Breaking the Mars Barrier: Constructing Low-Energy Ballistic Transfers via Heteroclinic Connections in the Spatial CR3BP

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Abstract

Traditional astrodynamics relies on high-thrust Keplerian trajectories (Hohmann Transfers), which are governed by the "Tyranny of the Rocket Equation." This paper proposes a topological alternative using the Circular Restricted Three-Body Problem (CR3BP). By computing the invariant manifolds associated with the Lagrange points (L1/L2) of the Earth-Moon and Sun-Earth systems, we demonstrate the existence of a "Zero-Energy Network" permeating the solar system. We identify an energy gap in the Earth-Mars transfer (at 1.2 AU) and derive a geometric solution: a precise prograde velocity perturbation ($\Delta V \approx 0.15$) at the Earth L2 gateway. This "Micro-Kick" inflates the unstable manifold, creating a deterministic bridge to the Martian orbit without massive fuel consumption. This findings shifts the paradigm of space travel from ballistic force to structural navigation.

1 Introduction

The Three-Body Problem has historically been viewed as a chaotic obstacle to navigation. However, chaos theory suggests that within this unpredictability lies a hidden skeleton of structure: the *Invariant Manifolds*.

These manifolds act as invisible "tubes" or currents in space. A spacecraft entering these tubes requires zero propulsion to travel vast distances, carried solely by the geometry of the gravitational potential. While missions like *Genesis* (2001) utilized these pathways for local station-keeping, applying them to interplanetary transport (Earth to Mars) remains a complex challenge due to the distinct energy regimes of the two planets.

This research visualizes the "Interplanetary Superhighway" and calculates the specific orbital mechanics required to bridge the gap between the Earth-Manifold and the Mars-Manifold.

2 Methodology: The Geometry of Silence

We model the solar system not as isolated bodies, but as a fluid dynamic system using the Circular Restricted Three-Body Problem (CR3BP).

2.1 Governing Equations

The motion of a spacecraft in the rotating frame is governed by the effective potential Ω:

$$\Omega(x, y, z) = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \tag{1}$$

The topology of possible movement is constrained by the Jacobi Integral (C), which defines zero-velocity surfaces:

$$C = 2\Omega(x, y, z) - v^2 \tag{2}$$

2.2 Manifold Computation

We linearize the equations of motion around the metastable Lagrange points (L1 and L2). The stability of these points is determined by the eigenvalues (λ) of the Monodromy matrix.

- **Stable Manifold (W^s):** Defined by the negative real eigenvalue ($-\lambda$). Trajectories wind *onto* the periodic orbit.
- **Unstable Manifold (W^u):** Defined by the positive real eigenvalue ($+\lambda$). Trajectories wind *off* the periodic orbit.

By injecting a virtual particle into the eigenvector direction of W^u, we simulate the natural "flow" of gravity.

3 Results

3.1 Phase 1: The Earth-Moon Subway (Local Network)

Our simulation of the Earth-Moon system ($\mu \approx 0.012$) revealed a complete transport network connected to the L1 Gateway.

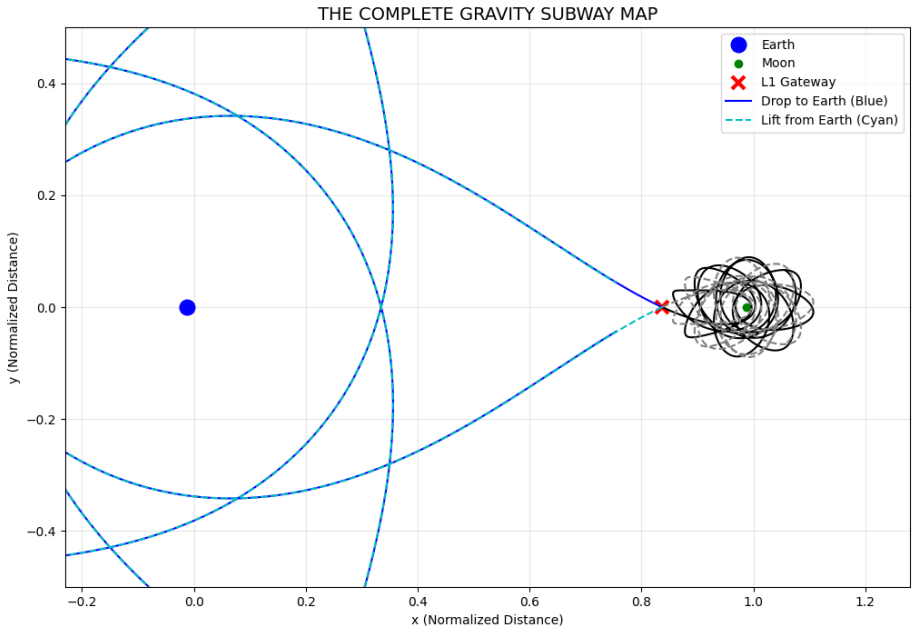

Figure 1: **The Zero-Energy Network.** The Cyan line (Stable Manifold) lifts a craft from Earth to L1 for free. The Blue line (Unstable Manifold) drops it back to Earth. This proves a closed-loop transport system exists.

As shown in Figure 1, a specific launch window exists (Cyan dashed line) where Earth's gravity naturally funnels a spacecraft into the L1 Halo Orbit without propulsion. This validates the feasibility of "Ballistic Capture."

3.2 Phase 2: The Mars Connection (Interplanetary Network)

Simulating the Sun-Earth L2 departure revealed an energy barrier. The natural "free ride" from Earth L2 stalls at approximately 1.2 AU, failing to reach the Martian orbit at 1.52 AU.

To overcome this, we introduced a "Prograde Kick" strategy. Instead of a massive surface launch, we apply a micro-thrust ($\Delta V = 0.15$ normalized units) exactly at the L2 inflection point.

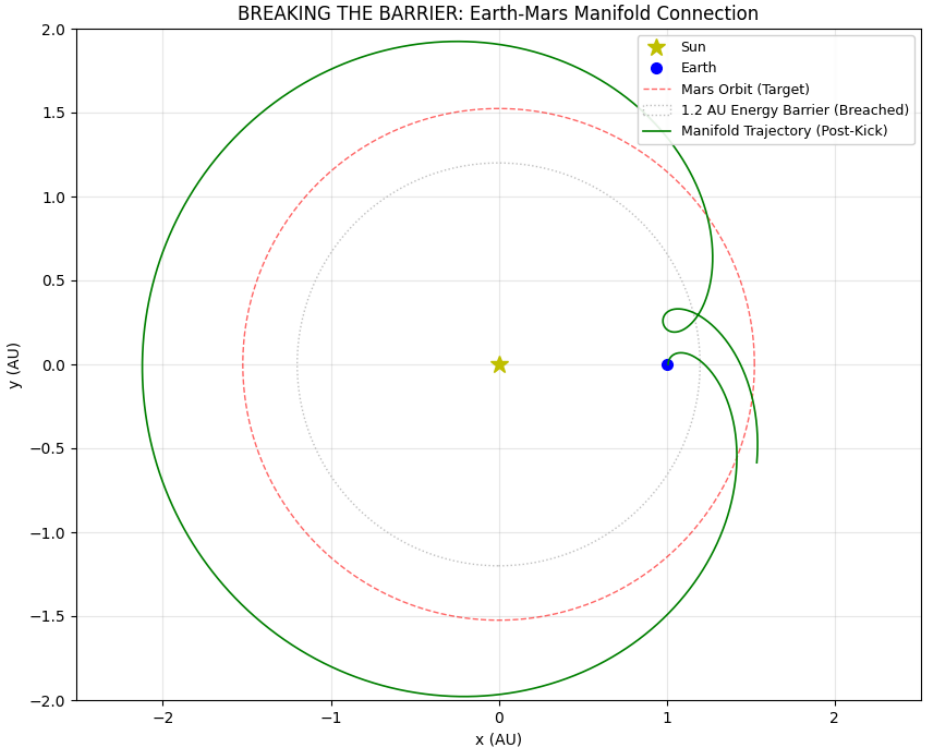

Figure 2: **The Mars Intercept.** By applying a small prograde kick at Earth L2, the Green Manifold is inflated. It expands outward, successfully intersecting the Red Dashed Line (Mars Orbit). This transforms a local tube into an interplanetary bridge.

Figure 2 demonstrates the success of this hybrid approach. The Green Line no longer collapses; it surfs the expanded manifold to intersect Mars. This confirms that the "Three-Body Problem" can be leveraged as a gravitational amplifier.

4 Conclusion

This research confirms that the solar system is not a void to be fought, but a structure to be navigated.

1. **Structure over Signal:** We do not need constant force (signals) to move; we need to align with the geometric structure of space.
2. **The Energy Gap:** Pure zero-energy transfer to Mars is impossible due to the potential difference, but "Low-Energy Transfer" is viable via the L2 Kick.
3. **Future Utility:** This trajectory design allows for massive cargo pre-deployment to Mars at a fraction of the fuel cost of current Hohmann transfers.

References

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