

- 1 Programming by contract
- 2 Verification
- 3 Floyd to Hoare

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Verifying program correctness

A program P *satisfies* a $(\text{pre}, \text{post})$ contract :

- P transforms a variable v from initial values v_0 and produces a final value $v_f : v_0 \xrightarrow{P} v_f$
- v_0 satisfies $\text{pre} : \text{pre}(v_0)$ and v_f satisfies $\text{post} : \text{post}(v_0, v_f)$
- $\text{pre}(v_0) \wedge v_0 \xrightarrow{P} v_f \Rightarrow \text{post}(v_0, v_f)$
- \mathbb{D} est le domaine RTE de V

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- \mathbb{D} est le domaine RTE de V

```
requires  $\text{pre}(v_0)$ 
ensures  $\text{post}(v_0, v_f)$ 
variables  $X$ 
begin
  0 :  $P_0(v_0, v)$ 
  instruction0
  ...
  i :  $P_i(v_0, v)$ 
  ...
  instructionf-1
  f :  $P_f(v_0, v)$ 
end
```

- $\text{pre}(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$
- $\text{pre}(v_0) \wedge P_f(v_0, v) \Rightarrow \text{post}(v_0, v)$
- For any pair of labels ℓ, ℓ' such that $\ell \longrightarrow \ell'$, one verifies that, pour any values $v, v' \in \text{MEMORY}$
$$\left(\begin{array}{l} \text{pre}(v_0) \wedge P_\ell(v_0, v) \\ \wedge \text{cond}_{\ell, \ell'}(v) \wedge v' = f_{\ell, \ell'}(v) \end{array} \right) \Rightarrow P_{\ell'}(v_0, v')$$

Contracts - Verification Conditions

```
contract P
variables v
requires  $pre(v_0)$ 
ensures  $post(v_0, v_f)$ 
begin
  0 :  $P_0(v_0, v)$ 
  S0
  ...
  i :  $P_i(v_0, v)$ 
  ...
  Sf-1
  f :  $P_f(v_0, v)$ 
end
```

Verification conditions are listed as follows :

```
contract P
variables v
requires  $pre(v_0)$ 
ensures  $post(v_0, v_f)$ 
begin
  0 :  $P_0(v_0, v)$ 
  S0
  ...
  i :  $P_i(v_0, v)$ 
  ...
  Sf-1
  f :  $P_f(v_0, v)$ 
end
```

- (initialisation)
 $pre(v_0) \wedge v = v_0 \Rightarrow P_0(v_0, v)$
- (finalisation)
 $pre(v_0) \wedge P_f(v_0, v) \Rightarrow post(v_0, v)$
- (induction)
For each labels pair ℓ, ℓ'
such that $\ell \longrightarrow \ell'$, one checks that,
for any value $v, v' \in \text{MEMORY}$
$$\left(\begin{array}{l} pre(v_0) \wedge P_\ell(v_0, v) \\ \wedge cond_{\ell, \ell'}(v) \wedge v' = f_{\ell, \ell'}(v) \end{array} \right) \Rightarrow P_{\ell'}(v_0, v')$$

Three kinds of verification conditions should be checked and we justify the method in the full version..

From PAP to Rodin ...

From PAP to Rodin ...

```

MACHINE  $M$ 
SEES  $C0$ 
VARIABLES
   $v, pc$ 
INVARIANTS
  typing :  $v \in D$ 
  control :  $pc \in L$ 
  ...
   $at\ell : pc = \ell \Rightarrow P_\ell(v0, v)$ 
  ...
   $th1 : pre(v0) \wedge v = v0 \Rightarrow P_0(v0, v)$ 
   $th2 : pre(v0) \wedge P_f(v0, v)$ 
            $\Rightarrow post(v0, v)$ 
  ...
END
  ...
END

```

From PAP to Rodin ...

```
MACHINE  $M$ 
SEES  $C0$ 
VARIABLES
   $v, pc$ 
INVARIANTS
  typing :  $v \in D$ 
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  ...
   $th1 : pre(v0) \wedge v = v0 \Rightarrow P_0(v0, v)$ 
   $th2 : pre(v0) \wedge P_f(v0, v)$ 
     $\Rightarrow post(v0, v)$ 
  ...
END
...
END
```

```
MACHINE  $M$ 
EVENTS
INITIALISATION
BEGIN
   $(pc, v) : \mid \left( \begin{array}{l} pc' = l0 \wedge v' = v0 \\ \wedge pre(v0) \end{array} \right)$ 
END
...
 $e(\ell, \ell')$ 
  WHEN
     $pc = \ell$ 
     $cond_{\ell, \ell'}(v)$ 
  THEN
     $pc := \ell'$ 
     $v := f_{\ell, \ell'}(v)$ 
  END
...
END
```

(Induction Principle (I))

A property $S(z_0, z)$ is a safety for an annotated program P if, and only if, there exists a property $I(z_0, z)$ satisfying :

- 1 $\forall z_0, z \in L \times D. \text{init}(z_0) \wedge z = z_0 \Rightarrow I(z_0, z)$
- 2 $\forall z_0, z, z' \in L \times D. \text{init}(z_0) \wedge I(z_0, z) \wedge (z \xrightarrow{P} z') \Rightarrow I(z_0, z')$
- 3 $\forall z_0, z \in L \times D. \text{init}(z_0) \wedge I(z_0, z) \Rightarrow S(z_0, z)$

(Induction Principle (II))

A property $S(\ell_0, x_0, \ell, x)$ is a safety property for an annotated program P if, and only if, there exists a property $I(\ell_0, x_0, \ell, x)$ satisfying :

- 1 $\forall \ell_0, \ell \in L, x_0 \in D. \ell_0 \in L_0 \wedge \text{pre}(x_0) \wedge x = x_0 \wedge \text{pc} = \ell_0 \Rightarrow J(\ell_0, x_0, \ell, x)$
- 2 $\forall \ell, \ell' \in L, x, x_0 \in D. \ell_0 \in L_0 \wedge \text{pre}(x_0) \wedge J(\ell_0, x_0, \ell, x) \wedge BA(e(\ell, \ell'), \ell, x, \ell', x') \Rightarrow J(\ell_0, x_0, \ell', x')$
- 3 $\forall \ell_0, \ell \in L, x_0, x \in D. \text{pre}(x_0) \wedge \ell_0 \in L_0 \wedge J(\ell_0, x_0, \ell, x) \Rightarrow S(\ell_0, x_0, \ell, x)$

(Induction Principle (II))

A property $S(\ell_0, x_0, \ell, x)$ is a safety property for an annotated program P if, and only if, there exists a property $I(\ell_0, x_0, \ell, x)$ satisfying :

- ① $\forall \ell_0, \ell \in L, x_0 \in D. \ell_0 \in L_0 \wedge pre(x_0) \wedge x = x_0 \wedge pc = \ell_0 \Rightarrow J(\ell_0, x_0, \ell, x)$
- ② $\forall \ell, \ell' \in L, x, x_0 \in D. \ell_0 \in L_0 \wedge pre(x_0) \wedge J(\ell_0, x_0, \ell, x) \wedge BA(e(\ell, \ell'), \ell, x, \ell', x') \Rightarrow J(\ell_0, x_0, \ell', x')$
- ③ $\forall \ell_0, \ell \in L, x_0, x \in D. pre(x_0) \wedge \ell_0 \in L_0 \wedge J(\ell_0, x_0, \ell, x) \Rightarrow S(\ell_0, x_0, \ell, x)$

(Induction Principle (III))

A property $S(x_0, \ell, x)$ is a safety for an annotated program P with one entry point if, and only if, there exists a property $I(x_0, \ell, x)$ satisfying :

- ① $\forall x_0 \in D. pre(x_0) \wedge x = x_0 \wedge \ell = \ell_0 \Rightarrow J(x_0, \ell, x)$
- ② $\forall \ell, \ell' \in L, x, x_0 \in D. pre(x_0) \wedge J(x_0, \ell, x) \wedge BA(e(\ell, \ell'), \ell, x, \ell', x') \Rightarrow J(x_0, \ell', x')$
- ③ $\forall \ell \in L, x_0, x \in D. pre(x_0) \wedge J(x_0, \ell, x) \Rightarrow S(x_0, \ell, x)$

(Soundness of the method)

If the initialisation init , the generalisation gen and the step induction are proved to be correct by the Rodin platform, the property $S(x_0, \ell, x)$ is a correct safety property for the program P . In particular, one can handle the partial correctness and the run time error safety properties.

(Soundness of the method)

If the initialisation init , the generalisation gen and the step induction are proved to be correct by the Rodin platform, the property $S(x_0, \ell, x)$ is a correct safety property for the program P . In particular, one can handle the partial correctness and the run time error safety properties.

- Contract and verification conditions are translated into Event-B and are discharged by Rodin and its provers.
- Verification conditions are derived from Floyd's method.
- Annotation as assertion

A short example

S

contract SIMPLE

variables x

requires $x_0 \in \mathbb{N}$

ensures $x_f = 0$

begin

$\ell_0 : \{0 \leq x \leq x_0 \wedge x_0 \in \mathbb{N}\}$

while $0 < x$ do

$\ell_1 : \{0 < x \wedge x \leq x_0 \wedge x_0 \in \mathbb{N}\}$

$x := x - 1;$

od

$\ell_2 : \{x = 0\}$ end

Event *Init*

THEN

$act1 : x := x_0$

$act2 : l := l_0$

Event *el0l1*

WHEN

$grd1 : l = l_0$

$grd2 : 0 < x$

THEN

$act1 : l := l_1$

INVARIANTS

$inv1 : x \in \mathbb{N}$

$inv2 : l \in L$

$inv3 : l = l_0 \Rightarrow$

$0 \leq x \wedge x \leq x_0 \wedge x_0 \in \mathbb{N}$

$inv4 : l = l_1 \Rightarrow$

$0 < x \wedge x \leq x_0 \wedge x_0 \in \mathbb{N}$

$inv5 : l = l_2 \Rightarrow x = 0$

$requires : x_0 \in \mathbb{N} \wedge x = x_0$

$\Rightarrow x = x_0 \wedge x_0 \in \mathbb{N}$

$ensures : x = 0 \wedge x = x_0$

$\Rightarrow x = 0$

Event *el0l2*

WHEN

$grd1 : l = l_0$

$grd2 : \neg(0 < x)$

THEN

$act1 : l := l_2$

Event *el1l0*

WHEN

$grd1 : l = l_1$

THEN

$act1 : l := l_0$

$act2 : x := x - 1$

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Summary

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Annotation of programs

$$\begin{array}{l} \ell : \{P_\ell(v)\} \\ \text{cond}_{\ell, \ell'}(v) \longrightarrow v := f_{\ell, \ell'}(v) \\ \ell' : \{P_{\ell'}(v)\} \end{array}$$
$$\begin{array}{l} e(\ell, \ell') \\ \text{WHEN} \\ \quad c = \ell \\ \quad \text{cond}_{\ell, \ell'}(v) \\ \text{THEN} \\ \quad c := \ell' \\ \quad v := f_{\ell, \ell'}(v) \\ \text{END} \end{array}$$
$$\begin{array}{l} \ell_0^1 : \{x = 0\} \\ \quad x := x + 1; \\ \ell_0^1 : \{x = 1\} \end{array}$$

- v is the state memory variable or list of memory variables; v includes the local variables and the results variables.
- c is a new variable which is modelling the control flow and its type is **LOCATIONS**.
- $e(\ell, \ell')$ is simulating the computation flow starting from ℓ and moving to ℓ' ; v is updated.

From annotations to invariants

INVARIANTS

$$inv_i : c \in \text{LOCATIONS}$$
$$inv_j : v \in Type$$

...

$$inv_k : c = \ell \Rightarrow P_\ell(v)$$
$$inv_m : c = \ell' \Rightarrow P_{\ell'}(v)$$

• • •

$$th_n : A(c, v)$$

- *Type* is the type of the variables v and is a set of possible values defined in the context C .
- The annotation is giving us for free the conditions satisfied by v when the control is in ℓ , (resp. in ℓ').
- $A(c, v)$ is a safety property that we are supposed to check and the case of Event-B, it is a theorem.

Partial correctness using Event-B models

For each pair of successive labels ℓ, ℓ' , the three statements are equivalent :

- $P_\ell(v) \wedge \text{cond}_{\ell, \ell'}(v) \wedge v' = f_{\ell, \ell'}(v) \Rightarrow P_{\ell'}(v')$
- $I(c, v) \wedge c = \ell \wedge \text{cond}_{\ell, \ell'}(v) \wedge c' = \ell' \wedge v' = f_{\ell, \ell'}(v) \Rightarrow (c' = \ell' \Rightarrow P_{\ell'}(v'))$
- $I(c, v) \wedge BA(e(\ell, \ell'))(c, v, c', v') \Rightarrow (c' = \ell' \Rightarrow P_{\ell'}(v'))$

L

et AA an annotated algorithm with precondition **pre**(AA)(v) and postcondition **post**(AA)(v₀, v). Let the context *C* and the machine *M* generated from AA using the construction given previously. We assume that ℓ_0 is the first label and ℓ_e is the last label. We add the following safety properties in the machine *M* :

- $c = \ell_0 \wedge \text{pre}(\text{AA})(v) \Rightarrow P_{\ell_0}(v)$
- $c = \ell_e \Rightarrow (P_{\ell_e}(v) \Rightarrow \text{post}(\text{AA})(v_0, v))$

If proof obligations are discharged, then the annotated algorithm AA is partially correct with respect to ist pre/post specification.

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From Floyd to Hoare

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From Floyd to Hoare

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- $\forall x_0. \text{pre}(x_0) \Rightarrow \forall x_f. x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
- $\forall x_0. \text{pre}(x_0) \Rightarrow [P] \text{post}(x_0, x_f)$
- wlp calculus is introduced

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- $[x := e]P(x) = P[x \mapsto e]$

From Floyd to Hoare

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- $[x := e]P(x) = P[x \mapsto e]$
- $[\text{if } b(x) \text{ then } S1 \text{ else } S2]P(x) = b(x) \wedge [S1]P(x) \vee \text{not } b(x) \wedge [S2]P(x)$

From Floyd to Hoare

- $\forall x_f, x_0. \text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
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- Frama-c uses the HOARE logic for defining the verification conditions as R. Leino in DAFNY.

From Floyd to Hoare

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- Frama-c uses the HOARE logic for defining the verification conditions as R. Leino in DAFNY.
- Questions of termination require the wp calculus ...