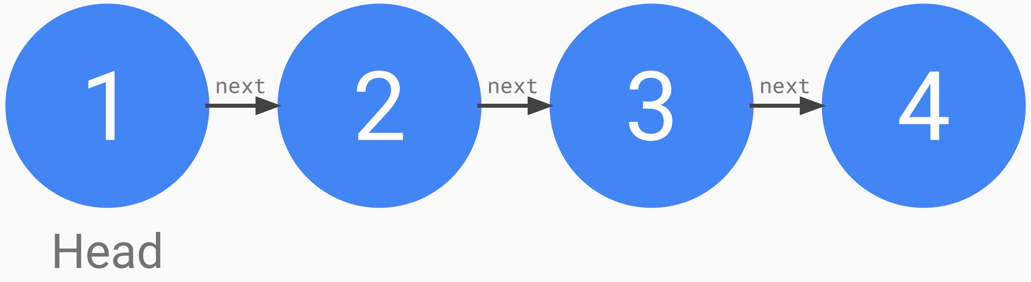
Linked lists and hashing

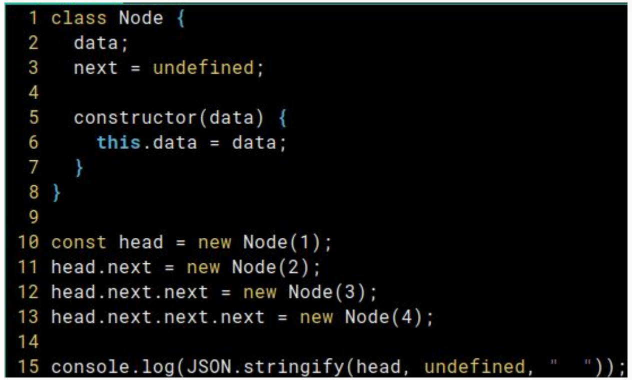
Code: https://github.com/mesmere/sasha-tutorial/tree/main/2024-02-25

Linked lists

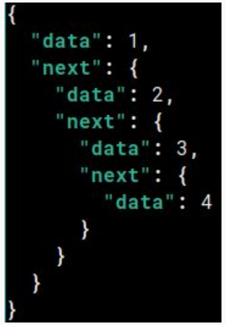
Linked lists - Introduction

- Efficient at (head) insertions and at deletions.
- Conventionally *never* used in JavaScript — but we'll do it anyway.





```
class Node {
  data;
  next = undefined;

  constructor(data) {
    this.data = data;
  }
}

const head = new Node(1);
head.next = new Node(2);
head.next.next = new Node(3);
head.next.next.next = new Node(4);

console.log(JSON.stringify(head, undefined, "  "));
```



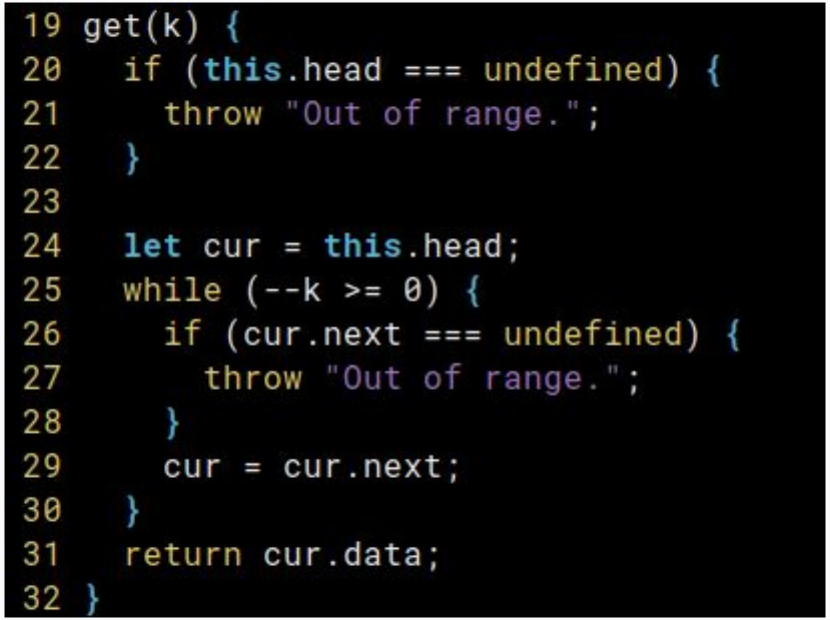
```
{
  "data": 1,
  "next": {
    "data": 2,
    "next": {
      "data": 3,
      "next": {
        "data": 4
      }
    }
  }
}
```

Linked lists - Traversing

Get the *k*th element from a linked list:

1. Start at the head.
2. Follow the `next` reference *k* times.

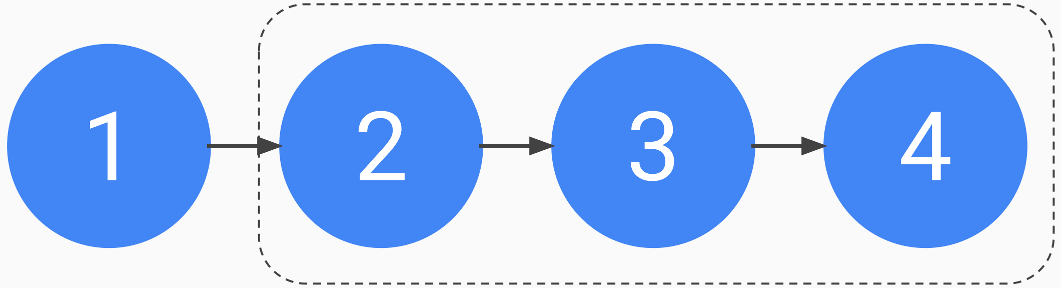
NB: `cur` is a node, not a numerical index.



```
get(k) {
  if (this.head === undefined) {
    throw "Out of range.";
  }

  let cur = this.head;
  while (--k >= 0) {
    if (cur.next === undefined) {
      throw "Out of range.";
    }
    cur = cur.next;
  }
  return cur.data;
}
```

Linked lists - Traversing (recursive)

Wait a second, lists have a recursive structure...

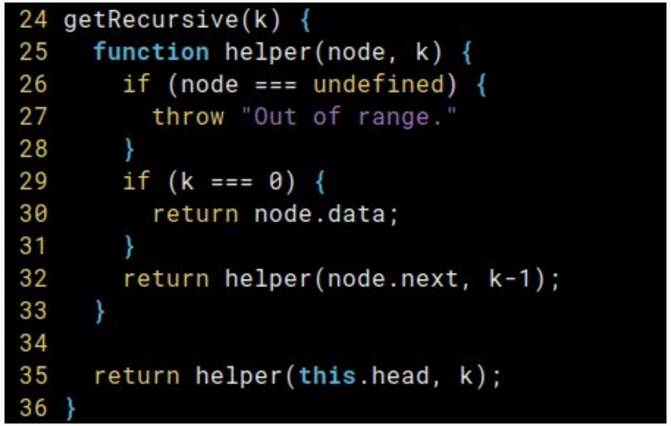


This part is itself a linked list with “2” at its head!

Linked lists - Traversing (recursive)

Get the *k*th element from a linked list recursively:

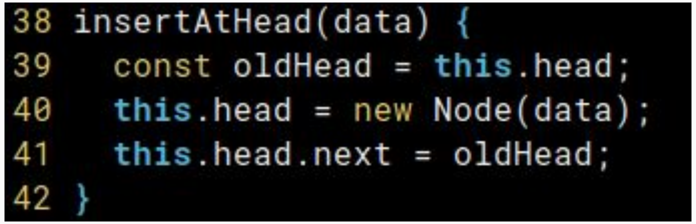
1. Start at the head.
2. Get the (*k*-1)th element from the “list” which is located at `head.next`.



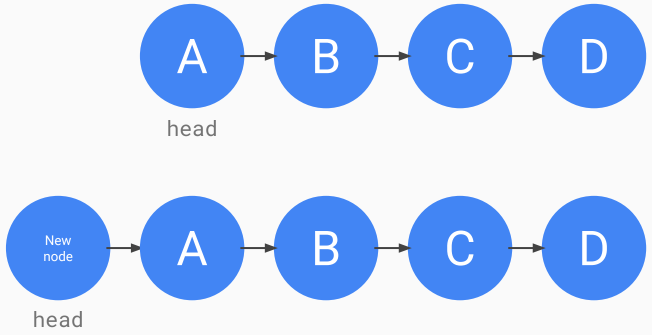
```
getRecursive(k) {
  function helper(node, k) {
    if (node === undefined) {
      throw "Out of range."
    }
    if (k === 0) {
      return node.data;
    }
    return helper(node.next, k-1);
  }

  return helper(this.head, k);
}
```

Linked lists (inserting)

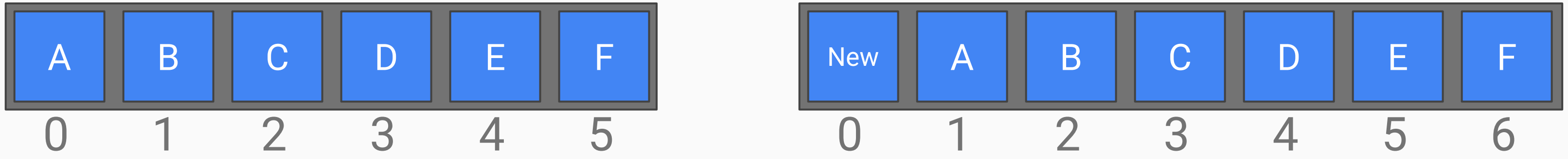
Operations at the head of the list are O(1):



```
insertAtHead(data) {
  const oldHead = this.head;
  this.head = new Node(data);
  this.head.next = oldHead;
}
```

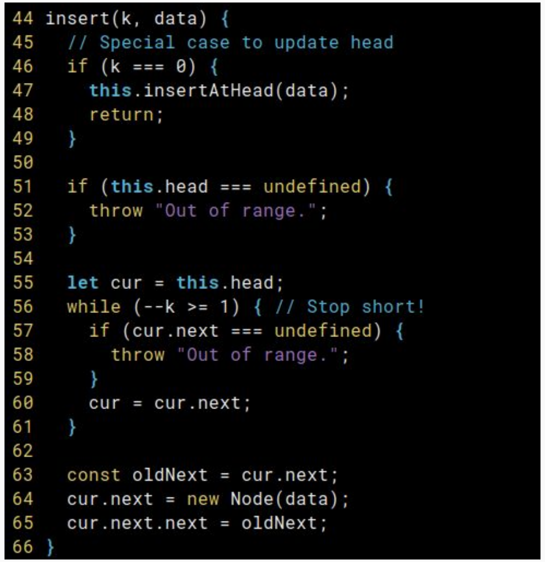


Compare this “O(n)” insertion at the head of an array (shift *n* elements):

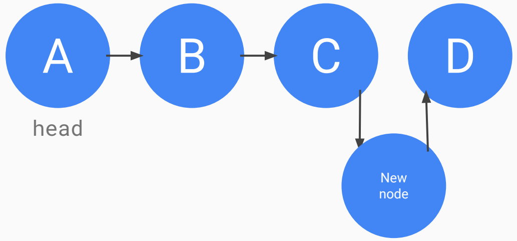


Linked lists (inserting)

To insert at the *k*th position in the list, we'll have to traverse to get there:



```
insert(k, data) {
  // Special case to update head
  if (k === 0) {
    this.insertAtHead(data);
    return;
  }

  if (this.head === undefined) {
    throw "Out of range.";
  }

  let cur = this.head;
  while (--k >= 1) { // Stop short!
    if (cur.next === undefined) {
      throw "Out of range.";
    }
    cur = cur.next;
  }

  const oldNext = cur.next;
  cur.next = new Node(data);
  cur.next.next = oldNext;
}
```



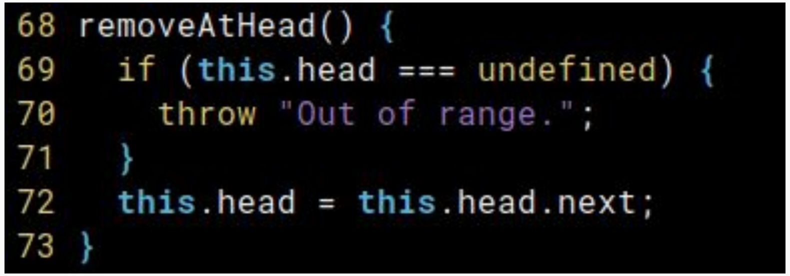
We stop with `cur` at the (*k*-1)th node…

…and set `cur.next` to our new node.

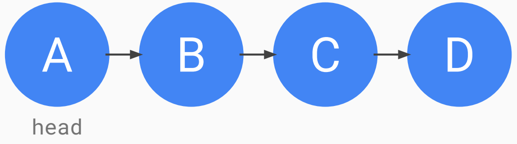
Linked lists (removing)

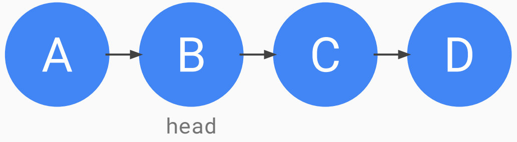
What about removing elements?

Again, operations at the head are easy:



```
removeAtHead() {
  if (this.head === undefined) {
    throw "Out of range.";
  }
  this.head = this.head.next;
}
```

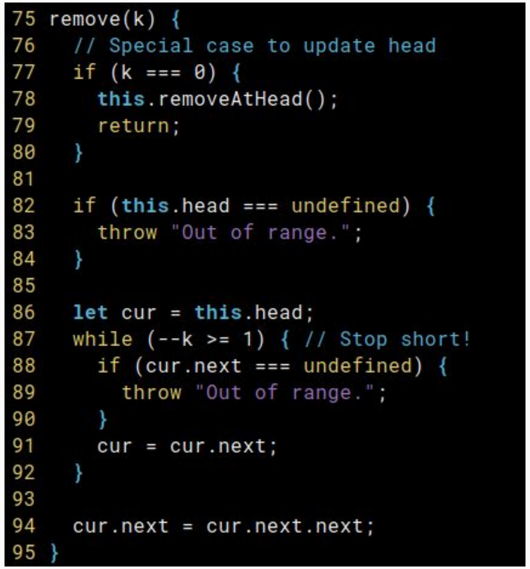




“A” is still pointing to “B”, but there’s no way to reach “A” now. The garbage collector will free up unreachable parts of the object graph eventually…

Linked lists (removing)

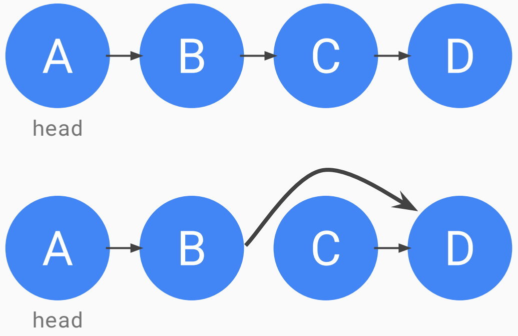
To remove the *k*th item in the list, we'll have to traverse to get there:



```javascript
remove(k) {
  // Special case to update head
  if (k === 0) {
    this.removeAtHead();
    return;
  }

  if (this.head === undefined) {
    throw "Out of range.";
  }

  let cur = this.head;
  while (--k >= 1) { // Stop short!
    if (cur.next === undefined) {
      throw "Out of range.";
    }
    cur = cur.next;
  }

  cur.next = cur.next.next;
}
```

Stop with `cur` at the (*k*-1)th node...

...and set `cur.next` to **its** `next`.



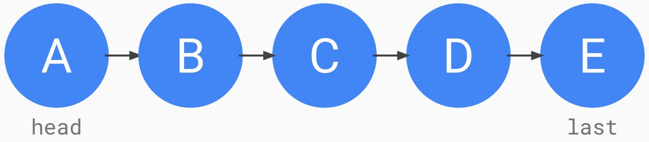
"C" is still pointing to "D", but, again, who cares?

Doubly-linked lists

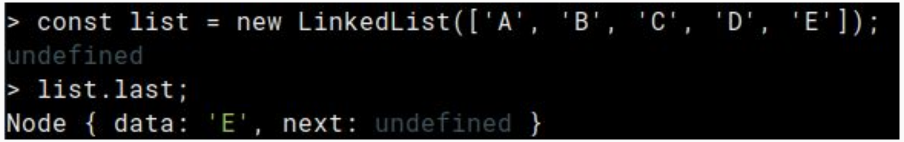
Problem: What if we're frequently doing operations at the end of the list? Ideally we wouldn't need to traverse all *n* elements every time.

Solution: In addition to head, keep another reference to the `last` element.

But how would we delete the last element?



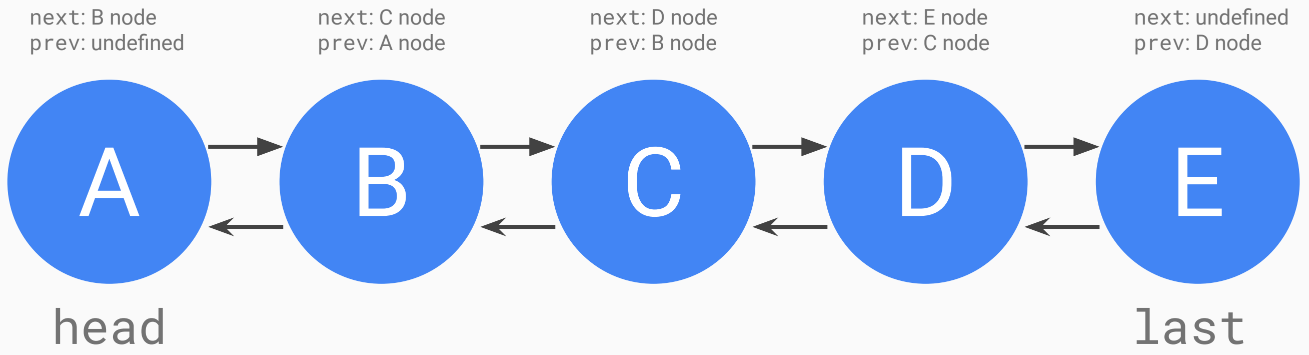
To delete "E" we need to set `D.next = undefined`
…but there's no way to get to "D" without traversing the list.



```
> const list = new LinkedList(['A', 'B', 'C', 'D', 'E']);
undefined
> list.last;
Node { data: 'E', next: undefined }
```

Doubly-linked lists

Simultaneously keep track of `next` and `prev` references on each node:



Now to delete node, we just set `node.prev.next = node.next`.

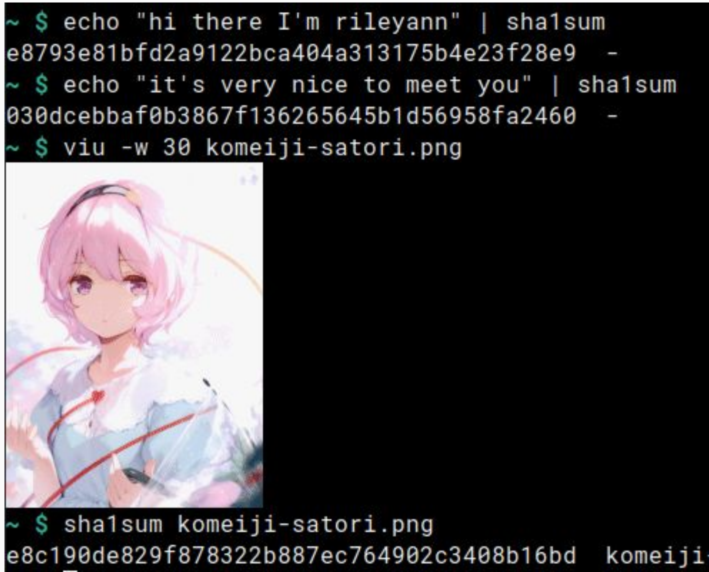
Hashing

Hashing

A **hash function** is a mathematical function which maps arbitrary input onto small numbers (“hashes”).

Example: sha1 is a hash function which takes any data as input and outputs a 20-byte number (conventionally rendered in hexadecimal).

We can feed it text or even a 5MB png file and it will *always* output 20 bytes.



```
~ $ echo "hi there I'm rileyann" | sha1sum
e8793e81bfd2a9122bca404a313175b4e23f28e9  -
~ $ echo "it's very nice to meet you" | sha1sum
030dcebbaf0b3867f136265645b1d56958fa2460  -
~ $ viu -w 30 komeiji-satori.png
~ $ sha1sum komeiji-satori.png
e8c190de829f878322b887ec764902c3408b16bd  komeiji
```

Hashing

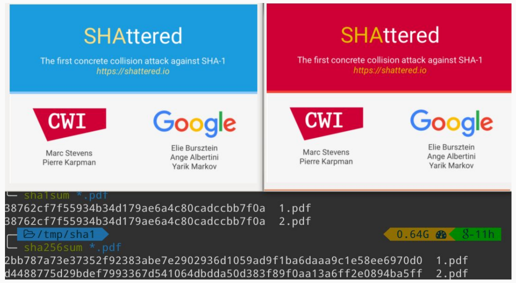
Good hash functions will be:

- **Deterministic.** They always output the same hash for the same input.
- **Sensitive to changes in the input.** A single bit changed in the input will usually change the hash, even if the input is large.
- **Good at distributing inputs *uniformly* across the possible outputs.** It's easy to write hash functions that produce distributions like this 🤢.
- **Fast.** People put a *lot* of effort into engineering the tradeoff between computational efficiency and the other properties.

Hashing

Sometimes hash functions need to stand up against a malicious attacker. In this kind of scenario these properties are also important:

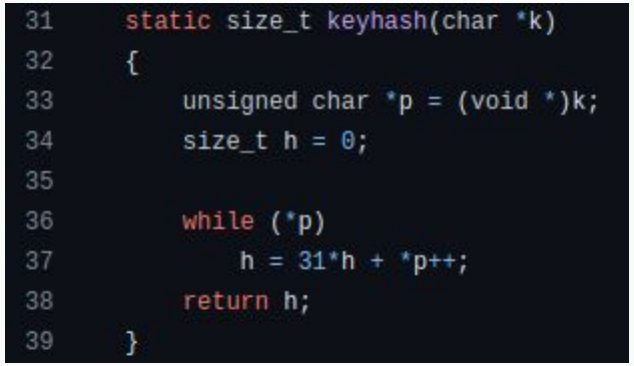
- **Hard to reverse.** We shouldn't be able to recover the input from the hash.
- **Resistant to collision.** We shouldn't be able to find two inputs that hash to the same value. E.g. a team at Google famously managed to craft two different PDFs with the *same* sha1 hash:



```
└ sha1sum *.pdf
38762cf7f55934b34d179ae6a4c80cadccbb7f0a  1.pdf
38762cf7f55934b34d179ae6a4c80cadccbb7f0a  2.pdf
┌ /tmp/sha1                                          0.64G   -11h
└ sha256sum *.pdf
2bb787a73e37352f92383abe7e2902936d1059ad9f1ba6daaa9c1e58ee6970d0  1.pdf
d4488775d29bdef7993367d541064dbdda50d383f89f0aa13a6ff2e0894ba5ff  2.pdf
```

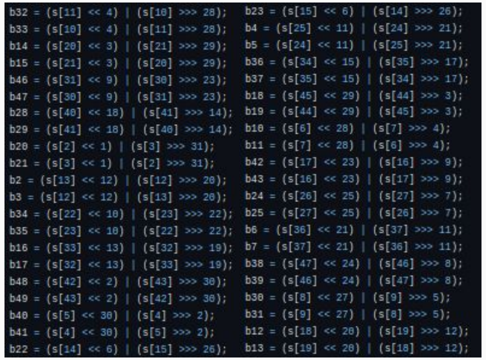
Hashing

In practice, *very poor* hash functions are often used in cases where speed is more important than the other properties.



```c
static size_t keyhash(char *k)
{
    unsigned char *p = (void *)k;
    size_t h = 0;

    while (*p)
        h = 31*h + *p++;
    return h;
}
```

musl libc (*fast*)



```
b32 = (s[11] << 4) | (s[10] >>> 28);   b23 = (s[15] << 6) | (s[14] >>> 26);
b33 = (s[10] << 4) | (s[11] >>> 28);   b4 = (s[25] << 11) | (s[24] >>> 21);
b14 = (s[20] << 3) | (s[21] >>> 29);   b5 = (s[24] << 11) | (s[25] >>> 21);
b15 = (s[21] << 3) | (s[20] >>> 29);   b36 = (s[34] << 15) | (s[35] >>> 17);
b46 = (s[31] << 9) | (s[30] >>> 23);   b37 = (s[35] << 15) | (s[34] >>> 17);
b47 = (s[30] << 9) | (s[31] >>> 23);   b18 = (s[45] << 29) | (s[44] >>> 3);
b28 = (s[40] << 18) | (s[41] >>> 14);  b19 = (s[44] << 29) | (s[45] >>> 3);
b29 = (s[41] << 18) | (s[40] >>> 14);  b10 = (s[6] << 28) | (s[7] >>> 4);
b20 = (s[2] << 1) | (s[3] >>> 31);     b11 = (s[7] << 28) | (s[6] >>> 4);
b21 = (s[3] << 1) | (s[2] >>> 31);     b42 = (s[17] << 23) | (s[16] >>> 9);
b2 = (s[13] << 12) | (s[12] >>> 20);   b43 = (s[16] << 23) | (s[17] >>> 9);
b3 = (s[12] << 12) | (s[13] >>> 20);   b24 = (s[26] << 25) | (s[27] >>> 7);
b34 = (s[22] << 10) | (s[23] >>> 22);  b25 = (s[27] << 25) | (s[26] >>> 7);
b35 = (s[23] << 10) | (s[22] >>> 22);  b6 = (s[36] << 21) | (s[37] >>> 11);
b16 = (s[33] << 13) | (s[32] >>> 19);  b7 = (s[37] << 21) | (s[36] >>> 11);
b17 = (s[32] << 13) | (s[33] >>> 19);  b38 = (s[47] << 24) | (s[46] >>> 8);
b48 = (s[42] << 2) | (s[43] >>> 30);   b39 = (s[46] << 24) | (s[47] >>> 8);
b49 = (s[43] << 2) | (s[42] >>> 30);   b30 = (s[8] << 27) | (s[9] >>> 5);
b40 = (s[5] << 30) | (s[4] >>> 2);     b31 = (s[9] << 27) | (s[8] >>> 5);
b41 = (s[4] << 30) | (s[5] >>> 2);     b12 = (s[18] << 20) | (s[19] >>> 12);
b22 = (s[14] << 6) | (s[15] >>> 26);   b13 = (s[19] << 20) | (s[18] >>> 12);
```

One small piece of the hash function used by Ethereum (*secure*)

Application - HMACs

The last time that Alice saw Bob in person she whispered a secret key in his ear: “`IMURS4EVER`”

One day, Alice wants to DM a message to Bob but it would be nice if:

1. Bob could be sure that Alice is sending a message, i.e. that she’s not asleep and it’s just some bored fed fucking with him. (*Authenticity*)
2. Bob could be sure that Alice’s original message hasn’t been maliciously altered in transit. (*Integrity*)

Solution: Along with the message send `sha3('IMURS4EVER'+message)`.

Application - HMACs

Bob receives a message and then some hash-looking string in a DM.

Since he already has the shared key `IMURS4EVER` he can validate the message by *himself* evaluating `sha3('IMURS4EVER'+message)`. If the result matches the hash from the DM, then the message is genuinely from Alice!

Why does this work?

- Any attacker intercepting their DMs **cannot reverse a hash function** to recover the input value `('IMURS4EVER'+message)` from the hash to reveal the key.
- Without the key, if an attacker alters the message they **can't provide the corresponding hash value** `sha3('IMURS4EVER'+evilMessage)`.

Application - Hash tables

Problem: Store a set of `keys` and `values` such that all of the `keys` are unique — and provide **efficient** lookup of the `value` that's associated with any given `key`.

(This is `Map` from JavaScript, or just regular JS objects.)

Naive solution: Store all of the `<key, value>` pairs in an array sorted by `key` and do binary search every time to look up the corresponding `value`.

Better solution: Allocate a big array and **hash the `keys`** to determine *exactly* where in the array their corresponding `values` should be located, with no* searching necessary. 😎

*ominous asterisk

Application - Hash tables (Learn by example!)

Let `hash(str)` = sum of the ASCII codes of the characters of `str` (so A=65, B=66, ...).

Our table will be an array of size n=10, so we'll be able to hold up to 10 key/value pairs.

Let's try inserting Key: 'MOON', Value: 'cheese'.

First calculate the hash of the key: `hash('MOON')` = 77+79+79+78 = 313.

We need to map this into our space of n=10 slots so we use the **modulus operator**:

`table[313%10] = 'cheese';` ✅ (a%b is the *remainder* of `a/b`)

Now `table` looks like
```
[undefined, undefined, undefined, 'cheese',
 undefined, undefined, undefined, undefined,
 undefined, undefined].
```

...In general, we set `table[hash(key)%n] = value`.

Application - Hash tables (Learn by example!)

Reminder: `table =`

```
[undefined, undefined, undefined, 'cheese',
 undefined, undefined, undefined, undefined,
 undefined, undefined].
```

To look up the value for the key 'MOON':

First calculate the hash of the key: `hash('MOON')` = 77+79+79+78 = 313.

Now `return table[313%10]`. This gets 'cheese' out of `table[3]` where we left it.

...In general, we `return table[hash(key)%n]`.

Application - Hash tables (Learn by example!)

Reminder: `table` = `[undefined, undefined, undefined, 'cheese', undefined, undefined, undefined, undefined, undefined, undefined].`

Now let's insert:

Key: 'NEPTUNE', Value: 'seawater'.

First calculate the hash of the key:

`hash('NEPTUNE')` = 78+69+80+84+85+78+69 = 543.

We evaluate `543%10` to figure out which index we'll store 'seawater' in.

Hold up! `543%10` is 3, and we **already have something in `table[3]`**. This is a **hash collision**. Maybe things aren't quite as simple as we thought…

Application - Hash tables redux

Our approach up to this point only works if every key we insert hashes to a different location in the table. This is up to pure chance and will *almost certainly* not remain true for long as the table starts to fill up.

Here are a few common approaches for handling hash collisions:

- **Linear probing**
- **Double hashing**
- **Chaining**

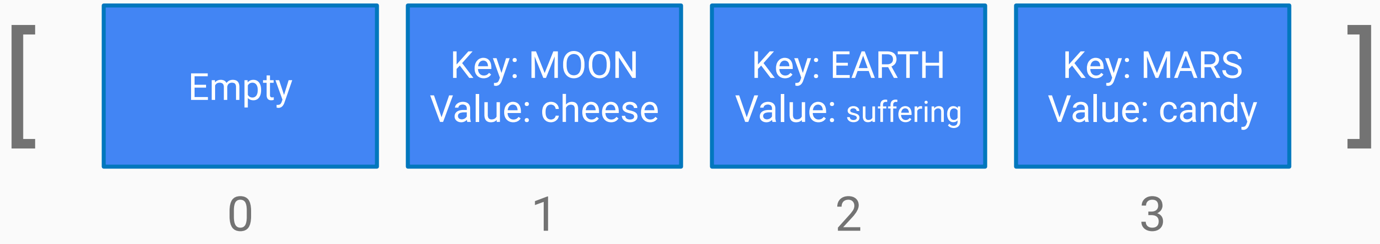
Application - Hash tables (linear probing)

In **linear probing**, when we insert into the table at a spot that’s already full, we just try the next spot… and the next… and the next… until we find a free space.

Similarly we when we look up a key, we have to check the spot determined by the hash function… and the next… and the next… until we find the key we’re looking for (or an empty space).

NB: Now we have to store the values *and* the keys in the table so that we can identify the correct value when we want to look them up later!)

Application - Hash tables (linear probing)

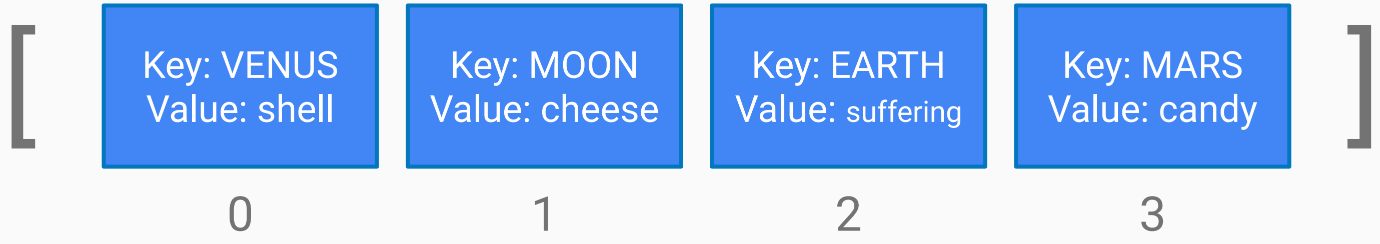


Say we're inserting **`<VENUS,shell>`** and **`hash('VENUS')%4 = 1`**.

We would like to insert at `table[1]` but it's full…
…so we try `table[2]` but it's full…
…so we try `table[3]` but it's full…
…so we wrap around and try `table[0]` and we find a space!

In the end, we put `<VENUS,shell>` into `table[0]`.

Application - Hash tables (linear probing)



We successfully inserted VENUS but we had to probe through the *entire array* to find an open space. 😬

Let's try retrieving the key MERCURY. Say that **`hash('MERCURY')%4 = 0`**.

We check `table[0]` and it's not MERCURY. But maybe that space was just full when we inserted MERCURY earlier? So we have to check `table[1]` too. Nope. Next we check `table[2]`. And `table[3]`. Finally we've checked them all and can say for sure that MERCURY is not in the table.

In fact, as the "load factor" of the table approaches 1 (all spaces full), *worst-case* insert and retrieve cost approaches O(n). Don't let your table get too full...

Application - Hash tables

When the table fills up (or the load factor crosses some predetermined threshold) you'll need to **resize the table**.

Basically, allocate a new array and reinsert everything again. **Every key will need to be re-hashed**, to account for the new array size.

Application - Hash tables (double hashing)

Double hashing makes use of two *different* hash functions, hash1 and hash2.

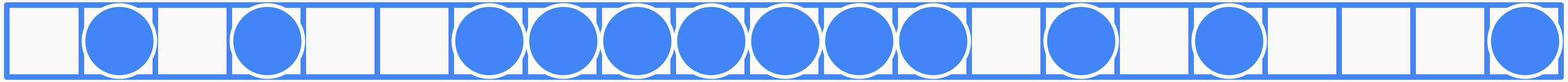
First we try to insert at `hash1(key)`%n. If that's full, we jump forward by `hash2(key)` spaces and try again. We keep jumping forward by increments of `hash2(key)` with each attempt.

This is the same as linear probing but with fixed increments of 1 replaced with increments of `hash2(key)`.

Application - Hash tables (double hashing)

Con:	Double hashing breaks **cache locality** by jumping all over the place.
Pro:	Double hashing mitigates the **clustering** phenomenon.

With linear probing, it's easy to accumulate "clusters" of filled spots:



The longer a "cluster" gets, the likelier it is that a newly-inserted key will hash somewhere inside it. Then the new key gets shunted to the end and the cluster grows even bigger, absorbing other clusters in the way and growing huge! 😳

Pretty soon our beautiful O(1) performance is wrecked from all the probing.

With double hashing we have *somewhere else to go* rather than growing the cluster. Note that since the "somewhere else" depends on hash2, we'll go off to *somewhere different* for each key that had collided under hash1.

Application - Hash tables

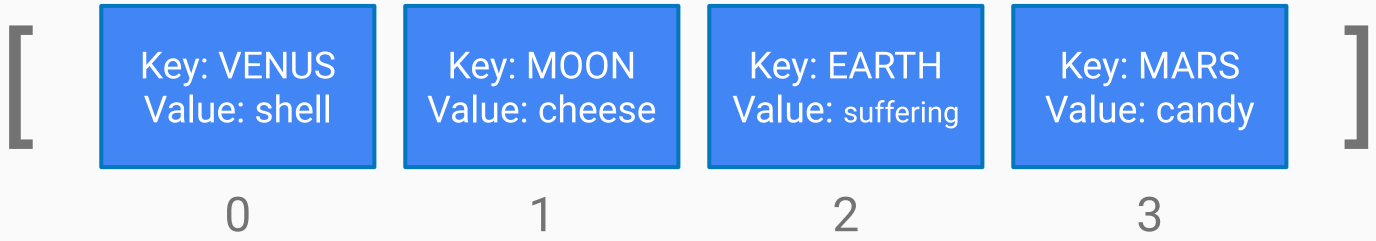
Linear probing and double hashing are called **open-addressing methods**. They’re very efficient, particularly linear probing (provided the load factor is low).

But there’s one critical operation we’ve overlooked so far: *deleting* items from the hash table. This is kind of a headache for open-addressing methods:

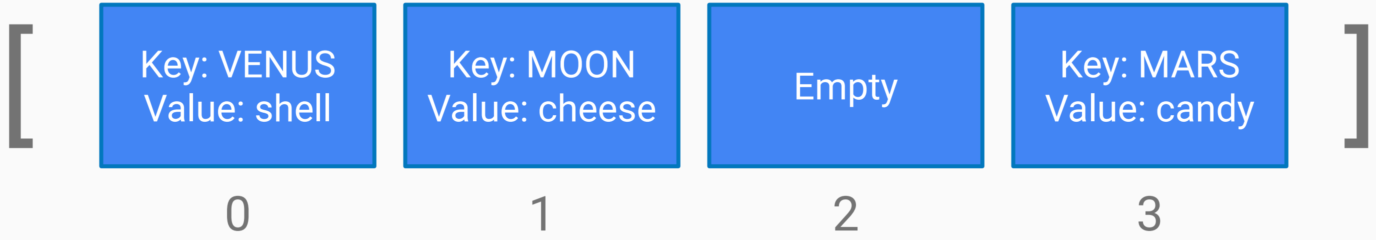
What happens if we delete an element from inside a “cluster” of collisions?

Oh. 😬

Application - Hash tables

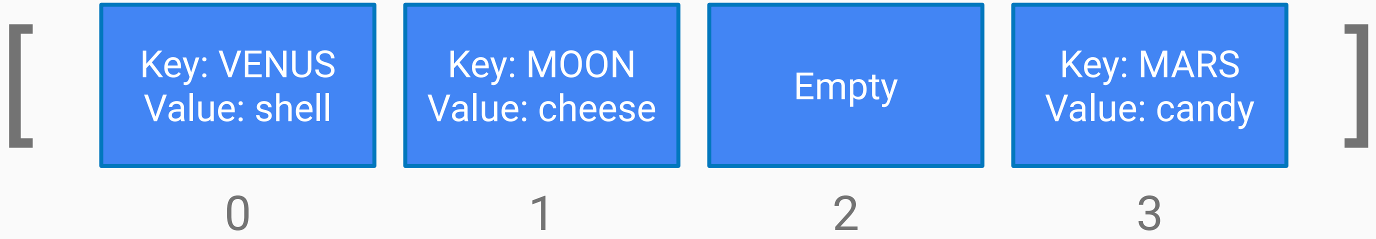


Remember that in this example `hash('VENUS')`%4 was 1 but we linearly probed to 2, then 3, then 0 to find a space for it. Say that now we delete EARTH.

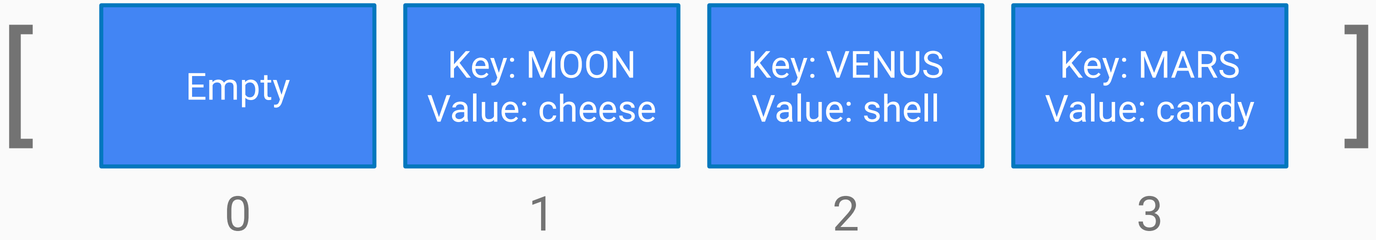


When we retrieve VENUS now, we'll look at `table[1]`, then `table[2]` and say we're done because it's empty. What do we do? We can't keep going; if our strategy were to probe through the entire table every time then we would get worst-case O(n) lookups even with a totally empty hash table! 👎

Application - Hash tables



The trick is that after deleting EARTH we probe ahead for a key that hashes to something that could *fill in that empty space*. For example, say that `hash('MARS')%4 = 3` so it's in the correct spot. We have to keep probing. Next up is VENUS, and we know that `hash('VENUS')%4 = 2` so let's move it:



Now we have *a new* empty space. We keep repeating the process until we reach an empty space or come back around to spot 2 again.

Application - Hash tables (chaining)

With **chaining**, each “spot” simply holds the head of a *linked list* of all of the key,value pairs that hashed to that spot.

Pros:	Simple to understand and implement! 🙂
Cons:	Linked lists are mad slow compared to one big `table` array. 🙁

To insert `<key,value>`, check the linked list at `table[hash(key)%n]` to see if key is already there- if so update its value, if not add `<key,value>` to the list.

To retrieve the value for `key`, look for `key` in the list at `table[hash(key)%n]`.