# Cours MALG & MOVEX

# Vérification d'une annotation

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# Ecriture du contrat

$$
\begin{array}{l}
\ell_1 : x = 3 \ \wedge \ y = z+x \ \wedge z = 2\cdot x \\
y := z+x \\
\ell_2 : x = 3 \ \wedge \ y = x+6
\end{array}
$$

On définit un contrat comme suit :

variables x, y, z
requires $x0 = 3 \wedge y0 = z0+x0 \wedge z0 = 2.x0$
ensures $x_f = 3 \wedge y_f = x_f+6$

begin
$\ell_1 : x = 3 \ \wedge \ y = z+x \ \wedge z = 2\cdot x$
$y := z+x$
$\ell_2 : x = 3 \ \wedge \ y = x+6$
end

## Conditions de vérification du contrat

On pose les assertions suivantes à partir de l'annotation :

- $pre(x_0, y_0, z_0) \stackrel{def}{=} x0 = 3 \wedge y0 = z0+x0 \wedge z0 = 2.x0$
- $prepost(x_0, y_0, z_0, x, y, z) \stackrel{def}{=} x = 3 \ \wedge \ y = x+6$
- $Q_1(x_0, y_0, z_0, x, y, z) \stackrel{def}{=} x = 3 \ \wedge \ y = z+x \ \wedge z = 2 \cdot x$
- $Q_2(x_0, y_0, z_0, x, y, z) \stackrel{def}{=} x = 3 \ \wedge \ y = x+6$

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On établit les trois cpnditions pour valider le contrat :

- (init) $pre(x_0, y_0, z_0) \wedge (x, y, z) = (x_0, y_0, z_0) \Rightarrow Q_1(x_0, y_0, z_0, x, y, z)$
- (concl) $pre(v_0) \wedge Q_2(x_0, y_0, z_0, x, y, z) \Rightarrow prepost(x_0, y_0, z_0, x, y, z)$
- (induct)
  $pre(x_0, y_0, z_0) \wedge Q_1(x_0, y_0, z_0, x, y, z) \wedge TRUE \wedge (x', y', z') = (x, z{+}x, z) \Rightarrow Q_2(x_0, y_0, z_0, x', y', z')$

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  - $x = 3$ est une hypothèse à gauche. Le séquent est valide.

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  - Réflexivité de l'égalité.