Tutorial Modelling Software-based Systems
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Tutorial 2 : Designing and verifying sequential algorithms using the Event-B modelling language
Dominique Méry
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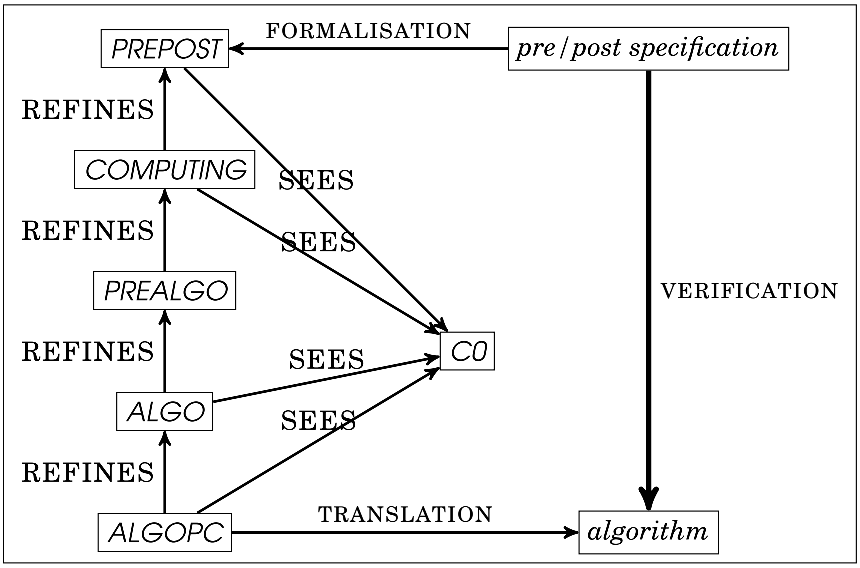
Exercice 1 *fx1-tut2.zip*
We consider a finite sequence of integers $v_1, \ldots, v_n$ where n is the length of the sequence and is supposed to be fixed. Write an Event B specification modelling the computation of the value of the summation of the sequence v. You should define cerafully v, n and the summation of a finite sequence of integers.

Exercice 2 *fx2-tut2.zip*



Apply the pattern for computing the value n^2 using the sequence $(n+1)^2 = n^2+n+n+1$.
Write a C function with annotation that you will check with Frama-c.

Exercice 3 *fx3-tut2.zip*
Develop an algorithmic solution with the pattern for the problem of finding the number of occurrences of a value v value v satisfying a condition CO in a table t of dimension n. dimension n. The table is assumed to have a value in an envelope V. seems V and that CO is a part of V.

Exercice 4 *fx4-tut2.zip*
Apply this pattern to find the index i of t such that $t(i) = v$. Write a C function that you will check with Frama-c.

Exercice 5 *fx5-tut2.zip*
Apply this pattern to compute x^3 using $(i+1)^3 = i^3+3i^2+3i+1$. We use the following sequences :

- $z_0 = 0$ *et* $\forall n \in \mathbb{N} : z_{n+1} = z_n+v_n+w_n$
- $v_0 = 0$ *et* $\forall n \in \mathbb{N} : v_{n+1} = v_n+t_n$
- $t_0 = 3$ *et* $\forall n \in \mathbb{N} : t_{n+1} = t_n+6$
- $w_0 = 1$ *et* $\forall n \in \mathbb{N} : w_{n+1} = w_n+3$
- $u_0 = 0$ *et* $\forall n \in \mathbb{N} : u_{n+1} = u_n+1$

$$\begin{pmatrix} z(i+1) \\ v(i+1) \\ t(i+1) \\ w(i+1) \\ u(i+1) \end{pmatrix} = \begin{pmatrix} z_i+v_i+w_i \\ v(i)+t(i) \\ t(i)+6 \\ w(i)+3 \\ u(i)+1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 01 & 0 & \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z(i) \\ v(i \\ t(i) \\ w(i) \\ u(i) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 6 \\ 3 \\ 1 \end{pmatrix}$$

Write a C function from the development and use Frama-c for checking it.

Exercice 6 *(tutorial in Maynooth NUI)*
The objective is to design a correct by construction algorithm in a programming language with annotations and proof tool as Frama-c or Dafny.
The problem to solve is the power function defined usually as follows in a classical matematical language $power = \lambda x, y.x^y$. For instance, the notation x^y is foubnd in the C programlming language and a function can be called for computing the power function.

Question 6.1 *Define an inductive statement of the power function by defining the sequence $p(x \mapsto y)$ where $x, y \in \mathbb{N}$. The function $powereb = lambdax, y.x^y$. is defined in the Event-B language and you should prove that $\forall x, y \in \mathbb{N}.p(x \mapsto y) = x^y$.*

Question 6.2 *Define a context POW0 and a prepost machine POW1 expressing the contract os the problem to solve. The problem is to define first the contract in your programming langauge.*

Question 6.3 *Develop a computing process according to the methodology of the refinement to get an algorithm.*

Question 6.4 *Check that the algorithm `power` satisfies the constract using the proof tool related to your programming language. Yiu can use the Event-B machines for deriving a possible loop invariant.*

Question 6.5 *Using the mathematical expression of the power function nameley matrhpower that you have defined in POW0, you can derive a recursive algorithm `int p(int a0,int b0)` which is using the inductive definition for computing the function but following a recursive schema. Derive an algorithm in your pet programming language and check the cobtract.*

Question 6.6 *The two resulting algorithm power and `p` are computing the same value for the same inputs. Using your pet programling language, write a function `check` which is calling `power` and `p` and returning 1 if the two values are equal. Write a contract for expressing this equivalence.*

Question 6.7 *a and b are two natural numbers and if we launch the function power for some values we obtain the following results :*

— $power(0, 0) = 1$
— $power(0, 1) = 1$
— $power(0, 2) = 1$
—
— $power(2, 0) = 1$
—
— $power(2, 1) = 2$
—
— $power(2, 2) = 4$

Hence,we obtain the following property $\forall y.y \in \mathbb{N} \Rightarrow power(0, y) = 1$. However, what does mean $power(a, b)$?
$power(a, b) = \underbrace{a\times \ldots \times a}_{b\ times}$ *and*

— $power(a, 1) = a$
— $power(a, 2) = a\times a$
— $power(a, b+c) = power(a, b)\times power(a, c)$
— $power(a, 1) = power(a, 0+1) = power(a, 0)\times power(a, 1) = power(a, 0)\times a = a : power(a, 0) = 1$, *when* $a \neq 0$.

Hence $\forall y.y \in \mathbb{N} \land y \neq 0 \Rightarrow power(y, 0) = 1$.
When $a = 0$, *we can give the following properties :*

— $power(0, 1) = 0$

— $power(0,2) = 0 \times 0 = 0$
— $power(0,0)$ ***means that there is no number or is developed as follow :*** $power(0,1) = power(0,0{+}1) : power(0,0) \times power(0,1)$ ***and*** $power(0,0) = 1$.

We summarize as follow :

— $\forall y . y \in \mathbb{N} \wedge y \neq 0 \Rightarrow power(y,0) = 1$.
— $\forall y . y \in \mathbb{N} \wedge y \neq 0 \Rightarrow power(0,y) = 1$.
— `power` ***is not defined for (0,0).***

Write two new theorems in POW0 for stating the two first theorems.

Write the same theorems for the function $\lambda x, y . x^y$ *and write the following properties*

$\forall y . y \in \mathbb{N} \Rightarrow y^0 = 1$
$\forall y . y \in \mathbb{N} \Rightarrow 0^y = 0$
$0^0 = 1$
$0^0 = 0$
$1 = 2$

What is your conclusion ?
Modify your context POW0 according to the sound definition and modify the contract.