Computational Physics Project 4

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1 Pendulum and Chaos

1.1 Introduction

In McLaughlin's work on the bifurcations and chaos of a driven pendulum, numerical computation methods are not discussed and the error of bifurcation points have an error of at least 10^{-5} seconds. In our work. we use the four-order Runge-Kutta method to calculate the motion of a driven oscillator. Errors will be given in the same notation as McLaughlin to be on the order of 10^{-6} seconds at maximum.

Our work starts with translating a program simulating the motion of a driven oscillator from Fortran 77 to Fortran 95. After translating the program, the large arrays or dimensions used were changed to arrays of size four. This was done to minimize memory used by the program, as only four values are needed per array in the fourth-order Runge-Kutta method. The next change that was made was to change when values of time, phi, and omega were written to the output file. Only the last point of the last 64 periods is written out. These points are taken only at the end of the computation so that the driving and dampening forces of the pendulum can balance out. With these changes, we can now investigate the transition values between bifurcation points.

1.2 Methods and Results

To begin testing for bifurcation points, the endpoints of McLaughlin's results were used. Calculations were run manually by oscillating back and forth between a driving value giving n points (denoted q_n) and a value giving $2n$ points (denoted q_{2n}), closing the distance between the two points until the desired error was reached. The dampening constant $k = 0.2$ and the force frequency $\Omega = 2$ for all calculations. Initial calculations were done with 100,000 periods and 200 points per period. After values were found, another round of computations with 200,000 periods and 400 points per period were done to confirm results. Figure 1 is a graph of the transition point between two and four points, which is included as an example of what the lower transition points look like. Figure 2 is the transition point between 16 and 32 points.

- Transition between q_1 and q_2: Reached as q approaches $k = 0.2$.
- Transition between q_2 and q_{1^*}: $0.674900825 \pm 5 \cdot 10^{-9}$.
- Transition between q_{1^*} and q_{2^*}: $0.790085 \pm 5 \cdot 10^{-6}$.

- Transition between q_{2^*} and q_4: $0.989250 \pm 5 \cdot 10^{-6}$.
- Transition between q_4 and q_8: $1.024185 \pm 5 \cdot 10^{-6}$.
- Transition between q_8 and q_{16}: $1.031223 \pm 5 \cdot 10^{-6}$.
- Transition between q_{16} and q_{32}: $1.032888099 \pm 5 \cdot 10^{-8}$

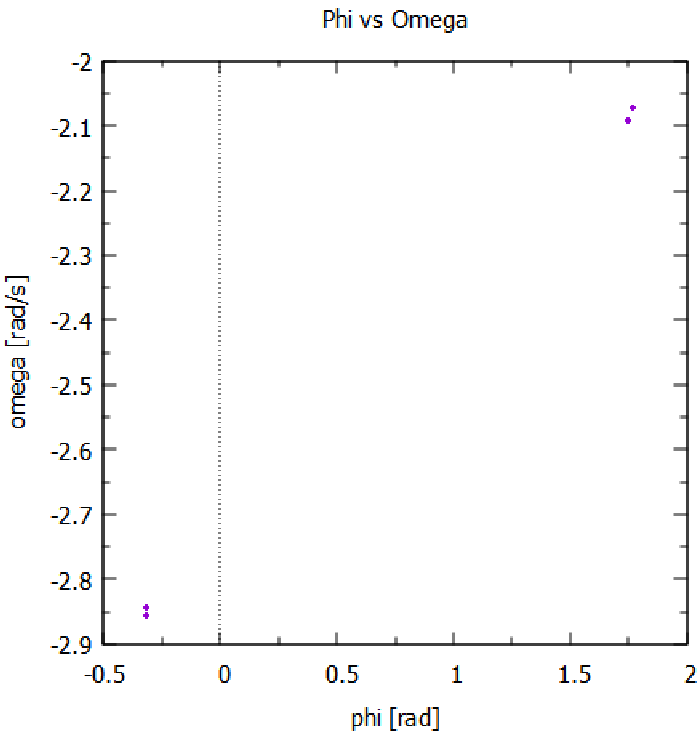

Figure 1: Transition between q_2 and q_4

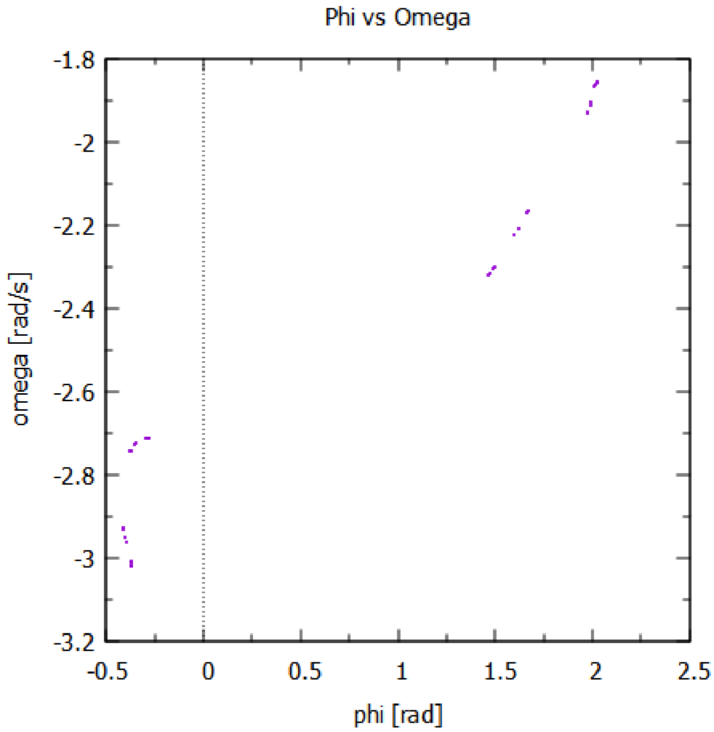

Figure 2: Transition between q_{16} and q_{32}

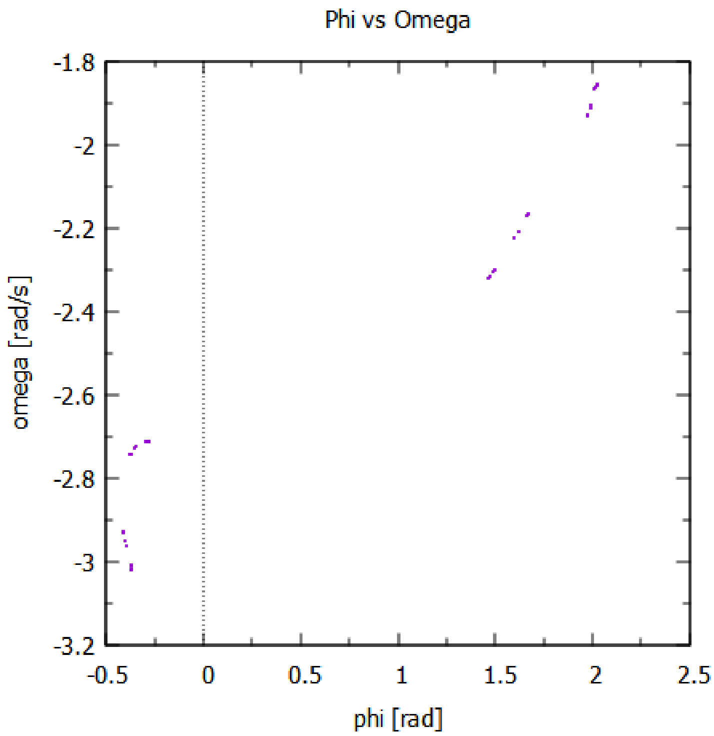

Figure 3: Replication of McLaughlin's Figure 3 with the same initial conditions

1.3 Conclusion

Compared to McLaughlin's results, our results were similar but deviated in certain ways. Our results include the unexpected transition values where there are two points, then one point, then two again. McLaughlin does not include these points and only starts counting two points at q values greater than this unexpected and unexplained transition point. All of our other values were within the same range as McLaughlin's error bars, except for the transition between 8 and 16 points. Our q_8 to q_{16} transition happened above McLaughlin's value. McLaughlin's Figure 3 was replicated with our computation method as well. Our error values were within the desired 10^{-6} units, as desired, meaning we achieved a greater accuracy than McLaughlin.

1.4 Sources

1 McLaughlin, John B. "Period-Doubling Bifurcations and Chaotic Motion for a Parametrically Forced Pendulum." Journal of Statistical Physics, vol. 24, no. 2, 1981, pp. 375–388., https://doi.org/10.1007/bf01013307.