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1. MLE and MAP Question 1.a

Probability Density Function (PDF): $P(x \mid \alpha) = \alpha(1-x)^{\alpha-1}$

where x lines in interval $[0, 1]$ and $\alpha > 0$

① MLE for the paremetr α based on dataset $X = \{x_1, x_2, \ldots x_n\}$

of flutternes measurments

↓

* So, we need to differentiate the log-likelihood

Function withr respect to α, se it equal 0, and solve for

α.

$X = \{x_1, x_2, \ldots, x_n\}$: $L(\alpha) = \sum_{i=1}^{n} \ln(p(x_i \mid a))$

For each observation x_i, we evaluate the log of the PDf

$\log(p(x_i \mid a)) = \lg(\alpha) + (\alpha - 1)\ln(1-x_i)$

- We sum up these log-likelihood terms for data points in the dataset

(Takin log of likelihood → simplifies comp, maximizing it equal to maximizing original likelihood

- $\frac{dL}{d\alpha} = \sum_{i=1}^{n} \left(\frac{1}{\alpha} - \ln(1-x_i)\right)$

(The MLE aims to find value of par α that max log L func)

- Our goal is find the critical point where der. eq. zero)

$\sum_{i=1}^{n} \left(\frac{1}{\alpha} - \log(1-x_i)\right) = 0 \implies \alpha = \frac{n}{\sum \ln(1-x_i)}$

This our Maximum Likelihood Estimate

1 MLE and MAP Question 1.6

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$P(x \mid \alpha) = \alpha(1-x)^{\alpha-1}$ $\quad L(\alpha) = \sum_{i=1}^{n} \alpha(1-x_i)^{\alpha-1}$

$P(\alpha) = \lambda \alpha^{\lambda-1} e^{-\lambda\alpha}$ where $\lambda > 0$

We need to find the value of $\lambda > 0$ that maximizes

$$L(\alpha) \cdot P(\alpha) = \lambda \alpha^{\lambda-1} e^{-\lambda\alpha} \cdot \sum_{i=1}^{n} (\alpha(1-x_i)^{\alpha-1}$$

But taking der of this is hard so lets write log funct → maximizing it is same with maximizing original

$$\ln(L(\alpha)) \cdot \ln((P(\alpha)) = \ln\lambda + (\lambda-1)\ln\alpha - \lambda\alpha$$
$$+ \sum_{i=1}^{n} \ln\alpha + (\alpha-1)\ln(1-x_i)$$

→ Taking the derivative of this and set it to zero

$$\frac{\lambda-1}{\alpha} - \lambda + \sum_{i=1}^{n} \frac{1}{\alpha} - \ln(1-x_i) = 0$$

$$\frac{\lambda-1+n}{\alpha} = \sum_{i=1}^{n} \ln(1-x_i)$$

$$\Rightarrow \alpha = \frac{\lambda-1+n}{\sum_{i=1}^{n} \ln(1-x_i)}$$