

Cours MALG & MOVEX

Vérification mécanisée de contrats (II) (The ANSI/ISO C Specification Language (ACSL))

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① Programs as Predicate

Transformers

② Annotations

③ Contracts

Extending C programming

language by contracts

Playing with variables

Ghost Variables

Logic Specification

① Programs as Predicate Transformers

② Annotations

③ Contracts

- Extending C programming language by contracts

- Playing with variables

- Ghost Variables

- Logic Specification

① Programs as Predicate Transformers

② Annotations

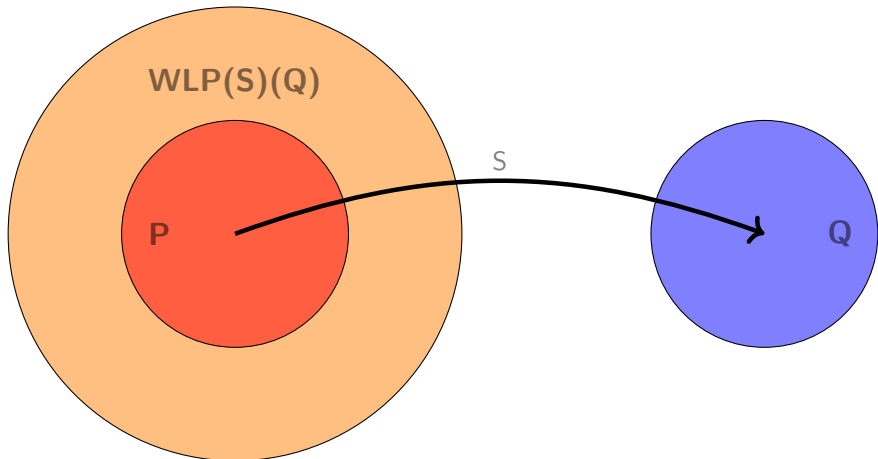
③ Contracts

Extending C programming language by contracts

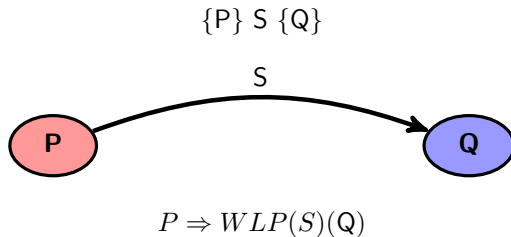
Playing with variables

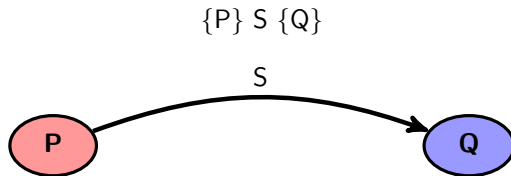
Ghost Variables

Logic Specification









$$P \Rightarrow WLP(S)(Q)$$

Computing $WLP(S)(Q)$?

Writing a simple contract

variables x

requires $x \geq 0 \wedge x \leq 10$;

ensures $\begin{cases} x \% 2 = 0 \Rightarrow 2 \cdot \text{result} = x +; \\ x \% 2 \neq 0 \Rightarrow 2 \cdot \text{result} = x - 1; \end{cases}$

begin

int y ;

$y = x / 2$;

return(y);

end

► result is the value returned by the command *return*(y).

► *return*(y) is equivalent to $\text{result} := y$.

(Writing a simple contract.)

Listing 1 – project-divers/annotation.c

```
/*@ requires x <= 0 && x >= 10;
   @ assigns \nothing;
   @ ensures x % 2 == 0 ==> 2*\result == x;
   @ ensures x % 2 != 0 ==> 2*\result == x-1;
   @*/
int annotation(int x)
{
    int y;
    y = x / 2;
    return(y);
}
```

(Writing a simple contract.)

Listing 2 – project-divers/annotationwp.c

```
/*@ requires 0 <= x && x <= 10;  
  @ assigns \nothing;  
  @ ensures x % 2 == 0 ==> 2*\result == x;  
  @ ensures x % 2 != 0 ==> 2*\result == x-1;  
  @*/  
int annotation(int x)  
{  
  /*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */  
  /*@ assert x % 2 != 0 ==> 2* (x / 2) == x-1; */  
  int y;  
  /*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */  
  /*@ assert x % 2 != 0 ==> 2* (x / 2) == x-1; */  
  y = x / 2;  
  /*@ assert x % 2 == 0 ==> 2*y == x; */  
  /*@ assert x % 2 != 0 ==> 2*y == x-1; */  
  return(y);  
  /*@ assert x % 2 == 0 ==> 2*y == x; */  
  /*@ assert x % 2 != 0 ==> 2*y == x-1; */  
}
```

Property to check

$$x \geq 0 \wedge x \leq 10 \Rightarrow \begin{cases} x \% 2 \neq 0 \Rightarrow 2 \cdot (x/2) = x-1 \\ x \% 2 = 0 \Rightarrow 2 \cdot (x/2) = x \end{cases}$$

(Checking the precondition.)

Listing 3 – project-divers/annotation0.c

```
/*@ requires x >= 0 && x < 0;  
  @ assigns \nothing;  
  @ ensures \result == 0;  
  @*/  
int annotation0(int x)  
{  
    int y;  
    y = y / (x-x);  
    return(y);  
}
```

(Checking the precondition.)

Listing 4 – project-divers/annotation0wp.c

```
/*@ requires x >= 0 && x < 0;  
  @ assigns \nothing;  
  @ ensures \result == 0;  
  @*/  
int annotation(int x)  
{  
  /*@ assert y / (x-x) == 0; */  
  int y;  
  /*@ assert y / (x-x) == 0; */  
  y = y / (x-x);  
  /*@ assert y == 0; */  
  return(y);  
  /*@ assert y == 0; */  
}
```

Property to check

$$x \geq 0 \wedge x < 0 \Rightarrow y / (x - x) = 0$$

Transformations of annotated programs (1)

```
//@ assert  $P(v0, v) :$ 
 $S1; S2$ 
//@ assert  $Q(v0, v) :$ 
```

- ▶ Applying the property :

$$wp(S1; S2)(A) = wp(S1)(wp(S2)(A))$$

```
//@ assert  $P(v0, v)$  :  
S1;  
//@ assert  $wp(S2)(Q(v0, v))$  :  
S2;  
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :
//@ assert  $xp(S1)(wp(S2)(Q(v0, v)))$  :
S1;
//@ assert  $wp(S2)(Q(v0, v))$  :
S2;
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
   $S1$   
ELSE  
   $S2$   
FI  
//@ assert  $Q(v0, v)$  :
```

- Applying the property :
 $wp(if(B, S1, S2))(A) =$
 $b \wedge wp(S1)(A) \vee \neg B \wedge$
 $wp(S2)(A).$

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
   $S1$   
ELSE  
   $S2$   
FI  
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
   $S1$   
  //@ assert  $Q(v0, v)$  :  
ELSE  
   $S2$   
  //@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

Transformations of annotated programs (2)

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
   $S1$   
//@ assert  $Q(v0, v)$  :  
ELSE  
   $S2$   
//@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

Transformations of annotated programs (2)

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
   $S1$   
//@ assert  $Q(v0, v)$  :  
ELSE  
   $S2$   
//@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
//@ assert  $B \wedge wp(S2)(Q(v0, v))$  :  
   $S1$   
//@ assert  $Q(v0, v)$  :  
ELSE  
//@ assert  $\neg B \wedge wp(S2)(Q(v0, v))$  :  
   $S2$   
//@ assert  $Q(v0, v)$  :
```


Transformations of annotated programs (2)

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
   $S1$   
  //@ assert  $Q(v0, v)$  :  
ELSE  
   $S2$   
  //@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
  //@ assert  $B \wedge wp(S2)(Q(v0, v))$  :  
   $S1$   
  //@ assert  $Q(v0, v)$  :  
ELSE  
  //@ assert  $\neg B \wedge wp(S2)(Q(v0, v))$  :  
   $S2$   
  //@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
  //@ assert  $b \wedge wp(S1)(Q(v0, v))$  :  
   $S1$   
  //@ assert  $Q(v0, v)$  :  
ELSE  
  //@ assert  $\neg b \wedge wp(S2)(Q(v0, v))$  :  
   $S2$   
  //@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

Transformations of annotated programs (2)

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
   $S1$   
  //@ assert  $Q(v0, v)$  :  
ELSE  
   $S2$   
  //@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
  //@ assert  $B \wedge wp(S2)(Q(v0, v))$  :  
   $S1$   
  //@ assert  $Q(v0, v)$  :  
ELSE  
  //@ assert  $\neg B \wedge wp(S2)(Q(v0, v))$  :  
   $S2$   
  //@ assert  $Q(v0, v)$  :
```

```
//@ assert  $P(v0, v)$  :  
IF  $B$  THEN  
  //@ assert  $b \wedge wp(S1)(Q(v0, v))$  :  
   $S1$   
  //@ assert  $Q(v0, v)$  :  
ELSE  
  //@ assert  $\neg b \wedge wp(S2)(Q(v0, v))$  :  
   $S2$   
  //@ assert  $Q(v0, v)$  :  
FI  
//@ assert  $Q(v0, v)$  :
```

- ▶ $b \wedge P(v0, v) \Rightarrow b \wedge wp(S1)(Q(v0, v))$
- ▶ $\neg b \wedge P(v0, v) \Rightarrow \neg b \wedge wp(S2)(Q(v0, v))$

```
//@ assert  $P(v0, v)$  :  
//@ loop invariant  $I(v0, v)$  :  
WHILE  $B$  THEN  
   $S$   
OD  
//@ assert  $Q(v0, v)$  :
```

- ▶ Applying the iteration rule of Hoare Logic :


```
//@ assert  $P(v0, v)$  :  
//@ loop invariant  $I(v0, v)$  :  
WHILE  $B$  THEN  
   $S$   
OD  
//@ assert  $Q(v0, v)$  :
```

- ▶ Applying the iteration rule of Hoare Logic :

```
//@ assert  $P(v0, v)$  :  
//@ loop invariant  $I(v0, v)$  :  
//@ assert  $I(v0, v)$  :  
WHILE  $B$  THEN  
  //@ assert  $b \wedge I(v0, v)$  :  
   $S$   
  //@ assert  $I(v0, v)$  :  
OD  
//@ assert  $Q(v0, v)$  :
```

- ▶ $b \wedge I(v0, v) \Rightarrow wp(S)(I(v0, v))$
- ▶ $P(v0, v) \Rightarrow I(v0, v)$
- ▶ $\neg b \wedge I(v0, v) \Rightarrow Q(v0, v)$

- ▶ Checking the preservation of invariant.
- ▶ Applying the wps on assertions according to startements.

- ▶ Assertions at a control point of the program

```
/*@ assert pred; */
```

```
//@ assert pred;
```

- ▶ Assertions at a control point of the program components.

```
/*@ for id1,id2, ..., idn: assert pred; */
```

(Incrementing a number)

Listing 5 – project-divers/compwp0.c

```
#define x0 5
/*@ assigns \nothing; */
int exemple() {
    int x=x0;
    //@ assert x == x0;
    x = x + 1;
    //@ assert x == x0+1;
    return x;
}
```

(Incrementing a number)

Listing 6 – project-divers/compwp0wp.c

```
#define x0 5
/*@ assigns \nothing; */
int exemple() {
    //@ assert x0 == x0;
    //@ assert x0+1 == x0+1;
    int x=x0;
    //@ assert x == x0;
    //@ assert x+1 == x0+1;
    x = x + 1;
    //@ assert x == x0+1;
    return x;
}
```


- requires
- assigns
- ensures
- decreases
- predicate
- logic
- lemma

- ▶ The calling function should guarantee the required condition or precondition introduced by the clauses *requires* $P1 \wedge \dots \wedge Pn$ at the calling point.
- ▶ The called function returns results that are ensured by the clause *ensures* $E1 \wedge \dots \wedge Em$; ensures clause expresses a relationship between the initial values of variables and the final values.
- ▶ initial values of a variable v is denoted $\backslash old(v)$
- ▶ The variables which are not in the set $L1 \cup \dots \cup Lp$ are not modified.

Listing 7 – contrat

```
/*@ requires P1;...;requires Pn;  
   @ assigns L1;...;assigns Lm;  
   @ ensures E1;...;ensures Ep;  
   @*/
```

(Division)

Listing 8 – project-divers/annotation.c

```
/*@ requires x <= 0 && x >= 10;  
   @ assigns \nothing;  
   @ ensures x % 2 == 0 ==> 2*\result == x;  
   @ ensures x % 2 != 0 ==> 2*\result == x-1;  
   @*/  
int annotation(int x)  
{  
    int y;  
    y = x / 2;  
    return(y);  
}
```

(Division)

Listing 9 – project-divers/annotationwp.c

```
/*@ requires 0 <= x && x <= 10;
   @ assigns \nothing;
   @ ensures x % 2 == 0 ==> 2*\result == x;
   @ ensures x % 2 != 0 ==> 2*\result == x-1;
   @*/
int annotation(int x)
{
  /*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
  /*@ assert x % 2 != 0 ==> 2* (x / 2) == x-1; */
  int y;
  /*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
  /*@ assert x % 2 != 0 ==> 2* (x / 2) == x-1; */
  y = x / 2;
  /*@ assert x % 2 == 0 ==> 2*y == x; */
  /*@ assert x % 2 != 0 ==> 2*y == x-1; */
  return(y);
  /*@ assert x % 2 == 0 ==> 2*y == x; */
  /*@ assert x % 2 != 0 ==> 2*y == x-1; */
}
```

Property to check

$$x \geq 0 \wedge x < 0; \Rightarrow \left(\begin{array}{l} x \% 2 = 0 \Rightarrow 2 \cdot (x/2) = x \\ x \% 2 \neq 0 \Rightarrow 2 \cdot (x/2) = x-1 \end{array} \right)$$

(Precondition)

Listing 10 – project-divers/annotation0.c

```
/*@ requires x >= 0 && x < 0;  
  @ assigns \nothing;  
  @ ensures \result == 0;  
  @*/  
int annotation0(int x)  
{  
  int y;  
  y = y / (x-x);  
  return(y);  
}
```

(Precondition)

Listing 11 – project-divers/annotation0wp.c

```
/*@ requires x >= 0 && x < 0;  
  @ assigns \nothing;  
  @ ensures \result == 0;  
  @*/  
int annotation(int x)  
{  
  /*@ assert y / (x-x) == 0; */  
  int y;  
  /*@ assert y / (x-x) == 0; */  
  y = y / (x-x);  
  /*@ assert y == 0; */  
  return(y);  
  /*@ assert y == 0; */  
}
```

Property to check

$$0 \leq x \wedge x \leq 10 \Rightarrow y/(x-x) = 0$$

Definition of a contract (specification)

- ▶ Define the mathematical function to compute (what to compute?)
- ▶ Define an inductive method for computing the mathematical function and using axioms.

(factorial what)

Listing 12 – project-factorial/factorial.h

```
#ifndef _A.H
#define _A.H
/*@ axiomatic mathfact {
  @ logic integer mathfact(integer n);
  @ axiom mathfact.1: mathfact(1) == 1;
  @ axiom mathfact_rec: \forall integer n; n > 1
  ==> mathfact(n) == n * mathfact(n-1);
  @ } */

/*@ requires n > 0;
    decreases n;
    ensures \result == mathfact(n);
    assigns \nothing;
  */
int codefact(int n);
#endif
```

Definition of a contract (programming)

- ▶ Define the program codefact for computing mathfact (How to compute?)
- ▶ Define the algorithm computing the function mathfact

(factorial how)

Listing 13 – project-factorial/factorial.c

```
#include "factorial.h"

int codefact(int n) {
    int y = 1;
    int x = n;
    /*@ loop invariant x >= 1 && x <= n && mathfact(n) == y * mathfact(x);
       loop assigns x, y;
       loop variant x;
    */
    while (x != 1) {
        y = y * x;
        x = x - 1;
    };
    return y;
}
```

Definition of a contract (approach)

- ▶ The specification of a function (mathfact) to compute requires to define it mathematically.
- ▶ The definition is stated in an axiomatic framework and is preferably inductive (mathfact) which is used in assertions or theorems or lemmas.
- ▶ The relationship between the computed value ($\backslash\text{result}$) and the mathematical value ($\text{mathfact}(n)$) is stated in the ensures clause :
$$\backslash\text{result} == \text{mathfact}(n)$$
- ▶ The main property to prove is $\text{codefact}(n) == \text{mathfact}(n)$: Calling codefact for n returns a value equal to $\text{mathfact}(n)$.



Listing 14 – contrat

```
/*@ requires P;  
@ behavior b1:  
  @ assumes A1;  
  @ requires R1 ;  
  @ assigns L1;  
  @ ensures E1;  
@ behavior b2:  
  @ assumes A2;  
  @ requires R2;  
  @ assigns L2;  
  @ ensures E2;  
@*/
```

(Pairs of integers)

Listing 15 – project-divers/structures.h

```
#ifndef _STRUCTURE_H
```

```
struct s {  
    int q;  
    int r;  
};
```

```
#endif
```

Division should not return silly expressions !

(Specification)

Listing 16 – project-divers/division.h

```
#ifndef _A_H
#define _A_H
#include "structures.h"
/*@ requires a >= 0 && b >= 0;
@ behavior b :
    @ assumes b == 0;
    @ assigns \nothing;
    @ ensures \result.q == -1 && \result.r == -1 ;
@ behavior B2:
    @ assumes b != 0;
    @ assigns \nothing;
    @ ensures 0 <= \result.r;
    @ ensures \result.r < b;
    @ ensures a == b * \result.q + \result.r;
*/
struct s division(int a, int b);
#endif
```

Division should not return silly expressions!

(Algorithm)

Listing 17 – project-divers/division.c

```
#include <stdio.h>
#include <stdlib.h>

#include "division.h"

struct s division(int a, int b)
{
    int rr = a;
    int qq = 0;
    struct s silly = {-1,-1};
    struct s resu;
    if (b == 0) {
        return silly;
    }
    else
    {
        /*@
        loop invariant
        ( a == b*qq + rr) &&
        rr >= 0;
        loop assigns rr,qq;
        loop variant rr;
        */
        while (rr >= b) { rr = rr - b; qq=qq+1;};
        resu.q = qq;
        resu.r = rr;
        return resu;
    }
}
```


Iteration Rule for PC

If $\{P \wedge B\}S\{P\}$, then $\{P\}\mathbf{while\ B\ do\ S\ od}\{P \wedge \neg B\}$.

- ▶ Prove $\{P \wedge B\}S\{P\}$ or $P \wedge B \Rightarrow \{S\}(P)$.
- ▶ By the iteration rule, we conclude that $\{P\}\mathbf{while\ B\ do\ S\ od}\{P \wedge \neg B\}$ without using WLP.
- ▶ Introduction of LOOP INVARIANTS in the notation.

Listing 18 – loop.c

```
/*@ loop invariant I1;  
    loop invariant I2;  
  
    ...  
    loop invariant In;  
    loop assigns X;  
    loop variant E;  
*/
```

(Invariant de boucle)

Listing 19 – project-divers/anno6.c

```
/*@ requires a >= 0 && b >= 0;  
    ensures 0 <= \result;  
    ensures \result < b;  
    ensures \exists integer k; a == k * b + \result;  
*/  
int rem(int a, int b) {  
    int r = a;  
    /*@  
        loop invariant  
        (\exists integer i; a == i * b + r) &&  
        r >= 0;  
        loop assigns r;  
    */  
    while (r >= b) { r = r - b;};  
    return r;  
}
```

Initial value of x $\backslash old(x)$

- ▶ $\backslash old(x)$ is the value of the variable when the function is called.
- ▶ It can be used in the postcondition of the *ensures* clause.

(Modifying variables while calling)

Listing 20 – project-divers/old1.c

```
/*@ requires \valid(a) && \valid(b);
   @ assigns *a,*b;
   @ ensures \at(*a,Pre) +2 == *a;
   @ ensures *b == \at(*b,Pre)+\at(*a,Pre)+2;

   @ ensures \result == 0;
*/
int old(int *a, int *b) {
    int x,y;
    x = *a;
    y = *b;
    x=x+2;
    y = y +x;

    *a = x;
    *b = y;
    return 0 ;
}
```

- ▶ $\backslash at(e, id)$ is the value of e at the control point id .
- ▶ id should occur before $\backslash at(e, id)$
- ▶ id is one of the possible expressions : Pre, Here, Old, Post, LoopEntry, LoopCurrent, Init
- ▶ $\backslash old(e)$ is equivalent to $\backslash at(e, Old)$

(label Pre)

Listing 21 – project-divers/at1.c

```
/*@
  requires \valid(a) && \valid(b);
  assigns *a,*b;
  ensures  *a == \old(*a)+2;
  ensures  *b == \old(*b)+\old(*a)+2;
*/
int at1(int *a, int *b) {
  //@ assert *a == \at(*a, Pre);
  *a = *a + 1;
  //@ assert *a == \at(*a, Pre)+1;
  *a = *a + 1;
  //@ assert *a == \at(*a, Pre)+2;
  *b = *b + *a;
  //@ assert *a == \at(*a, Pre)+2 && *b == \at(*b, Pre)+\at(*a, Pre)+2;
  return 0;
}
```


(autre label)

Listing 22 – project-divers/at2.c

```
void f (int n) {  
  for (int i = 0; i < n; i++) {  
    /*@ assert \at(i, LoopEntry) == 0; */  
    int j=0;  
    while (j++ < i) {  
      /*@ assert \at(j, LoopEntry) == 0; */  
      /*@ assert \at(j, LoopCurrent) + 1 == j; */  
    }  
  }  
}
```

(otherlabel)

Listing 23 – project-divers/change1.c

```
/*@ requires \valid(a) && *a >= 0;
   @ assigns *a;
   @ ensures *a == \old(*a)+2 && \result == 0;
*/
int change1(int *a)
{
    int x = *a;
    x = x + 2;
    *a = x;
    return 0;
}
```


- ▶ A variable called *ghost* allows to model a computed cvalue useful for stating a model property : the ghost variable is hidden for the computer but not for the model.
- ▶ It should not change the semantics of others variables and should not change the effective variables.

(Bug)

Listing 24 – project-divers/ghost2.c

```
int f (int x, int y) {  
    //@ghost int z=x+y;  
    switch (x) {  
    case 0: return y;  
    //@ ghost case 1: z=y;  
    // above statement is correct.  
    //@ ghost case 2: { z++; break; }  
    // invalid, would bypass the non-ghost default  
    default: y++; }  
    return y; }  
  
int g(int x) { //@ ghost int z=x;  
    if (x>0){return x;}  
    //@ ghost else { z++; return x; }  
    // invalid, would bypass the non-ghost return  
    return x+1; }
```

(Ghost variable)

Listing 25 – project-divers/ghost1.c

```
/*@ requires a >= 0 && b >= 0;  
    ensures 0 <= \result;  
    ensures \result < b;  
    ensures \exists integer k; a == k * b + \result; */  
int rem(int a, int b) {  
    int r = a;  
    /*@ ghost    int q=0;    */  
    /*@  
        loop invariant  
        a == q * b + r &&  
        r >= 0 && r <= a;  
        loop assigns r;  
        loop assigns q;  
    // loop variant r;  
    */  
    while (r >= b) {  
        r = r - b;  
    /*@ ghost    q = q+1;    */  
    };  
    return r;  
}
```



▶ predicate

(Predicate)

Listing 26 – project-divers/predicate1.c

```
/*@ predicate is_positive(integer x) = x > 0; */  
  
/*@ logic integer get_sign(real x) = @ x > 0.0 ? 1 : (x < 0.0 ? -1 : 0);  
*/  
/*@ logic integer max(int x, int y) = x > y ? x : y;  
*/
```

(Lemma)

Listing 27 – project-divers/lemma1.c

```
/*@ lemma div_mul_identity:  
@ \forall real x, real y; y != 0.0 => y*(x/y) == x; @*/  
  
/*@ lemma div_qr:  
@ \forall int a, int b; a >= 0 && b > 0 =>  
\exists int q, int r; a == b*q + r && 0 <= r && r < b; @*/
```

(Definition of fibonacci function)

Listing 28 – project-divers/predicate2.c

```
/*@ axiomatic mathfibonacci{  
  @ logic integer mathfib(integer n);  
  @ axiom mathfib0: mathfib(0) == 1;  
  @ axiom mathfib1: mathfib(1) == 1;  
  @ axiom mathfibrec: \forall integer n; n > 1  
    ==> mathfib(n) == mathfib(n-1)+mathfib(n-2);  
  @ } */
```

(Definition of gcd)

Listing 29 – project-divers/predicate3.c

```
/*@ inductive is_gcd(integer a, integer b, integer d) {  
  @ case gcd_zero:  
    @ \forall integer n; is_gcd(n,0,n);  
  @ case gcd_succ:  
    @ \forall integer a,b,d; is_gcd(b, a % b, d) ==> is_gcd(a,b,d); @}  
  @ */
```

(Definition of function odd/even)

Listing 30 – project-divers/predicate4.c

```
//@ predicate pair(integer x) = (x/2)*2==x;
//@ predicate impair(integer x) = (x/2)*2!=x;
//@ lemma ex: \forall integer a,b; a < b ==> 2*a < 2*b;

/*@ inductive is_gcd(integer a, integer b, integer c) {
  case zero: \forall integer n; is_gcd(n,0,n);
  case un: \forall integer u,v,w; u >= v ==> is_gcd(u-v,v,w);
  case deux: \forall integer u,v,w; u < v ==> is_gcd(u,v-u,w);
}
*/
```


- ▶ The termination is proved by showing that each loop terminates.
- ▶ Any loop is characterized by an expression $\text{expvariant}(x)$ called **variant** which should decrease each execution of the body :

$$\forall x_1, x_2. b(x_1) \wedge x_1 \xrightarrow{S} x_2 \Rightarrow \text{expvariant}(x_1) > \text{expvariant}(x_2)$$

(Variant)

Listing 31 – project-divers/variant1.c

```
/*@ requires n > 0;
    terminates n > 0;

    ensures \result == 0;
*/
int code(int n) {
    int x = n;
    /*@ loop invariant x >= 0 && x <= n;
        loop assigns x;
        loop variant x;
    */
    while (x != 0) {
        x = x - 1;
    };
    return x;
}
```

(Variant)

Listing 32 – project-divers/variant3.c

```
int f() {  
  int x = 0;  
  int y = 10;  
  /*@  
    loop invariant  
    0 <= x < 11 && x+y == 10;  
    loop variant y;  
  */  
  while (y > 0) {  
    x++;  
    y—;  
  }  
  return 0;  
}
```

(Variant)

Listing 33 – project-divers/variant4.c

```
g/*@ requires n <= 12;  
  @ decreases n;  
  @*/  
int fact(int n){  
    if (n <= 1) return 1;  
    return n*fact(n-1);  
}
```