

# Probability Theory

Oualid Merzouga

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# 1 Set Theory

Let  $C_1, C_2$  be countable sets and  $X_i : \Omega \longrightarrow \mathcal{C}$   $i \in \{1, 2\}$  be random variables.

$$P\{X_1 + X_2 \in B\} = \int_{\mathbb{R}} P_{X_1}(B - x_2) dP_{X_2}(x_2) \quad \text{Definition} \quad (1)$$

$$= \int_{\mathbb{R}} \sum_{c_i \in \mathcal{C}_{\infty}} p_{X_1}(c_i) \mathbb{1}_{\{B - x_2\}}(c_i) dP_{X_2}(x_2) \quad \text{Expanding } P_{X_1} \quad (2)$$

$$= \int_{\mathbb{R}} \sum_{c_i \in \mathcal{C}_{\infty}} p_{X_1}(c_i) \mathbb{1}_{\{B - c_i\}}(x_2) dP_{X_2}(x_2) \quad c_i \in B - x_2 \implies x_2 \in B - c_i \quad (3)$$

$$= \sum_{c_i \in \mathcal{C}_{\infty}} p_{X_1}(c_i) P_{X_2}(B - c_i) \quad (4)$$

$$= \sum_{c_i \in \mathcal{C}_{\infty}} \sum_{b_i \in \mathcal{C}_{\infty}} p_{X_1}(c_i) p_{X_2}(b_i) \mathbb{1}_{\{B - c_i\}}(b_i) \quad (5)$$

$$= \boxed{\sum_{c_i \in \mathcal{C}_{\infty}} \sum_{b_i \in \{B - c_i\}} p_{X_1}(c_i) p_{X_2}(b_i)} \quad (6)$$

## 1.1 Subsection

Structuring a document is easy!