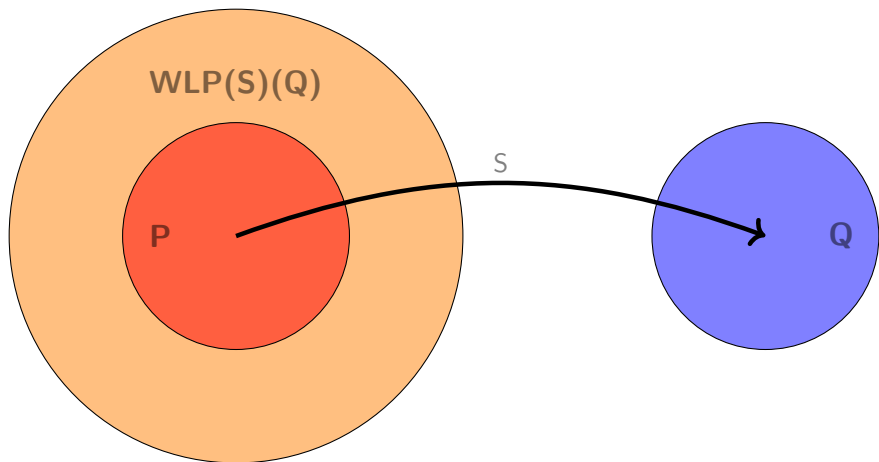
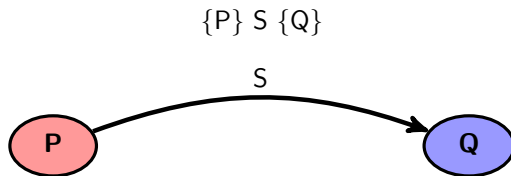
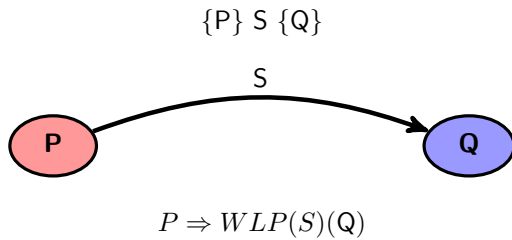
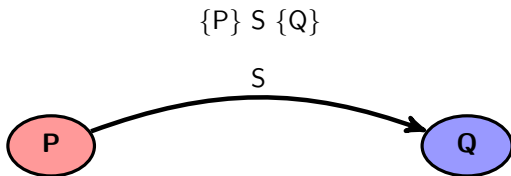


- 1 Programs as Predicate Transformers
- 2 Mechanizing the contract checking
- 3 Transforming predicates
 - Hoare Logic for PC
 - Examples in ACSL
 - Définition et propriétés du calcul wp
- 4 Using predicate transformers for checking contracts









$$P \Rightarrow WLP(S)(Q)$$

Computing $WLP(S)(Q)$?

A program P *satisfies* a contract $(x, \text{pre}, \text{post})$:

- ▶ P transforms a variable x from an initial value x_0 and produces a final value x_f : $x_0 \xrightarrow{P} x_f$
- ▶ x_0 satisfies pre : $\text{pre}(x_0)$ and x_f satisfies post : $\text{post}(x_0, x_f)$
- ▶ $\text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
- ▶ \mathbb{D} is the domain of x for RTE (No Run Time Errors) .

i

```
variables  $x : \mathbb{D}$   
requires  $\text{pre}(x_0)$   
ensures  $\text{post}(x_0, x_f)$   
[  
  begin  
     $0 : P_0(x_0, x)$   
    S  
     $f : P_f(x_0, x)$   
  end
```

A program P satisfies a contract $(x, \text{pre}, \text{post})$:

- ▶ P transforms a variable x from an initial value x_0 and produces a final value x_f : $x_0 \xrightarrow{P} x_f$
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i

variables $x : \mathbb{D}$
requires $\text{pre}(x_0)$
ensures $\text{post}(x_0, x_f)$

begin
 $0 : P_0(x_0, x)$
 S
 $f : P_f(x_0, x)$
end

- ▶ $\text{pre}(x_0) \wedge x = x_0 \Rightarrow P_0(x_0, x)$
- ▶ $\text{pre}(x_0) \wedge P_f(x_0, x) \Rightarrow \text{post}(x_0, x)$
- ▶ For any pair ℓ, ℓ' such that $\ell \longrightarrow \ell'$, we verify that for any values $x, x' \in \text{MEMORY}$
$$\left(\begin{array}{l} P_\ell(x_0, x) \\ \wedge \text{cond}_{\ell, \ell'}(x) \wedge x' = f_{\ell, \ell'}(x) \end{array} \right) \Rightarrow P_{\ell'}(x_0, x')$$
- ▶ For any pair m, n such that $m \longrightarrow n$, we verify that $\forall x, x' \in \text{MEMORY}$:
 $\text{pre}(x_0) \wedge P_m(x_0, x) \Rightarrow \text{DOM}(m, n)(x)$

- ▶ $pre(x_0) \wedge x = x_0 \Rightarrow P_0(x_0, x)$
- ▶ $pre(x_0) \wedge P_f(x_0, x) \Rightarrow post(x_0, x)$
- ▶ For any pair ℓ, ℓ' such that $\ell \longrightarrow \ell'$, we verify that for any values $x, x' \in \text{MEMORY}$
$$\left(\begin{array}{l} P_\ell(x_0, x) \\ \wedge cond_{\ell, \ell'}(x) \wedge x' = f_{\ell, \ell'}(x) \\ \Rightarrow P_{\ell'}(x_0, x') \end{array} \right),$$
- ▶ For any pair m, n such that $m \longrightarrow n$, we verify that $\forall x, x' \in \text{MEMORY} : pre(x_0) \wedge P_m(x_0, x) \Rightarrow \mathbf{DOM}(m, n)(x)$

Example $\mathbf{DOM}(m, n)(x)$

$DOM(\ell_0, \ell_1)(u) = u \in \text{minint}.. \text{maxint} \wedge 5 \in \text{minint}.. \text{maxint} \wedge u+5 \in$

$\text{minint}.. \text{maxint}$ where

$$\begin{array}{l} \ell_0 : P_{\ell_0}(u); \\ u := u+5; \\ \ell_1 : P_{\ell_0}(u); \end{array}$$

- ▶ A program P *produces* results or outputs from inputs according to a (operational or denotational) semantics
 - STATES is the set of states of P : $STATES = x \rightarrow \mathbb{Z}$ where x designate variables of P .
 - s_0 et s_f two states of STATES : $\mathcal{D}(P)(s_0) = s_f$ means that P is executed from the memory state s_0 and produces a final state s_f .
 - For any current state s of P , $s(x) = x$ for expressing the value of x in state s :

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 - STATES is the set of states of P : $STATES = x \rightarrow \mathbb{Z}$ where x designate variables of P .
 - s_0 et s_f two states of STATES : $\mathcal{D}(P)(s_0) = s_f$ means that P is executed from the memory state s_0 and produces a final state s_f .
 - For any current state s of P , $s(x) = x$ for expressing the value of x in state s :

$$s_0(x) = x_0, s_f(x) = x_f, s'(x) = x'$$

- $\mathcal{D}(P)(s_0) = s_f$ defines the computation nrelation over the set of states :

$$x_0 \xrightarrow{P} x_f$$

- ▶ A program P *satisfies* the contract $(x, \text{pre}, \text{post})$:
 - P transforms a variable x from a value x_0 and produces a value x_f :
$$x_0 \xrightarrow{P} x_f$$
 - x_0 satisfies pre : $\text{pre}(x_0)$
 - x_f satisfies post : $\text{post}(x_0, x_f)$
 - $\text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$

A program P *satisfies* a contract $(x, \text{pre}, \text{post})$:

- ▶ P transforms a variable x from an initial value x_0 and produces a final value x_f : $x_0 \xrightarrow{P} x_f$
- ▶ x_0 satisfies pre : $\text{pre}(x_0)$ and x_f satisfies post : $\text{post}(x_0, x_f)$
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- ▶ $\text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$

```
variables  $x : \mathbb{D}$   
requires  $\text{pre}(x_0)$   
ensures  $\text{post}(x_0, x_f)$   
[  
  begin  
     $0 : P_0(x_0, x)$   
    S  
     $f : P_f(x_0, x)$   
  end
```

A program P *satisfies* a contract $(x, \text{pre}, \text{post})$:

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```
variables  $x : \mathbb{D}$ 
requires  $\text{pre}(x_0)$ 
ensures  $\text{post}(x_0, x_f)$ 
[
  begin
     $0 : P_0(x_0, x)$ 
    S
     $f : P_f(x_0, x)$ 
  end
```

- ▶ $\text{pre}(x_0) \wedge x = x_0 \Rightarrow P_0(x_0, x)$
- ▶ $\text{pre}(x_0) \wedge P_f(x_0, x) \Rightarrow \text{post}(x_0, x)$
- ▶ For any pair ℓ, ℓ' such that $\ell \longrightarrow \ell'$, we verify that for any values $x, x' \in \text{MEMORY}$
$$\left(\begin{array}{c} P_\ell(x_0, x) \\ \wedge \text{cond}_{\ell, \ell'}(x) \wedge x' = f_{\ell, \ell'}(x) \end{array} \right) \Rightarrow P_{\ell'}(x_0, x')$$

requires $x_0 \geq 0$;
ensures $x_f = x_0 + 2$;
variables X

```
begin  
  int X = x0;  
  0 : x = x0  
  X = X + 2;  
  1 : x = x0 + 2  
end
```

- ▶ $x_0 \geq 0 \wedge x = x_0 \Rightarrow x = x_0$
- ▶ $x = x_0 + 2 \Rightarrow x = x_0 + 2$
- ▶ conditions de vérification $0 \longrightarrow 1$:
 $x = x_0 \wedge x' = x + 2 \Rightarrow x' = x_0 + 2$
- ▶ $(x_0 \geq 0, x == x_0, x != x_0)$
- ▶ $(x == x_0 + 2, x != x_0 + 2)$
- ▶ $(x == x_0, x \neq x + 2, x \neq x_0 + 2)$

Listing 1 – z3 en Python

```
from numbers import Real  
from z3 import *  
x = Real('x')  
xp = Real('xp')  
x0 = Real('x0')  
s = Solver()  
s.add(x0 >= 0, x == x0, x != x0)  
print(s.check())  
s.add(x == x0 + 2, x != x0 + 2)  
print(s.check())  
s.add(x == x0, xp == x + 2, xp != x0 + 2)  
print(s.check())
```


► $\forall x_0, x_f. \text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$

- ▶ $\forall x_0, x_f. \text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
- ▶ $\forall x_0, x. \text{pre}(x_0) \Rightarrow (x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x))$

- ▶ $\forall x_0, x_f. \text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
- ▶ $\forall x_0, x. \text{pre}(x_0) \Rightarrow (x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x))$
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- ▶ $\forall x_0, x. \text{pre}(x_0) \Rightarrow (x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x))$
- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow \forall x. x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x)$

- ▶ $\forall x_0, x_f. \text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
- ▶ $\forall x_0, x. \text{pre}(x_0) \Rightarrow (x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x))$
- ▶ $\forall x_0, x. \text{pre}(x_0) \Rightarrow (x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x))$
- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow \forall x. x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x)$
- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow \{P\} \text{post}(x_0, x)$

- ▶ $\forall x_0, x_f. \text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
- ▶ $\forall x_0, x. \text{pre}(x_0) \Rightarrow (x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x))$
- ▶ $\forall x_0, x. \text{pre}(x_0) \Rightarrow (x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x))$
- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow \forall x. x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x)$
- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow \{P\} \text{post}(x_0, x)$

Weakest Liberal Precondition of S for P

$$\{S\}P(x) \stackrel{\text{def}}{=} \forall x_f. x \xrightarrow{S} x_f \Rightarrow P(x_f)$$

- ▶ $\forall x_0, x_f. \text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
- ▶ $\forall x_0, x. \text{pre}(x_0) \Rightarrow (x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x))$
- ▶ $\forall x_0, x. \text{pre}(x_0) \Rightarrow (x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x))$
- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow \forall x. x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x)$
- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow \{P\} \text{post}(x_0, x)$

Weakest Liberal Precondition of S for P

$$\{S\}P(x) \stackrel{def}{=} \forall x_f.x \xrightarrow{S} x_f \Rightarrow P(x_f)$$

- $\{x := e\}P(x) = P[x \mapsto e]$
- $\{\text{if } b(x) \text{ then } S1 \text{ else } S2\}P(x) = b(x) \wedge \{S1\}P(x) \vee \text{not } b(x) \wedge \{S2\}P(x)$

- ▶ $WLP(S)(P(x))$ is another notation for $\{S\}P(x)$.
- ▶ $\{\text{while } b(x) \text{ do } S \text{ end}\}P(x) = \{w\}(P(x))$

- ▶ $WLP(S)(P(x))$ is another notation for $\{S\}P(x)$.
- ▶ $\{\text{while } b(x) \text{ do } S \text{ end}\}P(x) = \{w\}(P(x))$
- ▶ $\{\text{if } b(x) \text{ then } S; w \text{ else } skip \}P(x) =$

- ▶ $WLP(S)(P(x))$ is another notation for $\{S\}P(x)$.
- ▶ $\{\text{while } b(x) \text{ do } S \text{ end}\}P(x) = \{w\}(P(x))$
- ▶ $\{\text{if } b(x) \text{ then } S; w \text{ else } skip \}P(x) =$
- ▶ $b(x) \wedge \{S; w\}P(x) \vee \text{not } b(x) \wedge \{skip\}P(x) =$

Computing WLP function

- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow \{P\} \text{post}(x_0, x)$
- ▶ $\forall x_0. x = x_0 \wedge \text{pre}(x_0) \Rightarrow \{P\} \text{post}(x_0, x)$
- ▶ Hoare Triple : $\{ \text{pre}(x_0) \wedge x = x_0 \} P \{ \text{post}(x_0, x) \}$

Exemple de preuve $\{x = 1\} \mathbf{Z} := \mathbf{X}; \mathbf{X} := \mathbf{Y}; \mathbf{Y} := \mathbf{Z} \{y = 1\}$

- ▶ (1) $x = 1 \Rightarrow (z = 1)[x/z]$ (propriété logique)
- ▶ (2) $\{(z = 1)[x/z]\} \mathbf{Z} := \mathbf{X} \{z = 1\}$ (axiome d'affectation)
- ▶ (3) $\{x = 1\} \mathbf{Z} := \mathbf{X} \{z = 1\}$ (Règle de renforcement/affaiblissement avec (1) et (2))
- ▶ (4) $z = 1 \Rightarrow (z = 1)[y/x]$ (propriété logique)
- ▶ (5) $\{(z = 1)[y/x]\} \mathbf{X} := \mathbf{Y} \{z = 1\}$ (axiome d'affectation)
- ▶ (6) $\{z = 1\} \mathbf{X} := \mathbf{Y} \{z = 1\}$ (Règle de renforcement/affaiblissement avec (4) et (5))
- ▶ (7) $z = 1 \Rightarrow (y = 1)[z/y]$ (propriété logique)
- ▶ (8) $\{(z = 1)[x/z]\} \mathbf{Y} := \mathbf{Z} \{y = 1\}$ (axiome d'affectation)
- ▶ (9) $\{z = 1\} \mathbf{Y} := \mathbf{Z} \{y = 1\}$ (Règle de renforcement/affaiblissement avec (7) et (8))
- ▶ (10) $\{x = 1\} \mathbf{Z} := \mathbf{X}; \mathbf{X} := \mathbf{Y}; \{z = 1\}$ (Règle de composition avec 3 et 6)
- ▶ (11) $\{x = 1\} \mathbf{Z} := \mathbf{X}; \mathbf{X} := \mathbf{Y}; \mathbf{Y} := \mathbf{Z} \{y = 1\}$ (Règle de composition avec 11 et 9)

.....

⊠ Definition

$\{P\}\mathbf{S}\{Q\}$ est défini par $\forall s, t \in STATES : P(s) \wedge \mathcal{D}(S)(s) = t \Rightarrow Q(t)$

.....

.....

☺ Property Correction du système axiomatique des programmes commentés

- ▶ S'il existe une preuve construite avec les règles précédentes de $\{P\}\mathbf{S}\{Q\}$, alors $\{P\}\mathbf{S}\{Q\}$ est valide.
- ▶ Si $\{P'\}\mathbf{S}\{Q'\}$ est valide et si le langage d'assertions est suffisamment expressif, alors il existe une preuve construite avec les règles précédentes de $\{P\}\mathbf{S}\{Q\}$.

.....

⊠ Definition

Un langage d'assertions est la donnée d'un ensemble de prédicats et d'opérateurs de composition comme la disjonction et la conjonction ; il est muni d'une relation d'ordre partielle appelée implication. On le notera $(\text{PRED}, \Rightarrow, \mathbf{false}, \mathbf{true}, \wedge, \vee) : (\text{PRED}, \Rightarrow, \mathbf{false}, \mathbf{true}, \wedge, \vee)$ est un treillis complet.

- ▶ $\{P\}\mathbf{S}\{Q\}$
- ▶ $\forall s, t \in STATES : P(s) \wedge \mathcal{D}(S)(s) = t \Rightarrow Q(t)$
- ▶ $\forall s \in STATES : P(s) \Rightarrow (\forall t \in STATES : \mathcal{D}(S)(s) = t \Rightarrow Q(t))$

.....

Définition de wlp

$$wlp(S)(Q) \stackrel{def}{=} (\forall t \in STATES : \mathcal{D}(S)(s) = t \Rightarrow Q(t))$$

.....

$$wlp(S)(Q) \equiv \overline{(\exists t \in STATES : \mathcal{D}(S)(s) = t \wedge \overline{Q}(t))}$$

.....

Lien entre wp et wlp

- ▶ $loop(S) \equiv \overline{(\exists t \in STATES : \mathcal{D}(S)(s) = t)}$ (ensemble des états qui ne permettent pas à S de terminer)
 - ▶ $wp(S)(Q) \equiv wlp(S)(Q) \wedge \overline{loop(S)}$
-

.....

☒ Definition

$$WLP(S)(P) = \nu \lambda X. ((B \wedge wlp(BS)(X)) \vee (\neg B \wedge P))$$

.....

.....

☺ Property

▶ Si $P \Rightarrow Q$, then $wlp(S)(P) \Rightarrow wlp(S)(Q)$.

.....

⊠ Definition triplets de Hoare

$$\{P\}\mathbf{S}\{Q\} \stackrel{def}{=} P \Rightarrow wlp(S)(Q)$$

.....

.....

⊠ Définition triplets de Hoare

$$\{P\}\mathbf{S}\{Q\} \stackrel{def}{=} P \Rightarrow wlp(S)(Q)$$

.....

.....

⊠ Définition (Axiomes et règles d'inférence)

- ▶ Axiome d'affectation : $\{P(e/x)\}\mathbf{X} := \mathbf{E}(\mathbf{X})\{P\}$.
 - ▶ Axiome du saut : $\{P\}\mathbf{skip}\{P\}$.
 - ▶ Règle de composition : Si $\{P\}\mathbf{S}_1\{R\}$ et $\{R\}\mathbf{S}_2\{Q\}$, alors $\{P\}\mathbf{S}_1;\mathbf{S}_2\{Q\}$.
 - ▶ Si $\{P \wedge B\}\mathbf{S}_1\{Q\}$ et $\{P \wedge \neg B\}\mathbf{S}_2\{Q\}$, alors $\{P\}\mathbf{if\ B\ then\ S_1\ then\ S_2\ fi}\{Q\}$.
 - ▶ Si $\{P \wedge B\}\mathbf{S}\{P\}$, alors $\{P\}\mathbf{while\ B\ do\ S\ od}\{P \wedge \neg B\}$.
 - ▶ Règle de renforcement/affaiblissement : Si $P' \Rightarrow P$, $\{P\}\mathbf{S}\{Q\}$, $Q \Rightarrow Q'$, alors $\{P'\}\mathbf{S}\{Q'\}$.
-

Listing 3 – incrément de nombre

```
/*@ requires x0 >= 0;  
    assigns \nothing;  
    ensures \result == x0+2;  
    @*/
```

```
int exemple(int x0) {  
    int x=x0;  
    //@ assert x == x0;  
    x = x + 2;  
    //@ assert x == x0+2;  
    return x;  
}
```

requires $x_0 \geq 0$;
ensures $x_f = x_0 + 2$;
variables x

```
begin  
  int x = x0;  
  0 : x = x0  
  x := x + 2;  
  1 : x = x0 + 2  
end
```

Conditions de vérification 0 \rightarrow 1 :

- ▶ $x = x_0 \wedge x' = x + 2 \Rightarrow x' = x_0 + 2$
- ▶ $x = x_0 \Rightarrow (x' = x + 2 \Rightarrow x' = x_0 + 2)$
- ▶ $x = x_0 \Rightarrow (x + 2 = x_0 + 2)$
- ▶ $wp(x := x + 2)(x = x_0 + 2) = (x + 2 = x_0 + 2)$
- ▶ $x = x_0 \wedge x_0 \geq 0 \Rightarrow wp(x := x + 2)(x = x_0 + 2)$
- ▶ $x = x_0 \wedge x_0 \geq 0 \Rightarrow x + 2 = x_0 + 2$
- ▶ $x = x_0 \wedge x_0 \geq 0 \Rightarrow x_0 + 2 = x_0 + 2$

- ▶ $x_0 \geq 0 \wedge x = x_0 \Rightarrow x = x_0$
- ▶ $x = x_0 + 2 \Rightarrow x = x_0 + 2$
- ▶ $x = x_0 \Rightarrow wp(x := x + 2)(x = x_0 + 2)$



calcul de $wp(X := X + 2)(x = x_0 + 2)$

Listing 4 – incrément de nombre

```
/*@ requires x0 >= 0;  
    assigns \nothing;  
    ensures \result == x0+1;  
    @*/
```

```
int exemple(int x0) {  
    int x=x0;  
    //@ assert x == x0;  
    x = x + 2;  
    //@ assert x == x0+2;  
    return x;  
    //@ assert \result == x0+2;  
}
```

Listing 5 – incrément de nombre

```
/*@ requires x0 >= 0;  
    assigns \nothing;  
    ensures \result == x0;  
    @*/
```

```
int exemple(int x0) {  
    int x=x0;  
    //@ assert x == x0+1;  
    x = x + 2;  
    //@ assert x == x0+2;  
    return x;  
  
}
```

Opérateur WP

Soit $STATES$ l'ensemble des états sur l'ensemble X des variables. Soit S une instruction de programme sur X . Soit A une partie de $STATES$.

$s \in WP(S)(A)$, si la condition suivante est vérifiée :

$$\left(\begin{array}{l} \forall t \in STATES : \mathcal{D}(S)(s) = t \Rightarrow t \in A \\ \wedge \\ \exists t \in STATES : \mathcal{D}(S)(s) = t \end{array} \right)$$

- ▶ $WP(X := X+1)(A) = \{s \in STATES \mid s[X \mapsto s(X) \oplus 1] \in A\}$
- ▶ $WP(X := Y+1)(A) = \{s \in STATES \mid s[X \mapsto s(Y) \oplus 1] \in A\}$
- ▶ $WP(\text{while } X > 0 \text{ do } X := X-1 \text{ od})(A) = \{s \in STATES \mid (s(X) \leq 0) \vee (s(X) \in A \wedge s(X) < 0)\}$
- ▶ $WP(\text{while } x > 0 \text{ do } x := x+1 \text{ od})(A) = \{s \in STATES \mid (s(X) \in A \wedge s(X) \leq 0)\}$
- ▶ $WP(\text{while } x > 0 \text{ do } x := x+1 \text{ od})(\emptyset) = \emptyset$
- ▶ $WP(\text{while } x > 0 \text{ do } x := x+1 \text{ od})(STATES) = \{s \in STATES \mid s(X) < 0\}$

Propriétés

- ▶ WP est une fonction monotone pour l'inclusion d'ensembles de STATES.
- ▶ $WP(S)(\emptyset) = \emptyset$
- ▶ $WP(S)(A \cap B) = WP(S)(A) \cap WP(S)(B)$
- ▶ $WP(S)(A) \cup WP(S)(B) \subseteq WP(S)(A \cup B)$
- ▶ Si S est déterministe, $WP(S)(A \cup B) = WP(S)(A) \cup WP(S)(B)$
- ▶ WP est un opérateur avec le profil suivant
pour toute instruction S du langage de programmation,
 $WP(S) \in \mathcal{P}(\text{STATES}) \rightarrow \mathcal{P}(\text{STATES})$
- ▶ $(\mathcal{P}(\text{STATES}), \subseteq)$ est un treillis complet.
- ▶ $(Pred, \Rightarrow)$ est une structure où
 - (1) $Pred$ est une *extension* du langage d'expressions booléennes
 - (2) $Pred$ est une *intension* introduite comme un langage d'assertions
 - \Rightarrow est l'implication
 - $s \in A$ correspond une assertion P vraie en s notée $P(s)$.

S	$wp(S)(P)$
$X := E(X, D)$	$P[e(x, d)/x]$
SKIP	P
$S_1; S_2$	$wp(S_1)(wp(S_2)(P))$
IF B S_1 ELSE S_2 FI	$(B \Rightarrow wp(S_1)(P)) \wedge (\neg B \Rightarrow wp(S_2)(P))$
WHILE B DO S OD	$\mu.(\lambda X. (B \Rightarrow wp(S)(X)) \wedge (\neg B \Rightarrow P))$

- ▶ $wp(X := X+5)(x \geq 8) \stackrel{def}{=} x+5 \geq 8 \wedge x \geq 3$
- ▶ $wp(\text{WHILE } x > 1 \text{ DO } X := X+1 \text{ OD})(x = 4) = FALSE$
- ▶ $wp(\text{WHILE } x > 1 \text{ DO } X := X+1 \text{ OD})(x = 0) = x = 0$

.....
☒ Definition triplets de Hoare Correction Totale

$$[P]\mathbf{S}[Q] \stackrel{def}{=} P \Rightarrow wp(S)(Q)$$

.....

.....

⊠ Definition triplets de Hoare Correction Totale

$$[P]\mathbf{S}[Q] \stackrel{def}{=} P \Rightarrow wp(S)(Q)$$

.....

.....

⊠ Definition (Axiomes et règles d'inférence)

- ▶ Axiome d'affectation : $[P(e/x)]\mathbf{X} := \mathbf{E}(\mathbf{X})[P]$.
 - ▶ Axiome du saut : $[P]\mathbf{skip}[P]$.
 - ▶ Règle de composition : Si $[P]\mathbf{S}_1[R]$ et $[R]\mathbf{S}_2[Q]$, alors $[P]\mathbf{S}_1 ; \mathbf{S}_2[Q]$.
 - ▶ Si $[P \wedge B]\mathbf{S}_1[Q]$ et $[P \wedge \neg B]\mathbf{S}_2[Q]$, alors $[P]\mathbf{if\ B\ then\ S_1\ then\ S_2\ fi}[Q]$.
 - ▶ Si $[P(n+1)]\mathbf{S}[P(n)]$, $P(n+1) \Rightarrow b$, $P(0) \Rightarrow \neg b$, alors $[\exists n \in \mathbb{N}. P(n)]\mathbf{while\ B\ do\ S\ od}[P(0)]$.
 - ▶ Règle de renforcement/affaiblissement : Si $P' \Rightarrow P$, $[P]\mathbf{S}[Q]$, $Q \Rightarrow Q'$, alors $[P']\mathbf{S}[Q']$.
-

Si $P \Rightarrow wp(S)(Q)$, alors il existe une preuve de $[P]\mathbf{S}[Q]$ construites avec les règles ci-dessus,

- $P \Rightarrow wp(X := E(X))(Q) : P \Rightarrow Q(e/x)$ et $[Q(e/x)]\mathbf{X} := \mathbf{E}(\mathbf{X})[Q]$ constituent une preuve.
- $P \Rightarrow wp(\text{while})(Q) :$
 - On construit la suite de $P(n)$ en définissant $P(n) = W_n$.
 - On vérifie que cela vérifie la règle du while.

Verification of contract (I)

A program P *satisfies* a contract $(\text{pre}, \text{post})$:

- ▶ P transforms a variable x from an initial value x_0 and produces a final value x_f : $x_0 \xrightarrow{P} x_f$
- ▶ x_0 satisfait pre : $\text{pre}(x_0)$ and x_f satisfait post : $\text{post}(x_0, x_f)$
- ▶ $\text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$

requires $\text{pre}(x_0)$

ensures $\text{post}(x_0, x_f)$

variables X

begin

$0 : P_0(x_0, x)$

instruction₀

...

$i : P_i(x_0, x)$

...

instruction _{$f-1$}

$f : P_f(x_0, x)$

end

▶ $\text{pre}(x_0) \wedge x = x_0 \Rightarrow P_0(x_0, x)$

▶ $\text{pre}(x_0) \wedge P_f(x_0, x) \Rightarrow \text{post}(x_0, x)$

▶ For each pair ℓ, ℓ'
such that $\ell \longrightarrow \ell'$, one checks that
for any value $x, x' \in \text{MEMORY}$

$$\left(\begin{array}{l} \left(\text{pre}(x_0) \wedge P_\ell(x_0, x) \right) \\ \wedge \text{cond}_{\ell, \ell'}(x) \wedge x' = f_{\ell, \ell'}(x) \end{array} \right) \Rightarrow P_{\ell'}(x_0, x')$$

Verification du contrat (II)

A program P *satisfies* a contract (pre,post) :

- ▶ P transforms a variable x from an initial value x_0 and produces a final value x_f : $x_0 \xrightarrow{P} x_f$
- ▶ x_0 satisfait pre : $\text{pre}(x_0)$ and x_f satisfait post : $\text{post}(x_0, x_f)$
- ▶ $\text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$

requires $\text{pre}(x_0)$
ensures $\text{post}(x_0, x_f)$
variables X

```
begin  
  0 :  $P_0(x_0, x)$   
  instruction0  
  ...  
   $i : P_i(x_0, x)$   
  ...  
  instruction $f-1$   
   $f : P_f(x_0, x)$   
end
```

- ▶ $\forall x_f, x_0. \text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
- ▶ $\forall x_f, x_0. \text{pre}(x_0) \Rightarrow (x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f))$
- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow \forall x_f. (x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f))$
- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow \forall x. (x_0 \xrightarrow{P} x \Rightarrow \text{post}(x_0, x))$
- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow \text{WLP}(P)(\text{post}(x_0, x))$

Un programme P *satisfies* a contract $(\text{pre}, \text{post})$:

- ▶ P transforms a variable x from an initial value x_0 and produces a final value x_f : $x_0 \xrightarrow{P} x_f$
- ▶ x_0 satisfies pre : $\text{pre}(x_0)$ and x_f satisfies post : $\text{post}(x_0, x_f)$
- ▶ $\text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow WLP(P)(\text{post}(x_0, x))$

Un programme P *satisfies* a contract $(\text{pre}, \text{post})$:

- ▶ P transforms a variable x from an initial value x_0 and produces a final value x_f : $x_0 \xrightarrow{P} x_f$
- ▶ x_0 satisfies pre : $\text{pre}(x_0)$ and x_f satisfies post : $\text{post}(x_0, x_f)$
- ▶ $\text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$
- ▶ $\forall x_0. \text{pre}(x_0) \Rightarrow WLP(P)(\text{post}(x_0, x))$
- ▶ WLP is not computable ...
- ▶ Using Hoare logic in the WLP computing as suggested by Rustan Leino. de WLP.

Verification of contract (III)

A program P *satisfies* a contract $(\text{pre}, \text{post})$:

- ▶ P transforms a variable x from an initial value x_0 and produces a final value x_f : $x_0 \xrightarrow{P} x_f$
- ▶ x_0 satisfait pre : $\text{pre}(x_0)$ and x_f satisfait post : $\text{post}(x_0, x_f)$
- ▶ $\text{pre}(x_0) \wedge x_0 \xrightarrow{P} x_f \Rightarrow \text{post}(x_0, x_f)$

```
requires  $\text{pre}(x_0)$ 
ensures  $\text{post}(x_0, x_f)$ 
variables  $X$ 
begin
  /·@assert  $P_0(x_0, x)$ ·/
  T;
  /·@loop invariant  $I(x_0, x)$ ·/
  while  $B(x)$  do
    S
  od
  /·@assert  $P_f(x_0, x)$ ·/
end
```

- ▶ $x = x_0 \wedge \text{pre}(x_0) \Rightarrow P_0(x_0, x)$
- ▶ $\text{pre}(x_0) \wedge P_0(x_0, x) \Rightarrow \text{WLP}(T)(I(x_0, x))$
- ▶ $I(x_0, x) \wedge B(x) \Rightarrow \text{WLP}(S)(I(x_0, x))$
- ▶ $I(x_0, x) \wedge \neg B(x) \Rightarrow P_f(x_0, x)$

```
requires  $pre(x_0)$   
ensures  $post(x_0, x_f)$   
variables  $X$   
[ begin  
  /·@assert  $P_0(x_0, x)$ ·/  
  S1;  
  S2;  
  /·@assert  $P_f(x_0, x)$ ·/  
end
```

- ▶ $x =$
 $x_0 \wedge \text{pre}(x_0) \Rightarrow P_0(x_0, x)$
- ▶ $P_0(x_0, x) \Rightarrow$
 $WLP(S1; S2)(P_f(x_0, x))$

```
requires  $pre(x_0)$ 
ensures  $post(x_0, x_f)$ 
variables  $X$ 
begin
  /·@assert  $P_0(x_0, x)$ ·/
  if  $B(x)$  do
     $S1$ 
  else
     $S2$ 
  elfi
  /·@assert  $P_f(x_0, x)$ ·/
end
```

$$x = x_0 \wedge pre(x_0) \Rightarrow P_0(x_0, x)$$

$$\begin{aligned} P_0(x_0, x) \Rightarrow \\ B(x) \wedge WLP(S1)(P_f(x_0, x)) \\ \vee \\ \neg B(x) \wedge WLP(S2)(P_f(x_0, x)) \end{aligned}$$