

Modelling Software-based Systems

The Modelling Language Event-B

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General Summary

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 - Safety Properties of C Programs
 - Importance of Domain
 - Tracking bugs in C codes
- 3 Overview of formal techniques and formal methods
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- 7 The Event B modelling language
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- 9 Summary on Events

Current Summary

- Summary on Events

- Event B : <http://www.event-b.org/>
- Atelier B : <http://www.atelierb.eu/>
- RODIN Platform : <http://www.event-b.org/platform.html>
- EB2ALL Toolset : <http://eb2all.loria.fr>
- RIMEL project : <http://rimel.loria.fr>
- The Modelling Language Event-B and related topics as lectures notes, tutorials, models. <https://mery54.github.io/teaching/>
- **Using the Arche platform of UL and accessing the course MOSOS with password mery**

Current Summary

- Summary on Events

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y;
    // Seed the random number generator with the current time
    srand(time(NULL));
    // Generate a random number between 1 and 100
    x = rand() % 100 + 1;
    // Perform some calculations
    y = x / (100 - x);
    printf("Result: -%d\n", y);
    return 0;
}
```

```
int main(void)
{
    int __retres;
    int x;
    int y;
    time_t tmp;
    int tmp_0;
    tmp = time((time_t *)0);
    srand((unsigned int)tmp);
    { /* sequence */
        tmp_0 = rand();
        /*@ assert rte: signed_overflow: (int)(tmp_0 % 100) + 1 <= 214
        x = tmp_0 % 100 + 1;
    }
    /*@ assert rte: signed_overflow: 100 - x <= 2147483647; */
    /*@ assert rte: division_by_zero: (int)(100 - x) /= 0; */
    /*@ assert rte: signed_overflow: x / (int)(100 - x) <= 21474836
    y = x / (100 - x);
    printf(" Result: -%d\n", y); /* printf_va_1 */
    __retres = 0;
    return __retres;
}
```

```
// Heisenbug
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y, i=0;

    for (i = 0; i <= 100000; i++) {
        // Seed the random number generator with the current time
        srand(time(NULL));

        // Generate a random number between 1 and 100
        x = rand() % 100 + 1;
        printf(" Result: -x=- %d\n", x);
        // Perform some calculations
        y = x / (100 - x);

        printf(" Result: -i=%d - %d\n", i, y);
    }

    return 0;
}
```


RTE with frama-c but a modification

```
// Heisenbug
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y, i=0;

    for (i = 0; i <= 100; i++) {
        // Seed the random number generator with the current time
        srand(time(NULL)+i);

        // Generate a random number between 1 and 100
        x = rand() % 100 + 1;
        printf(" Result: -x=- %d\n", x);
        // Perform some calculations
        y = x / (100 - x);

        printf(" Result: -i=%d - %d\n", i, y);
    }

    return 0;
}
```

Our aim is to analyze what is implicit and what is explicit in formal modelling...

- **Semantics in modelling :**

- ▶ Semantics expressed by a *theory* (e.g. Event-B) used to formalize hardware and/or software systems
- ▶ Same theory is used for wide variety of heterogeneous systems

- **Semantics in domain :**

- ▶ Environment within which system evolve : application domain/context
- ▶ Information provided by domain is often associated while in operation
- ▶ Either assumed and omitted while formalising systems or hardcoded in formal models
- ▶ Same context is used for wide variety of heterogeneous systems



A case study for studying these properties

Nose Gear Velocity



- Estimated ground velocity of the aircraft should be available only if it is within 3 km/hr of the true velocity at some moment within past 3 seconds

Characterization of a System (I)

- NG velocity system :
 - ▶ **Hardware :**
 - *Electro-mechanical sensor* : detects rotations
 - *Two 16-bit counters* : Rotation counter, Milliseconds counter
 - *Interrupt service routine* : updates rotation counter and stores current time.
 - ▶ **Software :**
 - *Real-time operating system* : invokes update function every 500 ms
 - *16-bit global variable* : for recording rotation counter update time
 - *An update function* : estimates ground velocity of the aircraft.
- Input data available to the system :
 - ▶ *time* : in milliseconds
 - ▶ *distance* : in inches
 - ▶ *rotation angle* : in degrees
- Specified system performs velocity estimations in *imperial* unit system
- **Note** : expressed functional requirement is in *SI* unit system (km/hr).

What are the main properties to consider for formalization ?

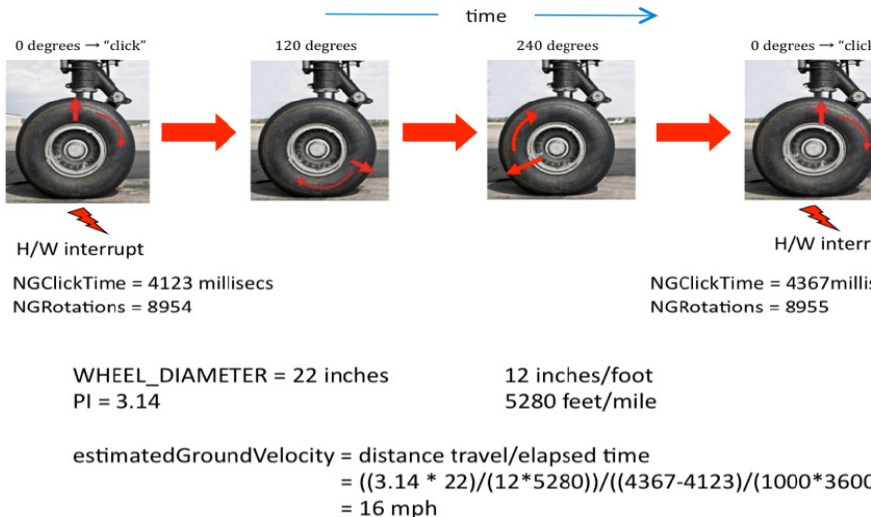
- Two different types of data :
 - ▶ counters with modulo semantics
 - ▶ non-negative values for time, distance, and velocity
- Two dimensions : *distance* and *time*
- Many units : distance (inches, kilometers, miles), time (milliseconds, hours), velocity (kph, mph)
- And interaction among components

How should we model ?

- Designer needs to consider units and conversions between them to manipulate the model
- **One approach** : Model units as *sets*, and conversions as constructed types – *projections*.
- Example :

- 1 $estimateVelocity \in \text{MILES} \times \text{HOURS} \rightarrow \text{MPH}$
- 2 $mphTokph \in \text{MPH} \rightarrow \text{KPH}$

Sample Velocity Estimation



Listing 1 – Bug bug0

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y;
    // Seed the random number generator with the current time
    srand(time(NULL));
    // Generate a random number between 1 and 100
    x = rand() % 100 + 1;
    // Perform some calculations
    y = x / (100 - x);
    printf("Result: %d\n", y);
    return 0;
}
```

Listing 2 – Bug bug00

```
// Heisenbug
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y, i=0;

    for (i = 0; i <= 100000; i++) {
        // Seed the random number generator with the current time
        srand(time(NULL));

        // Generate a random number between 1 and 100
        x = rand() % 100 + 1;
        printf("Result: -x=-%d\n", x);
        // Perform some calculations
        y = x / (100 - x);

        printf("Result: -i=%d -y=%d\n", i, y);
    }

    return 0;
}
```


Listing 3 – Bug bug000

```
// Heisenbug
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y, i=0;

    for (i = 0; i <= 100; i++) {
        // Seed the random number generator with the current time
        srand(time(NULL)+i);

        // Generate a random number between 1 and 100
        x = rand() % 100 + 1;
        printf("Result: -x=-%d\n", x);
        // Perform some calculations
        y = x / (100 - x);

        printf("Result: -i=%d -y=%d\n", i, y);
    }

    return 0;
}
```

Current Summary

- Summary on Events

- Distributed systems : web services, information systems, distributed algorithms ...
- Safety critical systems : medical devices, embedded systems, cyber-physical systems, ...
- Fault-tolerant systems : networks, communication infrastructure, ...
- Environments : heart, the glucose-insulin regulatory system, ...

- Abstraction and refinement of features, 2000 *with D. Cansell*
- Incremental Proof of the Producer/Consumer Property for the PCI Protocol, 2002 *with D. Cansell, G. Gopalkrishnan, S. Jones.*
- A Mechanically Proved and Incremental Development of IEEE 1394 Tree Identify Protocol, 2003, *with J.-R. Abrial and D. Cansell.*
- The challenge of QoS for digital television services-. *EBU Technical Review* (avril 2005) *with D. Abraham, D. Cansell, C. Proch.*
- -Formal and Incremental Construction of Distributed Algorithms : On the Distributed Reference Counting Algorithm, 2006 *with D. Cansell.*

- Refinement : A Constructive Approach to Formal Software Design for a Secure e-voting Interface-, 2007 *with D. Cansell and P. Gibson.*
- Incremental Parametric Development of Greedy Algorithms, 2007, *with D. Cansell.*
- System-on-Chip Design by Proof-based Refinement, 2009 *with D. Cansell and C. Proch*
- -A simple refinement-based method for constructing algorithms, 2009. *Alone.*
- -Refinement-based guidelines for algorithmic systems-. *Alone. International Journal of Software and Informatics* (2009),

- Cryptologic algorithms : Event B development, combining cryptologic properties, modeling attacks.
- Access control systems : relating policy models and Event B models like in RBAC, TMAC, ORBAC
- Distributed algorithms : integration of local computation models into Event B, tool B2VISIDIA, algorithms of naming, election etc
- Medical devices : modelling the pacemaker, interacting with cardiologists, ...
- Modelling self- \star systems
- Modelling medical devices item Modelling environments for medical devices : closed-loop modelling

Next modelling

- Modelling human-in-the-loop systems
- Modelling cyber-physical systems

General Approach

- Constructing a model of the system
- Elements for defining a formal or semi-formal model : syntax, semantics, verification, validation, documentation
- Mathematical structures : transition systems, temporal/modal/deontic/... logics,
- Validation of a model : tests, proofs, animation,...
- Modelling Techniques : state-based techniques
- Structure of a model : module, object, class,
- Design Patterns

Mathematical tools for modelling systems

- set theory : sets, relations, functions . . .
- transition systems
- predicate calculus
- decision procedures
- interactive theorem prover

Examples of modelling languages

- Z : set theory, predicate calculus, schemas.
- VDM : types, pre/post specification, invariant, operations
- B : set theory, predicate calculus, generalized substitution, abstract machines, refinement, implementation.
- RAISE : abstract data types, functions,
- TLA⁺ : set theory, modules, temporal logic of actions.
- UNITY : temporal logic, actions systems, superposition.
- UML
- JML and Spec[#] : programming by contract

Objectives of the modelling

- To get a better understanding of the current system : requirements, properties, cost, maintenance . . .
- To document the the system
- To systematize operations of modelling : reuse, parametrization
- To ensure the quality of the final product : safety, security issues
- To elaborate a contract between the customer and the designer

The Triptych Approach

$$\mathcal{D}, \mathcal{S} \longrightarrow \mathcal{R} \quad (1)$$

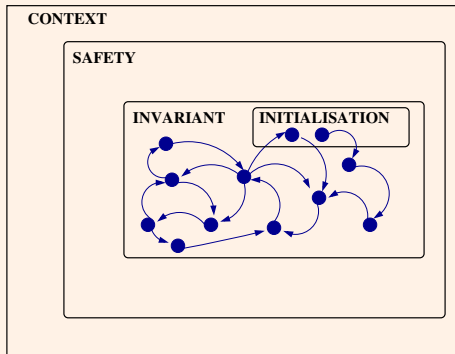
- \mathcal{R} requirements or system properties
- \mathcal{D} domain of the problem
- \mathcal{S} model of the system
- \longrightarrow relation of satisfaction

- Mathematical foundations of Models : syntax, semantics, pragmatics, theory, soundness.
- Mathematical reasoning is based on sound proof rules
- Common language for facilitating the communication.

Current Summary

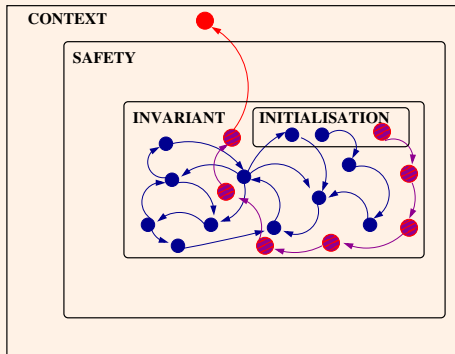
- Summary on Events

Observing the safe system



- The context defines the possible values
- Safety requirement means that *something bad will never happen*.
- Invariant defines the set of effective possible values
- Transitions modify state variables and maintains the invariant.

Observing the unsafe system



- Transitions modify state variables and **may not maintain** the invariant.
- ... and **may not guarantee** safety properties.

- Event B : <http://www.event-b.org/>
- Atelier B : <http://www.atelierb.eu/>
- RODIN Platform : <http://www.event-b.org/platform.html>

The Event B Method

- The Event B Method is invented by J.-R. Abrial from 1988 : abstract system, events, refinement, invariant.
- Atelier B and RODIN are supporting the Event B method
- An event is observed and triggered, when a guard is true
- Proof obligations are generated using the weakest-precondition semantics.
- A Event B model intends to model a reactive system.

Current Summary

- Summary on Events

Managing teachers, students, lectures and class rooms

- Modelling the access control of students for lectures given by teachers
- When a student is attending a lecture, he/she can not attend another lecture
- When a teacher is lecturing, he/she is not lecturing another session.
- A student can not be lecturing without a teacher and when he is not attending a lecture, he is outside the classroom.
- When a teacher is ending a lecture, every student which is attending, is leaving the class room.
- When a student is not attending a lecture, he is free.

First step : identification of sets, constants, properties

- Sets : students, teachers
- **Property 1** : When a student is attending a lecture, he/she can not attend another lecture
- **Property 2** : When a teacher is lecturing, he/she is not lecturing another session.
- **Property 3** : A student can not be lecturing without a teacher and when he is not attending a lecture, he is outside the classroom.
- **Property 4** : When a teacher is ending a lecture, every student which is attending, is leaving the class room.
- **Property 5** : When a student is not attending a lecture, he is free.

Second step : definition of state variables

- The system model should be able to record the lecturing teachers and the attending students.
- The system model should be enough expressive to state when a given student is attending a lecture given by whom.
- Variable **attending** records students which attended some lecture with a given teacher.
- Variable **islecturing** records teachers who are lecturing.
- Variable **pause** records students are not attending a lecture but are somewhere not in a lecture.

Third step : properties of state variables

Expression of the invariant

$$\begin{aligned} inv1 &: attending \in STUDENTS \rightarrow TEACHERS \\ inv2 &: islecturing \subseteq TEACHERS \\ inv3 &: \forall e \cdot e \in STUDENTS \wedge e \in dom(attending) \\ &\quad \Rightarrow attending(e) \in islecturing \\ inv4 &: pause \subseteq STUDENTS \\ inv5 &: pause \cap dom(attending) = \emptyset \\ inv6 &: pause \cup dom(attending) = STUDENTS \end{aligned}$$

Checking proof obligations!

Fourth step : Updating state variables

UseCases

- **EVENT** INITIALISATION : initializing state variables
- **EVENT** startingattending : a group of students is moving from *pause* to *lecture*
- **EVENT** teachergivinglecture : a teacher is starting a new lecture
- **EVENT** teacherendinglecture : a teacher is halting the lecture
- **EVENT** studentleavinglecture : a group of students is moving from *lecture* to *pause*

Fourth step : Updating state variables

EVENT INITIALISATION

BEGIN

$$act1 : attending := \emptyset$$
$$act2 : islecturing := \emptyset$$
$$act3 : pause := STUDENTS$$

END

Fourth step : Updating state variables

```
EVENT startingattending
ANY
  e e is a student
  p p is a teacher
WHERE
  grd1 :  $e \in STUDENTS$ 
  grd3 :  $p \in TEACHERS$ 
  grd4 :  $p \in islecturing$ 
  grd2 :  $e \notin dom(attending)$ 
THEN
  act1 :  $attending(e) := p$ 
  act2 :  $pause := pause \setminus \{e\}$ 
END
```

Fourth step : Updating state variables

```
EVENT teachergivinglecture
  ANY
     $p$ 
  WHERE
     $grd2 : p \in TEACHERS$ 
     $grd1 : p \notin islecturing$ 
  THEN
     $act1\ islecturing := islecturing \cup \{p\}$ 
  END
```

Fourth step : Updating state variables

```
EVENT teacherendinglecture
ANY
  p
WHERE
  grd1 :  $p \in TEACHERS$ 
  grd2 :  $p \in islecturing$ 
THEN
  act1 :  $islecturing := islecturing \setminus \{p\}$ 

  act3 :  $attending := attending \setminus \{f \mapsto q \mid \left( \begin{array}{l} f \in STUDENTS \\ \wedge q \in TEACHERS \\ \wedge f \mapsto q \in attending \\ \wedge q = p \end{array} \right)\}$ 

  act2 :  $pause := pause \cup \{f \mid f \in STUDENTS \wedge f \in attending^{-1}[\{p\}]\}$ 
END
```

Fourth step : Updating state variables

```
EVENT studentleavinglecture
  ANY
     $ge$ 
  WHERE
     $grd1 : ge \subseteq dom(attending)$ 
     $grd2 : ge \neq \emptyset$ 
  THEN
     $act1 : attending := ge \triangleleft attending$ 
     $act2 : pause := pause \cup ge$ 
  END
```

Mathematical tools for modelling systems

- **set theory : sets, relations, functions . . .**
- **transition systems**
- **predicate calculus**
- **decision procedures**
- **interactive theorem prover**

Current Summary

- Summary on Events

Modelling systems

- A **system** is **observed**
- Observation of things which are
 - ▶ either changing over the **time** (*variable*)
 - ▶ or stuttering over the **time** (*constant*)
- A system is characterized by a **state**
- A state is made up of contextual **constant informations** over the problem theory and of **modifiable flexible informations** over the system.

Changing state of system

A **flexible variable** x is observed at different instants :

$$x_0 \xrightarrow{\tau} x_1 \xrightarrow{\tau} x_2 \xrightarrow{\tau} x_3 \xrightarrow{\tau} \dots \xrightarrow{\tau} x_i \xrightarrow{\tau} x_{i+1} \xrightarrow{\tau} \dots$$

Changing state of system

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$$x_0 \xrightarrow{\tau} x_1 \xrightarrow{\tau} x_2 \xrightarrow{\tau} x_3 \xrightarrow{\tau} \dots \xrightarrow{\tau} x_i \xrightarrow{\tau} x_{i+1} \xrightarrow{\tau} \dots$$

τ hides effectives changes of state or actions or events

Changing state of system

A **flexible variable** x is observed at different instants :

$$x_0 \xrightarrow{\tau} x_1 \xrightarrow{\tau} x_2 \xrightarrow{\tau} x_3 \xrightarrow{\tau} \dots \xrightarrow{\tau} x_i \xrightarrow{\tau} x_{i+1} \xrightarrow{\tau} \dots$$

τ hides effectives changes of state or actions or events

$$x_0 \xrightarrow{\alpha_1} x_1 \xrightarrow{\alpha_2} x_2 \xrightarrow{\alpha_3} x_3 \xrightarrow{\alpha_4} \dots \xrightarrow{\alpha_i} x_i \xrightarrow{\alpha_{i+1}} x_{i+1} \xrightarrow{\alpha_{i+2}} \dots$$

Changing state of system

A **flexible variable** x is observed at different instants :

$$x_0 \xrightarrow{\tau} x_1 \xrightarrow{\tau} x_2 \xrightarrow{\tau} x_3 \xrightarrow{\tau} \dots \xrightarrow{\tau} x_i \xrightarrow{\tau} x_{i+1} \xrightarrow{\tau} \dots$$

τ hides effectives changes of state or actions or events

$$x_0 \xrightarrow{\alpha_1} x_1 \xrightarrow{\alpha_2} x_2 \xrightarrow{\alpha_3} x_3 \xrightarrow{\alpha_4} \dots \xrightarrow{\alpha_i} x_i \xrightarrow{\alpha_{i+1}} x_{i+1} \xrightarrow{\alpha_{i+2}} \dots$$

Occurrences of e τ can be added between two instants ie **stuttering steps** :

Changing state of system

A **flexible variable** x is observed at different instants :

$$x_0 \xrightarrow{\tau} x_1 \xrightarrow{\tau} x_2 \xrightarrow{\tau} x_3 \xrightarrow{\tau} \dots \xrightarrow{\tau} x_i \xrightarrow{\tau} x_{i+1} \xrightarrow{\tau} \dots$$

τ hides effective changes of state or actions or events

$$x_0 \xrightarrow{\alpha_1} x_1 \xrightarrow{\alpha_2} x_2 \xrightarrow{\alpha_3} x_3 \xrightarrow{\alpha_4} \dots \xrightarrow{\alpha_i} x_i \xrightarrow{\alpha_{i+1}} x_{i+1} \xrightarrow{\alpha_{i+2}} \dots$$

Occurrences of τ can be added between two instants ie **stuttering steps** :

$$x_0 \xrightarrow{\alpha_1} x_1 \xrightarrow{\alpha_2} x_2 \xrightarrow{\tau} x_2 \xrightarrow{\alpha_3} x_3 \xrightarrow{\alpha_4} \dots \xrightarrow{\alpha_i} x_i \xrightarrow{\tau} x_i \xrightarrow{\alpha_{i+1}} x_{i+1} \xrightarrow{\alpha_{i+2}} \dots$$

A **safety** property S over x states that something will not happen : $S(x)$
means that S holds for x

An **invariant** property I over x states a strong safety property

$$\begin{array}{ccccccc} x_0 & \xrightarrow{\alpha_1} & x_1 & \xrightarrow{\alpha_2} & x_2 & \xrightarrow{\tau} & x_2 & \xrightarrow{\alpha_3} & x_3 & \xrightarrow{\alpha_4} & \dots & \xrightarrow{\alpha_i} & x_i & \xrightarrow{\tau} & x_i & \xrightarrow{\alpha_{i+1}} & x_{i+1} & \xrightarrow{\alpha_{i+2}} & \dots \\ (S(x_0) & \xrightarrow{\alpha_1} & S(x_1) & \xrightarrow{\alpha_2} & S(x_2) & \xrightarrow{\tau} & S(x_2) & \xrightarrow{\alpha_3} & S(x_3) & \xrightarrow{\alpha_4} & \dots & \xrightarrow{\alpha_i} & S(x_i) & \xrightarrow{\tau} & \\ S(x_i) & \xrightarrow{\alpha_{i+1}} & S(x_{i+1}) & \xrightarrow{\alpha_{i+2}} & \dots & \end{array}$$

or equivalently $\forall i \in \mathbb{N} : S(x_i)$

Checking the relation

- You can check for every i in \mathbb{N} that $S(x_i)$ is true but it can be long if states are different
- You can compute an abstraction of the set of states
- You can try to prove and for instance the induction principle may be usefull
- So be carefull and improve your modelling before to run the checker
- Use the induction

State properties of a system

- A state property namely $P(x)$ is a first order predicate with free variables x , where x is a flexible variable.
- A flexible variable x has a current value x , a next value x' , an initial value x_0 and possibly a final value x_f .
- A predicate $P(x)$ is considered as a set of values v such that $P(v)$ holds : set-theoretical interpretation

Safety Property

A safety property states that nothing bad can happen.

Example

Safety Properties

- Partial correctness a component is correct with respect to a precondition and a postcondition.
- No Run Time Error any software action or event does not produce a run-time error as overflow, division by zero ...
- Mutual exclusion a set of processes share common resources, a printer is shared by users, ...
- Deadlock freedom the system is never blocked, there is always at least one next state, ...

- An action α over states is a relation between values of state variables **before** and values of variables **after**

$$\alpha(x, x') \text{ or } x \xrightarrow{\alpha} x'$$

- Flexible variable x has two values x and x' .
- Priming flexible variables is borrowed from TLA
- **Hypothesis 1** : **Values of x belongs to a set of values called VALUES** and defines the context of the system.
- **Hypothesis 2** : **Relations over x and x' belong to a set of relations $\{r_0, \dots, r_n\}$**

Operational model of a system

- A system \mathcal{S} is observed with respect to flexible variables x .
- Flexible variables x of \mathcal{S} are modified according to a finite set of relations over the set of values $\text{VALUES} : \{r_0, \dots, r_n\}$
- $\text{INIT}(x)$ denotes the set of possible initial values for x .

$$\mathcal{OMS} = (x, \mathbf{Values}, \mathbf{Init}(x), \{r_0, \dots, r_n\})$$

Safety and invariance of system

- **Hypothesis 3** : $\mathcal{OMS} = (x, \text{VALUES}, \text{INIT}(x), \{r_0, \dots, r_n\})$
- **Hypothesis 4** : $x \longrightarrow x' \triangleq (x \ r_0 \ x') \vee \dots \vee (x \ r_n \ x')$
- $I(x)$ is inductively invariant for a system called \mathcal{S} , if

$$\begin{cases} \forall x \in \text{VALUES} : \text{INIT}(x) \Rightarrow I(x) \\ \forall x, x' \in \text{VALUES} : I(x) \wedge x \longrightarrow x' \Rightarrow I(x') \end{cases}$$
 $I(x)$ is called an invariant in \mathcal{B}
- $Q(x)$ is a safety property for a system called \mathcal{S} , if

$$\forall x, y \in \text{VALUES} : \text{INIT}(x) \wedge x \xrightarrow{\star} y \Rightarrow Q(y)$$
 $Q(x)$ is called a theorem in \mathcal{B}

Modelling systems : first attempt

MODEL m

...

...

...

VARIABLES x **INVARIANT** $I(x)$ **THEOREMS** $Q(x)$ **INITIALISATION** $Init(x)$ **EVENTS** $\{r_0, \dots, r_n\}$ **END**

- A model has a name m
- Flexible variables x are declared
- $I(x)$ provides information over x
- $Q(x)$ provides information over x

- $\forall x_0, x \in \text{VALUES} : \text{INIT}(x_0) \wedge x_0 \xrightarrow{*} x \Rightarrow Q(x)$
- **Solution 1** Writing a procedure checking $\text{INIT}(x_0) \wedge x_0 \xrightarrow{*} x \Rightarrow Q(x)$ for each pair $x_0, x \in \text{VALUES}$, when VALUES is finite and small.
- **Solution 2** Writing a procedure checking $\text{INIT}(x_0) \wedge x_0 \xrightarrow{*} x \Rightarrow Q(x)$ for each pair $x_0, x \in \text{VALUES}$, by constructing an abstraction of VALUES .
- **Solution 3** Writing a proof for $\forall x_0, x \in \text{VALUES} : \text{INIT}(x_0) \wedge x_0 \xrightarrow{*} x \Rightarrow Q(x)$.

Defining an induction principle for an operational model

(I) $\forall x_0, x \in \mathbf{Values} : \mathbf{Init}(x_0) \wedge x_0 \xrightarrow{\star} x \Rightarrow \mathbf{Q}(x)$

if, and only if,

(II) there exists a state property $\mathbf{I}(x)$ such that :

$$\forall x_0, x, x' \in \mathbf{Values} : \begin{cases} (1) & \mathbf{Init}(x_0) \Rightarrow \mathbf{I}(x_0) \\ (2) & \mathbf{I}(x) \Rightarrow \mathbf{Q}(x) \\ (3) & \mathbf{I}(x) \wedge x \longrightarrow x' \Rightarrow \mathbf{I}(x') \end{cases}$$

if, and only if,

(III) there exists a state property $\mathbf{I}(x)$ such that :

$$\forall x_0, x, x' \in \mathbf{Values} : \begin{cases} (1) & \mathbf{Init}(x_0) \Rightarrow \mathbf{I}(x_0) \\ (2) & \mathbf{I}(x) \Rightarrow \mathbf{Q}(x) \\ (3) & \forall i \in \{0, \dots, n\} : \mathbf{I}(x) \wedge x \text{ } r_i \text{ } x' \Rightarrow \mathbf{I}(x') \end{cases}$$

Modelling systems : second attempt

MODEL

m

...

...

...

VARIABLES

x

INVARIANT

$I(x)$

THEOREMS

$Q(x)$

INITIALISATION

$Init(x)$

EVENTS

$\{r_0, \dots, r_n\}$

END

- $\forall x_0 \in \text{VALUES} : \text{INIT}(x_0) \Rightarrow I(x_0)$
- $\forall x, x' \in \text{VALUES} : \forall i \in \{0, \dots, n\} :$
$$I(x) \wedge x \text{ } r_i \text{ } x' \Rightarrow I(x')$$
- $\forall x \in \text{VALUES} : I(x) \Rightarrow Q(x)$

Modelling systems : last attempt ?

MODEL

m

?

?

?

VARIABLES

x

INVARIANT

$I(x)$

THEOREMS

$Q(x)$

INITIALISATION

$Init(x)$

EVENTS

$\{r_0, \dots, r_n\}$

END

- What are the environment of the proof for properties ?
- What are theories ?
- How are defining the static objects ?

Modelling systems : last attempt !

MODEL

m

$\Gamma(m)$

VARIABLES

x

INVARIANT

$I(x)$

THEOREMS

$Q(x)$

INITIALISATION

$Init(x)$

EVENTS

$\{r_0, \dots, r_n\}$

END

- $\Gamma(m)$ defines the static environment for the proofs related to m .
- $\Gamma(m) \vdash \forall x \in \text{VALUES} : \text{INIT}(x) \Rightarrow I(x)$
- $\forall i \in \{0, \dots, n\} :$
 $\Gamma(m) \vdash \forall x, x' \in \text{VALUES} : I(x) \wedge x \ r_i \ x' \Rightarrow I(x')$
- $\Gamma(m) \vdash \forall x \in \text{VALUES} : I(x) \Rightarrow Q(x)$

An **event system model** is made of

State **constants** and state **variables** constrained by a state **invariant**

A finite set of **events**

Proofs ensures the consistency between the invariant and the events

An event system model can be **refined**

Proofs must ensure the correctness of refinement

Modelling systems : Hello world !

MODEL FACTORIAL_EVENTS

Static Part *context*

CONSTANTS

factorial, m

AXIOMS

$$\begin{aligned} & m \in \mathbb{N} \wedge \text{factorial} \in \mathbb{N} \leftrightarrow \mathbb{N} \wedge 0 \mapsto 1 \in \text{factorial} \wedge \\ & \forall (n, fn). (n \mapsto fn \in \text{factorial} \Rightarrow n + 1 \mapsto (n + 1) * fn \in \text{factorial}) \wedge \\ & \forall f. \left(\begin{array}{l} f \in \mathbb{N} \leftrightarrow \mathbb{N} \wedge \\ 0 \mapsto 1 \in f \wedge \\ \forall (n, fn). (n \mapsto fn \in f \Rightarrow n + 1 \mapsto (n + 1) \times fn \in f) \\ \Rightarrow \\ \text{factorial} \subseteq f \end{array} \right) \end{aligned}$$

Dynamic Part *machine*

VARIABLES

result, ok

INVARIANT

$result \in \mathbb{N}$

$ok \in \mathbb{B}$

$ok = TRUE \Rightarrow result = \text{factorial}(n)$

THEOREMS

$\text{factorial} \in \mathbb{N} \longrightarrow \mathbb{N}$;

$\text{factorial}(0) = 1$;

$\forall n. (n \in \mathbb{N} \Rightarrow \text{factorial}(n + 1) = (n + 1) \times \text{factorial}(n))$

INITIALISATION

$result : \in \mathbb{N}$

$ok := FALSE$

EVENTS

computation = **ANY** $ok = FALSE$ **THEN** $result, ok := \text{factorial}(m), TRUE$ **END**

END

Modelling systems relations as events

MODEL

m

Static Part *context*

SETS

s

CONSTANTS

c

AXIOMS

$P(s, c)$

THEOREMS

$Q(s, c)$

Dynamic Part *machine*

VARIABLES

x

INVARIANT

$I(s, c, x)$

THEOREMS

$S(s, c, x)$

INITIALISATION

$Init(s, c, x)$

EVENTS

$\{r_1, \dots, r_n\}$

END

- $\Gamma(m)$ defines the static environment for the proofs related to m from s , c and $P(s, c)$ and $\Gamma(m)$ is defined from the static part.
- $\Gamma(m) \vdash Q(s, c)$
- $\Gamma(m) \vdash \forall x, x' \in \text{VALUES} : \text{INIT}(s, c, x) \Rightarrow I(s, c, x)$
- $\forall i \in \{1, \dots, n\} :$
 $\Gamma(m) \vdash \forall x, x' \in \text{VALUES} :$
 $I(s, c, x) \wedge x \ r_i \ x' \Rightarrow I(s, c, x')$
- $\Gamma(m) \vdash \forall x, x' \in \text{VALUES} : I(s, c, x) \Rightarrow S(s, c, x)$

Modelling systems relations as events

CONTEXT

context_name

SETS

s

CONSTANTS

c

AXIOMS

$P(s, c)$

THEOREMS

$Q(s, c)$

MACHINE

m

SEES

context_name

VARIABLES

x

INVARIANT

$I(s, c, x)$

THEOREMS

$S(s, c, x)$

INITIALISATION

$Init(s, c, x)$

EVENTS

$\{e_1, \dots, e_n\}$

END

- $\Gamma(m)$ defines the static environment for the proofs related to m from s , c and $P(s, c)$ and $\Gamma(m)$ is defined from the static part.
- $\Gamma(m) \vdash Q(s, c)$
- $\Gamma(m) \vdash \forall x, x' \in \text{VALUES} : \text{INIT}(s, c, x) \Rightarrow I(s, c, x)$
- $\forall i \in \{0, \dots, n\} : r_i(x, x') \triangleq BA(e_i)(s, c, x, x')$
- $\forall i \in \{0, \dots, n\} :$
 $\Gamma(m) \vdash \forall x, x' \in \text{VALUES} :$
 $I(x) \wedge BA(e_i)(s, c, x, x') \Rightarrow I(s, c, x')$
- $\Gamma(m) \vdash \forall x, x' \in \text{VALUES} : I(x) \Rightarrow S(s, c, x)$

step 1 : Understanding the **problem** to solve

step 2 : Organizing requirements and extracting properties

step 3 : Writing a first very **abstract** system model

step 4 : Consulting the requirements and **adding** a new detail in the current model by **refinement**

step 5 : Either the model is enough detailed and the process stops, or the model is not yet enough concrete and the step 4 is repeated.

Current Summary

- 1 Documentation
- 2 Introduction by Problems
- 3 Overview of formal techniques and formal methods
- 4 Modelling Language
- 5 A Simple Example : Management of Students and Teachers
- 6 Modelling state-based systems
- 7 The Event B modelling language
- 8 Examples of Event B models
- 9 Summary on Events

Expressing models in the event B notation

- Models are defined in two ways :
 - ▶ an abstract machine
 - ▶ a refinement of an existing model
- Models use **constants** which are defined in structures called **contexts**
- B structures are related by the three possible relations :
 - ▶ the **sees** relationship for expressing the use of constants, sets satisfying axioms and theorems.
 - ▶ the **extends** relationship for expressing the extension of contexts by adding new constants and new sets
 - ▶ the **refines** relationship stating that a B model is refined by another one.

Machines

- **REFINES**
- **SEES** a context
- **VARIABLES** of the model
- **INVARIANTS** satisfied by the variables
- **THEOREMS** satisfied by the variables
- **EVENTS** modifying the variables
- **VARIANT**

Contexts

- **EXTENDS** another context
- **SETS** declares new sets
- **CONSTANTS** define a list of constants
- **AXIOMS** define the properties of constants and sets
- **THEOREMS** list the theorems which should be derived from axioms

MACHINE

m

REFINES

am

SEES

c

VARIABLES

u

INVARIANTS

I(u)

THEOREMS

Q(u)

VARIANT

< variant >

EVENTS

< event >

END

- $\Gamma(m)$: environment for the machine m defined by the context c
- $\Gamma(m) \vdash \forall u \in \text{VALUES} : \text{INIT}(u) \Rightarrow I(u)$
- For each event e in E :
 $\Gamma(m) \vdash \forall u, u' \in \text{VALUES} : I(u) \wedge BA(e)(u, u') \Rightarrow I(u')$
- $\Gamma(m) \vdash \forall u \in \text{VALUES} : I(u) \Rightarrow Q(u)$

CONTEXTS c **EXTENDS** ac **SETS** s **CONSTANTS** c **AXIOMS** $ax1 : \dots$ **THEOREMS** $th1 : \dots$ **END**

- $ac : c$ is extending ac and add new features
- s : sets are defined either by intension or by extension
- c : constants are defined and
- axioms characterize constants and sets
- theorems are derived from axioms in the current context

Event : E	Before-After Predicate
BEGIN $x : P(x, x')$ END	$P(x, x')$
WHEN $G(x)$ THEN $x : P(x, x')$ END	$G(x) \wedge P(x, x')$
ANY t WHERE $G(t, x)$ THEN $x : P(x, x', t)$ END	$\exists t \cdot (G(t, x) \wedge P(x, x', t))$

Guards of event

Event : E	Guard : $\text{grd}(E)$
BEGIN S END	$TRUE$
WHEN $G(x)$ THEN T END	$G(x)$
ANY t WHERE $G(t, x)$ THEN T END	$\exists t. G(t, x)$

Proof obligations for a B model

	Proof obligation
(INV1)	$\Gamma(s, c) \vdash Init(x) \Rightarrow I(x)$
(INV2)	$\Gamma(s, c) \vdash I(x) \wedge BA(e)(x, x') \Rightarrow I(x')$
(DEAD)	$\Gamma(s, c) \vdash I(x) \Rightarrow (\text{grd}(e_1) \vee \dots \text{grd}(e_n))$
(SAFE)	$\Gamma(s, c) \vdash I(x) \Rightarrow A(x)$
(FIS)	$\Gamma(s, c) \vdash I(x) \wedge \text{grd}(E) \Rightarrow \exists x' \cdot P(x, x')$

Current Summary

- ## Summary on Events

The factorial model

CONTEXT

fonctions

CONSTANTS

factorial, n

AXIOMS

ax1 : $n \in \mathbb{N}$

ax2 : $factorial \in \mathbb{N} \leftrightarrow \mathbb{N}$

ax3 : $0 \mapsto 1 \in factorial$

ax4 : $\forall(i, fn).(i \mapsto fn \in factorial \Rightarrow i + 1 \mapsto (i + 1) * fi \in factorial) \wedge$

$$\forall f \cdot \left(\begin{array}{l} f \in \mathbb{N} \leftrightarrow \mathbb{N} \wedge \\ 0 \mapsto 1 \in f \wedge \\ \forall(n, fn).(n \mapsto fn \in f \Rightarrow n + 1 \mapsto (n + 1) \times fn \in f) \\ \Rightarrow \\ factorial \subseteq f \end{array} \right)$$

END

The factorial model

MACHINE

specification

SEES *fonctions*

VARIABLES

resultat

INVARIANT

resultat $\in \mathbb{N}$

THEOREMS

th1 : *factorial* $\in \mathbb{N} \rightarrow \mathbb{N}$;

th2 : *factorial*(0) = 1 ;

th3 : $\forall n. (n \in \mathbb{N} \Rightarrow \text{factorial}(n+1) = (n+1) \times \text{factorial}(n))$

INITIALISATION

resultat : $\in \mathbb{N}$

EVENTS

computing1 = **BEGIN** *resultat* := *factorial*(*n*) **END**

END

Communications between agents

MACHINE *agents*

SEES *data*

VARIABLES

sent

got

lost

INVARIANTS

inv1 : $sent \subseteq AGENTS \times AGENTS$

inv2 : $got \subseteq AGENTS \times AGENTS$

inv4 : $(got \cup lost) \subseteq sent$

inv6 : $lost \subseteq AGENTS \times AGENTS$

inv7 : $got \cap lost = \emptyset$

INITIALISATION

BEGIN

act1 : $sent := \emptyset$

act2 : $got := \emptyset$

act4 : $lost := \emptyset$

END

Communications between agents

EVENT sending a message

ANY

 a, b

WHERE

$$grd11 : a \in AGENTS$$
$$grd12 : b \in AGENTS$$
$$grd1 : a \mapsto b \notin sent$$

THEN

$$act11 : sent := sent \cup \{a \mapsto b\}$$

END

EVENT getting a message

ANY

 a, b

WHERE

$$grd11 : a \in AGENTS$$
$$grd12 : b \in AGENTS$$
$$grd13 : a \mapsto b \in sent \setminus (got \cup lost)$$

THEN

$$act11 : got := got \cup \{a \mapsto b\}$$

END

Communications between agents

```
EVENT loosing a message  
ANY  
  a  
  b  
WHERE grd1 : a ∈ AGENTS  
        grd2 : b ∈ AGENTS  
        grd3 : a ↦ b ∈ sent \ (got ∪ lost)  
THEN  
  act1 : lost := lost ∪ {a ↦ b}  
END
```

```
CONTEXTS  
  data  
SETS  
  MESSAGES  
  AGENTS  
  DATA  
CONSTANTS  
  n  
  infile  
AXIOMS  
  axm1 : n ∈ ℕ  
  axm2 : n ≠ 0  
  axm3 : infile ∈ 1 .. n → DATA  
END
```

Current Summary

Summary on Events

General form of an event

```
EVENT e
ANY t
WHERE
   $G(c, s, t, x)$ 
THEN
   $x : |(P(c, s, t, x, x'))|$ 
END
```

- c et s are constantes and visible sets by e
- x is a state variable or a list of variables
- $G(c, s, t, x)$ is the condition for observing e .
- $P(c, s, t, x, x')$ is the assertion for the relation over x and x' .
- $BA(e)(c, s, x, x')$ is the *before-after* relationship for e and is defined by $\exists t. G(c, s, t, x) \wedge P(c, s, t, x, x')$.

General form of proof obligations for an event e

Proofs obligations are simplified when they are generated by the module called POG and goals in sequents as $\Gamma \vdash G$:

- ① $\Gamma \vdash G_1 \wedge G_2$ is decomposed into the two sequents $(1) \Gamma \vdash G_1$
 $(2) \Gamma \vdash G_2$
- ② $\Gamma \vdash G_1 \Rightarrow G_2$ is transformed into the sequent $\Gamma, G_1 \vdash G_2$

Proof obligations in Rodin

- $INIT/I/INV : C(s, c), INIT(c, s, x) \vdash I(c, s, x)$
- $e/I/INV : C(s, c), I(c, s, x), G(c, s, t, x), P(c, s, t, x, x') \vdash I(c, s, x')$
- $e/act/FIS : C(s, c), I(c, s, x), G(c, s, t, x) \vdash \exists x'. P(c, s, t, x, x')$

- U