Modelling Software-based Systems

Lecture 6

Validation, Verification and Proof Tools for Event-B for Correct-by-Construction

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General Summary

Current Summary

Summary

Annotation of programs

$$\ell : \{P_\ell(v)\}$$
$$\quad cond_{\ell,\ell'}(v) \longrightarrow v := f_{\ell,\ell'}(v)$$
$$\ell' : \{P_{\ell'}(v)\}$$

$$e(\ell, \ell')$$
WHEN
$$\quad c = \ell$$
$$\quad cond_{\ell,\ell'}(v)$$
THEN
$$\quad c := \ell'$$
$$\quad v := f_{\ell,\ell'}(v)$$
END

$$\ell_0^1 : \{x = 0\}$$
$$\quad x := x + 1;$$
$$\ell_0^1 : \{x = 1\}$$

- v is the state meory variable or list of memory variables ; v includes the local variables and the results variables.
- c is a new variable which is modelling the control flow and its type is LOCATIONS.
- $e(\ell, \ell')$ is simulating the computation flow starting from ℓ and moving to ℓ' ; v is updated.

From annotations to invariants

INVARIANTS

$inv_i : c \in \text{LOCATIONS}$

$inv_j : v \in Type$

$\ldots$

$inv_k : c = \ell \Rightarrow P_\ell(v)$

$inv_m : c = \ell' \Rightarrow P_{\ell'}(v)$

$\ldots$

$th_n : A(c, v)$

- $Type$ is the type of the variables v and is a set of possible values defined in the context C.
- The annotation is giving us for free the conditions satisfied by v when the control is in ℓ, (resp. in ℓ').
- $A(c, v)$ is a safety property that we are supposed to check and the case of Event-B, it is a theorem.

Partial correctness using Event-B models

For each pair of successive labels ℓ, ℓ', the three statements are equivalent :

- $P_\ell(v) \wedge cond_{\ell,\ell'}(v) \wedge v' = f_{\ell,\ell'}(v) \Rightarrow P_{\ell'}(v')$
- $I(c,v) \wedge c = \ell \wedge cond_{\ell,\ell'}(v) \wedge c' = \ell' \wedge v' = f_{\ell,\ell'}(v) \Rightarrow (c' = \ell' \Rightarrow P_{\ell'}(v'))$
- $I(c,v) \wedge BA(e(\ell,\ell'))(c,v,c',v') \Rightarrow (c' = \ell' \Rightarrow P_{\ell'}(v'))$

L

et AA an annotated algorithm with precondition **pre**(AA)(v) and postcondition **post**(AA)(v_0,v). Let the context C and the machine M generated from AA using the construction given previously. We assume that ℓ_0 is the first label and ℓ_e is the last label. We add the following safety properties in the machine M :

- $c = \ell_0 \wedge \mathbf{pre}(\mathrm{AA})(v) \Rightarrow P_{\ell_0}(v)$
- $c = \ell_e \Rightarrow (P_{\ell_e}(v) \Rightarrow \mathbf{post}(\mathrm{AA})(v_0, v)$

If proof obligations are discharged, then the annotated algorithm AA is partially correct with respect to ist pre/post specification.

Summary

Current Summary

Proof Management

- The panel called *proof control* is providing tools for *discharging* proof obligations
- Proof obligations are stated as follows : $\Gamma \vdash P$: *When assertions of* Γ *are supposed to hold, then* P *does hold*
- Problem : Checking that the sequent $\Gamma \vdash P$ is valid.
- How : Applying automatic procedures as pp, pr etc
- . . . or helping the proof tool to build the *proof tree*

Main Limits of PP (Predicate Prover)

- Quite sensitive to useless hypotheses.
- Unable to automatically abstract some statements.
- Sometimes poor management of equalities.
- Needs some creative hints

Creative Hints

- Proposing clever lemma : the user may have interesting ideas because he/she knows the problem to solve.
- Suggesting a proof by case.
- Proposing a witness for an existential goal.
- Proposing a universal hypothesis instanciation.
- etc.

Sequents and Inference rules

- Sequent (where $list\_of\_hypotheses$ might be empty).

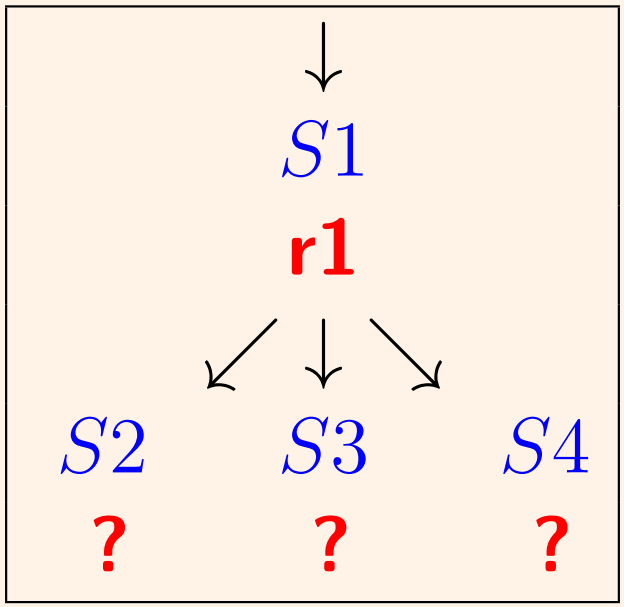
$$list\_of\_hypotheses \quad \vdash \quad conclusion$$

- Inference rule (where $list\_of\_sequents$ might be empty).

$$\frac{list\_of\_sequents}{sequent}$$

- Backwards application of inf. rules $\rightsquigarrow$ proof tree.

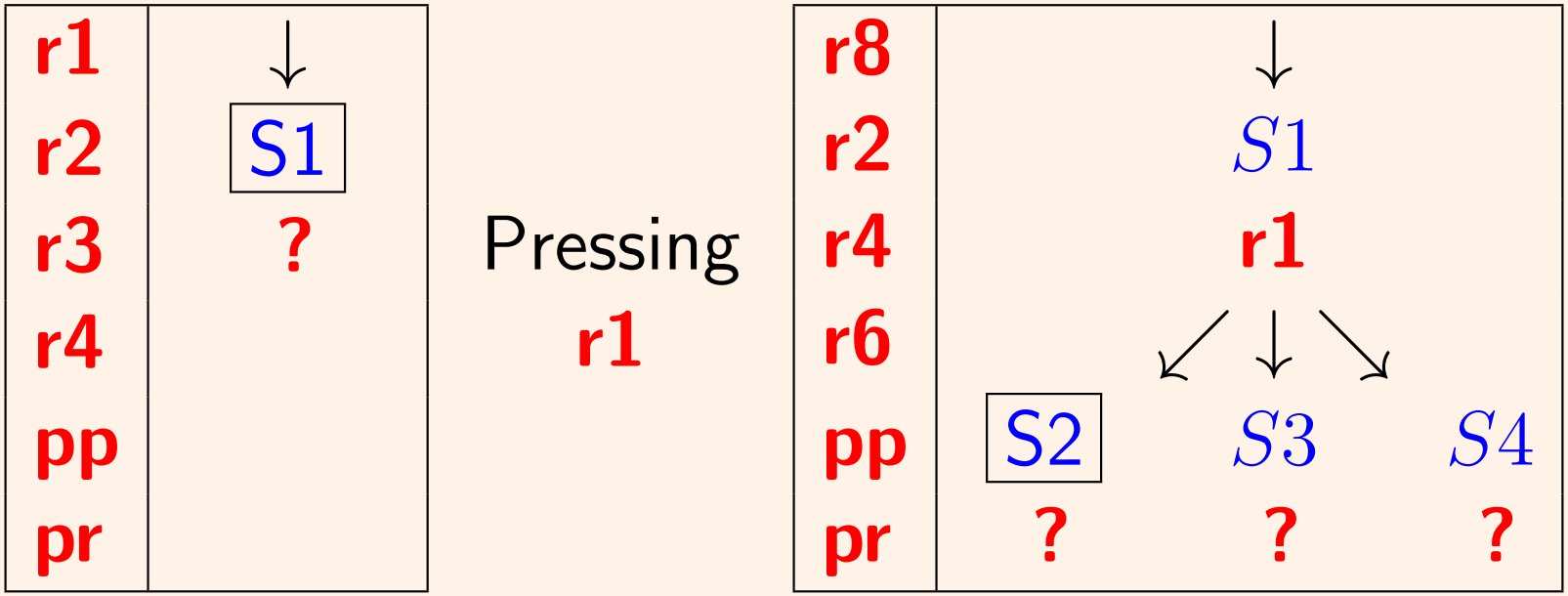
The Proof Tree



- Rule **r1** is applied to sequent S1.

- Producing sequents S2, S3, and S4.

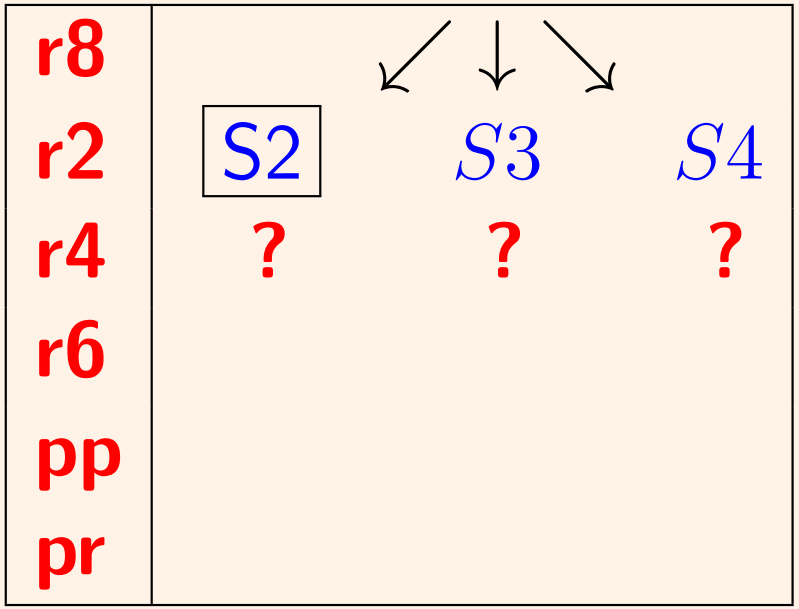
Interface : Tree and Palette of Rule Buttons



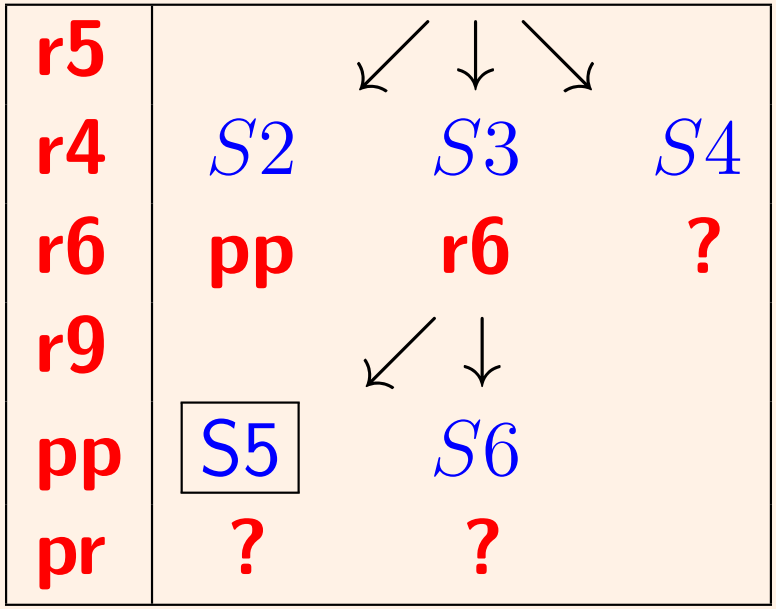
- The proof tree is constructed interactively.

- The palette evolves as the sequent of interest varies.

Difficulty : Size of Window



pp
then
r6



- Our tentative "interface" does not work.

More Precise Form of Sequent

$$H \vdash L \Longrightarrow G$$

- Most of the time the conclusion has an implicative form
- G is called the goal (it is a non-conjunctive predicate)
- L are called the local hypotheses (might be empty)

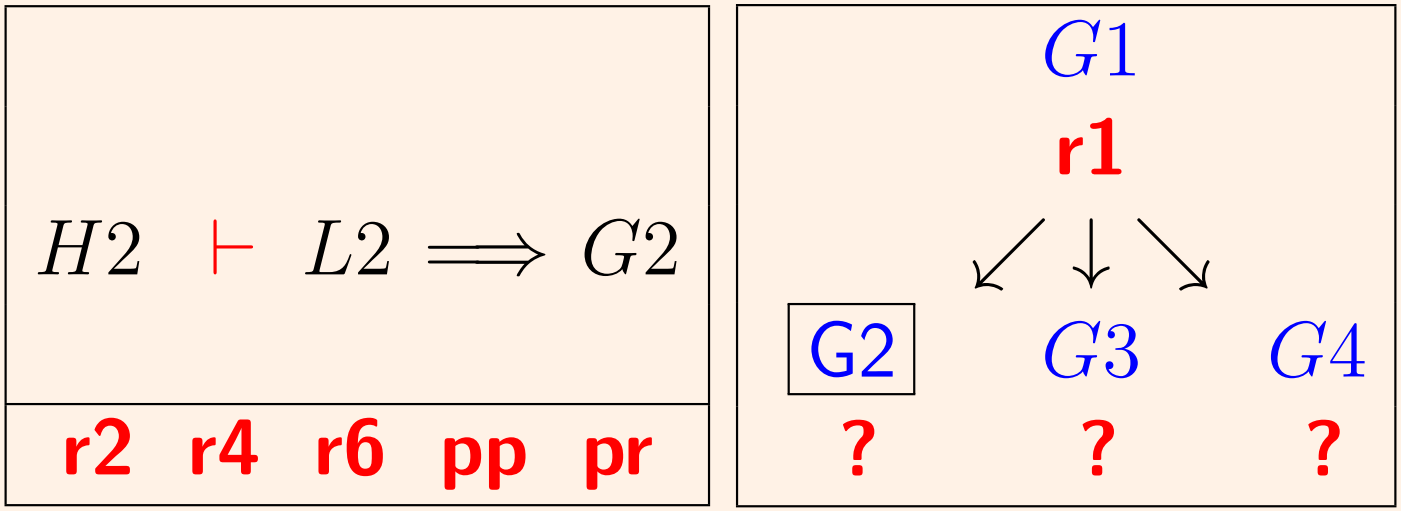
$$H \vdash I(x) \wedge G(x) \wedge P(x, x') \Longrightarrow I(x')$$

Tentative Solution : Using Two Windows

- Tree window with simplified sequents (goals only)

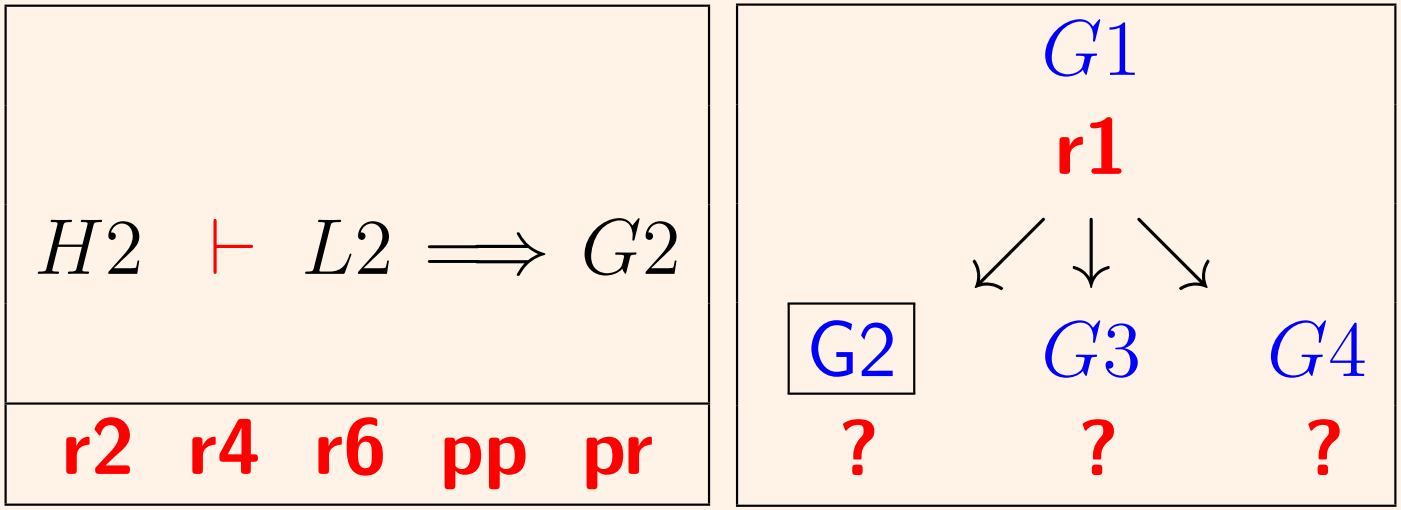
- Sequent window containing the

full sequent of interest

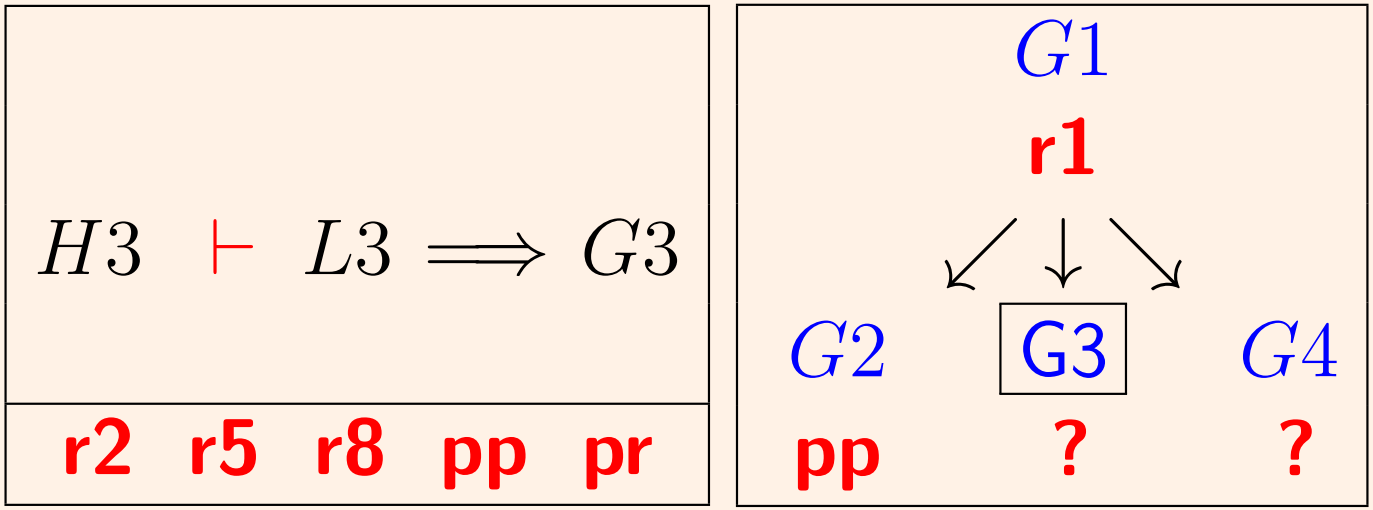


Sequent Window Tree Window

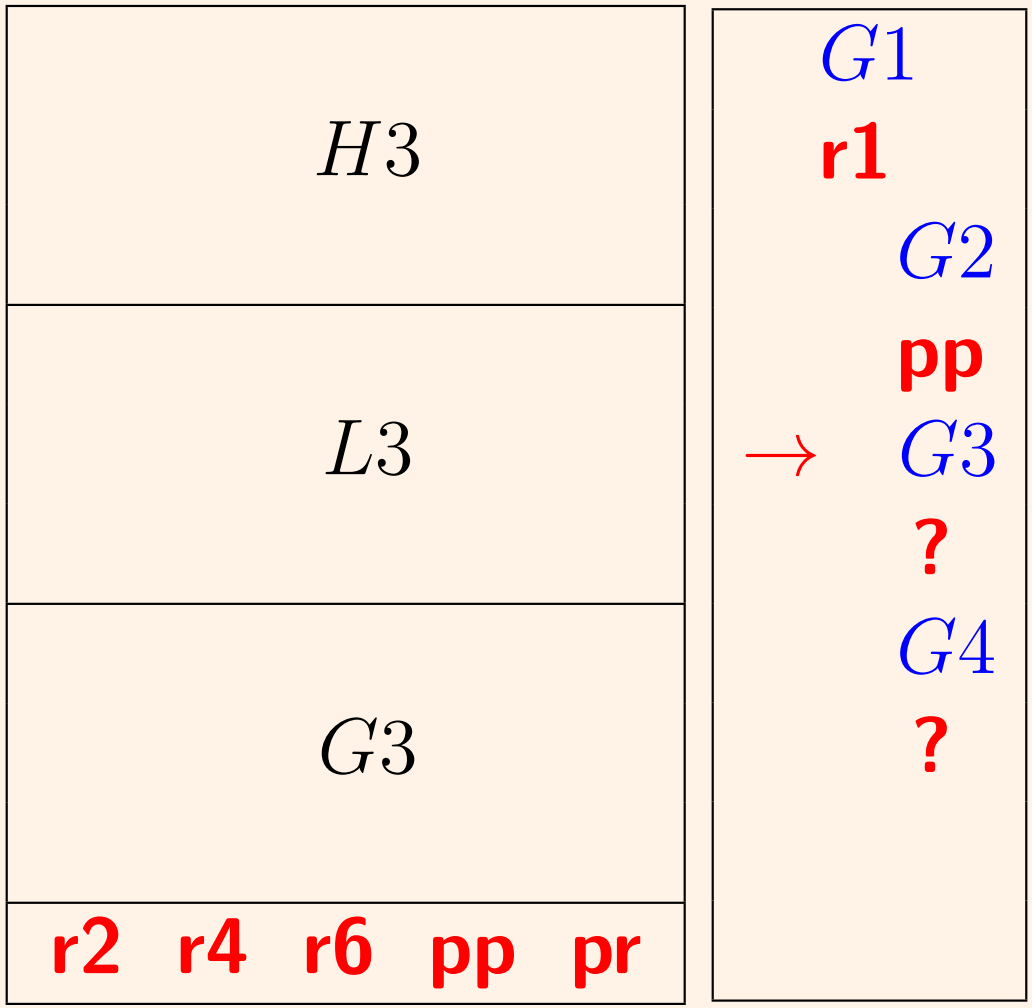
One Proof Step



Pressing **pp**



More Realistic Windows



More Elaborate Sequents

$$\underbrace{hidden \;;\; searched \;;\; cached}_{list\_of\_hypotheses} \vdash \underbrace{local \Longrightarrow goal}_{conclusion}$$

$$\begin{cases} hidden & \text{Not visible on the sequent window} \\ searched & \text{Visible after some search in } hidden \\ cached & \text{Visible but not part of the conclusion any m} \\ local & \text{Visible and part of the conclusion} \end{cases}$$

Current Summary

Context and Objectives

Context

Developing life-cycle methodology and a set of associated techniques and tools to develop the highly critical systems using formal techniques from requirements analysis to code generation.

Objectives

- To establish a unified theory for the critical system development.
- To Build a set of tools for supporting new development life-cycle methodology.
- To develop a closed-loop system for verification purpose.
- Graphical based refinement technique to handle the complexity of the system.
- To satisfy requirements and metrics for certifiable assurance and safety.
- To support evidence-based certification.

Main Contributions

Development Methodology

- Formal Methods based Critical System Development Life-Cycle

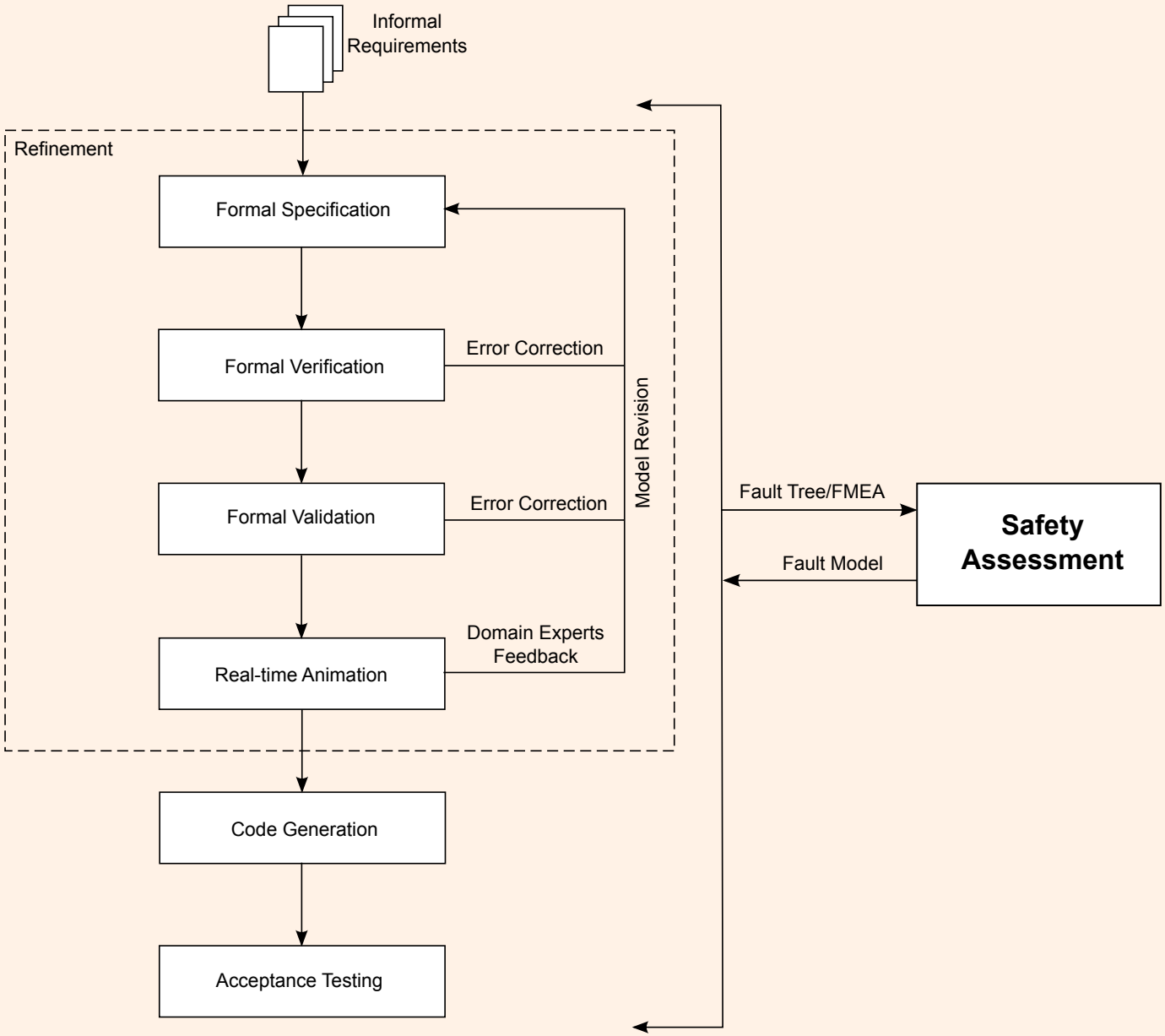
Techniques and Tools

- Real-Time Animator
- Refinement Chart
- Automatic Code Generator : EB2ALL
- Formal Logic Based Heart-Model

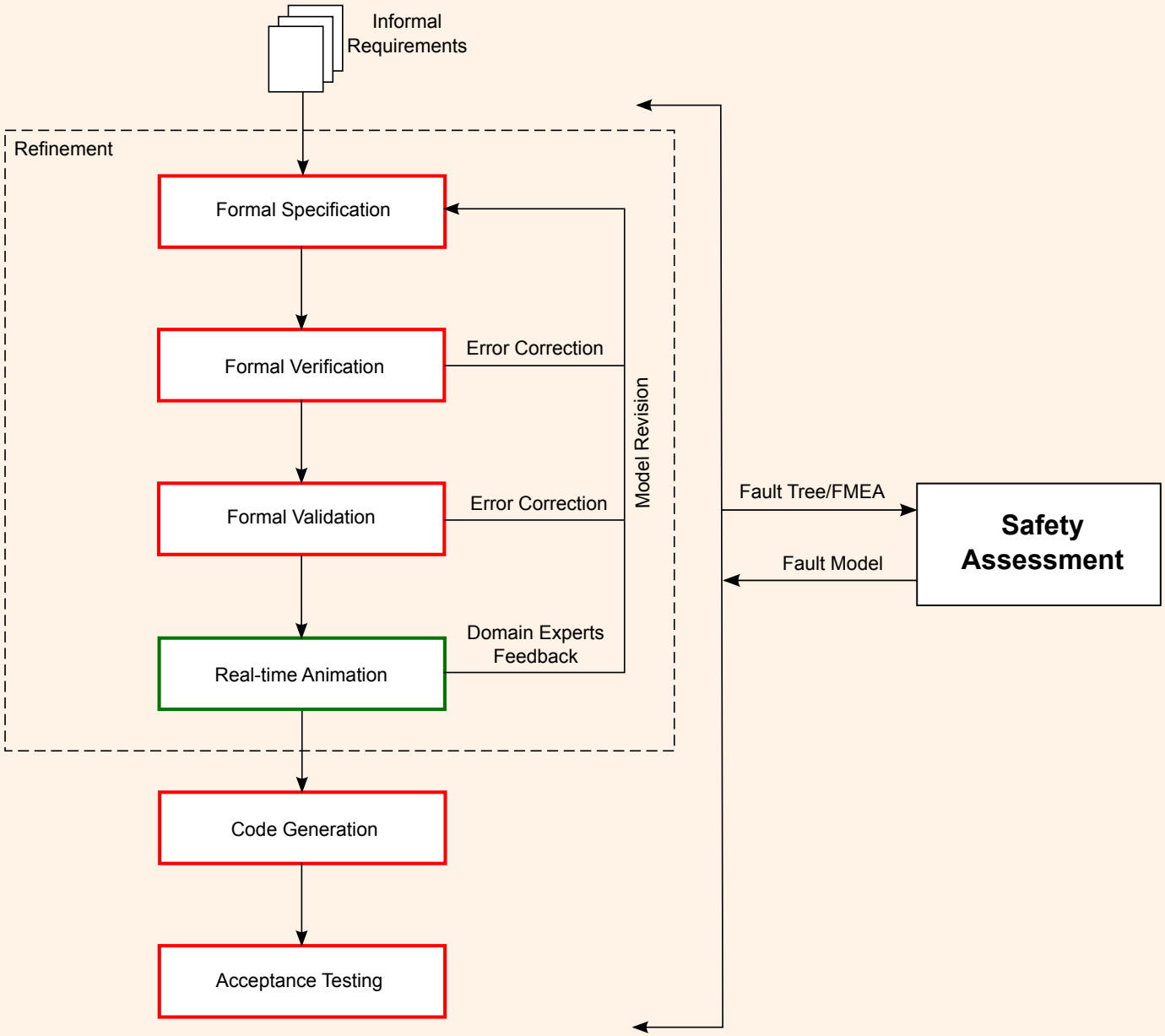
Case Studies

- The Cardiac Pacemaker

Critical System Development Life-Cycle Methodology



Critical System Development Life-Cycle Methodology



Real-Time Animator

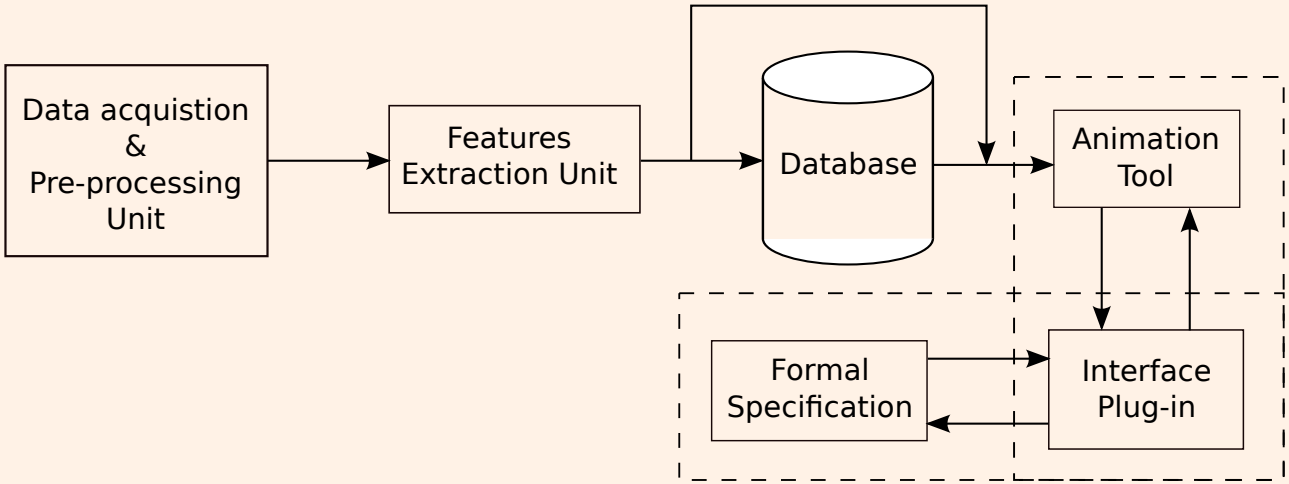
What is Real-time Animator ?

- Visual representation of formal model using real time data set.

Why should we use Formal Model Animator ?

- To validate system behavior according to the stakeholders
- To express formal models for non-mathematical domain experts
- To discover the error in the early stage of system development (Traceability)

Proposed Architecture



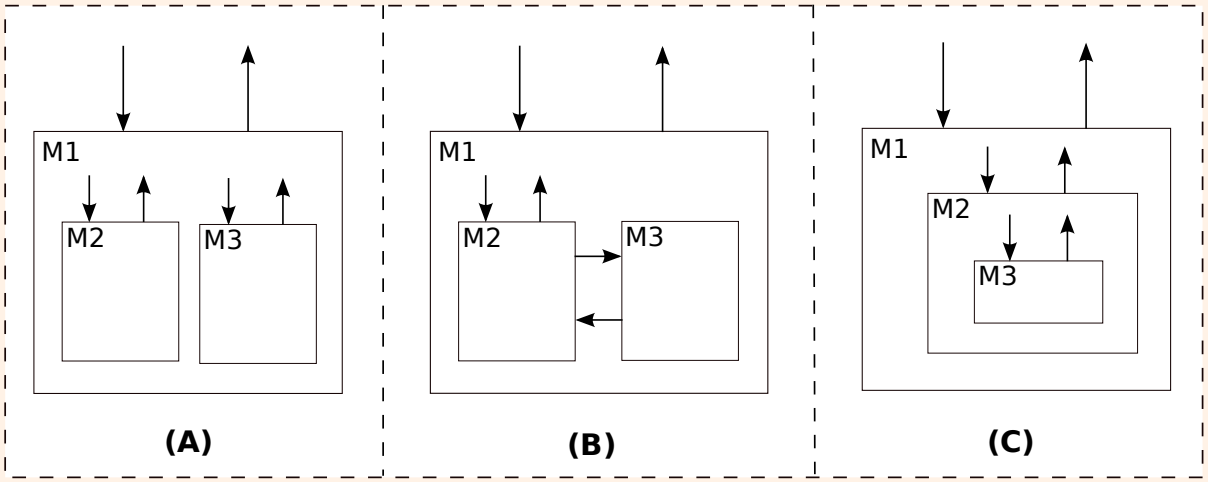
Refinement Chart

- Refinement chart is a graphical representation of a complex system using layering approach.
- Integration and features based code structuring of different components of the systems.

Basic rules of refinement chart

- Parallel Refinement
- Sequential Refinement
- Nested Refinement

$$M1 \sqsubseteq (M2 \parallel M3 \parallel ..... \parallel M_{n-1} \parallel M_n)$$
$$M1 \sqsubseteq (M2 \succ M3 \succ ..... \succ M_{n-1} \succ M_n)$$
$$(M1 \sqsubseteq M2,\ M2 \sqsubseteq M3, ... ,\ M_{n-1} \sqsubseteq M_n)$$



$$M1 \sqsubseteq (M2 \parallel M3)$$
$$M1 \sqsubseteq (M2 \succ M3)$$
$$(M1 \sqsubseteq M2,\ M2 \sqsubseteq M3)$$

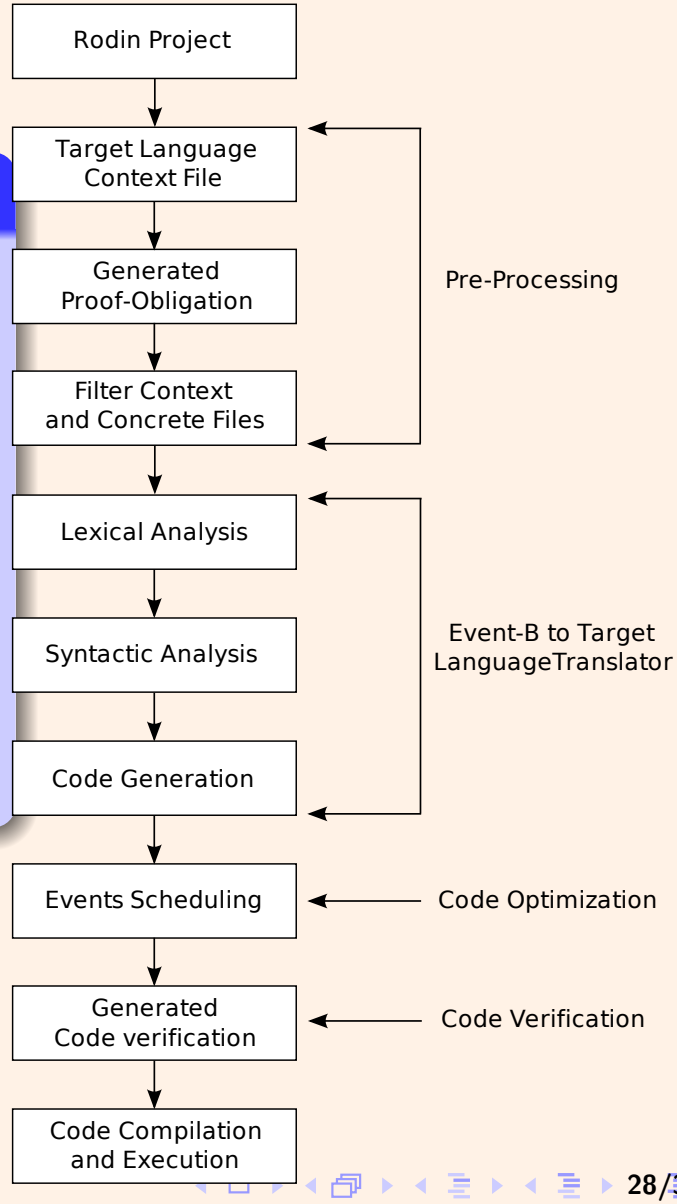
EB2ALL Code Generation Tool

EB2ALL

- Automatic code generation from Event-B formal specification to multiple languages (C, C++, Java, C#) to make it executable.
- A module for Rodin (Deploy is an European Commission FP7 project)
- It is more than 20000 lines of code
- More than 1000 downloads

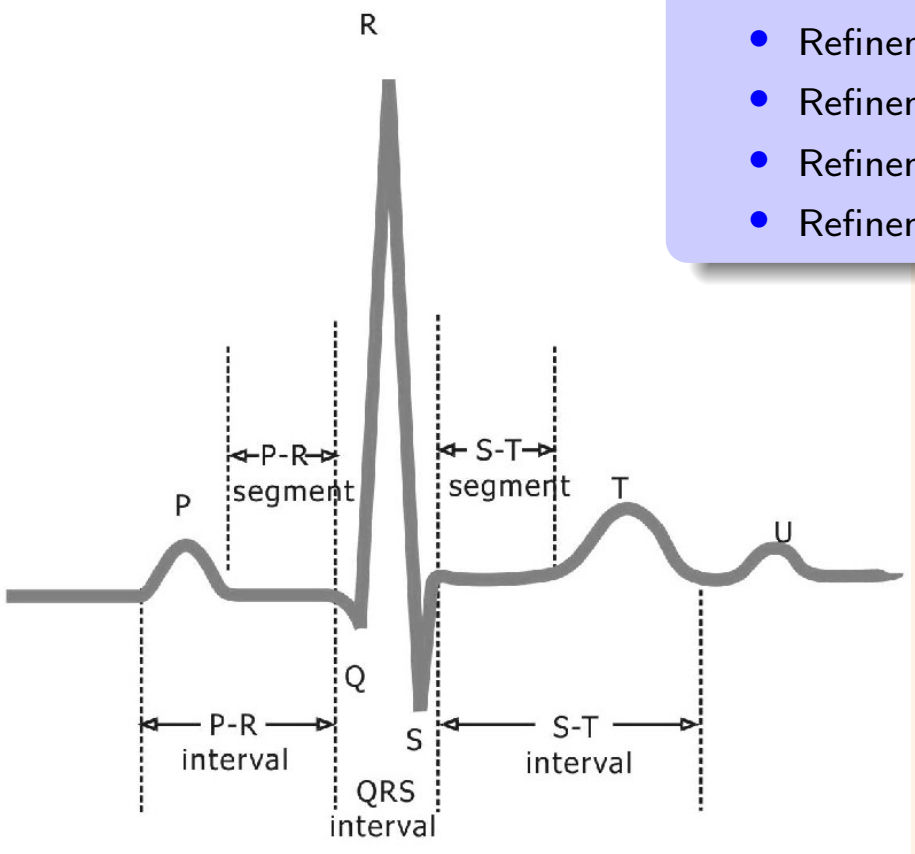
Download : http://eb2all.loria.fr/

EB2ALL Tool Architecture



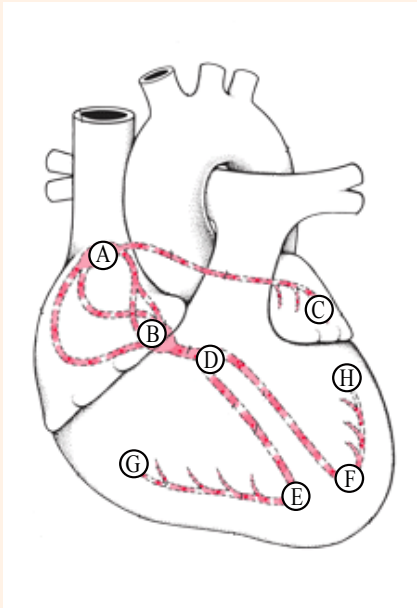
Formal Logic Based Heart-Model

Electrocardiogram (ECG)



Formal Development

- Abstract Model
- Refinement 1 : Introducing Steps in the Propagation
- Refinement 2 : Impulse Propagation
- Refinement 3 : Perturbation the Conduction
- Refinement 4 : Getting a Cellular Model



Landmarks on the Electrical Conduction of Heart

Current Summary

Steps of the project

- Reading the description of the problem to solve.
- List the safety properties as partial correctness, mutual exclusion, ... and properties on sets, constants, ...
- Identify the state variables for the first machine
- Define inductive definitions if necessary and state correctness properties as $\forall n.n\mathbb{N} \Rightarrow n * n = v(n)$.
- List Use Cases : for instance, adding a new user, updating a a phone number, recording a movement ...
- First expression of a context and a machine.

Documentation

- **Property**
  - *the set of people is non empty and finite*
  - $people \neq \varnothing \wedge finute(people)$