Modelling Software-based Systems

Lecture 2 Proof Obligation Generation

Telecom Nancy

Dominique Méry
Telecom Nancy,Université de Lorraine

27 mars 2025(11:41 P.M.)

dominique.mery@loria.fr

General Summary

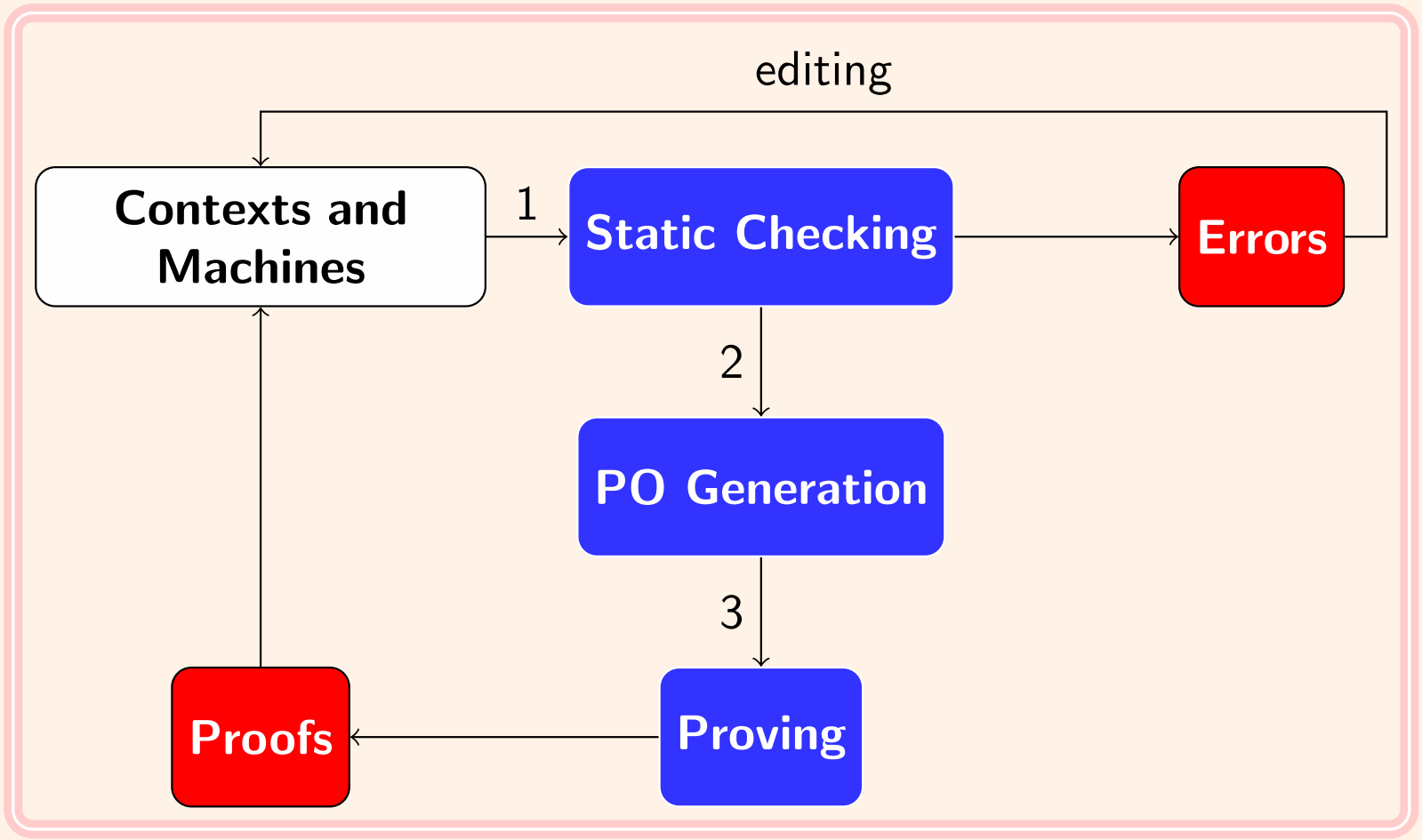
1. Overview of machines, contexts and proof obligations

2. Proof Obligations for Contexts and Machines
   - PO thm/THM (context)
   - PO th/THM (machine)
   - PO evt/inv/INV
   - PO evt/act/FIS

3. Proof Obligations for Refinement
   - PO evt/grd/GRD
   - PO evt/act/SIM
   - PO evt/NAT
   - PO NAT
   - PO evt/VAR (arithmetic)
   - PO evt/VAR (set-theoretic)
   - PO evt/x/WFIS

Current Summary

Analysis of the Event-B Models



Machines en Event B

MACHINE
m
REFINES
am
SEES
c
VARIABLES
u
INVARIANTS
$I(s, c, u)$
THEOREMS
$Q(s, c, u)$
VARIANT
$exp(s, c, u)$
EVENTS
$INITIALIZATION$
$\ldots$
e
$\ldots$
END

Machines en Event B

MACHINE
m
REFINES
am
SEES
c
VARIABLES
u
INVARIANTS
$I(s, c, u)$
THEOREMS
$Q(s, c, u)$
VARIANT
$exp(s, c, u)$
EVENTS
$INITIALIZATION$
$\ldots$
e
$\ldots$
END

- $\Gamma(m)$: environment for the machine m defined by the context c and it provides a list of seen axioms $Ax(s, c)$ and a list of seen theorems $Th(s, c)$ for the sets s and constants c.
- $\Gamma(m) \vdash \forall u.\textsc{Init}(s, c, u) \Rightarrow \mathrm{I}(s, c, u)$
- For each event e in E :
  $\Gamma(m) \vdash \forall u, u'.\mathrm{I}(s, c, u) \wedge BA(e)(u, u') \Rightarrow \mathrm{I}(u')$
- For each event e in E :
  $\Gamma(m) \vdash \forall u.\mathrm{I}(s, c, u) \wedge GRD(e)(s, c, u) \Rightarrow \exists u'.BA(e)(u, u')$
- $\Gamma(m) \vdash \forall u.\mathrm{I}(s, c, u) \Rightarrow \mathrm{Q}(s, c, u)$
- Generated proof obligations are derived from those conditions.

Checking the well formation of Event-B expressions

- Event-B expressions are contexts, machines, properties, equations, set-theoretical expressions ...
- e is an Event-B expression and $\mathsf{wd}(e)$ is a logical property expressing the well definition of e.
- $\mathsf{wd}(1 = 2) \triangleq \mathsf{wd}(1) \wedge \mathsf{wd}(2)$
- $\mathsf{wd}(a/b) \triangleq b \neq 0 \wedge \mathsf{wd}(a) \wedge \mathsf{wd}(b)$
- $\mathsf{wd}(f(g)) \triangleq g \in dom(f) \wedge f \in A \nrightarrow B$

Current Summary

PO th/THM

CONTEXTS
c
EXTENDS
ac
SETS
s
CONSTANTS
c
AXIOMS
$Ax(s, c)$
THEOREMS
$th_1 : P_1(s, c)$
$\ldots$
$th_n : P_n(s, c)$
$th : P(s, c)$
$\ldots$
END

s	*seen sets*
c	*seen constants*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*previous proved theorems*
	$PTh(s, c) = \{P_i(s, c) \mid i\ 1..n\}$
$P(s, c)$	*property over s and c*

PO th/THM

$Ax(s, c), Th(s, c) \vdash P(s, c)$

PO th/THM (machine)

MACHINE
m
...
VARIABLES
u
INVARIANTS
$I(s, c, u)$
THEOREMS
$Q(s, c, u)$
$th : P(s, c, u)$
...
END

s	*seen sets*
c	*seen constants*
u	*variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*invariants*
$Q(s, c, u)$	*theorems*
$P(s, c, u)$	*property over s,c and u*

PO th/THM

$$Ax(s, c), Th(s, c), I(s, c, u) \vdash P(s, c, u)$$

PO evt/inv/INV

EVENT evt
  ANY x **WHERE**
    $G(x, s, c, u)$
  THEN
    $u :| BAP(x, s, c, u, u')$
  END

$BA(\text{evt}) \ \widehat{=}$

$$\exists x. \left(\begin{array}{l} \wedge\ G(x, s, c, u) \\ \wedge\ BAP(x, s, c, u, u') \end{array} \right)$$

$GRD(\text{evt}) \ \widehat{=}\ G(x, s, c, u)$

$ACT(\text{evt}) \ \widehat{=}\ BAP(x, s, c, u, u')$

s	*seen sets*
c	*seen constants*
u	*variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*invariants*
$Q(s, c, u)$	*theorems*
evt	*event name*
x	*event parameter*
$G(x, s, c, u)$	*event guard*
$BAP(x, s, c, u, u')$	*event before-after predicate*
$inv : inv(s, c, u')$	*specific modified invariant*

PO evt/inv/INV

$$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), BAP(x, s, c, u, u') \vdash inv(s, c, u')$$

PO Q/THM $Ax(s, c), Th(s, c), I(s, c, u) \vdash Q(s, c, u)$

PO evt/act/FIS

EVENT evt
  ANY x **WHERE**
    $G(x, s, c, u)$
  THEN
    $u :\mid BAP(x, s, c, u, u')$
  END

$BA(\text{evt}) \mathrel{\widehat{=}}$
$\left(\begin{array}{l} \wedge\ G(x, s, c, u) \\ \wedge\ BAP(x, s, c, u, u') \end{array} \right)$
$GRD(\text{evt}) \mathrel{\widehat{=}} G(x, s, c, u)$
$ACT(\text{evt}) \mathrel{\widehat{=}}$
$BAP(x, s, c, u, u')$

s	*seen sets*
c	*seen constants*
u	*variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*invariants*
$Q(s, c, u)$	*theorems*
evt	*event name*
x	*event parameter*
$G(x, s, c, u)$	*event guard*
$BAP(x, s, c, u, u')$	*event before-after predicate*

PO evt/act/FIS

$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), \vdash \exists u'.BAP(x, s, c, u, u')$

Current Summary

PO evt/grd/GRD

EVENT ae
ANY x **WHERE**
$G(x, s, c, u)$
THEN
$u :| ABAP(x, s, c, u, u')$
END

EVENT ce
REFINES
ae
ANY y **WHERE**
$H(y, s, c, v)$
WITH
$x : W(x, y, s, c, v)$
THEN
$v :| CBAP(y, s, c, v, v')$
END

s	*seen sets*
c	*seen constants*
u, v	*abstract and concrete variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*abstract invariants*
$J(s, c, u, v)$	*concrete invariants*
$Q(s, c, u)$, $R(s, c, u, v)$	*abstract and concrete theorems*
ae, ce	*abstract and concrete event name*
x,y	*event parameters*
$G(x, s, c, u)$	*abstract event guard*
$H(y, s, c, v)$	*concrete event guard*
$ABAP(x, s, c, u, u')$	*abstract event before-after predic*
$CBAP(x, s, c, u, u')$	*concrete event before-after predic*
W(x,y,s,c,v)	witness predicate

PO evt/grd/GRD

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), W(x, y, s, c, v), H(y, s, c, v), \vdash$
$G(x, s, c, u, u')$

PO evt/act/SIM

EVENT ae
  ANY x **WHERE**
    $G(x, s, c, u)$
  THEN
    $u :| ABAP(x, s, c, u, u')$
  END

EVENT ce
  REFINES
    ae
  ANY y **WHERE**
    $H(y, s, c, v)$
  WITH
    $x : WP(x, y, s, c, v)$
    $u' : WV(y, u', s, c, v)$
  THEN
    $v :| CBAP(y, s, c, v, v')$
  END

s	*seen sets*
c	*seen constants*
u, v	*abstract and concrete variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*abstract invariants*
$J(s, c, u, v)$	*concrete invariants*
$Q(s, c, u)$, $R(s, c, u, v)$	*abstract and concrete theorems*
ae, ce	*abstract and concrete event name*
x,y	*event parameters*
$G(x, s, c, u)$	*abstract event guard*
$H(y, s, c, v)$	*concrete event guard*
$ABAP(x, s, c, u, u')$	*abstract event before-after predic*
$CBAP(x, s, c, u, u')$	*concrete event before-after predic*
$WP(x, y, s, c, v)$	witness parameter predicate
$WV(y, u', s, c, v)$	witness variable predicate

PO evt/act/SIM

$$\left(\begin{array}{l} Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v) \\ WP(x, y, s, c, v), WV(y, u', s, c, v) \\ H(y, s, c, v), CBAP(y, s, c, v, v') \end{array}\right) \vdash ABAP(x, s, c, u, u')$$

PO evt/act/SIM

EVENT ae
  ANY x **WHERE**
    $G(x, s, c, u)$
  THEN
    $u :| BAP(x, s, c, u, u')$
  END
...
VARIANT
    $exp(s, c, u)$

s	*seen sets*
c	*seen constants*
u, v	*abstract and concrete variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*abstract invariants*
$J(s, c, u, v)$	*concrete invariants*
$Q(s, c, u)$, $R(s, c, u, v)$	*abstract and concrete theorems*
evt, ce	*event name*
x	*event parameters*
$G(x, s, c, u)$	*abstract event guard*
$BAP(x, s, c, u, u')$	*event before-after predicate*
$exp(s, c, u)$	*aritthmetic expression*

PO evt/NAT

$$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u) \vdash exp(s, c, u) \in \mathbb{N}$$

PO evt/act/SIM

EVENT ae
  ANY x **WHERE**
    $G(x, s, c, u)$
  THEN
    $u :| BAP(x, s, c, u, u')$
  END
...
VARIANT
    $exp(s, c, u)$

s	*seen sets*
c	*seen constants*
u, v	*abstract and concrete variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*abstract invariants*
$J(s, c, u, v)$	*concrete invariants*
$Q(s, c, u)$, $R(s, c, u, v)$	*abstract and concrete theorems*
evt, ce	*event name*
x	*event parameters*
$G(x, s, c, u)$	*abstract event guard*
$BAP(x, s, c, u, u')$	*event before-after predicate*
$setexp(s, c, u)$	*set expression*

PO evt/NAT $Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u) \vdash$
$finite(setexp(s, c, u))$

PO evt/VAR

EVENT ae
  ANY x **WHERE**
    $G(x, s, c, u)$
  THEN
    $u :\mid BAP(x, s, c, u, u')$
  END
...
VARIANT
    $exp(s, c, u)$

s	*seen sets*
c	*seen constants*
u, v	*abstract and concrete variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*abstract invariants*
$J(s, c, u, v)$	*concrete invariants*
$Q(s, c, u)$, $R(s, c, u, v)$	*abstract and concrete theorems*
evt, ce	*event name*
x	*event parameters*
$G(x, s, c, u)$	*abstract event guard*
$BAP(x, s, c, u, u')$	*event before-after predicate*
$exp(s, c, u)$	*aritthmetic expression*

PO evt/VAR

$$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u), BAP(x, s, c, u, u') \vdash exp(s, c, u') < exp(s, c, u)$$

PO evt/VAR

EVENT ae
  ANY x **WHERE**
    $G(x, s, c, u)$
  THEN
    $u :\mid BAP(x, s, c, u, u')$
  END
...
VARIANT
    $setexp(s, c, u)$

s	*seen sets*
c	*seen constants*
u, v	*abstract and concrete variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*abstract invariants*
$J(s, c, u, v)$	*concrete invariants*
$Q(s, c, u)$, $R(s, c, u, v)$	*abstract and concrete theorems*
evt, ce	*event name*
x	*event parameters*
$G(x, s, c, u)$	*abstract event guard*
$BAP(x, s, c, u, u')$	*event before-after predicate*
$setexp(s, c, u)$	*set-theoretic expression*

PO evt/VAR

$$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u), BAP(x, s, c, u, u') \vdash setexp(s, c, u') \subset setexp(s, c, u)$$

PO evt/x/WFIS

EVENT ae
ANY x **WHERE**
$G(x, s, c, u)$
THEN
$u : |ABAP(x, s, c, u, u')$
END

EVENT ce
REFINES
ae
ANY y **WHERE**
$H(y, s, c, v)$
WITH
$x : WP(x, y, s, c, v)$
$u' : WV(y, u', s, c, v)$
THEN
$v : |CBAP(y, s, c, v, v')$
END

s	*seen sets*
c	*seen constants*
u, v	*abstract and concrete variables*
$Ax(s, c)$	*seen axioms*
$Th(s, c)$	*seen theorems*
$I(s, c, u)$	*abstract invariants*
$J(s, c, u, v)$	*concrete invariants*
$Q(s, c, u)$, $R(s, c, u, v)$	*abstract and concrete theorems*
ae, ce	*abstract and concrete event name*
x,y	*event parameters*
$G(x, s, c, u)$	*abstract event guard*
$H(y, s, c, v)$	*concrete event guard*
$ABAP(x, s, c, u, u')$	*abstract event before-after predic*
$CBAP(x, s, c, u, u')$	*concrete event before-after predic*
$WP(x, y, s, c, v)$	witness parameter predicate
$WV(y, u', s, c, v)$	witness variable predicate

PO evt/x/WFIS

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), H(y, s, c, v) \vdash$
$\exists x.WP(x, y, s, c, v)$