Modelling Software-based Systems

Lecture 6

Validation, Verification and Proof Tools for Event-B for Correct-by-Construction

Telecom Nancy (IL et LE)

Dominique Méry
Telecom Nancy,Université de Lorraine

24 novembre 2025

dominique.mery@loria.fr

General Summary

Current Summary

Summary

Annotation of programs

$\ell : \{P_\ell(v)\}$
$\quad cond_{\ell,\ell'}(v) \longrightarrow v := f_{\ell,\ell'}(v)$
$\ell' : \{P_{\ell'}(v)\}$

$\ell_0^1 : \{x = 0\}$
$\quad x := x + 1;$
$\ell_0^1 : \{x = 1\}$

$e(\ell, \ell')$
WHEN
$\quad c = \ell$
$\quad cond_{\ell,\ell'}(v)$
THEN
$\quad c := \ell'$
$\quad v := f_{\ell,\ell'}(v)$
END

- v is the state meory variable or list of memory variables ; v includes the local variables and the results variables.
- c is a new variable which is modelling the control flow and its type is LOCATIONS.
- $e(\ell, \ell')$ is simulating the computation flow starting from ℓ and moving to ℓ' ; v is updated.

INVARIANTS

$inv_i : c \in \textsc{Locations}$
$inv_j : v \in Type$
$\ldots$
$inv_k : c = \ell \Rightarrow P_\ell(v)$
$inv_m : c = \ell' \Rightarrow P_{\ell'}(v)$
$\ldots$
$th_n : A(c, v)$

- $Type$ is the type of the variables v and is a set of possible values defined in the context C.
- The annotation is giving us for free the conditions satisfied by v when the control is in ℓ, (resp. in ℓ').
- $A(c, v)$ is a safety property that we are supposed to check and the case of Event-B, it is a theorem.

Partial correctness using Event-B models

For each pair of successive labels ℓ, ℓ', the three statements are equivalent :

- $P_\ell(v) \wedge cond_{\ell,\ell'}(v) \wedge v' = f_{\ell,\ell'}(v) \Rightarrow P_{\ell'}(v')$
- $I(c,v) \wedge c = \ell \wedge cond_{\ell,\ell'}(v) \wedge c' = \ell' \wedge v' = f_{\ell,\ell'}(v) \Rightarrow (c' = \ell' \Rightarrow P_{\ell'}(v'))$
- $I(c,v) \wedge BA(e(\ell,\ell'))(c,v,c',v') \Rightarrow (c' = \ell' \Rightarrow P_{\ell'}(v'))$

L

et AA an annotated algorithm with precondition **pre**(AA)(v) and postcondition **post**(AA)(v_0,v). Let the context C and the machine M generated from AA using the construction given previously. We assume that ℓ_0 is the first label and ℓ_e is the last label. We add the following safety properties in the machine M :

- $c = \ell_0 \wedge \textbf{pre}(\mathrm{AA})(v) \Rightarrow P_{\ell_0}(v)$
- $c = \ell_e \Rightarrow (P_{\ell_e}(v) \Rightarrow \textbf{post}(\mathrm{AA})(v_0, v)$

If proof obligations are discharged, then the annotated algorithm AA is partially correct with respect to ist pre/post specification.

Summary

Current Summary

- The panel called *proof control* is providing tools for *discharging* proof obligations
- Proof obligations are stated as follows : $\Gamma \vdash P$: *When assertions of* Γ *are supposed to hold, then* P *does hold*
- Problem : Checking that the sequent $\Gamma \vdash P$ is valid.
- How : Applying automatic procedures as pp, pr etc
- . . . or helping the proof tool to build the *proof tree*

Main Limits of PP (Predicate Prover)

- Quite sensitive to useless hypotheses.
- Unable to automatically abstract some statements.
- Sometimes poor management of equalities.
- Needs some creative hints

- Proposing clever lemma : the user may have interesting ideas because he/she knows the problem to solve.
- Suggesting a proof by case.
- Proposing a witness for an existential goal.
- Proposing a universal hypothesis instanciation.
- etc.

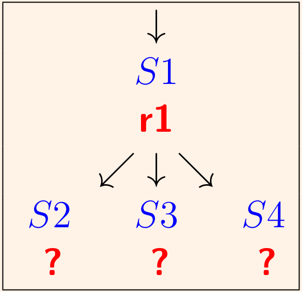
- Sequent (where $list\_of\_hypotheses$ might be empty).

$$\boxed{list\_of\_hypotheses \quad \vdash \quad conclusion}$$

- Inference rule (where $list\_of\_sequents$ might be empty).

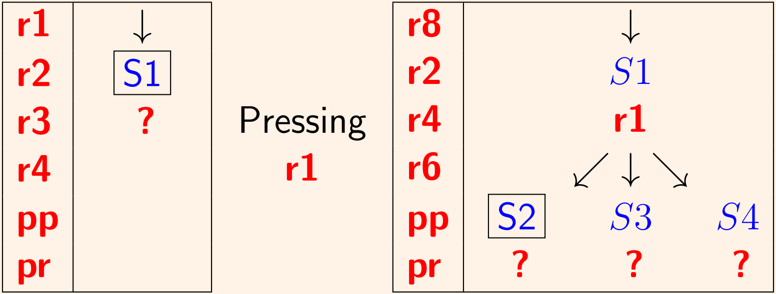
$$\boxed{\frac{list\_of\_sequents}{sequent}}$$

- Backwards application of inf. rules $\leadsto$ proof tree.



- Rule **r1** is applied to sequent S1.

- Producing sequents S2, S3, and S4.



- The proof tree is constructed interactively.

- The palette evolves as the $\boxed{\text{sequent of interest}}$ varies.

r8		↙ ↓ ↘	
r2	$\boxed{\mathrm{S2}}$	$S3$	$S4$
r4	**?**	**?**	**?**
r6			
pp			
pr			

pp
then
r6

r5		↙ ↓ ↘	
r4	$S2$	$S3$	$S4$
r6	**pp**	**r6**	**?**
r9		↙ ↓	
pp	$\boxed{\mathrm{S5}}$	$S6$	
pr	**?**	**?**	

- Our tentative "interface" does not work.

More Precise Form of Sequent

$$H \vdash L \implies G$$

- Most of the time the conclusion has an implicative form
- G is called the goal (it is a non-conjunctive predicate)
- L are called the local hypotheses (might be empty)

$$H \vdash I(x) \wedge G(x) \wedge P(x, x') \implies I(x')$$

Tentative Solution : Using Two Windows

- Tree window with simplified sequents (goals only)

- Sequent window containing the $\boxed{\text{full sequent of interest}}$

Sequent Window	Tree Window
$H2 \vdash L2 \Longrightarrow G2$	$G1$ **r1** $\swarrow \; \downarrow \; \searrow$ $\boxed{\text{G2}}$ $G3$ $G4$ **?** **?** **?**
r2 r4 r6 pp pr	

$$H2 \vdash L2 \Longrightarrow G2$$

r2 r4 r6 pp pr

$$\begin{array}{ccc} & G1 & \\ & \textbf{r1} & \\ \swarrow & \downarrow & \searrow \\ \boxed{\mathrm{G2}} & G3 & G4 \\ \textbf{?} & \textbf{?} & \textbf{?} \end{array}$$

Pressing **pp**

$$H3 \vdash L3 \Longrightarrow G3$$

r2 r5 r8 pp pr

$$\begin{array}{ccc} & G1 & \\ & \textbf{r1} & \\ \swarrow & \downarrow & \searrow \\ G2 & \boxed{\mathrm{G3}} & G4 \\ \textbf{pp} & \textbf{?} & \textbf{?} \end{array}$$

More Realistic Windows

$H3$
$L3$
$G3$
r2 r4 r6 pp pr

$G1$
r1
$G2$
pp
$\rightarrow$ $G3$
?
$G4$
?

$$\underbrace{hidden \;\; ; \;\; searched \;\; ; \;\; cached}_{list\_of\_hypotheses} \;\; \vdash \;\; \underbrace{local \; \Longrightarrow \; goal}_{conclusion}$$

$hidden$	Not visible on the sequent window
$searched$	Visible after some search in $hidden$
$cached$	Visible but not part of the conclusion any m
$local$	Visible and part of the conclusion

Current Summary

Context and Objectives

Context

Developing life-cycle methodology and a set of associated techniques and tools to develop the highly critical systems using formal techniques from requirements analysis to code generation.

Objectives

- To establish a unified theory for the critical system development.
- To Build a set of tools for supporting new development life-cycle methodology.
- To develop a closed-loop system for verification purpose.
- Graphical based refinement technique to handle the complexity of the system.
- To satisfy requirements and metrics for certifiable assurance and safety.
- To support evidence-based certification.

Development Methodology

- Formal Methods based Critical System Development Life-Cycle

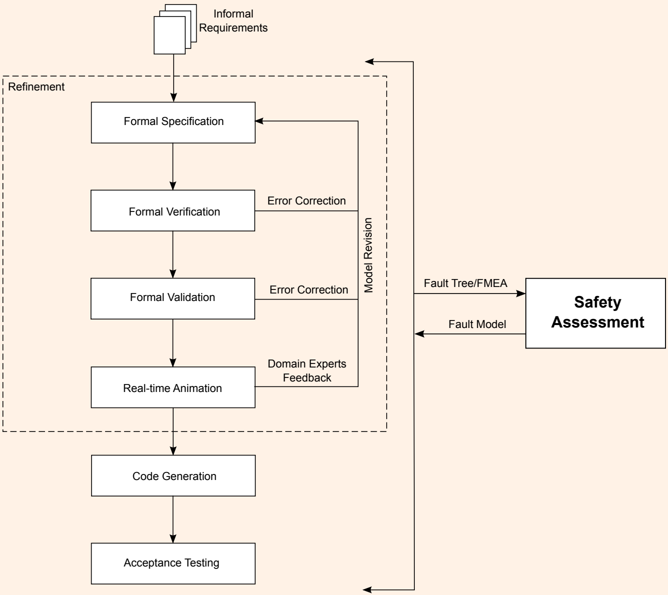
Techniques and Tools

- Real-Time Animator
- Refinement Chart
- Automatic Code Generator : EB2ALL
- Formal Logic Based Heart-Model

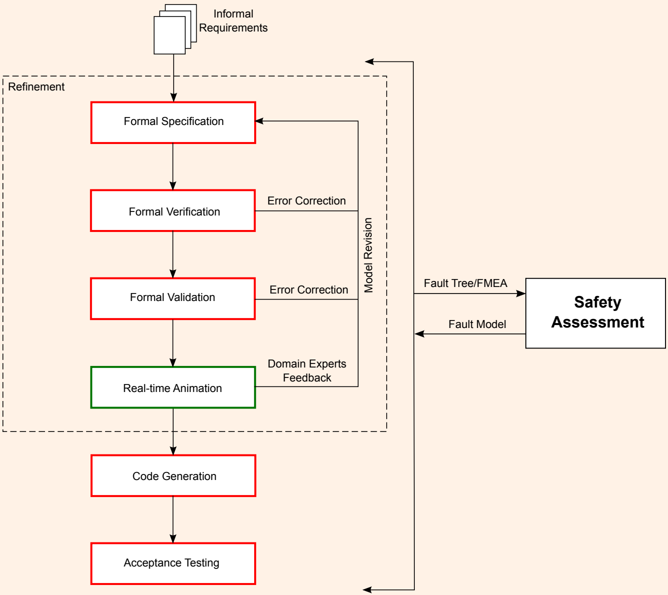
Case Studies

- The Cardiac Pacemaker

Critical System Development Life-Cycle Methodology



Critical System Development Life-Cycle Methodology



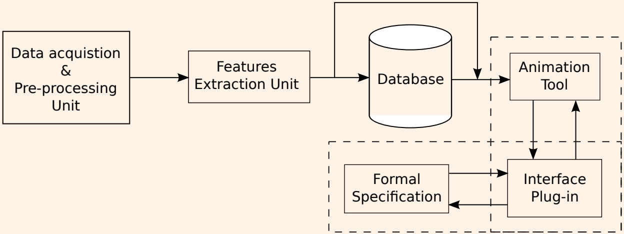
What is Real-time Animator ?

- Visual representation of formal model using real time data set.

Why should we use Formal Model Animator ?

- To validate system behavior according to the stakeholders
- To express formal models for non-mathematical domain experts
- To discover the error in the early stage of system development (Traceability)

Proposed Architecture

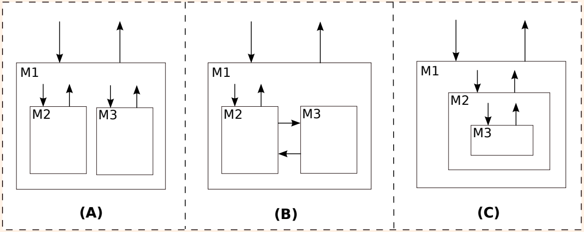


- Refinement chart is a graphical representation of a complex system using layering approach.
- Integration and features based code structuring of different components of the systems.

Basic rules of refinement chart

- Parallel Refinement
- Sequential Refinement
- Nested Refinement

$$M1 \sqsubseteq (M2 \parallel M3 \parallel \ldots\ldots \parallel M_{n-1} \parallel M_n)$$
$$M1 \sqsubseteq (M2 \succ M3 \succ \ldots\ldots \succ M_{n-1} \succ M_n)$$
$$(M1 \sqsubseteq M2,\ M2 \sqsubseteq M3, \ldots\ldots,\ M_{n-1} \sqsubseteq M_n)$$



$$M1 \sqsubseteq (M2 \parallel M3)$$
$$M1 \sqsubseteq (M2 \succ M3)$$
$$(M1 \sqsubseteq M2,\ M2 \sqsubseteq M3)$$

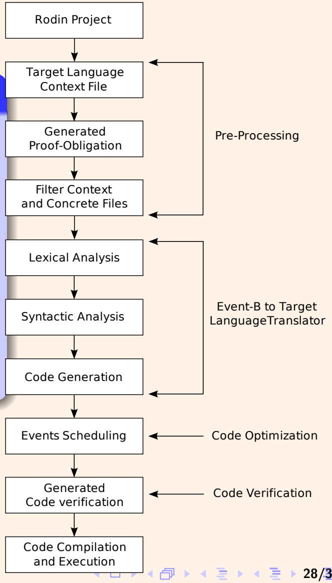
EB2ALL Code Generation Tool

EB2ALL

- Automatic code generation from Event-B formal specification to multiple languages (C, C++, Java, C#) to make it executable.
- A module for Rodin (Deploy is an European Commission FP7 project)
- It is more than 20000 lines of code
- More than 1000 downloads

Download : http://eb2all.loria.fr/

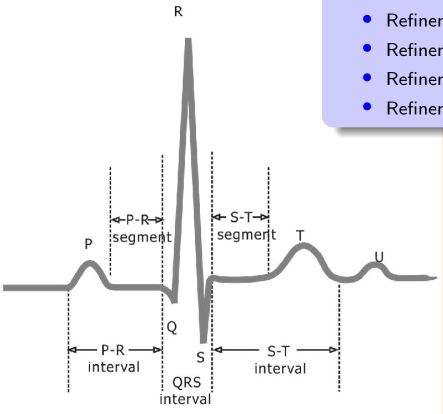
EB2ALL Tool Architecture

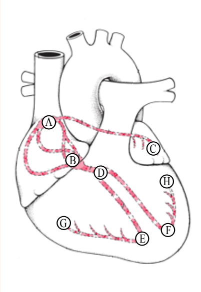


Formal Development

- Abstract Model
- Refinement 1 : Introducing Steps in the Propagation
- Refinement 2 : Impulse Propagation
- Refinement 3 : Perturbation the Conduction
- Refinement 4 : Getting a Cellular Model

Electrocardiogram (ECG)





Landmarks on the Electrical Conduction of Heart

Current Summary

Steps of the project

- Reading the description of the problem to solve.
- List the safety properties as partial correctness, mutual exclusion, ... and properties on sets, constants, ...
- Identify the state variables for the first machine
- Define inductive definitions if necessary and state correctness properties as $\forall n.n\mathbb{N} \Rightarrow n * n = v(n)$.
- List Use Cases : for instance, adding a new user, updating a a phone number, recording a movement ...
- First expression of a context and a machine.

- **Property**
  - *the set of people is non empty and finite*
  - $people \neq \varnothing \wedge finute(people)$