```python
import numpy as np

print('##############################################################')
print('                      Question 1a                              ')
print('##############################################################')
print(' ')

a = [[1, 0, 0, 0],
     [0, 2, 3, 0],
     [1, 0, 0, 5]]
J = np.array(a)
print(np.linalg.inv(np.dot(J,J.T)))


print('##############################################################')
print('                      Question 1c                              ')
print('##############################################################')
print(' ')

T1 = np.array([2, 4, 6, 5])
T2 = np.array([2, 5, 6, 5])
pseudoInv = np.dot(np.linalg.inv(np.dot(J,J.T)),J)
print(np.dot(pseudoInv,T1))
print(np.dot(pseudoInv,T2))
```

Output

```
ak@ubuntu16:~/Dynamics$ python3 Constraints.py
##############################################################
                      Question 1a
##############################################################

[[ 1.04        0.         -0.04      ]
 [ 0.          0.07692308  0.        ]
 [-0.04        0.          0.04      ]]
##############################################################
                      Question 1c
##############################################################

[1. 2. 1.]
[1.         2.15384615 1.        ]
```

# Homework

Saturday, November 23, 2019 11:57 AM

1.) $\tau = J^T F$

Considering $J_\infty = \begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 0 & 5 \end{bmatrix}$ $\tau_1 = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 5 \end{bmatrix}$ $\tau_2 = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 5 \end{bmatrix}$

a) $\tau = J^T F$

$\therefore \quad J\tau = JJ^T F$

$\therefore \quad (JJ^T)^{-1} J\tau = (JJ^T)^{-1} JJ^T F$

$(JJ^T)^{-1} J \tau = F$

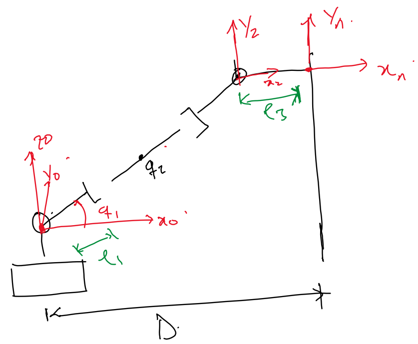
Consider $J_\infty = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 1 & 0 & 6 & 5 \end{bmatrix}$

Finding whether $(JJ^T)^{-1}$ exist $\rightsquigarrow$ Please refer to code

$(JJ^T)^{-1}$ exist, Therefore $(JJ^T)^{-1}$ is full rank

Please refer to code.

2.




$q_2$ has restriction $\therefore q_{2min} < q_2 < q_{2max}$

Distance Constraint:

$(l_1 + q_2)\cos(q_1) + l_3 = D$ $\rightsquigarrow$ holonomic Constraint. Restricts the points the end effector can go to.

This can be written as

$(l_1 + q_2)\cos(q_1) + l_3 - D = 0$

This eqn is of the form

$h(q) = 0 \quad \therefore$ Holonomic Constraint.

Min/Max Constraint.

$l_3 \leq D$

$q_1 = \cos^{-1}\left(\dfrac{D - l_3}{l_1 + q_2}\right)$

$\therefore \quad q_{1min} = \cos^{-1}\left(\frac{D - \ell_3}{\ell_1 + q_{2min}}\right).$

$q_{1max} = \cos^{-1}\left(\frac{D - \ell_3}{\ell_1 + q_{2max}}\right).$

$0 \leq q_{1min} \leq q_1 \leq q_{1max} \leq 90^\circ$

$\ell_3 > D.$

$q_1 = \cos^{-1}\left(\frac{D - \ell_3}{\ell_1 + q_2}\right)$

$\therefore \quad q_{1min} = \cos^{-1}\left(\frac{D - \ell_3}{\ell_1 + q_{2max}}\right)$

$\therefore \quad q_{1max} = \cos^{-1}\left(\frac{D - \ell_3}{\ell_1 + q_{2min}}\right)$

$90 < q_{1min} \leq q_1 \leq q_{1max} < 180$

$\rightsquigarrow$ This constraint can't be written in the form: $h(q) = 0$.
$\therefore$ Non holonomic.

Rotation Constraint.

We fix joint $q_3$ at some angle. Therefore, angular velocity $= 0 \quad \Longrightarrow \dot{q}_3 = 0$. $\rightsquigarrow$ Holonomic b/cos it is integrable.

$\Longrightarrow q_3 = c$

and this is of the form $h(q) = 0$ where $h(q) = q_3 - c$

NB, the value of $c$ depends on the frames used.
In ~~this~~ case, we choose body attached fixed frames (That is frames that do not rotate as joints are activated but may be translated) as shown in the diagram above.
$\therefore$ In that case, $c = 0 \Rightarrow q_3 = 0$

3.

$$\frac{\partial \gamma(q)\, a_k}{\partial q_j} = \frac{\partial \gamma(q)\, a_j}{\partial q_k} \quad \text{for } j \neq k$$

$$\therefore \to \frac{\partial \gamma(q)\, a_1}{\partial q_j} = \frac{\partial \gamma(q) \cdot 1}{d q_1} = \frac{\partial \gamma(q)\, \rho \sin(q_3)}{\partial q_1} =$$

$$= \frac{\partial \gamma(q)\, \rho \cos q_3}{d q_1} = \frac{\partial \gamma(q) \cos(q_3)}{\partial q_1} = 0.$$

3

$$\frac{\partial \gamma(q)}{\partial q_3} = \frac{\partial \gamma(q)\, p \sin q_5}{\partial q_2}$$

3

$$\frac{\partial \gamma(q)}{\partial q_4} = \frac{\partial \gamma(q)\, p \cos q_3}{\partial q_2}$$

$$\frac{\partial \gamma(q)}{\partial q_3} = p \sin q_5 \frac{\partial \gamma(q)}{\partial q_2} \quad (1)$$

$$\frac{\partial \gamma(q)}{\partial q_4} = p \cos q_3 \frac{\partial \gamma(q)}{\partial q_2} \quad (2)$$

$$\frac{\partial \gamma(q)}{\partial q_5} = \cos q_5 \frac{\partial \gamma(q)}{\partial q_2} \quad (3)$$

$$p \sin q_5 \frac{\partial \gamma(q)}{\partial q_4} = p \cos q_3 \frac{\partial \gamma(q)}{\partial q_3} - \gamma(q) p \sin(q_3) \quad (4)$$

$$\frac{\partial \gamma(q)\, p \sin q_5}{\partial q_5} = \frac{\partial \gamma(q)\, p \cos q}{\partial q_3}$$

$$p \sin q_5 \frac{\partial \gamma(q)}{\partial q_5} + \gamma(q) p \cos(q_5) = \cos q_5 \frac{\partial \gamma(q)}{\partial q_3} \quad (5)$$

$$\frac{\partial \gamma(q)\, p \sin q_5}{\partial q_4} = \frac{\partial \gamma(q) \cos q_5}{\partial q_3}$$

$$p \cos q_3 \frac{\partial \gamma(q)}{\partial q_5} = \cos q_5 \frac{\partial \gamma(q)}{\partial q_4} \quad (6)$$

$$\frac{\partial \gamma(q)\, p \cos q}{\partial q_5} = \frac{\partial \gamma(q) \cos q_5}{\partial q_4}$$

From eqn 4

$$\Rightarrow \quad p \sin q_5 \frac{\partial \gamma(q)}{\partial q_4} = p \cos(q_3) \frac{\partial \gamma(q)}{\partial q_3} - \gamma(q) p \sin(q_3)$$

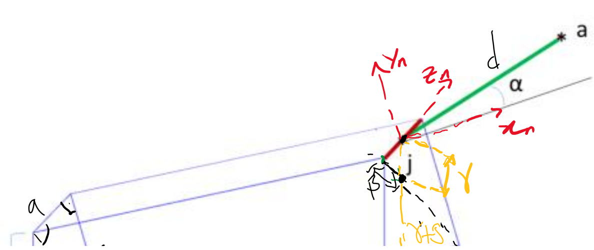
Substituting $\frac{\partial \gamma(q)}{\partial q_4}$ and $\frac{\partial \gamma(q)}{\partial q_3}$ from eqn 1 and 2, we get

$$\Rightarrow \quad \cancel{(p \sin q_5)(p \cos q_3) \frac{\partial \gamma(q)}{\partial q_2}} = \cancel{(p \cos(q_3))(p \sin(q_5)) \frac{\partial \gamma(q)}{\partial q_2}} - \gamma(q) p \sin(q_3)$$

$$\Rightarrow \quad -\gamma(q) p \sin(q_3) = 0$$

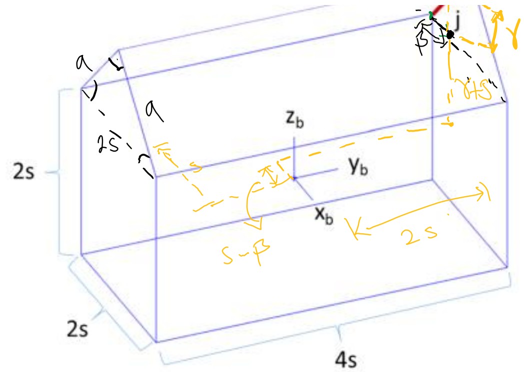
$$\Rightarrow \quad \gamma(q) = 0 \text{ since } -p\sin(q_3) \text{ is not always zero}$$

4).




point a relative to frame n.

$^n$ [ d cos α ]




frame n.

$$p^n = \begin{bmatrix} d\cos\alpha \\ d\sin\alpha \\ 0 \end{bmatrix}$$

From pythagoras

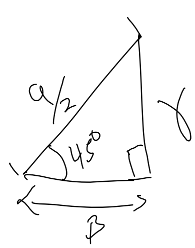
$$\therefore \quad a^2 + a^2 = (2s)^2$$

$$\cancel{2}a^2 = \cancel{4}^{2}s^2 \implies a = \sqrt{2}\,s$$

The angle between the second link (rod) and revolute (red) axis is assumed to be $90°$ as we are looking for maximum value for d so that the building doesn't tip over.

Position of frame n wrt to frame b.

From $\triangle$ as shown in diagram.

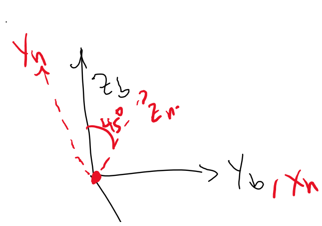


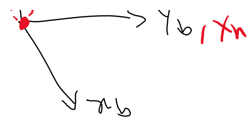

$$\therefore \quad \beta = \frac{a}{2}\cos(45) = \frac{\sqrt{2}}{2}s\,\frac{\sqrt{2}}{2} = \frac{1}{2}s$$

$$\& \quad \gamma = \frac{a}{2}\sin(45) = \frac{\sqrt{2}}{2}s\,\frac{\sqrt{2}}{2} = \frac{1}{2}s$$

$$\therefore \quad \text{position} \rightsquigarrow \begin{bmatrix} -0.5s \\ 2s \\ 1.5s \end{bmatrix}$$

Orientation of frame n wrt to frame b






$$Z_n^0 = \begin{bmatrix} \cos 45 \\ 0 \\ \sin 45 \end{bmatrix} \quad Y_n^0 = \begin{bmatrix} -\cos 45 \\ 0 \\ \sin 45 \end{bmatrix} \quad X_n^0 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

$$\therefore R_n^0 = \begin{bmatrix} 0 & -\cos 45 & \cos 45 \\ 1 & 0 & 0 \\ 0 & \sin 45 & \sin 45 \end{bmatrix}$$

$$\therefore H_n^0 = \begin{bmatrix} 0 & -\cos 45 & \cos 45 & -0.5 \\ 1 & 0 & 0 & 2 \\ 0 & \sin 45 & \sin 45 & 1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Getting pt a in base frame.

$$P^0 = H_n^0 P^n$$

$$= \begin{bmatrix} 0 & -\cos 45 & \cos 45 & -0.5 \\ 1 & 0 & 0 & 2 \\ 0 & \sin 45 & \sin 45 & 1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d\cos\alpha \\ d\sin\alpha \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\sqrt{2}/2 & \sqrt{2}/2 & -0.5 \\ 1 & 0 & 0 & 2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 & 1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d\cos\alpha \\ d\sin\alpha \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore P^0 = \begin{bmatrix} -\frac{\sqrt{2}}{2} d\sin\alpha - 0.5 \\ d\cos\alpha + 2 \\ \frac{\sqrt{2}}{2} d\sin\alpha + 1.5 \end{bmatrix}.$$

$$\therefore Com = \frac{m_1 P_1 + m_2 P_2}{m_1 + m_2}$$

where $m_1 \rightarrow$ mass of base link 1.
$m_2 \rightarrow$ mass of base link 2.
$P_1 \rightarrow$ Position of base link 1

$\overline{\phantom{xx}}$
$m_1 + m_2$

$m_2 \rightsquigarrow$ mass ...

$P_1 \rightsquigarrow$ Position of base link1

$P_2 \rightsquigarrow$ Positn of base link2.

Using base link as frame of measurement.

$P_1 = (0,0,0) \rightsquigarrow$ since position of the center of mass is at $(0,0,0)$

$$\therefore \; Com = \frac{m_2 P_2}{m_1 + m_2} = \frac{m_2}{m_1 + m_2} \left\langle -\tfrac{\sqrt{2}}{2} d\sin\alpha - 0.5,\; d\cos\alpha + 2,\; \tfrac{\sqrt{2}}{2} d\sin\alpha + 1.5 \right\rangle$$

For Stability,

$com_s = \langle -0.5, \; 2, \; 1.5 \rangle$. No torque about pt.

compare com and $com_s$ and solving for m

choosing 2nd coordinate we have,

$$\therefore \; \frac{m_2}{m_1 + m_2} d\cos\alpha + 2 = 2$$

$$m_2 \, d\cos\alpha + 2 = 2m_1 + 2m_2$$

$$\therefore \; m_2 \leq \frac{2m_1}{d\cos\alpha + 2 - 2}.$$