

# Modelling Software-based Systems

## Lecture 5 Checking contracts with Event-B

Master Informatique

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# General Summary

① Programming by contract

② Verification

③ Floyd to Hoare

## ① Programming by contract

② Verification

③ Floyd to Hoare

# Verifying program correctness

A program  $P$  *satisfies* a (pre,post) contract :

- $P$  transforms a variable  $v$  from initial values  $v_0$  and produces a final value  $v_f$  :  $v_0 \xrightarrow{P} v_f$
- $v_0$  satisfies pre :  $\text{pre}(v_0)$  and  $v_f$  satisfies post :  $\text{post}(v_0, v_f)$
- $\text{pre}(v_0) \wedge v_0 \xrightarrow{P} v_f \Rightarrow \text{post}(v_0, v_f)$
- $\mathbb{D}$  est le domaine RTE de V

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```
requires pre(v0)
ensures post(v0, vf)
variables X
begin
  0 : P0(v0, v)
  instruction0
  ...
  i : Pi(v0, v)
  ...
  instructionf-1
  f : Pf(v0, v)
end
```

- $\text{pre}(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$
- $\text{pre}(v_0) \wedge P_f(v_0, v) \Rightarrow \text{post}(v_0, v)$
- For any pair of labels  $\ell, \ell'$  such that  $\ell \longrightarrow \ell'$ , one verifies that, pour any values  $v, v' \in \text{MEMORY}$ 
$$\left( \left( \begin{array}{l} \text{pre}(v_0) \wedge P_\ell(v_0, v) \\ \wedge \text{cond}_{\ell, \ell'}(v) \wedge v' = f_{\ell, \ell'}(v) \end{array} \right) \right) \Rightarrow P_{\ell'}(v_0, v')$$

# Contracts - Verification Conditions

```
contract P
variables v
requires pre( $v_0$ )
ensures post( $v_0, v_f$ )
begin
  0 :  $P_0(v_0, v)$ 
  s0
  ...
  i :  $P_i(v_0, v)$ 
  ...
  sf-1
  f :  $P_f(v_0, v)$ 
end
```

# Contracts - Verification Conditions

Verification conditions are listed as follows :

```
contract P
variables v
requires pre(v0)
ensures post(v0, vf)
begin
  0 : P0(v0, v)
  s0
  ...
  i : Pi(v0, v)
  ...
  sf-1
  f : Pf(v0, v)
end
```

- (initialisation)  
 $pre(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$
- (finalisation)  
 $pre(v_0) \wedge P_f(v_0, v) \Rightarrow post(v_0, v)$
- (induction)  
For each labels pair  $\ell, \ell'$   
such that  $\ell \longrightarrow \ell'$ , one checks that,  
for any value  $v, v' \in \text{MEMORY}$   
$$\left( \begin{array}{l} pre(v_0) \wedge P_\ell(v_0, v) \\ \wedge cond_{\ell, \ell'}(v) \wedge v' = f_{\ell, \ell'}(v) \end{array} \right) \Rightarrow P_{\ell'}(v_0, v')$$

Three kinds of verification conditions should be checked and we justify the method in the full version..

# From PAP to Rodin ...

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MACHINE  $M$

SEES  $C_0$

VARIABLES

$v, pc$

INVARIANTS

typing :  $v \in D$

control :  $pc \in L$

...

at $\ell$  :  $pc = \ell \Rightarrow P_\ell(v_0, v)$

...

th1 :  $pre(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$

th2 :  $pre(v_0) \wedge P_f(v_0, v) \Rightarrow post(v_0, v)$

...

END

...

END

# From PAP to Rodin . . .

MACHINE  $M$   
SEES  $C_0$   
VARIABLES

$v, pc$   
**INVARIANTS**

typing :  $v \in D$   
control :  $pc \in L$

...  
 $at\ell : pc = \ell \Rightarrow P_\ell(v_0, v)$

...  
 $th1 : pre(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$   
 $th2 : pre(v_0) \wedge P_f(v_0, v)$   
 $\Rightarrow post(v_0, v)$

...  
**END**

...  
**END**

MACHINE  $M$   
**EVENTS**  
**INITIALISATION**  
**BEGIN**

$(pc, v) : | \left( \begin{array}{l} pc' = l_0 \wedge v' = v_0 \\ \wedge pre(v_0) \end{array} \right)$   
**END**

...  
 $e(\ell, \ell')$   
**WHEN**

$pc = \ell$   
 $cond_{\ell, \ell'}(v)$   
**THEN**  
 $pc := \ell'$   
 $v := f_{\ell, \ell'}(v)$   
**END**

...  
**END**

## (Induction Principle (I))

A property  $S(z_0, z)$  is a safety for an annotated program P if, and only if, there exists a property  $I(z_0, z)$  satisfying :

- ①  $\forall z_0, z \in L \times D. init(z_0) \wedge z = z_0 \Rightarrow I(z_0, z)$
- ②  $\forall z_0, z, z' \in L \times D. init(z_0) \wedge I(z_0, z) \wedge (z \xrightarrow{P} z') \Rightarrow I(z_0, z')$
- ③  $\forall z_0, z \in L \times D. init(z_0) \wedge I(z_0, z) \Rightarrow S(z_0, z)$

## (Induction Principle (II))

A property  $S(\ell_0, x_0, \ell, x)$  is a safety property for an annotated program P if, and only if, there exists a property  $I(\ell_0, x_0, \ell, x)$  satisfying :

- ①  $\forall \ell_0 \in L, x_0 \in D. \ell_0 \in L_0 \wedge pre(x_0) \wedge x = x_0 \wedge pc = \ell_0 \Rightarrow J(\ell_0, x_0, \ell, x)$
- ②  $\forall \ell, \ell' \in L, x, x_0 \in D. \ell_0 \in L_0 \wedge pre(x_0) \wedge J(\ell_0, x_0, \ell, x) \wedge BA(e(\ell, \ell'), )(\ell, x, \ell', x') \Rightarrow J(\ell_0, x_0, \ell', x')$
- ③  $\forall \ell_0, \ell \in L, x_0, x \in D. pre(x_0) \wedge \ell_0 \in L_0 \wedge J(\ell_0, x_0, \ell, x) \Rightarrow S(\ell_0, x_0, \ell, x)$

## (Induction Principle (II))

A property  $S(\ell_0, x_0, \ell, x)$  is a safety property for an annotated program P if, and only if, there exists a property  $I(\ell_0, x_0, \ell, x)$  satisfying :

- ①  $\forall \ell_0 \in L, x_0 \in D. \ell_0 \in L_0 \wedge pre(x_0) \wedge x = x_0 \wedge pc = \ell_0 \Rightarrow J(\ell_0, x_0, \ell, x)$
- ②  $\forall \ell, \ell' \in L, x, x_0 \in D. \ell_0 \in L_0 \wedge pre(x_0) \wedge J(\ell_0, x_0, \ell, x) \wedge BA(e(\ell, \ell'), )(\ell, x, \ell', x') \Rightarrow J(\ell_0, x_0, \ell', x')$
- ③  $\forall \ell_0, \ell \in L, x_0, x \in D. pre(x_0) \wedge \ell_0 \in L_0 \wedge J(\ell_0, x_0, \ell, x) \Rightarrow S(\ell_0, x_0, \ell, x)$

## (Induction Principle (III))

A property  $S(x_0, \ell, x)$  is a safety for an annotated program P with one entry point if, and only if, there exists a property  $I(x_0, \ell, x)$  satisfying :

- ①  $\forall x_0 \in D. pre(x_0) \wedge x = x_0 \wedge \ell = \ell_0 \Rightarrow J(x_0, \ell, x)$
- ②  $\forall \ell, \ell' \in L, x, x_0 \in D. pre(x_0) \wedge J(x_0, \ell, x) \wedge BA(e(\ell, \ell'), )(\ell, x, \ell', x') \Rightarrow J(x_0, \ell', x')$
- ③  $\forall \ell \in L, x_0, x \in D. pre(x_0) \wedge J(x_0, \ell, x) \Rightarrow S(x_0, \ell, x)$



## (Soundness of the method)

If the initialisation init, the generalisation gen and the step induction are proved to be correct by the Rodin platform, the property  $S(x_0, \ell, x)$  is a correct safety property for the program P. In particular, one can handle the partial correctness and the run time error safety properties.

## (Soundness of the method)

If the initialisation init, the generalisation gen and the step induction are proved to be correct by the Rodin platform, the property  $S(x_0, \ell, x)$  is a correct safety property for the program P. In particular, one can handle the partial correctness and the run time error safety properties.

- Contract and verification conditions are translated into Event-B and are discharged by Rodin and its provers.
- Verification conditions are derived from Floyd's method.
- Annotation as assertion

# A short example

s

```
contract SIMPLE
variables x
requires  $x_0 \in \mathbb{N}$ 
ensures  $x_f = 0$ 
begin
 $\ell_0 : \{0 \leq x \leq x_0 \wedge x_0 \in \mathbb{N}\}$ 
while  $0 < x$  do
     $\ell_1 : \{0 < x \wedge x \leq x_0 \wedge x_0 \in \mathbb{N}\}$ 
     $x := x - 1;$ 
od
 $\ell_2 : \{x = 0\}$  end
```

```
Event Init
THEN
    act1 :  $x := x_0$ 
    act2 :  $l := l_0$ 
```

```
Event el0l1
WHEN
    grd1 :  $l = l_0$ 
    grd2 :  $0 < x$ 
THEN
    act1 :  $l := l_1$ 
```

## INVARIANTS

```
inv1 :  $x \in \mathbb{N}$ 
inv2 :  $l \in L$ 
inv3 :  $l = l_0 \Rightarrow$ 
 $0 \leq x \wedge x \leq x_0 \wedge x_0 \in \mathbb{N}$ 
inv4 :  $l = l_1 \Rightarrow$ 
 $0 < x \wedge x \leq x_0 \wedge x_0 \in \mathbb{N}$ 
inv5 :  $l = l_2 \Rightarrow x = 0$ 
requires :  $x_0 \in \mathbb{N} \wedge x = x_0$ 
 $\Rightarrow x = x_0 \wedge x_0 \in \mathbb{N}$ 
ensures :  $x = 0 \wedge x = x_0$ 
 $\Rightarrow x = 0$ 
```

## Event el0l2

```
WHEN
    grd1 :  $l = l_0$ 
    grd2 :  $\neg(0 < x)$ 
THEN
    act1 :  $l := l_2$ 
```

## Event el1l0

```
WHEN
    grd1 :  $l = l_1$ 
THEN
    act1 :  $l := l_0$ 
    act2 :  $x := x - 1$ 
```

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# Annotation of programs

$$\begin{aligned}\ell : \{P_\ell(v)\} \\ cond_{\ell, \ell'}(v) \longrightarrow v := f_{\ell, \ell'}(v) \\ \ell' : \{P_{\ell'}(v)\}\end{aligned}$$
$$\begin{aligned}\ell_0^1 : \{x = 0\} \\ x := x + 1; \\ \ell_0^1 : \{x = 1\}\end{aligned}$$
$$\begin{aligned}e(\ell, \ell') \\ \text{WHEN} \\ c = \ell \\ cond_{\ell, \ell'}(v) \\ \text{THEN} \\ c := \ell' \\ v := f_{\ell, \ell'}(v) \\ \text{END}\end{aligned}$$

- $v$  is the state memory variable or list of memory variables ;  $v$  includes the local variables and the results variables.
- $c$  is a new variable which is modelling the control flow and its type is LOCATIONS.
- $e(\ell, \ell')$  is simulating the computation flow starting from  $\ell$  and moving to  $\ell'$  ;  $v$  is updated.

# From annotations to invariants

## INVARIANTS

$inv_i : c \in \text{LOCATIONS}$

$inv_j : v \in Type$

...

$inv_k : c = \ell \Rightarrow P_\ell(v)$

$inv_m : c = \ell' \Rightarrow P_{\ell'}(v)$

...

$th_n : A(c, v)$

- $Type$  is the type of the variables  $v$  and is a set of possible values defined in the context  $C$ .
- The annotation is giving us for free the conditions satisfied by  $v$  when the control is in  $\ell$ , (resp. in  $\ell'$ ).
- $A(c, v)$  is a safety property that we are supposed to check and the case of Event-B, it is a theorem.

# Partial correctness using Event-B models

For each pair of successive labels  $\ell, \ell'$ , the three statements are equivalent :

- $P_\ell(v) \wedge cond_{\ell,\ell'}(v) \wedge v' = f_{\ell,\ell'}(v) \Rightarrow P_{\ell'}(v')$
- $I(c, v) \wedge c = \ell \wedge cond_{\ell,\ell'}(v) \wedge c' = \ell' \wedge v' = f_{\ell,\ell'}(v) \Rightarrow (c' = \ell' \Rightarrow P_{\ell'}(v'))$
- $I(c, v) \wedge BA(e(\ell, \ell'))(c, v, c', v') \Rightarrow (c' = \ell' \Rightarrow P_{\ell'}(v'))$

L

et AA an annotated algorithm with precondition  $\mathbf{pre}(\text{AA})(v)$  and postcondition  $\mathbf{post}(\text{AA})(v_0, v)$ . Let the context  $C$  and the machine  $M$  generated from AA using the construction given previously. We assume that  $\ell_0$  is the first label and  $\ell_e$  is the last label. We add the following safety properties in the machine  $M$  :

- $c = \ell_0 \wedge \mathbf{pre}(\text{AA})(v) \Rightarrow P_{\ell_0}(v)$
- $c = \ell_e \Rightarrow (P_{\ell_e}(v) \Rightarrow \mathbf{post}(\text{AA})(v_0, v))$

If proof obligations are discharged, then the annotated algorithm AA is partially correct with respect to its pre/post specification.



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- $\forall x_f, x_0. \text{pre}(x_0) \wedge x_0 \xrightarrow{\text{P}} x_f \Rightarrow \text{post}(x_0, x_f)$

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- $\forall x_f, x_0. \text{pre}(x_0) \wedge x_0 \xrightarrow{\text{P}} x_f \Rightarrow \text{post}(x_0, x_f)$
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- $\forall x_f, x_0. \text{pre}(x_0) \Rightarrow x_0 \xrightarrow{\text{P}} x_f \Rightarrow \text{post}(x_0, x_f)$
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- $\forall x_0. \text{pre}(x_0) \Rightarrow \forall x_f. x_0 \xrightarrow{\text{P}} x_f \Rightarrow \text{post}(x_0, x_f)$

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- $\forall x_0. \text{pre}(x_0) \Rightarrow [P]\text{post}(x_0, x_f)$
- wlp calculus is introduced

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- $[x := e]P(x) = P[x \mapsto e]$

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- $\forall x_0. \text{pre}(x_0) \Rightarrow [P]\text{post}(x_0, x_f)$
- wlp calculus is introduced
- $[x := e]P(x) = P[x \mapsto e]$
- $[\text{if } b(x) \text{ then } S1 \text{ else } S2]P(x) = b(x) \wedge [S1]P(x) \vee \text{not } b(x) [S2]P(x)$

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- $\forall x_f, x_0. \text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
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- Frama-c uses the HOARE logic for defining the verification conditions as R. Leino in DAFNY.

# From Floyd to Hoare

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- wlp calculus is introduced
- $[x := e]P(x) = P[x \mapsto e]$
- $[\text{if } b(x) \text{ then } S1 \text{ else } S2 ]P(x) = b(x) \wedge [S1]P(x) \vee \neg b(x) [S2]P(x)$
- Frama-c uses the HOARE logic for defining the verification conditions as R. Leino in DAFNY.
- Questions of termination require the wp calculus . . .