Tutorial Modelling Software-based Systems
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Tutorial 2 : Designing and verifying sequential algorithms using the Event-B modelling language
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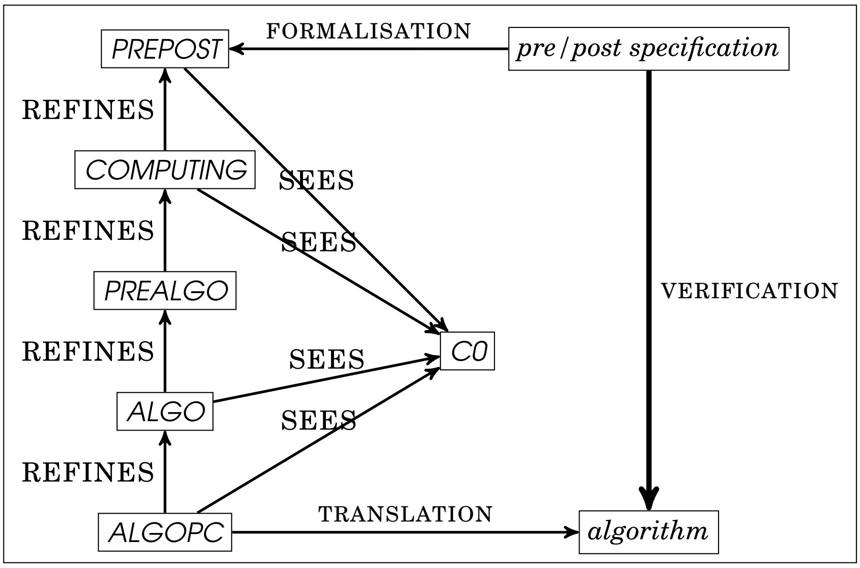
Exercice 1 *fx1-tut2.zip*
We consider a finite sequence of integers $v_1, \dots, v_n$ where n is the length of the sequence and is supposed to be fixed. Write an Event B specification modelling the computation of the value of the summation of the sequence v. You should define cerafully v, n and the summation of a finite sequence of integers.

Exercice 2 *fx2-tut2.zip*



Apply the pattern for computing the value n^2 using the sequence $(n+1)^2 = n^2+n+n+1$.
Write a C function with annotation that you will check with Frama-c.

Exercice 3 *fx3-tut2.zip*
Develop an algorithmic solution with the pattern for the problem of finding the number of occurrences of a value v value v satisfying a condition CO in a table t of dimension n. dimension n. The table is assumed to have a value in an envelope V. seems V and that CO is a part of V.

Exercice 4 *fx4-tut2.zip*
Apply this pattern to find the index i of t such that $t(i) = v$. Write a C function that you will check with Frama-c.

Exercice 5 *fx5-tut2.zip*
Apply this pattern to compute x^3 using $(i+1)^3 = i^3+3i^2+3i+1$. We use the following sequences :

- $z_0 = 0$ *et* $\forall n \in \mathbb{N} : z_{n+1} = z_n+v_n+w_n$
- $v_0 = 0$ *et* $\forall n \in \mathbb{N} : v_{n+1} = v_n+t_n$
- $t_0 = 3$ *et* $\forall n \in \mathbb{N} : t_{n+1} = t_n+6$
- $w_0 = 1$ *et* $\forall n \in \mathbb{N} : w_{n+1} = w_n+3$
- $u_0 = 0$ *et* $\forall n \in \mathbb{N} : u_{n+1} = u_n+1$

$$\begin{pmatrix} z(i+1) \\ v(i+1) \\ t(i+1) \\ w(i+1) \\ u(i+1) \end{pmatrix} = \begin{pmatrix} z_i+v_i+w_i \\ v(i)+t(i) \\ t(i)+6 \\ w(i)+3 \\ u(i)+1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 01 & 0 & \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z(i) \\ v(i \\ t(i) \\ w(i) \\ u(i) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 6 \\ 3 \\ 1 \end{pmatrix}$$

Write a C function from the development and use Frama-c for checking it.

Exercice 6 $\star$
The objective is to design a correct by construction algorithm in a programming language with annotations and proof tool as Frama-c or Dafny.
The problem to solve is the power function defined usually as follows in a classical matematical language $power = \lambda x, y.x^y$. *For instance, the notation* x^y *is foubnd in the C programlming language and a function can be called for computing the power function.*

Question 6.1 *Define an inductive statement of the power function by defining the sequence* $p(x \mapsto y)$ *where* $x, y \in \mathbb{N}$. *The function* $powereb = lambdax, y.x^y$. *is defined in the Event-B language and you should prove that* $\forall x, y \in \mathbb{N}.p(x \mapsto y) = x^y$.

Question 6.2 *Define a context* POW0 *and a prepost machine* POW1 *expressing the contract os the problem to solve. The problem is to define first the contract in your programming langauge.*

Question 6.3 *Develop a computing process according to the methodology of the refinement to get an algorithm.*

Question 6.4 *Check that the algroithm satisfies the constract using the proof tool related to your programling language.*