//@ assert I(v0,v) :
OD
//@ assert Q(v0,v) :
```

Transformations of annotated programs (3)

//@ assert $P(v0, v)$:
//@ loop invariant $I(v0, v)$:
WHILE B THEN
  S
OD
//@ assert $Q(v0, v)$:

- Applying the iteration rule of Hoare Logic :

//@ assert $P(v0, v)$:
//@ loop invariant $I(v0, v)$:
//@ assert $I(v0, v)$:
WHILE B THEN
//@ assert $b \wedge I(v0, v)$:
  S
//@ assert $I(v0, v)$:
OD
//@ assert $Q(v0, v)$:

- $b \wedge I(v0, v) \Rightarrow wp(S)(I(v0, v))$
- $P(v0, v) \Rightarrow I(v0, v))$
- $\neg b \wedge I(v0, v) \Rightarrow Q(v0, v)$

Summary of transformations

- Checking the preservation of invariant.
- Applying the wps on assertions according to startements.

Assertions

- Assertions at a control point of the program

  ```
  /*@  assert pred; */

  //@  assert pred;
  ```

- Assertions at a control point of the program components.

  ```
  /*@  for id1,id2, ..., idn: assert pred; */
  ```

Verification using WLP

(Incrementing a number)

Listing 5 – project-divers/compwp0.c

```
#define x0 5
/*@ assigns \nothing;*/
int  exemple() {
  int x=x0;
  //@ assert x == x0;
  x = x + 1;
//@ assert  x== x0+1;
return x;
}
```

(Incrementing a number)

Listing 6 – project-divers/compwp0wp.c

```
#define x0 5
/*@ assigns \nothing; */
int  exemple() {
  //@ assert x0 == x0;
//@ assert  x0+1 == x0+1;
  int x=x0;
  //@ assert x == x0;
//@ assert  x+1 == x0+1;
  x = x + 1;
//@ assert  x== x0+1;
return x;
}
```

Sommaire des annotations et autres assertions

- requires
- assigns
- ensures
- decreases
- predicate
- logic
- lemma

Programming by contract

- The calling function should garantee the required condition or precondition introduced by the clauses requires $P1 \wedge \ldots \wedge Pn$ at the calling point.
- The called function returns results that are ensured by the clause ensures $E1 \wedge \ldots \wedge Em$; ensures clause exporess a relatrionship between the initial values of variables and the final values.
- initial values of a variable v is denoted $\backslash old(v)$
- The variables which are not in the set $L1 \cup \ldots \cup Lp$ are not modified.

Listing 7 – contrat

```
/*@ requires P1;...; requires Pn;
  @ assigns L1;...; assigns Lm;
  @ ensures E1;...; ensures Ep;
  @*/
```

Examples of contract (1)

(Division)

Listing 8 – project-divers/annotation.c

```
/*@ requires x <= 0 && x >= 10;
  @ assigns \nothing;
  @ ensures  x % 2 == 0 ==> 2*\result == x;
  @ ensures  x % 2 != 0 ==> 2*\result == x-1;
  @*/
int annotation(int x)
{
  int y;
  y  = x / 2;
  return(y);
  }
```

Examples of contract (1)

(Division)

Listing 9 – project-divers/annotationwp.c

```
/*@ requires 0 <= x  && x <=  10;
  @ assigns \nothing;
  @ ensures  x % 2 == 0 ==> 2*\result == x;
  @ ensures  x % 2 != 0 ==> 2*\result == x-1;
  @*/
int annotation(int x)
{
/*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
/*@ assert  x % 2 != 0 ==> 2* (x / 2) == x-1; */
  int y;
/*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
/*@ assert  x % 2 != 0 ==> 2* (x / 2) == x-1; */
  y  = x / 2;
/*@ assert x % 2 == 0 ==> 2*y == x; */
/*@ assert  x % 2 != 0 ==> 2*y == x-1; */
  return(y);
/*@ assert x % 2 == 0 ==> 2*y == x; */
/*@ assert  x % 2 != 0 ==> 2*y == x-1; */

  }
```

Property to check

$$x \geq 0 \wedge x < 0; \Rightarrow \left(\begin{array}{lcl} x \;\%\; 2 & = & 0 \Rightarrow 2{\cdot}(x/2) = x \\ x \;\%\; 2 & \neq & 0 \Rightarrow 2{\cdot}(x/2) = x{-}1 \end{array} \right)$$

Examples of contract (2)

(Precondition)

Listing 10 – project-divers/annotation0.c

```
g/*@ requires x >= 0 && x <   0;
  @ assigns \nothing;
  @ ensures \result == 0;
  @*/
int annotation(int x)
{
  int y;
  y  =  y / (x-x);
  return(y);
  }
```

Examples of contract (2)

(Precondition)

Listing 11 – project-divers/annotation0wp.c

```
/*@ requires x >= 0 && x <  0;
  @ assigns \nothing;
  @ ensures \result == 0;
  @*/
int annotation(int x)
{
  /*@ assert y / (x-x) == 0; */
  int y;
  /*@ assert y / (x-x) == 0; */
  y  =  y / (x-x);
  /*@ assert y == 0; */
  return(y);
  /*@ assert y == 0; */
  }
```

Examples of contract (2)

Property to check

$0 \leq x \wedge x \leq 10 \Rightarrow y/(x-x) = 0$

Definition of a contract (specification)

- Define the mathematical fucntion to compute (what to compute ?)
- Define an inductive method for computing the mathematical function and using axioms.

(facctorial what)

Listing 12 – project-factorial/factorial.h

```
#ifndef _A_H
#define _A_H
/*@ axiomatic mathfact {
  @ logic integer mathfact(integer n);
  @ axiom mathfact_1: mathfact(1) == 1;
  @ axiom mathfact_rec: \forall integer n; n > 1
  ==> mathfact(n) == n * mathfact(n-1);
  @ } */

/*@ requires n > 0;
   decreases n;
   ensures \result == mathfact(n);
   assigns \nothing;
*/
int codefact(int n);
#endif
```

Definition of a contract (programming)

- Define the program codefact for computing mathfact (How to compute ?)
- Define the algorithm computing the function mathfact

(facctorial how)

Listing 13 – project-factorial/factorial.c

```
#include "factorial.h"

int codefact(int n) {
  int y = 1;
  int x = n;
  /*@ loop invariant x >= 1 && x <= n && mathfact(n) == y * mathfact(x);
    loop assigns x, y;
    loop variant x;
   */
  while (x != 1) {
    y = y * x;
    x = x - 1;
  };
  return y;

}
```

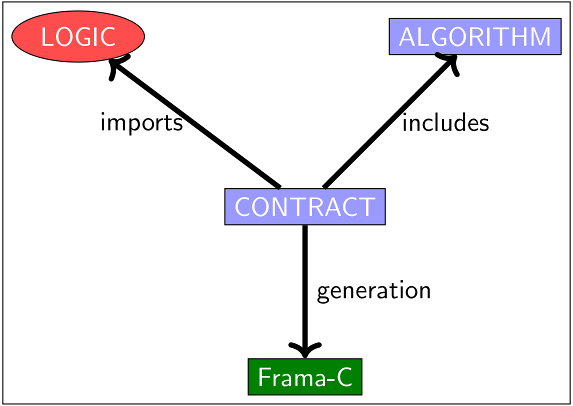
Definition of a contract (approach)

- The specification of a function (mathfact) to compute requires to define it mathematically.
- The definition is stated in an axtiomatic framework and is preferably inductive (mathfact) which is used in assrtions or theorems or lemmas.
- The relationship between the ciomputed value (\result) and the mathematical value (mathfact(n)) is stated in the ensures clause :

$$\backslash\text{result} == \text{mathfact(n)}$$

- The main property to prove is codefact(n)==mathfact(n) : Calling codefact for n returns a value equal to mathfact(n).

Tryptic Pattern



Contracts with named behaviours

Listing 14 – contrat

```
/*@ requires P;
@ behavior b1:
  @ assumes A1;
  @ requires R1 ;
  @ assigns L1;
  @ ensures E1;
@ behavior b2:
  @ assumes A2;
  @ requires R2;
  @ assigns L2;
  @ ensures E2;
@*/
```

Division shpuld not return silly expressions !

(Pairs of integers)

Listing 15 – project-divers/structures.h

```
#ifndef _STRUCTURE_H


struct s {
    int q;
    int r;
};

#endif
```

Division shpuld not return silly expressions !

(Specification)

Listing 16 – project-divers/division.h

```
#ifndef _A_H
#define _A_H
#include "structures.h"
/*@ requires a >= 0 && b >= 0;
@ behavior b :
  @ assumes b == 0;
  @ assigns \nothing;
  @ ensures \result.q == -1  && \result.r == -1 ;
@ behavior B2:
  @ assumes b != 0;
  @ assigns \nothing;
  @ ensures 0 <= \result.r;
  @ ensures \result.r < b;
  @ ensures  a ==  b * \result.q + \result.r;
*/
struct s  division(int a, int b);
#endif
```

Division shpuld not return silly expressions !

(Algorithm)

Listing 17 – project-divers/division.c

```
#include <stdio.h>
#include <stdlib.h>

#include "division.h"

struct s  division(int a, int b)
{  int rr = a;
   int qq = 0;
   struct s  silly = {-1,-1};
   struct s resu;
   if (b == 0) {
     return silly;
   }
   else
     {
  /*@
    loop invariant
    ( a ==  b*qq + rr) &&
    rr >= 0;
    loop assigns rr,qq;
    loop variant rr;
   */
   while (rr >= b) { rr = rr - b; qq=qq+1;};
   resu.q= qq;
   resu.r = rr;
  return resu;
}
}
```

Iteration Rule for PC

If $\{P \wedge B\}\mathbf{S}\{P\}$, then $\{P\}$**while B do S od**$\{P \wedge \neg B\}$.

- Prove $\{P \wedge B\}\mathbf{S}\{P\}$ or $P \wedge B \Rightarrow \{\mathbf{S}\}(P)$.
- By the iteration rule, we conclude that $\{P\}$**while B do S od**$\{P \wedge \neg B\}$ without using WLP.
- Introduction of LOOP INVARIANTS in the notation.

Listing 18 – loop.c

```
/*@ loop invariant I1;
    loop invariant I2;
    ...
    loop invariant In;
    loop assigns X;
    loop variant E;
  */
```

(Invariant de boucle)

Listing 19 – project-divers/anno6.c

```
/*@ requires a >= 0 && b >= 0;
  ensures 0 <= \result;
  ensures \result < b;
  ensures \exists integer k; a == k * b + \result;
*/
int rem(int a, int b) {
  int r = a;
  /*@
    loop invariant
    (\exists integer i; a == i * b + r) &&
    r >= 0;
    loop assigns r;
   */
  while (r >= b) { r = r - b;};
  return r;
}
```

Initial value of x $\backslash old(x)$

- $\backslash old(x)$ is the value of the variable when the function is called.
- It can be used in the postcondition of the *ensures* clause.

(Modifying variables while calling)

Listing 20 – project-divers/old1.c

```
/*@ requires \valid(a) && \valid(b);
   @ assigns *a,*b;
    @ ensures  *b == \old(*b)+\old(*a)+2;
    @ ensures  *a == \old(*a)+2;
    @ ensures  \result == 0;
*/
int old(int *a, int *b) {
  int x,y;
  x = *a;
  y = *b;
  x = x +1;
   x = x +1;
   y = y +x;
  *a = x;
  *b = y;
  return 0 ;
}
```

Expression $\backslash at(e, id)$

- $\backslash at(e, id)$ is the value of e at the control point id.
- id should occur before $\backslash at(e, id)$
- id is one of the possible expressions : Pre, Here, Old, Post, LoopEntry, LoopCurrent, Init
- $\backslash old(e)$ is equivalent to $\backslash at(e, Old)$

Exemple pour $\backslash at(e, id)$

(label Pre)

Listing 21 – project-divers/at1.c

```
/*@
  requires \valid(a) && \valid(b);
  assigns *a,*b;
  ensures  *a == \old(*a)+2;
  ensures  *b == \old(*b)+\old(*a)+2;
*/
int at1(int *a, int *b) {
//@ assert *a == \at(*a,Pre);
  *a = *a +1;
//@ assert *a == \at(*a,Pre)+1;
  *a = *a +1;
//@ assert *a == \at(*a,Pre)+2;
  *b = *b +*a;
//@ assert *a == \at(*a,Pre)+2 && *b == \at(*b,Pre)+\at(*a,Pre)+2;
  return 0;
}
```

Example for $\backslash at(e, id)$

(autre label)

Listing 22 – project-divers/at2.c

```
void f (int n) {
for (int i = 0; i < n; i++) {
/*@ assert \at(i,LoopEntry) == 0; */
int j=0;
while (j++ < i) {
/*@ assert \at(j,LoopEntry) == 0; */
/*@ assert \at(j,LoopCurrent) + 1 == j; */
} }
}
```

Example for $\backslash at(e, id)$

(otherlabel)

Listing 23 – project-divers/change1.c

```
/*@ requires \valid(a) && *a >= 0;
  @ assigns *a;
  @ ensures   *a == \old(*a)+2 && \result == 0;
*/
int  change1(int *a)
{  int x = *a;
   x = x + 2;
   *a = x;
   return 0;
}
```

Ghost variable

- A variable called *ghost* llows to model a computed cvalue useful for stating a model property : the gost variable is hidden for the computer but not for the model.
- It should not chnage the semantics of others variables and should not change the effective variables.

Wrong use of ghost variable

(Bug)

Listing 24 – project-divers/ghost2.c

```
int f (int x, int y) {
  //@ghost int z=x+y;
switch (x) {
case 0: return y;
//@ ghost case 1: z=y;
// above statement is correct.
//@ ghost case 2: { z++; break; }
// invalid, would bypass the non-ghost default
default: y++; }
return y; }

int g(int x) { //@ ghost int z=x;
if (x>0){return x;}
//@ ghost else { z++; return x; }
// invalid, would bypass the non-ghost return
return x+1; }
```

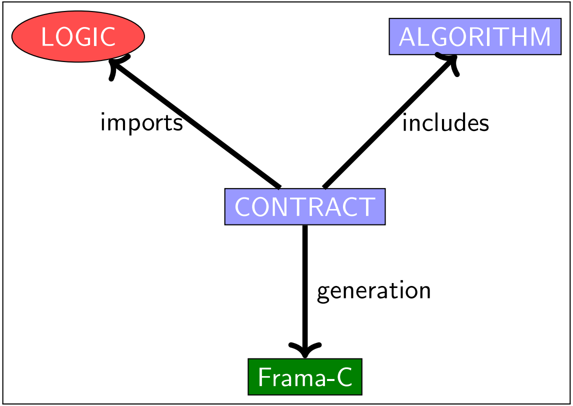
Programmation par contrat

(Ghost variable)

Listing 25 – project-divers/ghost1.c

```
/*@ requires a >= 0 && b >= 0;
  ensures 0 <= \result;
  ensures \result < b;
  ensures \exists integer k; a == k * b + \result; */
int rem(int a, int b) {
  int r = a;
 /*@ ghost   int q=0;    */
  /*@
    loop invariant
    a == q * b + r &&
    r >= 0 &&  r <= a;
    loop assigns r;
    loop assigns q;
// loop variant r;
*/
  while (r >= b) {
    r = r - b;
/*@ ghost     q = q+1;    */
  };
  return r;
}
```

Tryptic Pattern



Defining domain properties in logical theory

- predicate

Predicates - Logic - Lemma (1)

(Predicate)

Listing 26 – project-divers/predicate1.c

```
/*@ predicate is_positive(integer x) = x > 0; */

/*@ logic integer get_sign(real x) =   @ x>0.0?1:(x<0.0?-1:0);
*/
/*@ logic integer max(int x,int y) =    x>=y?x:y;
*/
```

(Lemma)

Listing 27 – project-divers/lemma1.c

```
/*@ lemma div_mul_identity:
@ \forall real x, real y; y != 0.0 ==> y*(x/y) == x; @*/

/*@  lemma div_qr:
@ \forall int a, int b; a >= 0 && b >0 ==>
\exists  int q, int r; a == b*q +r && 0<=r && r <b; @*/
```

Predicates - Logic - Lemma (2)

(Definition of fibonacci function)

Listing 28 – project-divers/predicate2.c

```
/*@ axiomatic mathfibonacci{
  @ logic integer mathfib(integer n);
  @ axiom mathfib0: mathfib(0) == 1;
  @ axiom mathfib1: mathfib(1) == 1;
  @ axiom mathfibrec: \forall integer n; n > 1
  ==> mathfib(n) ==  mathfib(n-1)+mathfib(n-2);
  @ } */
```

(Definition of gcd)

Listing 29 – project-divers/predicate3.c

```
/*@ inductive is_gcd(integer a, integer b, integer d) {
@ case gcd_zero:
@ \forall integer n; is_gcd(n,0,n);
@ case gcd_succ:
@ \forall integer a,b,d; is_gcd(b, a % b, d) ==> is_gcd(a,b,d); @}
@*/
```

Predicates - Logic - Lemma (3)

(Definition of function odd/even)

Listing 30 – project-divers/predicate4.c

```
//@ predicate pair(integer x) =  (x/2)*2==x;
//@ predicate impair(integer x) =  (x/2)*2!=x;
//@ lemma ex: \forall integer a,b; a < b ==> 2*a < 2*b;

/*@ inductive   is_gcd(integer a,integer b , integer c) {
  case zero:   \forall integer n; is_gcd(n,0,n);
   case un:   \forall integer u,v,w; u >= v ==>  is_gcd(u-v,v,w);
   case deux:   \forall integer u,v,w; u <  v ==>  is_gcd(u,v-u,w);
   }
*/
```

Loop termination

- The termination is proved by shoiwing that eaxg loop terminates.
- Any loop is characterized by an expression `expvariant(x)` called `variant` which should decrease each execution of the body :
  $\forall x_1, x_2 . b(x_1) \wedge x_1 \xrightarrow{\mathsf{S}} x_2 \Rightarrow \mathsf{expvariant}(x_1) > \mathsf{expvariant}(x_2)$

(Variant)

Listing 31 – project-divers/variant1.c

```
/*@ requires n > 0;
  terminates n > 0;

  ensures \result == 0;
*/
int code(int n) {
  int x = n;
  /*@ loop invariant x >= 0 &&  x <= n;
    loop assigns x;
    loop variant x;
   */
  while (x != 0) {
    x = x - 1;
  };
  return x;
}
```

Example of loop variant

(Variant)

Listing 32 – project-divers/variant3.c

```
int f() {
int x = 0;
int y = 10;
/*@
    loop invariant
    0 <= x < 11 && x+y == 10;
    loop variant y;
  */
while (y > 0) {
  x++;
  y--;
}
 return 0;
}
```

Recursive Call

(Variant)

Listing 33 – project-divers/variant4.c

```
g/*@ requires n <= 12;
  @ decreases n;
  @*/
int fact(int n){
  if (n <= 1) return 1;
  return n*fact(n-1);
}
```