



Content of the presentation

- 1 Study objectives**
- 2 Motivation**
- 3 The Peruvian Stock Market**
- 4 Methodology**
- 5 Results**
- 6 Conclusions**
- 7 Recommendations**

GENERAL OBJECTIVE

Applying mathematical tools, operations research, and simulation to determine efficient portfolios of stocks traded in the Lima Stock Exchange (BVL).

SPECIFIC OBJECTIVES

- Apply Markowitz's portfolio theory and propose a simulation tool based on copulas to determine the optimal investment weights for each selected stock, analyzing and comparing the results.
- Conduct a statistical analysis of the selected stocks, contrasting it with the assumptions of the Markowitz model and the stylized facts of financial asset returns.
- Analyze the effect of diversification as a tool for risk reduction in an investment portfolio.

Equity market

Potential for high returns
Subject to higher risks

Stock Market



Purchase and sale
of securities



Uncertainty



- How to manage such risk?
- What should be the investment strategy?
- In which assets should I invest, and how much should I invest in each one?



Tools for making
intelligent decisions



Design efficient
portfolios according to
the level of risk
aversion





Lima Stock Exchange (BVL)

- Ensure a fair, transparent, and efficient market
- Facilitator of transactions
- Centralization of buying and selling of securities

Importance

- Offers a variety of investment alternatives
- The investor becomes a partial owner of large companies
- Promotes the economic development of the country.



Supervisory and regulatory body of the Peruvian stock market



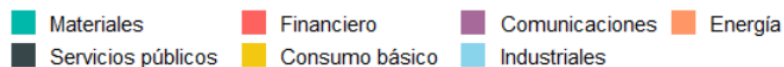
Legal entity intermediary of the stock market. Performs buying and selling operations

Use of the **R** programming language.

A Stock selection

- 30 stocks traded in the BVL
- Price download from 06/2007 to 07/2020 (monthly data) - Bloomberg

Distribución de acciones, según sector al que pertenecen



B Descriptive analysis

Data from 07/2007 to 06/2017 (10 years) were used.

- 1 Descriptive statistics and indicators
- 2 Analysis of the correlation matrix
- 3 Normality tests
- 4 Autocorrelation analysis

C Modeling and optimization

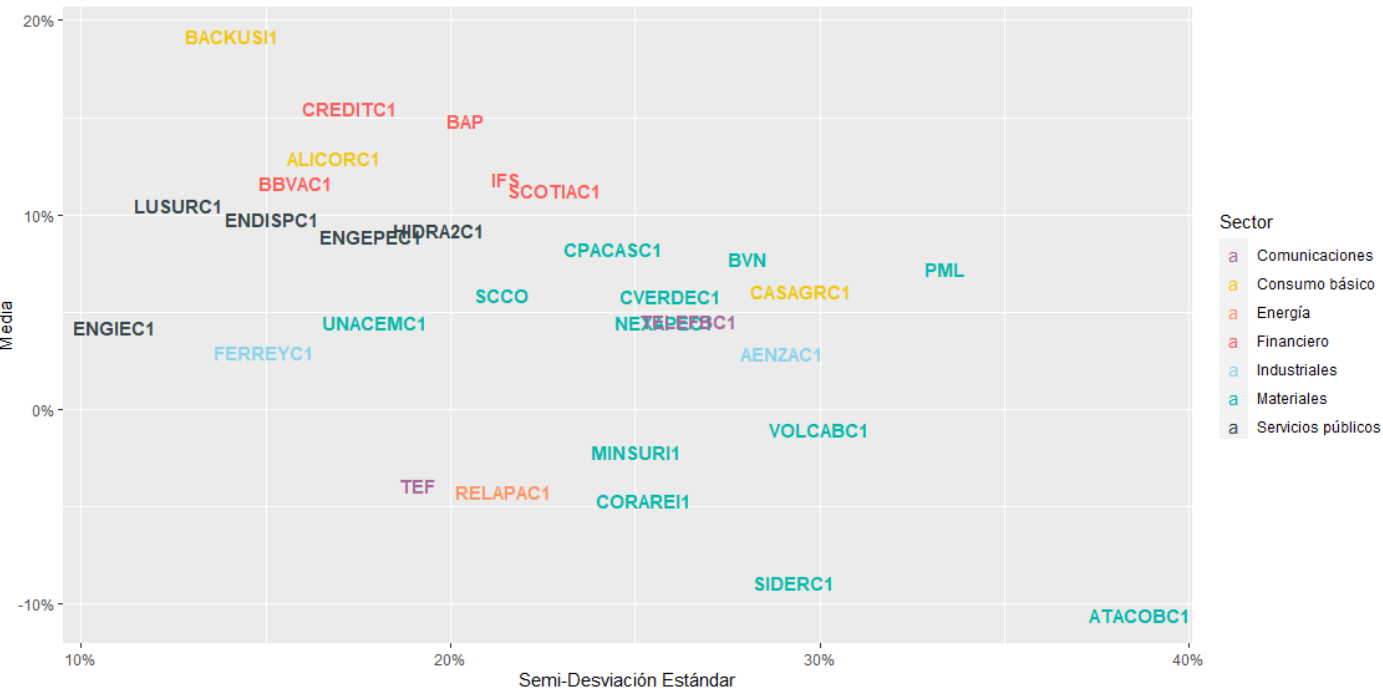
- 1 Markowitz model
- 2 Simulation model based on copulas

D Backtesting and results analysis

E Conclusions and recommendations

Descriptive statistics and indicators

Gráfico de dispersión - Semi-Desviación Estándar vs Media de los rendimientos



Ranking using Sortino ratio

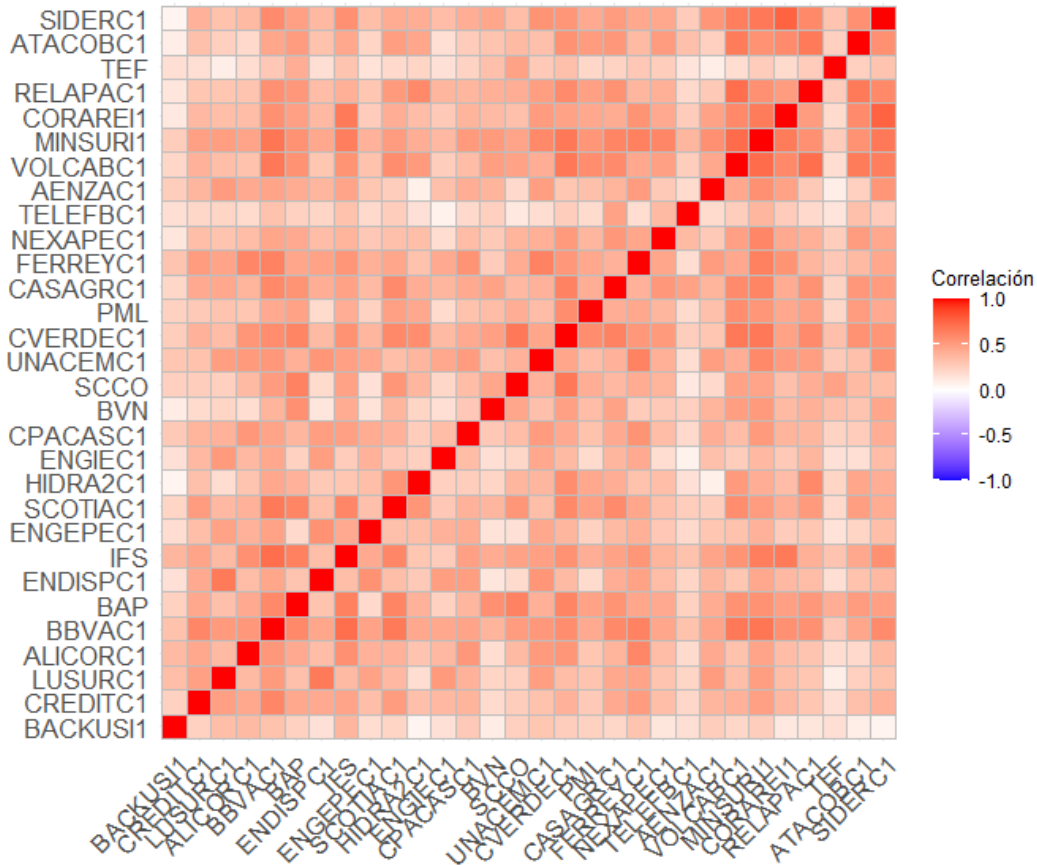
- ✓ **BACKUSI1:** 1.36
- ✓ **CREDITC1:** 0.90
- ✓ **LUSURC1:** 0.88
- ✗ **SIDERC1:** -0.30
- ✗ **ATACOBC1:** -0.27
- ✗ **TEF:** -0.20

Calculation of indicators (Mean, Standard Deviation, Semi-Standard Deviation, Sharpe Ratio, Sortino Ratio, Skewness, Kurtosis, Percentiles, etc.).

23 stocks with positive mean

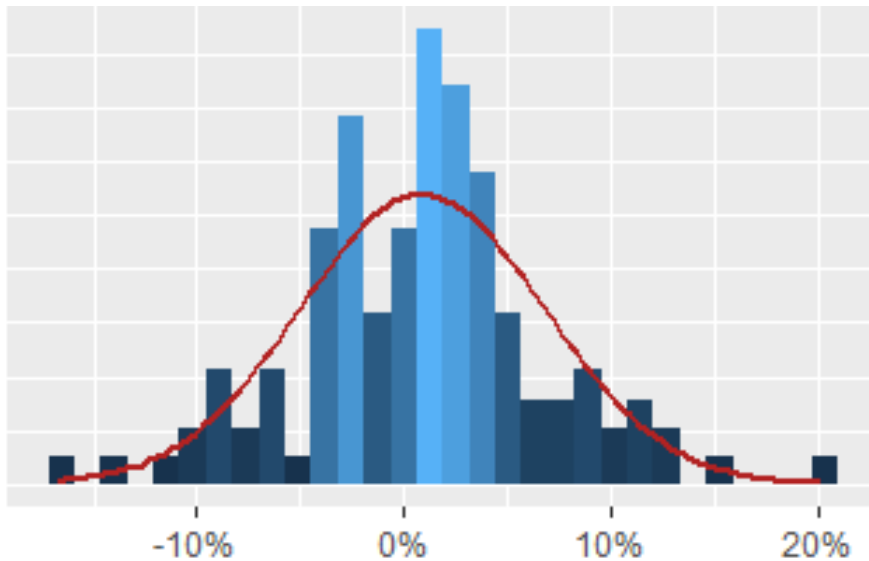
7 stocks with negative mean

Correlation matrix



- Symmetric matrix 30x30
- No pair of stocks presents negative correlation
- Higher correlation between assets of the same economic sector
- Stocks with low correlation compared to the rest: TELEFBC1, TEF, BACKUSI1

Normality tests



100% of the stocks exhibit leptokurtosis

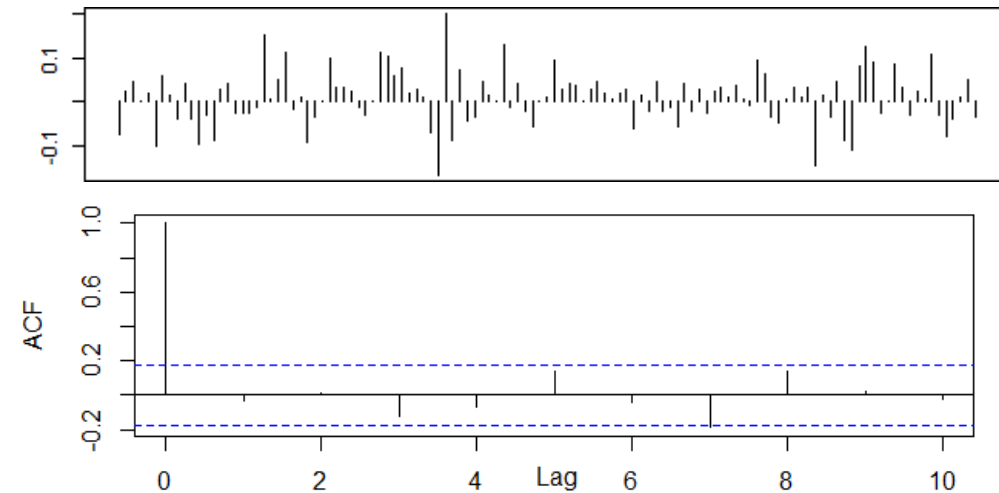
Goodness-of-fit tests ($\alpha=5\%$)

Number of tests where H_0 is not rejected:

- Anderson-Darling (6)
- Lilliefors (7)
- Jarque-Bera (6)

In general, they do not meet normality

Autocorrelation analysis



Evaluate if the returns and absolute returns exhibit an i.i.d. (independent and identically distributed) process.

Ljung-Box tests ($\alpha=5\%$)

Number of tests where H_0 is not rejected:

- Returns (22)
- Absolute returns (23)
- Both criteria (18)

Considerable number of stocks pass the tests: monthly data periodicity.

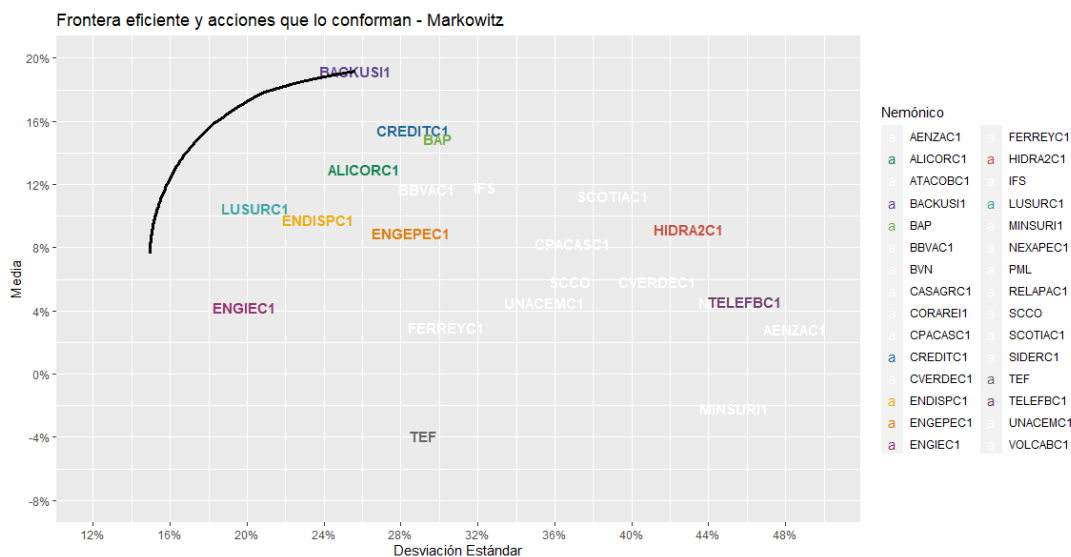
Markowitz model

$$\begin{aligned}
 & \text{Min } \sigma(R_P) \\
 & \text{s.a.} \\
 & E(R_P) = R_P^* \\
 & \sum_{i=1}^n w_i = 1 \\
 & w_i \geq 0, \forall w_i
 \end{aligned}$$

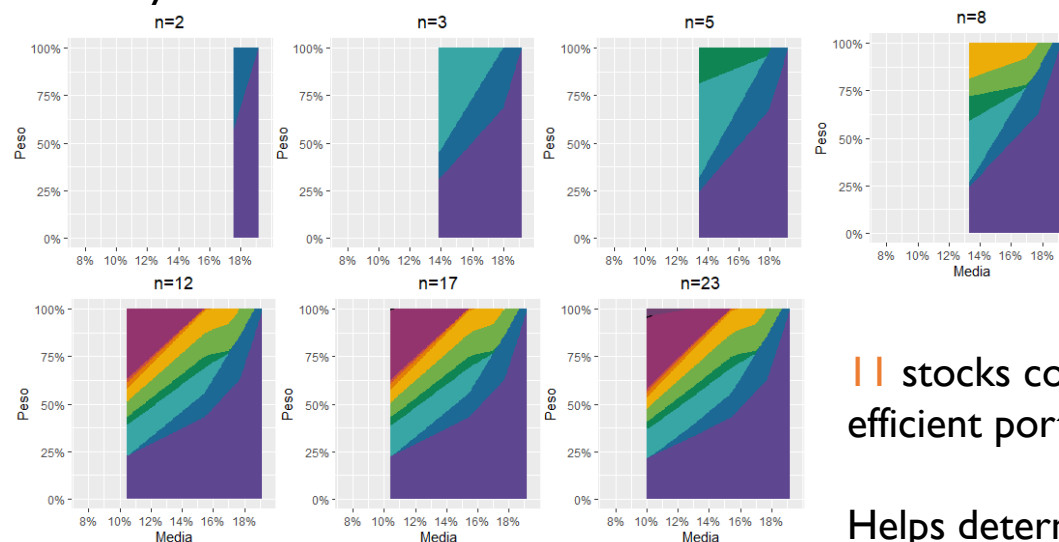
$$\sigma(R_P) = \sqrt{\sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1, j \neq i}^n w_i w_j \sigma_{ij}}$$

$$E(R_P) = \sum_{i=1}^n w_i * E(R_i)$$

Optimization of 100 portfolios, with no short sales allowed. Portfolio returns ranging from the global minimum variance portfolio to the maximum return portfolio.



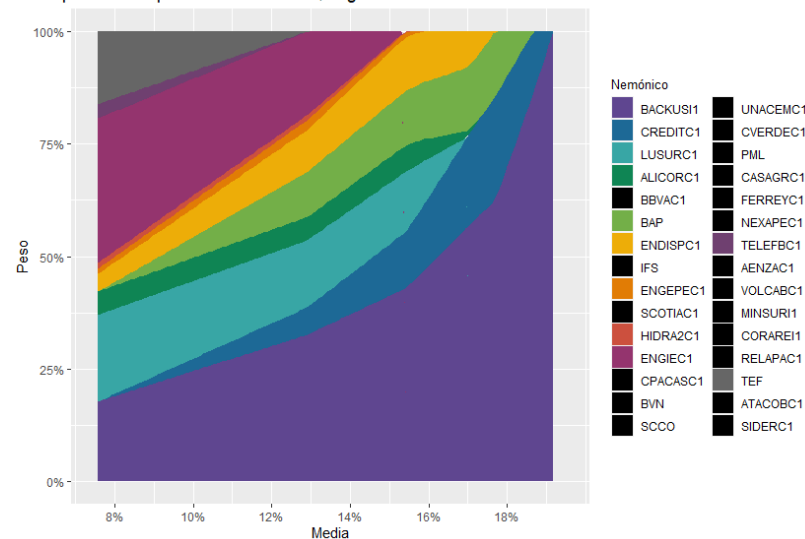
Analysis of the diversification effect:



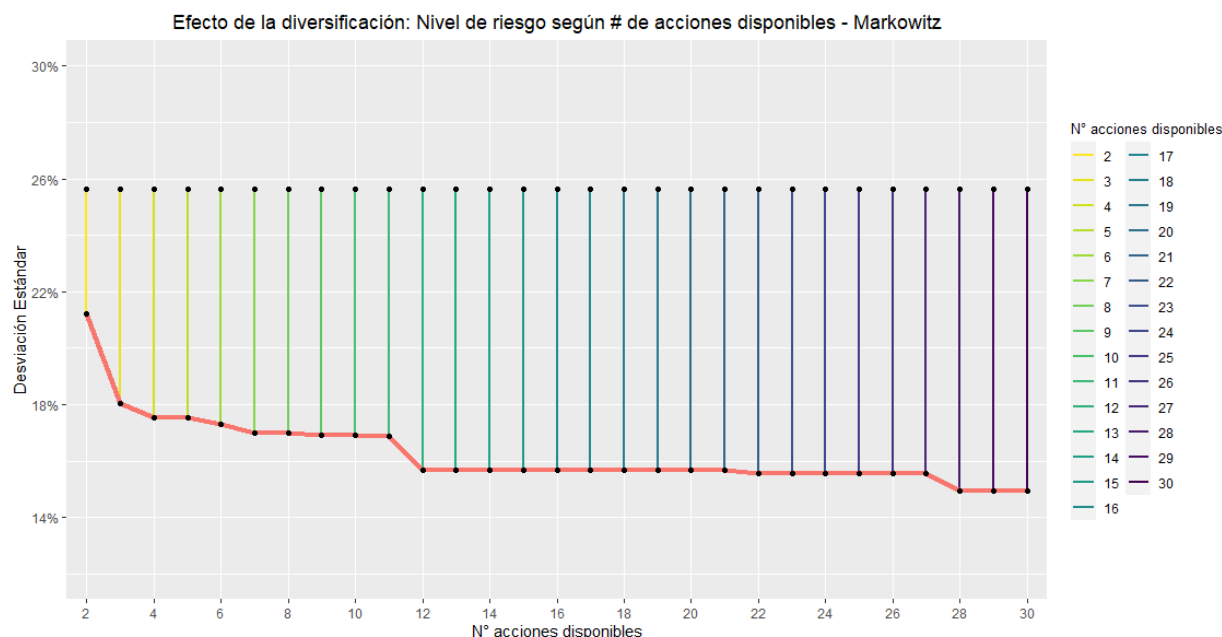
|| stocks comprise some efficient portfolio.

Helps determine investment strategy: portfolio composition in response to an increase in the level of return.

Composición de portafolios eficientes, según nivel de rentabilidad - Markowitz

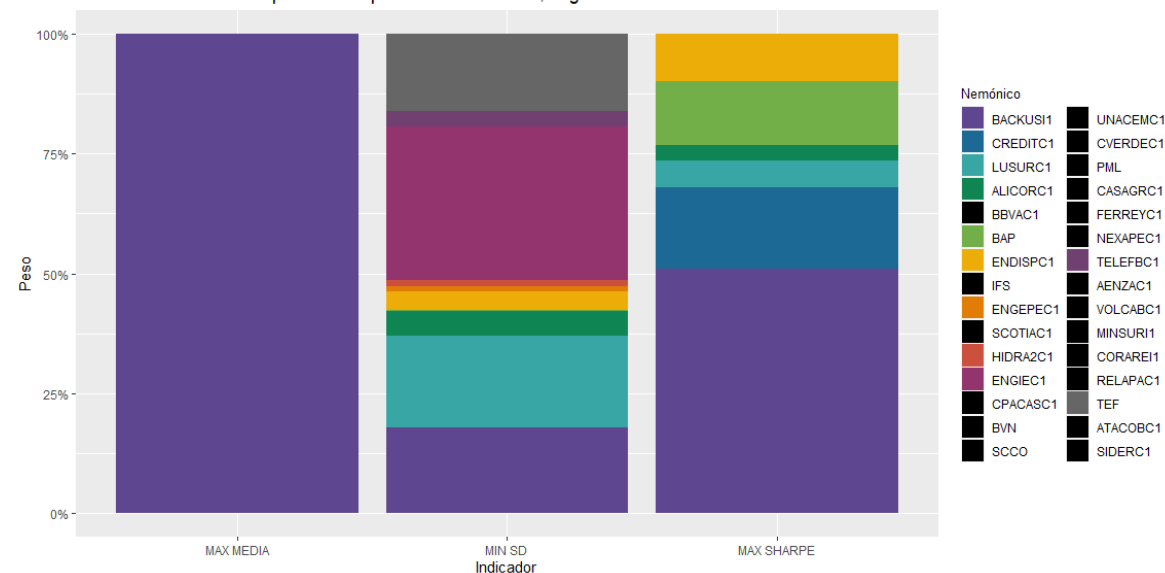


Markowitz model



- The minimum-risk portfolio is composed of the largest number of stocks. The same model suggests diversification.
- As the return increases (at the expense of bearing a higher level of risk), the number of stocks decreases until it becomes a concentrated portfolio.

Composición de portafolios eficientes, según indicador - Markowitz



Markowitz – In Sample

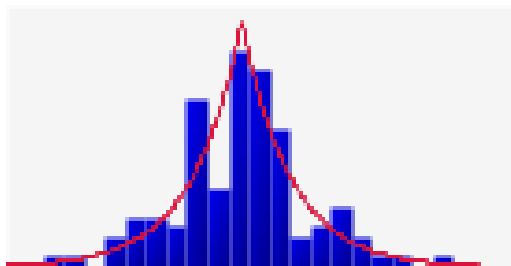
	MAX MEAN	MIN SD	MAX SHARPE
Annual. Geom. Mean	19.2%	7.6%	16.4%
Annual. Std. Dev.	25.6%	15.0%	18.8%
Annual. Sharpe (Rf=0%)	0.75	0.51	0.87

Simulation model based on copulas

Estimation of marginal probability distributions and their parameters

Anderson-Darling criterion

Distribution	Number of modelos stocks
Log-Logistic	17
Laplace	8
Normal	3
Erlang	1
Logistic	1

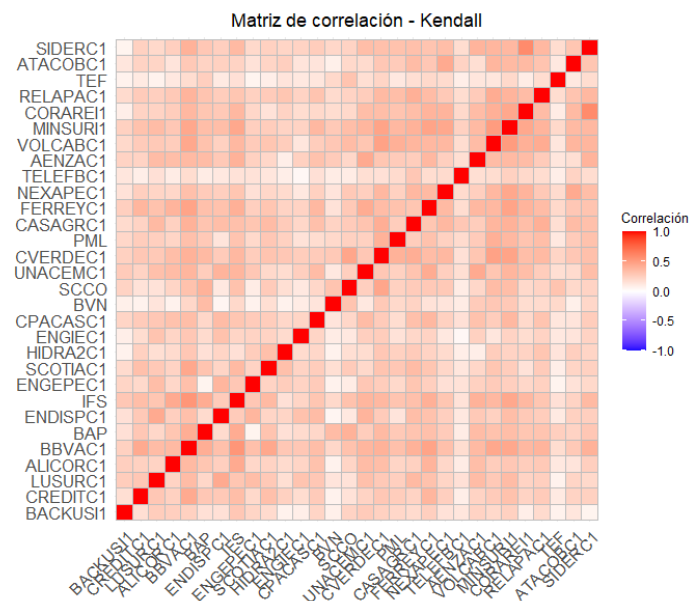


Copula estimation

(BIC, AIC, average log-likelihood, Goodness of Fit)

Copula t ($\nu = 9$)

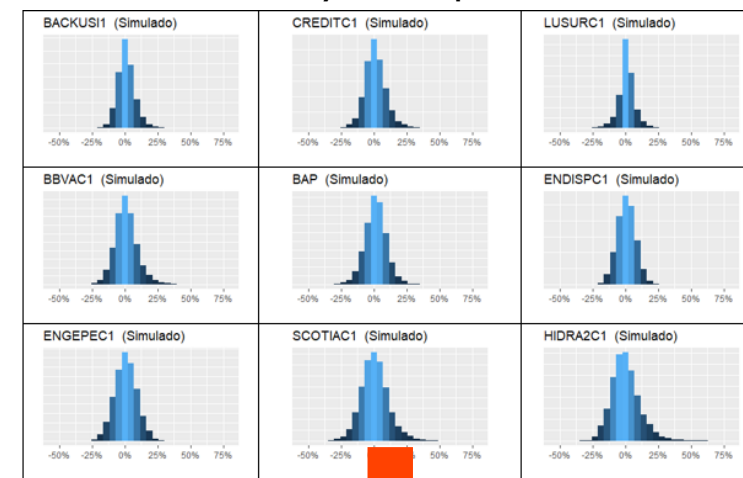
Correlation matrix



Data simulation

10,000 scenarios

Validation of simulated data and structure set by the copula.

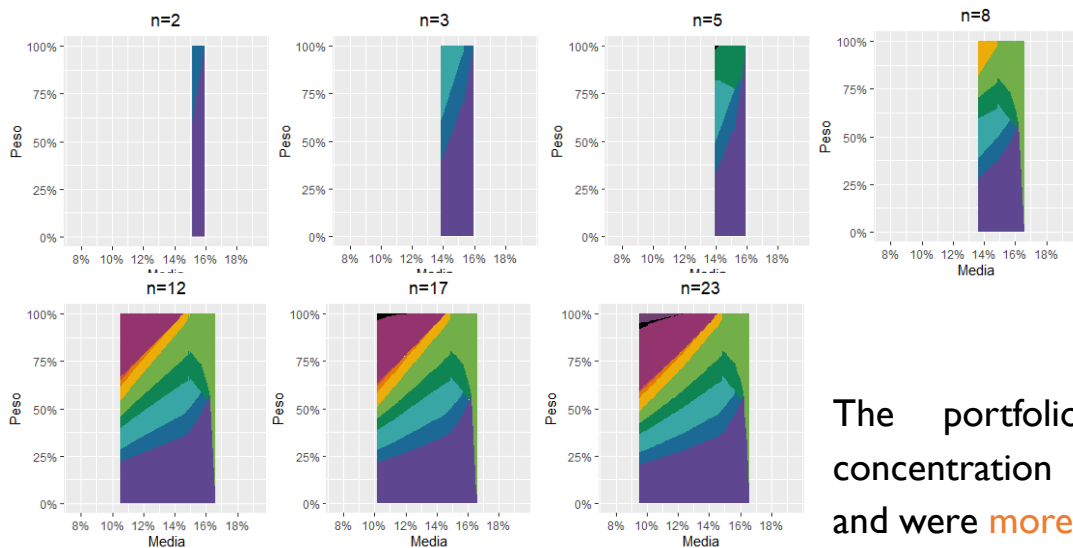


Optimization

Methodology analogous to that developed in the Markowitz Model.

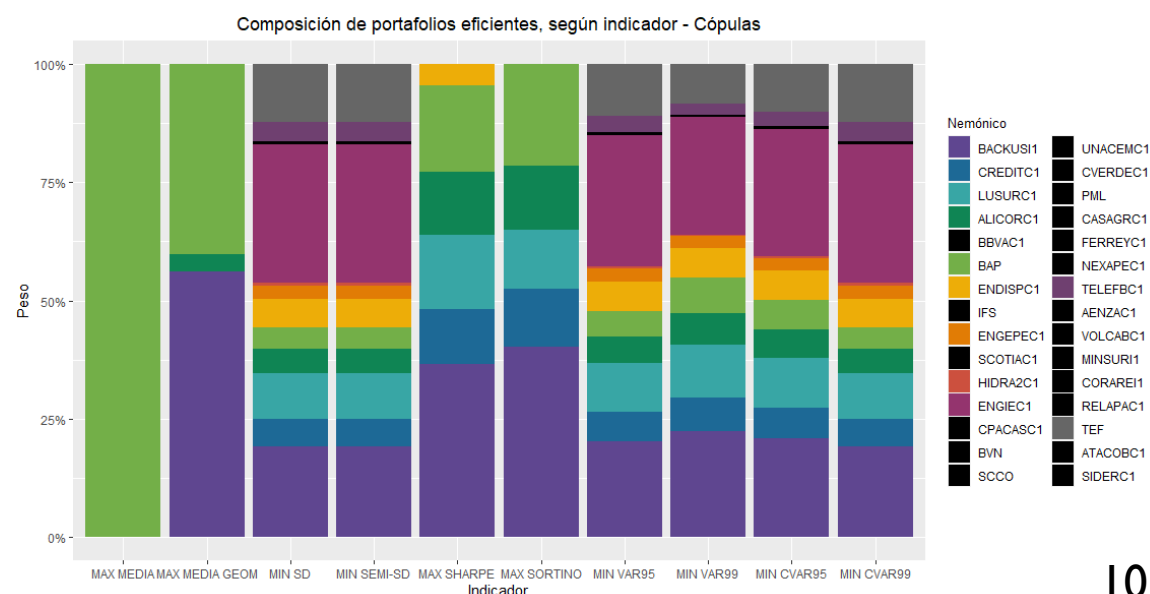
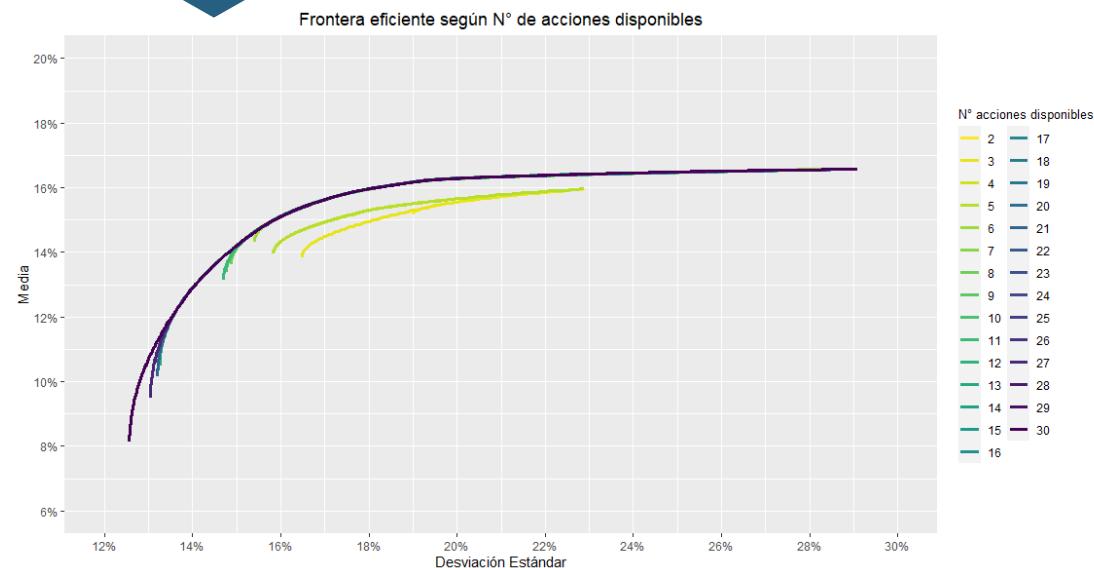
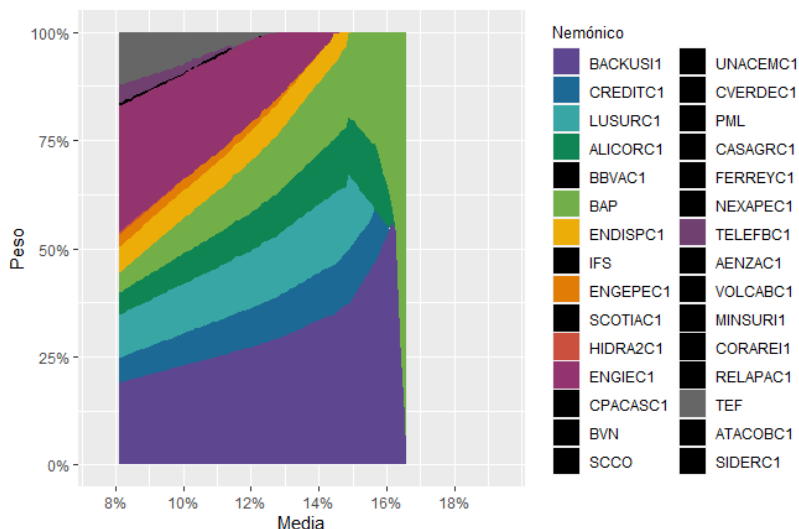
Max. Mean, Max. Geom. Mean,
Min. SD, Min Semi-SD
Min. VaR95, Min. VaR99
Min. CVaR95, Min. CVaR99
Max. Sharpe Ratio ($R_f=0$)
Max. Sortino Ratio ($R_f=0$)

Simulation model based on copulas



The portfolios had lower concentration in BACKUS11 and were more diversified.

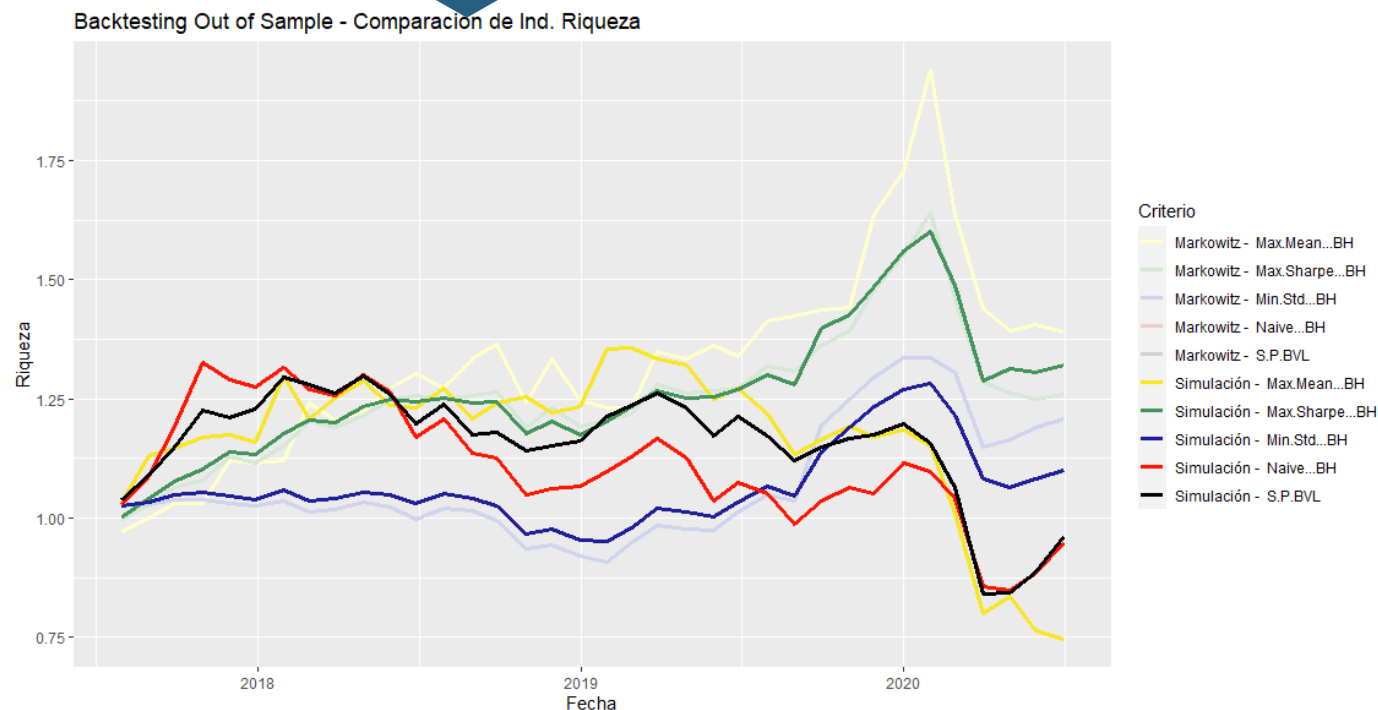
The method allowed modeling of the leptokurtosis of returns, according to the adjusted statistical distributions.



Backtesting

- **Split-sample evaluation:**
 - Estimation period: 07/2007 – 06/2017
 - Evaluation period: 07/2017 – 06/ 2020
- **Backtesting using the wealth index**
- **Rebalancing strategies:** Buy & Hold, monthly rebalancing, quarterly rebalancing.
- **Benchmarks:** Naive diversified portfolio and S&P/BVL Peru General Index.

The optimized portfolios had significantly superior in-sample and out-of-sample performance compared to the benchmarks, which ended up with negative returns due to the impact of the pandemic.



	Benchmarks		Markowitz			Simulation		
	NAIVE	S&P/BVL	MAX MEAN	MIN SD	MAX SHARPE	MAX MEAN	MIN SD	MAX SHARPE
Annual. Geom. Mean.	-1.8%	-1.3%	11.6%	6.5%	8.0%	-9.3%	3.2%	9.7%
Annual. Std. Dev.	19.5%	18.2%	21.2%	14.1%	13.8%	21.0%	11.7%	13.0%
Annual. Sharpe (Rf=0%)	-0.09	-0.07	0.55	0.46	0.58	-0.44	0.28	0.74

Conclusions

- Non-normality in stock returns in the Peruvian stock market (leptokurtic distribution). Under monthly periodicity, most stock returns were independently distributed.
- Lower-risk portfolios were composed of a greater number of assets, demonstrating the diversification effect. Results among the models were consistent and followed the same trend.
- The Markowitz model was extremely efficient but less robust. The simulation method was computationally demanding but more flexible, resulting in less concentrated portfolios with similar performance.
- Backtesting plays a fundamental role in portfolio optimization.
- In-sample and out-of-sample results of optimized portfolios were significantly superior to the benchmarks.
- The most plausible objective function was to maximize risk-adjusted returns. The importance of model development lies in making intelligent decisions in the allocation of financial resources

Recommendations

- Expand scope: include a larger number of stocks in the optimization process, explore other stock exchanges, include other financial instruments, etc.
- Explore other models that allow for eliminating assumptions or modeling more accurately (robust parameter estimation).
- Invest in a diversified portfolio, with risk-free or low-risk assets to mitigate the impact of a systemic crisis.
- Conduct continuous backtesting and monitor results, analyzing the suitability of the strategy. Do not solely rely on the model as a blind tool. Supplement with fundamental analysis.





THANKS