Computational Physics Project 5

Jack Messerli-Wallace

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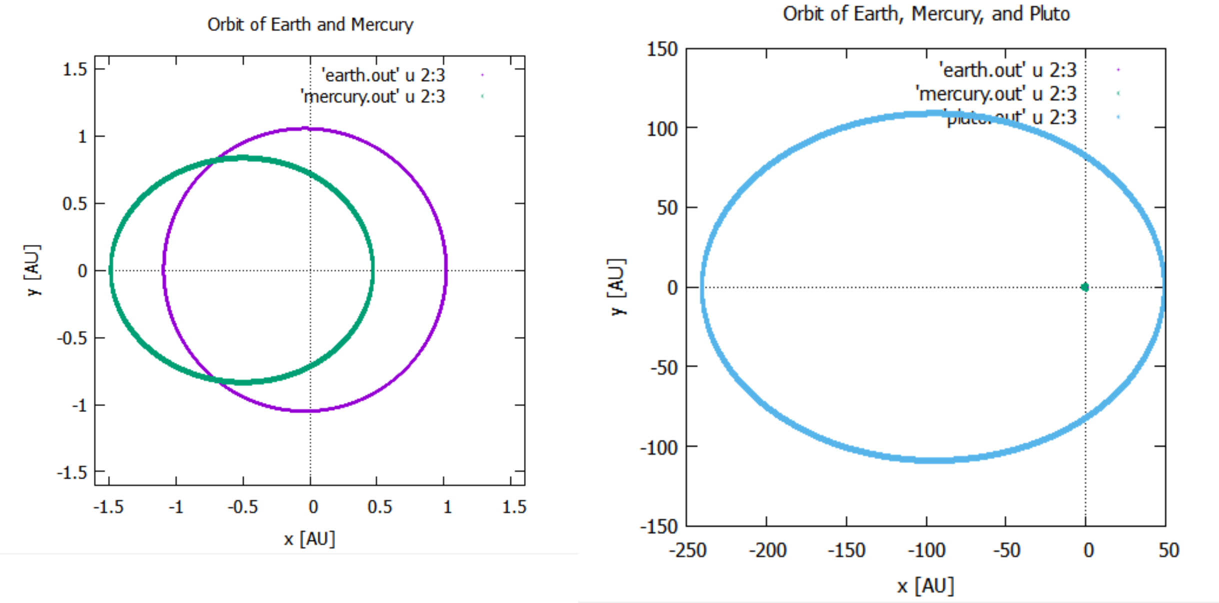
1 The Solar System

1.1 Introduction

In this work, we will investigate the Kepler two-body problem and the Kepler three-body problem. We will add features to existing programs to examine how changing nature impacts orbits and to more accurately model the planet's orbit around the sun.

1.2 Two-body Orbits

We were tasked with modifying the `kepler-demo.f90` program to read in the eccentricity of the orbit and initial velocity, to change β (the coefficient of the force law), and to add a switch to change the computation method. The existing program was modified to do so. The eccentricity was implemented in the initial position of the earth, where the perihelion is taken to be the initial x-position. The initial velocity is computed from the eccentricity as well. The input for initial velocity and eccentricity is only used when another input factor, `planet_switch`, does not match with the correlated value for Earth, Mercury, or Pluto. These three planets have set masses, eccentricities, and semi-major axis values. Figure 1 is a plot of the orbits of Earth and Mercury, while figure 2 is a plot of Earth, Mercury and Pluto's orbits. The discussed orbital constants are pulled from our textbook. All three planets are set to orbit ten times. Earth and Mercury use a time-step of 0.00001 seconds while Pluto uses a time-step of 0.0001 seconds. Note that we use two different graphs (in figure 1) as opposed to one because the size of Pluto's orbit is much larger than that of Earth and Mercury.



(a) Earth and Mercury's Orbit

(b) Pluto's Orbit

Figure 1: Orbit of Earth, Mercury, and Pluto

Note that the orbit lines of Mercury and Pluto are not of uniform thickness, as the orbit seems to spread as the absolute value of the y-position increases. This is less noticeable with Earth's orbit.

1.3 Changing β and Computation Method for the Two-Body Problem

As mentioned in the previous section, an input to change β (the coefficient of the gravitational force law) and an input to change the computation method were implemented. First, we note the differences between using the Euler computation method versus the Euler-Cromer method.

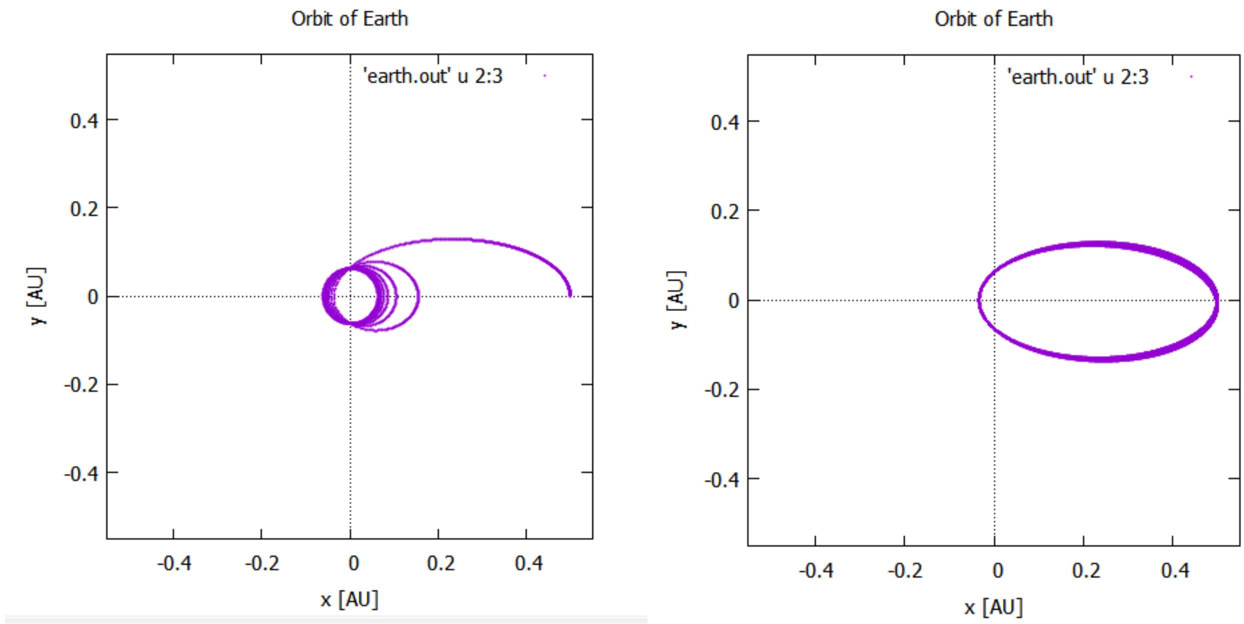


Figure 2: Earth's orbit calculated via Euler (left) and Euler-Cromer (right)

Note that the left graph of figure 3 (Euler method) gives an inconsistent orbit, while the right graph (Euler-Cromer) is ellipsoidal. The right graph still shows inconsistencies as the absolute value of the y-position increases, as seen in previous results.

The Euler-Cromer method is used in our exploration of changing β. Figure 4 contains two plots of the same data. In the left plot the entire orbit is seen, while in the right plot, only the area close to the starting position is shown. This is to highlight the contrast in orbit between different cases of β.

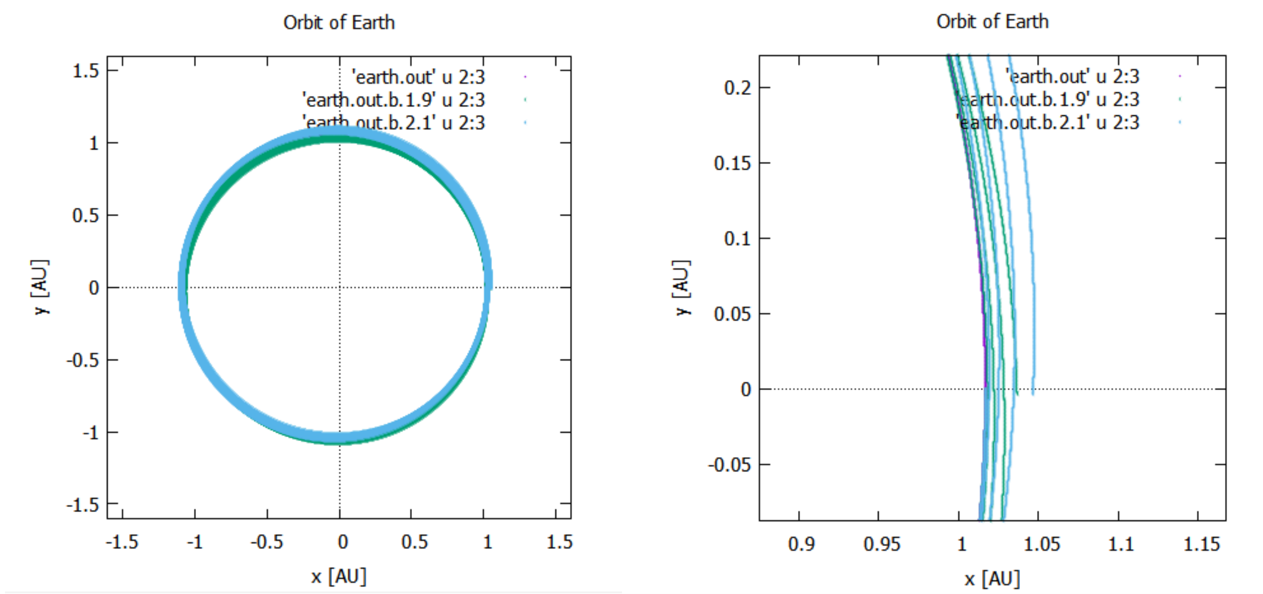


Figure 3: Earth's orbit calculated with varying β values

In figure 4, note that the orbits with $\beta = 1.9, 2.1$ are not consistent like the orbit with $\beta = 2.0$. Changing β changes what velocity is needed to maintain stable orbit.

1.4 Three-body

In the program `threebody-final.f90`, mass ratios are used to correctly determine initial velocities and scale forces. Figure 5 and 6 shows what happens to the orbit of Earth when the mass of Jupiter is increased.

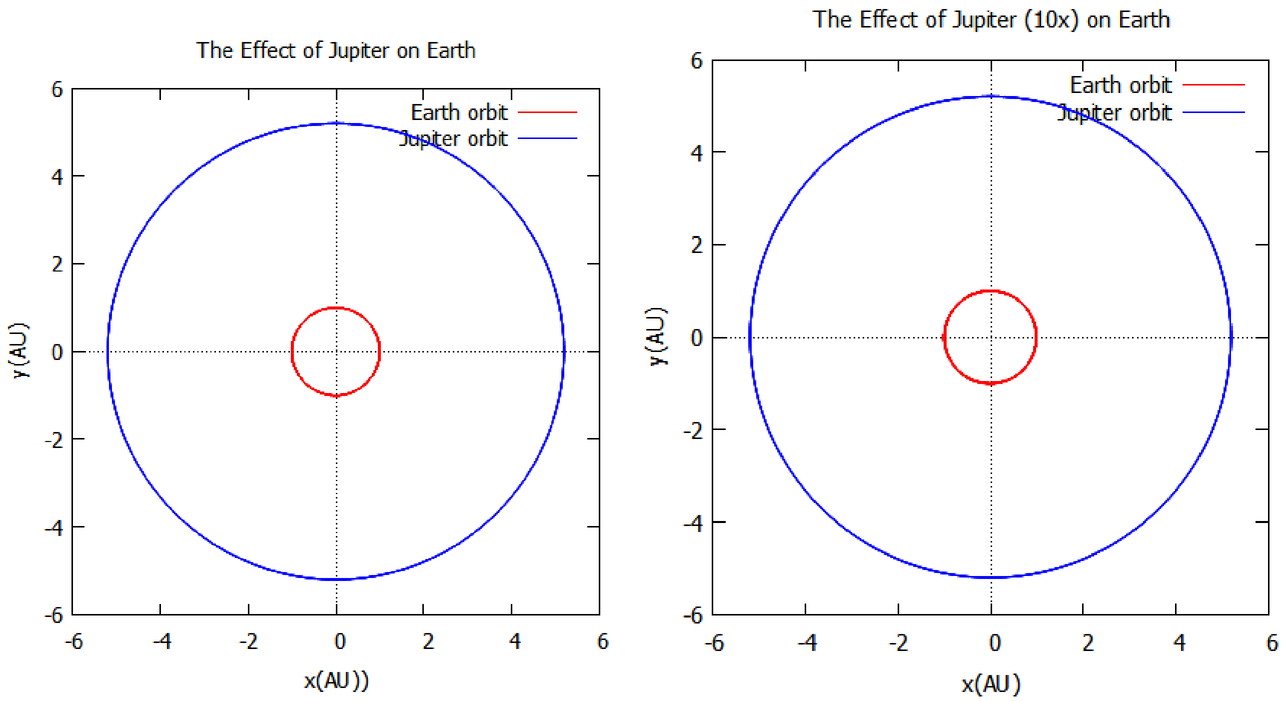


Figure 4: Jupiter's actual mass (left) and 10-times Jupiter's mass (right)

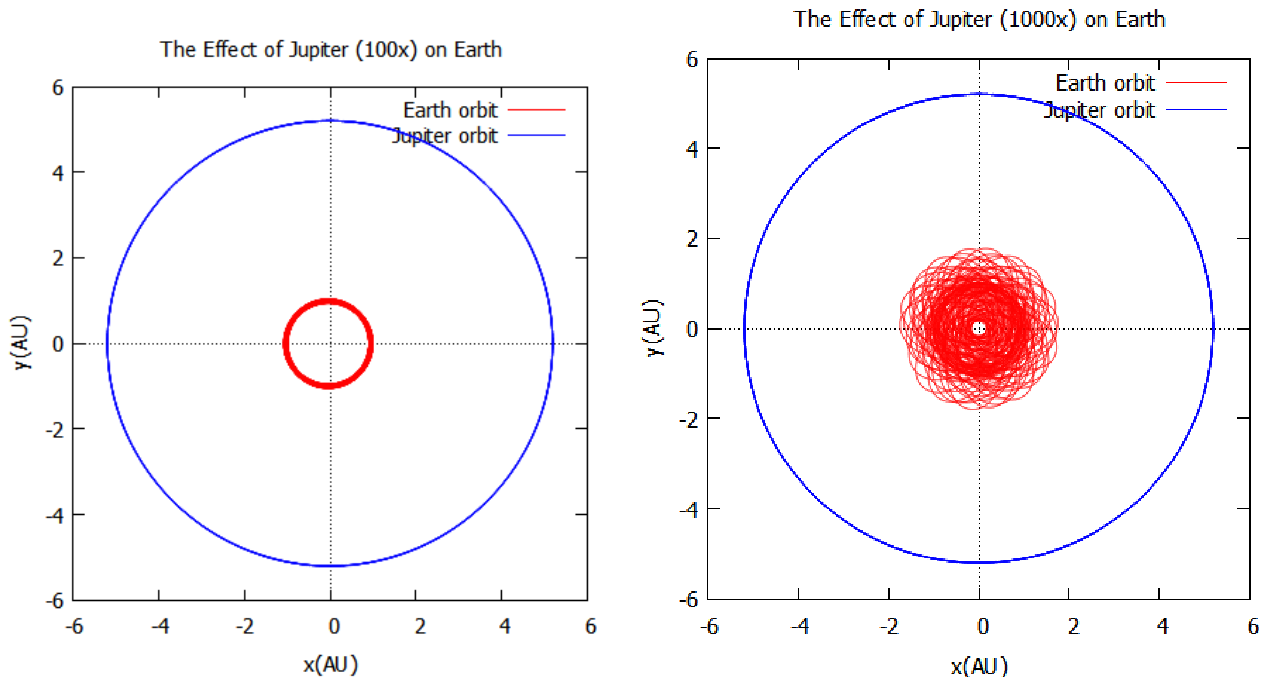


Figure 5: 100-times Jupiter's mass (left) and 1000-times Jupiter's mass (right)

Note that in all of the graphs in figures 5 and 6, Jupiter's orbit stays the same. As Jupiter's mass increases, the consistency of Earth's orbit decreases, as can be seen by the red line denoting Earth's orbit getting thicker. When Jupiter's mass is 1000-times larger than its actual mass, Earth's orbit becomes erratic.

1.5 Full Three-Body Calculation

The next task we completed was to implement the actual center of mass of the three-body system along with the motion of the Sun. This was done by first calculating the Sun's initial position (which lies on the x-axis) via the center of mass located at the origin. We also calculate the initial velocity of the sun (which is in the negative y-direction) via the momentum of the center of mass is zero. The Sun's motion is then added to the `calculate` subroutine. Note that the distance between the bodies was also adjusted to include the Sun, as prior to this the Sun was assumed to be at the origin.

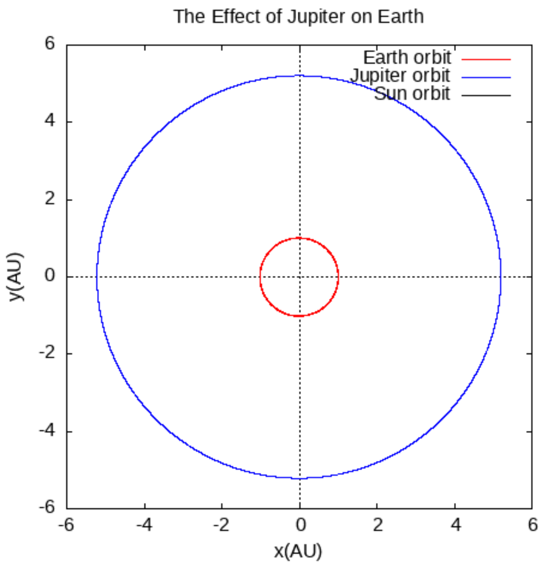


Figure 6: Orbit of the three bodies including the center of mass

Note that in figure 6, which includes the center of mass calculation, the Sun's motion is not visible. Figure 7 pictures just the Sun's movement.

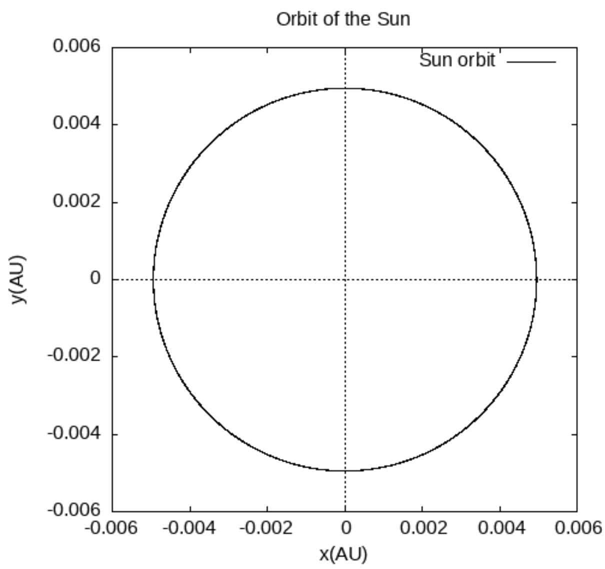


Figure 7: Motion of the Sun

Note that the motion of the Sun is spherical. Next is to increase the mass of Jupiter to examine the effects on the planet's orbits.

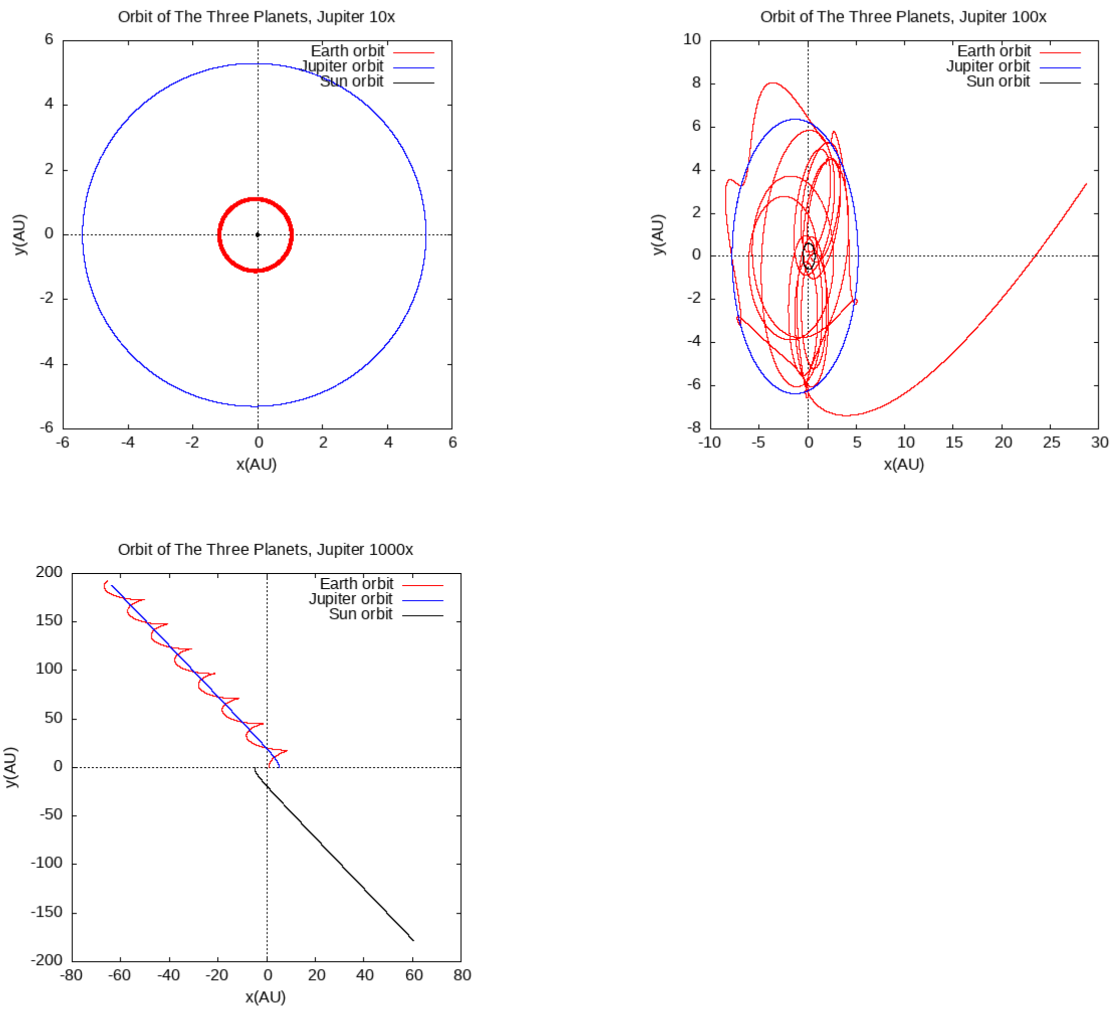


Figure 8: 10-times Jupiter's mass (top left), 100-times Jupiter's mass (top right), and 1000-times Jupiter's mass (bottom left)

Note that the Earth is ejected when Jupiter's mass is 100 and 1000-times larger than its actual value.

1.6 Conclusion

In our work, we investigated the effects of changing the inverse-square law, computation method, and center of mass implementation. In the two-body problem, changing the coefficient for the gravitational force law causes massive changes in the orbit of Earth around the Sun. Changing the computation method for the two-body problem either made convergence possible or impossible. We found that the Euler method did not work for the two-body problem while the Euler-Cromer method did work. The given three-body code gave approximate plots in the approximation where the Sun's mass is considered infinite. Changing the mass of Jupiter caused Earth's orbit to change considerably. After the integration of the center of mass of the three-body system (and the movement of the Sun), the base orbit of all planets was very similar to the approximated version. Changing the mass of Jupiter in this case caused major changes in Earth and the Sun's orbit.