Solusi Tutorial 1 Fisika Dasar 2A ITB

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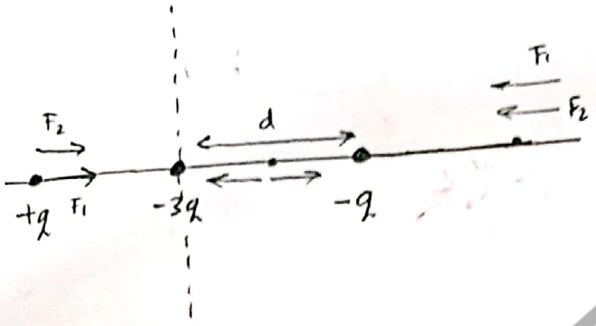
A. Pertanyaan

①



Kita coba ketiga daerah,

• $x<0$

$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 \neq 0$

• $0<x<d$ (antara kedua muatan)

$\vec{F}_{tot} = \vec{F}_1 - \vec{F}_2 = 0$

• $x>d$

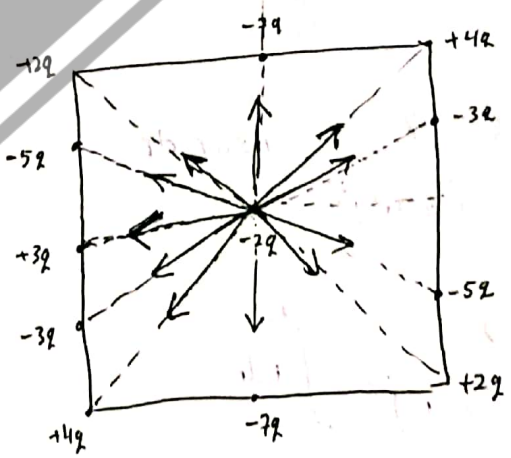
$F_{tot} = -\vec{F}_1 - \vec{F}_2 \neq 0$

a) Diantara kedua muatan

b) muatan positif

c) kesetimbangan tidak stabil (karena tidak berada di tengah-tengah)

②



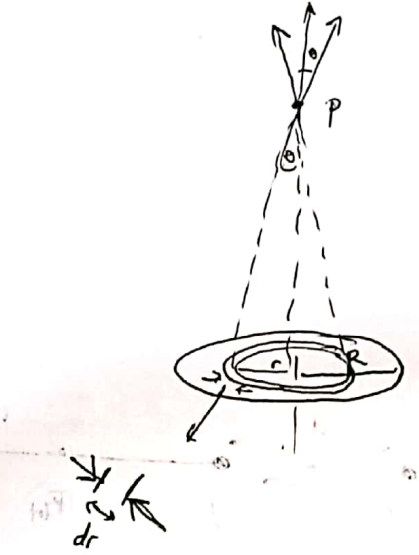
Berdasarkan simetri, maka kedua pasangan simetri akan saling meniadakan.

Sehingga,

$$\vec{F}_{sisa} = k\frac{(2q)(3q)}{d^2}(-\hat{i})$$

$$\vec{F} = 6\frac{kq^2}{d^2} \text{ arah kiri}$$

(3)



luas = $A = \pi r^2$

partisi luas,

$dA = 2\pi r\, dr$

rapat muatan σ,

$$\sigma = \frac{q}{A}$$

$$q = \sigma A$$

$$dq = \sigma\, dA$$

$$dq = \sigma\,(2\pi r\, dr)$$

maka: $\vec{E}_p = \frac{1}{4\pi\varepsilon_0}\left[\int \frac{dq}{r^2}\cos\theta\,\hat{j} + \int \frac{dq}{r^2}\sin\theta\,\hat{i}\right]$

$\int \frac{dq}{r^2}\sin\theta\,\hat{i} = 0$ → saling meniadakan

$$\vec{E}_p = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r^2}\cos\theta\,\hat{j}$$

$$= \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, dA}{\left(z^2+r^2\right)^{1/2}}\cdot\frac{z}{\left(z^2+r^2\right)^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\,\sigma\,\frac{z}{\left(z^2+r^2\right)^{3/2}}\int 2\pi r\, dr$$

$$\vec{E}_p = \frac{2\pi\sigma z}{4\pi\varepsilon_0\left(z^2+r^2\right)^{3/2}}\int_0^R r\, dr$$

=

(3)

$$\vec{E_p} = \frac{2\pi\sigma z}{4\pi\varepsilon_0} \int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}$$

misalkan : $u = z^2+r^2$

$du = 2r\,dr$

maka : $\vec{E_p} = \frac{\pi\sigma z}{4\pi\varepsilon_0} \int \frac{du}{u^{3/2}}$

$$= \frac{\sigma z}{4\varepsilon_0} \int u^{-3/2}\,du$$

$$= \frac{2\sigma z}{4\varepsilon_0} \left[-u^{-1/2}\right]$$

$$= \frac{2\sigma z}{4\varepsilon_0} \left[-\frac{1}{\sqrt{z^2+r^2}}\right]_0^R$$

$$= \frac{\sigma z}{2\varepsilon_0} \left[-\frac{1}{\sqrt{z^2+R^2}} + \frac{1}{z}\right]$$

$$\vec{E_p} = \frac{\sigma}{2\varepsilon_0} \left[1 - \frac{z}{\sqrt{z^2+R^2}}\right] \hat{j}$$

a) untuk $r = R$, maka

$$E = \frac{\sigma}{2\varepsilon_0} \left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

b) untuk $r = 2R$, maka :

$$E = \frac{\sigma}{2\varepsilon_0} \left[1 - \frac{z}{\sqrt{z^2+4R^2}}\right]$$

(3) untuk kondisi (c)

$$E = E_{(b)} - E_{(a)}$$

$$E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+4R^2}}\right) - \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

Sehingga jawabannya :

$E_a > E_b > E_c$ atau urutannya a, b, c

(4) a) Fluks medan listriknya :

$$\Phi = \vec{E} \cdot \vec{A}$$

$$= (4\hat{i}) \cdot (2\hat{i} + 3\hat{j})$$

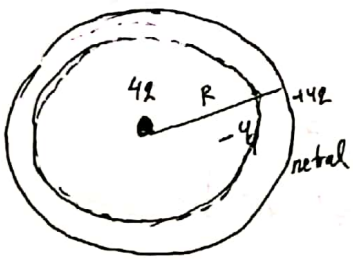
$$\Phi = 8 + 0 = 8 \ Nm^2/C$$

b) fluks medan listrik :

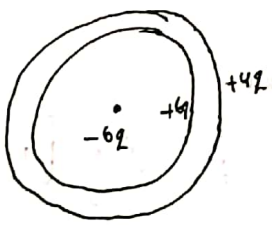
$$\Phi = \vec{E} \cdot \vec{A}$$

$$\Phi = (4\hat{k}) \cdot (2\hat{i} + 3\hat{j}) = 0$$

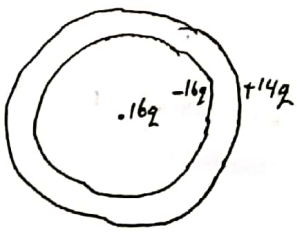
(5) keadaan 1



keadaan 2



keadaan 3

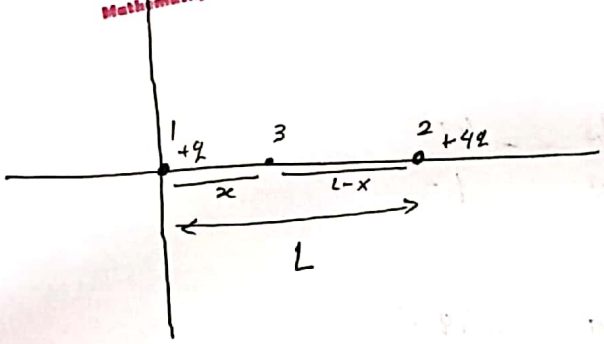


a) 2, 1, 3 b) 3, 2 = 1

B. SOAL

Dibuat: oleh wawan K

①



Agar F = 0, maka partikel 3 harus ditempatkan antara 1 dan 2

maka : $F_3 = F_{31} - F_{32}$

$$F_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{qq_3}{x^2} - \frac{4qq_3}{(L-x)^2}\right) = 0$$

$$\frac{qq_3}{x^2} = \frac{4qq_3}{(L-x)^2}$$

$$\frac{1}{x^2} = \frac{4}{(L-x)^2}$$

$$\frac{1}{x} = \frac{2}{L-x}$$

$$x = \frac{L}{3}, \quad x = \frac{9\,cm}{3} = 3\,cm$$

b) dengan cara seperti (a), koordinatnya $q_3 = 0$.

c) Gaya pada q adalah:

$$F_q = -\frac{1}{4\pi\varepsilon_0}\left(\frac{qq_3}{x^2} + \frac{4q^2}{L^2}\right)$$

tanda dipilih negatif. Sehingga q bergerak ke kiri. kita membutuhkan $F_q = 0$

Agar ketiga partikel diam di tempat.

sehingga :

$$0 = -\frac{1}{4\pi\varepsilon_0}\left(\frac{qq_3}{x^2} + \frac{4q^2}{L^2}\right)$$

$$\frac{qq_3}{x^2} = -\frac{4q^2}{L^2}$$

$$q_3 = -\frac{4qx^2}{L^2}$$

$$q_3 = -\frac{4}{9}q \rightarrow \boxed{\frac{q_3}{q} = -\frac{4}{9} = -0{,}444}$$

dimana $x = \frac{L}{3}$, kita juga dapat membuktikan gaya pada $4q$ akan hilang.

Bukti :

$$F_{4q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4qq_3}{(L-x)^2}\right)$$

$$F_{4q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4(-4/9)q^2}{(4/9)L^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} - \frac{4q^2}{L^2}\right) = 0$$

(2) a) Besar gaya antara ion positif adalah :

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq}{r^2} = k\frac{q^2}{r^2}, \text{ maka}$$

$$q = r\sqrt{\frac{F}{k}} = (5\times10^{-10}\,m)\sqrt{\frac{3{,}7\times10^{-9}\,N}{8{,}99\times10^{9}\,Nm^2/C^2}} = 3{,}2\times10^{-19}\,C$$

b) misalkan n adalah jumlah elektron yang hilang tiap ion, kemudian $ne = q$

$$n = \frac{q}{e} = \frac{3{,}2\times10^{-19}\,C}{1{,}6\times10^{-19}\,C} = 2$$

(3) ·) karena grafik melewati nol ($F=0$), q_1 harus bertanda positif. agar $F_2 net = 0$.

·) Saat $r = 0{,}40$ m maka nilai $F_{2net} = 0$.

Sekarang, nilai asimtot dari gaya menghasilkan besar dan tanda q_2 :

Saat $x \to \infty \to q_3$ bergerak ke kanan

$$F_2 = F_{12} + F_{23}$$
$$= 0$$

$$F_{23} = k\frac{q_2 q_3}{x_{23}^2} = k\frac{q_2 q_3}{\infty}$$

$$F_{23} = 0$$

$$1{,}5 \times 10^{-25} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \qquad r = 0{,}40\ m$$

$$q_2 = \frac{1{,}5 \times 10^{-25}}{9 \times 10^9 (q_1)} \times r^2$$

$$= \frac{1{,}5 \times 10^{-25}}{9 \times 10^9 (8 \times 1{,}6 \times 10^{-19})} \times (0{,}4)^2$$

$$q_2 = 2{,}086 \times 10^{-18}\ C = 13e$$

(4) untuk kemungkinan medan menghilang pada saat $x > 0$, dua medan (akibat q_1 dan q_2) harus berlawanan arah untuk $x > 0$. Dengan lokasi pada gambar, kita simpulkan muatan berlawanan tanda. kemudian, karena medan neto berarah lebih kuat kekiri untuk positif x (kecil) dimana sangat dekat dengan q_2.

maka $q_2 \to$ muatan negatif.

$q_1 \to$ muatan positif

$q_1 = 4\mathcal{E}$ dan $q_2 = -\mathcal{E}$

(4) $E_{net} = E_1 + E_2$

$$E_{net} = \frac{4\varepsilon}{4\pi\varepsilon_0 (L+x)^2} - \frac{\varepsilon}{4\pi\varepsilon_0 x^2}$$

untuk sepanjang sumbu x positif

$E_{net} = 0$, pada $x = 20$ cm (lihat gambar), sehingga $L = 20$ cm

a) kita diferensialkan E_{net} terhadap x dan sama dengan nol, agar E_{net} max

$$\frac{dE_{net}}{dx} = 0$$

$$\frac{d}{dx}\left(\frac{4\varepsilon}{4\pi\varepsilon_0 (L+x)^2} - \frac{\varepsilon}{4\pi\varepsilon_0 x^2} \right) = 0$$

$$\frac{d}{dx}\left(\frac{\varepsilon}{\pi\varepsilon_0} (L+x)^{-2} - \frac{\varepsilon}{4\pi\varepsilon_0} x^{-2} \right) = 0$$

$$(-2)(L+x)^{-3} \frac{\varepsilon}{\pi\varepsilon_0} - (-2)(x^{-3}) \frac{\varepsilon}{4\pi\varepsilon_0} = 0$$

$$(L+x)^{-3} = \frac{x^{-3}}{4}$$

$$\frac{1}{(L+x)^3} = \frac{1}{4x^3}$$

$$\frac{1}{L+x} = \frac{1}{4^{\frac{1}{3}} x} \rightarrow 4^{\frac{1}{3}} x = L + x$$

$$4^{1/3} x - x = L$$

$$\therefore x = \frac{L}{0.587} = 34 \text{ cm}$$

(4) b) Sekarang kita tuliskan $\mathcal{E} = 3e$

dimana $e = 1{,}60 \times 10^{-19} C$

dan menghitung nilai E_{net} pada nilai x (dikonversi ke meter), telah didapatkan dari no (a).

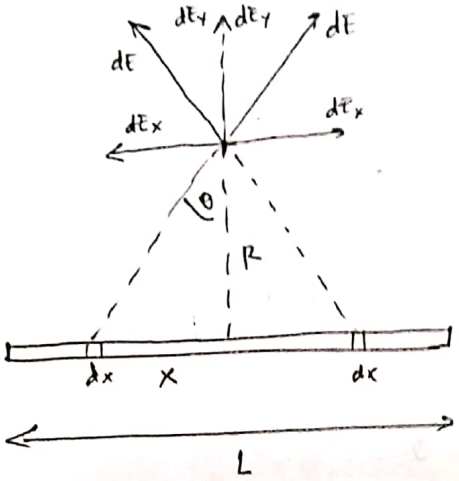
$x = 34\ cm = 0{,}34\ m.$

Sehingga $E_{net\ maksimum} = \dfrac{4(3e)}{4\pi\varepsilon_0 (L+x)^2} - \dfrac{\mathcal{E}}{4\pi\varepsilon_0 x^2}$

$= \dfrac{4(3 \times 1{,}6 \times 10^{-19})}{(0{,}2+0{,}34)^2} \times 9 \times 10^9 - \dfrac{3 \times 1{,}6 \times 10^{-19}}{(0{,}34)^2} \times 9 \times 10^9$

$$E_{net\ max} = 2{,}2 \times 10^{-8}\ N/C$$

(5)



Ambil partisi dx, maka $\lambda = \frac{q}{L} \rightarrow dq = \lambda dL$

$dq = \lambda dx$

Medan listrik akibat distribusi muatan,

$$\vec{E_p} = \int dE_x \hat{i} + \int dE_y \hat{j}$$

komponen x saling meniadakan, maka:

medan listrik di titik P.

$$\vec{E_p} = \int dE_y \hat{j}$$

$$= \frac{1}{4\pi\epsilon_0} \int \frac{dq}{\left((x^2+R^2)^{1/2}\right)^2} \cos\theta \ \hat{j}$$

$$= \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dx}{(x^2+R^2)} \cdot \frac{R}{(x^2+R^2)^{1/2}} \hat{j}$$

$$\vec{E_p} = \frac{\lambda R}{4\pi\epsilon_0} \int \frac{dx}{(x^2+R^2)^{3/2}} \hat{j}$$

Misalkan : $x = R \tan\theta$

$dx = R \sec^2\theta \, d\theta$

(5)

$$\vec{E}_p = \frac{\lambda R}{4\pi\epsilon_0}\int \frac{R\sec^2\theta\, d\theta}{(R^2+R^2\tan^2\theta)^{3/2}}\hat{j}$$

$(1+\tan^2\theta) = \sec^2\theta$

$$\rightarrow (R^2(1+\tan^2\theta))^{3/2} = [R^2(\sec^2\theta)]^{3/2}$$

$$= \frac{\lambda R^2}{4\pi\epsilon_0 R^3}\cdot\int \frac{\sec^2\theta\, d\theta}{\sec^3\theta}\hat{j}$$

$$\vec{E}_p = \frac{\lambda}{4\pi\epsilon_0 R}\int_{-\theta}^{\theta} \cos\theta\, d\theta\, \hat{j}$$

Batas integral dari $(-\theta \rightarrow \theta)$, karena titik P berada di tengah-tengah batang.

maka:

$$E_p = \frac{\lambda}{4\pi\epsilon_0 R}\sin\theta\Big|_{-\theta}^{\theta}\hat{j} = \frac{Q/L}{4\pi\epsilon_0 R}(\sin\theta - \sin(-\theta))\hat{j}$$

$$\vec{E}_p = \frac{Q}{4\pi\epsilon_0 RL}(2\sin\theta)\hat{j}$$

$$= \frac{Q}{4\pi\epsilon_0 RL}\cdot 2\frac{L/2}{\sqrt{\frac{L^2}{4}+R^2}}\hat{j}$$

$$= \frac{Q}{4\pi\epsilon_0 R}\frac{1}{\sqrt{L^2/4+R^2}}$$

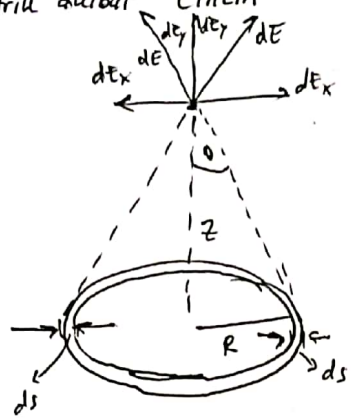
$$= \frac{Q}{2\pi\epsilon_0 R}\frac{1}{\sqrt{L^2+4R^2}}$$

$$= \frac{7{,}81\times10^{-12}}{2(3{,}14)(8{,}85\times10^{-12})}\cdot\frac{1}{\sqrt{(0{,}145)^2+4(0{,}06)^2}}$$

$$|\vec{E}_p| = 12{,}4\ N/C$$

Arah ke sumbu y positif

(6) Medan listrik akibat Cincin



Kita cari medan listrik di suatu titik P yang terletak sejauh z dari pusat cincin, akibat elemen ds.

Sehingga :

$$\vec{E}_p = \int dE_x \hat{i} + \int dE_y \hat{j}$$

↳ Saling meniadakan

Sehingga :

$$\vec{E}_p = \int dE_y \hat{j}$$

$$= \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2} \cos\theta \, \hat{j} \qquad ds = r\,d\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda\, ds}{(z^2+R^2)} \cdot \frac{z}{(z^2+R^2)^{1/2}}$$

$$= \frac{\lambda R z}{4\pi\varepsilon_0} \frac{1}{(z^2+R^2)^{3/2}} \int_0^{2\pi} d\theta \, \hat{j}$$

$$= \frac{\lambda R z}{4\pi\varepsilon_0} \frac{(2\pi)}{(z^2+R^2)^{3/2}} \hat{j}$$

$$= \frac{\left(\frac{q}{2\pi R}\right) 2\pi R \; (z)}{4\pi\varepsilon_0 \, (z^2+R^2)^{3/2}} \hat{j}$$

$$\vec{E}_p = \frac{qz}{4\pi\varepsilon_0} \frac{1}{(z^2+R^2)^{3/2}} \hat{j}$$

Sehingga, medan listrik di P akibat dua cincin,

$$\vec{E}_p = \vec{E}_{\text{cincin kecil}} + \vec{E}_{\text{cincin besar}}$$

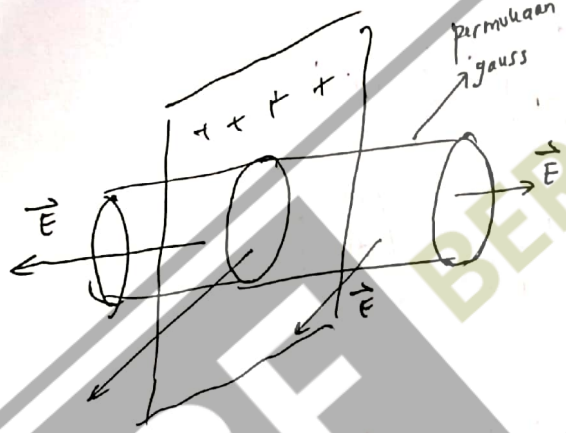
(6) $\vec{E}_p = \frac{q_2}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} - \frac{q_2}{4\pi\varepsilon_0 (z^2+(3R)^2)^{3/2}} = 0$

$$q = -\left(\frac{13}{5}\right)^{3/2} Q$$

$$\boxed{q = -4{,}19\, Q}$$

(7) Medan listrik di titik P diakibatkan oleh distribusi muatan lembaran luas dengan rapat muatan $\sigma = 4{,}5 \times 10^{-12}\ C/m^2$, ditambah lubang kecil dengan rapat muatan $-\sigma$

- Medan listrik akibat lembaran luas.



$$\oint E\,dA = \frac{q}{\varepsilon_0}$$

$$E(2A) = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Medan listrik akibat lubang kecil (cekung) adalah (lihat no 3 bagian pertanyaan)

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

Sehingga medan listrik di P.

$$\vec{E}_p = \vec{E}_{lemb} + \vec{E}_{lubang\ kecil}$$

$$= \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{k} + \frac{-\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{k} = \frac{\sigma z}{2\varepsilon_0\left(\sqrt{z^2+R^2}\right)}\hat{k}$$

$$\vec{E}_p = 0{,}208\ N/C\ \hat{k}$$

(8) kita tentukan muatan total bola dengan menguji nilai maksimum :

$E = 5 \times 10^7 \ N/C$ seperti pada gambar.

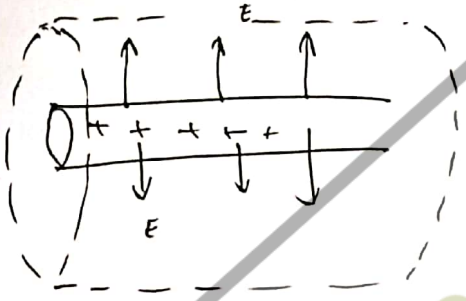
(yang terjadi saat $r = 0{,}020 \ m$)

dari $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

$$q = 4\pi\epsilon_0 E r^2 = \frac{(0{,}020)^2 (5 \times 10^7)}{8{,}99 \times 10^9}$$

$$\boxed{q = 2{,}2 \times 10^{-6} \ C}$$

(9)



permukaan Gauss

Medan listrik di sekitar permukaan luar silinder :

$$\oint E \, dA = \frac{q_{enc}}{\epsilon_0}$$

$$E(2\pi r L) = \frac{\lambda L}{\epsilon_0}$$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \hat{r}}$$

•) Medan listrik di dalam (yakni antara Silinder pejal dan silinder tipis (luar))

pada saat $r = 3{,}5 \ cm$ adalah :

$$E_{dalam} = \frac{\lambda}{2\pi\epsilon_0 r} = 1000 \ N/C$$

↳ lihat grafik

Untuk memperoleh rapat muatan lapisan tipis, maka :

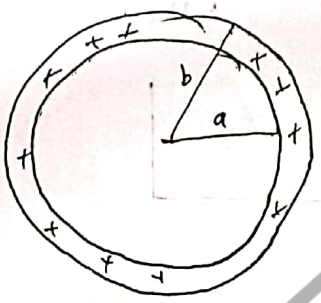
(9)

$$E_{luar} - E_{dalam} = \left(\frac{\lambda}{2\pi\epsilon_0 r} + \frac{\lambda'}{2\pi\epsilon_0 r}\right) - \frac{\lambda}{2\pi\epsilon_0 r}$$

Saat $r = 0{,}035$ m

$$-2000 - 1000 = \frac{\lambda'}{2\pi\epsilon_0 r} \rightarrow \lambda' = -5{,}8 \times 10^{-9}\ C/m$$

(10) Bola berongga $\rho = 1{,}84 \times 10^{-9}\ C/m^3$



a) Saat $r = 0$, $\oint E dA = \frac{q_{enc}}{\epsilon_0}$

$$E(4\pi r^2) = \frac{0}{\epsilon_0}$$

$$E = 0$$

b) Saat $r = \frac{a}{2}$, masih dalam rongga, $E = 0$

c) Saat $r = a$, masih dalam rongga $E = 0$

d) untuk daerah $a \leq r \leq b$, muatan yg dilingkupinya,

$$\rho = \frac{Q}{V} \rightarrow q = \rho V$$

$$q' = \rho \cdot (V_{gauss})$$

$$q_{enc} = \rho\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right)$$

d) Medan listrik adalah, $(a \le r \le b)$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{enc}}{r^2} = \frac{\rho}{4\pi\varepsilon_0 r^2}\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right)$$

$$E = \frac{\rho}{3\varepsilon_0}\left(\frac{r^3 - a^3}{r^2}\right)$$

untuk $r = 1{,}5a$, maka :

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\frac{(1{,}5a)^3 - a^3}{(1{,}5a)^2}$$

$$= \frac{\rho a}{3\varepsilon_0}\left(\frac{2{,}375}{2{,}25}\right)$$

$$= \frac{1{,}84\times10^{-9}\,(0{,}100)}{3\,(8{,}85\times10^{-12})}\left(\frac{2{,}375}{2{,}25}\right)$$

$$\vec{E} = 7{,}32\ N/C$$

e) Untuk $r = b = 2a$, medan listriknya,

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2a)^3 - a^3}{(2a)^2}$$

$$= \frac{\rho a}{3\varepsilon_0}\left(\frac{7}{4}\right) = \frac{(1{,}84\times10^{-9})\,(0{,}100)}{3\,(8{,}85\times10^{-12})}\left(\frac{7}{4}\right)$$

$$E = 12{,}1\ N/C$$

f) untuk $r \geq b$, maka :

$$E = \frac{q_{total}}{4\pi\varepsilon_0}(r^2)$$

$$E = \frac{\rho}{3\varepsilon_0}\frac{b^3 - a^3}{r^2}$$

Untuk $r = 3b = 6a$, maka medan listriknya :

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2a)^3 - a^3}{(6a)^2}$$

$$= \frac{\rho a}{3\varepsilon_0}\left(\frac{7}{36}\right)$$

$$= \frac{(1{,}84 \times 10^{-9})\ (0{,}100\ m)\left(\frac{7}{36}\right)}{3(8{,}85 \times 10^{-12})}$$

$$E = 1{,}35\ N/C$$

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Good luck