

# Cours MALG & MOVEX

## MALG

### Vérification mécanisée de contrats (III) (The ANSI/ISO C Specification Language (ACSL))

Dominique Méry

Telecom Nancy, Université de Lorraine

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## ① Contracts

- Extending C programming language by contracts

- Playing with variables

- Ghost Variables

## ② Generation of Verification Conditions

- WP calculus in Frama-c

- First annotation

- Second annotation

## ③ Memory Models in Frama-c

## ④ Logic Specification

## ⑤ Organisation of the verification process

## ⑥ Conclusion

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## 6 Conclusion

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## 6 Conclusion

## Summary on annotations and assertions

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- ▶ requires
- ▶ assigns
- ▶ ensures
- ▶ decreases
- ▶ predicate
- ▶ logic
- ▶ lemma

- ▶ The calling function should guarantee the required condition or precondition introduced by the clauses requires  $P1 \wedge \dots \wedge Pn$  at the calling point.
- ▶ The called function returns results that are ensured by the clause ensures  $E1 \wedge \dots \wedge Em$ ; ensures clause expresses a relationship between the initial values of variables and the final values.
- ▶ initial values of a variable  $v$  is denoted  $\backslash old(v)$
- ▶ The variables which are not in the set  $L1 \cup \dots \cup Lp$  are not modified.

### Listing 1 – contrat

```
/*@ requires P1;...;requires Pn;  
  @ assigns L1;...;assigns Lm;  
  @ ensures E1;...;ensures Ep;  
  @*/
```

(Division)

### Listing 2 – project-divers/annotation.c

```
/*@ requires x <= 0 && x >= 10;  
  @ assigns \nothing;  
  @ ensures x % 2 == 0 ==> 2*\result == x;  
  @ ensures x % 2 != 0 ==> 2*\result == x-1;  
  @*/  
int annotation(int x)  
{  
    int y;  
    y = x / 2;  
    return(y);  
}
```

(Division)

## Listing 3 – project-divers/annotationwp.c

```
/*@ requires 0 <= x && x <= 10;
   @ assigns \nothing;
   @ ensures x % 2 == 0 ==> 2*\result == x;
   @ ensures x % 2 != 0 ==> 2*\result == x-1;
   @*/
int annotation(int x)
{
  /*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
  /*@ assert x % 2 != 0 ==> 2* (x / 2) == x-1; */
  int y;
  /*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
  /*@ assert x % 2 != 0 ==> 2* (x / 2) == x-1; */
  y = x / 2;
  /*@ assert x % 2 == 0 ==> 2*y == x; */
  /*@ assert x % 2 != 0 ==> 2*y == x-1; */
  return(y);
  /*@ assert x % 2 == 0 ==> 2*y == x; */
  /*@ assert x % 2 != 0 ==> 2*y == x-1; */
}
```



*Property to check*

$$x \geq 0 \wedge x < 0; \Rightarrow \left( \begin{array}{l} x \% 2 = 0 \Rightarrow 2 \cdot (x/2) = x \\ x \% 2 \neq 0 \Rightarrow 2 \cdot (x/2) = x-1 \end{array} \right)$$

(Precondition)

### Listing 4 – project-divers/annotation0.c

```
/*@ requires x >= 0 && x < 0;  
   @ assigns \nothing;  
   @ ensures \result == 0;  
   @*/  
int annotation0(int x)  
{  
    int y;  
    y = y / (x-x);  
    return(y);  
}
```

(Precondition)

### Listing 5 – project-divers/annotation0wp.c

```
/*@ requires x >= 0 && x < 0;  
  @ assigns \nothing;  
  @ ensures \result == 0;  
  @*/  
int annotation(int x)  
{  
  /*@ assert y / (x-x) == 0; */  
  int y;  
  /*@ assert y / (x-x) == 0; */  
  y = y / (x-x);  
  /*@ assert y == 0; */  
  return(y);  
  /*@ assert y == 0; */  
}
```

*Property to check*

$$0 \leq x \wedge x \leq 10 \Rightarrow y/(x-x) = 0$$

## Definition of a contract (specification)

- ▶ Define the mathematical function to compute (what to compute?)
- ▶ Define an inductive method for computing the mathematical function and using axioms.

(factorial what)

### Listing 6 – project-factorial/factorial.h

```
#ifndef _A.H
#define _A.H
/*@ axiomatic mathfact {
  @ logic integer mathfact(integer n);
  @ axiom mathfact_1: mathfact(1) == 1;
  @ axiom mathfact_rec: \forall integer n; n > 1
    ==> mathfact(n) == n * mathfact(n-1);
  @ } */

/*@ requires n > 0;
    decreases n;
    ensures \result == mathfact(n);
    assigns \nothing;
*/
int codefact(int n);
#endif
```

## Definition of a contract (programming)

- ▶ Define the program codefact for computing mathfact (How to compute?)
- ▶ Define the algorithm computing the function mathfact

(factorial how )

### Listing 7 – project-factorial/factorial.c

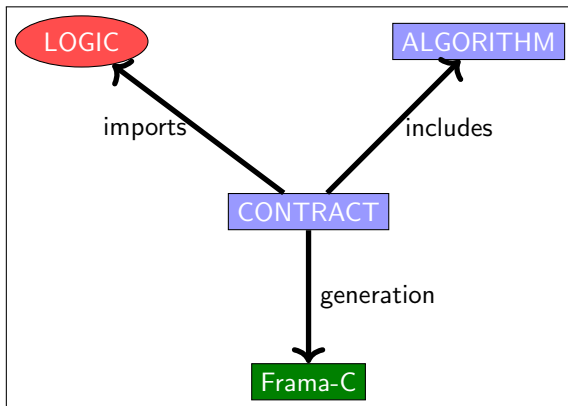
```
#include "factorial.h"

int codefact(int n) {
    int y = 1;
    int x = n;
    /*@ loop invariant x >= 1 && x <= n && mathfact(n) == y * mathfact(x);
       loop assigns x, y;
       loop variant x;
    */
    while (x != 1) {
        y = y * x;
        x = x - 1;
    };
    return y;
}
```

## Definition of a contract (approach)

---

- ▶ The specification of a function ( $\text{mathfact}$ ) to compute requires to define it mathematically.
- ▶ The definition is stated in an axiomatic framework and is preferably inductive ( $\text{mathfact}$ ) which is used in assertions or theorems or lemmas.
- ▶ The relationship between the computed value ( $\backslash\text{result}$ ) and the mathematical value ( $\text{mathfact}(n)$ ) is stated in the ensures clause :  
$$\backslash\text{result} == \text{mathfact}(n)$$
- ▶ The main property to prove is  $\text{codefact}(n) == \text{mathfact}(n)$  : Calling  $\text{codefact}$  for  $n$  returns a value equal to  $\text{mathfact}(n)$ .





### Listing 8 – contrat

```
/*@ requires P;  
@ behavior b1:  
  @ assumes A1;  
  @ requires R1 ;  
  @ assigns L1;  
  @ ensures E1;  
@ behavior b2:  
  @ assumes A2;  
  @ requires R2;  
  @ assigns L2;  
  @ ensures E2;  
@*/
```

(Pairs of integers)

## Listing 9 – project-divers/structures.h

```
#ifndef _STRUCTURE_H

struct s {
    int q;
    int r;
};

#endif
```

# Division should not return silly expressions!

(Specification)

## Listing 10 – project-divers/division.h

```
#ifndef _A_H
#define _A_H
#include "structures.h"
/*@ requires a >= 0 && b >= 0;
@ behavior b :
  @ assumes b == 0;
  @ assigns \nothing;
  @ ensures \result.q == -1 && \result.r == -1 ;
@ behavior B2:
  @ assumes b != 0;
  @ assigns \nothing;
  @ ensures 0 <= \result.r;
  @ ensures \result.r < b;
  @ ensures a == b * \result.q + \result.r;
*/
struct s division(int a, int b);
#endif
```

# Division should not return silly expressions!

(Algorithm)

## Listing 11 – project-divers/division.c

```
#include <stdio.h>
#include <stdlib.h>

#include "division.h"

struct s division(int a, int b)
{
    int rr = a;
    int qq = 0;
    struct s silly = {-1,-1};
    struct s resu;
    if (b == 0) {
        return silly;
    }
    else
    {
        /*@
        loop invariant
        ( a == b*qq + rr) &&
        rr >= 0;
        loop assigns rr,qq;
        loop variant rr;
        */
        while (rr >= b) { rr = rr - b; qq=qq+1;};
        resu.q = qq;
        resu.r = rr;
        return resu;
    }
}
```

### Iteration Rule for PC

If  $\{P \wedge B\} \mathbf{S} \{P\}$ , then  $\{P\} \mathbf{while\ B\ do\ S\ od} \{P \wedge \neg B\}$ .

- ▶ Prove  $\{P \wedge B\} \mathbf{S} \{P\}$  or  $P \wedge B \Rightarrow \{\mathbf{S}\}(P)$ .
- ▶ By the iteration rule, we conclude that  $\{P\} \mathbf{while\ B\ do\ S\ od} \{P \wedge \neg B\}$  without using WLP.
- ▶ Introduction of LOOP INVARIANTS in the notation.

Listing 12 – loop.c

```
/*@ loop invariant I1;  
    loop invariant I2;  
    ...  
    loop invariant In;  
    loop assigns X;  
    loop variant E;  
*/
```

(Invariant de boucle)

## Listing 13 – project-divers/anno6.c

```
/*@ requires a >= 0 && b >= 0;  
    ensures 0 <= \result;  
    ensures \result < b;  
    ensures \exists integer k; a == k * b + \result;  
*/  
int rem(int a, int b) {  
    int r = a;  
    /*@  
        loop invariant  
        (\exists integer i; a == i * b + r) &&  
        r >= 0;  
        loop assigns r;  
    */  
    while (r >= b) { r = r - b;};  
    return r;  
}
```

- ▶  $\backslash old(x)$  is the value of the variable when the function is called.
- ▶ It can be used in the postcondition of the *ensures* clause.

(Modifying variables while calling)

### Listing 14 – project-divers/old1.c

```
/*@ requires \valid(a) && \valid(b);
   @ assigns *a,*b;
   @ ensures  *a == \at(*a,Pre) +2;
   @ ensures  *b == \at(*b,Pre)+\at(*a,Pre)+2;

       @ ensures \result == 0;
*/
int old(int *a, int *b) {
    int x,y;
    x = *a;
    y = *b;
    x=x+2;
    y = y +x;

    *a = x;
    *b = y;
    return 0 ;
}
```

- ▶  $\backslash at(e, id)$  is the value of  $e$  at the control point  $id$ .
- ▶  $id$  should occur before  $\backslash at(e, id)$
- ▶  $id$  is one of the possible expressions : Pre, Here, Old, Post, LoopEntry, LoopCurrent, Init
- ▶  $\backslash old(e)$  is equivalent to  $\backslash at(e, Old)$



(label Pre)

### Listing 15 – project-divers/at1.c

```
/*@
  requires \valid(a) && \valid(b);
  assigns *a,*b;
  ensures  *a == \old(*a)+2;
  ensures  *b == \old(*b)+\old(*a)+2;
*/
int at1(int *a, int *b) {
  //@ assert *a == \at(*a, Pre);
  *a = *a + 1;
  //@ assert *a == \at(*a, Pre)+1;
  *a = *a + 1;
  //@ assert *a == \at(*a, Pre)+2;
  *b = *b + *a;
  //@ assert *a == \at(*a, Pre)+2 && *b == \at(*b, Pre)+\at(*a, Pre)+2;
  return 0;
}
```



(autre label)

### Listing 16 – project-divers/at2.c

```
void f (int n) {  
  for (int i = 0; i < n; i++) {  
    /*@ assert \at(i, LoopEntry) == 0; */  
    int j=0;  
    while (j++ < i) {  
      /*@ assert \at(j, LoopEntry) == 0; */  
      /*@ assert \at(j, LoopCurrent) + 1 == j; */  
    } }  
}
```

(otherlabel)

### Listing 17 – project-divers/change1.c

```
/*@ requires \valid(a) && *a >= 0;  
   @ assigns *a;  
   @ ensures  *a == \old(*a)+2 && \result == 0;  
*/  
int change1(int *a)  
{   int x = *a;  
    x = x + 2;  
    *a = x;  
    return 0;  
}
```

- ▶ A variable called *ghost* allows to model a computed value useful for stating a model property : the ghost variable is hidden for the computer but not for the model.
- ▶ It should not change the semantics of others variables and should not change the effective variables.

(Bug)

### Listing 18 – project-divers/ghost2.c

```
int f (int x, int y) {  
    //@ghost int z=x+y;  
    switch (x) {  
    case 0: return y;  
    //@ ghost case 1: z=y;  
    // above statement is correct.  
    //@ ghost case 2: { z++; break; }  
    // invalid, would bypass the non-ghost default  
    default: y++; }  
    return y; }  
  
int g(int x) { //@ ghost int z=x;  
    if (x>0){return x;}  
    //@ ghost else { z++; return x; }  
    // invalid, would bypass the non-ghost return  
    return x+1; }
```

(Ghost variable)

## Listing 19 – project-divers/ghost1.c

```
/*@ requires a >= 0 && b >= 0;
   ensures 0 <= \result;
   ensures \result < b;
   ensures \exists integer k; a == k * b + \result; */
int rem(int a, int b) {
    int r = a;
    /*@ ghost    int q=0;    */
    /*@
       loop invariant
       a == q * b + r &&
       r >= 0 && r <= a;
       loop assigns r;
       loop assigns q;
    // loop variant r;
    */
    while (r >= b) {
        r = r - b;
    /*@ ghost    q = q+1;    */
    };
    return r;
}
```

### Listing 20 – an1.c

```
//@ assert l1:  $P(x)$ ;  
  x = e(x);  
//@ assert l2:  $Q(x)$ ;
```



### Listing 21 – an1.c

```
//@ assert l1: P(x);  
  x = e(x);  
//@ assert l2: Q(x);
```

►  $P(x) \Rightarrow WP(x := e(x))(Q(x))$

### Listing 22 – an1.c

```
//@ assert l1: P(x);  
  x = e(x);  
//@ assert l2: Q(x);
```

- ▶  $P(x) \Rightarrow WP(x := e(x))(Q(x))$
- ▶  $P(x) \Rightarrow Q[x \mapsto e(x)]$

### Listing 23 – an1.c

```
//@ assert l1: P(x);  
  x = e(x);  
//@ assert l2: Q(x);
```

- ▶  $P(x) \Rightarrow WP(x := e(x))(Q(x))$
- ▶  $P(x) \Rightarrow Q[x \mapsto e(x)]$
- ▶  $P(x1) \Rightarrow Q[x \mapsto e(x1)]$  (renaming of free occurrences of  $x$  by  $x1$ )

### Listing 24 – an1.c

```
//@ assert l1: P(x);  
  x = e(x);  
//@ assert l2: Q(x);
```

- ▶  $P(x) \Rightarrow WP(x := e(x))(Q(x))$
- ▶  $P(x) \Rightarrow Q[x \mapsto e(x)]$
- ▶  $P(x1) \Rightarrow Q[x \mapsto e(x1)]$  (renaming of free occurrences of  $x$  by  $x1$ )
- ▶  $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$

### Listing 25 – an1.c

```
//@ assert l1: P(x);  
  x = e(x);  
//@ assert l2: Q(x);
```

- ▶  $P(x) \Rightarrow WP(x := e(x))(Q(x))$
- ▶  $P(x) \Rightarrow Q[x \mapsto e(x)]$
- ▶  $P(x1) \Rightarrow Q[x \mapsto e(x1)]$  (renaming of free occurrences of  $x$  by  $x1$ )
- ▶  $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- ▶  $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$

### Listing 26 – an1.c

```
//@ assert l1: P(x);  
  x = e(x);  
//@ assert l2: Q(x);
```

- ▶  $P(x) \Rightarrow WP(x := e(x))(Q(x))$
- ▶  $P(x) \Rightarrow Q[x \mapsto e(x)]$
- ▶  $P(x1) \Rightarrow Q[x \mapsto e(x1)]$  (renaming of free occurrences of  $x$  by  $x1$ )
- ▶  $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- ▶  $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- ▶  $\vdash P(x1) \wedge x = e(x1) \Rightarrow Q(x)$

### Listing 27 – an1.c

```
//@ assert l1: P(x);  
  x = e(x);  
//@ assert l2: Q(x);
```

- ▶  $P(x) \Rightarrow WP(x := e(x))(Q(x))$
- ▶  $P(x) \Rightarrow Q[x \mapsto e(x)]$
- ▶  $P(x1) \Rightarrow Q[x \mapsto e(x1)]$  (renaming of free occurrences of  $x$  by  $x1$ )
- ▶  $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- ▶  $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- ▶  $\vdash P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- ▶  $P(x1) \wedge x = e(x1) \vdash Q(x)$

### Listing 28 – an1.c

```
//@ assert l1: P(x);  
  x = e(x);  
//@ assert l2: Q(x);
```

- ▶  $P(x) \Rightarrow WP(x := e(x))(Q(x))$
- ▶  $P(x) \Rightarrow Q[x \mapsto e(x)]$
- ▶  $P(x1) \Rightarrow Q[x \mapsto e(x1)]$  (renaming of free occurrences of  $x$  by  $x1$ )
- ▶  $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- ▶  $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- ▶  $\vdash P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- ▶  $P(x1) \wedge x = e(x1) \vdash Q(x)$ 
  - Assume {
  - $P(x1)$
  - ▶  $x = e(x1)$
  - }
  - Prove:  $Q(x)$



Listing 29 – an1.c

```
void ex(void) {  
    int x=12,y=24;  
    //@ assert l1:  $2*x == y$ ;  
    x = x+1;  
    //@ assert l2:  $y == 2*(x-1)$ ;  
}
```

```
[kernel] Parsing an1.c (with preprocessing)
[wp] Running WP plugin...
[wp] Warning: Missing RTE guards
[wp] 2 goals scheduled
[wp] Proved goals:      4 / 4
    Terminating:      1
    Unreachable:        1
    Qed:                 2
```

Goal Assertion 'l1' (file an1.c, line 3):

```
Assume {  
  Type: is_sint32(x) /\ is_sint32(y).  
  (* Initializer *)  
  Init: x = 12.  
  (* Initializer *)  
  Init: y = 24.  
}  
Prove: (2 * x) = y.  
Prover Qed returns Valid
```

Goal Assertion 'l2' (file an1.c, line 5):

```
Assume {  
  Type: is_sint32(x) /\ is_sint32(x_1) /\ is_sint32(y).  
  (* Initializer *)  
  Init: x_1 = 12.  
  (* Initializer *)  
  Init: y = 24.  
  (* Assertion 'l1' *)  
  Have: (2 * x_1) = y.  
  Have: (1 + x_1) = x.  
}  
Prove: (2 + y) = (2 * x).  
Prover Qed returns Valid
```

```
[kernel] Parsing an1.c (with preprocessing)
[wp] Running WP plugin...
[wp] Warning: Missing RTE guards
[wp] 2 goals scheduled
[wp] Proved goals:      4 / 4
    Terminating:      1
    Unreachable:       1
    Qed:                2
```

Listing 30 – an2.c

```
void ex(void) {  
    int x=12,y=24;  
    //@ assert l1:  $2*x == y$ ;  
    x = x+1;  
    //@ assert l2:  $y == 2*(x-1)$ ;  
    x = x+2;  
    //@ assert l3:  $y+6 == 2*x$ ;  
}
```

```
[kernel] Parsing an2.c (with preprocessing)
[wp] Running WP plugin...
[wp] Warning: Missing RTE guards
[wp] 3 goals scheduled
[wp] Proved goals:      5 / 5
    Terminating:      1
    Unreachable:       1
    Qed:                3
```

Goal Assertion 'l1' (file an2.c, line 3):

```
Assume {  
  Type: is_sint32(x) /\ is_sint32(y).  
  (* Initializer *)  
  Init: x = 12.  
  (* Initializer *)  
  Init: y = 24.  
}  
Prove: (2 * x) = y.  
Prover Qed returns Valid
```



Goal Assertion 'l2' (file an2.c, line 5):

```
Assume {  
  Type: is_sint32(x) /\ is_sint32(x_1) /\ is_sint32(y).  
  (* Initializer *)  
  Init: x_1 = 12.  
  (* Initializer *)  
  Init: y = 24.  
  (* Assertion 'l1' *)  
  Have: (2 * x_1) = y.  
  Have: (1 + x_1) = x.  
}  
Prove: (2 + y) = (2 * x).  
Prover Qed returns Valid
```

Goal Assertion 'l3' (file an2.c, line 7):

Assume {

Type: is\_sint32(x) /\ is\_sint32(x\_1) /\ is\_sint32(x\_2) /\ is\_s  
(\* Initializer \*)

Init: x\_2 = 12.

(\* Initializer \*)

Init: y = 24.

(\* Assertion 'l1' \*)

Have: (2 \* x\_2) = y.

Have: (1 + x\_2) = x\_1.

(\* Assertion 'l2' \*)

Have: (2 + y) = (2 \* x\_1).

Have: (2 + x\_1) = x.

}

Prove: (6 + y) = (2 \* x).

Prover Qed returns Valid

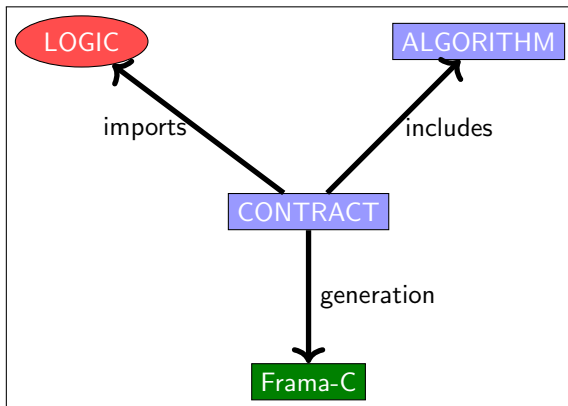
## Listing 31 – an2bis.c

```
void ex(void) {  
    int x=12,y=24;  
    //@ assert l1:  $2*x == y$ ;  
    x = x+1;  
    //@ assert l2:  $y == 2*(x-1)$ ;  
    x = x+2;  
    //@ assert l3:  $y+6 == 2*x$ ;  
}
```

- ▶ Hoare Model : `-wp hoare` is the option of `frama-c`
- ▶ Typed Model : default model is typed model

- ▶ It simply maps each C variable to one pure logical variable.
- ▶ Heap cannot be represented in this model, and expressions such as `*p` cannot be translated at all.
- ▶ You can still represent pointer values, but you cannot read or write the heap through pointers.

- ▶ The default model for WP plug-in.
- ▶ Heap values are stored in several separated global arrays, one for each atomic type (integers, floats, pointers) and an additional one for memory allocation.
- ▶ Pointer values are translated into an index into these arrays.
- ▶ all C integer types are represented by mathematical integers and each pointer type to a given type is represented by a specific logical abstract datatype.



► predicate



(Predicate)

## Listing 32 – project-divers/predicate1.c

```
/*@ predicate is_positive(integer x) = x > 0; */  
  
/*@ logic integer get_sign(real x) = @ x > 0.0 ? 1 : (x < 0.0 ? -1 : 0);  
*/  
/*@ logic integer max(int x, int y) = x > y ? x : y;  
*/
```

(Lemma)

## Listing 33 – project-divers/lemma1.c

```
/*@ lemma div_mul_identity:  
@ \forall real x, real y; y != 0.0  $\Rightarrow$  y*(x/y) = x; @*/  
  
/*@ lemma div_qr:  
@ \forall int a, int b; a >= 0 && b > 0  $\Rightarrow$   
\exists int q, int r; a = b*q + r && 0 <= r && r < b; @*/
```

(Definition of fibonacci function)

### Listing 34 – project-divers/predicate2.c

```
/*@ axiomatic mathfibonacci{  
  @ logic integer mathfib(integer n);  
  @ axiom mathfib0: mathfib(0) == 1;  
  @ axiom mathfib1: mathfib(1) == 1;  
  @ axiom mathfibrec: \forall integer n; n > 1  
    ==> mathfib(n) == mathfib(n-1)+mathfib(n-2);  
  @ } */
```

(Definition of gcd)

### Listing 35 – project-divers/predicate3.c

```
/*@ inductive is_gcd(integer a, integer b, integer d) {  
  @ case gcd.zero:  
  @ \forall integer n; is_gcd(n,0,n);  
  @ case gcd.succ:  
  @ \forall integer a,b,d; is_gcd(b, a % b, d) ==> is_gcd(a,b,d); @}  
/*@
```

(Definition of function odd/even)

### Listing 36 – project-divers/predicate4.c

```
//@ predicate pair(integer x) = (x/2)*2==x;
//@ predicate impair(integer x) = (x/2)*2!=x;
//@ lemma ex: \forall integer a,b; a < b ==> 2*a < 2*b;

/*@ inductive is_gcd(integer a, integer b, integer c) {
  case zero: \forall integer n; is_gcd(n,0,n);
  case un: \forall integer u,v,w; u >= v ==> is_gcd(u-v,v,w);
  case deux: \forall integer u,v,w; u < v ==> is_gcd(u,v-u,w);
}
*/
```

- ▶ The termination is proved by showing that each loop terminates.
- ▶ Any loop is characterized by an expression  $\text{expvariant}(x)$  called **variant** which should decrease each execution of the body :

$$\forall x_1, x_2. b(x_1) \wedge x_1 \xrightarrow{S} x_2 \Rightarrow \text{expvariant}(x_1) > \text{expvariant}(x_2)$$

(Variant)

### Listing 37 – project-divers/variant1.c

```
/*@ requires n > 0;
   terminates n > 0;

   ensures \result == 0;
*/
int code(int n) {
  int x = n;
  /*@ loop invariant x >= 0 && x <= n;
     loop assigns x;
     loop variant x;
  */
  while (x != 0) {
    x = x - 1;
  };
  return x;
}
```

(Variant)

### Listing 38 – project-divers/variant3.c

```
int f() {  
  int x = 0;  
  int y = 10;  
  /*@  
    loop invariant  
    0 <= x < 11 && x+y == 10;  
    loop variant y;  
  */  
  while (y > 0) {  
    x++;  
    y--;  
  }  
  return 0;  
}
```

(Variant)

## Listing 39 – project-divers/variant4.c

```
g/*@ requires n <= 12;  
  @ decreases n;  
  @*/  
int fact(int n){  
    if (n <= 1) return 1;  
    return n*fact(n-1);  
}
```

- ▶ – lemma : 1 VC
- ▶ – axiom : no VC (admitted with no proof)
- ▶ – ensures : 1 VC
- ▶ – exits : 1 VC
- ▶ – disjoint : 1 VC
- ▶ – complete : 1 VC
- ▶ – requires : 1 VC for each call
- ▶ – terminates : 1 VC for each call, 1 VC for each loop without "loop variant"
- ▶ – decreases : 1 VC for each recursive call
- ▶ – assigns : 1 VC for each assigned lvalue
- ▶ – admit : no VC (admitted with no proof)
- ▶ – assert/check : 1 VC
- ▶ – loop invariant : 2 VCs (established, preserved)
- ▶ – loop variant (integer) : 2 VCs (positive, decreasing)
- ▶ – loop variant (general measure) : 1 VC (the measure is assumed to be well-founded) – loop assigns : 1 VC for each assigned lvalue within the loop

- ▶ Defining the mathematical function to compute *mathf*
- ▶ Stating the postcondition using the mathematical function
- ▶ Evaluating the inductive sequence  $u_i$  computing the function *mathf*
- ▶  $\forall i \in \mathbb{N} : u_i = \text{mathf}(i)$
- ▶ Evaluating relationship among variables.



(power2.h)

## Listing 40 – project-powers/power21.h

```
#ifndef _A.H
#define _A.H
// Definition of the mathematical function mathpower2
/*@ axiomatic mathpower2 {
  @ logic integer mathpower2(integer n);
  @ axiom mathpower2_0: mathpower2(0) == 0;
  @ axiom mathpower2_rec: \forall integer n; n > 0
    => mathpower2(n) == mathpower2(n-1) + n+n+1;
  @ } */
/*@ axiomatic matheven {
  @ logic integer matheven(integer n);
  @ axiom matheven_0: matheven(0) == 0;
  @ axiom matheven_rec: \forall integer n; n > 0
    => matheven(n) == matheven(n-1) + 2;
  @ } */
// We define v and w in a one shot axiomatic definition
/*@ axiomatic vw {
  @ logic integer v(integer n);
  @ logic integer w(integer n);
  @ axiom v_0: v(0) == 0;
  @ axiom w_0: w(0) == 0;
  @ axiom v_rec: \forall integer n; n > 0
    => v(n) == v(n-1) + n+n+1 && w(n) == w(n-1) + 2;
  @ } */
```

(power2.h)

## Listing 41 – project-powers/power22.h

```
/*@ lemma propw:  
@ \forallall int n; n >= 0 => w(n) == n+n; @*/  
/*@ lemma propv:  
@ \forallall int n; n >= 0 => v(n) == n*n; @*/  
/*@ lemma prop1:  
@ \forallall int n; n >= 0 => matheven(n) == n+n; @*/  
/*@ lemma prop2:  
@ \forallall int n; n >= 0 => mathpower2(n) == n*n; @*/  
/*@ axiomatic auxmath {  
  @ lemma rule1: \forallall int n; n > 0 => n*n == (n-1)*(n-1)+2*(n-1)+1;  
  @ } */  
/*@ requires 0 <= x;  
    assigns \nothing;  
    ensures \result == x*x;  
*/  
int power2(int x);  
#endif
```

(power2.h)

## Listing 42 – project-powers/power2.c

```
#include <limits.h>
#include "power2.h"

int power2(int x)
{
    int r, k, cv, cw, or, ok, ocv, ocw;
    r=0; k=0; cv=0; cw=0; or=0; ok=k; ocv=cv; ocw=cw;
    /*@ loop invariant 0 <= cv && 0 <= cw && 0 <= k;
       @ loop invariant cv == k*k;
       @ loop invariant k <= x;
       @ loop invariant cw == 2*k;
       @ loop invariant 4*cv == cw*cw;
       @ loop assigns k, cv, cw, or, ok, ocv, ocw; */
    while (k<x)
    {
        ok=k; ocv=cv; ocw=cw;
        k=ok+1;
        cv=ocv+ocw+1;
        cw=ocw+2;
    }
    r=cv;
    return (r);
}
```

- ▶ Plugin -rte adds specific assertions for each variable
- ▶ Systematic checking of RTE.

- ▶ Defining domain properties (axioms, lemmas, proofs)
- ▶ Defining loop invariants (typing, equation, ...)
- ▶ Analyzing inductive properties
- ▶ Identifying inputs (*requires*) and outputs (*ensures*)