

Modelling Software-based Systems

Lecture 2

Proof Obligation Generation

Master Informatique

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8 décembre 2025
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General Summary

- ① Overview of machines, contexts
and proof obligations
- ② Proof Obligations for Contexts
and Machines

PO thm/THM (context)

PO thm/THM (machine)

PO evt/inv/INV

PO evt/act/FIS

PO evt/NAT

PO NAT

PO evt/VAR (arithmetic)

PO evt/VAR (set-theoretic)

- ③ Proof Obligations for Refinement

PO evt/grd/GRD

PO evt/act/SIM

PO evt/NAT

PO NAT

PO evt/VAR (arithmetic)

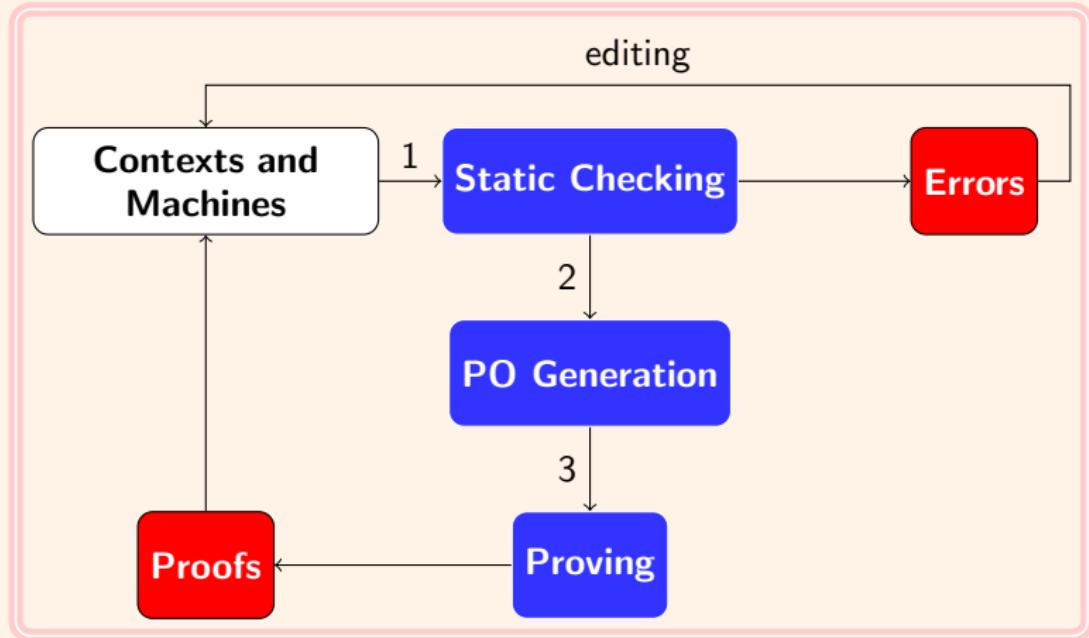
PO evt/VAR (set-theoretic)

PO evt/x/WFIS

Current Summary

- ① Overview of machines, contexts and proof obligations
- ② Proof Obligations for Contexts and Machines
- ③ Proof Obligations for Refinement

Analysis of the Event-B Models



Machines en Event B

```
MACHINE
  m
REFINES
  am
SEES
  c
VARIABLES
  u
INVARIANTS
  I(s, c, u)
THEOREMS
  Q(s, c, u)
VARIANT
  exp(s, c, u)
EVENTS
  INITIALIZATION
  ...
  e
  ...
END
```

Machines en Event B

```
MACHINE
  m
REFINES
  am
SEES
  c
VARIABLES
  u
INVARIANTS
  I(s, c, u)
THEOREMS
  Q(s, c, u)
VARIANT
  exp(s, c, u)
EVENTS
  INITIALIZATION
  ...
  e
  ...
END
```

- $\Gamma(m)$: environment for the machine m defined by the context c and it provides a list of seen axioms $Ax(s, c)$ and a list of seen theorems $Th(s, c)$ for the sets s and constants c .
- $\Gamma(m) \vdash \forall u.\text{INIT}(s, c, u) \Rightarrow \text{I}(s, c, u)$
- For each event e in E :
 $\Gamma(m) \vdash \forall u, u'.\text{I}(s, c, u) \wedge BA(e)(u, u') \Rightarrow \text{I}(u')$
- For each event e in E :
 $\Gamma(m) \vdash \forall u.\text{I}(s, c, u) \wedge GRD(e)(s, c, u) \Rightarrow \exists u'.BA(e)(u, u')$
- $\Gamma(m) \vdash \forall u.\text{I}(s, c, u) \Rightarrow Q(s, c, u)$
- Generated proof obligations are derived from those conditions.

Three kinds of events

Events are divided into three kinds of events :

- An event is **ordinary** and, when it is observed, it modifies variables according to a guard and an action.
- An event is **anticipated** and, when it is observed, it means that something is observed but later in the further refinement.
- An event is **convergent** and, when it is observed, it decreases a variant which is member of naturals or is a set..

Checking the well formation of Event-B expressions

- Event-B expressions are contexts, machines, properties, equations, set-theoretical expressions ...
- e is an Event-B expression and $\text{wd}(e)$ is a logical property expressing the well definition of e .
- $\text{wd}(1 = 2) \triangleq \text{wd}(1) \wedge \text{wd}(2)$
- $\text{wd}(a/b) \triangleq b \neq 0 \wedge \text{wd}(a) \wedge \text{wd}(b)$
- $\text{wd}(f(g)) \triangleq g \in \text{dom}(f) \wedge f \in A \rightarrow B$

- ① Overview of machines, contexts and proof obligations
- ② Proof Obligations for Contexts and Machines
- ③ Proof Obligations for Refinement

PO thm/THM (context)

CONTEXTS

$\stackrel{c}{\text{EXTENDS}}$

$\stackrel{ac}{\text{SETS}}$

$\stackrel{s}{\text{CONSTANTS}}$

$\stackrel{c}{\text{AXIOMS}}$

$Ax(s, c)$

THEOREMS

$th_1 : P_1(s, c)$

...

$th_n : P_n(s, c)$

$th : P(s, c)$

...

END

s	<i>seen sets</i>
c	<i>seen constants</i>
$Ax(s, c)$	<i>seen axioms</i>
$Th(s, c)$	<i>previous proved theorems</i>
$P(s, c)$	$Th(s, c) = \{P_i(s, c) i \in 1..n\}$ <i>property over s and c</i>

PO th/THM

$Ax(s, c), Th(s, c) \vdash P(s, c)$

PO thm/THM (machine)

MACHINE

m

VARIABLES

u

INVARIANTS

$I(s, c, u)$

THEOREMS

$Q(s, c, u)$

$th : P(s, c, u)$

...

END

s	<i>seen sets</i>
c	<i>seen constants</i>
u	<i>variables</i>
$Ax(s, c)$	<i>seen axioms</i>
$Th(s, c)$	<i>seen theorems</i>
$I(s, c, u)$	<i>invariants</i>
$Q(s, c, u)$	<i>theorems</i>
$P(s, c, u)$	<i>property over s, c and u</i>

PO th/THM

$$Ax(s, c), Th(s, c), I(s, c, u) \vdash P(s, c, u)$$

PO evt/inv/INV

```
EVENT evt
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
```

$$\begin{aligned}BA(\text{evt}) &\hat{=} \\ \exists x. & \left(\begin{array}{l} \wedge G(x, s, c, u) \\ \wedge BAP(x, s, c, u, u') \end{array} \right) \\ GRD(\text{evt}) &\hat{=} G(x, s, c, u) \\ ACT(\text{evt}) &\hat{=} BAP(x, s, c, u, u')\end{aligned}$$

<i>s</i>	<i>seen sets</i>
<i>c</i>	<i>seen constants</i>
<i>u</i>	<i>variables</i>
<i>Ax(s, c)</i>	<i>seen axioms</i>
<i>Th(s, c)</i>	<i>seen theorems</i>
<i>I(s, c, u)</i>	<i>invariants</i>
<i>Q(s, c, u)</i>	<i>theorems</i>
<i>evt</i>	<i>event name</i>
<i>x</i>	<i>event parameter</i>
<i>G(x, s, c, u)</i>	<i>event guard</i>
<i>BAP(x, s, c, u, u')</i>	<i>event before-after predicate</i>
<i>inv : inv(s, c, u')</i>	<i>specific modified invariant</i>

PO evt/inv/INV

$$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), BAP(x, s, c, u, u') \vdash inv(s, c, u')$$

PO evt/act/FIS

```
EVENT evt
ANY x WHERE
  G(x, s, c, u)
THEN
  u : |BAP(x, s, c, u, u')
END
```

$BA(\text{evt}) \hat{=} \left(\begin{array}{l} \wedge G(x, s, c, u) \\ \wedge BAP(x, s, c, u, u') \end{array} \right)$

$GRD(\text{evt}) \hat{=} G(x, s, c, u)$

$ACT(\text{evt}) \hat{=} G(x, s, c, u)$

$BAP(x, s, c, u, u')$

s	<i>seen sets</i>
c	<i>seen constants</i>
u	<i>variables</i>
$Ax(s, c)$	<i>seen axioms</i>
$Th(s, c)$	<i>seen theorems</i>
$I(s, c, u)$	<i>invariants</i>
$Q(s, c, u)$	<i>theorems</i>
evt	<i>event name</i>
x	<i>event parameter</i>
$G(x, s, c, u)$	<i>event guard</i>
$BAP(x, s, c, u, u')$	<i>event before-after predicate</i>

PO evt/act/FIS

$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), \vdash \exists u'. BAP(x, s, c, u, u')$

PO evt/NAT

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
...
VARIANT
  exp(s, c, u)
```

<i>s</i>	<i>seen sets</i>
<i>c</i>	<i>seen constants</i>
<i>u</i>	<i>abstract variables</i>
$Ax(s, c)$	<i>seen axioms</i>
$Th(s, c)$	<i>seen theorems</i>
$I(s, c, u)$	<i>abstract invariants</i>
$Q(s, c, u), R(s, c, u, v)$	<i>abstract and concrete theorems</i>
evt, ce	<i>event name</i>
<i>x</i>	<i>event parameters</i>
$G(x, s, c, u)$	<i>abstract event guard</i>
$BAP(x, s, c, u, u')$	<i>event before-after predicate</i>
$exp(s, c, u)$	<i>arithmetric expression</i>

PO evt/NAT $Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u) \vdash exp(s, c, u) \in \mathbb{N}$

PO evt/NAT

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
  ...
VARIANT
  exp(s, c, u)
```

s	<i>seen sets</i>
c	<i>seen constants</i>
u, v	<i>abstract variables</i>
$Ax(s, c)$	<i>seen axioms</i>
$Th(s, c)$	<i>seen theorems</i>
$I(s, c, u)$	<i>abstract invariants</i>
$Q(s, c, u), R(s, c, u, v)$	<i>abstract and concrete theorems</i>
evt, ce	<i>event name</i>
x	<i>event parameters</i>
$G(x, s, c, u)$	<i>abstract event guard</i>
$BAP(x, s, c, u, u')$	<i>event before-after predicate</i>
$setexp(s, c, u)$	<i>set expression</i>

PO evt/NAT

$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u) \vdash finite(setexp(s, c, u))$

PO evt/VAR

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
  ...
VARIANT
  exp(s, c, u)
```

s	<i>seen sets</i>
c	<i>seen constants</i>
u, v	<i>abstract and concrete variables</i>
$Ax(s, c)$	<i>seen axioms</i>
$Th(s, c)$	<i>seen theorems</i>
$I(s, c, u)$	<i>abstract invariants</i>
evt	<i>event name</i>
x	<i>event parameters</i>
$G(x, s, c, u)$	<i>abstract event guard</i>
$BAP(x, s, c, u, u')$	<i>event before-after predicate</i>
$exp(s, c, u)$	<i>arithmetric expression</i>

PO evt/VAR

$$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), BAP(x, s, c, u, u') \vdash \\ exp(s, c, u') < exp(s, c, u)$$

PO evt/VAR

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
...
VARIANT
  setexp(s, c, u)
```

<i>s</i>	<i>seen sets</i>
<i>c</i>	<i>seen constants</i>
<i>u, v</i>	<i>abstract and concrete variables</i>
$Ax(s, c)$	<i>seen axioms</i>
$Th(s, c)$	<i>seen theorems</i>
$I(s, c, u)$	<i>abstract invariants</i>
evt	<i>event name</i>
x	<i>event parameters</i>
$G(x, s, c, u)$	<i>abstract event guard</i>
$BAP(x, s, c, u, u')$	<i>event before-after predicate</i>
$setexp(s, c, u)$	<i>set-theoretic expression</i>

PO evt/VAR

$$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), BAP(x, s, c, u, u') \vdash \\ setexp(s, c, u') \subset setexp(s, c, u)$$

- ① Overview of machines, contexts and proof obligations
- ② Proof Obligations for Contexts and Machines
- ③ Proof Obligations for Refinement

PO evt/grd/GRD

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |ABAP(x, s, c, u, u')
  END
```

```
EVENT ce
  REFINES
    ae
  ANY y WHERE
    H(y, s, c, v)
  WITH
    x : W(x, y, s, c, v)
  THEN
    v : |CBAP(y, s, c, v, v')
  END
```

s	<i>seen sets</i>
c	<i>seen constants</i>
u, v	<i>abstract and concrete variables</i>
$Ax(s, c)$	<i>seen axioms</i>
$Th(s, c)$	<i>seen theorems</i>
$I(s, c, u)$	<i>abstract invariants</i>
$J(s, c, u, v)$	<i>concrete invariants</i>
$Q(s, c, u), R(s, c, u, v)$	<i>abstract and concrete theorems</i>
ae, ce	<i>abstract and concrete event names</i>
x, y	<i>event parameters</i>
$G(x, s, c, u)$	<i>abstract event guard</i>
$H(y, s, c, v)$	<i>concrete event guard</i>
$ABAP(x, s, c, u, u')$	<i>abstract event before-after predicate</i>
$CBAP(x, s, c, u, u')$	<i>concrete event before-after predicate</i>
$W(x, y, s, c, v)$	witness predicate

PO evt/grd/GRD

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), W(x, y, s, c, v), H(y, s, c, v), \vdash$
 $G(x, s, c, u, u')$

PO evt/act/SIM

```

EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |ABAP(x, s, c, u, u')
  END

EVENT ce
  REFINES
    ae
  ANY y WHERE
    H(y, s, c, v)
  WITH
    x : WP(x, y, s, c, v)
    u' : WV(y, u', s, c, v)
  THEN
    v : |CBAP(y, s, c, v, v')
  END

```

<i>s</i>	<i>seen sets</i>
<i>c</i>	<i>seen constants</i>
<i>u, v</i>	<i>abstract and concrete variables</i>
<i>Ax(s, c)</i>	<i>seen axioms</i>
<i>Th(s, c)</i>	<i>seen theorems</i>
<i>I(s, c, u)</i>	<i>abstract invariants</i>
<i>J(s, c, u, v)</i>	<i>concrete invariants</i>
<i>Q(s, c, u), R(s, c, u, v)</i>	<i>abstract and concrete theorems</i>
<i>ae, ce</i>	<i>abstract and concrete event names</i>
<i>x, y</i>	<i>event parameters</i>
<i>G(x, s, c, u)</i>	<i>abstract event guard</i>
<i>H(y, s, c, v)</i>	<i>concrete event guard</i>
<i>ABAP(x, s, c, u, u')</i>	<i>abstract event before-after predicate</i>
<i>CBAP(x, s, c, u, u')</i>	<i>concrete event before-after predicate</i>
<i>WP(x, y, s, c, v)</i>	<i>witness parameter predicate</i>
<i>WV(y, u', s, c, v)</i>	<i>witness variable predicate</i>

PO evt/act/SIM

$$\left(\begin{array}{l}
 Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v) \\
 WP(x, y, s, c, v), WV(y, u', s, c, v) \\
 H(y, s, c, v), CBAP(y, s, c, v, v')
 \end{array} \right) \vdash ABAP(x, s, c, u, u')$$

```

EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
...
VARIANT
  exp(s, c, u)

```

<i>s</i>	<i>seen sets</i>
<i>c</i>	<i>seen constants</i>
<i>u, v</i>	<i>abstract and concrete variables</i>
$Ax(s, c)$	<i>seen axioms</i>
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$J(s, c, u, v)$	<i>concrete invariants</i>
$Q(s, c, u), R(s, c, u, v)$	<i>abstract and concrete theorems</i>
evt, ce	<i>event name</i>
x	<i>event parameters</i>
$G(x, s, c, u)$	<i>abstract event guard</i>
$BAP(x, s, c, u, u')$	<i>event before-after predicate</i>
$exp(s, c, u)$	<i>arithmetric expression</i>

PO evt/NAT

$$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u) \vdash exp(s, c, u) \in \mathbb{N}$$

PO evt/act/SIM

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
...
VARIANT
  exp(s, c, u)
```

s	<i>seen sets</i>
c	<i>seen constants</i>
u, v	<i>abstract and concrete variables</i>
$Ax(s, c)$	<i>seen axioms</i>
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$Q(s, c, u), R(s, c, u, v)$	<i>abstract and concrete theorems</i>
evt, ce	<i>event name</i>
x	<i>event parameters</i>
$G(x, s, c, u)$	<i>abstract event guard</i>
$BAP(x, s, c, u, u')$	<i>event before-after predicate</i>
$setexp(s, c, u)$	<i>set expression</i>

PO evt/NAT $Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u) \vdash$
 $finite(setexp(s, c, u))$

PO evt/VAR

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
  ...
VARIANT
  exp(s, c, u)
```

s	<i>seen sets</i>
c	<i>seen constants</i>
u, v	<i>abstract and concrete variables</i>
$Ax(s, c)$	<i>seen axioms</i>
$Th(s, c)$	<i>seen theorems</i>
$I(s, c, u)$	<i>abstract invariants</i>
$J(s, c, u, v)$	<i>concrete invariants</i>
$Q(s, c, u), R(s, c, u, v)$	<i>abstract and concrete theorems</i>
evt, ce	<i>event name</i>
x	<i>event parameters</i>
$G(x, s, c, u)$	<i>abstract event guard</i>
$BAP(x, s, c, u, u')$	<i>event before-after predicate</i>
$exp(s, c, u)$	<i>arithmetric expression</i>

PO evt/VAR

$$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u), BAP(x, s, c, u, u') \vdash \\ exp(s, c, u') < exp(s, c, u)$$

PO evt/VAR

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |BAP(x, s, c, u, u')
  END
  ...
VARIANT
  setexp(s, c, u)
```

s	<i>seen sets</i>
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u, v	<i>abstract and concrete variables</i>
$Ax(s, c)$	<i>seen axioms</i>
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$I(s, c, u)$	<i>abstract invariants</i>
$J(s, c, u, v)$	<i>concrete invariants</i>
$Q(s, c, u), R(s, c, u, v)$	<i>abstract and concrete theorems</i>
evt, ce	<i>event name</i>
x	<i>event parameters</i>
$G(x, s, c, u)$	<i>abstract event guard</i>
$BAP(x, s, c, u, u')$	<i>event before-after predicate</i>
$setexp(s, c, u)$	<i>set-theoretic expression</i>

PO evt/VAR

$$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u), BAP(x, s, c, u, u') \vdash \\ setexp(s, c, u') \subset setexp(s, c, u)$$

PO evt/x/WFIS

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u : |ABAP(x, s, c, u, u')
  END
```

```
EVENT ce
  REFINES
    ae
  ANY y WHERE
    H(y, s, c, v)
  WITH
    x : WP(x, y, s, c, v)
    u' : WV(y, u', s, c, v)
  THEN
    v : |CBAP(y, s, c, v, v')
  END
```

<i>s</i>	<i>seen sets</i>
<i>c</i>	<i>seen constants</i>
<i>u, v</i>	<i>abstract and concrete variables</i>
$Ax(s, c)$	<i>seen axioms</i>
$Th(s, c)$	<i>seen theorems</i>
$I(s, c, u)$	<i>abstract invariants</i>
$J(s, c, u, v)$	<i>concrete invariants</i>
$Q(s, c, u), R(s, c, u, v)$	<i>abstract and concrete theorems</i>
<i>ae, ce</i>	<i>abstract and concrete event names</i>
<i>x, y</i>	<i>event parameters</i>
$G(x, s, c, u)$	<i>abstract event guard</i>
$H(y, s, c, v)$	<i>concrete event guard</i>
$ABAP(x, s, c, u, u')$	<i>abstract event before-after predicate</i>
$CBAP(x, s, c, u, u')$	<i>concrete event before-after predicate</i>
$WP(x, y, s, c, v)$	witness parameter predicate
$WV(y, u', s, c, v)$	witness variable predicate

PO evt/x/WFIS

$$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), H(y, s, c, v) \vdash \\ \exists x. WP(x, y, s, c, v)$$