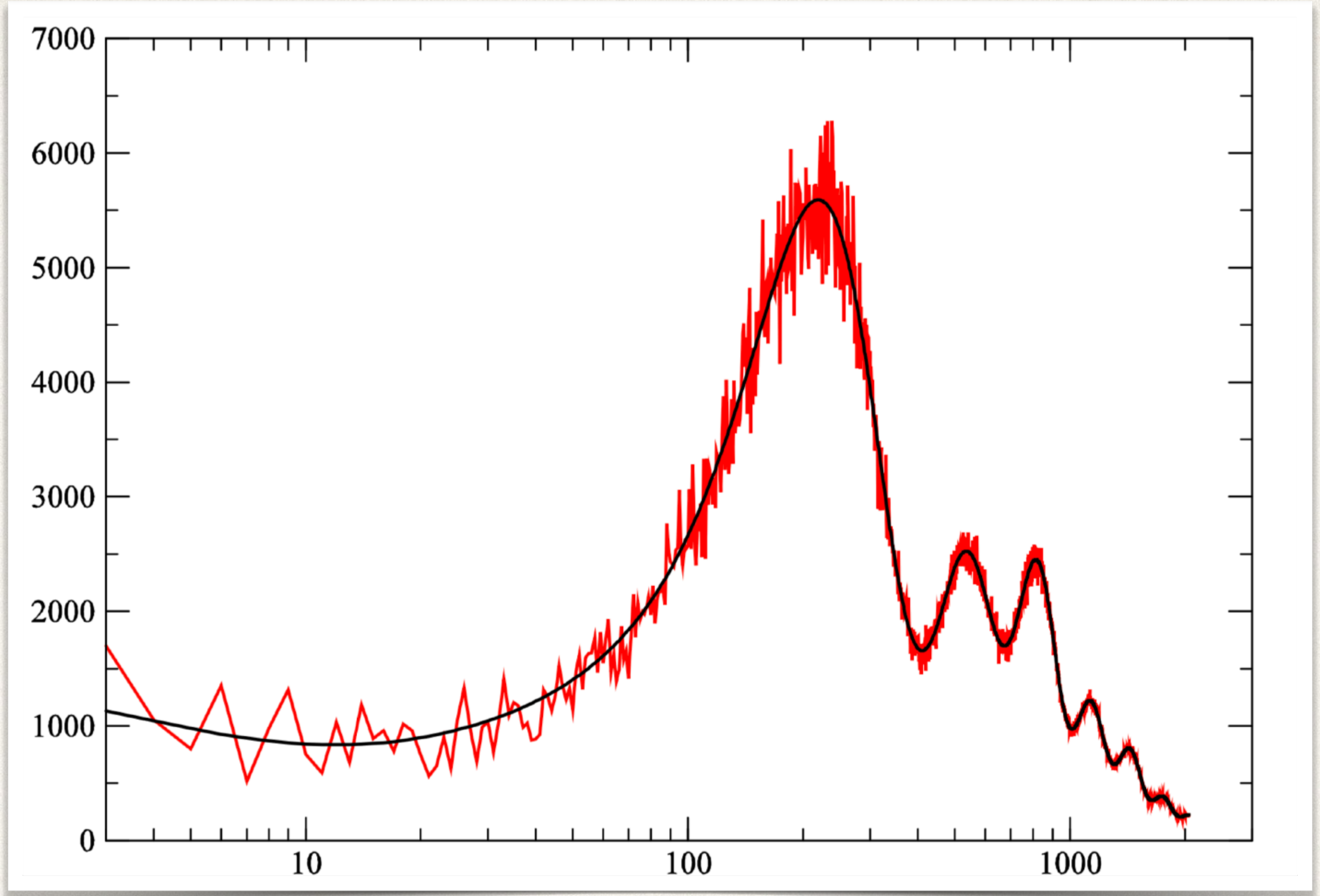

Writing our own CMB code

Part II

Jul 1st

- In this part we are going to write our own CMB code.

- Our code will implement the two fluid approximation of Seljak which is based on the simple observation that before recombination of photons and baryons are coupled strongly to each other via Thompson Scattering .

- We can treat them as a single fluid and since dark matter and this fluid don't coupled directly to each other the stress tensor of each of them are separately conserved.

$$\nabla_\mu T_i^{\mu\nu} = 0$$

- By solving this we can get the fluid equation of motion.

- The dynamics of scaler perturbations is then governed by continuity equation for density fluctuations and Euler equation for the divergence of the velocity field.

$$\delta = \frac{\delta\rho}{\rho} \qquad \theta = \overrightarrow{\nabla} . \overrightarrow{v}$$

- For a single uncoupled fluid with the equation of state and sound speed ,

$$\omega = \frac{P}{\rho} \qquad\qquad c_s^2 = \frac{\delta P}{\delta \rho}$$

- These dynamic equation become

$$\delta' = -(1+\omega)(\theta - 3\phi') - 3H(c_s^2 - \omega)\delta$$

$$\theta' = -H(1-3\omega)\theta - \frac{\omega'}{1+\omega}\theta + \frac{c_s^2}{1+\omega}k^2\delta + k^2\phi$$

- And the metric perturbation in Newtonian gauge

$$ds^2 = a^2(\tau)[-(1+2\phi)d\tau^2 + (1-2\phi)d\vec{x}^2]$$

- By taking $\overrightarrow{v}$ in the form of

$$\overrightarrow{v} = -iv\hat{k}$$

- Which implies

$$\theta = kv$$

- For cold dark matter we have

$$\omega = c_s^2 = 0$$

- So the fluid equations for cold dark matter become

$$\delta'_c = -kv_c + 3\phi' \qquad v'_c = -Hv_c + k\phi$$

- Now for photons we know they contribute a pressure to the baryon-photon fluid

$$p_\nu = \frac{1}{3}\rho_\nu$$

- So the effective equation of state and sound speed of this fluid are

$$\omega = \frac{1}{3+4R} \qquad c_s^2 = \frac{1}{3(1+R)}$$

- Where R is

$$R = \frac{3}{4}\frac{\overline{\rho_b}}{\overline{\rho_\nu}}$$

- In the tight coupling approximation we have

$$v_\nu = v_b \qquad \delta_\nu = \frac{3}{4}\delta_b$$

- So the photon evolution equation (dynamic equation) become

$$\delta'_\nu = -\frac{4}{3}kv_\nu + 4\phi'$$

$$v'_\nu = \frac{-Rv_\nu + \frac{1}{4}k\delta_\nu + (1+R)k\phi}{1+R}$$

- So these are our fluid equations, now we have to be supplemented by the linearized Einstein equation for the gravitational potential, so we have

$$k(\phi' + H\phi) = 4\pi G a^2 \sum_i (\overline{\rho}_i + \overline{p}_i)v_i$$

- And from Friedman equation from background we have

$$H^2 = (\frac{a'}{a})^2 = \frac{8\pi G a^2}{3}\sum_i \overline{\rho_i}$$

- And this equation has an analytic solution

$$y = \frac{a}{a_{eq}} = (\alpha x)^2 + 2\alpha x \qquad x = \frac{\tau}{\tau_r} \qquad \alpha = \sqrt{\frac{a_{rec}}{a_{eq}}}$$

- For numerical convenience we rescale time and momentum to

$$x = \frac{\tau}{\tau_r} \qquad \kappa = k\tau_r$$

- And also we define

$$\eta = \tau_r H$$

- By these definitions our fluid equation become

$$\delta'_c = -kv_c + 3\phi'$$

$$v'_c = -Hv_c + k\phi$$

$$\delta'_\nu = -\frac{4}{3}kv_\nu + 4\phi'$$

$$v'_\nu = \frac{-\frac{3}{4}y(\frac{\Omega_b}{\Omega_m})\eta v_\nu + \frac{1}{4}\kappa\delta_{nu}}{1 + \frac{3}{4}y(\frac{\Omega_b}{\Omega_m})} + \kappa\phi$$

$$\phi' = -\eta\phi + \frac{3\eta^2}{2\kappa}\frac{v_\nu(\frac{3}{4} + y - y(\frac{\Omega_c}{\Omega_m})) + v_c y(\frac{\Omega_c}{\Omega_m})}{1 + y}$$

- Now by setting initial conditions

$$\phi(x_i) = 1$$

$$\delta_\nu = -2\phi(x_i)$$

$$\delta_c = \frac{3}{4}\delta_\nu$$

$$v_\nu = -\frac{1}{4}\frac{\kappa}{\eta}\delta_\nu$$

$$v_c = v_\nu$$

- We can solve this coupled differential equation easily.

- Assuming instantaneous recombination we have the following relation between these perturbations and observed CMB anisotropies,

$$\Theta(k\,.\,\overrightarrow{n}) = [\phi + \frac{1}{4}\delta_\nu](\tau_{rec}) + \overrightarrow{n}\,.\,\overrightarrow{v_\nu}(\tau_{rec}) + 2\int_{\tau_{rec}}^{\tau_0} \phi'(\tau)d\tau$$

- We need to relate $\Theta(\vec{n})$ to $\Theta(k)$ .First we note that our assumption of instantaneous recombination implies

$$\Theta(\vec{n}) = \int dr \Theta(\vec{x}) \delta(r - r_*)$$

- Where the integral is over conformal distance and r_* is the angular diameter distance between us and the last-scattering surface. Here, $\Theta(\vec{x})$ is the real space temperature field relate to our solution via

$$\Theta(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}.\vec{x}} \zeta_{\vec{k}} \Theta(k)$$

- So we have

$$\Theta(\overrightarrow{n}) = \int \frac{d^3k}{(2\pi)^3} e^{i\overrightarrow{k}.r_* n} \zeta_{\overrightarrow{k}} \Theta(k)$$

- Where

$$e^{i\overrightarrow{k}.r_* n} = 4\pi \sum_{lm} i^l j_l(kr_*) Y^*_{lm}(\overrightarrow{k}) Y_{lm}(\overrightarrow{n})$$

- We find the CMB angular power spectrum

$$C_l^{TT} = \frac{2}{\pi}\int k^2 dk P_\zeta(k)\Delta_{Tl}^2(k)$$

- Where

$$\Delta_{Tl}(k) = (\phi + \frac{1}{4}\delta_\nu)j_l(k[\tau_0 - \tau_{rec}]) + v_\nu j'_l(k[\tau_0 - \tau_{rec}]) + 2\int_{\tau_{rec}}^{\tau_0} d\tau j_l(k[\tau_0 - \tau_{rec}])\frac{\phi'(\tau)}{\phi'(\tau_0)}$$

- By assuming perfect tight coupling (mean free path = 0) and instantaneous recombination, our solution is still missing some important physics including the effects of a finite mean free path for photons and a finite duration of recombination would lead us to the damping of small fluctuations , we could rectify this by adding viscosity directly if FEM. From Seljak paper we take a simpler approach, we introduce a high momentum cutoff by defining

$$C_l^{TT} \approx \frac{2}{\pi} \int k^2 dk P_\zeta(k) D(k) \Delta_{Tl}^2(k)$$

- Where

$$D(k) = e^{(-\frac{k}{k_D})^2}$$

✤ Plotting power spectrum by changing density parameter