



Cours MALG & MOVEX

Vérification d'une annotation

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$$\ell_1 : x = 3 \ \land \ y = z + x \ \land z = 2 \cdot x$$

$$y := z + x$$

$$\ell_2 : x = 3 \ \land \ y = x + 6$$

On définit un contrat comme suit :

variables x, y, z
$$\begin{array}{l} \text{requires } x0=3 \wedge y0=z0+x0 \wedge z0=2.x0 \\ \text{ensures } x_f=3 \wedge y_f=x_f+6 \\ \\ \begin{bmatrix} \text{begin} \\ \ell_1: x=3 \ \wedge \ y=z+x \ \wedge z=2\cdot x \\ y:=z+x \\ \ell_2: x=3 \ \wedge \ y=x+6 \\ \text{end} \\ \end{array}$$

On pose les assertions suivantes à partir de l'annotation :

- $ightharpoonup pre(x_0, y_0, z_0) \stackrel{def}{=} x0 = 3 \land y0 = z0 + x0 \land z0 = 2.x0$
- $ightharpoonup prepost(x_0, y_0, z_0, x, y, z) \stackrel{def}{=} x = 3 \land y = x + 6$
- $ightharpoonup Q_1(x_0, y_0, z_0, x, y, z) \stackrel{def}{=} x = 3 \land y = z + x \land z = 2 \cdot x$
- $Q_2(x_0, y_0, z_0, x, y, z) \stackrel{def}{=} x = 3 \land y = x + 6$

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On établit les trois cpnditions pour valider le contrat :

- ightharpoonup (init) $pre(x_0, y_0, z_0) \land (x, y, z) = (x_0, y_0, z_0) \Rightarrow Q_1(x_0, y_0, z_0, x, y, z)$
- ► (concl) $pre(v_0) \land Q_2(x_0, y_0, z_0, x, y, z) \Rightarrow prepost(x_0, y_0, z_0, x, y, z)$
- (induct) $pre(x_0, y_0, z_0) \wedge Q_1(x_0, y_0, z_0, x, y, z) \wedge TRUE \wedge (x', y', z') = (x, z+x, z) \Rightarrow Q_2(x_0, y_0, z_0, x', y', z')$

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 - x = 3 est une hypothèse à gauche. Le séquent est valide.

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- $x_0 = 3, y_0 = z_0 + x_0, z_0 = 2.x_0, x = 3, y = z + x, z = 0$ $2 \cdot x$, TRUE, $(x', y', z') = (x, z+x, z) \vdash y' = x'+6$
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 - $2 \cdot x$, TRUE, $(x', y', z') = (x, z+x, z) \vdash 2 \cdot x + x = x+6$

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 - x0 = 3, y0 = z0+x0, z0 = 2.x0, x = 3, y = z+x, z =
 - $2 \cdot x, TRUE, (x', y', z') = (x, z + x, z) \vdash 2.3 + 3 = 3 + 6$

- $x0 = 3, y0 = z0+x0, z0 = 2.x0, x = 3, y = z+x, z = 2.x, TRUE, (x', y', z') = (x, z+x, z) \vdash y' = x'+6$
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 - x0 = 3, y0 = z0+x0, z0 = 2.x0, x = 3, y = z+x, z = 2.x. TRUE. (x', y', z') = (x. z+x, z) + 2.x+x = x+6
 - x0 = 3, y0 = z0+x0, z0 = 2x0, z0 = 3, y0 = z0+x0, z0 = 2x0, z0
 - $2 \cdot x$, TRUE, $(x', y', z') = (x, z+x, z) \vdash 2.3+3 = 3+6$
 - $x0 = 3, y0 = z0+x0, z0 = 2.x0, x = 3, y = z+x, z = 2.x, TRUE, (x', y', z') = (x, z+x, z) \vdash 9 = 9$

- $x0 = 3, y0 = z0+x0, z0 = 2.x0, x = 3, y = z+x, z = 2.x, TRUE, (x', y', z') = (x, z+x, z) \vdash y' = x'+6$
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 - x0 = 3, y0 = z0 + x0, z0 = 2.x0, x = 3, y = z + x, z = 2.x. TRUE. (x', y', z') = (x. z + x, z) + 2.x + x = x + 6
 - x0 = 3, y0 = z0+x0, $z0 = (x, z+x, z) \vdash z \cdot x+x = x+c$
 - $2 \cdot x$, TRUE, $(x', y', z') = (x, z+x, z) \vdash 2.3+3 = 3+6$
 - $x0 = 3, y0 = z0+x0, z0 = 2.x0, x = 3, y = z+x, z = 2.x, TRUE, (x', y', z') = (x, z+x, z) \vdash 9 = 9$
 - Réflexivité de l'égalité.