# Modelling Software-based Systems

## Lecture 2

## Proof Obligation Generation

Master Informatique

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# General Summary

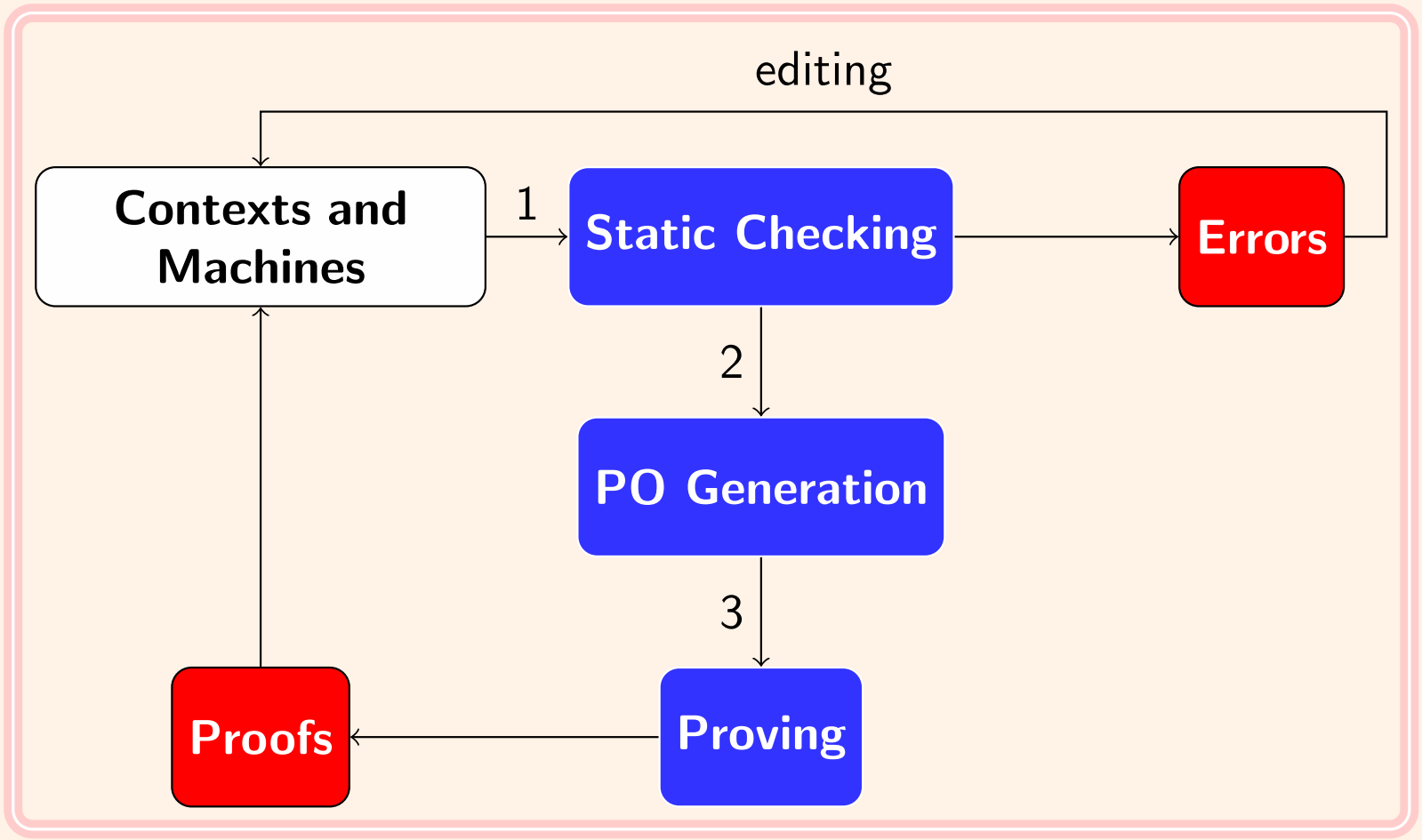
1. Overview of machines, contexts and proof obligations

2. Proof Obligations for Contexts and Machines
   - PO thm/THM (context)
   - PO thm/THM (machine)
   - PO evt/inv/INV
   - PO evt/act/FIS

3. Proof Obligations for Refinement
   - PO evt/grd/GRD
   - PO evt/act/SIM
   - PO evt/NAT
   - PO NAT
   - PO evt/VAR (arithmetic)
   - PO evt/VAR (set-theoretic)
   - PO evt/x/WFIS

# Current Summary

# Analysis of the Event-B Models



# Machines en Event B

MACHINE
  $m$
REFINES
  $am$
SEES
  $c$
VARIABLES
  $u$
INVARIANTS
  $I(s, c, u)$
THEOREMS
  $Q(s, c, u)$
VARIANT
  $exp(s, c, u)$
EVENTS
  $INITIALIZATION$
  $\ldots$
  $e$
  $\ldots$
END

# Machines en Event B

MACHINE
$m$
REFINES
$am$
SEES
$c$
VARIABLES
$u$
INVARIANTS
$I(s, c, u)$
THEOREMS
$Q(s, c, u)$
VARIANT
$exp(s, c, u)$
EVENTS
$INITIALIZATION$
$\ldots$
$e$
$\ldots$
END

- $\Gamma(m)$ : environment for the machine $m$ defined by the context $c$ and it provides a list of seen axioms $Ax(s, c)$ and a list of seen theorems $Th(s, c)$ for the sets s and constants c.
- $\Gamma(m) \vdash \forall u.\textsc{Init}(s, c, u) \Rightarrow \mathrm{I}(s, c, u)$
- For each event $e$ in $E$ :
  $\Gamma(m) \vdash \forall u, u'.\mathrm{I}(s, c, u) \wedge BA(e)(u, u') \Rightarrow \mathrm{I}(u')$
- For each event $e$ in $E$ :
  $\Gamma(m) \vdash \forall u.\mathrm{I}(s, c, u) \wedge GRD(e)(s, c, u) \Rightarrow \exists u'.BA(e)(u, u')$
- $\Gamma(m) \vdash \forall u.\mathrm{I}(s, c, u) \Rightarrow \mathrm{Q}(s, c, u)$
- Generated proof obligations are derived from those conditions.

# Current Summary

# PO thm/THM

CONTEXTS
$c$
EXTENDS
$ac$
SETS
$s$
CONSTANTS
$c$
AXIOMS
$Ax(s,c)$
THEOREMS
$th_1 : P_1(s,c)$
$\ldots$
$th_n : P_n(s,c)$
$th : P(s,c)$
$\ldots$
END

| | |
|---|---|
| $s$ | *seen sets* |
| $c$ | *seen constants* |
| $Ax(s,c)$ | *seen axioms* |
| $Th(s,c)$ | *previous proved theorems* |
| | $Th(s,c) = \{P_i(s,c) \mid i\ 1..n\}$ |
| $P(s,c)$ | *property over s and c* |

**PO** th/THM

$Ax(s,c), Th(s,c) \vdash P(s,c)$

# PO thm/THM (machine)

MACHINE
$m$
...
VARIABLES
$u$
INVARIANTS
$I(s, c, u)$
THEOREMS
$Q(s, c, u)$
$th : P(s, c, u)$
...
END

| | |
|---|---|
| $s$ | *seen sets* |
| $c$ | *seen constants* |
| $u$ | *variables* |
| $Ax(s, c)$ | *seen axioms* |
| $Th(s, c)$ | *seen theorems* |
| $I(s, c, u)$ | *invariants* |
| $Q(s, c, u)$ | *theorems* |
| $P(s, c, u)$ | *property over s,c and u* |

**PO** thm/THM

$Ax(s, c), Th(s, c), I(s, c, u) \vdash P(s, c, u)$

# PO evt/inv/INV

EVENT evt
ANY $x$ WHERE
$G(x, s, c, u)$
THEN
$u :| BAP(x, s, c, u, u')$
END

$BA(\text{evt}) \mathrel{\widehat{=}}$
$\exists x. \left( \begin{array}{l} \wedge\ G(x, s, c, u) \\ \wedge\ BAP(x, s, c, u, u') \end{array} \right)$
$GRD(\text{evt}) \mathrel{\widehat{=}} G(x, s, c, u)$
$ACT(\text{evt}) \mathrel{\widehat{=}} BAP(x, s, c, u, u')$

| | |
|---|---|
| $s$ | *seen sets* |
| $c$ | *seen constants* |
| $u$ | *variables* |
| $Ax(s, c)$ | *seen axioms* |
| $Th(s, c)$ | *seen theorems* |
| $I(s, c, u)$ | *invariants* |
| $Q(s, c, u)$ | *theorems* |
| evt | *event name* |
| x | *event parameter* |
| $G(x, s, c, u)$ | *event guard* |
| $BAP(x, s, c, u, u')$ | *event before-after predicate* |
| $inv : inv(s, c, u')$ | *specific modified invariant* |

**PO** evt/inv/INV

$$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), BAP(x, s, c, u, u') \vdash inv(s, c, u')$$

**PO** Q/THM $Ax(s, c), Th(s, c), I(s, c, u) \vdash Q(s, c, u)$

# PO evt/act/FIS

EVENT evt
  ANY $x$ WHERE
    $G(x, s, c, u)$
  THEN
    $u :\mid BAP(x, s, c, u, u')$
  END

$BA(\text{evt}) \mathrel{\widehat{=}}$
$\left( \begin{array}{l} \wedge\ G(x, s, c, u) \\ \wedge\ BAP(x, s, c, u, u') \end{array} \right)$
$GRD(\text{evt}) \mathrel{\widehat{=}} G(x, s, c, u)$
$ACT(\text{evt}) \mathrel{\widehat{=}}$
$BAP(x, s, c, u, u')$

| | |
|---|---|
| $s$ | *seen sets* |
| $c$ | *seen constants* |
| $u$ | *variables* |
| $Ax(s, c)$ | *seen axioms* |
| $Th(s, c)$ | *seen theorems* |
| $I(s, c, u)$ | *invariants* |
| $Q(s, c, u)$ | *theorems* |
| evt | *event name* |
| x | *event parameter* |
| $G(x, s, c, u)$ | *event guard* |
| $BAP(x, s, c, u, u')$ | *event before-after predicate* |

**PO** evt/act/FIS

$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), \vdash \exists u'.BAP(x, s, c, u, u')$

# Current Summary

# PO evt/grd/GRD

EVENT ae
ANY $x$ WHERE
$G(x, s, c, u)$
THEN
$u :| ABAP(x, s, c, u, u')$
END

EVENT ce
REFINES
$ae$
ANY $y$ WHERE
$H(y, s, c, v)$
WITH
$x : W(x, y, s, c, v)$
THEN
$v :| CBAP(y, s, c, v, v')$
END

| | |
|---|---|
| $s$ | *seen sets* |
| $c$ | *seen constants* |
| $u, v$ | *abstract and concrete variables* |
| $Ax(s, c)$ | *seen axioms* |
| $Th(s, c)$ | *seen theorems* |
| $I(s, c, u)$ | *abstract invariants* |
| $J(s, c, u, v)$ | *concrete invariants* |
| $Q(s, c, u)$, $R(s, c, u, v)$ | *abstract and concrete theorems* |
| ae, ce | *abstract and concrete event name* |
| x,y | *event parameters* |
| $G(x, s, c, u)$ | *abstract event guard* |
| $H(y, s, c, v)$ | *concrete event guard* |
| $ABAP(x, s, c, u, u')$ | *abstract event before-after predic* |
| $CBAP(x, s, c, u, u')$ | *concrete event before-after predic* |
| W(x,y,s,c,v) | witness predicate |

**PO** evt/grd/GRD

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), W(x, y, s, c, v), H(y, s, c, v), \vdash$
$G(x, s, c, u, u')$

# PO evt/act/SIM

```
EVENT ae
  ANY x WHERE
    G(x, s, c, u)
  THEN
    u :| ABAP(x, s, c, u, u')
  END
```

```
EVENT ce
  REFINES
    ae
  ANY y WHERE
    H(y, s, c, v)
  WITH
    x : WP(x, y, s, c, v)
    u' : WV(y, u', s, c, v)
  THEN
    v :| CBAP(y, s, c, v, v')
  END
```

| | |
|---|---|
| $s$ | *seen sets* |
| $c$ | *seen constants* |
| $u, v$ | *abstract and concrete variables* |
| $Ax(s, c)$ | *seen axioms* |
| $Th(s, c)$ | *seen theorems* |
| $I(s, c, u)$ | *abstract invariants* |
| $J(s, c, u, v)$ | *concrete invariants* |
| $Q(s, c, u)$, $R(s, c, u, v)$ | *abstract and concrete theorems* |
| ae, ce | *abstract and concrete event name* |
| x,y | *event parameters* |
| $G(x, s, c, u)$ | *abstract event guard* |
| $H(y, s, c, v)$ | *concrete event guard* |
| $ABAP(x, s, c, u, u')$ | *abstract event before-after predic* |
| $CBAP(x, s, c, u, u')$ | *concrete event before-after predic* |
| $WP(x, y, s, c, v)$ | witness parameter predicate |
| $WV(y, u', s, c, v)$ | witness variable predicate |

**PO** evt/act/SIM

$$\left(\begin{array}{l} Ax(s,c), Th(s,c), I(s,c,u), J(s,c,u,v) \\ WP(x,y,s,c,v), WV(y,u',s,c,v) \\ H(y,s,c,v), CBAP(y,s,c,v,v') \end{array}\right) \vdash ABAP(x,s,c,u,u')$$

# PO evt/act/SIM

EVENT ae
  ANY $x$ WHERE
    $G(x, s, c, u)$
  THEN
    $u :\mid BAP(x, s, c, u, u')$
  END
...
VARIANT
    $exp(s, c, u)$

| | |
|---|---|
| $s$ | *seen sets* |
| $c$ | *seen constants* |
| $u, v$ | *abstract and concrete variables* |
| $Ax(s, c)$ | *seen axioms* |
| $Th(s, c)$ | *seen theorems* |
| $I(s, c, u)$ | *abstract invariants* |
| $J(s, c, u, v)$ | *concrete invariants* |
| $Q(s, c, u)$, $R(s, c, u, v)$ | *abstract and concrete theorems* |
| evt, ce | *event name* |
| x | *event parameters* |
| $G(x, s, c, u)$ | *abstract event guard* |
| $BAP(x, s, c, u, u')$ | *event before-after predicate* |
| $exp(s, c, u)$ | *aritthmetic expression* |

**PO** evt/NAT

$$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u) \vdash exp(s, c, u) \in \mathbb{N}$$

# PO evt/act/SIM

EVENT ae
  ANY $x$ WHERE
    $G(x, s, c, u)$
  THEN
    $u :| BAP(x, s, c, u, u')$
  END
...
VARIANT
    $exp(s, c, u)$

| | |
|---|---|
| $s$ | *seen sets* |
| $c$ | *seen constants* |
| $u, v$ | *abstract and concrete variables* |
| $Ax(s, c)$ | *seen axioms* |
| $Th(s, c)$ | *seen theorems* |
| $I(s, c, u)$ | *abstract invariants* |
| $J(s, c, u, v)$ | *concrete invariants* |
| $Q(s, c, u)$, $R(s, c, u, v)$ | *abstract and concrete theorems* |
| evt, ce | *event name* |
| x | *event parameters* |
| $G(x, s, c, u)$ | *abstract event guard* |
| $BAP(x, s, c, u, u')$ | *event before-after predicate* |
| $setexp(s, c, u)$ | *set expression* |

**PO** evt/NAT $Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u) \vdash$
$finite(setexp(s, c, u))$

# PO evt/VAR

EVENT ae
  ANY $x$ WHERE
    $G(x, s, c, u)$
  THEN
    $u :\mid BAP(x, s, c, u, u')$
  END
...
VARIANT
    $exp(s, c, u)$

| | |
|---|---|
| $s$ | *seen sets* |
| $c$ | *seen constants* |
| $u, v$ | *abstract and concrete variables* |
| $Ax(s, c)$ | *seen axioms* |
| $Th(s, c)$ | *seen theorems* |
| $I(s, c, u)$ | *abstract invariants* |
| $J(s, c, u, v)$ | *concrete invariants* |
| $Q(s, c, u)$, $R(s, c, u, v)$ | *abstract and concrete theorems* |
| evt, ce | *event name* |
| x | *event parameters* |
| $G(x, s, c, u)$ | *abstract event guard* |
| $BAP(x, s, c, u, u')$ | *event before-after predicate* |
| $exp(s, c, u)$ | *aritthmetic expression* |

**PO** evt/VAR

$$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u), BAP(x, s, c, u, u') \vdash exp(s, c, u') < exp(s, c, u)$$

# PO evt/VAR

EVENT ae
  ANY $x$ WHERE
    $G(x, s, c, u)$
  THEN
    $u :\mid BAP(x, s, c, u, u')$
  END
...
VARIANT
    $setexp(s, c, u)$

| | |
|---|---|
| $s$ | *seen sets* |
| $c$ | *seen constants* |
| $u, v$ | *abstract and concrete variables* |
| $Ax(s, c)$ | *seen axioms* |
| $Th(s, c)$ | *seen theorems* |
| $I(s, c, u)$ | *abstract invariants* |
| $J(s, c, u, v)$ | *concrete invariants* |
| $Q(s, c, u)$, $R(s, c, u, v)$ | *abstract and concrete theorems* |
| evt, ce | *event name* |
| x | *event parameters* |
| $G(x, s, c, u)$ | *abstract event guard* |
| $BAP(x, s, c, u, u')$ | *event before-after predicate* |
| $setexp(s, c, u)$ | *set-theoretic expression* |

**PO** evt/VAR

$$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u), BAP(x, s, c, u, u') \vdash setexp(s, c, u') \subset setexp(s, c, u)$$

# PO evt/x/WFIS

EVENT ae
ANY $x$ WHERE
$G(x, s, c, u)$
THEN
$u :| ABAP(x, s, c, u, u')$
END

EVENT ce
REFINES
$ae$
ANY $y$ WHERE
$H(y, s, c, v)$
WITH
$x : WP(x, y, s, c, v)$
$u' : WV(y, u', s, c, v)$
THEN
$v :| CBAP(y, s, c, v, v')$
END

| | |
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| $ABAP(x, s, c, u, u')$ | *abstract event before-after predic* |
| $CBAP(x, s, c, u, u')$ | *concrete event before-after predic* |
| $WP(x, y, s, c, v)$ | witness parameter predicate |
| $WV(y, u', s, c, v)$ | witness variable predicate |

**PO** evt/x/WFIS

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), H(y, s, c, v) \vdash$
$\exists x. WP(x, y, s, c, v)$