Computational Physics Project 1

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1 Random Walk in 2D

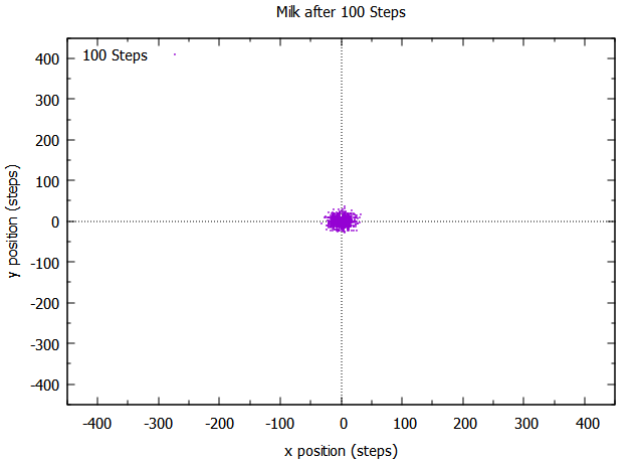
1.1 Introduction

To model the diffusion of milk in a cup of coffee as a simple set of particles, we assume that the motion of the milk particles is random and that the milk particles only spread on the surface of the milk. These assumptions reduce the model to a 2D random walk, where the 1D walk discussed in class can be extended.

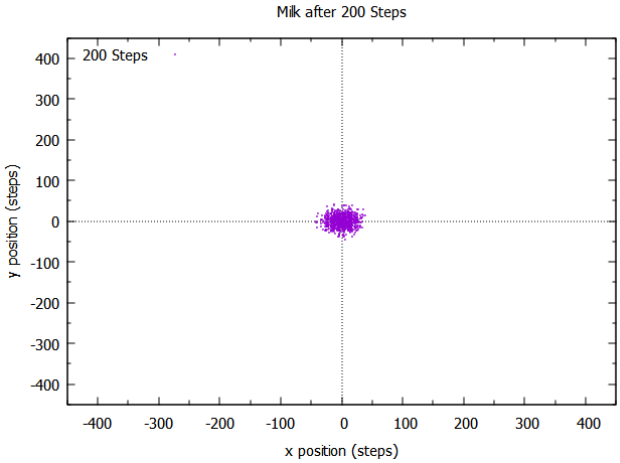
Note that for the 1D walk to be changed to a 2D walk, several changes must be made. First, we must define variables to include separate x and y positions. It is important to note that each component has its own random number call, such that the movement in one axis does not always match the other. Another change is to the distance calculation, where the Euclidean Distance ($r_i = \sqrt{x_i^2 + y_i^2}$) is used.

To create gnuplot graphs for each checkpoint, the position of each milk particle is written to a separate file during each checkpoint step. This means that, for example, at step 100, the x and y component of each point is written to a file called 'step.100'. This process is repeated for the remaining checkpoints, then the files are used in a gnuplot file titled 'plot-2D.pl', whose graphs are below.

1.2 Gnuplot Graphs of Checkpoints

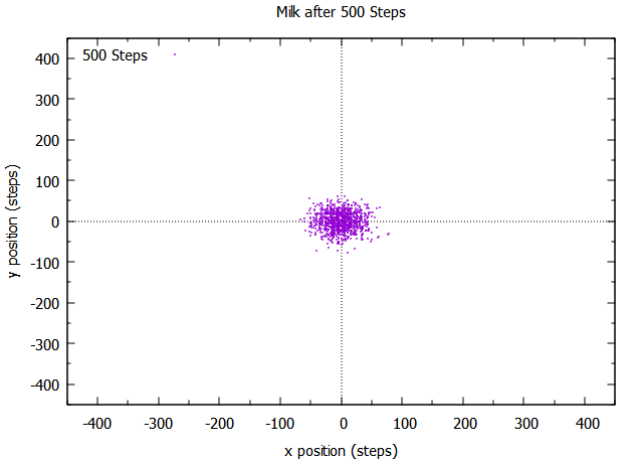


(a) Simulation after 100 steps

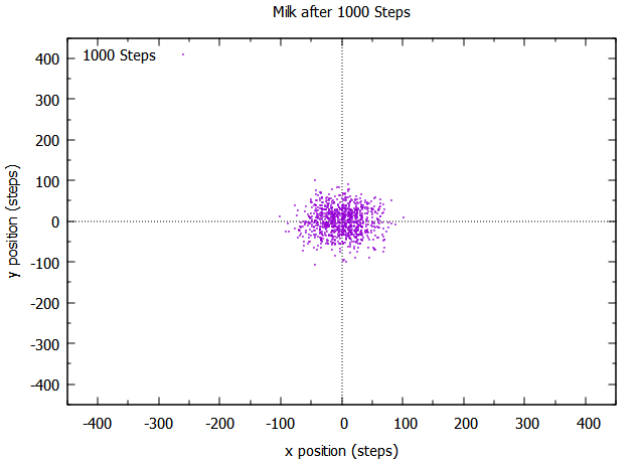


(b) Simulation after 200 steps

Figure 1: 100 and 200 step checkpoints

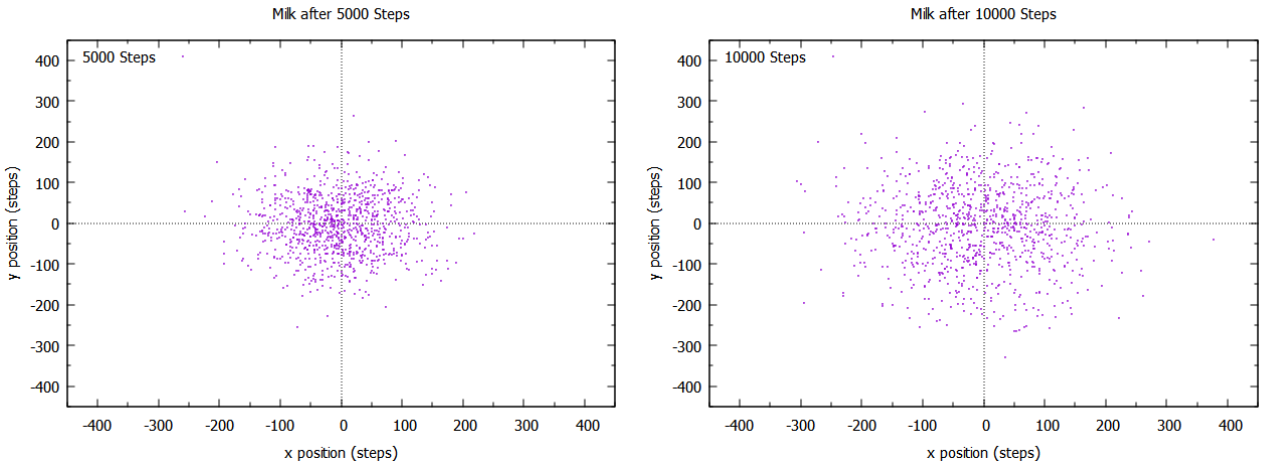


(a) Simulation after 500 steps



(b) Simulation after 1000 steps

Figure 2: 500 and 1000 step checkpoints



(a) Simulation after 5000 steps

(b) Simulation after 10000 steps

Figure 3: 5000 and 10000 step checkpoints

1.3 Conclusion

According to the simulation run that created the preceding images, the average distance that the particle moved away from the origin after 10,000 steps is 122.86 units. This result is logical as the expected distance of a randomized walk away from the origin is about $\sqrt{n}$, where n is the number of steps.