

# Modelling Software-based Systems

## Lecture 4 Correctness by Construction with the Modelling Language Event-B using the Refinement

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# General Summary

- 1 Correctness by Construction
- 2 The refinement of models
- 3 Example of the clock
- 4 Summary of the refinement
- 5 Example of the factorial function refined into an algorithm
- 6 Designing a algorithm for computing the factorial function
- 7 Review of Event-B
- 8 Intermezzo on the Event B modelling notation
- 9 Transformations of Event-B models
- 10 Conclusion
- 11 Summary

# Current Summary

## Correctness by Construction

- Correctness by Construction is a method of building software-based systems with **demonstrable correctness** for security- and safety-critical applications.
- Correctness by Construction advocates a **step-wise refinement** process from specification to code using tools for checking and transforming models.
- Correctness by Construction is an approach to software/system construction
  - ▶ starting with an abstract model of the problem.
  - ▶ progressively adding details in a step-wise and checked fashion.
  - ▶ each step guarantees and proves the correctness of the new concrete model with respect to requirements

# The Cleanroom Method as CbC

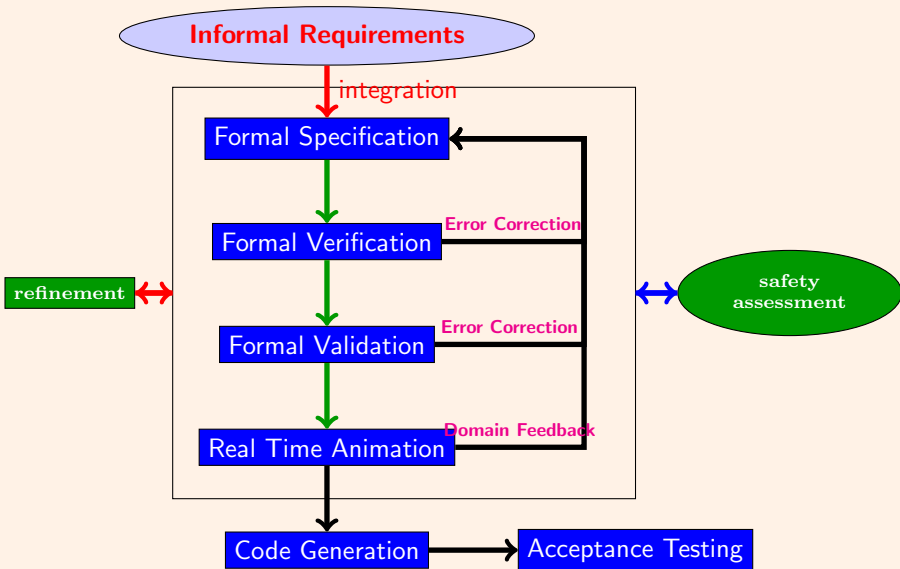
- The **Cleanroom** method, developed by Harlan Mills and his colleagues at IBM and elsewhere, attempts to do for software what cleanroom fabrication does for semiconductors : to achieve quality by keeping defects out during fabrication.
- In semiconductors, **dirt** or **dust** that is allowed to **contaminate** a chip as it is being made cannot possibly be removed later.
- But we try to do the equivalent when we write programs that are full of bugs, and then attempt to remove them all using debugging.

# The Cleanroom Method as CbC

The Cleanroom method, then, uses a number of techniques to develop software carefully, in a well-controlled way, so as to avoid or eliminate as many defects as possible before the software is ever executed. Elements of the method are :

- specification of all components of the software at all levels ;
- stepwise refinement using constructs called "box structures" ;
- verification of all components by the development team ;
- statistical quality control by independent certification testing ;
- no unit testing, no execution at all prior to certification testing.

# Critical System Development Life-Cycle Methodology



## Current Summary



# Problems for Modelling systems

- Systems are generally very complex
- Invariant should be strong enough for proving safety properties
- Problems for modelling : finding suitable mathematical structures, listing events or actions of the system, proving proof obligations, ...

## Solution : refining models

- To understand more and more the system
- To distribute the complexity of the system
- To distribute the difficulties of the proof
- To improve explanations
- Validation (step by step)
- Refinement (invariant & behavior)

## definition

Let  $x$  be the abstract variable (or list of variables) and  $I(s, c, x)$  the abstract invariant,  $y$  the concrete variable (or list of variables) and  $J(s, c, x, y)$  the concrete invariant.

Let  $c$  be a concrete event observing the variable  $y$  and  $a$  an event observing the variable  $x$  and preserving  $I(s, c, x)$ .

Event  $c$  refines event  $a$  with respect to  $x$ ,  $I(s, c, x)$ ,  $y$  and  $J(s, c, x, y)$ , if

$$AX(s, c) \vdash I(s, c, x) \wedge J(s, c, x, y) \Rightarrow [c](\neg[a](\neg J(s, c, x, y)))$$

# Abstract event refined by a concret event

$$\begin{array}{c}
 \begin{array}{l}
 \text{a} \\
 \left\{ \begin{array}{l}
 \text{ANY } u \text{ WHERE} \\
 \quad G(u, s, c, x) \\
 \text{THEN} \\
 \quad x : |ABAP(u, s, c, x, x') \\
 \text{END}
 \end{array} \right.
 \end{array}
 \stackrel{\text{def}}{=}
 \begin{array}{l}
 \text{c} \\
 \left\{ \begin{array}{l}
 \text{ANY } v \text{ WHERE} \\
 \quad H(v, s, c, y) \\
 \text{WITNESS} \\
 \quad u : WP(u, s, c, v, y) \\
 \quad x' : WV(v, s, c, y', x') \\
 \text{THEN} \\
 \quad y : |CBAP(v, s, c, y, y') \\
 \text{END}
 \end{array} \right.
 \end{array}
 \stackrel{\text{def}}{=}
 \end{array}$$

The two events  $a$  and  $c$  are normalised by a relationship called  $BA(e)(s, c, x, x')$ , which simplifies the notations used.

The two events  $a$  and  $c$  are equivalent to events of the following normalized form :

- $a$  is equivalent to  
begin  $x : |(\exists u. G(u, s, c, x) \wedge ABAP(u, s, c, x, x'))$  end
- $c$  is equivalent to  
begin  $y : |(\exists v. H(v, s, c, y) \wedge CBAP(v, s, c, y, y'))$  end

# Explanations for the refinement

(Hypothesis)

$$(1) \quad AX(s, c) \vdash I(s, c, x) \wedge J(s, c, x, y) \Rightarrow [c](\neg[a](\neg J(s, c, x, y)))$$

*equivalent to*

( Definition of  $[a]$  :  $[a](\neg J(s, c, x, y)) \equiv$

$$\forall x'. (\exists u. G(u, s, c, x) \wedge ABAP(u, s, c, x, x')) \Rightarrow \neg J(s, c, x', y))$$

$$(2) \quad AX(s, c) \vdash I(s, c, x) \wedge J(s, c, x, y) \Rightarrow [c](\neg(\forall x'. (\exists u. G(u, s, c, x) \wedge ABAP(u, s, c, x, x')) \Rightarrow \neg J(s, c, x', y)))$$

*equivalent to*

(Transformation by simplification of logical connectives)

$$(3) \quad AX(s, c) \vdash I(s, c, x) \wedge J(s, c, x, y) \Rightarrow [c](\exists x'. (\exists u. G(u, s, c, x) \wedge ABAP(u, s, c, x, x')) \wedge J(s, c, x', y))$$

*equivalent to*

( Definition of  $[c]$ )

(4)  $AX(s, c) \vdash$

$$I(s, c, x) \wedge J(s, c, x, y) \Rightarrow (\forall y'. (\exists v. H(v, s, c, x) \wedge CBAP(v, s, c, y, y')) \Rightarrow ((\exists x'. (\exists u. G(u, s, c, x) \wedge ABAP(u, s, c, x, x')) \wedge J(s, c, x', y'))))$$

*equivalent to*

(Transformation by quantifier elimination  $\forall$ )

(5)  $AX(s, c) \vdash$

$$I(s, c, x) \wedge J(s, c, x, y) \Rightarrow (\exists v. H(v, s, c, y) \wedge CBAP(v, s, c, y, y')) \Rightarrow ((\exists x'. (\exists u. G(u, s, c, x) \wedge ABAP(u, s, c, x, x')) \wedge J(s, c, x', y')))$$

*equivalent to*

(Transformation by elimination of connector  $\wedge$ )

(6)  $AX(s, c) \vdash$

$$I(s, c, x) \wedge J(s, c, x, y) \wedge (\exists v. H(v, s, c, y) \wedge CBAP(v, s, c, y, y')) \Rightarrow ((\exists x'. (\exists u. G(u, s, c, x) \wedge ABAP(u, s, c, x, x')) \Rightarrow J(s, c, x', y')))$$

*equivalent to*

(Transformation by elimination of quantifier  $\exists$ )

(7)

$$AX(s, c) \vdash I(s, c, x) \wedge J(s, c, x, y) \wedge H(v, s, c, y) \wedge CBAP(v, s, c, y, y') \Rightarrow ((\exists x'. (\exists u. G(u, s, c, x) \wedge ABAP(u, s, c, x, x')) \wedge J(s, c, x', y')))$$

*equivalent to*

(Transformation by property of quantifier  $\exists$ )

(8)

$$AX(s, c) \vdash I(s, c, x) \wedge J(s, c, x, y) \wedge H(v, s, c, y) \wedge CBAP(v, s, c, y, y') \Rightarrow ((\exists x'. ((\exists u. G(u, s, c, x) \wedge ABAP(u, s, c, x, x')) \wedge J(s, c, x', y'))))$$

*equivalent to*

(Transformation by elimination of  $\wedge$ )

(9)

- ①  $AX(s, c) \vdash I(s, c, x) \wedge J(s, c, x, y) \wedge H(v, s, c, y) \wedge$   
 $CBAP(v, s, c, y, y') \Rightarrow (((\exists u. G(u, s, c, x)))$
- ②  $AX(s, c) \vdash$   
 $I(s, c, x) \wedge J(s, c, x, y) \wedge H(v, s, c, x) \wedge CBAP(v, s, c, y, y') \Rightarrow$   
 $((\exists x'. \exists u. (ABAP(u, s, c, x, x')) \wedge J(s, c, x', y'))))$



## property refinement between events (II)

Let  $x$  be the abstract variable (or list of variables) and  $I(s, c, x)$  the abstract invariant,  $y$  the concrete variable (or list of variables) and  $J(s, c, x, y)$  the concrete invariant. the concrete invariant.

Let  $c$  be a concrete event observing the variable  $y$  and  $a$  an event observing the variable  $x$  and preserving  $I(s, c, x)$ .

Event  $c$  refines event  $a$  with respect to  $x$ ,  $I(s, c, x)$ ,  $y$  and  $J(s, c, x, y)$  if, and only if,

- ① (GRD)  $AX(s, c) \vdash I(s, c, x) \wedge J(s, c, x, y) \wedge H(v, s, c, x) \wedge CBAP(v, s, c, y, y') \Rightarrow \exists u. G(u, s, c, x)$
- ② (SIM)  $AX(s, c) \vdash I(s, c, x) \wedge J(s, c, x, y) \wedge H(v, s, c, x) \wedge CBAP(v, s, c, y, y') \Rightarrow ((\exists x'. \exists u. ABAP(u, s, c, x, x') \wedge J(s, c, x', y')))$

## Proof obligations for Event-B refinement

- (INIT)  $AX(s, c), CInit(s, c, y') \vdash \exists x'. (AInit(s, c, x') \wedge J(s, c, x', y'))$
- (GRD)  
 $AX(s, c), I(s, c, x), J(s, c, x, y), H(v, s, c, x), CBAP(v, s, c, y, y') \vdash$   
 $((\exists u. G(u, s, c, x)))$
- (GRD-WIT)  
 $AX(s, c), I(s, c, x), J(s, c, x, y), H(v, s, c, x), CBAP(v, s, c, y, y'),$   
 $WP(u, s, c, v, y) \vdash G(u, s, c, x)$
- (SIM)  $AX(s, c), I(s, c, x), J(s, c, x, y), H(v, s, c, x), CBAP(v, s, c, y, y') \vdash$   
 $((\exists x'. (\exists u. ABAP(u, s, c, x, x')) \wedge J(s, c, x', y')))$
- (SIM-WIT)  
 $AX(s, c), I(s, c, x), J(s, c, x, y), H(v, s, c, x), CBAP(v, s, c, y, y'),$   
 $WP(u, s, c, v, y), WV(v, s, c, y, x') \vdash ABAP(u, s, c, x, x') \wedge J(s, c, x', y')$
- (WFIS-P)  
 $AX(s, c) \vdash I(s, c, x) \wedge J(s, c, x, y) \wedge H(v, s, c, x) \wedge CBAP(v, s, c, y, y') \vdash$   
 $\exists u. WP(u, s, c, v, y)$
- (WFIS-V)  
 $AX(s, c) \vdash I(s, c, x) \wedge J(s, c, x, y) \wedge H(v, s, c, x) \wedge CBAP(v, s, c, y, y') \vdash$   
 $\exists x'. WV(v, s, c, y, x')$
- (TH)  $AX(s, c) \vdash I(s, c, x) \wedge J(s, c, x, y) \vdash SAFE_1(s, c, x, y)$

**MACHINE** CM    **REFINES** AM

**SEES** E

**VARIABLES**  $y$

**INVARIANTS**

$jnv_1 : J_1(s, c, x, y)$

...

$jnv_r : J_r(s, c, x, y)$

**THEOREMS**

$th_1 : SAFE_1(s, c, x, y)$

...

$th_n : SAFE_n(s, c, x, y)$

**VARIANTS**

$var_1 : varexp_1(s, c, y)$

...

$var_t : varexp_t(s, c, y)$

**EVENTS**

**EVENT** initialisation

**BEGIN**

$y : |(CInit(s, c, y'))$

**END**

...

**EVENT** c    **REFINES** a

**ANY** v **WHERE**

$H(v, s, c, y)$

**WITNESS**

$u : WP(u, s, c, v, y)$

$x' : WV(v, s, c, y', x')$

**THEN**

$y : |CBAP(v, s, c, y, y')$

**END**

...

**END**

- The machine CM is a model describing a set of events  $E(\text{CM})$  modifying the  $y$  variable declared in the clause **VARIABLES**.

- A clause **REFINES** indicates that the CM machine refines a AM machine and  $E(\text{AM})$  is the set of abstract events in AM.

- A particular event defines the initialisation of variable  $y$  according to the relationship  $CInit(s, c, y')$ .

- The property “Event c refines event a with respect to  $x$ ,  $I(s, c, x)$ ,  $y$  and  $J(s, c, x, y)$ ” is denoted by the expression c refines a. Events a and c are attached to two machines AM and CM; the invariant attached to each event is the invariant of its machine.

- A clause **INVARIANTS** describes the inductive invariant  $J(s, c, x, y)$  that this machine is assumed to respect provided that the associated verification conditions are shown to be valid in the theory induced by the context  $E$  mentioned by the clause **SEES**.  $J(s, c, x, y)$  is the gluing invariant linking the variable  $y$  to the variable  $x$ .
- The clause **THEOREMS** introduces the list of safety properties derived in the theory. These properties relate to the variables  $y$  and  $x$  and must be proved valid. It is possible to add theorems about sets and constants; this can help the proofs to be carried out during the verification process.
- To conclude this description, we would like to add that events can carry very important information for the proof process, in particular for proposing witnesses during event refinement. Furthermore, each event has a status (ordinary, convergent, anticipated) which is important in the production of verification conditions. The clause **VARIANTS** is linked to events of convergent and anticipated status. The event  $c$  (concrete) explicitly refines an event  $a$  of the AM machine.

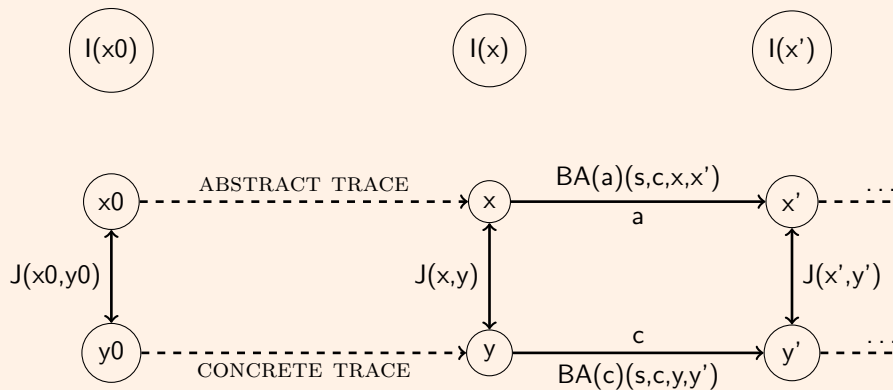
## definition

The machine CM refines the machine AM , if any event c of CM refines an event a of AM :

$$\forall c.c \in E(\text{CM}) \Rightarrow \exists a.a \in E(\text{AM}) \wedge e \text{ refines } a.$$

- Each machine has an event skip which does not modify the machine's variables.
- A concrete event  $c$  can refine an event skip whose effect is not to modify  $x$  in the abstract machine AM.
- The invariant of AM is  $I(s, c, x)$  and that the initialisation of AM is  $AInit(s, c, x')$ .
- The proof witnesses are used to give properties of the parameter  $u$  and the variable  $x$  which have disappeared in the machine CM but for which the user must give an expression according to the state of CM.

# Refinement between two machines



## Current Summary

## Example of a clock

- A machine  $M_1$  models hours or a machine  $M_1$  reports observations of hours



## Example of a clock

- A machine M1 models hours or a machine M1 reports observations of hours
- and a machine M2 reports hours and minutes.
- A very special case of refinement called *superposition* and the proof is fairly straightforward.

CONTEXT  $C$

CONSTANTS  $H M$

AXIOMS

$@axm1 \ H = 0..23$

$@axm2 \ M = 0..59$

*end*

MACHINE  $M1$  SEES  $C$

VARIABLES  $h$

INVARIANTS

$@inv1\ h \in H$

EVENTS

EVENT  $INITIALISATION$

then

$@act1\ h : \in H$

end

EVENT  $h1$

where

$@grd1\ h < 23$

then

$@act1\ h := h + 1$

end

EVENT  $h2$

where

$@grd1\ h = 23$

then

$@act1\ h := 0$

end

*end*

```
MACHINE M2
  REFINES M1
  SEES C
```

```
VARIABLES h m
```

```
INVARIANTS
```

```
  @inv1 m ∈ M
```

```
  theorem @inv2 h ∈ H
```

```
EVENTS
```

```
  EVENT INITIALISATION
```

```
    then
```

```
      @act1 h : ∈ H
```

```
      @act2 m : ∈ M
```

```
    end
```

```
  EVENT h1m1
```

```
    where
```

```
      @grd1 h < 23
```

```
      @grd2 m < 59
```

```
    then
```

```
      @act2 m := m + 1
```

```
    end
```

```
EVENT h1m2 REFINES h1
```

```
  where
```

```
    @grd1 h < 23
```

```
    @grd2 m = 59
```

```
  then
```

```
    @act1 h := h + 1
```

```
    @act2 m := 0
```

```
  end
```

```
EVENT h2m1 REFINES h2
```

```
  where
```

```
    @grd1 h = 23
```

```
    @grd2 m = 59
```

```
  then
```

```
    @act1 h := 0
```

```
    @act2 m := 0
```

```
  end
```

```
EVENT h2m2
```

```
  where
```

```
    @grd1 h = 23
```

```
    @grd2 m < 59
```

```
  then
```

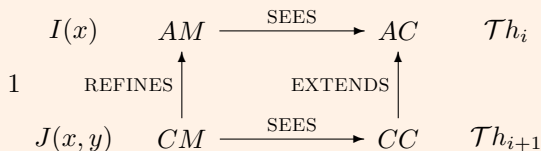
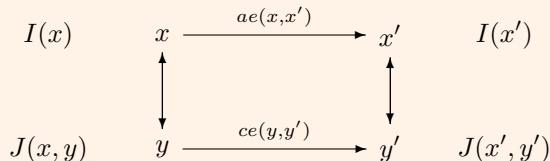
```
    @act1 m := m + 1
```

```
  end
```

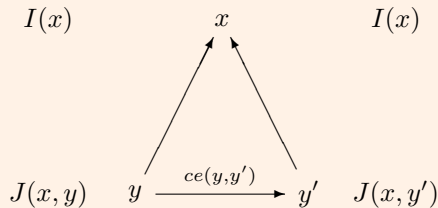
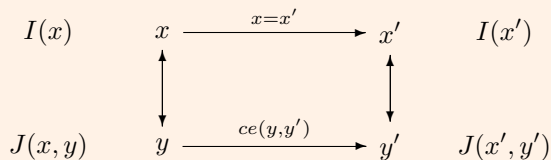
```
end
```

## Current Summary

# Refinement of a model by another one (I)



# Refinement of a model by another one (II)



(REF1) : refinement of initial conditions

$$\text{INITC}(y) \Rightarrow \exists x * (\text{INIT}(x) \wedge \mathbf{J}(x, y)) :$$

The initial condition of the refinement model imply that there exists an abstract value in the abstract model such that that value satisfies the initial conditions of the abstract one and implies the new invariant of the refinement model.



(REF2) : refinement of events

$$I(x) \wedge J(x, y) \wedge ce(y, y') \Rightarrow \exists x'. (ae(x, x') \wedge J(x', y')) :$$

The invariant in the refinement model is preserved by the refined event and the activation of the refined event triggers the corresponding abstract event.

(REF3) : refinement of stuttering steps

$$I(x) \wedge J(x, y) \wedge ce(y, y') \Rightarrow J(x, y') :$$

The invariant in the refinement model is preserved by the refined event but the event of the refinement model is a new event which was not visible in the abstract model ; the new event refines *skip*.

(REF4) : Refinement does not introduce more blocking states

$$I(x) \wedge J(x,y) \wedge (G_1(x) \vee \dots \vee G_n(x)) \Rightarrow H_1(y) \vee \dots \vee H_k(y) :$$

The guards of events in the refinement model are strengthened and we have to prove that the refinement model is not more blocked than the abstract.

(REF5) : Well-definedness of variant

$$I(x) \wedge J(x, y) \Rightarrow V(y) \in \mathbb{N}$$

(REF6) : Well behaviour of new events

$$I(x) \wedge J(x, y) \wedge ce(y, y') \Rightarrow V(y') < V(y) :$$

**New events should not block forever abstract ones.**

(REF7) : Feasibility of refined events

$$\Gamma(s, c) \vdash I(x) \wedge J(x, y) \wedge \text{grad}(E) \Rightarrow \exists y' \cdot P(y, y')$$

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# Designing a algorithm for computing $\lambda x.x!$

## Computing $\lambda x.x!$

The problem is to derive an algorithm which is computing the function  $\lambda x.x!$ .

```
#ifndef _A_H
#define _A_H
/*@ axiomatic mathfact {
    @ logic integer mathfact(integer n);
    @ axiom mathfact_1: mathfact(0) == 1;
    @ axiom mathfact_rec: \forall integer n; n >= 1
    ==> mathfact(n) == n * mathfact(n-1);
    @ } */

/*@ requires n >= 0;
    ensures \result == mathfact(n);
*/
int codefact(int n);
#endif
```

# Context for computing $\lambda x.x!$ .

CONTEXT *A – functions*

CONSTANTS *factorial n*

AXIOMS

@axm1  $n \in \mathbb{N}$

@axm2  $\text{factorial} \in \mathbb{N} \leftrightarrow \mathbb{N}$

@axm3  $0 \mapsto 1 \in \text{factorial}$

@axm4  $\forall a. \forall b. a \in \mathbb{N} \wedge b \in \mathbb{N} \wedge a \mapsto b \in \text{factorial}$   
 $\Rightarrow a+1 \mapsto (a+1) * b \in \text{factorial}$

@axm5  $\forall f. f \in \mathbb{N} \leftrightarrow \mathbb{N} \wedge 0 \mapsto 1 \in f$   
 $\wedge (\forall a, b. a \in \mathbb{N} \wedge b \in \mathbb{N} \wedge a \mapsto b \in f$   
 $\Rightarrow a+1 \mapsto (a+1) * b \in f)$   
 $\Rightarrow \text{factorial} \subseteq f$

theorem @th1  $\text{factorial} \in \mathbb{N} \rightarrow \mathbb{N}$

theorem @th2  $\text{factorial}(0) = 1$

theorem @th3  $\forall u. u \in \mathbb{N} \wedge u \neq 0 \Rightarrow \text{factorial}(u) = u * \text{factorial}(u - 1)$

@axm6  $n > 3$

end

## Machine B-prepost for stating the pre/post specification

```

MACHINE  $B - \text{prepost}$  SEES  $A - \text{functions}$ 
VARIABLES  $r$ 
INVARIANTS
  @inv1  $r \in \mathbb{Z}$ 
EVENTS
  EVENT INITIALISATION
    then
      @act1  $r := 0$ 
    end
  EVENT computing1
    then
      @act1  $r := \text{factorial}(n)$ 
    end
end
end

```

- Defining variables and invariant
- $r$  is the variable for the result.
- $n$  is the constant containing the input of the process.

## Machine C-computing for stating the computing process

```

MACHINE C – computing   REFINES B – prepost
  SEES A – functions
  VARIABLES r fac x
  INVARIANTS
    @inv1 fac ∈ ℕ → ℕ
    @inv2 dom(fac) ⊆ 0..n
    @inv3 dom(fac) ≠ ∅
    @inv4 ∀ i.i ∈ dom(fac) ⇒ fac(i) = factorial(i)
    @inv5 x ∈ dom(fac)
    @inv6 dom(fac) = 0..x
  EVENT INITIALISATION REFINES INITIALISATION
  then
    @act1 r : ∈ ℤ
    @act2 fac := { 0 ↦ 1 }
    @act3 x := 0
  end

```

- Two new variables  $x$  and  $fac$  are introduced for storing the sequence  $factorial$  by iterating over  $x$
- Condition of termination is that  $n \in dom(fac)$
- $fac(i) = factorial(i)$  is expressing the relationship between computed values and mathematically defined values of the sequence.

## Machine C-computing for stating the computing process

```

EVENT computing2 REFINES computing1
  where
    @grd1  $n \in \text{dom}(fac)$ 
    then
      @act1  $r := fac(n)$ 
    end
  end

```

```

convergent EVENT step2
  where
    @grd11  $x \in \text{dom}(fac)$ 
    @grd12  $x + 1 \notin \text{dom}(fac)$ 
    @grd13  $n \notin \text{dom}(fac)$ 
  then
    @act11  $fac(x + 1) := (x + 1) * fac(x)$ 
    @act1  $x := x + 1$ 
  end
  VARIANT  $0..n \setminus \text{dom}(fac)$ 

```

- the event final is controlled by the condition  $n \in \text{dom}(vv)$  meaning that we have finally reached the computing goal.
- SIM proof obligations are generated.
- the event step-computing is refining iteration and when it observed, the variant is decreasing.
- it refines skip

# Machine D-prealgo for getting an algorithmic process

MACHINE *D - prealgo* REFINES *C - computing*

SEES *A - functions*

VARIABLES *r vfac cfac fac x*

INVARIANTS

@inv1 *vfac*  $\in \mathbb{N}$

@inv2 *cfac*  $\in \mathbb{N}$

@inv3 *cfac*  $\leq n$

@inv4 *cfac*  $\geq 0$

@inv6 *cfac*  $\in \text{dom}(fac)$

@inv5 *vfac* = *fac*(*cfac*)

@inv7 *cfac* + 1  $\notin \text{dom}(fac)$

@inv8  $\text{dom}(fac) = 0..cfac$

@inv9 *x* = *cfac*

EVENT *INITIALISATION*

then

@act1 *r* :  $\in \mathbb{N}$

@act2 *fac* := { 0  $\mapsto$  1 }

@act3 *cfac* := 0

@act4 *vfac* := 1

@act5 *x* := 0

end

- Two new variables are introduced for storing really useful data namely the last computed values of the two sequences.
- Obviously, *vfac* and *cfac* satisfy  $vfac = fac(cfac)$
- Previous properties of abstract variables are safety properties which are no more to be reproved, thanks to refinement.

# Machine D-prealgo for getting an algorithmic process

```
EVENT computing3 REFINES computing2
  where
    @grd2 cfac = n
    then
      @act1 r := vfac
    end
  end

convergent EVENT step3 REFINES step2
  where
    @grd1 cfac ≠ n
    then
      @act1 vfac := (cfac + 1) * vfac
      @act2 cfac := cfac + 1
      @act3 fac(cfac + 1) := (cfac + 1) * fac(cfac)
      @act4 x := x + 1
    end
  end
```

- The two events SIMulate the abstract events.
- However, the guards are strengthened and are made closer to an implementation :  
 $cfac < n$  implies  $n \notin \text{dom}(fac)$  and  
 $cfac = n$  implies that  $n \in \text{dom}(fac)$ .

# Machine E-algo for getting an algorithmic machine

VARIABLES  $r \text{ } vfac \text{ } cfac$

INVARIANTS

*theorem* @thm1  $vfac = factorial(cfac)$

@inv1  $r \in \mathbb{N}$

@computation\_inv1  $fac \in \mathbb{N} \rightarrow \mathbb{N}$

@inv2  $dom(fac) \subseteq 0..n$

@inv3  $dom(fac) \neq \emptyset$

@inv4  $\forall i. i \in dom(fac) \Rightarrow fac(i) = factorial(i)$

@inv5  $x \in dom(fac)$

@inv6  $dom(fac) = 0..x$

@algorithm\_inv1  $vfac \in \mathbb{N}$

@algorithm\_inv2  $cfac \in \mathbb{N}$

@algorithm\_inv3  $cfac \leq n$

@algorithm\_inv4  $cfac \geq 0$

@algorithm\_inv6  $cfac \in dom(fac)$

@algorithm\_inv5  $vfac = fac(cfac)$

@inv7  $cfac + 1 \notin dom(fac)$

@inv8  $dom(fac) = 0..cfac$

@inv9  $x = cfac$

VARIANT  $n - cfac$

- The variables *fac* is now hidden and they disappear from the machine.
- It is playing the role of model variables as ghost variables.
- Invariants and safety properties are preserved through refinement.



# Machine E-algo for getting an algorithmic machine

- Assignments of  $fac$  are removed.

## Current Summary

## Simple Form of an Event

- An event of the **simple** form is denoted by :

```

< event_name > ≡
WHEN
    < condition >
THEN
    < action >
END

```

where

- $\langle event\_name \rangle$  is an identifier
- $\langle condition \rangle$  is the firing condition of the event
- $\langle action \rangle$  is a generalized substitution (parallel "assignment")

# Non-deterministic Form of an Event

- An event of the **non-deterministic** form is denoted by :

```
< event_name >  $\hat{=}$   
  ANY < variable > WHERE  
    < condition >  
  THEN  
    < action >  
  END
```

where

- < *event\_name* > is an identifier
- < *variable* > is a (list of) variable(s)
- < *condition* > is the firing condition of the event
- < *action* > is a generalized substitution (**parallel** “assignment”)

# Shape of a Generalized Substitution

A generalized substitution can be

- **Simple** assignment :  $x := E$
- **Generalized** assignment :  $x : P(x, x')$
- **Set** assignment :  $x : \in S$
- **Parallel** composition : 
$$\begin{array}{c} T \\ \dots \\ U \end{array}$$

$$\text{INVARIANT} \wedge \text{GUARD} \\ \implies \\ \text{ACTION} \text{ establishes INVARIANT}$$

# Invariant Preservation Verification (1)

- Given an event of the simple form :

```

EVENT EVENT ≡
  WHEN
     $G(x)$ 
  THEN
     $x := E(x)$ 
  END

```

and invariant  $I(x)$  to be preserved, the statement to prove is :

$$I(x) \wedge G(x) \implies I(E(x))$$

# Invariant Preservation Verification (2)

- Given an event of the simple form :

**EVENT** EVENT  $\triangleq$   
**WHEN**  
     $G(x)$   
**THEN**  
     $x : |P(x, x')$   
**END**

and invariant  $I(x)$  to be preserved, the statement to prove is :

$$I(x) \wedge G(x) \wedge P(x, x') \implies I(x')$$



# Invariant Preservation Verification (3)

- Given an event of the simple form :

**EVENT** EVENT  $\hat{=}$   
**WHEN**  
     $G(x)$   
**THEN**  
     $x \in S(x)$   
**END**

and invariant  $I(x)$  to be preserved, the statement to prove is :

$$I(x) \wedge G(x) \wedge x' \in S(x) \implies I(x')$$

# Invariant Preservation Verification (4)

- Given an event of the non-deterministic form :

```
EVENT EVENT  $\hat{=}$   
  ANY  $v$  WHERE  
     $G(x, v)$   
  THEN  
     $x := E(x, v)$   
  END
```

and invariant  $I(x)$  to be preserved, the statement to prove is :

$$I(x) \wedge G(x, v) \implies I(E(x, v))$$



## Refinement Technique (2)

# Correct Refinement Verification (1)

- Given an **abstract** and a corresponding **concrete** event

```
EVENT ea ≐  
  WHEN  
    G(x)  
  THEN  
    x := E(x)  
  END
```

```
EVENT ec ≐  
  WHEN  
    H(y)  
  THEN  
    y := F(y)  
  END
```

and invariants  $I(x)$  and  $J(x, y)$ , the statement to prove is :

$$I(x) \wedge J(x, y) \wedge H(y) \implies G(x) \wedge J(E(x), F(y))$$

# Correct Refinement Verification (2)

- Given an **abstract** and a corresponding **concrete** event

**EVENT**  $ea \hat{=}$   
**ANY**  $v$  **WHERE**  
     $G(x, v)$   
**THEN**  
     $x := E(x, v)$   
**END**

**EVENT**  $ec \hat{=}$   
**ANY**  $w$  **WHERE**  
     $H(y, w)$   
**THEN**  
     $y := F(y, w)$   
**END**

$$\begin{aligned} & I(x) \wedge J(x, y) \wedge H(y, w) \\ \implies & \exists v \cdot (G(x, v) \wedge J(E(x, v), F(y, w))) \end{aligned}$$

# Correct Refinement Verification (3)

- Given a NEW event

```
EVENT EVENT  $\hat{=}$   
  WHEN  
     $H(y)$   
  THEN  
     $y := F(y)$   
  END
```

and invariants  $I(x)$  and  $J(x, y)$ , the statement to prove is :

$$I(x) \wedge J(x, y) \wedge H(y) \implies J(x, F(y))$$

# Current Summary

- 1 Correctness by Construction
- 2 The refinement of models
- 3 Example of the clock
- 4 Summary of the refinement
- 5 Example of the factorial function refined into an algorithm
- 6 Designing a algorithm for computing the factorial function
- 7 Review of Event-B
- 8 Intermezzo on the Event B modelling notation

- ## 9 Transformations of Event-B models



# General form of proof obligations for an event $e$

- $INIT/I/INV : C(s, c), INIT(c, s, x) \vdash I(c, s, x)$
- $e/I/INV : C(s, c), I(c, s, x), G(c, s, t, x), P(c, s, t, x, x') \vdash I(c, s, x')$
- $e/act/FIS : C(s, c), I(c, s, x), G(c, s, t, x) \vdash$
- $e/act/WD : C(s, c), I(c, s, x), G(c, s, t, x) \vdash \exists x'. P(c, s, t, x, x')$

Well-definedness of an Axiom	m / WD	m is the axiom name
Well-definedness of a Derived Axiom	m / WD	m is the axiom name
Derived Axiom	m / THM	m is the axiom name
Well-definedness of an Invariant	v / WD	v is the invariant name
Well-definedness of a Derived Invariant	m / WD	m is the invariant name
Well-definedness of an event Guard	t / d / WD	t is the event name d is the action name
Well-definedness of an event Action	t / d / WD	t is the event name d is the action name
Feasibility of a non-det. event Action	t / d / FIS	t is the event name d is the action name
Derived Invariant	m / THM	m is the invariant name
Invariant Establishment	INIT. / v / INV	v is the invariant name
Invariant Preservation	t / v / INV	t is the event name v is the invariant name

## Current Summary

## 9 Transformations of Event-B models

```
WHEN
  P
  Q
THEN
  S
END
```

```
WHEN
  P
   $\neg Q$ 
THEN
  T
END
```

*are merged into*

```
WHEN
  P
THEN
  WHILE  Q  DO
    S
  END;
  T
END
```

## Side Conditions :

- P must be invariant under S.
- The first event must have been introduced at one refinement step below the second one.
- Special Case : If P is missing the resulting "event" has no guard

```
WHEN
  P
  Q
THEN
  S
END
```

```
WHEN
  P
  ¬Q
THEN
  T
END
```

*are merged into*

```
WHEN
  P
THEN
  IF Q THEN S
  ELSE T
END;
END
```

## Side Conditions :

- The disjunctive negation of the previous side conditions
- Special Case : If P is missing the resulting "event" has no guard

## Deriving an algorithm

```
precondition    :  $n \in \mathbb{N}$ 
```

**postcondition** :  $result = factorial(n)$

**local variables :**  $vfac, cfac \in \mathbb{N}$

$$cfac := 0; vfac := 1; result := \mathbb{N};$$
**while**  $cfac \neq n$  **do**

<b>Invariant</b>	$: vfac = fac(cfac)$
------------------	----------------------

$$vfac := (cfac + 1) * vfac; cfac := cfac + 1;$$

•

```

result := vfac;

```

## Translating the machine E-algo to an algorithm

```
#include <limits.h>
#include "factorial.h"

int codefact(int n) {
    int cfac=0;
    int vfac= 1;
    /*@ loop invariant cfac >= 0 && cfac <= n && mathfact(cfac) ==
        loop assigns cfac, vfac;
        loop variant n-cfac;
    */
    while (cfac != n) {
        vfac = (cfac+1)*vfac;
        cfac  = cfac + 1;
    };
    return vfac;
}
```

## Current Summary



## Conclusion

- Refinement helps in discovering invariants
- Refinement helps in proving invariants
- The choice of the *good* abstraction is not very simple and is a challenge by itself

## Current Summary

## Summary on refinement

- Refining means making models more deterministic
- Refining means adding new variable and new events
- Refining is simulating
- Refining preserves safety properties of the refined model.
- The very abstract model is crucial.
- The process should be incremental to make proofs easier for the proof tool.
- Problem : Preserving the liveness properties