



Modelling Software-based Systems Lecture 2

Correctness by Construction with the Modelling Language Event-B using the Refinement

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General Summary

- Correctness by Construction
- 2 The refinement of models
- 3 TOP CM
- 4 Example of the factorial function refined into an algorithm
- **5** Summary on Event-B
- 6 Intermezzo on the Event B modelling notation
- ♠ Transformations of EVENT B models
- Conclusion
- The Inductive Paradigm
- Summary

Current Summary

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Correctness by Construction

- Correctness by Construction is a method of building software -based systems with demonstrable correctness for security- and safety-critical applications.
- Correctness by Construction advocates a step-wise refinement process from specification to code using tools for checking and transforming models.
- Correctness by Construction is an approach to software/system construction
 - starting with an abstract model of the problem.
 - progressively adding details in a step-wise and checked fashion.
 - each step guarantees and proves the correctness of the new concrete model with respect to requirements

The Cleanroom Method as CbC

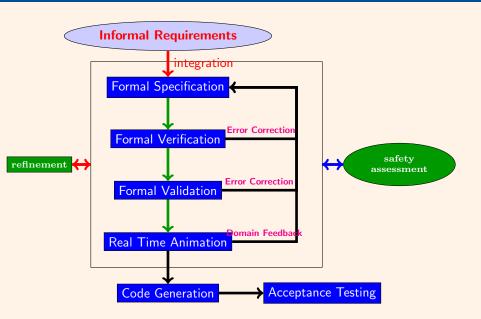
- The Cleanroom method, developed by Harlan Mills and his colleagues at IBM and elsewhere, attempts to do for software what cleanroom fabrication does for semiconductors: to achieve quality by keeping defects out during fabrication.
- In semiconductors, dirt or dust that is allowed to contaminate a chip as it is being made cannot possibly be removed later.
- But we try to do the equivalent when we write programs that are full of bugs, and then attempt to remove them all using debugging.

The Cleanroom Method as CbC

The Cleanroom method, then, uses a number of techniques to develop software carefully, in a well-controlled way, so as to avoid or eliminate as many defects as possible before the software is ever executed. Elements of the method are:

- specification of all components of the software at all levels;
- stepwise refinement using constructs called "box structures";
- verification of all components by the development team;
- statistical quality control by independent certification testing;
- no unit testing, no execution at all prior to certification testing.

Critical System Development Life-Cycle Methodology



Overview of Methodology

- Informal Requirements: Restricted form of natural language.
- Formal Specification: Modeling language like Event-B, Z, ASM, VDM, TLA+...
- Formal Verification: Theorem Prover Tools like PVS, Z3, SAT, SMT Solver...
- Formal Validation: Model Checker Tools like ProB, UPPAAL, SPIN, SMV...
- Real-time Animation : Our proposed approach ... Real-Time Animator ...
- Code Generation: Our proposed approach ... EB2ALL: EB2C, EB2C++, EB2J, EB2C# ...
- Acceptance Testing: Failure Mode, Effects and Critically analysis(FMEA and FMEA), System Hazard Analyses(SHA)

Case Studies

- Colin Boyd and Anish Mathuria. Protocols Authentication and Key Establisment. Springer 2003.
- C. C. Marquezan and L. Z. Granville. Self-* and P2P for Network Management - Design Principles and Case Studies. Springer Briefs in Computer Science. Springer, 2012.
- Pacemaker Challenge Contribution

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Problems for Modelling systems

- Systems are generally very complex
- Invariant should be strong enough for proving safety properties
- Problems for modelling: finding suitable mathematical structures, listing events or actions of the system, proving proof obligations, . . .

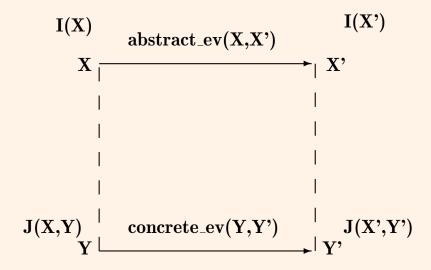
Solution : refining models

- To understand more and more the system
- To distribute the complexity of the system
- To distribute the difficulties of the proof
- To improve explanations
- Validation (step by step)
- Refinement (invariant & behavior)

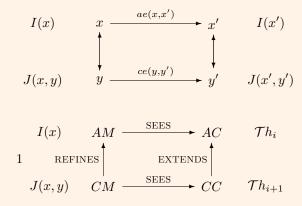
Refinement of models

- we can add more details (like superposition),
- we can add new events (we can observe more transformations),
- each new event refines SKIP
- no deadlock
- abstract events occur (new events decrease something)

Refinement of a model by another one



Refinement of a model by another one (I)



Refinement of a model by another one (II)



(REF1): refinement of initial conditions

$$Initc(y) \Rightarrow \exists x * (Init(x) \land J(x,y))$$
:

The initial condition of the refinement model imply that there exists an abstract value in the abstract model such that that value satisfies the initial conditions of the abstract one and implies the new invariant of the refinement model.

(REF2): refinement of events

$$I(x) \wedge J(x,y) \wedge ce(y,y') \Rightarrow \exists x'.(ae(x,x') \wedge J(x',y'))$$
:

The invariant in the refinement model is preserved by the refined event and the activation of the refined event triggers the corresponding abstract event.

(REF3): refinement of stuttering steps

$$I(x) \wedge J(x,y) \wedge ce(y,y') \Rightarrow J(x,y')$$
:

The invariant in the refinement model is preserved by the refined event but the event of the refinement model is a new event which was not visible in the abstract model; the new event refines skip.

(REF4): Refinement does not introduce more blocking states

$$I(x) \wedge J(x,y) \wedge (G_1(x) \vee \ldots \vee G_n(x)) \Rightarrow H_1(y) \vee \ldots \vee H_k(y)$$
:

The guards of events in the refinement model are strengthened and we have to prove that the refinement model is not more blocked than the abstract.

(REF5): Well-definedness of variant

$$I(x) \wedge J(x,y) \Rightarrow V(y) \in \mathbb{N}$$

(REF6): Well behaviour of new events

$$I(x) \wedge J(x,y) \wedge ce(y,y') \Rightarrow V(y') < V(y)$$
:

New events should not block forever abstract ones.

(REF7): Feasibility of refined events

$$\Gamma(s,c) \; \vdash \; I(x) \; \land \; J(x,y) \; \land \; grd\left(E\right) \; \Rightarrow \; \exists y' \cdot P(y,y')$$

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The factorial model

```
CONTEXT fonctions CONSTANTS factorial, n AXIOMS n \in \mathbb{N} \land factorial \in \mathbb{N} \leftrightarrow \mathbb{N} \land 0 \mapsto 1 \in factorial \land \forall (i,fn).(i \mapsto fn \in factorial \Rightarrow i+1 \mapsto (i+1)*fi \in factorial) \land f \in \mathbb{N} \leftrightarrow \mathbb{N} \land 0 \mapsto 1 \in f \land \forall (n,fn).(n \mapsto fn \in f \Rightarrow n+1 \mapsto (n+1) \times fn \in f) \Rightarrow factorial \subseteq f
```

The factorial model

```
MACHINE
  specification
SEES fonctions
VARIABLES
  result at
INVARIANT
  resultat \in \mathbb{N}
THEOREMS
  factorial \in \mathbb{N} \longrightarrow \mathbb{N};
  factorial(0) = 1;
  \forall n.(n \in \mathbb{N} \Rightarrow factorial(n+1) = (n+1) \times factorial(n))
INITIALISATION
  resultat :\in \mathbb{N}
EVENTS
  computing1 = BEGIN \ resultat := factorial(n) \ END
END
```

Refining specification by computation

```
MACHINE computation
REFINES specification
SEES fonctions
VARIABLES resultat, fac, x
INVARIANTS
  inv1: fac \in \mathbb{N} \to \mathbb{N}
  inv2: dom(fac) \subseteq 0 \dots n
  inv4: dom(fac) \neq \emptyset
  inv5: \forall i.i \in dom(fac) \Rightarrow fac(i) = factorial(i)
  inv3: x \in dom(fac)
  inv6: dom(fac) = 0 \dots x
EVENTS
EVENT INITIALISATION
  BEGIN
    act1: resultat : \in \mathbb{N}
    act2: fac := \{0 \mapsto 1\}
    act3: x := 0
  END
EVENT computing2 REFINES EVENT computing1
  WHEN
    grd1: n \in dom(fac)
  THEN
    act1: resultat := fac(n)
  FND
END
```

Refining specification by computation

```
MACHINE computation
REFINES specification
SEES fonctions
VARIABLES resultat, fac, x
INVARIANTS
  inv1: fac \in \mathbb{N} \to \mathbb{N}
  inv2: dom(fac) \subseteq 0 \dots n
  inv4:dom(fac) \neq \emptyset
  inv5: \forall i.i \in dom(fac) \Rightarrow fac(i) = factorial(i)
  inv3: x \in dom(fac)
  inv6: dom(fac) = 0 \dots x
EVENTS
FVFNT event2
  WHEN
    grd11: x \in dom(fac)
    grd12: x + 1 \notin dom(fac)
    qrd13: n \notin dom(fac)
  THEN
    act11 : fac(x+1) := (x+1) * fac(x)
    act1 : x := x + 1
  FND
END
```

Refining computation by algorithm

```
EVENTS
EVENT INITIALISATION
 BEGIN
   act1: resultat : \in \mathbb{N}
   act2: fac := \{0 \mapsto 1\}
   act3: cfac := 0
   act4: vfac := 1
   act5 : x := 0
 END
EVENT computing3 REFINES EVENT computing2
 WHEN qrd2: cfac = n
 THEN act1: resultat := v fac
 END
EVENT event3 REFINES EVENT event2
 WHEN grd1: cfac \neq n
 THEN
   act1: vfac := (cfac + 1) * vfac
   act2: cfac := cfac + 1
   act3: fac(cfac+1) := (cfac+1) * fac(cfac)
   act4: x := x + 1
 END
END
```

Refining computation by algorithm

```
\begin{array}{lll} \textbf{MACHINE} & algorithm & \textbf{REFINES} & computation \\ \textbf{SEES} & fonctions \\ \textbf{VARIABLES} & resultat, vfac, & cfac, fac, x \\ \textbf{INVARIANTS} \\ & inv1: vfac \in \mathbb{N} \\ & inv2: cfac \in \mathbb{N} \\ & inv3: cfac \leq n \\ & inv4: cfac \geq 0 \\ & inv6: cfac \in dom(fac) \\ & inv5: vfac = fac(cfac) \\ & inv7: cfac + 1 \notin dom(fac) \\ & inv8: dom(fac) = 0 \dots cfac \\ & inv9: x = cfac \\ \end{array}
```

Refining algorithm by simplealgorithm

```
MACHINE simpleal gorithm REFINES algorithm
SEES fonctions
VARIABLES resultat, vfac, cfac
THEOREMS thm1: vfac = factorial(cfac)
FVFNTS
EVENT INITIALISATION
  BEGIN
    act1: resultat : \in \mathbb{N}
   act3: cfac := 0
    act4: vfac := 1
  FND
EVENT computing4 REFINES EVENT computing3
  WHFN
    qrd2: cfac = n
  THEN
    act1: resultat := v fac
  END
EVENT event4 REFINES EVENT event3
  WHFN
   qrd1: cfac \neq n
  THEN
    act1: vfac := (cfac + 1) * vfac
    act2: cfac := cfac + 1
  FND
END
```

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Simple Form of an Event

- An event of the simple form is denoted by :

```
 \begin{array}{l} < event\_name > \; \widehat{=} \\ \textbf{WHEN} \\ < condition > \\ \textbf{THEN} \\ < action > \\ \textbf{END} \end{array}
```

where

- $< event_name >$ is an identifier
- < condition > is the firing condition of the event
- < action > is a generalized substitution (parallel "assignment")

Non-deterministic Form of an Event

- An event of the non-deterministic form is denoted by :

```
 < event\_name > \triangleq \\ \mathbf{ANY} < variable > \mathbf{WHERE} \\ < condition > \\ \mathbf{THEN} \\ < action > \\ \mathbf{END}
```

where

- $< event_name >$ is an identifier
- < variable > is a (list of) variable(s)
- < condition > is the firing condition of the event
- < action > is a generalized substitution (parallel "assignment")

Shape of a Generalized Substitution

A generalized substitution can be

```
- Simple assignment : x := E
```

- Generalized assignment : x: P(x, x')

```
- Set assignment : x :\in S
```

T

- Parallel composition : ·

Invariant Preservation Verification (0)

INVARIANT ∧ GUARD ⇒
ACTION establishes INVARIANT

Invariant Preservation Verification (1)

- Given an event of the simple form :

```
 \begin{array}{c} \textbf{EVENT EVENT} & \cong \\ \textbf{WHEN} \\ G(x) \\ \textbf{THEN} \\ x := E(x) \\ \textbf{END} \end{array}
```

$$| I(x) \wedge G(x) \implies I(E(x))$$

Invariant Preservation Verification (2)

- Given an event of the simple form :

```
 \begin{array}{c} \textbf{EVENT EVENT} & \widehat{=} \\ \textbf{WHEN} \\ G(x) \\ \textbf{THEN} \\ x: |P(x,x') \\ \textbf{END} \end{array}
```

$$I(x) \wedge G(x) \wedge P(x,x') \implies I(x')$$

Invariant Preservation Verification (3)

- Given an event of the simple form :

```
\begin{array}{ll} \textbf{EVENT EVENT} & \widehat{=} \\ \textbf{WHEN} \\ G(x) \\ \textbf{THEN} \\ x : \in S(x) \\ \textbf{END} \end{array}
```

$$I(x) \wedge G(x) \wedge x' \in S(x) \implies I(x')$$

Invariant Preservation Verification (4)

- Given an event of the non-deterministic form :

```
 \begin{array}{c} \textbf{EVENT EVENT} \; \stackrel{\triangle}{=} \\ \textbf{ANY} \; v \; \textbf{WHERE} \\ G(x,v) \\ \textbf{THEN} \\ x := E(x,v) \\ \textbf{END} \\ \end{array}
```

$$I(x) \wedge G(x,v) \implies I(E(x,v))$$

Refinement Technique (1)

- Abstract models works with variables \boldsymbol{x} , and concrete one with \boldsymbol{y}
- A gluing invariant J(x,y) links both sets of vrbls
- Each abstract event is refined by concrete one (see below)

Refinement Technique (2)

- Some new events may appear : they refine "skip"
- Concrete events must not block more often than the abstract ones
- The set of new event alone must always block eventually

Correct Refinement Verification (1)

- Given an abstract and a corresponding concrete event

```
\begin{array}{l} \textbf{EVENT ea} & \widehat{=} \\ \textbf{WHEN} \\ G(x) \\ \textbf{THEN} \\ x := E(x) \\ \textbf{END} \end{array}
```

```
\begin{array}{l} {\rm EVENT\ ec} & \cong \\ {\rm WHEN} & \\ H(y) \\ {\rm THEN} & \\ y := F(y) \\ {\rm END} & \end{array}
```

and invariants I(x) and J(x,y), the statement to prove is :

$$I(x) \wedge J(x,y) \wedge H(y) \Longrightarrow G(x) \wedge J(E(x),F(y))$$

Correct Refinement Verification (2)

- Given an abstract and a corresponding concrete event

```
\begin{array}{ll} \mathsf{EVENT} \ \mathsf{ea} & \widehat{=} \\ \mathbf{ANY} \ v \ \mathbf{WHERE} \\ G(x,v) \\ \mathbf{THEN} \\ x \coloneqq E(x,v) \\ \mathbf{END} \end{array}
```

```
 \begin{array}{c} \textbf{EVENT ec} & \cong \\ \textbf{ANY } w \textbf{ WHERE} \\ H(y,w) \\ \textbf{THEN} \\ y := F(y,w) \\ \textbf{END} \end{array}
```

$$\begin{array}{cccc} I(x) & \wedge & J(x,y) & \wedge & H(y,w) \\ \Longrightarrow & \\ \exists v \cdot (G(x,v) & \wedge & J(E(x,v),F(y,w))) \end{array}$$

Correct Refinement Verification (3)

- Given a NEW event

```
\begin{array}{ll} \textbf{EVENT EVENT} & \widehat{=} \\ \textbf{WHEN} \\ H(y) \\ \textbf{THEN} \\ y := F(y) \\ \textbf{END} \end{array}
```

and invariants I(x) and J(x,y), the statement to prove is :

$$I(x) \wedge J(x,y) \wedge H(y) \implies J(x,F(y))$$

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General form of proof obligations for an event e

- $INIT/I/INV : C(s,c), INIT(c,s,x) \vdash I(c,s,x)$
- $\bullet \ \ e/I/INV: C(s,c), I(c,s,x), G(c,s,t,x), P(c,s,t,x,x') \vdash I(c,s,x')$
- $e/act/FIS : C(s,c), I(c,s,x), G(c,s,t,x) \vdash$
- e/act/WD : $C(s,c), I(c,s,x), G(c,s,t,x) \vdash \exists x'. P(c,s,t,x,x')$

Well-definedness of an Axiom	m / WD	m is the axiom name	
Well-definedness of a Derived Axiom	m / WD	m is the axiom name	
Derived Axiom	m / THM	m is the axiom name	
Well-definedness of an Invariant	v / WD	v is the invariant name	
Well-definedness of a Derived Invariant	m / WD	m is the invariant name	
Well-definedness of an event Guard	t / d / WD	t is the event name d is the action name	
Well-definedness of an event Action	t / d / WD	t is the event name d is the action name	
Feasibility of a non-det. event Action	t / d / FIS	t is the event name d is the action name	
Derived Invariant	m / THM	m is the invariant name	
Invariant Establishment	INIT. / v / INV	v is the invariant name	
Invariant Preservation	t / v / INV t is the event name v is th invariant name		

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algorithms by J.-R. Abrial

 $\begin{array}{c} \textbf{WHEN} \\ P \\ Q \\ \textbf{THEN} \\ S \\ \textbf{END} \end{array}$

 $\begin{array}{c} \textbf{WHEN} \\ P \\ \neg Q \\ \textbf{THEN} \\ T \\ \textbf{END} \end{array}$

are merged into

```
\begin{array}{c} \textbf{WHEN} \\ P \\ \textbf{THEN} \\ \textbf{WHILE} \quad Q \quad \textbf{DO} \\ S \\ \textbf{END}; \\ T \\ \textbf{END} \end{array}
```

Side Conditions:

- P must be invariant under S.
- The first event must have been introduced at one refinement step below the second one.
- Special Case : If P is missing the resulting "event" has no guard

algorithms by J.-R. Abrial

 $\begin{array}{c} \textbf{WHEN} \\ P \\ Q \\ \textbf{THEN} \\ S \\ \textbf{END} \end{array}$

 $\begin{array}{c} \textbf{WHEN} \\ P \\ \neg Q \\ \textbf{THEN} \\ T \\ \textbf{END} \end{array}$

are merged into

 $\begin{array}{c} \textbf{WHEN} \\ P \\ \textbf{THEN} \\ \textbf{IF} \quad Q \quad \textbf{THEN} \quad S \\ \textbf{ELSE} \quad T \\ \textbf{END}; \\ \textbf{END} \end{array}$

Side Conditions:

- The disjunctive negation of the previous side conditions
- Special Case : If P is missing the resulting "event" has no guard

Applying the rule for the while

Applying the INITIALISATION rule

EVENT INITIALISATION BEGIN

 $\begin{array}{l} init \\ resultat :\in \mathbb{N}; \\ cfac := 0; \\ vfac := 1; \end{array}$

Deriving an algorithm

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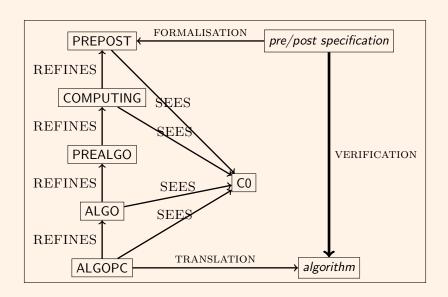
Conclusion

- Refinement helps in discovering invariants
- Refinement helps in proving invariants
- The choice of the good abstraction is not very simple and is a challenge by itself

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The Iterative Pattern



General context C0

```
CONTEXT C0

SETS

U

CONSTANTS

x, v, d0, f, D

AXIOMS

axm1: x \in \mathbb{N}

axm25: D \subseteq U

axm24: f \in D \rightarrow D

axm23: d0 \in D

axm23: v \in \mathbb{N} \rightarrow D

axm3: v(0) = d0

axm4: \forall n \cdot n \in \mathbb{N} \Rightarrow v(n+1) = f(v(n))

th1: Q(d_0, d) \equiv (d = v(x))
```

- the sequence v expresses the post-condition $Q(d_0,d)$ with the precondition $P(d_0)$.
- $Q(d_0, d)$ is equivalent to d = v(x).
- The theorem th1 should be proved in the context C0. he

General PREPOST Machine

- The theorem th1 is validating the definition of the result r to compute.
- The event computing is expressing the contract of the given problem.
- it by a very simple problem that is the computation of the function n^2 using the addition operator.

First Refinement COMPUTING : Inductive Computation

$\begin{aligned} & \textbf{EVENT INITIALISATION} \\ & \textbf{BEGIN} \\ & act1: r: \in D \\ & act3: vv: = \{0 \mapsto d0\} \\ & act5: k: = 0 \\ & \textbf{END} \end{aligned}$

INITIALISATION is initializing the variables with respect to the initial values of the sequences of the context.

First Refinement COMPUTING: Inductive Computation

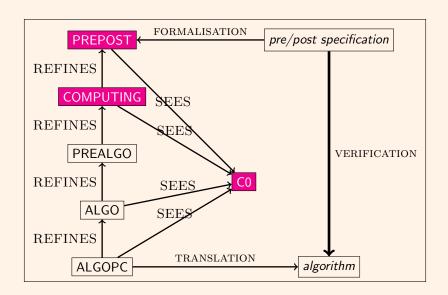
computing is imply observing that the result is computed simulating the sequence vv.

First Refinement COMPUTING: Inductive Computation

```
\begin{array}{l} \textbf{EVENT step} \\ \textbf{WHEN} \\ grd1: x \notin dom(vv) \\ \textbf{THEN} \\ act2: vv(k+1) := f(vv(k)) \\ act4: k := k+1 \\ \textbf{END} \end{array}
```

step is *simulating* the computation of the values of the sequence vv as a model computation.

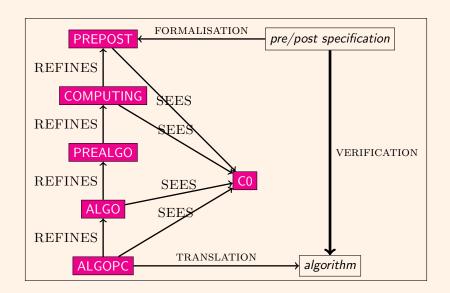
The Iterative Pattern



Completing the machines

- PREALGO : adding new variables for pointing out the necessary values to store cvv
- \bullet ALGO : hiding the model variables storing the unnecessary values of sequence vv
- ALGOPC; adding control variable c

The Iterative Pattern



Listing 1 – Function derived from pattern for the sequence v

Comments

- The produced algorithm can be now checked using another proof environment as for instance Frama-C.
- The inductive property of the invariant is clearly verified and is easily derived from the Event-B machines.
- The verification is not required, since the system is correct by construction but it is a checking of the process itself
- the project called ITERATIVE-PATTERN;
- the project is the pattern itself
- The invariants of the Event-B models can be reused in the verification using Frama-C, for instance, and the verification of the resulting algorithm is a confirmation of the translation.

Listing 2 - Function derived from pattern power3

```
#include < limits .h>
/*@ requires 0 \le x;
     requires x*x*x <= INT_MAX ;
     ensures \result ==x*x*x;
int power3(int x)
{int r.ocz.cz.cv.cu.ocv.cw.ocw.ct.oct.ocu.k.ok:
  cz=0:cv=0:cw=1:ct=3:cu=0: ocw=cw:ocz=cz:
  oct=ct:ocv=cv:ocu=cu:k=0:ok=k:
       /*@ loop invariant cz == k*k*k:
         @ loop invariant cu == k:
         Q loop invariant cv+ct==3*(cu+1)*(cu+1):
         @ loop invariant cz+cv+cw==3*(cu+1)*(cu+1)*(cu+1);
         @ loop invariant cv 3*cu*cu:
         @ loop invariant cw == 3*cu+1:
         @ loop invariant k <= x;
         @ loop assigns ct,oct,cu,ocu,cz,ocz,k,cv,cw,r,ok;
         @ loop assigns ocv.ocw: */
  while (k<x)
          ocz=cz:ok=k:ocv=cv:ocw=cw:oct=ct:ocu=cu:
          cz=ocz+ocv+ocw:
          cv=ocv+oct:
          ct = oct + 6:
          cw = ocw + 3:
          cu=ocu+1:
          k = 0k + 1:
  r=cz; return (r); }
```

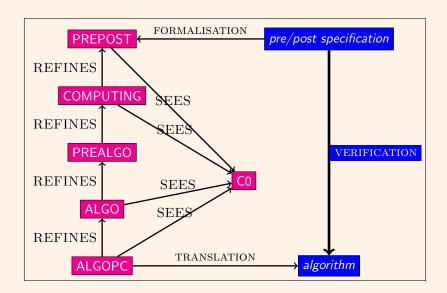
Summary for proof obligations

Name	Total	Automatic	Interactive
ex-induction	40	36	4
C0	2	0	2
PREPOST	4	4	0
COMPUTING	16	14	2
PREALGO	9	9	0
ALGO	6	6	0
ALGOPC	3	3	0

Summary

- The loop invariant is inductive but Frama-C does not prove it completely.
- Not the case with the RODIN platform which is able to discharge the whole set of proof obligations.
- However, the Event-B model is using auxiliary knowledge over sequences used for defining the computing process.
- The most difficult theorem is to prove that $\forall n \in \mathbb{N} : z_n = n * n * n$.

The Iterative Pattern



Current Summary

- ① Correctness by Construction
- 2 The refinement of models
- **3** TOP CM
- Example of the factorial function refined into an algorithm
- **5** Summary on Event-B
- 6 Intermezzo on the Event B modelling notation
- **7** Transformations of EVENT B models
- 8 Conclusion

Summary on refinement

- Refining means making models more deterministic
- Refining means adding new variable and new events
- Refining is simulating
- Refining preserves safety properties of the refined model.
- The very abstract model is crucial.
- The process should be incremental to make proofs easier for the proof tool.
- Problem : Preserving the liveness properties