MBIC13 - Decoding

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Cognitive Neuroscience - Maastricht University

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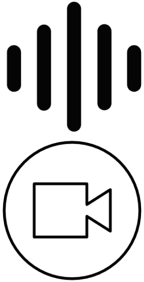
Decoding Information in Neuroscience



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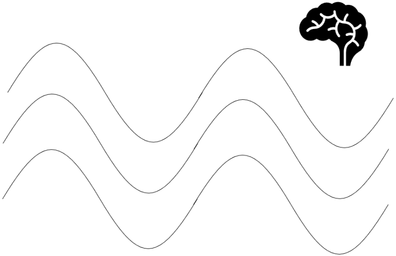

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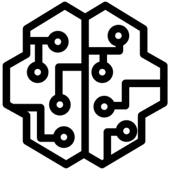

P(C | X) = ? [decoder]

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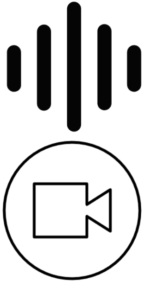

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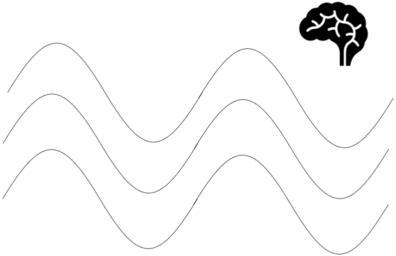

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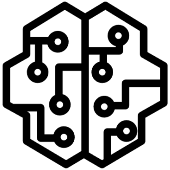

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Decoding ≡ Supervised Machine Learning

Typical ML Pipeline

1. Data handling of missing values and outliers
2. Data normalization (*Standard Scaling*, *Min-Max Scaling*)
3. Feature selection or extraction (*ANOVA*, *PCA*, *Lasso*, *Clustering*, ...)
4. Model selection (*Logistic Regression*, *Support Vector Machine*, *Decision Trees*, ...)
5. Model training (a.k.a. fitting)
6. Model evaluation using cross validation (*K-Fold*, *Leave-One-Out*) and statistical testing

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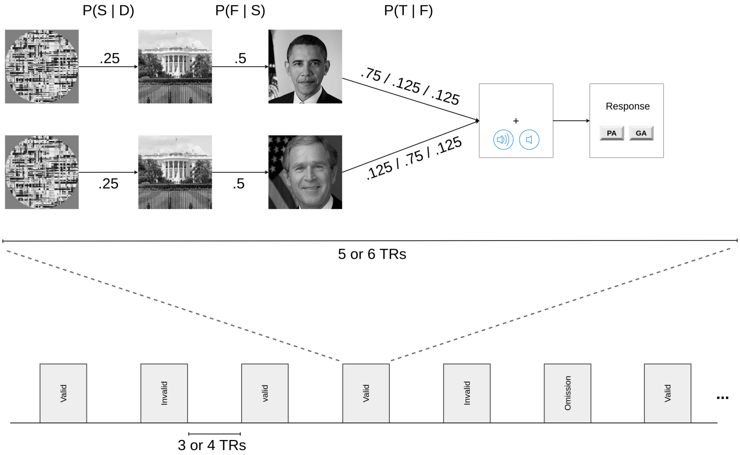
Case Study



P(S | D)
P(F | S)
P(T | F)
.25
.5
.75 / .125 / .125
.25
.5
.125 / .75 / .125
+
Response
PA
GA
5 or 6 TRs
Valid
Invalid
valid
Valid
Invalid
Omission
Valid
...
3 or 4 TRs

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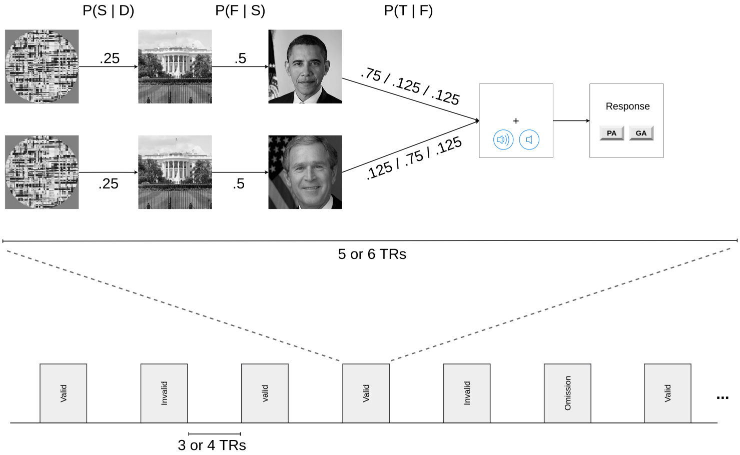

Our goal here is to decode / classify whether information in the brain code (as measured by BOLD changes) relates more to PA or GA, or whether the trial is congruent or incongruent. Thus we are interested in **binary** classification!

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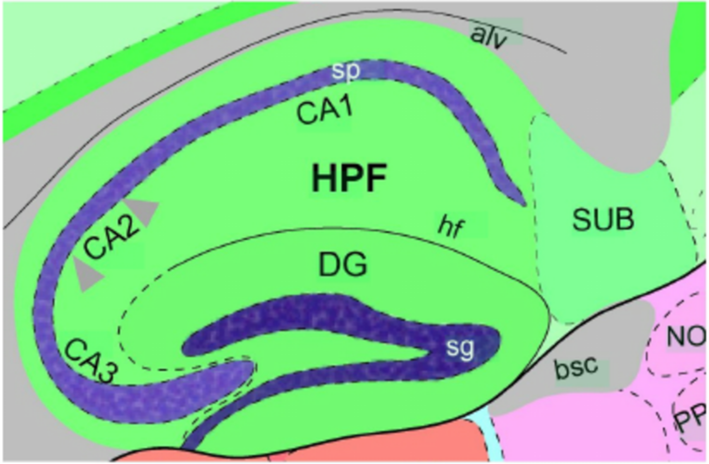

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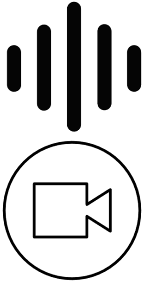

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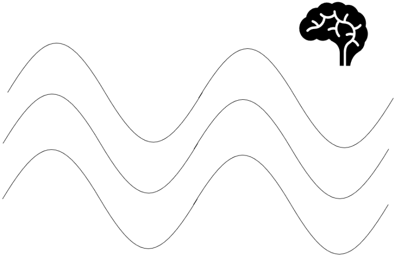

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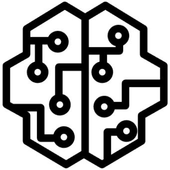

$P(C \mid X) = ?$ [decoder]

How can we estimate the posterior $P(C|X)$?

Bayesian Decision Theory

Bayes Theorem

$$P(C|X) = \frac{P(X|C)P(C)}{P(X)}$$

Assign the data features X to the class C_1 if

$$P(C_1|X) > P(C_2|X)$$

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This seems difficult, but we can approximate it with a generalized linear model!

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Logistic Regression

from a Bayesian perspective

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Goal: What is the most probable class given the features?

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assume $f(X) \approx f(X; W) = w_0 + w_1x_1 + \cdots + w_nx_n$. Thus

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Define loss as the binary cross entropy between prediction and true classes, and fit weights using gradient descent over this loss.
BCE is the NLL of the Bernoulli dist.:

$$\mathcal{L}_{BCE}(y, \hat{y}) = y \log \hat{y} + (1 - y) \log(1 - \hat{y})$$

Let's do some decoding...

But first, are there any questions?