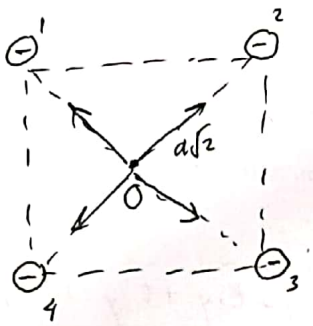
Solusi Tutorial 2 Fisika Dasar 2A ITB

oleh: Wawan K

A. Pertanyaan

① (i) Untuk situasi (a)



② medan listrik di O

$E_{di\,O} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$

- karena $\vec{E}_2$ berlawan dengan $\vec{E}_4$, sehingga saling meniadakan
- $\vec{E}_1$ dan $\vec{E}_3$ saling berlawanan, dan meniadakan

maka $\vec{E}_O = 0$

Jadi, medan listrik di O adalah nol.

③ untuk potensial di O

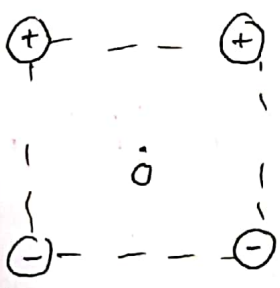
$V_O = V_1 + V_2 + V_3 + V_4$

$= -\frac{kq}{r} - \frac{kq}{r} - \frac{kq}{r} - \frac{kq}{r}$

$V_O = -4\frac{kq}{r}$

$V_O \neq 0$

(ii) untuk situasi (b)



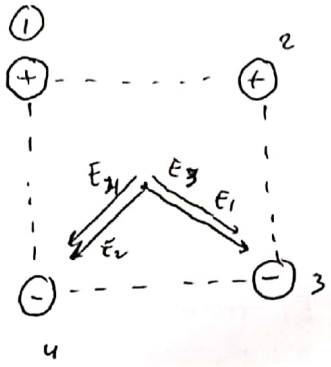
③ Potensial di O

$V_O = V_+ + V_+ + V_- + V_-$

$= \frac{kq}{r} + \frac{kq}{r} - \frac{kq}{r} - \frac{kq}{r}$

$= 0$

(1) ·) medan listrik di O



$\vec{E}_O = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$

·) komponen arah x saling meniadakan,

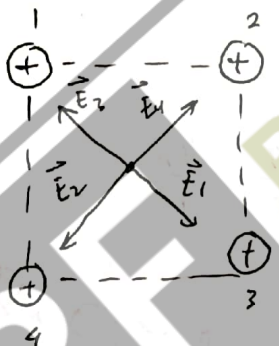
$E_{1x} + E_{3x} - E_{2x} - E_{4x} = 0$

·) komponen arah y saling menjumlahkan

$\vec{E}_O = \vec{E}_{1y} + \vec{E}_{2y} + \vec{E}_{3y} + \vec{E}_{4y}$

Jadi $\vec{E}_o \neq 0$

(iii) untuk situasi (c)



·) Medan listrik di O

$\vec{E}_O = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$

·) $\vec{E}_1$ dan $\vec{E}_3$ saling meniadakan (besar sama dan arahnya berlawanan)

·) $\vec{E}_2$ dan $\vec{E}_4$ saling meniadakan,

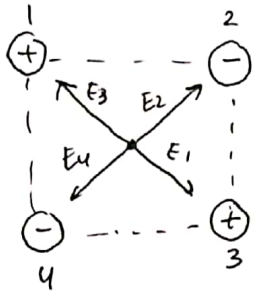
Sehingga $\vec{E}_O = 0$

· Potensial listrik di O

$$V_O = V_1 + V_2 + V_3 + V_4 = \frac{kq}{r} + \frac{kq}{r} + \frac{kq}{r} + \frac{kq}{r}$$

$$V_o = \frac{4kq}{r} \neq 0$$

① (iv) Situasi (d)



a) medan listrik di O

$\vec{E}_0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$

- $\vec{E}_1$ dan $\vec{E}_3$ saling meniadakan (besar sama, arah berlawanan.
- $\vec{E}_2$ dan $\vec{E}_4$ saling meniadakan

Sehingga $\vec{E}_0 = 0$

- Potensial listrik di O

$$V_0 = V_1 + V_2 + V_3 + V_4$$

$$V_0 = \frac{kq}{r} - \frac{kq}{r} + \frac{kq}{r} - \frac{kq}{r}$$

$$V_0 = 0$$

Jadi, jawaban nya adalah situasi (d), $E = 0$ dan $V = 0$

② Beda potensial listrik dapat kita tuliskan

$$\Delta V = -\int E\,dx$$

Untuk potensial di tak hingga, nilainya adalah nol, sehingga:

$$V(x) - V(\infty) = -\int E\,dx$$

$$V(x) - 0 = -\int E\,dx$$

$$V(x) = -\int E\,dx \rightarrow dV(x) = -E\,dx$$

$$\boxed{E = -\frac{dV(x)}{dx}}$$

(2) Sehingga medan listrik ada kaitan dengan potensial dan jarak.

$$E(x) = -\frac{dV(x)}{dx}$$

yang tidak lain adalah turunan V terhadap x (jarak)

minus gradien dari

.) Dalam grafik potensial terhadap x (jarak), medan listrik merupakan kurva tersebut.

.) Daerah A-B

$$\text{gradien} = \frac{\Delta y}{\Delta x}$$

$$E = -\frac{\Delta V}{\Delta x} = -\frac{(V_B - V_A)}{\Delta x} = 0$$

.) Daerah B-C

$$\text{gradien} = -\frac{\Delta y}{\Delta x}$$

$$E = -\frac{\Delta V}{\Delta x} = -\frac{(3-5)}{0{,}4-0{,}2} = \frac{+2}{\frac{2}{10}} = 10 \text{ V/m}$$

.) Daerah C-D

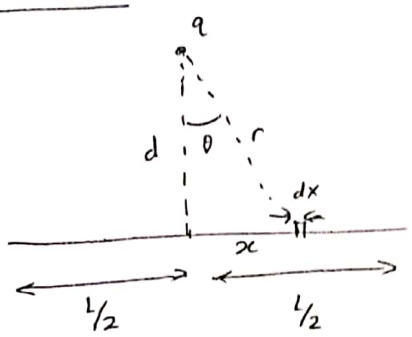
$$E = -\frac{\Delta V}{\Delta x} = -\frac{(-2)}{0{,}4} = 5 \text{ V/m}$$

Jadi, urutan besar medan listrik, $|E|_{AB} = 0$, $|E|_{BC} = 10$, $|E|_{CD} = 5$

Jadi, jawaban nya $\boxed{|E|_{BC} > |E|_{CD} > |E|_{AB}}$

(3) Jawaban nya adalah : a, b, c

Bukti : (i) Polensial di (a)



$$V_a = k\int \frac{dx}{r} = k\int \frac{\lambda\, dx}{\sqrt{d^2+x^2}}$$

kita lihat hubungan :

$$\tan\theta = \frac{x}{d}$$

$$x = d\tan\theta$$

$$dx = d\sec^2\theta\, d\theta$$

$$V_a = k\lambda \int \frac{d\sec^2\theta\, d\theta}{(d^2+d^2\tan^2\theta)^{1/2}}$$

$$V_a = \frac{k\lambda d}{d}\int \frac{\sec^2\theta\, d\theta}{(1+\tan^2\theta)^{1/2}}$$

$$V_a = k\lambda \int_{-\theta}^{\theta} \sec\theta\, d\theta$$

$$V_a = 2k\lambda \int_{0}^{\theta} \sec\theta\, d\theta = 2k\lambda \ln\left|\sec\theta + \tan\theta\right|_0^{\theta}$$

$$V_a = 2k\lambda \ln\left|\sec\theta + \tan\theta\right|$$

$$V_a = 2k\lambda \ln\left|\frac{\sqrt{\frac{L^2}{4}+d^2}}{d} + \frac{L/2}{d}\right|$$

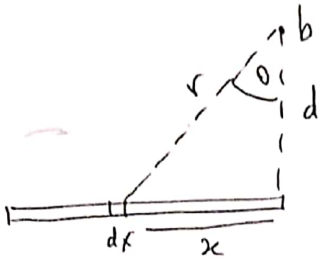
$$V_a = 2k\lambda \ln\left|\frac{\sqrt{\frac{L^2}{4}+d^2} + \frac{L}{2}}{d}\right|$$

misalkan $L = 1\ m$

$d = 0{,}4\ m$

maka : $V_a = 2k\lambda \ln|2{,}85| = 2{,}095\, k\lambda$

(3) (ii) Potensial di (b)



kita lihat: $\tan\theta = \frac{x}{d}$

$x = d\tan\theta$

$dx = d\sec^2\theta\, d\theta$

$$V_b = k\int \frac{\lambda dx}{r}$$

$$V_b = k\lambda \int \frac{d\sec^2\theta\, d\theta}{(d^2+x^2)^{1/2}}$$

$$V_b = k\lambda \int \frac{d\sec^2\theta\, d\theta}{(d^2+d^2\tan^2\theta)^{1/2}}$$

$$V_b = \frac{k\lambda d}{d}\int \frac{\sec^2\theta\, d\theta}{\sec\theta}$$

$$V_b = k\lambda \int_0^{\theta} \sec\theta\, d\theta$$

$$V_b = k\lambda \ln\left|\sec\theta + \tan\theta\right|_0^{\theta}$$

$$V_b = k\lambda \ln\left|\frac{\sqrt{L^2+d^2}}{d} + \frac{L}{d}\right|$$

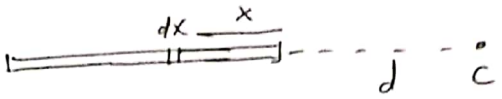
$$V_b = k\lambda \ln\left|\frac{\sqrt{L^2+d^2}+L}{d}\right|$$

misal $L = 1$

$d = 0{,}4$

maka $V_b = 1{,}647k\lambda$

(3) ·) Potensial di titik C



$V_C = k\lambda \int \frac{dx}{r}$

misalkan : $u = x+d$

$du = dx$

$= k\lambda \int \frac{dx}{(x+d)}$

$= k\lambda \int \frac{du}{u}$

$= k\lambda \ln u = k\lambda \ln (d+x)_0^L$

$V_C = k\lambda \ln \left(\frac{d+L}{d}\right)$

$V_C = k\lambda \ln \left(1+\frac{L}{d}\right)$

misal : $L = 1$

$d = 0{,}4$

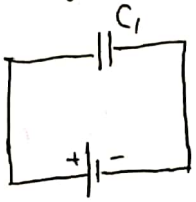
$V_C = k\lambda \ln \left(1+\frac{1}{0{,}4}\right)$

$V_C = 1{,}25\, k\lambda$

Jadi, Jawabannya adalah :

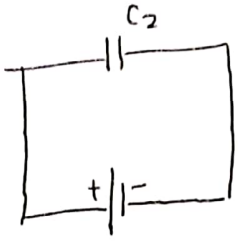
$$V_a > V_b > V_C$$

(4) (i) konfigurasi C_1 dengan baterai



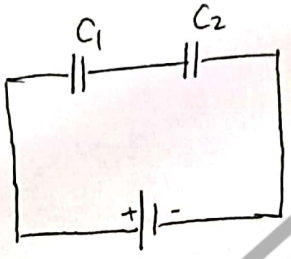
muatannya : $Q_1 = C_1 V$

(4) (ii) konfigurasi C_2 dengan baterai



$$Q_2 = C_2 V$$

(iii) konfigurasi C_1 dan C_2 disusun seri dengan baterai



C_1 dan C_2 dapat diganti menjadi $C_{ekivalen}$.

$$\frac{1}{C_{ekiv}} = \frac{1}{C_1} + \frac{1}{C_2}$$

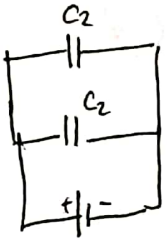
$$\frac{1}{C_{ekiv}} = \frac{C_1 + C_2}{C_1 C_2}$$

$$C_{ekiv} = \frac{C_1 C_2}{C_1 + C_2}$$

muatan totalnya: $Q_{tot} = C_{ekiv} \cdot V$

$$Q_3 = \left(\frac{C_1 C_2}{C_1 + C}\right) V$$

(iv) konfigurasi C_1 dan C_2 disusun paralel



$$C_{ekiv} = C_1 + C_2$$

$$Q_{tot} = C_{ekiv} \cdot V = (C_1 + C_2) V$$

$$Q_4 = (C_1 + C_2) V$$

(4) Sehingga Q pada masing-masing konfigurasi,

(i) $Q_1 = C_1 V$

(ii) $Q_2 = C_2 V$

(iii) $Q_3 = \left(\frac{C_1 C_2}{C_1 + C_2}\right) V$

(iv) $Q_4 = (C_1 + C_2) V$

misalkan : $C_1 = 6\mu F$

$C_2 = 2\mu F$

maka : $Q_1 = 6V \cdot 10^{-6}$

$Q_2 = 2V \cdot 10^{-6}$

$Q_3 = \frac{12 \cdot 10^{-12}}{8 \cdot 10^{-6}} V$

$Q_3 = 1{,}5V \cdot 10^{-6}$ dan $Q_4 = 8V \cdot 10^{-6}$

Jadi urutan nya $Q_4 > Q_1 > Q_2 > Q_3$

Sehingga jawaban nya : Paralel, C_1 sendiri, C_2 sendiri, Seri

(5) a) $C = \frac{\varepsilon_0 A}{d}$, jika ada bahan dielektrik, maka $C' = \frac{k \varepsilon_0 A}{d}$

Jadi $C \sim k$ (kapasitansi akan bertambah)

b) $C = \frac{Q}{V} \rightarrow C \sim Q$

C meningkat maka Q meningkat, Jadi muatan bertambah

(5) c) Perbedaan Potensial

- Medan listrik awal = E_0
- Medan listrik setelah adanya dielektrik,

$$E' = \frac{E_0}{\kappa}$$

$\Delta E = E' - E_0$ = berkurang (negatif)

$\Delta E = \frac{E_0}{\kappa} - E_0$ = berkurang

$V = E.d$, $\Delta V = \Delta E.d$

$\Delta V \sim \Delta E$, jadi, perbedaan potensial akan berkurang.

d) $\Delta U = q\Delta V$ $\Delta V \rightarrow$ berkurang

$\Delta U \sim \Delta V \rightarrow$ berkurang

Jadi, energi potensial akan berkurang

B. SOAL

dibuat oleh : Wawan K

① Ampere adalah Coulomb/detik. Sehingga :

$$84\ A.h = \left(84\ \frac{C.jam}{s}\right) 3600\ \frac{s}{jam} = 3\times10^{5}\ C$$

Perubahan energi potensial $\Delta U = q\,\Delta V$

$$= (3\times10^{5}\ C)(12V)$$

$$\Delta U = 3{,}6\times10^{6}\ J$$

Jadi, besarnya perubahan energi potensial adalah $|\Delta U| = 3{,}6\times10^{6}\ J$

② Usaha yang dilakukan oleh medan listrik

$$W = \int_i^f \vec{F}.d\vec{s}$$

$$= \int_i^f q_0 \vec{E}.d\vec{s}$$

$$= \int_i^f q_0 \left(\frac{\sigma}{2\epsilon_0}\right) dz$$

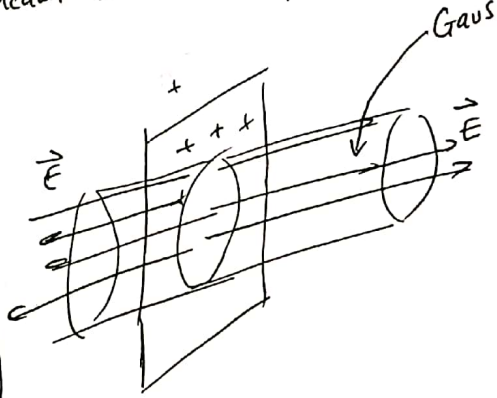
$$= \frac{q_0 \sigma}{2\epsilon_0} \int_0^d dz$$

$$= \frac{1{,}6\times10^{-19}\ (5{,}8\times10^{-12})(0{,}0356)}{2(8{,}85\times10^{-12})}$$

$$W = 1{,}87\times10^{-21}\ J$$

Catatan :

Medan listrik akibat plat non-konduktor:



$$\Phi = \int E\,dA = \frac{q_{enc}}{\epsilon_0}$$

$$\Phi_1 + \Phi_2 = \frac{q_{enc}}{\epsilon_0}$$

$$EA + EA = \frac{\sigma A}{\epsilon_0}$$

$$2EA = \frac{\sigma A}{\epsilon_0} \rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

(3) a) Ketika muatan q_2 berada jauh di tak hingga, maka potensial di titik asal koordinat (0,0) hanya akibat muatan q_1, maka:

$$V_1 = \frac{q_1}{4\pi\epsilon_0 d} = 5{,}76 \times 10^{-7}\ V$$

$$\frac{q_1}{d} \times 9 \times 10^9 = 5{,}76 \times 10^{-7}$$

$$\frac{q_1}{d} = 6{,}41 \times 10^{-17}\ C/m$$

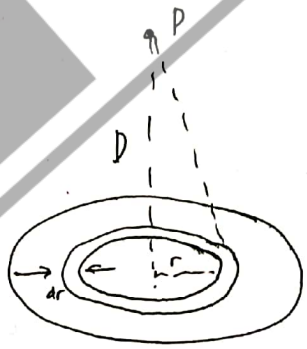
b) Dari grafik, kita ketahui bahwa $V = 0$ ketika q_2 terletak di $x = 0{,}080m$

$V_{net} = 0$ atau $V_1 + V_2 = 0$ sehingga:

$$0 = \frac{kq_2}{0{,}08} + \frac{kq_1}{d}$$

$$q_2 = -\left(\frac{q_1}{d}\right)(0{,}08) = -5{,}13 \times 10^{-18}\ C = -32e$$

(4)



$A = \pi r^2$

$dA = 2\pi r dr$

Potensial oleh sebagian dq adalah:

$$dV = k\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{\sigma dA}{r}$$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{\sigma 2\pi r dr}{\sqrt{r^2+D^2}} = \frac{\sigma r dr}{2\epsilon_0\sqrt{r^2+D^2}}$$

(4) Potensial total di P adalah :

$$V = \int dV$$

$$= \int \frac{\sigma r\, dr}{2\varepsilon_0 \sqrt{r^2+D^2}}$$

$$V = \frac{\sigma}{2\varepsilon_0} \int_0^R \frac{r\,dr}{\sqrt{r^2+D^2}} = \frac{\sigma}{2\varepsilon_0} \sqrt{r^2+D^2} \Big|_0^R = \frac{\sigma}{2\varepsilon_0} \left[\sqrt{R^2+D^2} - D \right]$$

Potensial akibat $\frac{1}{4}$ bagian Cakram

$$V = \frac{V}{4} = \frac{\sigma}{8\varepsilon_0} \left[\sqrt{R^2+D^2} - D \right]$$

$$V = \frac{7{,}73 \times 10^{-15}}{8(8{,}85 \times 10^{-12})} \left[\sqrt{(0{,}640)^2 + (0{,}259)^2} - 0{,}259 \right]$$

$$\boxed{V = 4{,}71 \times 10^{-5} \text{ Volt}}$$

(5) Energi Potensial dua bola :

$$U = \frac{q^2}{4\pi\varepsilon_0 d} = \frac{(8{,}99 \times 10^{9})(5 \times 10^{-6})^2}{1} = 0{,}225 \text{ J}$$

a) Energi kekal, dan energi potensial awal adalah $U = 0{,}225$ J.

b) energi kinetik awal adalah nol, karena bola mula-mula diam.

(5) -) Energi potensial akhir nol karena kedua bola terpisah menjauh.

$$EM_i = EM_f$$

$$U_i + K_i = U_f + K_f$$

$$U + 0 = 0 + K$$

$$U = K$$

$$U = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$$

- Pada sistem ini, momentum kekal, maka :

$$P_i = P_f$$

$$0 = m_A V_A + m_B V_B$$

$$V_B = -\left(\frac{m_A}{m_B}\right) V_A$$

Kemudian,

$$U = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B \left(-\frac{m_A}{m_B} V_A\right)^2$$

$$U = \frac{1}{2} m_A V_A^2 + \frac{1}{2} \frac{m_A^2}{m_B} V_A^2$$

$$U = \frac{1}{2} \left(\frac{m_A}{m_B}\right) \left(m_A + m_B\right) V_A^2$$

$$\frac{2U m_B}{m_A \left(m_A + m_B\right)} = V_A^2$$

(5) $$V_A = \sqrt{\frac{2U m_B}{m_A(m_A+m_B)}} = \sqrt{\frac{2\,(0{,}225)\,(10\times10^{-3})}{(5\times10^{-3})(5\times10^{-3}+10\times10^{-3})}} = 7{,}75\ m/s$$

kemudian, $$V_B = -\frac{m_A}{m_B}V_A = -\left(\frac{5\times10^{-3}}{10\times10^{-3}}\right)(7{,}75) = -3{,}87\ m/s$$

(6) kapasitor bola dapat kita hitung kapasitansinya,

•) dengan menggunakan hukum Gauss,

$$\oint E\,dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\,4\pi r^2\varepsilon_0 = q$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Beda potensial,

$$V = \int_{-}^{+} E\,ds = -\frac{q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$V = \frac{q}{4\pi\varepsilon_0}\,\frac{b-a}{ab}$$

$$C = \frac{q}{V} = \frac{q}{\frac{q}{4\pi\varepsilon_0}\frac{(b-a)}{ab}} = 4\pi\varepsilon_0\frac{ab}{(b-a)}$$

Sehingga : $$C = 4\pi\varepsilon_0\frac{ab}{(b-a)} = \frac{40\times10^{-3}\times38\times10^{-3}}{9\times10^{9}\,(40\times10^{-3}-38\times10^{-3})} = 84{,}5\,pF$$

(6) Kita ketahui bahwa $C = \frac{\varepsilon_0 A}{d}$

atau $C = \frac{\varepsilon_0 A}{(b-a)}$

Sehingga, $A = \frac{C(b-a)}{\varepsilon_0} = \frac{84{,}5 \times 10^{-12}\left(40\times10^{-3} - 38\times10^{-3}\right)}{8{,}85\times10^{-12}}$

$$\boxed{A = 191\ cm^2}$$

(7) ·) Pertama-tama kapasitansi ekivalen C_3 dan C_5 disusun seri,

$$C_{p1} = \frac{4\mu F}{2} = 2\mu F$$

Kemudian, kombinasi ini dihubungkan paralel dengan C_2 dan C_4 dan menghasilkan kapasitansi ekivalen

$$C_{p2} = C_{p1} + C_2 + C_4$$
$$= 2\mu F + 2\mu F + 2\mu F$$
$$C_{ekiv_2} = 6\mu F$$

-) C_6 dan C_3 disusun paralel,

maka $C_{ekiv\,3} = 3\mu F + 3\mu F = 6\mu F$

·) Terakhir $C_{ekiv\,2}$ dan $C_{ekiv\,3}$ disusun seri, $\frac{1}{C_{ekiv}} = \frac{1}{C_{ek2}} + \frac{1}{C_{ek3}}$

$$C_{ekivalen} = \frac{(C_{ek2})(C_{ek3})}{(C_{ek2}) + (C_{ek3})} = \frac{(6\mu F)(6\mu F)}{6\mu F + 6\mu F} = 3\mu F$$

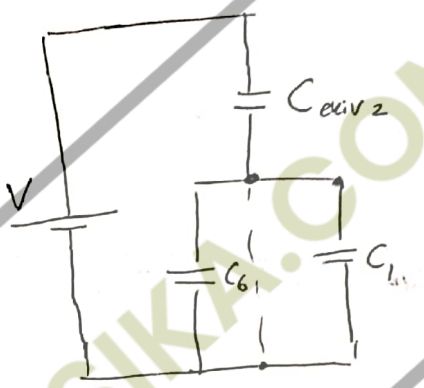
(7) muatan total yang tersimpan pada $C_{ekivalen}$

$$q = C_{ekiv}\, V$$

$$= 3\mu F\ (20V)$$

$$q_{tot} = 6 \times 10^{-5} C$$

•) Beda potensial melalui C_1 diberikan oleh :



•) Tegangan drop melalui V_1

$$V_1 = \left(\frac{C_{eq_2}}{C_{eq_2} + C_{eq_3}} \right) V$$

$$V_1 = \frac{6\mu F\ (20V)}{6\mu F + 6\mu F} = 10V$$

•) beda potensial melalui C_2

$$V_2 = V - V_1 = 20V - 10V = 10V$$

•) karena beda potensial V_2 dibagi dengan sama antara C_3 dan kapasitor lain $4\mu F$ yang dihubung seri, sehingga $V_3 = \frac{V_2}{2} = \frac{10V}{2} = 5V$

(7) Jadi, $q_3 = C_3 V_3 = (4\mu F)(5V) = 2 \times 10^{-5} C$

(8) Setelah ditutup $V_{ab} = \frac{Q}{C_{eq}}$, $C_{eq} = C_1 + C_2 = 4 \times 10^{-6} F$

maka: $q_1 = C_1 V = 10^{-6}(100) = 10^{-4} C$

$q_2 = C_2 V = 3 \times 10^{-6}(100) = 3.10^{-4} C$

Q total pada $C_{ekivalen} = q_2 - q_1 = 3.10^{-4} C - 10^{-4} C = 2.10^{-4} C$

$$V_{ab} = \frac{Q_{eq}}{C_{eq}} = \frac{2.10^{-4}}{4.10^{-6}} = 50 V$$

(9) $E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{V}{R}$

rapat energi. $u = \frac{1}{2}\varepsilon_0 E^2$

$= \frac{1}{2}\varepsilon_0 \left(\frac{V}{R}\right)^2$

$= \frac{1}{2}(8{,}85 \times 10^{-12})\left(\frac{8000}{0105}\right)^2$

$u = 0{,}11 \ J/m^3$

(10) Kita ketahui bahwa,

$C = \frac{\kappa \varepsilon_0 A}{d}$, karena kapasitor tersebut diisi oleh tiga material maka kita asumsikan terdapat 3 buah kapasitor dengan nilai sebagai berikut :

(10) •) $C_1 = \dfrac{k_1 \varepsilon_0 \left(\frac{A}{2}\right)}{2d} = \dfrac{\varepsilon_0 A k_1}{4d}$

$$C_2 = \frac{k_2 \varepsilon_0 \left(\frac{A}{2}\right)}{d} = \frac{\varepsilon_0 A k_2}{2d}$$

$$C_3 = \frac{\varepsilon_0 A k_3}{d}$$

•) C_2 dan C_3 dihubungkan seri (kita peroleh C_{23} → pengganti) dan C_{23} disusun paralel dengan C_1, maka kapasitansi total nya,

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{\varepsilon_0 A k_1}{4d} + \frac{\left(\frac{\varepsilon_0 A}{d}\right)^2 \frac{k_2}{2}\left(k_3\right)}{\left(\frac{\varepsilon_0 A}{d}\right)\left(\frac{k_2}{2} + k_3\right)}$$

$$C = \frac{\varepsilon_0 A k_1}{4d} + \frac{\frac{\varepsilon_0 A}{d}\,\frac{k_2 k_3}{2}}{\frac{k_2}{2} + k_3}$$

$$= \frac{\varepsilon_0 A}{d}\left(\frac{k_1}{4} + \frac{\frac{k_2 k_3}{2}}{\frac{k_2}{2} + k_3}\right)$$

$$= 8{,}85 \times 10^{-12}\left(1215 \times 10^{-4}\right)\left(\frac{21}{4} + \frac{\frac{42.58}{2}}{\frac{42}{2} + 58}\right)$$

$$= 110{,}625 \times 10^{-16}\left(5{,}25 + \frac{1218}{79}\right)$$

$$C = 2{,}286 \times 10^{-13}\ F$$