Cours MALG & MOVEX

Vérification mécanisée de contrats (III) (The ANSI/ISO C Specification Language (ACSL))

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Outline

1 Contracts

2 Generation of Verification Conditions

3 Memory Models in Frama-c

4 Logic Specification

5 Organisation of the verification process

6 Conclusion

Summary on annotations and assertions

- requires
- assigns
- ensures
- decreases
- predicate
- logic
- lemma

Programming by contract

▶ The calling function should garantee the required condition or precondition introduced by the clauses requires $P1 \wedge \ldots \wedge Pn$ at the calling point.

▶ The called function returns results that are ensured by the clause ensures $E1 \wedge \ldots \wedge Em$; ensures clause exporess a relationship between the initial values of variables and the final values.

▶ initial values of a variable v is denoted $\backslash old(v)$

▶ The variables which are not in the set $L1 \cup \ldots \cup Lp$ are not modified.

Listing 1 – contrat

```
/*@ requires P1;...; requires Pn;
  @ assigns L1;...; assigns Lm;
  @ ensures E1;...; ensures Ep;
  @*/
```

Examples of contract (1)

(Division)

Listing 2 – project-divers/annotation.c

```c
/*@ requires  x <= 0 && x >= 10;
  @ assigns \nothing;
  @ ensures   x % 2 == 0 ==> 2*\result == x;
  @ ensures   x % 2 != 0 ==> 2*\result == x-1;
  @*/
int annotation(int x)
{
  int y;
  y = x / 2;
  return(y);
  }
```

Examples of contract (1)

Listing 3 – project-divers/annotationwp.c

```
/*@ requires  0 <= x  && x <= 10;
  @ assigns \nothing;
  @ ensures  x % 2 == 0 ==> 2*\result == x;
  @ ensures  x % 2 != 0 ==> 2*\result == x-1;
  @*/
int annotation(int x)
{
/*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
/*@ assert   x % 2 != 0 ==> 2* (x / 2) == x-1; */
  int y;
/*@ assert x % 2 == 0 ==> 2* (x / 2) == x; */
/*@ assert   x % 2 != 0 ==> 2* (x / 2) == x-1; */
  y  = x / 2;
/*@ assert x % 2 == 0 ==> 2*y == x; */
/*@ assert   x % 2 != 0 ==> 2*y == x-1; */
  return(y);
/*@ assert x % 2 == 0 ==> 2*y == x; */
/*@ assert   x % 2 != 0 ==> 2*y == x-1; */

  }
```

Property to check

$$x \geq 0 \wedge x < 0; \Rightarrow \left(\begin{array}{l} x \; \% \; 2 \;\; = \;\; 0 \Rightarrow 2 \cdot (x/2) = x \\ x \; \% \; 2 \;\; \neq \;\; 0 \Rightarrow 2 \cdot (x/2) = x{-}1 \end{array} \right)$$

(Precondition)

Listing 4 – project-divers/annotation0.c

```
/*@ requires x >= 0 && x <  0;
   @ assigns \nothing;
   @ ensures \result == 0;
   @*/
int annotation0(int x)
{
   int y;
   y =  y / (x-x);
   return(y);
   }
```

(Precondition)

Listing 5 – project-divers/annotation0wp.c

```
/*@ requires x >= 0 && x <  0;
  @ assigns \nothing;
  @ ensures \result == 0;
  @*/
int annotation(int x)
{
  /*@ assert y / (x-x) == 0; */
  int y;
  /*@ assert y / (x-x) == 0; */
  y = y / (x-x);
  /*@ assert y == 0; */
  return(y);
  /*@ assert y == 0; */
}
```

Property to check
$0 \leq x \wedge x \leq 10 \Rightarrow y/(x-x) = 0$

Definition of a contract (specification)

▶ Define the mathematical fucntion to compute (what to compute ?)

▶ Define an inductive method for computing the mathematical function and using axioms.

(facctorial what)

Listing 6 – project-factorial/factorial.h

```
#ifndef _A_H
#define _A_H
/*@ axiomatic mathfact {
  @ logic integer mathfact(integer n);
  @ axiom mathfact_1: mathfact(1) == 1;
  @ axiom mathfact_rec: \forall integer n; n > 1
  ==> mathfact(n) == n * mathfact(n-1);
  @ } */

/*@ requires n > 0;
    decreases n;
    ensures \result == mathfact(n);
    assigns \nothing;
*/
int codefact(int n);
#endif
```

Definition of a contract (programming)

▶ Define the program codefact for computing mathfact (How to compute ?)

▶ Define the algorithm computing the function mathfact

```
(facctorial how )

                    Listing 7 – project-factorial/factorial.c

#include "factorial.h"

int codefact(int n) {
    int y = 1;
    int x = n;
    /*@ loop invariant x >= 1 && x <= n && mathfact(n) == y * mathfact(x);
      loop assigns x, y;
      loop variant x;
     */
    while (x != 1) {
        y = y * x;
        x = x - 1;
    };
    return y;

}
```

Definition of a contract (approach)

▶ The specification of a function (mathfact) to compute requires to define it mathematically.

▶ The definition is stated in an axtiomatic framework and is preferably inductive (mathfact) which is used in assrtions or theorems or lemmas.

▶ The relationship between the ciomputed value (\result) and the mathematical value (mathfact(n)) is stated in the ensures clause :

$$\result == mathfact(n)$$

▶ The main property to prove is codefact(n)==mathfact(n) : Calling codefact for n returns a value equal to mathfact(n).

Contracts with named behaviours
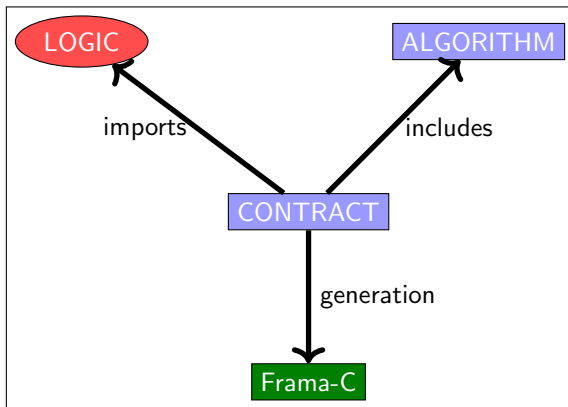
Listing 8 – contrat

```
/*@ requires P;
@ behavior b1:
  @ assumes A1;
  @ requires R1 ;
  @ assigns L1;
  @ ensures E1;
@ behavior b2:
  @ assumes A2;
  @ requires R2;
  @ assigns L2;
  @ ensures E2;
@*/
```

Division shpuld not return silly expressions !

(Pairs of integers)

Listing 9 – project-divers/structures.h

```
#ifndef _STRUCTURE_H

struct s {
    int q;
    int r;
};

#endif
```

Division shpuld not return silly expressions !

(Specification)

Listing 10 – project-divers/division.h

```
#ifndef _A_H
#define _A_H
#include "structures.h"
/*@ requires a >= 0 && b >= 0;
@ behavior b :
   @ assumes b == 0;
   @ assigns \nothing;
   @ ensures \result.q == -1  && \result.r == -1 ;
@ behavior B2:
   @ assumes b != 0;
   @ assigns \nothing;
   @ ensures 0 <= \result.r;
   @ ensures \result.r < b;
   @ ensures  a == b * \result.q + \result.r;
*/
struct s  division(int a, int b);
#endif
```

Division shpuld not return silly expressions !

Listing 11 – project-divers/division.c

```c
#include <stdio.h>
#include <stdlib.h>

#include "division.h"

struct s  division(int a, int b)
{   int rr = a;
    int qq = 0;
    struct s  silly = {-1,-1};
    struct s resu;
    if (b == 0) {
      return silly;
    }
    else
      {
  /*@
    loop invariant
    ( a == b*qq + rr ) &&
    rr >= 0;
    loop assigns rr,qq;
    loop variant rr;
  */
    while (rr >= b) { rr = rr - b; qq=qq+1;};
    resu.q= qq;
    resu.r = rr;
  return resu;
}
}
```

Management of loop

Iteration Rule for PC

If $\{P \wedge B\}\mathbf{S}\{P\}$, then $\{P\}$**while B do S od**$\{P \wedge \neg B\}$.

▶ Prove $\{P \wedge B\}\mathbf{S}\{P\}$ or $P \wedge B \Rightarrow \{\mathbf{S}\}(P)$.

▶ By the iteration rule, we conclude that
  $\{P\}$**while B do S od**$\{P \wedge \neg B\}$ without using WLP.

▶ Introduction of LOOP INVARIANTS in the notation.

Listing 12 – loop.c

```
/*@ loop invariant I1;
    loop invariant I2;
    ...
    loop invariant In;
    loop assigns X;
    loop variant E;
  */
```

(Invariant de boucle)

Listing 13 – project-divers/anno6.c

```c
/*@ requires a >= 0 && b >= 0;
   ensures 0 <= \result;
   ensures \result < b;
   ensures \exists integer k; a == k * b + \result;
*/
int rem(int a, int b) {
   int r = a;
   /*@
     loop invariant
     (\exists integer i; a == i * b + r) &&
     r >= 0;
     loop assigns r;
    */
   while (r >= b) { r = r - b; };
   return r;
}
```

Initial value of x $\backslash old(x)$

▶ $\backslash old(x)$ is the value of the variable when the function is called.
▶ It can be used in the postcondition of the *ensures* clause.

(Modifying variables while calling)

Listing 14 – project-divers/old1.c

```
/*@ requires \valid(a) && \valid(b);
  @ assigns *a,*b;
  @ ensures   *a == \at(*a,Pre) +2;
  @ ensures   *b == \at(*b,Pre)+\at(*a,Pre)+2;

  @ ensures   \result == 0;
*/
int old(int *a, int *b) {
  int x,y;
  x = *a;
  y = *b;
  x=x+2;
  y = y +x;

  *a = x;
  *b = y;
  return 0 ;
}
```

- $\backslash at(e, id)$ is the value of e at the control point id.
- id should occur before $\backslash at(e, id)$
- id is one of the possible expressions : Pre, Here, Old, Post, LoopEntry, LoopCurrent, Init
- $\backslash old(e)$ is equivalent to $\backslash at(e, Old)$

Exemple pour $\backslash at(e, id)$

(label Pre)

Listing 15 – project-divers/at1.c

```c
/*@
    requires \valid(a) && \valid(b);
    assigns *a,*b;
    ensures  *a == \old(*a)+2;
    ensures  *b == \old(*b)+\old(*a)+2;
*/
int at1(int *a, int *b) {
//@ assert *a == \at(*a,Pre);
    *a = *a +1;
//@ assert *a == \at(*a,Pre)+1;
    *a = *a +1;
//@ assert *a == \at(*a,Pre)+2;
    *b = *b +*a;
//@ assert *a == \at(*a,Pre)+2 && *b == \at(*b,Pre)+\at(*a,Pre)+2;
    return 0;
}
```

–

Example for $\at(e, id)$

Listing 16 – project-divers/at2.c

```
void f (int n) {
for (int i = 0; i < n; i++) {
/*@ assert \at(i, LoopEntry) == 0; */
int j=0;
while (j++ < i) {
/*@ assert \at(j, LoopEntry) == 0; */
/*@ assert \at(j, LoopCurrent) + 1 == j; */
} }
}
```

(otherlabel)

Listing 17 – project-divers/change1.c

```c
/*@ requires \valid(a) && *a >= 0;
  @ assigns *a;
  @ ensures  *a == \old(*a)+2 && \result == 0;
*/
int change1(int *a)
{   int x = *a;
    x = x + 2;
    *a = x;
    return 0;
}
```

Ghost variable

▶ A variable called *ghost* allows to model a computed value useful for stating a model property : the ghost variable is hidden for the computer but not for the model.

▶ It should not change the semantics of others variables and should not change the effective variables.

Wrong use of ghost variable

(Bug)

Listing 18 – project-divers/ghost2.c

```
int f (int x, int y) {
   //@ghost int z=x+y;
switch (x) {
case 0: return y;
//@ ghost case 1: z=y;
// above statement is correct.
//@ ghost case 2: { z++; break; }
// invalid, would bypass the non—ghost default
default: y++; }
return y; }

int g(int x) { //@ ghost int z=x;
if (x>0){return x;}
//@ ghost else { z++; return x; }
// invalid, would bypass the non—ghost return
return x+1; }
```

(Ghost variable)

Listing 19 – project-divers/ghost1.c

```c
/*@ requires a >= 0 && b >= 0;
    ensures 0 <= \result;
    ensures \result < b;
    ensures \exists integer k; a == k * b + \result; */
int rem(int a, int b) {
    int r = a;
  /*@ ghost    int q=0;    */
    /*@
       loop invariant
       a == q * b + r &&
       r >= 0 &&   r <= a;
       loop assigns r;
       loop assigns q;
// loop variant r;
*/
    while (r >= b) {
       r = r - b;
/*@ ghost     q = q+1;    */
    };
    return r;
}
```

Listing 20 – an1.c

```
//@ assert l1: P(x);
  x = e(x);
//@ assert l2: Q(x);
```

Listing 21 – an1.c

```
//@ assert l1: P(x);
  x = e(x);
//@ assert l2: Q(x);
```

▶ $P(x) \Rightarrow WP(x := e(x))(Q(x))$

Listing 22 – an1.c

```
//@ assert  l1 : P(x);
    x = e(x);
//@ assert  l2 : Q(x);
```

▶ $P(x) \Rightarrow WP(x := e(x))(Q(x))$
▶ $P(x) \Rightarrow Q[x \mapsto e(x)]$

Listing 23 – an1.c

```
//@ assert l1: P(x);
  x = e(x);
//@ assert l2: Q(x);
```

- ▶ $P(x) \Rightarrow WP(x := e(x))(Q(x))$
- ▶ $P(x) \Rightarrow Q[x \mapsto e(x)]$
- ▶ $P(x1) \Rightarrow Q[x \mapsto e(x1)]$ (renaming of free occurences of x by x1)

Listing 24 – an1.c

```
//@ assert l1: P(x);
   x = e(x);
//@ assert l2: Q(x);
```

- $P(x) \Rightarrow WP(x := e(x))(Q(x))$
- $P(x) \Rightarrow Q[x \mapsto e(x))]$
- $P(x1) \Rightarrow Q[x \mapsto e(x1))]$ (renaming of free occurences of x by x1)
- $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$

Verification Condition for Assignment

<div align="center">Listing 25 – an1.c</div>

```
//@ assert l1: P(x);
  x = e(x);
//@ assert l2: Q(x);
```

- $P(x) \Rightarrow WP(x := e(x))(Q(x))$
- $P(x) \Rightarrow Q[x \mapsto e(x)]$
- $P(x1) \Rightarrow Q[x \mapsto e(x1)]$ (renaming of free occurences of x by x1)
- $P(x1) \land x = e(x1) \Rightarrow Q(x)$
- $P(x1) \land x = e(x1) \Rightarrow Q(x)$

Listing 26 – an1.c

```
//@ assert l1: P(x);
   x = e(x);
//@ assert l2: Q(x);
```

- $P(x) \Rightarrow WP(x := e(x))(Q(x))$
- $P(x) \Rightarrow Q[x \mapsto e(x)]$
- $P(x1) \Rightarrow Q[x \mapsto e(x1)]$ (renaming of free occurences of x by x1)
- $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- $\vdash P(x1) \wedge x = e(x1) \Rightarrow Q(x)$

Listing 27 – an1.c

```
//@ assert l1: P(x);
  x = e(x);
//@ assert l2: Q(x);
```

▶ $P(x) \Rightarrow WP(x := e(x))(Q(x))$
▶ $P(x) \Rightarrow Q[x \mapsto e(x))]$
▶ $P(x1) \Rightarrow Q[x \mapsto e(x1))]$ (renaming of free occurences of x by x1)
▶ $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
▶ $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
▶ $\vdash P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
▶ $P(x1) \wedge x = e(x1) \vdash Q(x)$

Verification Condition for Assignment

Listing 28 – an1.c

```
//@ assert l1: P(x);
  x = e(x);
//@ assert l2: Q(x);
```

- $P(x) \Rightarrow WP(x := e(x))(Q(x))$
- $P(x) \Rightarrow Q[x \mapsto e(x)]$
- $P(x1) \Rightarrow Q[x \mapsto e(x1)]$ (renaming of free occurences of x by x1)
- $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- $P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- $\vdash P(x1) \wedge x = e(x1) \Rightarrow Q(x)$
- $P(x1) \wedge x = e(x1) \vdash Q(x)$
  ```
  Assume {
  ```
  $P(x1)$
- $x = e(x1)$
  ```
  }
  Prove:
  ```
  $Q(x)$

Listing 29 – an1.c

```
void ex(void) {
  int x=12,y=24;
  //@ assert l1: 2*x == y;
  x = x+1;
  //@ assert l2:  y == 2*(x-1);
}
```

Annotation simple(I)

```
[kernel] Parsing an1.c (with preprocessing)
[wp] Running WP plugin...
[wp] Warning: Missing RTE guards
[wp] 2 goals scheduled
[wp] Proved goals:    4 / 4
  Terminating:      1
  Unreachable:      1
  Qed:              2
```

```
Goal Assertion 'l1' (file an1.c, line 3):
Assume {
  Type: is_sint32(x) /\ is_sint32(y).
  (* Initializer *)
  Init: x = 12.
  (* Initializer *)
  Init: y = 24.
}
Prove: (2 * x) = y.
Prover Qed returns Valid
```

Annotation simple (I)

```
-----------------------------------------------------------

Goal Assertion 'l2' (file an1.c, line 5):
Assume {
  Type: is_sint32(x) /\ is_sint32(x_1) /\ is_sint32(y).
  (* Initializer *)
  Init: x_1 = 12.
  (* Initializer *)
  Init: y = 24.
  (* Assertion 'l1' *)
  Have: (2 * x_1) = y.
  Have: (1 + x_1) = x.
}
Prove: (2 + y) = (2 * x).
Prover Qed returns Valid
```

Annotation simple(I)

```
[kernel] Parsing an1.c (with preprocessing)
[wp] Running WP plugin...
[wp] Warning: Missing RTE guards
[wp] 2 goals scheduled
[wp] Proved goals:    4 / 4
  Terminating:      1
  Unreachable:      1
  Qed:              2
```

Listing 30 – an2.c

```
void ex(void) {
    int x=12,y=24;
    //@ assert l1:  2*x == y;
    x = x+1;
    //@ assert l2:   y == 2*(x-1);
    x = x+2;
    //@ assert l3:   y+6 == 2*x;
}
```

Annotation simple (II)

```
[kernel] Parsing an2.c (with preprocessing)
[wp] Running WP plugin...
[wp] Warning: Missing RTE guards
[wp] 3 goals scheduled
[wp] Proved goals:    5 / 5
  Terminating:      1
  Unreachable:      1
  Qed:              3
```

```
Goal Assertion 'l1' (file an2.c, line 3):
Assume {
  Type: is_sint32(x) /\ is_sint32(y).
  (* Initializer *)
  Init: x = 12.
  (* Initializer *)
  Init: y = 24.
}
Prove: (2 * x) = y.
Prover Qed returns Valid
```

Annotation simple (ii)

```
Goal Assertion 'l2' (file an2.c, line 5):
Assume {
  Type: is_sint32(x) /\ is_sint32(x_1) /\ is_sint32(y).
  (* Initializer *)
  Init: x_1 = 12.
  (* Initializer *)
  Init: y = 24.
  (* Assertion 'l1' *)
  Have: (2 * x_1) = y.
  Have: (1 + x_1) = x.
}
Prove: (2 + y) = (2 * x).
Prover Qed returns Valid
```

Annotation simple (ii)
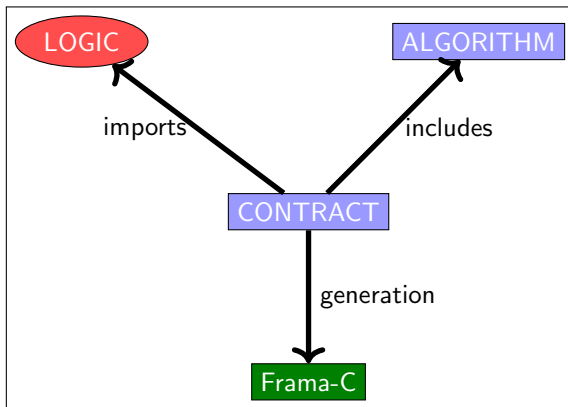
```
Goal Assertion 'l3' (file an2.c, line 7):
Assume {
  Type: is_sint32(x) /\ is_sint32(x_1) /\ is_sint32(x_2) /\ is_s
  (* Initializer *)
  Init: x_2 = 12.
  (* Initializer *)
  Init: y = 24.
  (* Assertion 'l1' *)
  Have: (2 * x_2) = y.
  Have: (1 + x_2) = x_1.
  (* Assertion 'l2' *)
  Have: (2 + y) = (2 * x_1).
  Have: (2 + x_1) = x.
}
Prove: (6 + y) = (2 * x).
Prover Qed returns Valid
```

Listing 31 – an2bis.c

```
void ex(void) {
  int x=12,y=24;
  //@ assert l1: 2*x == y;
  x = x+1;
  //@ assert l2:  y == 2*(x-1);
   x = x+2;
  //@ assert l3:  y+6 == 2*x;
}
```

▶ Hoare Model : `-wp hoare` is the option of frama-c

▶ Typed Model : default model is typed model

▶ It simply maps each C variable to one pure logical variable.

▶ Heap cannot be represented in this model, and expressions such as *p cannot be translated at all.

▶ You can still represent pointer values, but you cannot read or write the heap through pointers.

Typed Model

▶ The default model for WP plug-in.

▶ Heap values are stored in several separated global arrays, one for each atomic type (integers, floats, pointers) and an additional one for memory allocation.

▶ Pointer values are translated into an index into these arrays.

▶ all C integer types are represented by mathematical integers and each pointer type to a given type is represented by a specific logical abstract datatype.

▶ predicate

Predicates - Logic - Lemma (1)

(Predicate)

Listing 32 – project-divers/predicate1.c

```
/*@ predicate is_positive(integer x) = x > 0; */

/*@ logic integer get_sign(real x) =     @ x > 0.0?1:(x < 0.0?−1:0);
*/
/*@ logic integer max(int x, int y) =     x>=y?x:y;
*/
```

(Lemma)

Listing 33 – project-divers/lemma1.c

```
/*@ lemma div_mul_identity:
@ \forall real x, real y; y != 0.0 ==> y*(x/y) == x; @*/

/*@  lemma div_qr:
@ \forall int a, int b; a >= 0 && b >0 ==>
\exists  int q, int r; a == b*q +r && 0<=r && r <b; @*/
```

(Definition of fibonacci function)

Listing 34 – project-divers/predicate2.c

```
/*@ axiomatic mathfibonacci{
  @ logic integer mathfib(integer n);
  @ axiom mathfib0: mathfib(0) == 1;
  @ axiom mathfib1: mathfib(1) == 1;
  @ axiom mathfibrec: \forall integer n; n > 1
  ==> mathfib(n) == mathfib(n-1)+mathfib(n-2);
  @ } */
```

(Definition of gcd)

Listing 35 – project-divers/predicate3.c

```
/*@ inductive is_gcd(integer a, integer b, integer d) {
@ case gcd_zero:
@ \forall integer n; is_gcd(n,0,n);
@ case gcd_succ:
@ \forall integer a,b,d; is_gcd(b, a % b, d) ==> is_gcd(a,b,d); @}
@*/
```

(Definition of function odd/even)

Listing 36 – project-divers/predicate4.c

```
//@ predicate pair(integer x) = (x/2)*2==x;
//@ predicate impair(integer x) = (x/2)*2!=x;
//@ lemma ex: \forall integer a,b; a < b ==> 2*a < 2*b;

/*@ inductive   is_gcd(integer a,integer b , integer c) {
  case zero:   \forall integer n; is_gcd(n,0,n);
    case un:   \forall integer u,v,w; u >= v ==>  is_gcd(u-v,v,w);
    case deux:   \forall integer u,v,w; u <  v ==>  is_gcd(u,v-u,w);
    }
*/
```

Loop termination

▶ The termination is proved by shoiwing that eaxg loop terminates.

▶ Any loop is characterized by an expression `expvariant(x)` called `variant` which should decrease each execution of the body :

$$\forall x_1, x_2.b(x_1) \wedge x_1 \xrightarrow{\text{S}} x_2 \Rightarrow \text{expvariant}(x_1) > \text{expvariant}(x_2)$$

(Variant)

Listing 37 – project-divers/variant1.c

```c
/*@ requires n > 0;
  terminates n > 0;

  ensures \result == 0;
*/
int code(int n) {
  int x = n;
  /*@ loop invariant x >= 0 && x <= n;
    loop assigns x;
    loop variant x;
  */
  while (x != 0) {
    x = x - 1;
  };
  return x;
}
```

Example of loop variant

(Variant)

Listing 38 – project-divers/variant3.c

```c
int f() {
int x = 0;
int y = 10;
/*@
    loop invariant
    0 <= x < 11 && x+y == 10;
    loop variant y;
    */
while (y > 0) {
  x++;
  y--;
}
 return 0;
}
```

(Variant)

Listing 39 – project-divers/variant4.c

```
g/*@ requires n <= 12;
  @ decreases n;
  @*/
int fact(int n){
  if (n <= 1) return 1;
  return n*fact(n−1);
}
```

- ► – lemma : 1 VC
- ► – axiom : no VC (admitted with no proof)
- ► – ensures : 1 VC
- ► – exits : 1 VC
- ► – disjoint : 1 VC
- ► – complete : 1 VC
- ► – requires : 1 VC for each call
- ► – terminates : 1 VC for each call, 1 VC for each loop without "loop variant"
- ► – decreases : 1 VC for each recursive call
- ► – assigns : 1 VC for each assigned lvalue
- ► – admit : no VC (admitted with no proof)
- ► – assert/check : 1 VC
- ► – loop invariant : 2 VCs (established, preserved)
- ► – loop variant (integer) : 2 VCs (positive, decreasing)
- ► – loop variant (general measure) : 1 VC (the measure is assumed to be well-founded) – loop assigns : 1 VC for each assigned lvalue within the loop

Main steps

▶ Defining th mathematical function to compute *mathf*

▶ Stating the postcondition using the mathematical function

▶ Evaluating the inductive sequence u_i computing the function *mathf*

▶ $\forall i \in \mathbb{N} : u_i = marthf(i)$

▶ Evaluating relationship among variables.

Factorial function

(power2.h)

Listing 40 – project-powers/power21.h

```
#ifndef _A_H
#define _A_H
// Definition of the mathematical function mathpower2
/*@ axiomatic mathpower2 {
  @ logic integer mathpower2(integer n);
  @ axiom mathpower2_0: mathpower2(0) == 0;
  @ axiom mathpower2_rec: \forall integer n; n > 0
  ==> mathpower2(n) == mathpower2(n-1) + n+n+1;
  @ } */
/*@ axiomatic matheven {
  @ logic integer matheven(integer n);
  @ axiom matheven_0: matheven(0) == 0;
  @ axiom mathpeven_rec: \forall integer n; n > 0
  ==> matheven(n) == matheven(n-1) + 2;
  @ } */
// We define v and w in a one shot axiomatic definition
/*@ axiomatic vw {
  @ logic integer v(integer n);
  @ logic integer w(integer n);
  @ axiom v_0: v(0) == 0;
  @ axiom w_0: w(0) == 0;
  @ axiom v_rec: \forall integer n; n > 0
  ==> v(n) == v(n-1) +n+n+1 &&  w(n) == w(n-1) + 2;
  @ } */
```

Factorial function

Listing 41 – project-powers/power22.h

```
/*@ lemma propw :
@ \forall int n; n >= 0 ==> w(n) == n+n; @*/
/*@ lemma propv :
@ \forall int n; n >= 0 ==> v(n) == n*n; @*/
/*@ lemma prop1 :
@ \forall int n; n >= 0 ==> matheven(n) == n+n; @*/
/*@ lemma prop2 :
@ \forall int n; n >= 0 ==> mathpower2(n) == n*n; @*/
/*@ axiomatic auxmath {
  @ lemma rule1 : \forall int n; n >0 ==> n*n == (n−1)*(n−1)+2*(n−1)+1;
  @ } */
/*@   requires   0 <= x;
      assigns \nothing ;
      ensures \result == x*x;
*/
int power2(int x);
#endif
```

Factorial function

Listing 42 – project-powers/power2.c

```c
#include <limits.h>
#include "power2.h"

int power2(int x)
{ int   r,k,cv,cw,or,ok,ocv,ocw;
    r=0;k=0;cv=0;cw=0;or=0;ok=k;ocv=cv;ocw=cw;
        /*@ loop invariant  0 <= cv && 0 <= cw && 0 <= k;
          @ loop invariant  cv == k*k;
          @ loop invariant  k  <= x;
          @ loop invariant  cw  == 2*k;
          @ loop invariant  4*cv  == cw*cw;
          @ loop assigns  k,cv,cw,or,ok,ocv,ocw; */
    while (k<x)
      {
      ok=k;ocv=cv;ocw=cw;
        k=ok+1;
        cv=ocv+ocw+1;
        cw=ocw+2;

      }
    r=cv;
    return(r);
}
```

▶ Plugin -rte adds specifi assertions for each variable
▶ Systematic checking of RTE.

- ▶ Defining domain properties (axioms, lemmas, proofs)
- ▶ Defining loop invariants (typing, equation, . . .)
- ▶ Analyzing inductive properties
- ▶ Identifying inputs (*requires*) and outputs (*ensures*)