
Cours MVSI

Modélisation et Vérification des Systèmes Informatiques

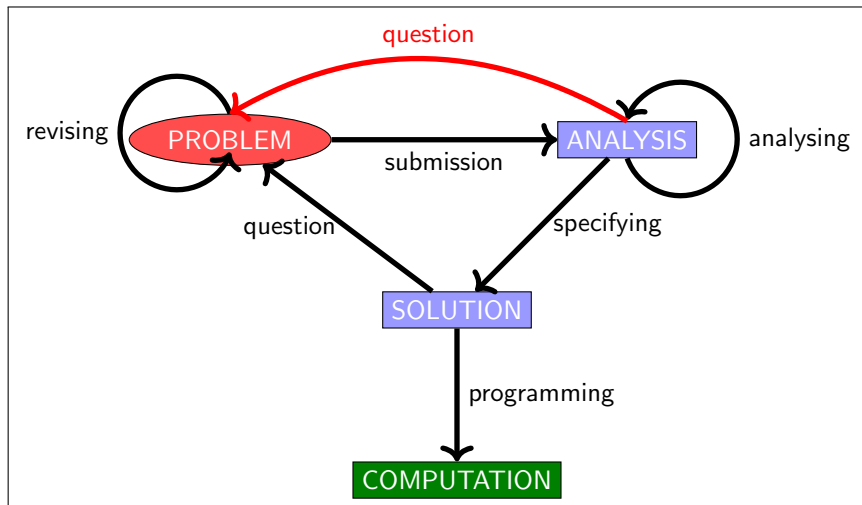
Overview of the course

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Année universitaire 2025-2026

- ① Tracking bugs in C codes
- ② Introduction by Example
 - Detecting overflows in computations
 - Computing the velocity of an aircraft on the ground
- ③ Verification of program properties
- ④ Topics of course

Problem versus Solution



Listing 1 – Bug bug0

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y;
    // Seed the random number generator with the current time
    srand(time(NULL));
    // Generate a random number between 1 and 100
    x = rand() % 100 + 1;
    // Perform some calculations
    y = x / (100 - x);
    printf("Result: -%d\n", y);
    return 0;
}
```

Listing 2 – Bug bug0

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y;
    // Seed the random number generator with the current time
    srand(time(NULL));
    // Generate a random number between 1 and 100
    x = rand() % 100 + 1;
    // Perform some calculations
    y = x / (100 - x);
    printf("Result: %d\n", y);
    return 0;
}
```

bug0.c prints Result : w

Listing 3 – Bug bug00

```
// Heisenbug
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y, i=0;

    for (i = 0; i <= 100000; i++) {
        // Seed the random number generator with the current time
        srand(time(NULL));

        // Generate a random number between 1 and 100
        x = rand() % 100 + 1;
        printf("Result: -x=--%d\n", x);
        // Perform some calculations
        y = x / (100 - x);

        printf("Result: -i=%d--and-y=%d\n", i, y);
    }

    return 0;
}
```

Listing 4 – Bug bug00

```
// Heisenbug
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y, i=0;

    for (i = 0; i <= 100000; i++) {
        // Seed the random number generator with the current time
        srand(time(NULL));

        // Generate a random number between 1 and 100
        x = rand() % 100 + 1;
        printf("Result: -x=-%d\n", x);
        // Perform some calculations
        y = x / (100 - x);

        printf("Result: -i=%d--and-y=%d\n", i, y);
    }

    return 0;
}
```

**bug00.c prints ... Result : x= w1;
Result : i=100000 w2**

Listing 5 – Bug bug000

```
// Heisenbug
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y, i=0;

    for (i = 0; i <= 200000; i++) {
        // Seed the random number generator with the current time
        srand(time(NULL)+i);

        // Generate a random number between 1 and 100
        x = rand() % 100 + 1;
        printf("Result: -x=-%d\n", x);
        // Perform some calculations
        y = x / (100 - x);

        printf("Result: -i=%d -y=%d\n", i, y);
    }

    return 0;
}
```


Listing 6 – Bug bug000

```
// Heisenbug
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

int main() {
    int x, y, i=0;

    for (i = 0; i <= 200000; i++) {
        // Seed the random number generator with the current time
        srand(time(NULL)+i);

        // Generate a random number between 1 and 100
        x = rand() % 100 + 1;
        printf("Result: -x=-%d\n", x);
        // Perform some calculations
        y = x / (100 - x);

        printf("Result: -i=%d -y=%d\n", i, y);
    }

    return 0;
}
```

Result: x= 70

Result: i=200000 2

Listing 7 – Bug bug1

```
#include <stdio.h>

int main() {
    int numbers[5] = {1, 2, 3, 4, 5};
    int sum = 0;

    // Attempt to calculate the sum of numbers in the array
    for (int i = 0; i <= 5; i++) {
        sum += numbers[i];
    }

    printf("Sum: %d\n", sum);
    sum = numbers[0];
    printf("Sum: %d\n", sum);

    return 0;
}
```

Listing 8 – Bug bug1

```
#include <stdio.h>

int main() {
    int numbers[5] = {1, 2, 3, 4, 5};
    int sum = 0;

    // Attempt to calculate the sum of numbers in the array
    for (int i = 0; i <= 5; i++) {
        sum += numbers[i];
    }

    printf("Sum: %d\n", sum);
    sum = numbers[0];
    printf("Sum: %d\n", sum);

    return 0;
}
```

Sum: 16

Sum: 1

Listing 9 – Bug bug2

```
#include <stdio.h>

int main() {
    int x = 5;
    int y = 3;

    // Bug 1: Incorrect variable in the printf statement
    printf("The-value-of-x-is:-%d\n", y);

    // Bug 2: Infinite loop
    while (x > 0) {
        printf("x-is-greater-than-0\n");
    }

    return 0;
}
```

Listing 10 – Bug bug2

```
#include <stdio.h>

int main() {
    int x = 5;
    int y = 3;

    // Bug 1: Incorrect variable in the printf statement
    printf("The-value-of-x-is:-%d\n", y);

    // Bug 2: Infinite loop
    while (x > 0) {
        printf("x-is-greater-than-0\n");
    }

    return 0;
}
```

Infinite loop . . .

Listing 11 – Bug bug7

```
#include <stdio.h>
#include <limits.h>
int average(int a, int b)
{
    return ((a+b)/2);
}

int main()
{
    int x, y;
    x=INT_MAX; y=INT_MAX;
    printf(" Average -- for -%d- and -%d- is -%d\n" , x, y,
        average(x, y));
    return 0;
}
```

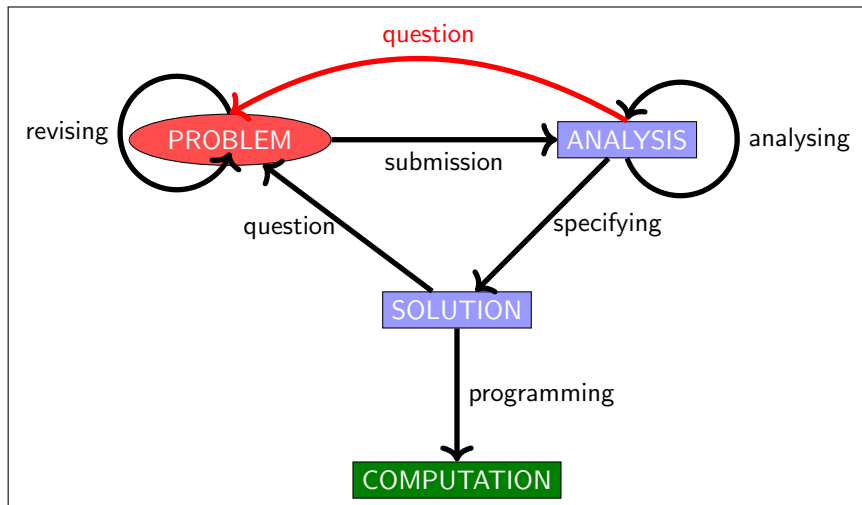
Listing 12 – Bug bug7

```
#include <stdio.h>
#include <limits.h>
int average(int a, int b)
{
    return ((a+b)/2);
}

int main()
{
    int x, y;
    x=INT_MAX; y=INT_MAX;
    printf(" Average -- for %d - and %d - is %d\n" , x, y,
        average(x, y));
    return 0;
}
```

Average for 2147483647 and
2147483647 is -1

Problem versus Solution



Execution produces a result

Average for 2147483647 and 2147483647 is -1

Execution produces a result

Average for 2147483647 and 2147483647 is -1

Using frama-c produces a required annotation

```
int average(int a, int b)
{
    int __retres;
    /*@ assert rte: signed_overflow: -2147483648 <= a + b; */
    /*@ assert rte: signed_overflow: a + b <= 2147483647; */
    __retres = (a + b) / 2;
    return __retres;
}
```

Listing 14 – Function average.....

```
#include <stdio.h>
#include <limits.h>
/*@ requires 0 <= a;
    requires a <= INT_MAX ;
    requires 0 <= b;
    requires b <= INT_MAX ;
    requires 0 <= a+b;
    requires a+b <= INT_MAX ;
    ensures \result <= INT_MAX;
*/
int average(int a,int b)
{
    return((a+b)/2);
}

int main()
{
    int x,y;
    x=INT_MAX / 2;y=INT_MAX / 2;
    // printf("Average for %d and %d is %d\n",x,y,
    //      );
    return average(x,y);
}
```

Nose Gear Velocity



- ▶ Estimated ground velocity of the aircraft should be available only if it is within 3 km/hr of the true velocity at some moment within past 3 seconds

▶ NG velocity system :

- **Hardware :**

- ▶ *Electro-mechanical sensor* : detects rotations
- ▶ *Two 16-bit counters* : Rotation counter, Milliseconds counter
- ▶ *Interrupt service routine* : updates rotation counter and stores current time.

- **Software :**

- ▶ *Real-time operating system* : invokes update function every 500 ms
- ▶ *16-bit global variable* : for recording rotation counter update time
- ▶ *An update function* : estimates ground velocity of the aircraft.

▶ Input data available to the system :

- *time* : in milliseconds
- *distance* : in inches
- *rotation angle* : in degrees

▶ Specified system performs velocity estimations in *imperial* unit system

▶ **Note** : expressed functional requirement is in *SI* unit system (km/hr).

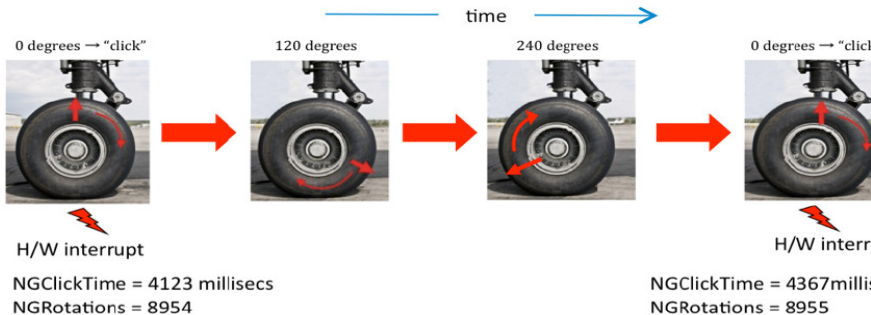
What are the main properties to consider for formalization ?

- ▶ Two different types of data :
 - counters with modulo semantics
 - non-negative values for time, distance, and velocity
- ▶ Two dimensions : *distance* and *time*
- ▶ Many units : distance (inches, kilometers, miles), time (milliseconds, hours), velocity (kph, mph)
- ▶ And interaction among components

How should we model ?

- ▶ Designer needs to consider units and conversions between them to manipulate the model
- ▶ One approach : Model units as *sets*, and conversions as constructed types – *projections*.
- ▶ Example :
 - 1 $estimateVelocity \in \text{MILES} \times \text{HOURS} \rightarrow \text{MPH}$
 - 2 $mphTokph \in \text{MPH} \rightarrow \text{KPH}$

Sample Velocity Estimation



WHEEL_DIAMETER = 22 inches
PI = 3.14

12 inches/foot
5280 feet/mile

$$\begin{aligned}\text{estimatedGroundVelocity} &= \text{distance travel/elapsed time} \\ &= ((3.14 * 22)/((12*5280)))/((4367-4123)/(1000*3600)) \\ &= 16 \text{ mph}\end{aligned}$$

Safety Property

- ▶ Storing the number of `NGClick` in a `n`-bit variable `VNGClick`
- ▶ Integers are denoted by the set *Int* and is simply defined by the interval $Int \hat{=} INT_MIN..INT_MAX$.
- ▶ Safety requirement :
The value of `VNGClick` is always in the range of implementation `Int` or equivalently $VNGClick \in Int$
- ▶ $Length = \pi \cdot diameter \cdot VNGClick$ (mathematical property)
- ▶ $Length \leq 6000$ (domain property)
- ▶ $\pi \cdot diameter \cdot VNGClick \leq 6000$
- ▶ $VNGClick \leq 6000 / (\pi \cdot diameter)$
- ▶ if $n=8$, then $2^7 - 1 = 127$ and
 $6000 / (\pi \cdot [22, inch]) = 6000 / (\pi \cdot 55, 88) = 6000 / (3, 24 \cdot [55, 88, cm]) = 6000 / (3, 24 \cdot 0.5588) \approx 3419$ and the condition of safety can not be satisfied in any situation.
- ▶ if $n=16$, then $2^{15} - 1 = 65535$ and $6000 / (\pi \cdot [22, inch]) \approx 3419$ and the condition of safety can be satisfied in any situation since
 $3419 \leq 65535$

Safety Property

- ▶ Storing the number of `NGClick` in a `n`-bit variable `VNGClick`
- ▶ Integers are denoted by the set *Int* and is simply defined by the interval $Int \hat{=} INT_MIN..INT_MAX$.
- ▶ Safety requirement :
*The value of `VNGClick` is always in the range of implementation *Int* or equivalently $VNGClick \in Int$*

$$RTE_VNGClick : 0 \leq vNGClick \leq INT_MAX \quad (1)$$

- ▶ The current value of `VNGClick` is always bounded by the two values 0 and `INT_MAX`.

- ▶ Validation : *Are we building the right product*
- ▶ Verification : *Are we building the process right?*

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- ▶ Proving automatically $\text{REACHABLE}(M) \subseteq A$: approximating semantics of programs

Decidability or Undecidability

- A problem $x \in P$ is generally stated by the function $\chi_{x \in P}$ where $\chi_{x \in P}(u) = 1$, if $P(u)$ is true and $\chi_{x \in P}(u) = 0$, if $P(u)$ is false :
- Problem 1 : $x \in 0..n$ where $n \in \mathbb{N}$
 - Problem 1 : $w \in \mathcal{L}(G)$ where G is a grammar over the finite set of alphabet symbols Σ and $\mathcal{L}(G) \subseteq \Sigma^*$.

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- ▶ A problem $x \in P$ is decidable, when the function $\chi_{x \in P}$ is computable or more precisely the function can be computed by a program
- ▶ Problem of the correctness of a program :
 - Assume that \mathcal{F} is the set of unary function over natural numbers : $\mathcal{F} = \mathbb{N} \rightarrow \mathbb{N}$.
 - $\mathcal{C} \subseteq \mathcal{F}$: the set of computable (or programmable) functions is \mathcal{C}
 - $f \in \mathcal{C} = \{\Phi_0, \Phi_1, \dots, \Phi_n, \dots\}$: the set of computable functions is denumerable.
 - The problem $x \in \text{dom}(\Phi_y)$ is not decidable and it expresses the correctness of programs.

Implicite versus explicite

- ▶ Ecrire $101 = 5$ peut avoir une signification

- ▶ Un programme P *produit* des résultats à partir de données en accord avec une sémantique :
 - STATES est l'ensemble de tous les états de P : $STATES = X \rightarrow \mathbb{Z}$ où X désigne les variables de P .
 - s_0 et s_f deux états de STATES : $\mathcal{D}(P)(s_0) = s_f$ signifie que P est exécuté à partir d'un état s_0 et produit un état s_f .
 - Pour un état s de P courant, on notera $s(X) = x$ pour distinguer la valeur de la variable X et sa valeur courante en s :

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 - Pour un état s de P courant, on notera $s(X) = x$ pour distinguer la valeur de la variable X et sa valeur courante en s :

$$s_0(X) = x_0, s_f(X) = x_f, s'(X) = x'$$

Un programme P *remplit* un contrat $(pre, post)$:

- ▶ P transforme une variable x à partir d'une valeur initiale x_0 et produisant une valeur finale x_f : $x_0 \xrightarrow{P} x_f$
- ▶ x_0 satisfait pre : $pre(x_0)$ and x_f satisfait $post$: $post(x_0, x_f)$
- ▶ $pre(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow post(x_0, x_f)$

requires $pre(x_0)$

ensures $post(x_0, x_f)$

variables X

begin

$0 : P_0(x_0, x)$

instruction₀

...

$i : P_i(x_0, x)$

...

instruction _{$f-1$}

$f : P_f(x_0, x)$

end

▶ $pre(x_0) \wedge x = x_0 \Rightarrow P_0(x_0, x)$

▶ $P_f(x_0, x) \Rightarrow post(x_0, x)$

▶ some conditions for verification
related to pairs $\ell \longrightarrow \ell'$



Asserted Program $\{P\} S \{Q\}$



► $\{P\} S \{Q\}$: *asserted program*



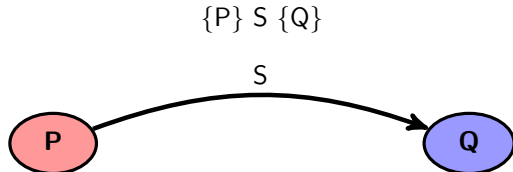
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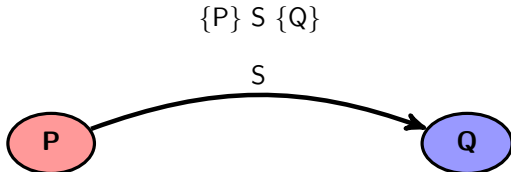
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Predicate Transformer

$WP(S)(Q)$ is the Weakest-Precondition of S for Q and is a predicate transformer but $WP(S)(.)$ is not a computable function over the set of predicates.

- ▶ Découpage de l'unité : 10 cours de 2 h

Esquisse des cours, TDs et TPs

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- ▶ Outils

