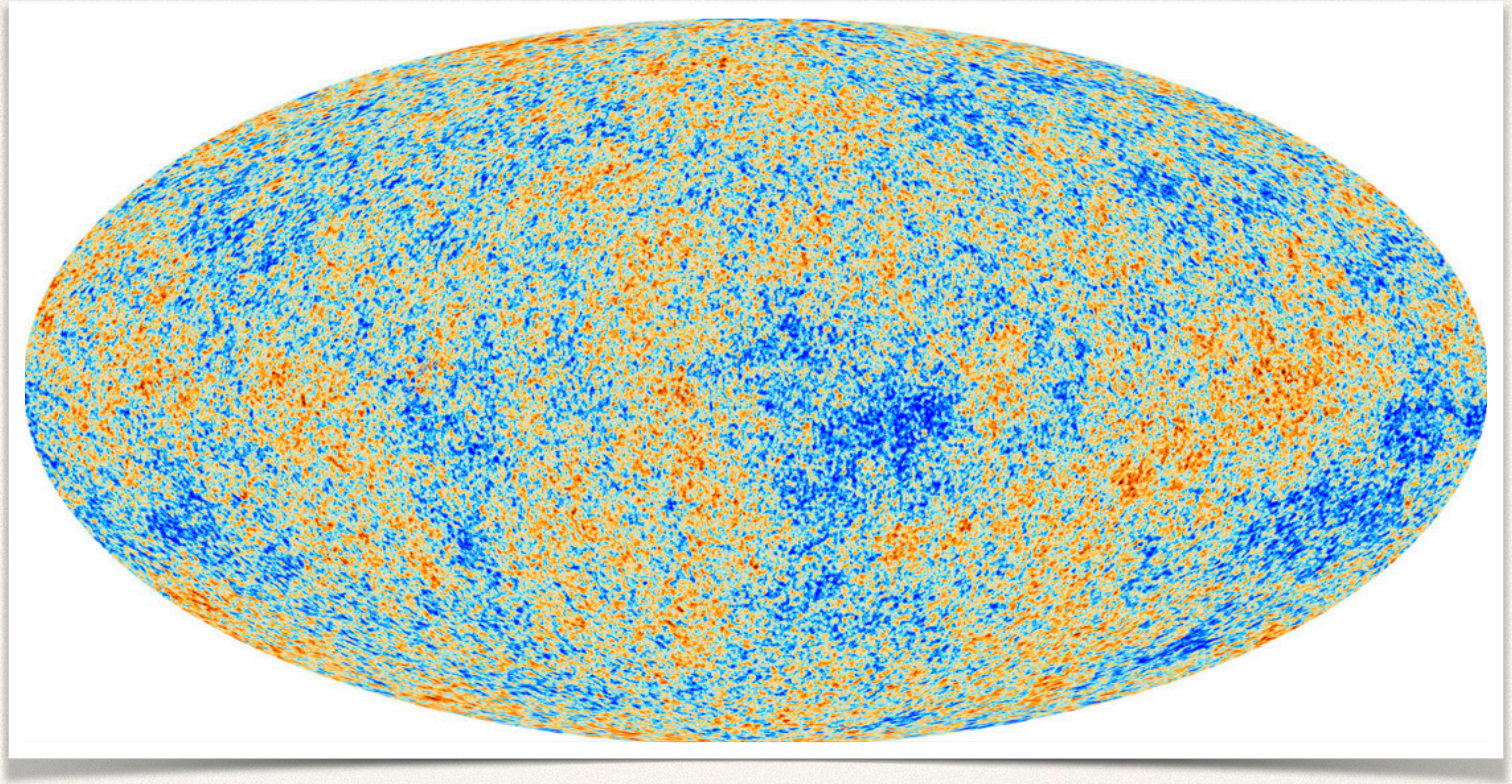


Writing our own CMB code

Part I

Jul 1st

Observations

- COBE : detected anisotropies on largest scales($l \sim 40$)
- WMAP : detected anisotropies on smaller scales($l \sim 1000$)
- Planck : ($l \sim 2000$)

- Multipole moments of temperature field

$$\Theta_{lm}=\int d\hat{n}\, Y_{lm}^{*}(\hat{n})\, \Theta(\hat{n})$$

→monopole(l=0): Θ

→dipole(l=1):v_{γ}

- Power spectrum

$$\Delta_T^2=\frac{l(l+1)}{2\pi}\, c_l T^2$$

Perfect fluid

- before redshift $z_* \approx 10^3$ the cosmological plasma was a tightly coupled photon-baryon fluid

- We neglect the effects of baryon and gravity

- Perturbation in this fluid can be described by continuity and Euler equations of fluid dynamics

- Continuity equation

$$\dot{\Theta} = \frac{-1}{3} k v_\gamma$$

- Euler equation

$$\dot{v_\gamma} = k\Theta$$

derivative with respect to conformal time : $\eta = \int \frac{dt}{a(t)}$

$\ddot{\Theta} + \frac{1}{3}kv_\gamma$=0 → $\ddot{\Theta}$+$c_s^2 k^2\Theta$=0

→Where c_s^2=$\sqrt{\dot{p}/\dot{\rho}}$=$1/\sqrt{3}$

- Pressure gradients act like a restoring force and the system oscillates at the speed of sound
- Temperature oscillations represent heating and cooling of the fluid
- At recombination

$$\Theta(\eta_*)=\Theta(0)\cos(ks_*)$$

→where s=$\int c_s\, d\eta \approx \eta/\sqrt{3}$

Initial condition

- Whence Θ(0)?!
- The inflation era was a time when the universe was driven by energy with negative pressure that potential energy of a scalar field provided this kind of energy
- Curvature fluctuations(Φ) are space-space and Newtonian potential(Ψ) is time-time
- Approximately Ψ≈-Φ
- Scale invariant $\Delta_\Phi^2 \equiv \frac{k^3}{2\pi^2} P_\Phi(k) \propto k^{n_s-1}$

- $\frac{\delta t}{t} = \Psi$
- a=$t^{2/[1+(^{p}/_{\rho})]}$
- Fractional change in CMB temperature

  $$\Theta=\frac{-\delta a}{a} = -\frac{2}{3}(1+\frac{p}{\rho})^{-1}\frac{\delta t}{t}$$

- In radiation domination era where $p = \frac{\rho}{3}$: $\Theta=\frac{-\Psi}{2}$
- In matter dominated era where p=0 : $\Theta=\frac{-2\Psi}{3}$
- Initial temperature perturbation is linked with initial gravitational potential perturbation

Gravity

- Newtonian potential and spatial curvature change the acoustic peaks because they provide a gravitational force on oscillator
- This shifts the equilibrium to $\Theta+\Psi=0$
- Gravity changes continuity equation :

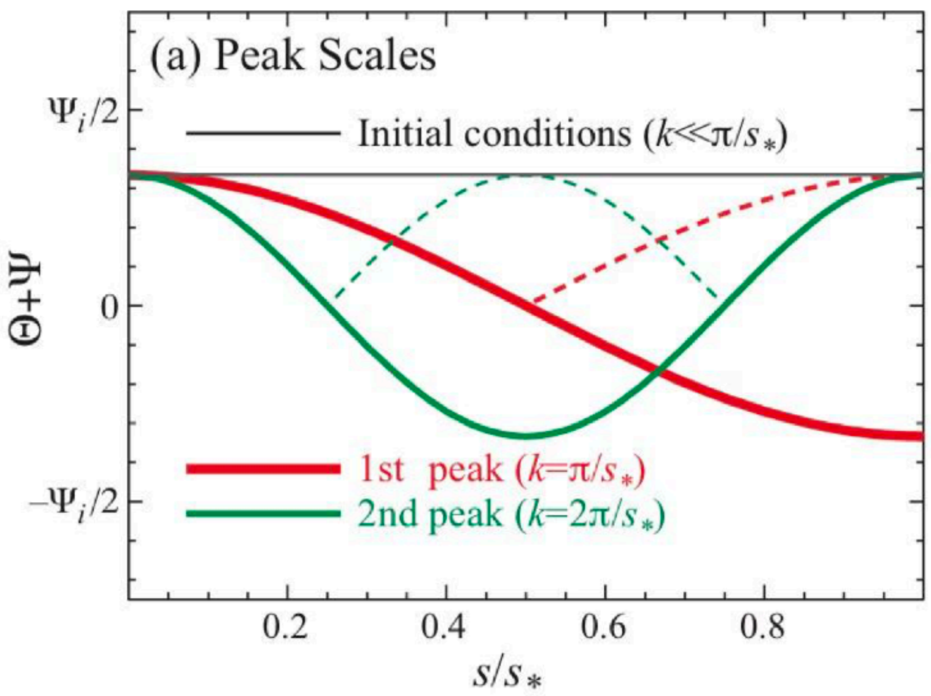
$$\dot{\Theta}=\frac{-1}{3}kv_{\gamma}-\dot{\Phi}$$

And Euler equation :

$$\dot{v_{\gamma}}=k\Phi+k\Psi$$

And we have :

$$\ddot{\Theta}+c_s^2k^2\Theta=\frac{-k^2}{3}\Psi-\ddot{\Phi}$$



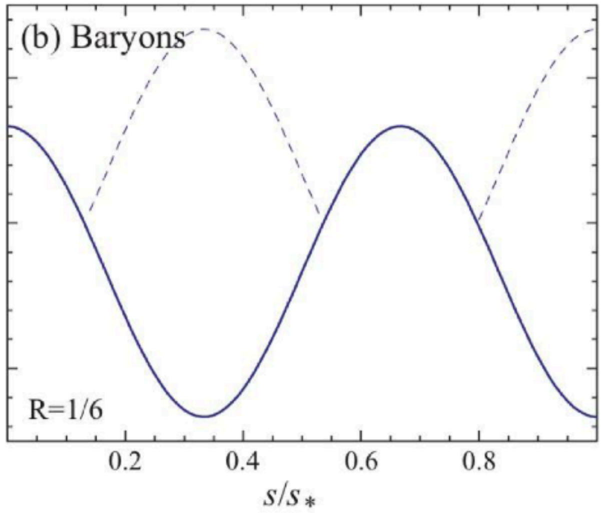
- With Φ+Ψ as effective temperature :

$$[\Phi+\Psi](\eta)=[\Phi+\Psi](\eta_{md})\cos(ks)$$

Where η_{md} is start of the matter dominated era

- Photons climb out of the potential well suffer a gravitational redshift of : $\frac{\Delta T}{T} = \Psi$
- Acoustic oscillations arise by compression due to infall of the fluid into gravitational potential well and rarefaction by pressure

Baryon



- Photon-baryon momentum ratio : R=$^{(p_b+\rho_b)}/_{(p_\gamma+\rho_\gamma)}$
- The oscillator equation :

$$c_s^2 \frac{d}{d\eta}(c_s^{-2}\dot{\Theta}) + c_s^2 k^2 \Theta = \frac{-k^2}{3}\Psi - c_s^2 \frac{d}{d\eta}\left(c_s^{-2}\dot{\Phi}\right)$$

Where $c_s = {}^{1}/_{\sqrt{3(1+R)}}$

- Baryons lower the sound speed which decreases the sound horizon
- $[\Theta+(1+R)\Psi](\eta)=[\Theta+(1+R)\Psi](\eta_{md})\cos(ks)$
- Baryons enhance the amplitude of the oscillations and shift equilibrium to $\Theta=-(1+R)\Psi$

Radiation

- Radiation Is dominant above the redshift of equality : $Z_{eq} = 2.4{\times}10^4\Omega\ _0h^2$
- Radiation makes gravitational force evolve with time and drives the acoustic oscillations
- The fluid is maximally compressed with no gravitational potential to fight as it turn back

Damping

- The photon-baryon fluid has imperfections (shear viscosity and heat conduction) which damp the oscillations
- The equations of motion of the system in full form :

$$\dot{\Theta}=\frac{-1}{3}kv_\gamma-\dot{\Phi} \qquad \dot{\delta_b}=-kv_b-3\dot{\Phi}$$

$$\dot{v_\gamma}=k(\Theta+\Phi)-\frac{k}{6}\pi_\gamma-\dot{\tau}\left(v_\gamma-v_b\right)$$

$$\dot{v_\gamma}=-\frac{\dot{a}}{a}v_b+k\Psi+\dot{\tau}\left(v_\gamma-v_b\right)/R$$

The final oscillator equation

$$c_s^2\frac{d}{d\eta}(c_s^{-2}\dot{\Theta})+\frac{c_s^2k^2}{\dot{\tau}}[A_v+A_h]\dot{\Theta}+c_s^2k^2\Theta=\frac{-k^2}{3}\Psi-c_s^2\frac{d}{d\eta}\left(c_s^{-2}\dot{\Phi}\right)$$

- $A_v = 16/15$ derived from Boltzman equation
- $A_h = R^2/1 + R$ heat conduction coefficient
- We expect the inhomogeneities to be damped by a exponential factor of order $e^{-k^2\eta/\dot{\tau}}$