COMP417 Lecture 5

Learning the Network: SLP with Multiple Outputs

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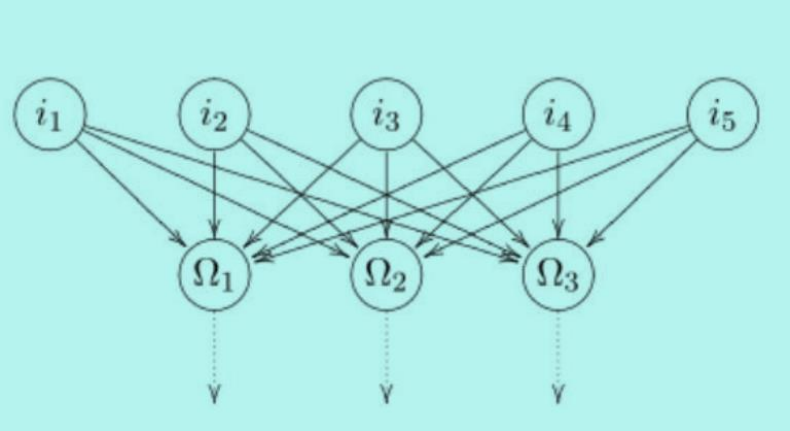
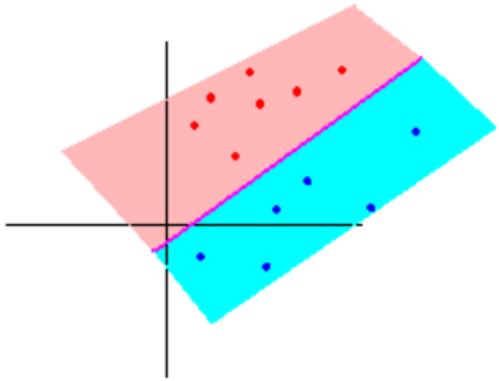
Recap: Learning the perceptron

- Two methods:
  - The perceptron learning algorithm
  - The delta rule





- Before using SLP, make sure the data is linearly separable
  - Visualize the data (not possible for more than 2 features)

The perceptron learning algorithm

```
1: while ∃p ∈ P and error too large do
2:   Input p into the network, calculate output y {P set of training patterns}
3:   for all output neurons Ω do
4:     if y_Ω = t_Ω then
5:       Output is okay, no correction of weights
6:     else
7:       if y_Ω = 0 then
8:         for all input neurons i do
9:           w_{i,Ω} := w_{i,Ω} + o_i {...increase weight towards Ω by o_i}
10:        end for
11:      end if
12:      if y_Ω = 1 then
13:        for all input neurons i do
14:          w_{i,Ω} := w_{i,Ω} − o_i {...decrease weight towards Ω by o_i}
15:        end for
16:      end if
17:    end if
18:  end for
19: end while
```

The delta rule

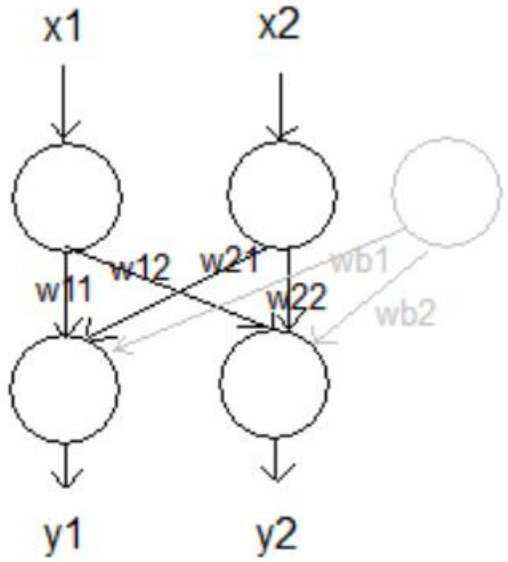
- Same as the previous example. Just updating weights is different

$$w_{i,\Omega} := w_{i,\Omega} + \eta\, o_i\, (t_\Omega - y_\Omega)$$

- For previous example:
  - w1 := w1 + η * x1 * (t – y)
  - w2 := w2 + η * x2 * (t – y)
  - w_bias := w_bias + η * bias * (t – y)

Example: SLP with multiple outputs

x1	x2	t1	t2
0	0	0	0
0	1	1	0
1	0	1	0
1	1	1	1



The two output neurons can have different threshold values

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0									0	0
0	1									1	0
1	0									1	0
1	1									1	1

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1			0	0
0	1									1	0
1	0									1	0
1	1									1	1

Initial weights

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0		0	0
0	1									1	0
1	0									1	0
1	1									1	1

net1 = w11 * x1 + w21 * x2 + b1
net1 = 0.1 * 0 + 0.2 * 0 - 0.2 = -0.2

Assume threshold1 = 0.1,
threshold2 = 1

net1 >= 0.1 ? → y = 1
else? → y = 0

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1									1	0
1	0									1	0
1	1									1	1

net2 = w12 * x1 + w22 * x2 + b2
net2 = 0.2 * 0 + 0.3 * 0 + 1 = 1

net2 >= 1 ? → y = 1
else? → y = 0

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2						1	0
1	0									1	0
1	1									1	1

y1 is OK

don't change its weights

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2						1	0
1	0									1	0
1	1									1	1

y2 is wrong

update its weights:
(We can either use perceptron learning algorithm or delta rule)

Assume we are using delta rule, $\eta = 0.1$

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9			1	0
1	0									1	0
1	1									1	1

w12 := w12 + 0.1 * x1 * (t2 – y2) → 0.2 + 0.1 * 0 * -1 → 0.2
w22 := w22 + 0.1 * x2 * (t2 – y2) → 0.3 + 0.1 * 0 * -1 → 0.3
wb2 := wb2 + 0.1 * (t2 – y2) → 1 + 0.1 * -1 → 0.9

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9	0		1	0
1	0									1	0
1	1									1	1

calculate net1, y1

net1 = 0.1 * 0 + 0.2 * 1 – 0.2 = 0

net1 < 0.1 → y1 = 0

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9	0	1	1	0
1	0									1	0
1	1									1	1

calculate net2, y2

net2 = 0.2 * 0 + 0.3 * 1 + 0.9 = 1.2 net2 >= 1 → y2 = 1

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9	0	1	1	0
1	0	0.1	0.3	-0.1						1	0
1	1									1	1

Update weights of y1

w11 = 0.1 + 0.1 * 0 * (1 - 0) = 0.1
w21 = 0.2 + 0.1 * 1 * (1 – 0) = 0.3
wb1 = -0.2 + 0.1 * (1 – 0) = -0.1

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9	0	1	1	0
1	0	0.1	0.3	-0.1	0.2	0.2	0.8			1	0
1	1									1	1

Update weights of y2

w12 = 0.2 + 0.1 * 0 * (0 - 1) = 0.2
w22 = 0.3 + 0.1 * 1 * (0 – 1) = 0.2
wb2 = 0.9 + 0.1 * (0 – 1) = 0.8

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9	0	1	1	0
1	0	0.1	0.3	-0.1	0.2	0.2	0.8	0		1	0
1	1									1	1

Calculate net1 and y1

net1 = 0.1 * 1 + 0.3 * 0 – 0.1 = 0 → y1 = 0

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9	0	1	1	0
1	0	0.1	0.3	-0.1	0.2	0.2	0.8	0	1	1	0
1	1									1	1

Calculate net2 and y2

net2 = 0.2 * 1 + 0.2 * 0 + 0.8 = 1 → y2 = 1

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9	0	1	1	0
1	0	0.1	0.3	-0.1	0.2	0.2	0.8	0	1	1	0
1	1	0.2	0.3	0						1	1

Update weights of y1

w11 = 0.1 + 0.1 * 1 * (1 - 0) = 0.2
w21 = 0.3 + 0.1 * 0 * (1 – 0) = 0.3
wb1 = -0.1 + 0.1 * (1 – 0) = 0

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9	0	1	1	0
1	0	0.1	0.3	-0.1	0.2	0.2	0.8	0	1	1	0
1	1	0.2	0.3	0	0.1	0.2	0.7			1	1

Update weights of y2

w12 = 0.2 + 0.1 * 1 * (0 - 1) = 0.1
w22 = 0.2 + 0.1 * 0 * (0 - 1) = 0.2
wb2 = 0.8 + 0.1 * (0 - 1) = 0.7

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9	0	1	1	0
1	0	0.1	0.3	-0.1	0.2	0.2	0.8	0	1	1	0
1	1	0.2	0.3	0	0.1	0.2	0.7	1		1	1

Calculate net1 and y1

net1 = 0.2 * 1 + 0.3 * 1 + 0 = 0.5 → y1 = 1

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9	0	1	1	0
1	0	0.1	0.3	-0.1	0.2	0.2	0.8	0	1	1	0
1	1	0.2	0.3	0	0.1	0.2	0.7	1	1	1	1

Calculate net2 and y2

net2 = 0.1 * 1 + 0.2 * 0.7 + 0 = 1 → y2 = 1

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9	0	1	1	0
1	0	0.1	0.3	-0.1	0.2	0.2	0.8	0	1	1	0
1	1	0.2	0.3	0	0.1	0.2	0.7	1	1	1	1
Next weights		0.2	0.3	0	0.1	0.2	0.7				

both y1 and y2 are OK

Don't update weights

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.1	0.2	-0.2	0.2	0.3	1	0	1	0	0
0	1	0.1	0.2	-0.2	0.2	0.3	0.9	0	1	1	0
1	0	0.1	0.3	-0.1	0.2	0.2	0.8	0	1	1	0
1	1	0.2	0.3	0	0.1	0.2	0.7	1	1	1	1
Next weights		0.2	0.3	0	0.1	0.2	0.7				

We need to run another epoch

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.2	0.3	0	0.1	0.2	0.7			0	0
0	1									1	0
1	0									1	0
1	1									1	1

Second epoch

Example: SLP with multiple outputs

x1	x2	w11	w21	wb1	w12	w22	wb2	y1	y2	t1	t2
0	0	0.2	0.3	0	0.1	0.2	0.7	0	0	0	0
0	1	0.2	0.3	0	0.1	0.2	0.7	1	0	1	0
1	0	0.2	0.3	0	0.1	0.2	0.7	1	0	1	0
1	1	0.2	0.3	0	0.1	0.2	0.7	1	1	1	1
Next weights		0.2	0.3	0	0.1	0.2	0.7				

Second epoch