Planar Motion (Small Oscillations)

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1 Small Oscillation With Two degrees of freedom

In order to understand this motion let's write Hamiltonian of System, By setting m = 1 we might have :

$$H = \frac{1}{2}\dot{x}_1^2 + \frac{1}{2}\dot{x}_2^2 + U(x_1, x_2) \tag{1}$$

With knowing the Oscillations Potential Energy We can write :

$$H = \frac{1}{2}(\dot{x}_1^2 + \dot{x}_2^2) + \frac{1}{2}\omega_1 x_1^2 + \frac{1}{2}\omega_2 x_2^2 \tag{2}$$

We can consider the relative angular velocity so we rewrite the H as :

$$H = \frac{1}{2}(\dot{x}_1^2 + \dot{x}_2^2) + \frac{1}{2}(x_1^2 + \omega x_2^2) \tag{3}$$

Because the constraint and Lagrangian are time-independent and generalized potential is absent we can say :

$$E = \frac{1}{2}(\dot{x}_1^2 + \dot{x}_2^2 + x_1^2 + \omega x_2^2) \tag{4}$$

where ;

$$\ddot{x}_1 = -x_1 \tag{5}$$

$$\ddot{x}_2 = -\omega^2 x_2 \tag{6}$$

From the law of conservation of energy all motion will take place inside the ellipse $U(x_1, x_2) \leq E$. The general solution for oscillators for our equations might be,

$$x_1 = A_1 Sin(t + \phi_1) \tag{7}$$

$$x_2 = A_2 Sin(\omega t + \phi_2) \tag{8}$$

The orthogonal projection of the sinusoid wound around the cylinder onto the x_1, x_2 plane gives the desired orbit called a *Lissajous figure*.
So we are going to plot some of these orbits.

1.1 Behaviour of Lissajous by changing ω for $\Delta\phi = 0$

As you see below with $\Delta\phi = 0$ by changing variable ω we should have different Lissajous as:

If $\omega = 2, 4, 6, ...$ the final curve is closed and independent to time means by changing time the shape may not change.

If $\omega = 1, 3, 5, ...$ the final curve isn't closed but it's independent to time like previous one.

If $\omega = irrational$ the final curve depends to time and over time the curve shape changes and final shape over enough time might be dense

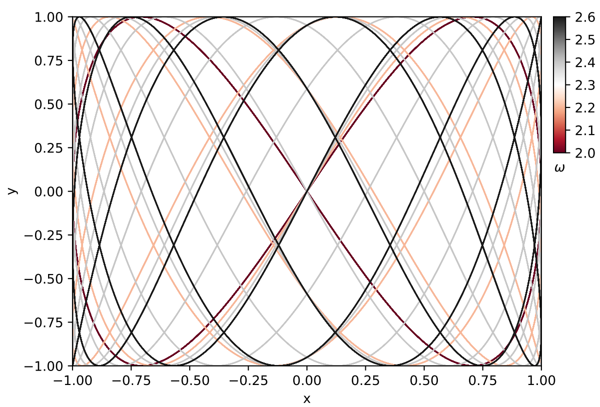

Figure 1: Lissajous By changing ω from t = 0 to t = 100

1.2 Behaviour of Lissajous by changing $\Delta\phi$ for $\omega = cons.$

By changing $\Delta\phi$ the center of curve may change it's position but other things like closed(open) or time-dependency of a curve determined by ω ,For instance if $\omega = 2$ then the curve is closed now by changing ϕ_2 the center of curve relocate.

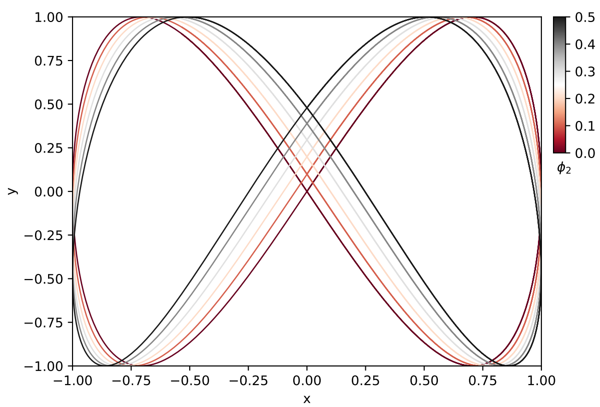

Figure 2: Lissajous By changing $\Delta\phi$ with $\omega = 2$

1.3 Behaviour of Lissajous by changing ω and $\Delta\phi$

It's a combination of section 1.2 and 1.3, by changing ω the we change the substance of curve and by changing $\Delta\phi$ we change the position of center and whole curve.