

9-Month Progress Presentation

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Brzowski's Derivative

$$\text{der}_a(\mathbf{0}) \Rightarrow \mathbf{0}$$

$$\text{der}_a(\mathbf{1}) \Rightarrow \mathbf{0}$$

$$\text{der}_a(c) \Rightarrow \begin{cases} \mathbf{1} & \text{if } a = c, \\ \mathbf{0} & \text{otherwise.} \end{cases}$$

$$\text{der}_a(r_1 + r_2) \Rightarrow \text{der}_a(r_1) + \text{der}_a(r_2)$$

$$\text{der}_a(r_1 \cdot r_2) \Rightarrow \text{der}_a(r_1) \cdot r_2 + \begin{cases} \text{der}_a(r_2) & \text{if } \text{nullable}(r_1), \\ \mathbf{0} & \text{otherwise.} \end{cases}$$

$$\text{der}_a(r^*) \Rightarrow \text{der}_a(r) \cdot r^*$$

Brzowski's derivatives

Matching Example

Regular expression $(ab + ba)$ **String** ba

$$\begin{aligned} \text{der}_b r &= \text{der}_b (ab + ba) \\ &= \text{der}_b (ab) + \text{der}_b (ba) \\ &= (\text{der}_b a) \cdot b + (\text{der}_b b) \cdot a \\ &= \mathbf{0} \cdot b + \mathbf{1} \cdot a \end{aligned}$$

$$\begin{aligned} \text{der}_a (\text{der}_b r) &= \text{der}_a (\mathbf{0} \cdot b + \mathbf{1} \cdot a) \\ &= \text{der}_a (\mathbf{0} \cdot b) + \text{der}_a (\mathbf{1} \cdot a) \\ &= \text{der}_a (\mathbf{0}) \cdot b + (\text{der}_a (\mathbf{1}) \cdot a + \text{der}_a a) \\ &= \mathbf{0} \cdot b + (\mathbf{0} \cdot a + \mathbf{1}) \end{aligned}$$

Size Explosion

Regular expression $((a)^* + (aa)^* + (aaa)^* + (aaaa)^* + (aaaaa)^*)^*$ **String** $\underbrace{a \dots a}_n$

$$\begin{aligned} \text{der}_a r &= \text{der}_a (((a)^* + (aa)^* + (aaa)^* + (aaaa)^* + (aaaaa)^*)^*) \\ &= \text{der}_a ((a)^* + (aa)^* + (aaa)^* + (aaaa)^* + (aaaaa)^*) \cdot r^* \\ &= ((a)^* + (a \cdot (aa)^*) + (aa \cdot (aaa)^*) + \dots) \cdot r^* \end{aligned}$$

$$\begin{aligned} \text{der}_a (\text{der}_a r) &= \text{der}_a ((a)^* + (a \cdot (aa)^*) + (aa \cdot (aaa)^*) + \dots) \cdot r^* \\ &\quad + \text{der}_a r^* \\ &= ((a)^* + (aa)^* + (a \cdot (aaa)^*) + \dots) \cdot r^* \\ &\quad + ((a)^* + (a \cdot (aa)^*) + (aa \cdot (aaa)^*) + \dots) \cdot r^* \end{aligned}$$

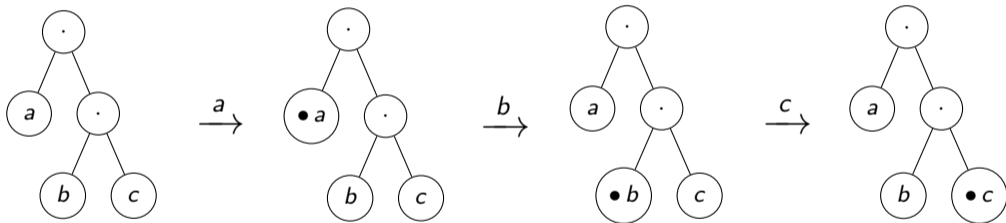
$$\begin{aligned} \text{der}_a (\text{der}_a (\text{der}_a r)) &= (\text{der}_a (((a)^* + ((aa)^*) + (a \cdot (aaa)^*) + \dots) \cdot r^*) \\ &\quad + \text{der}_a r^*) + (\text{der}_a (((a)^* + (a \cdot (aa)^*) + (aa \cdot (aaa)^*) \\ &\quad + \dots) \cdot r^*) + \text{der}_a r^*) \end{aligned}$$

Without simplifications: $n = 4 \Rightarrow 985$, $n = 10 \Rightarrow 65,353$.

With simplifications: $n = 4 \Rightarrow 315$, $n = 10 \Rightarrow 777$.

Marked Approach: Fischer et al. / Asperti et al.

Regular expression $a \cdot (b \cdot c)$ String abc



Our Definition of *shifts*

$$\text{shifts}(ms, 0) \Rightarrow []$$

$$\text{shifts}(ms, 1) \Rightarrow []$$

$$\text{shifts}(ms, c) \Rightarrow [\bullet s \mid \bullet c :: s \in ms]$$

$$\text{shifts}(ms, r_1 + r_2) \Rightarrow \text{shifts}(ms, r_1) @ \text{shifts}(ms, r_2)$$

$$\text{shifts}(ms, r_1 \cdot r_2) \Rightarrow \text{let } ms' = \text{shifts}(ms, r_1) \text{ in}$$

$$\begin{cases} \text{shifts}(ms' @ ms, r_2) @ ms' & \text{if } \text{nullable}(r_1) \wedge \text{nullable}(r_2), \\ \text{shifts}(ms' @ ms, r_2) & \text{if } \text{nullable}(r_1), \\ \text{shifts}(ms', r_2) @ ms' & \text{if } \text{nullable}(r_2), \\ \text{shifts}(ms', r_2) & \text{otherwise.} \end{cases}$$

$$\text{shifts}(ms, r^*) \Rightarrow \text{let } ms' = \text{shifts}(ms, r) \text{ in}$$

$$\text{if } ms' = [] \text{ then } [] \text{ else } \text{shifts}(ms', r^*) @ ms'$$

Matching Example

Regular expression $(a + (a \cdot c))$ **String** ac

1. $|_{[\bullet ac]} (a + (a \cdot c))$
2. $((|_{[\bullet ac]} a) + (|_{[\bullet ac]} (a \cdot c)))$
3. $(a |_{[\bullet c]}) + ((a \cdot c) |_{[\bullet []]})$
 $= [_{\bullet c, \bullet []}]$

Future Work

- ▶ **Extend the matcher.** Add support for bounded repetitions, intersection, lookahead, and lookbehind operators.
- ▶ **POSIX value extraction.**
- ▶ **Formalisation in Isabelle/HOL.**
- ▶ **Exploring context-free generalisation.** If time allows, we aim to extend the marked approach to context-free grammars, as done for derivatives.

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