



## 1 Overview of machines, contexts and proof obligations

## ② Proof Obligations for Contexts and Machines

PO thm/THM (context)

PO th/THM (machine)

PO evt/inv/INV

PO evt/act/FIS

### ③ Proof Obligations for Refinement

PO evt/grd/GRD

PO evt/act/SIM

PO evt/NAT

PO NAT

PO evt/VAR (arithmetic)

PO evt/VAR (set-theoretic)

PO evt/x/WFIS

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# Analysis of the Event-B Models



**MACHINE**

*m*

**REFINES**

*am*

**SEES**

*c*

**VARIABLES**

*u*

**INVARIANTS**

$I(s, c, u)$

**THEOREMS**

$Q(s, c, u)$

**VARIANT**

$exp(s, c, u)$

**EVENTS**

*INITIALIZATION*

...

*e*

...

**END**

**MACHINE**

*m*

**REFINES**

*am*

**SEES**

*c*

**VARIABLES**

*u*

**INVARIANTS**

$I(s, c, u)$

**THEOREMS**

$Q(s, c, u)$

**VARIANT**

$exp(s, c, u)$

**EVENTS**

*INITIALIZATION*

...

*e*

...

**END**

- $\Gamma(m)$  : environment for the machine  $m$  defined by the context  $c$  and it provides a list of seen axioms  $Ax(s, c)$  and a list of seen theorems  $Th(s, c)$  for the sets  $s$  and constants  $c$ .
- $\Gamma(m) \vdash \forall u. \text{INIT}(s, c, u) \Rightarrow I(s, c, u)$
- For each event  $e$  in  $E$  :  
 $\Gamma(m) \vdash \forall u, u'. I(s, c, u) \wedge BA(e)(u, u') \Rightarrow I(u')$
- For each event  $e$  in  $E$  :  
 $\Gamma(m) \vdash \forall u. I(s, c, u) \wedge GRD(e)(s, c, u) \Rightarrow \exists u'. BA(e)(u, u')$
- $\Gamma(m) \vdash \forall u. I(s, c, u) \Rightarrow Q(s, c, u)$
- Generated proof obligations are derived from those conditions.

## Checking the well formation of Event-B expressions

- Event-B expressions are contexts, machines, properties, equations, set-theoretical expressions ...
- $e$  is an Event-B expression and  $\text{wd}(e)$  is a logical property expressing the well definition of  $e$ .
- $\text{wd}(1 = 2) \triangleq \text{wd}(1) \wedge \text{wd}(2)$
- $\text{wd}(a/b) \triangleq b \neq 0 \wedge \text{wd}(a) \wedge \text{wd}(b)$
- $\text{wd}(f(g)) \triangleq g \in \text{dom}(f) \wedge f \in A \rightarrow B$

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## CONTEXTS

$c$

## EXTENDS

$ac$

## SETS

$s$

## CONSTANTS

$c$

## AXIOMS

$Ax(s, c)$

## THEOREMS

$th_1 : P_1(s, c)$

...

$th_n : P_n(s, c)$

$th : P(s, c)$

...

## END

$s$

*seen sets*

$c$

*seen constants*

$Ax(s, c)$

*seen axioms*

$Th(s, c)$

*previous proved theorems*

$PTh(s, c) = \{P_i(s, c) | i \ 1..n\}$

$P(s, c)$

*property over  $s$  and  $c$*

## PO th/THM

$Ax(s, c), Th(s, c) \vdash P(s, c)$

# PO th/THM (machine)

## MACHINE

$m$

...

## VARIABLES

$u$

## INVARIANTS

$I(s, c, u)$

## THEOREMS

$Q(s, c, u)$

$th : P(s, c, u)$

...

## END

$s$

*seen sets*

$c$

*seen constants*

$u$

*variables*

$Ax(s, c)$

*seen axioms*

$Th(s, c)$

*seen theorems*

$I(s, c, u)$

*invariants*

$Q(s, c, u)$

*theorems*

$P(s, c, u)$

*property over  $s, c$  and  $u$*

## PO th/THM

$Ax(s, c), Th(s, c), I(s, c, u) \vdash P(s, c, u)$

# PO evt/inv/INV

**EVENT** evt

**ANY**  $x$  **WHERE**

$G(x, s, c, u)$

**THEN**

$u : \mid BAP(x, s, c, u, u')$

**END**

$BA(\text{evt}) \hat{=}$

$\exists x. \left( \begin{array}{l} \wedge G(x, s, c, u) \\ \wedge BAP(x, s, c, u, u') \end{array} \right)$

$GRD(\text{evt}) \hat{=} G(x, s, c, u)$

$ACT(\text{evt}) \hat{=} BAP(x, s, c, u, u')$

$s$

$c$

$u$

$Ax(s, c)$

$Th(s, c)$

$I(s, c, u)$

$Q(s, c, u)$

evt

$x$

$G(x, s, c, u)$

$BAP(x, s, c, u, u')$

$inv : inv(s, c, u')$

*seen sets*

*seen constants*

*variables*

*seen axioms*

*seen theorems*

*invariants*

*theorems*

*event name*

*event parameter*

*event guard*

*event before-after predicate*

*specific modified invariant*

**PO** evt/inv/INV

$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), BAP(x, s, c, u, u') \vdash inv(s, c, u')$

**PO** Q/THM  $Ax(s, c), Th(s, c), I(s, c, u) \vdash Q(s, c, u)$

**EVENT** evt

**ANY**  $x$  **WHERE**

$G(x, s, c, u)$

**THEN**

$u : |BAP(x, s, c, u, u')$

**END**

$BA(\text{evt}) \hat{=}$

$\left( \begin{array}{l} \wedge G(x, s, c, u) \\ \wedge BAP(x, s, c, u, u') \end{array} \right)$

$GRD(\text{evt}) \hat{=} G(x, s, c, u)$

$ACT(\text{evt}) \hat{=}$

$BAP(x, s, c, u, u')$

$s$

$c$

$u$

$Ax(s, c)$

$Th(s, c)$

$I(s, c, u)$

$Q(s, c, u)$

evt

$x$

$G(x, s, c, u)$

$BAP(x, s, c, u, u')$

*seen sets*

*seen constants*

*variables*

*seen axioms*

*seen theorems*

*invariants*

*theorems*

*event name*

*event parameter*

*event guard*

*event before-after predicate*

**PO** evt/act/FIS

$Ax(s, c), Th(s, c), I(s, c, u), G(x, s, c, u), \vdash \exists u'. BAP(x, s, c, u, u')$

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# PO evt/grd/GRD

**EVENT** <sub>ae</sub>

**ANY**  $x$  **WHERE**

$G(x, s, c, u)$

**THEN**

$u : |ABAP(x, s, c, u, u')$

**END**

**EVENT** <sub>ce</sub>

**REFINES**

<sub>ae</sub>

**ANY**  $y$  **WHERE**

$H(y, s, c, v)$

**WITH**

$x : W(x, y, s, c, v)$

**THEN**

$v : |CBAP(y, s, c, v, v')$

**END**

$s$

$c$

$u, v$

$Ax(s, c)$

$Th(s, c)$

$I(s, c, u)$

$J(s, c, u, v)$

$Q(s, c, u), R(s, c, u, v)$

$ae, ce$

$x, y$

$G(x, s, c, u)$

$H(y, s, c, v)$

$ABAP(x, s, c, u, u')$

$CBAP(x, s, c, u, u')$

$W(x, y, s, c, v)$

*seen sets*

*seen constants*

*abstract and concrete variables*

*seen axioms*

*seen theorems*

*abstract invariants*

*concrete invariants*

*abstract and concrete theorems*

*abstract and concrete event name*

*event parameters*

*abstract event guard*

*concrete event guard*

*abstract event before-after predic*

*concrete event before-after predic*

*witness predicate*

**PO** evt/grd/GRD

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), W(x, y, s, c, v), H(y, s, c, v), \vdash$   
 $G(x, s, c, u, u')$

# PO evt/act/SIM

**EVENT**  $ae$

**ANY**  $x$  **WHERE**

$G(x, s, c, u)$

**THEN**

$u : |ABAP(x, s, c, u, u')$

**END**

**EVENT**  $ce$

**REFINES**

$ae$

**ANY**  $y$  **WHERE**

$H(y, s, c, v)$

**WITH**

$x : WP(x, y, s, c, v)$

$u' : WV(y, u', s, c, v)$

**THEN**

$v : |CBAP(y, s, c, v, v')$

**END**

$s$

$c$

$u, v$

$Ax(s, c)$

$Th(s, c)$

$I(s, c, u)$

$J(s, c, u, v)$

$Q(s, c, u), R(s, c, u, v)$

$ae, ce$

$x, y$

$G(x, s, c, u)$

$H(y, s, c, v)$

$ABAP(x, s, c, u, u')$

$CBAP(x, s, c, u, u')$

$WP(x, y, s, c, v)$

$WV(y, u', s, c, v)$

*seen sets*

*seen constants*

*abstract and concrete variables*

*seen axioms*

*seen theorems*

*abstract invariants*

*concrete invariants*

*abstract and concrete theorems*

*abstract and concrete event names*

*event parameters*

*abstract event guard*

*concrete event guard*

*abstract event before-after predicate*

*concrete event before-after predicate*

*witness parameter predicate*

*witness variable predicate*

**PO** evt/act/SIM

$$\left( \begin{array}{l} Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v) \\ WP(x, y, s, c, v), WV(y, u', s, c, v) \\ H(y, s, c, v), CBAP(y, s, c, v, v') \end{array} \right) \vdash ABAP(x, s, c, u, u')$$

```
EVENT ae
  ANY x WHERE
     $G(x, s, c, u)$ 
  THEN
     $u : |BAP(x, s, c, u, u')$ 
  END
...
VARIANT
   $exp(s, c, u)$ 
```

$s$   
 $c$   
 $u, v$   
 $Ax(s, c)$   
 $Th(s, c)$   
 $I(s, c, u)$   
 $J(s, c, u, v)$   
 $Q(s, c, u), R(s, c, u, v)$   
evt, ce  
 $x$   
 $G(x, s, c, u)$   
 $BAP(x, s, c, u, u')$   
 $exp(s, c, u)$

*seen sets*  
*seen constants*  
*abstract and concrete variables*  
*seen axioms*  
*seen theorems*  
*abstract invariants*  
*concrete invariants*  
*abstract and concrete theorems*  
*event name*  
*event parameters*  
*abstract event guard*  
*event before-after predicate*  
*arithmetic expression*

**PO** evt/NAT

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u) \vdash exp(s, c, u) \in \mathbb{N}$



```

EVENT ae
  ANY x WHERE
     $G(x, s, c, u)$ 
  THEN
     $u : |BAP(x, s, c, u, u')$ 
  END
  ...
VARIANT
     $exp(s, c, u)$ 

```

$s$   
 $c$   
 $u, v$   
 $Ax(s, c)$   
 $Th(s, c)$   
 $I(s, c, u)$   
 $J(s, c, u, v)$   
 $Q(s, c, u), R(s, c, u, v)$   
 evt, ce  
 $x$   
 $G(x, s, c, u)$   
 $BAP(x, s, c, u, u')$   
 $setexp(s, c, u)$

*seen sets*  
*seen constants*  
*abstract and concrete variables*  
*seen axioms*  
*seen theorems*  
*abstract invariants*  
*concrete invariants*  
*abstract and concrete theorems*  
*event name*  
*event parameters*  
*abstract event guard*  
*event before-after predicate*  
*set expression*

**PO** evt/NAT  $Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u) \vdash$   
 $finite(setexp(s, c, u))$

```

EVENT ae
  ANY x WHERE
     $G(x, s, c, u)$ 
  THEN
     $u : \mid BAP(x, s, c, u, u')$ 
  END
...
VARIANT
   $exp(s, c, u)$ 
    
```

$s$   
 $c$   
 $u, v$   
 $Ax(s, c)$   
 $Th(s, c)$   
 $I(s, c, u)$   
 $J(s, c, u, v)$   
 $Q(s, c, u), R(s, c, u, v)$   
 evt, ce  
 $x$   
 $G(x, s, c, u)$   
 $BAP(x, s, c, u, u')$   
 $exp(s, c, u)$

*seen sets*  
*seen constants*  
*abstract and concrete variables*  
*seen axioms*  
*seen theorems*  
*abstract invariants*  
*concrete invariants*  
*abstract and concrete theorems*  
*event name*  
*event parameters*  
*abstract event guard*  
*event before-after predicate*  
*ariththmic expression*

## PO evt/VAR

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u), BAP(x, s, c, u, u') \vdash$   
 $exp(s, c, u') < exp(s, c, u)$

```

EVENT ae
  ANY x WHERE
     $G(x, s, c, u)$ 
  THEN
     $u : \mid BAP(x, s, c, u, u')$ 
  END
...
VARIANT
   $setexp(s, c, u)$ 

```

$s$   
 $c$   
 $u, v$   
 $Ax(s, c)$   
 $Th(s, c)$   
 $I(s, c, u)$   
 $J(s, c, u, v)$   
 $Q(s, c, u), R(s, c, u, v)$   
 evt, ce  
 $x$   
 $G(x, s, c, u)$   
 $BAP(x, s, c, u, u')$   
 $setexp(s, c, u)$

*seen sets*  
*seen constants*  
*abstract and concrete variables*  
*seen axioms*  
*seen theorems*  
*abstract invariants*  
*concrete invariants*  
*abstract and concrete theorems*  
*event name*  
*event parameters*  
*abstract event guard*  
*event before-after predicate*  
*set-theoretic expression*

## PO evt/VAR

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), G(x, s, c, u), BAP(x, s, c, u, u') \vdash$   
 $setexp(s, c, u') \subset setexp(s, c, u)$

**EVENT**  $ae$

**ANY**  $x$  **WHERE**

$G(x, s, c, u)$

**THEN**

$u : |ABAP(x, s, c, u, u')$

**END**

**EVENT**  $ce$

**REFINES**

$ae$

**ANY**  $y$  **WHERE**

$H(y, s, c, v)$

**WITH**

$x : WP(x, y, s, c, v)$

$u' : WV(y, u', s, c, v)$

**THEN**

$v : |CBAP(y, s, c, v, v')$

**END**

$s$

$c$

$u, v$

$Ax(s, c)$

$Th(s, c)$

$I(s, c, u)$

$J(s, c, u, v)$

$Q(s, c, u), R(s, c, u, v)$

$ae, ce$

$x, y$

$G(x, s, c, u)$

$H(y, s, c, v)$

$ABAP(x, s, c, u, u')$

$CBAP(x, s, c, u, u')$

$WP(x, y, s, c, v)$

$WV(y, u', s, c, v)$

*seen sets*

*seen constants*

*abstract and concrete variables*

*seen axioms*

*seen theorems*

*abstract invariants*

*concrete invariants*

*abstract and concrete theorems*

*abstract and concrete event names*

*event parameters*

*abstract event guard*

*concrete event guard*

*abstract event before-after predicate*

*concrete event before-after predicate*

*witness parameter predicate*

*witness variable predicate*

**PO** evt/x/WFIS

$Ax(s, c), Th(s, c), I(s, c, u), J(s, c, u, v), H(y, s, c, v) \vdash$

$\exists x. WP(x, y, s, c, v)$