
Cours MALG & MOVEX

Vérification mécanisée de contrats (II) (The ANSI/ISO C Specification Language (ACSL))

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- ① Programs as Predicate Transformers
- ② Annotations
- ③ Contracts

Extending C programming
language by contracts
Playing with variables
Ghost Variables

① Programs as Predicate Transformers

② Annotations

③ Contracts

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1 Programs as Predicate Transformers

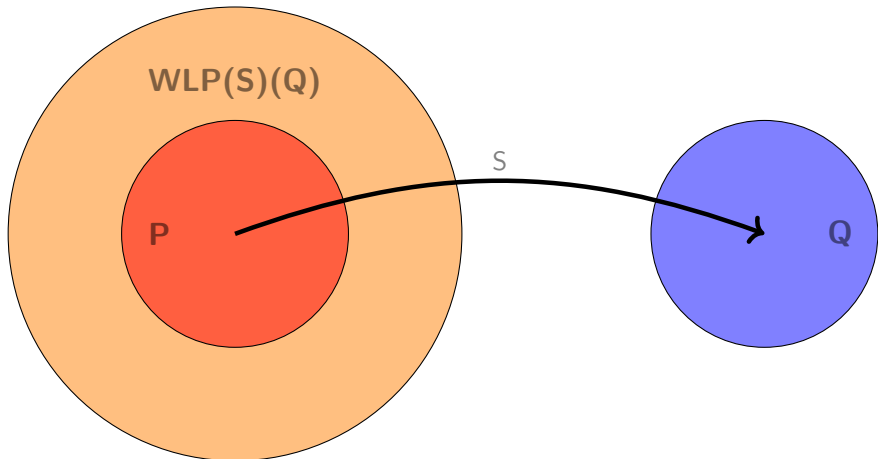
2 Annotations

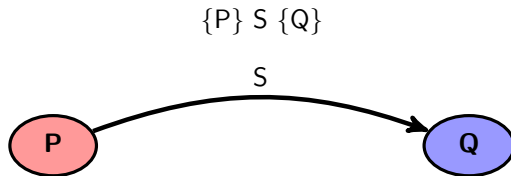
3 Contracts

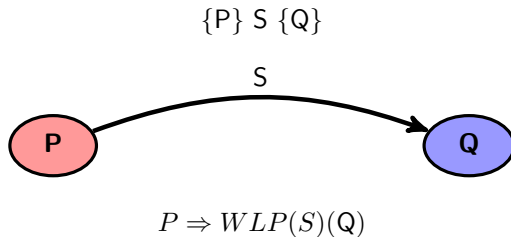
Extending C programming language by contracts

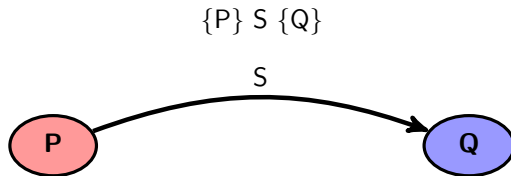
Playing with variables

Ghost Variables









$$P \Rightarrow WLP(S)(Q)$$

Computing $WLP(S)(Q)$?

① Programs as Predicate Transformers

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Writing a simple contract

variables x

requires $x \geq 0 \wedge x \leq 10$;

ensures $\begin{cases} x \% 2 = 0 \Rightarrow 2 \cdot \text{result} = x +; \\ x \% 2 \neq 0 \Rightarrow 2 \cdot \text{result} = x - 1; \end{cases}$

begin

int y ;

$y = x / 2$;

return(y);

end

► result is the value returned by the command *return*(y).

► *return*(y) is equivalent to $\text{result} := y$.

(Writing a simple contract.)

Listing 1 – project-divers/annotation.c

```
/*@ requires x >= 0 && x <= 10;
   @ assigns \nothing;
   @ ensures x % 2 == 0 ==> 2*\result == x;
   @ ensures x % 2 != 0 ==> 2*\result == x-1;
   @*/
int annotation(int x)
{
    int y;
    y = x / 2;
    return(y);
}
```

(Writing a simple contract.)

Listing 2 – project-divers/annotationwp.c

```
/*@ requires 0 <= x && x <= 10;
   @ assigns \nothing;
   @ ensures x % 2 == 0 ==> 2*\result == x;
   @ ensures x % 2 != 0 ==> 2*\result == x-1;
   @*/
int annotation(int x)
{
  /*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
  /*@ assert x % 2 != 0 ==> 2* (x / 2) == x-1; */
  int y;
  /*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
  /*@ assert x % 2 != 0 ==> 2* (x / 2) == x-1; */
  y = x / 2;
  /*@ assert x % 2 == 0 ==> 2*y == x; */
  /*@ assert x % 2 != 0 ==> 2*y == x-1; */
  return(y);
  /*@ assert x % 2 == 0 ==> 2*y == x; */
  /*@ assert x % 2 != 0 ==> 2*y == x-1; */
}
```

Property to check

$$x \geq 0 \wedge x \leq 10 \Rightarrow \begin{cases} x \% 2 \neq 0 \Rightarrow 2 \cdot (x/2) = x-1 \\ x \% 2 = 0 \Rightarrow 2 \cdot (x/2) = x \end{cases}$$

(Checking the precondition.)

Listing 3 – project-divers/annotation0.c

```
/*@ requires x >= 0 && x < 0;  
  @ assigns \nothing;  
  @ ensures \result == 0;  
  @*/  
int annotation0(int x)  
{  
  int y;  
  y = y / (x-x);  
  return(y);  
}
```

(Checking the precondition.)

Listing 4 – project-divers/annotation0wp.c

```
/*@ requires x >= 0 && x < 0;  
  @ assigns \nothing;  
  @ ensures \result == 0;  
  @*/  
int annotation(int x)  
{  
  /*@ assert y / (x-x) == 0; */  
  int y;  
  /*@ assert y / (x-x) == 0; */  
  y = y / (x-x);  
  /*@ assert y == 0; */  
  return(y);  
  /*@ assert y == 0; */  
}
```

Property to check

$$x \geq 0 \wedge x < 0 \Rightarrow y / (x - x) = 0$$

Transformations of annotated programs (1)

```
//@ assert  $P(v0, v)$  :  
 $S1; S2$   
//@ assert  $Q(v0, v)$  :
```

- ▶ Applying the property :
 $wp(S1; S2)(A) =$
 $wp(S1)(wp(S2)(A))$

```
//@ assert  $P(v0, v)$  :  
 $S1$ ;  
//@ assert  $wp(S2)(Q(v0, v))$  :  
 $S2$ ;  
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
//@ assert  $xp(S1)(wp(S2)(Q(v0, v)))$  :  
 $S1$ ;  
//@ assert  $wp(S2)(Q(v0, v))$  :  
 $S2$ ;  
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
     $S1$   
ELSE  
     $S2$   
FI  
//@ assert  $Q(v0, v)$  :
```

- Applying the property :
 $wp(if(B, S1, S2))(A) =$
 $b \wedge wp(S1)(A) \vee \neg b \wedge$
 $wp(S2)(A).$

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
     $S1$   
ELSE  
     $S2$   
FI  
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
     $S1$   
//@ assert  $Q(v0, v)$  :  
ELSE  
     $S2$   
//@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```


Transformations of annotated programs (2)

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
   $S1$   
//@ assert  $Q(v0, v)$  :  
ELSE  
   $S2$   
//@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

Transformations of annotated programs (2)

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
   $S1$   
//@ assert  $Q(v0, v)$  :  
ELSE  
   $S2$   
//@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
//@ assert  $B \wedge wp(S2)(Q(v0, v))$  :  
   $S1$   
//@ assert  $Q(v0, v)$  :  
ELSE  
//@ assert  $\neg B \wedge wp(S2)(Q(v0, v))$  :  
   $S2$   
//@ assert  $Q(v0, v)$  :
```

Transformations of annotated programs (2)

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
   $S1$   
  //@ assert  $Q(v0, v)$  :  
ELSE  
   $S2$   
  //@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
  //@ assert  $B \wedge wp(S2)(Q(v0, v))$  :  
   $S1$   
  //@ assert  $Q(v0, v)$  :  
ELSE  
  //@ assert  $\neg B \wedge wp(S2)(Q(v0, v))$  :  
   $S2$   
  //@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
  //@ assert  $b \wedge wp(S1)(Q(v0, v))$  :  
   $S1$   
  //@ assert  $Q(v0, v)$  :  
ELSE  
  //@ assert  $\neg b \wedge wp(S2)(Q(v0, v))$  :  
   $S2$   
  //@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

Transformations of annotated programs (2)

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
   $S1$   
//@ assert  $Q(v0, v)$  :  
ELSE  
   $S2$   
//@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
//@ assert  $B \wedge wp(S2)(Q(v0, v))$  :  
   $S1$   
//@ assert  $Q(v0, v)$  :  
ELSE  
//@ assert  $\neg B \wedge wp(S2)(Q(v0, v))$  :  
   $S2$   
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
//@ assert  $b \wedge wp(S1)(Q(v0, v))$  :  
   $S1$   
//@ assert  $Q(v0, v)$  :  
ELSE  
//@ assert  $\neg b \wedge wp(S2)(Q(v0, v))$  :  
   $S2$   
//@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

- ▶ $b \wedge P(v0, v) \Rightarrow b \wedge wp(S1)(Q(v0, v))$
- ▶ $\neg b \wedge P(v0, v) \Rightarrow \neg b \wedge wp(S2)(Q(v0, v))$

```
//@ assert  $P(v0, v)$  :  
//@ loop invariant  $I(v0, v)$  :  
WHILE  $B$  THEN  
   $S$   
OD  
//@ assert  $Q(v0, v)$  :
```

- ▶ Applying the iteration rule of Hoare Logic :

```
//@ assert  $P(v0, v)$  :  
//@ loop invariant  $I(v0, v)$  :  
WHILE  $B$  THEN  
   $S$   
OD  
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
//@ loop invariant  $I(v0, v)$  :  
//@ assert  $I(v0, v)$  :  
WHILE  $B$  THEN  
  //@ assert  $b \wedge I(v0, v)$  :  
   $S$   
  //@ assert  $I(v0, v)$  :  
OD  
//@ assert  $Q(v0, v)$  :
```

- Applying the iteration rule of Hoare Logic :

```
//@ assert  $P(v0, v)$  :  
//@ loop invariant  $I(v0, v)$  :  
WHILE  $B$  THEN  
   $S$   
OD  
//@ assert  $Q(v0, v)$  :
```

- ▶ Applying the iteration rule of Hoare Logic :

```
//@ assert  $P(v0, v)$  :  
//@ loop invariant  $I(v0, v)$  :  
//@ assert  $I(v0, v)$  :  
WHILE  $B$  THEN  
  //@ assert  $b \wedge I(v0, v)$  :  
   $S$   
  //@ assert  $I(v0, v)$  :  
OD  
//@ assert  $Q(v0, v)$  :
```

- ▶ $b \wedge I(v0, v) \Rightarrow wp(S)(I(v0, v))$
- ▶ $P(v0, v) \Rightarrow I(v0, v)$
- ▶ $\neg b \wedge I(v0, v) \Rightarrow Q(v0, v)$

- ▶ Checking the preservation of invariant.
- ▶ Applying the wps on assertions according to startements.

- ▶ Assertions at a control point of the program

```
/*@  assert pred; */
```

```
//@  assert pred;
```

- ▶ Assertions at a control point of the program components.

```
/*@  for id1,id2, ..., idn: assert pred; */
```

(Incrementing a number)

Listing 5 – project-divers/compwp0.c

```
#define x0 5
/*@ assigns \nothing; */
int exemple() {
    int x=x0;
    //@ assert x == x0;
    x = x + 1;
    //@ assert x == x0+1;
    return x;
}
```

(Incrementing a number)

Listing 6 – project-divers/compwp0wp.c

```
#define x0 5
/*@ assigns \nothing; */
int exemple() {
    //@ assert x0 == x0;
    //@ assert x0+1 == x0+1;
    int x=x0;
    //@ assert x == x0;
    //@ assert x+1 == x0+1;
    x = x + 1;
    //@ assert x == x0+1;
    return x;
}
```

Summary on annotations and assertions

- ▶ requires
- ▶ assigns
- ▶ ensures
- ▶ decreases
- ▶ predicate
- ▶ logic
- ▶ lemma

① Programs as Predicate Transformers

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2 Annotations

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- ▶ The calling function should guarantee the required condition or precondition introduced by the clauses *requires* $P1 \wedge \dots \wedge Pn$ at the calling point.
- ▶ The called function returns results that are ensured by the clause *ensures* $E1 \wedge \dots \wedge Em$; ensures clause expresses a relationship between the initial values of variables and the final values.
- ▶ initial values of a variable v is denoted $\backslash old(v)$
- ▶ The variables which are not in the set $L1 \cup \dots \cup Lp$ are not modified.

Listing 7 – contrat

```
/*@ requires P1;...;requires Pn;  
  @ assigns L1;...;assigns Lm;  
  @ ensures E1;...;ensures Ep;  
  @*/
```

(Division)

Listing 8 – project-divers/annotation.c

```
/*@ requires x >= 0 && x <= 10;  
  @ assigns \nothing;  
  @ ensures x % 2 == 0 ==> 2*\result == x;  
  @ ensures x % 2 != 0 ==> 2*\result == x-1;  
  @*/  
int annotation(int x)  
{  
    int y;  
    y = x / 2;  
    return(y);  
}
```

(Division)

Listing 9 – project-divers/annotationwp.c

```
/*@ requires 0 <= x && x <= 10;
   @ assigns \nothing;
   @ ensures x % 2 == 0 ==> 2*\result == x;
   @ ensures x % 2 != 0 ==> 2*\result == x-1;
   @*/
int annotation(int x)
{
  /*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
  /*@ assert x % 2 != 0 ==> 2* (x / 2) == x-1; */
  int y;
  /*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
  /*@ assert x % 2 != 0 ==> 2* (x / 2) == x-1; */
  y = x / 2;
  /*@ assert x % 2 == 0 ==> 2*y == x; */
  /*@ assert x % 2 != 0 ==> 2*y == x-1; */
  return(y);
  /*@ assert x % 2 == 0 ==> 2*y == x; */
  /*@ assert x % 2 != 0 ==> 2*y == x-1; */
}
```


Property to check

$$x \geq 0 \wedge x < 0; \Rightarrow \left(\begin{array}{l} x \% 2 = 0 \Rightarrow 2 \cdot (x/2) = x \\ x \% 2 \neq 0 \Rightarrow 2 \cdot (x/2) = x-1 \end{array} \right)$$

(Precondition)

Listing 10 – project-divers/annotation0.c

```
/*@ requires x >= 0 && x < 0;  
  @ assigns \nothing;  
  @ ensures \result == 0;  
  @*/  
int annotation0(int x)  
{  
  int y;  
  y = y / (x-x);  
  return(y);  
}
```

(Precondition)

Listing 11 – project-divers/annotation0wp.c

```
/*@ requires x >= 0 && x < 0;  
  @ assigns \nothing;  
  @ ensures \result == 0;  
  @*/  
int annotation(int x)  
{  
  /*@ assert y / (x-x) == 0; */  
  int y;  
  /*@ assert y / (x-x) == 0; */  
  y = y / (x-x);  
  /*@ assert y == 0; */  
  return(y);  
  /*@ assert y == 0; */  
}
```

Property to check

$$0 \leq x \wedge x \leq 10 \Rightarrow y/(x-x) = 0$$

Definition of a contract (specification)

- ▶ Define the mathematical function to compute (what to compute?)
- ▶ Define an inductive method for computing the mathematical function and using axioms.

(factorial what)

Listing 12 – project-factorial/factorial.h

```
#ifndef _A.H
#define _A.H
/*@ axiomatic mathfact {
    @ logic integer mathfact(integer n);
    @ axiom mathfact.1: mathfact(1) == 1;
    @ axiom mathfact_rec: \forall integer n; n > 1
    ==> mathfact(n) == n * mathfact(n-1);
    @ } */

/*@ requires n > 0;
    decreases n;
    ensures \result == mathfact(n);
    assigns \nothing;
*/
int codefact(int n);
#endif
```

Definition of a contract (programming)

- ▶ Define the program codefact for computing mathfact (How to compute?)
- ▶ Define the algorithm computing the function mathfact

(factorial how)

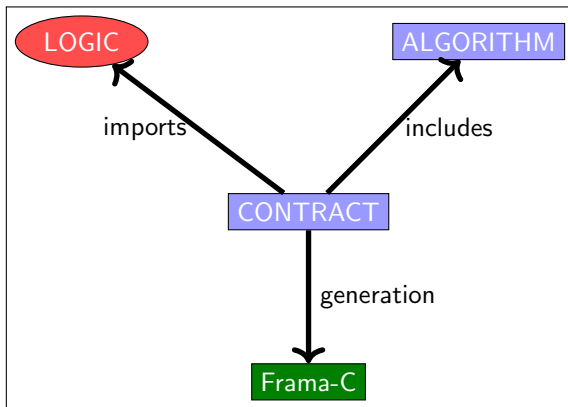
Listing 13 – project-factorial/factorial.c

```
#include "factorial.h"

int codefact(int n) {
    int y = 1;
    int x = n;
    /*@ loop invariant x >= 1 && x <= n && mathfact(n) == y * mathfact(x);
       loop assigns x, y;
       loop variant x;
    */
    while (x != 1) {
        y = y * x;
        x = x - 1;
    };
    return y;
}
```

Definition of a contract (approach)

- ▶ The specification of a function (`mathfact`) to compute requires to define it mathematically.
- ▶ The definition is stated in an axiomatic framework and is preferably inductive (`mathfact`) which is used in assertions or theorems or lemmas.
- ▶ The relationship between the computed value (`\result`) and the mathematical value (`mathfact(n)`) is stated in the ensures clause :
$$\backslash\text{result} == \text{mathfact}(n)$$
- ▶ The main property to prove is `codefact(n) == mathfact(n)` : Calling `codefact` for `n` returns a value equal to `mathfact(n)`.



Listing 14 – contrat

```
/*@ requires P;  
@ behavior b1:  
  @ assumes A1;  
  @ requires R1 ;  
  @ assigns L1;  
  @ ensures E1;  
@ behavior b2:  
  @ assumes A2;  
  @ requires R2;  
  @ assigns L2;  
  @ ensures E2;  
@*/
```

(Pairs of integers)

Listing 15 – project-divers/structures.h

```
#ifndef _STRUCTURE_H
```

```
struct s {  
    int q;  
    int r;  
};
```

```
#endif
```

Division should not return silly expressions!

(Specification)

Listing 16 – project-divers/division.h

```
#ifndef _A_H
#define _A_H
#include "structures.h"
/*@ requires a >= 0 && b >= 0;
@ behavior b :
  @ assumes b == 0;
  @ assigns \nothing;
  @ ensures \result.q == -1 && \result.r == -1 ;
@ behavior B2:
  @ assumes b != 0;
  @ assigns \nothing;
  @ ensures 0 <= \result.r;
  @ ensures \result.r < b;
  @ ensures a == b * \result.q + \result.r;
*/
struct s division(int a, int b);
#endif
```

Division should not return silly expressions!

(Algorithm)

Listing 17 – project-divers/division.c

```
#include <stdio.h>
#include <stdlib.h>
#include "division.h"

struct s division(int a, int b)
{
    int rr = a;
    int qq = 0;
    struct s silly = {-1,-1};
    struct s resu;
    if (b == 0) {
        return silly;
    }
    else
    {
        /*@
        loop invariant
        ( a == b*qq + rr) &&
        rr >= 0;
        loop assigns rr,qq;
        loop variant rr;
        */
        while (rr >= b) { rr = rr - b; qq=qq+1;};
        resu.q = qq;
        resu.r = rr;
        return resu;
    }
}
```

Iteration Rule for PC

If $\{P \wedge B\}S\{P\}$, then $\{P\}\mathbf{while\ B\ do\ S\ od}\{P \wedge \neg B\}$.

- ▶ Prove $\{P \wedge B\}S\{P\}$ or $P \wedge B \Rightarrow \{S\}(P)$.
- ▶ By the iteration rule, we conclude that $\{P\}\mathbf{while\ B\ do\ S\ od}\{P \wedge \neg B\}$ without using WLP.
- ▶ Introduction of LOOP INVARIANTS in the notation.

Listing 18 – loop.c

```
/*@ loop invariant I1;  
    loop invariant I2;  
    ...  
    loop invariant In;  
    loop assigns X;  
    loop variant E;  
*/
```

(Invariant de boucle)

Listing 19 – project-divers/anno6.c

```
/*@ requires a >= 0 && b >= 0;  
    ensures 0 <= \result;  
    ensures \result < b;  
    ensures \exists integer k; a == k * b + \result;  
*/  
int rem(int a, int b) {  
    int r = a;  
    /*@  
        loop invariant  
        (\exists integer i; a == i * b + r) &&  
        r >= 0;  
        loop assigns r;  
    */  
    while (r >= b) { r = r - b; };  
    return r;  
}
```

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Playing with variables

Ghost Variables

- ▶ $\backslash old(x)$ is the value of the variable when the function is called.
- ▶ It can be used in the postcondition of the *ensures* clause.

(Modifying variables while calling)

Listing 20 – project-divers/old1.c

```
/*@ requires \valid(a) && \valid(b);  
   @ assigns *a,*b;  
   @ ensures  *a == \at(*a,Pre) +2;  
   @ ensures  *b == \at(*b,Pre)+\at(*a,Pre)+2;  
  
   @ ensures  \result == 0;  
*/  
int old(int *a, int *b) {  
    int x,y;  
    x = *a;  
    y = *b;  
    x=x+2;  
    y = y +x;  
  
    *a = x;  
    *b = y;  
    return 0 ;  
}
```


- ▶ $\backslash at(e, id)$ is the value of e at the control point id .
- ▶ id should occur before $\backslash at(e, id)$
- ▶ id is one of the possible expressions : Pre, Here, Old, Post, LoopEntry, LoopCurrent, Init
- ▶ $\backslash old(e)$ is equivalent to $\backslash at(e, Old)$

(label Pre)

Listing 21 – project-divers/at1.c

```
/*@
  requires \valid(a) && \valid(b);
  assigns *a,*b;
  ensures  *a == \old(*a)+2;
  ensures  *b == \old(*b)+\old(*a)+2;
*/
int at1(int *a, int *b) {
  //@ assert *a == \at(*a, Pre);
  *a = *a + 1;
  //@ assert *a == \at(*a, Pre)+1;
  *a = *a + 1;
  //@ assert *a == \at(*a, Pre)+2;
  *b = *b + *a;
  //@ assert *a == \at(*a, Pre)+2 && *b == \at(*b, Pre)+\at(*a, Pre)+2;
  return 0;
}
```


(autre label)

Listing 22 – project-divers/at2.c

```
void f (int n) {  
  for (int i = 0; i < n; i++) {  
    /*@ assert \at(i, LoopEntry) == 0; */  
    int j=0;  
    while (j++ < i) {  
      /*@ assert \at(j, LoopEntry) == 0; */  
      /*@ assert \at(j, LoopCurrent) + 1 == j; */  
    }  
  }  
}
```

(otherlabel)

Listing 23 – project-divers/change1.c

```
/*@ requires \valid(a) && *a >= 0;
   @ assigns *a;
   @ ensures *a == \old(*a)+2 && \result == 0;
*/
int change1(int *a)
{
    int x = *a;
    x = x + 2;
    *a = x;
    return 0;
}
```

① Programs as Predicate Transformers

② Annotations

③ Contracts

Extending C programming language by contracts

Playing with variables

Ghost Variables

- ▶ A variable called *ghost* allows to model a computed value useful for stating a model property : the ghost variable is hidden for the computer but not for the model.
- ▶ It should not change the semantics of others variables and should not change the effective variables.

(Bug)

Listing 24 – project-divers/ghost2.c

```
int f (int x, int y) {  
    //@ghost int z=x+y;  
    switch (x) {  
    case 0: return y;  
    //@ ghost case 1: z=y;  
    // above statement is correct.  
    //@ ghost case 2: { z++; break; }  
    // invalid, would bypass the non-ghost default  
    default: y++; }  
    return y; }  
  
int g(int x) { //@ ghost int z=x;  
    if (x>0){return x;}  
    //@ ghost else { z++; return x; }  
    // invalid, would bypass the non-ghost return  
    return x+1; }
```


(Ghost variable)

Listing 25 – project-divers/ghost1.c

```
/*@ requires a >= 0 && b >= 0;  
    ensures 0 <= \result;  
    ensures \result < b;  
    ensures \exists integer k; a == k * b + \result; */  
int rem(int a, int b) {  
    int r = a;  
    /*@ ghost    int q=0;    */  
    /*@  
        loop invariant  
        a == q * b + r &&  
        r >= 0 && r <= a;  
        loop assigns r;  
        loop assigns q;  
    // loop variant r;  
    */  
    while (r >= b) {  
        r = r - b;  
    /*@ ghost    q = q+1;    */  
    };  
    return r;  
}
```