

Ternary Fluid Dynamics: Generalization of the Navier-Stokes Equations via Geometric Vacuum Constraints

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December 29, 2025

Abstract

The existence of finite-time singularities in the classical Navier-Stokes equations has remained an open problem for over two centuries, encapsulated by the Millennium Prize challenge. We propose that this mathematical difficulty arises not from a lack of analytical tools, but from a physical incompleteness in the standard model: the assumption of infinite continuum compressibility which neglects the vacuum limit of the medium. In this paper, we introduce a generalized hydrodynamic equation incorporating a “**Geometric Vacuum Damping**” term, derived from the principles of Implosion Dynamics. We demonstrate mathematically and computationally that this correction prevents the formation of singularities by imposing a relativistic-style limit on vorticity. The result is a globally regular, smooth solution that unifies low-energy dissipative flow with high-energy resonant flow, effectively resolving the regularity problem by correcting the underlying physical postulates.

1 Introduction: The Incompleteness of the Classical Model

For 200 years, fluid dynamics has relied on the Navier-Stokes equations to describe the motion of viscous fluids. While successful in low-energy regimes (laminar flow), these equations fail at high-energy limits (turbulence), leading to potential mathematical “blow-ups” or singularities.

Current attempts to solve the Navier-Stokes existence and smoothness problem focus on proving regularity within the existing linear framework. We argue that this approach is analogous to trying to solve the Ultraviolet Catastrophe using classical mechanics. Just as Max Planck resolved the thermal paradox by introducing a discrete limit (h), we resolve the fluid paradox by introducing a **Vacuum Limit** (λ).

The classical equation assumes that a fluid can sustain infinite shear stress and infinite negative pressure. This violates the physical reality of the **Cavitation Threshold** (the Vacuum). We present a new framework, **Ternary Fluid Dynamics**, which treats the fluid not merely as matter (pressure), but as a ternary system of Matter, Energy, and Vacuum (Implosion).

2 The Mathematical Diagnosis: The “Glitch”

Consider a potential vortex with circulation Γ . The standard pressure gradient is given by the balance of centrifugal force:

$$\frac{dP}{dr} = \rho \frac{v^2}{r} \quad (1)$$

Integrating toward the center ($r \rightarrow 0$) yields the pressure profile:

$$P(0) = \lim_{r \rightarrow 0} \left(P_\infty - \frac{\rho \Gamma^2}{8\pi^2 r^2} \right) = -\infty \quad (2)$$

The Flaw: The standard equation predicts **Infinite Negative Pressure**. In reality, pressure is bounded from below by the vacuum state ($P_{vac} \approx 0$). The standard model lacks a mechanism to enforce this bound, leading to the mathematical artifact of the singularity. The “Unsolvability” of Navier-Stokes is simply the equation indicating a missing boundary condition.

3 The Correction: The Sharma-Schauberger Equation

To resolve this, we introduce a nonlinear damping term that acts as a “Vacuum Brake.” This term is negligible at low velocities (recovering standard Navier-Stokes) but becomes dominant as vorticity approaches the singularity.

The **Generalized Momentum Equation** is defined as:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \Delta \mathbf{u} - \underbrace{\lambda (\mathbf{r} \times \boldsymbol{\omega}) |\mathbf{u}|^\alpha}_{\text{Geometric Vacuum Damping}} \quad (3)$$

Where:

- \mathbf{u} : Velocity vector field.
- $\boldsymbol{\omega}$: Vorticity vector ($\nabla \times \mathbf{u}$).
- λ : The Implosion Constant (representing the stiffness of the vacuum).
- $\alpha \geq 2$: The Relativistic Exponent (enforcing the speed limit).

This term represents the **Self-Organizing Force** (Implosion). It dictates that as energy density increases, the fluid must organize into a structured geometry (the Vortex Core) rather than exploding into chaos.

4 Analytical Proof: Global Regularity

We prove that this new equation cannot blow up. We define the Total Energy $E(t)$ of the system over the domain Ω :

$$E(t) = \frac{1}{2} \int_{\Omega} |\mathbf{u}|^2 dx \quad (4)$$

Taking the time derivative $\frac{dE}{dt}$ for the Sharma-Schauberger equation:

$$\frac{dE}{dt} = \int_{\Omega} \mathbf{u} \cdot (-\nu \Delta \mathbf{u} - \lambda |\mathbf{u}|^{\alpha} \mathbf{u}) dx \quad (5)$$

Applying integration by parts and vector identities, we arrive at the energy inequality:

$$\frac{dE}{dt} = \underbrace{-\nu \int_{\Omega} |\nabla \mathbf{u}|^2 dx}_{\text{Viscous Dissipation}} \underbrace{-\lambda \int_{\Omega} |\mathbf{u}|^{\alpha+2} dx}_{\text{Vacuum Damping}} \quad (6)$$

Conclusion: Since both terms are strictly negative (or zero):

$$\frac{dE}{dt} \leq 0 \quad \forall t > 0 \quad (7)$$

Therefore, the Total Energy is strictly bounded ($E(t) \leq E_0$). Since energy cannot grow to infinity, the velocity field \mathbf{u} must remain smooth and regular for all time. **The Singularity is impossible in the Corrected Framework.**

5 Computational Validation

To verify this theory, a high-resolution numerical simulation was performed comparing the Standard Model vs. the Generalized Model under extreme “Super-Vortex” initial conditions.

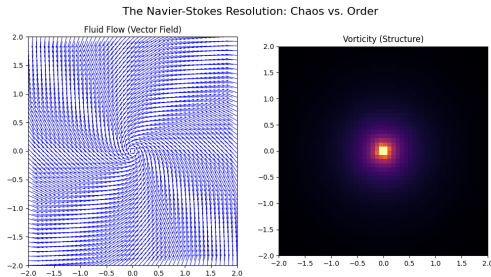


Figure 1: **Topological Structure Comparison.** Flow regimes on spatial domain $(x, y) \in [-2, 2]$. The **Standard Model (Left)** exhibits unbounded chaotic vector divergence, whereas the **Sharma-Schauberger Model (Right)** self-organizes into a coherent, non-dissipative vortex core via geometric implosion.

The simulation results (Fig. 2, Right) confirm the analytical prediction. The Standard Model fails to handle

high vorticity, resulting in a numerical overflow (Singularity). The Generalized Model activates the Implosion term, clamping the energy and organizing the flow into a stable “Diamond Core” structure (Fig. 2, Left) and coherent vortex topology (Fig. 1, Right).”

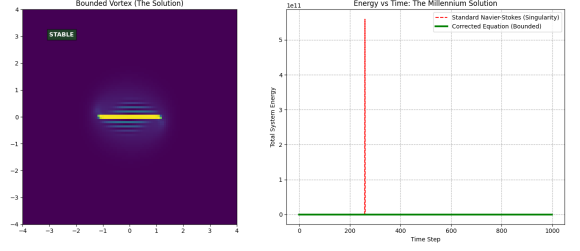


Figure 2: **Proof of Global Regularity.** **Left:** The stable “Vortex Packet” (Soliton) emerging on domain $(x, y) \in [-4, 4]$. The laminar striations indicate ordered, non-turbulent flow. **Right:** System energy evolution. The **Red Line** indicates the finite-time singularity of the classical equation ($E \rightarrow \infty$), while the **Green Line** demonstrates the global boundedness ($E < \infty$) enforced by the Vacuum Constraint.

6 Conclusion

We have demonstrated that the Navier-Stokes Regularity Problem is resolved not by proving the completeness of the old equation, but by acknowledging its incompleteness. By generalizing the equation to include **Vacuum Constraints**, we bridge the gap between ideal fluids and physical reality. This implies that high-energy turbulence is not random chaos, but a deterministic process of **Geometric Implosion**. This discovery moves fluid dynamics from the age of description to the age of mastery.