

**Exercise 1** *contract-annotations*

For each case, define a contract for checking the soundness or the unsoundness of the annotation.

$\forall x, y, x', y'. P_\ell(x, y) \wedge \text{cond}_{\ell, \ell'}(x, y) \wedge (x', y') = f_{\ell, \ell'}(x, y) \Rightarrow P_{\ell'}(x', y')$

You will use a context and a machine for expressing these conditions.

—  $\ell_1 : x = 10 \wedge y = z + x \wedge z = 2 \cdot x$   
 $y := z + x$   
 $\ell_2 : x = 10 \wedge y = x + 2 \cdot 10$

— We assume that  $p$  is a prime number.

$\ell_1 : x = 2^p \wedge y = 2^{p+1} \wedge x \cdot y = 2^{2 \cdot p + 1}$   
 $x := y + x + 2^x$   
 $\ell_2 : x = 5 \cdot 2^p \wedge y = 2^{p+1}$

—  $\ell_1 : x = 1 \wedge y = 12$   
 $x := 2 \cdot y$   
 $\ell_2 : x = 1 \wedge y = 24$

—  $\ell_1 : x = 11 \wedge y = 13$   
 $z := x; x := y; y := z;$   
 $\ell_2 : x = 26/2 \wedge y = 33/3$

**Exercise 2** *(contract-simple)*

Let the following partially annotated algorithm :

**precondition** :  $x = x_0 \wedge x_0 \in \mathbb{N}$   
**postcondition** :  $x = 0$   
 $\ell_0 : \{x = x_0 \wedge x_0 \in \mathbb{N}\}$   
**while**  $0 < x$  **do**  
     $\ell_1 : \{0 < x \leq x_0 \wedge x_0 \in \mathbb{N}\}$   
     $x := x - 1;$   
     $\ell_2 : \{0 \leq x \leq x_0 \wedge x_0 \in \mathbb{N}\}$   
;  
 $\ell_3 : \{x = 0\}$

**Algorithme 1:** Exercice 2

**Question 2.1** Translate each transition  $\ell, \ell'$  into an event modifying the variables according to the statements.

**Question 2.2** Define an invariant attaching to each label an assertion satisfied at the control point.

**Question 2.3** Verify proof obligations and deduce that the algorithm is partially correct.

**Question 2.4** Prove that the algorithm has no runtime error.

**Exercise 3** (contract-squareroot)

Let the following annotated invariant.

```

precondition :  $x \in \mathbb{N}$ 
postcondition :  $z^2 \leq x \wedge x < (z+1)^2$ 
local variables :  $y_1, y_2, y_3 \in \mathbb{N}$ 

pre :  $\{x \in \mathbb{N}\}$ 
post :  $\{z \cdot z \leq x \wedge x < (z+1) \cdot (z+1)\}$ 
 $\ell_0$  :  $\{x \in \mathbb{N} \wedge z \in \mathbb{Z} \wedge y_1 \in \mathbb{Z} \wedge y_2 \in \mathbb{Z} \wedge y_3 \in \mathbb{Z}\}$ 
 $(y_1, y_2, y_3) := (0, 1, 1);$ 
 $\ell_1$  :  $\{y_2 = (y_1+1) \cdot (y_1+1) \wedge y_3 = 2 \cdot y_1 + 1 \wedge y_1 \cdot y_1 \leq x\}$ 
while  $y_2 \leq x$  do
   $\ell_2$  :  $\{y_2 = (y_1+1) \cdot (y_1+1) \wedge y_3 = 2 \cdot y_1 + 1 \wedge y_2 \leq x\}$ 
   $(y_1, y_2, y_3) := (y_1+1, y_2+y_3+2, y_3+2);$ 
   $\ell_3$  :  $\{y_2 = (y_1+1) \cdot (y_1+1) \wedge y_3 = 2 \cdot y_1 + 1 \wedge y_1 \cdot y_1 \leq x\}$ 
;
 $\ell_4$  :  $\{y_2 = (y_1+1) \cdot (y_1+1) \wedge y_3 = 2 \cdot y_1 + 1 \wedge y_1 \cdot y_1 \leq x \wedge x < y_2\}$ 
 $z := y_1;$ 
 $\ell_5$  :  $\{y_2 = (y_1+1) \cdot (y_1+1) \wedge y_3 = 2 \cdot y_1 + 1 \wedge y_1 \cdot y_1 \leq x \wedge x < y_2 \wedge z = y_1 \wedge z \cdot z \leq x \wedge x < (z+1) \cdot (z+1)\}$ 

```

**Algorithme 2:** squareroot annotée Exercice ??

**Question 3.1** Translate each transition  $\ell, \ell'$  into an event modifying the variables according to the statements.

**Question 3.2** Define an invariant attaching to each label an assertion satisfied at the control point.

**Question 3.3** Verify proof obligations and deduce that the algorithm is partially correct.

**Question 3.4** Prove that the algorithm has no runtime error.

**Exercise 4** (contract-maximum)

Soit l'algorithme suivant annoté partiellement :

**Question 4.1** Translate each transition  $\ell, \ell'$  into an event modifying the variables according to the statements.

**Question 4.2** Define an invariant attaching to each label an assertion satisfied at the control point.

**Question 4.3** Verify proof obligations and deduce that the algorithm is partially correct.

**Question 4.4** Prove that the algorithm has no runtime error.

/\* algorithme de calcul du maximum avec une boucle while de l'exercice ?? \*/

**precondition** :  $\left( \begin{array}{l} n \in \mathbb{N} \wedge \\ n \neq 0 \wedge \\ f \in 0..n-1 \rightarrow \mathbb{N} \end{array} \right)$

**postcondition** :  $\left( \begin{array}{l} m \in \mathbb{N} \wedge \\ m \in \text{ran}(f) \wedge \\ (\forall j \cdot j \in 0..n-1 \Rightarrow f(j) \leq m) \end{array} \right)$

**local variables** :  $i \in \mathbb{Z}$

$\ell_0 : \left\{ \left( \begin{array}{l} n \in \mathbb{N} \wedge \\ n \neq 0 \wedge \\ f \in 0..n-1 \rightarrow \mathbb{N} \end{array} \right) \wedge i \in \mathbb{Z} \wedge m \in \mathbb{Z} \right\}$

$m := f(0);$

$\ell_1 : \left\{ \left( \begin{array}{l} n \in \mathbb{N} \wedge \\ n \neq 0 \wedge \\ f \in 0..n-1 \rightarrow \mathbb{N} \end{array} \right) \wedge i \in \mathbb{Z} \wedge m = f(0) \right\}$

$i := 1;$

$\ell_2 : \left\{ \left( \begin{array}{l} n \in \mathbb{N} \wedge \\ n \neq 0 \wedge \\ f \in 0..n-1 \rightarrow \mathbb{N} \end{array} \right) \wedge i = 1 \wedge \left( \begin{array}{l} m \in \mathbb{N} \wedge \\ m \in \text{ran}(f[0..i-1]) \wedge \\ (\forall j \cdot j \in 0..i-1 \Rightarrow f(j) \leq m) \end{array} \right) \right\}$

**while**  $i < n$  **do**

$\ell_3 : \left\{ \left( \begin{array}{l} n \in \mathbb{N} \wedge \\ n \neq 0 \wedge \\ f \in 0..n-1 \rightarrow \mathbb{N} \end{array} \right) \wedge i \in 1..n-1 \wedge \left( \begin{array}{l} m \in \mathbb{N} \wedge \\ m \in \text{ran}(f[0..i-1]) \wedge \\ (\forall j \cdot j \in 0..i-1 \Rightarrow f(j) \leq m) \end{array} \right) \right\}$

**if**  $f(i) > m$  **then**

$\ell_4 : \left\{ \left( \begin{array}{l} n \in \mathbb{N} \wedge \\ n \neq 0 \wedge \\ f \in 0..n-1 \rightarrow \mathbb{N} \end{array} \right) \wedge i \in 1..n-1 \wedge \left( \begin{array}{l} m \in \mathbb{N} \wedge \\ m \in \text{ran}(f[0..i-1]) \wedge \\ (\forall j \cdot j \in 0..i-1 \Rightarrow f(j) \leq m) \end{array} \right) \wedge \right.$

$f(i) > m \}$

$m := f(i);$

$\ell_5 : \left\{ \left( \begin{array}{l} n \in \mathbb{N} \wedge \\ n \neq 0 \wedge \\ f \in 0..n-1 \rightarrow \mathbb{N} \end{array} \right) \wedge i \in 1..n-1 \wedge \left( \begin{array}{l} m \in \mathbb{N} \wedge \\ m \in \text{ran}(f[0..i]) \wedge \\ (\forall j \cdot j \in 0..i \Rightarrow f(j) \leq m) \end{array} \right) \right\}$

;

$\ell_6 : \left\{ \left( \begin{array}{l} n \in \mathbb{N} \wedge \\ n \neq 0 \wedge \\ f \in 0..n-1 \rightarrow \mathbb{N} \end{array} \right) \wedge i \in \mathbb{Z} \wedge i \in 1..n-1 \wedge \left( \begin{array}{l} m \in \mathbb{N} \wedge \\ m \in \text{ran}(f[0..i]) \wedge \\ (\forall j \cdot j \in 0..i \Rightarrow f(j) \leq m) \end{array} \right) \right\}$

$i++;$

$\ell_7 : \left\{ \left( \begin{array}{l} n \in \mathbb{N} \wedge \\ n \neq 0 \wedge \\ f \in 0..n-1 \rightarrow \mathbb{N} \end{array} \right) \wedge i \in 2..n \wedge \left( \begin{array}{l} m \in \mathbb{N} \wedge \\ m \in \text{ran}(f[0..i-1]) \wedge \\ (\forall j \cdot j \in 0..i-1 \Rightarrow f(j) \leq m) \end{array} \right) \right\}$

;

$\ell_8 : \left\{ \left( \begin{array}{l} n \in \mathbb{N} \wedge \\ n \neq 0 \wedge \\ f \in 0..n-1 \rightarrow \mathbb{N} \end{array} \right) \wedge i = n \wedge \left( \begin{array}{l} m \in \mathbb{N} \wedge \\ m \in \text{ran}(f) \wedge \\ (\forall j \cdot j \in 0..n-1 \Rightarrow f(j) \leq m) \end{array} \right) \right\}$

**Algorithme 3:** Algorithme du manimum d'une liste annoté Exercice 4