

Modelling Software-based Systems

Lecture 5 Checking contracts with Event-B

Master Informatique

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25 novembre 2025
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General Summary

① Programming by contract

② Verification

③ Floyd to Hoare

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Verifying program correctness

A program P *satisfies* a (pre,post) contract :

- P transforms a variable v from initial values v_0 and produces a final value v_f : $v_0 \xrightarrow{P} v_f$
- v_0 satisfies pre : $\text{pre}(v_0)$ and v_f satisfies post : $\text{post}(v_0, v_f)$
- $\text{pre}(v_0) \wedge v_0 \xrightarrow{P} v_f \Rightarrow \text{post}(v_0, v_f)$
- \mathbb{D} est le domaine RTE de V

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```
requires pre(v0)
ensures post(v0, vf)
variables X
begin
  0 : P0(v0, v)
  instruction0
  ...
  i : Pi(v0, v)
  ...
  instructionf-1
  f : Pf(v0, v)
end
```

- $\text{pre}(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$
- $\text{pre}(v_0) \wedge P_f(v_0, v) \Rightarrow \text{post}(v_0, v)$
- For any pair of labels ℓ, ℓ' such that $\ell \longrightarrow \ell'$, one verifies that, pour any values $v, v' \in \text{MEMORY}$
$$\left(\left(\begin{array}{l} \text{pre}(v_0) \wedge P_\ell(v_0, v) \\ \wedge \text{cond}_{\ell, \ell'}(v) \wedge v' = f_{\ell, \ell'}(v) \end{array} \right) \right) \Rightarrow P_{\ell'}(v_0, v')$$

Contracts - Verification Conditions

```
contract P
variables v
requires pre( $v_0$ )
ensures post( $v_0, v_f$ )
begin
  0 :  $P_0(v_0, v)$ 
  s0
  ...
  i :  $P_i(v_0, v)$ 
  ...
  sf-1
  f :  $P_f(v_0, v)$ 
end
```

Contracts - Verification Conditions

Verification conditions are listed as follows :

```
contract P
variables v
requires pre(v0)
ensures post(v0, vf)
begin
  0 : P0(v0, v)
  s0
  ...
  i : Pi(v0, v)
  ...
  sf-1
  f : Pf(v0, v)
end
```

- (initialisation)
 $pre(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$
- (finalisation)
 $pre(v_0) \wedge P_f(v_0, v) \Rightarrow post(v_0, v)$
- (induction)
For each labels pair ℓ, ℓ'
such that $\ell \rightarrow \ell'$, one checks that,
for any value $v, v' \in \text{MEMORY}$
$$\left(\begin{array}{l} pre(v_0) \wedge P_\ell(v_0, v) \\ \wedge cond_{\ell, \ell'}(v) \wedge v' = f_{\ell, \ell'}(v) \end{array} \right) \Rightarrow P_{\ell'}(v_0, v')$$

Three kinds of verification conditions should be checked and we justify the method in the full version..

From PAP to Rodin ...

From PAP to Rodin . . .

MACHINE M

SEES C_0

VARIABLES

v, pc

INVARIANTS

typing : $v \in D$

control : $pc \in L$

...

at ℓ : $pc = \ell \Rightarrow P_\ell(v_0, v)$

...

th1 : $pre(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$

th2 : $pre(v_0) \wedge P_f(v_0, v) \Rightarrow post(v_0, v)$

...

END

...

END

From PAP to Rodin . . .

MACHINE M
SEES C_0
VARIABLES

v, pc
INVARIANTS

typing : $v \in D$
control : $pc \in L$

...
 $at\ell : pc = \ell \Rightarrow P_\ell(v_0, v)$

...
 $th1 : pre(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$
 $th2 : pre(v_0) \wedge P_f(v_0, v)$
 $\Rightarrow post(v_0, v)$

...
END

...
END

MACHINE M
EVENTS
INITIALISATION
BEGIN

$(pc, v) : | \left(\begin{array}{l} pc' = l_0 \wedge v' = v_0 \\ \wedge pre(v_0) \end{array} \right)$
END

...
 $e(\ell, \ell')$
WHEN

$pc = \ell$
 $cond_{\ell, \ell'}(v)$
THEN
 $pc := \ell'$
 $v := f_{\ell, \ell'}(v)$
END

...
END

(Induction Principle (I))

A property $S(z_0, z)$ is a safety for an annotated program P if, and only if, there exists a property $I(z_0, z)$ satisfying :

- ① $\forall z_0, z \in L \times D. init(z_0) \wedge z = z_0 \Rightarrow I(z_0, z)$
- ② $\forall z_0, z, z' \in L \times D. init(z_0) \wedge I(z_0, z) \wedge (z \xrightarrow{P} z') \Rightarrow I(z_0, z')$
- ③ $\forall z_0, z \in L \times D. init(z_0) \wedge I(z_0, z) \Rightarrow S(z_0, z)$

(Induction Principle (II))

A property $S(\ell_0, x_0, \ell, x)$ is a safety property for an annotated program P if, and only if, there exists a property $I(\ell_0, x_0, \ell, x)$ satisfying :

- ① $\forall \ell_0 \in L, x_0 \in D. \ell_0 \in L_0 \wedge pre(x_0) \wedge x = x_0 \wedge pc = \ell_0 \Rightarrow J(\ell_0, x_0, \ell, x)$
- ② $\forall \ell, \ell' \in L, x, x_0 \in D. \ell_0 \in L_0 \wedge pre(x_0) \wedge J(\ell_0, x_0, \ell, x) \wedge BA(e(\ell, \ell'),)(\ell, x, \ell', x') \Rightarrow J(\ell_0, x_0, \ell', x')$
- ③ $\forall \ell_0, \ell \in L, x_0, x \in D. pre(x_0) \wedge \ell_0 \in L_0 \wedge J(\ell_0, x_0, \ell, x) \Rightarrow S(\ell_0, x_0, \ell, x)$

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A property $S(\ell_0, x_0, \ell, x)$ is a safety property for an annotated program P if, and only if, there exists a property $I(\ell_0, x_0, \ell, x)$ satisfying :

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- ② $\forall \ell, \ell' \in L, x, x_0 \in D. \ell_0 \in L_0 \wedge pre(x_0) \wedge J(\ell_0, x_0, \ell, x) \wedge BA(e(\ell, \ell'),)(\ell, x, \ell', x') \Rightarrow J(\ell_0, x_0, \ell', x')$
- ③ $\forall \ell_0, \ell \in L, x_0, x \in D. pre(x_0) \wedge \ell_0 \in L_0 \wedge J(\ell_0, x_0, \ell, x) \Rightarrow S(\ell_0, x_0, \ell, x)$

(Induction Principle (III))

A property $S(x_0, \ell, x)$ is a safety for an annotated program P with one entry point if, and only if, there exists a property $I(x_0, \ell, x)$ satisfying :

- ① $\forall x_0 \in D. pre(x_0) \wedge x = x_0 \wedge \ell = \ell_0 \Rightarrow J(x_0, \ell, x)$
- ② $\forall \ell, \ell' \in L, x, x_0 \in D. pre(x_0) \wedge J(x_0, \ell, x) \wedge BA(e(\ell, \ell'),)(\ell, x, \ell', x') \Rightarrow J(x_0, \ell', x')$
- ③ $\forall \ell \in L, x_0, x \in D. pre(x_0) \wedge J(x_0, \ell, x) \Rightarrow S(x_0, \ell, x)$



(Soundness of the method)

If the initialisation init, the generalisation gen and the step induction are proved to be correct by the Rodin platform, the property $S(x_0, \ell, x)$ is a correct safety property for the program P. In particular, one can handle the partial correctness and the run time error safety properties.

(Soundness of the method)

If the initialisation init, the generalisation gen and the step induction are proved to be correct by the Rodin platform, the property $S(x_0, \ell, x)$ is a correct safety property for the program P. In particular, one can handle the partial correctness and the run time error safety properties.

- Contract and verification conditions are translated into Event-B and are discharged by Rodin and its provers.
- Verification conditions are derived from Floyd's method.
- Annotation as assertion

A short example

s

```
contract SIMPLE
variables x
requires  $x_0 \in \mathbb{N}$ 
ensures  $x_f = 0$ 
begin
 $\ell_0 : \{0 \leq x \leq x_0 \wedge x_0 \in \mathbb{N}\}$ 
while  $0 < x$  do
     $\ell_1 : \{0 < x \wedge x \leq x_0 \wedge x_0 \in \mathbb{N}\}$ 
     $x := x - 1;$ 
od
 $\ell_2 : \{x = 0\}$  end
```

```
Event Init
THEN
    act1 :  $x := x_0$ 
    act2 :  $l := l_0$ 
```

```
Event el0l1
WHEN
    grd1 :  $l = l_0$ 
    grd2 :  $0 < x$ 
THEN
    act1 :  $l := l_1$ 
```

INVARIANTS

```
inv1 :  $x \in \mathbb{N}$ 
inv2 :  $l \in L$ 
inv3 :  $l = l_0 \Rightarrow$ 
 $0 \leq x \wedge x \leq x_0 \wedge x_0 \in \mathbb{N}$ 
inv4 :  $l = l_1 \Rightarrow$ 
 $0 < x \wedge x \leq x_0 \wedge x_0 \in \mathbb{N}$ 
inv5 :  $l = l_2 \Rightarrow x = 0$ 
requires :  $x_0 \in \mathbb{N} \wedge x = x_0$ 
 $\Rightarrow x = x_0 \wedge x_0 \in \mathbb{N}$ 
ensures :  $x = 0 \wedge x = x_0$ 
 $\Rightarrow x = 0$ 
```

Event el0l2

```
WHEN
    grd1 :  $l = l_0$ 
    grd2 :  $\neg(0 < x)$ 
THEN
    act1 :  $l := l_2$ 
```

Event el1l0

```
WHEN
    grd1 :  $l = l_1$ 
THEN
    act1 :  $l := l_0$ 
    act2 :  $x := x - 1$ 
```

Current Summary

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Annotation of programs

$$\begin{aligned}\ell : \{P_\ell(v)\} \\ cond_{\ell,\ell'}(v) \longrightarrow v := f_{\ell,\ell'}(v) \\ \ell' : \{P_{\ell'}(v)\}\end{aligned}$$
$$\begin{aligned}\ell_0^1 : \{x = 0\} \\ x := x + 1; \\ \ell_0^1 : \{x = 1\}\end{aligned}$$
$$\begin{aligned}e(\ell, \ell') \\ \text{WHEN} \\ c = \ell \\ cond_{\ell,\ell'}(v) \\ \text{THEN} \\ c := \ell' \\ v := f_{\ell,\ell'}(v) \\ \text{END}\end{aligned}$$

- v is the state memory variable or list of memory variables ; v includes the local variables and the results variables.
- c is a new variable which is modelling the control flow and its type is LOCATIONS.
- $e(\ell, \ell')$ is simulating the computation flow starting from ℓ and moving to ℓ' ; v is updated.

From annotations to invariants

INVARIANTS

$inv_i : c \in \text{LOCATIONS}$

$inv_j : v \in Type$

...

$inv_k : c = \ell \Rightarrow P_\ell(v)$

$inv_m : c = \ell' \Rightarrow P_{\ell'}(v)$

...

$th_n : A(c, v)$

- $Type$ is the type of the variables v and is a set of possible values defined in the context C .
- The annotation is giving us for free the conditions satisfied by v when the control is in ℓ , (resp. in ℓ').
- $A(c, v)$ is a safety property that we are supposed to check and the case of Event-B, it is a theorem.

Partial correctness using Event-B models

For each pair of successive labels ℓ, ℓ' , the three statements are equivalent :

- $P_\ell(v) \wedge cond_{\ell,\ell'}(v) \wedge v' = f_{\ell,\ell'}(v) \Rightarrow P_{\ell'}(v')$
- $I(c, v) \wedge c = \ell \wedge cond_{\ell,\ell'}(v) \wedge c' = \ell' \wedge v' = f_{\ell,\ell'}(v) \Rightarrow (c' = \ell' \Rightarrow P_{\ell'}(v'))$
- $I(c, v) \wedge BA(e(\ell, \ell'))(c, v, c', v') \Rightarrow (c' = \ell' \Rightarrow P_{\ell'}(v'))$

L

et AA an annotated algorithm with precondition $\mathbf{pre}(\text{AA})(v)$ and postcondition $\mathbf{post}(\text{AA})(v_0, v)$. Let the context C and the machine M generated from AA using the construction given previously. We assume that ℓ_0 is the first label and ℓ_e is the last label. We add the following safety properties in the machine M :

- $c = \ell_0 \wedge \mathbf{pre}(\text{AA})(v) \Rightarrow P_{\ell_0}(v)$
- $c = \ell_e \Rightarrow (P_{\ell_e}(v) \Rightarrow \mathbf{post}(\text{AA})(v_0, v))$

If proof obligations are discharged, then the annotated algorithm AA is partially correct with respect to its pre/post specification.



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- $\forall x_f, x_0. \text{pre}(x_0) \wedge x_0 \xrightarrow{\text{P}} x_f \Rightarrow \text{post}(x_0, x_f)$

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- $\forall x_f, x_0. \text{pre}(x_0) \Rightarrow x_0 \xrightarrow{\text{P}} x_f \Rightarrow \text{post}(x_0, x_f)$

From Floyd to Hoare

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- $\forall x_f, x_0. \text{pre}(x_0) \Rightarrow x_0 \xrightarrow{\text{P}} x_f \Rightarrow \text{post}(x_0, x_f)$
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- $\forall x_f, x_0. \text{pre}(x_0) \Rightarrow x_0 \xrightarrow{\text{P}} x_f \Rightarrow \text{post}(x_0, x_f)$
- $\forall x_f, x_0. \text{pre}(x_0) \Rightarrow x_0 \xrightarrow{\text{P}} x_f \Rightarrow \text{post}(x_0, x_f)$
- $\forall x_0. \text{pre}(x_0) \Rightarrow \forall x_f. x_0 \xrightarrow{\text{P}} x_f \Rightarrow \text{post}(x_0, x_f)$

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- $\forall x_0. \text{pre}(x_0) \Rightarrow [P]\text{post}(x_0, x_f)$
- wlp calculus is introduced

From Floyd to Hoare

- $\forall x_f, x_0. \text{pre}(x_0) \wedge x_0 \xrightarrow{\text{P}} x_f \Rightarrow \text{post}(x_0, x_f)$
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- $[x := e]P(x) = P[x \mapsto e]$

From Floyd to Hoare

- $\forall x_f, x_0. \text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
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- $\forall x_0. \text{pre}(x_0) \Rightarrow [P]\text{post}(x_0, x_f)$
- wlp calculus is introduced
- $[x := e]P(x) = P[x \mapsto e]$
- $[\text{if } b(x) \text{ then } S1 \text{ else } S2]P(x) = b(x) \wedge [S1]P(x) \vee \text{not } b(x) [S2]P(x)$

From Floyd to Hoare

- $\forall x_f, x_0. \text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
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- $\forall x_0. \text{pre}(x_0) \Rightarrow \forall x_f. x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
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- Frama-c uses the HOARE logic for defining the verification conditions as R. Leino in DAFNY.

From Floyd to Hoare

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- $\forall x_0. \text{pre}(x_0) \Rightarrow [P]\text{post}(x_0, x_f)$
- wlp calculus is introduced
- $[x := e]P(x) = P[x \mapsto e]$
- $[\text{if } b(x) \text{ then } S1 \text{ else } S2]P(x) = b(x) \wedge [S1]P(x) \vee \neg b(x) [S2]P(x)$
- Frama-c uses the HOARE logic for defining the verification conditions as R. Leino in DAFNY.
- Questions of termination require the wp calculus . . .