# Precalculus: Chapter 5 Notes

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## 1 Exponents

### Laws of Exponents

If $s$, $t$, $a$, and $b$ are real numbers with $a > 0$ and $b > 0$, then

$$a^s \cdot a^t = a^{s+t} \qquad (a^s)^t = a^{st} \qquad (ab)^s = a^s \cdot b^s \tag{1.1}$$

$$1^s = 1 \qquad a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s \qquad a^0 = 1 \tag{1.2}$$

### Exponential Functions

An **exponential function** is a function of the form

$$f(x) = Ca^x \tag{1.3}$$

where $a \in \mathbb{R}$, $a > 0$, $a \neq 1$, and $C \neq 0$ is a real number. The domain of $f$ is $\mathbb{R}$. The base $a$ is the **growth factor**, and because $f(0) = Ca^0 = C$, we call $C$ the **initial value**.

For $f(x) = Ca^x$, where $a > 0$ and $a \neq 1$, if $x \in \mathbb{R}$, then

$$\frac{f(x+1)}{f(x)} = a \qquad \text{or} \qquad f(x+1) = af(x) \tag{1.4}$$

### The number $e$

$$e = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n \tag{1.5}$$

### Solving Exponential Equations

Use this property, expressing each side of the equation using the same base:

$$\text{If } a^u = a^v, \text{ then } u = v \tag{1.6}$$

# 2 Logarithms

$\log_a x$ represents the exponent to which $a$ must be raised to obtain $x$.

$$y = \log_a x \text{ if and only if } x = a^y \tag{2.1}$$
$$y = \ln x \text{ if and only if } x = e^y \tag{2.2}$$
$$y = \log x \text{ if and only if } x = 10^y \tag{2.3}$$

The domain of $y = \log_a x$ is $\{x|x > 0\}$.

## Logarithmic Functions

The logarithmic function is the inverse of the exponential function.

$$\text{If } f(x) = a^x, \text{ then } f^{-1}(x) = \log_a x \tag{2.4}$$

The domain of $f^{-1}$ is the range of $f$. The range of $f^{-1}$ is the domain of $f$.

## Properties of Logarithms

In the following properties, $M$, $N$, and $a$ are positive real numbers, $a \neq 1$, and $r \in \mathbb{R}$.

$$\log_a 1 = 0 \qquad log_a a = 1 \tag{2.5}$$
$$a^{\log_a M} = M \qquad \log_a a^r = r \tag{2.6}$$
$$\log_a M^r = r \log_a M \qquad a^x = e^{x \ln a} \tag{2.7}$$

$$\log_a(MN) = \log_a M + \log_a N \qquad \log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N \tag{2.8}$$

$$\text{If } M = N, \text{ then } \log_a M = \log_a N \qquad \text{If } \log_a M = \log_a N, \text{ then } M = N \tag{2.9}$$

## Change of Base Formula

If $a \neq 1$, $b \neq 1$, and $M$ are positive real numbers, then

$$\log_a M = \frac{\log_b M}{\log_b a} \qquad \text{so,} \qquad \log_a M = \frac{\log M}{\log a} \qquad \text{and} \qquad \log_a M = \frac{\ln M}{\ln a} \tag{2.10}$$