Computational Physics Project 2

Jack Messerli-Wallace

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1 Shooting a Cannon Ball

1.1 Introduction

To model the projectile motion of a cannonball, we first make a set of assumptions to make the model simpler, then get rid of assumptions to increase model accuracy and complexity. The first model tested assumed air drag was linear and that the cannonball stopped at a certain height h. In our case, $h = 200$ meters, meaning the cannonball trajectory ends when the y-component of the velocity is negative and the x position reaches 200 meters.

1.2 Height Optimization

The changes made to the original program to get to this point include redefining all variables as double precision and using implicit none at the beginning of the program and in all subroutines. We also added additional comments to improve the readability of the program, added the x and y speeds to the output file, and added the functionality of the height difference as described. To find the optimal angle, another input parameter was added. If the "test" parameter is equal to one, the angle input argument will be ignored and a full test of angles between $10.00°$ and $80.00°$ will be examined. Note that the angle step is $0.01°$.

The given input parameters used to find the optimal angle to reach 200 meters was a starting speed of 700 meters per second and a 0.1 second time-step. With linear air resistance equal to $4.0 \cdot 10^{-5}$ B2/m, as given in the text, the optimal angle is $39.17°$, giving the farthest distance of 21957.05 meters in the x direction.

1.3 Height-Dependent Air Drag

Now we examine the impact of changing the air resistance calculation from the linear model to the two altitude-dependent models described in our text. To implement these two equations (equation 2.23 and 2.24), another input parameter was added. If air_drag_type is zero, then equation 2.23 will be used. If air_drag_type is one, then equation 2.24 will be used. All other values will correspond to the linear air resistance used before. Below are the trajectory graphs of a cannonball shot under both equations with varying angles.

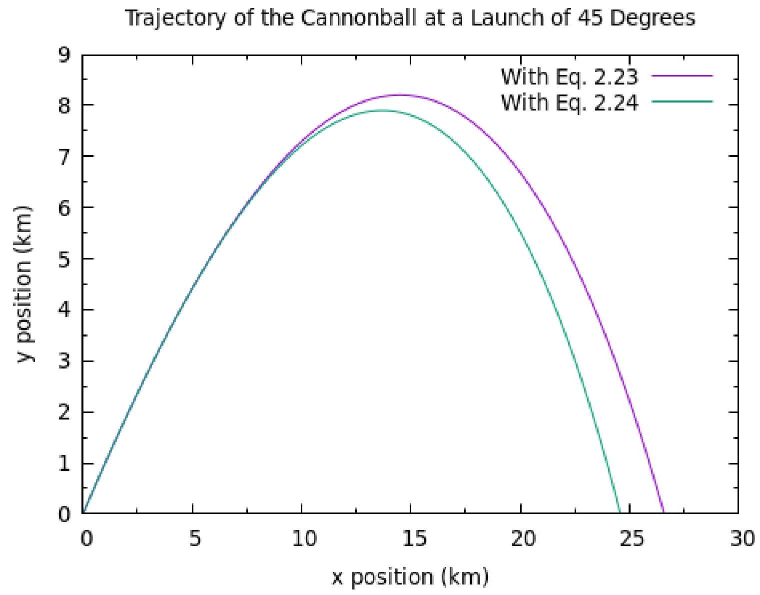


Figure 1: Cannonball shot at $45°$

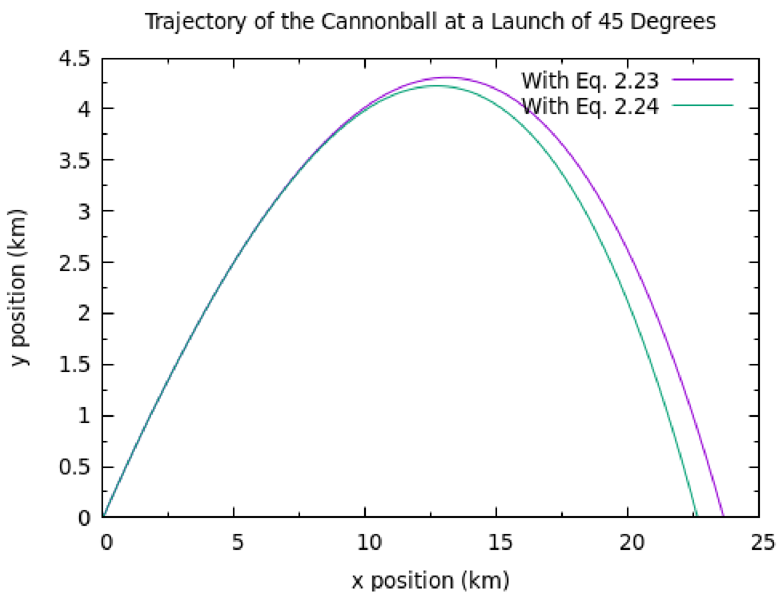
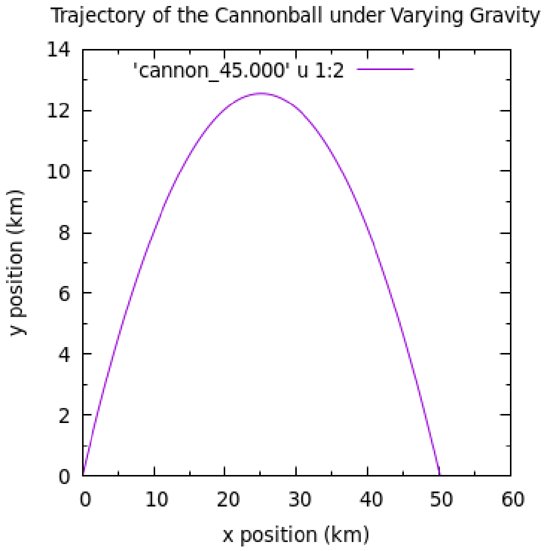


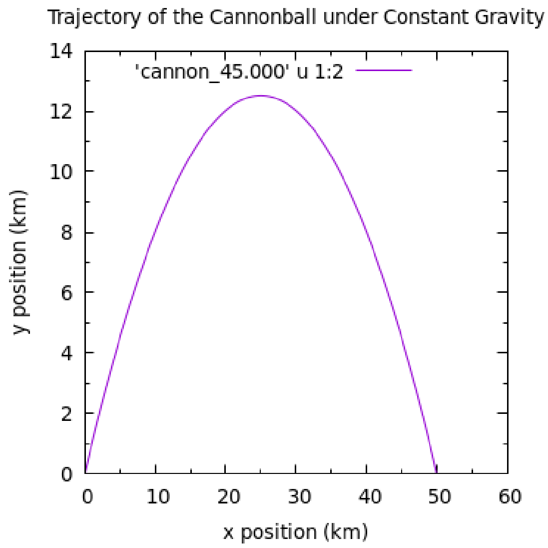
Figure 2: Cannonball shot at $30°$

1.4 Acceleration Due to Gravity

Next is to implement the varying of gravity with altitude. Again, a parameter is added to determine if the gravitational force should be constant or if it should vary. If gravity_adjustment is zero, constant gravity will be used, otherwise varying gravity will be used. Below are the graphs of a few angles with constant and varying gravitational force.

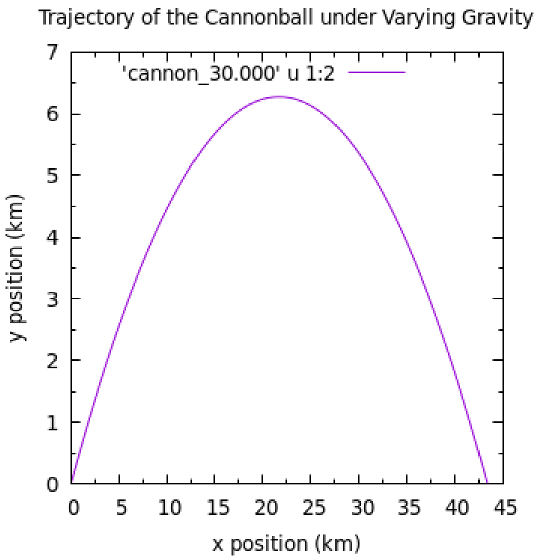


(a) 45° under varying gravity

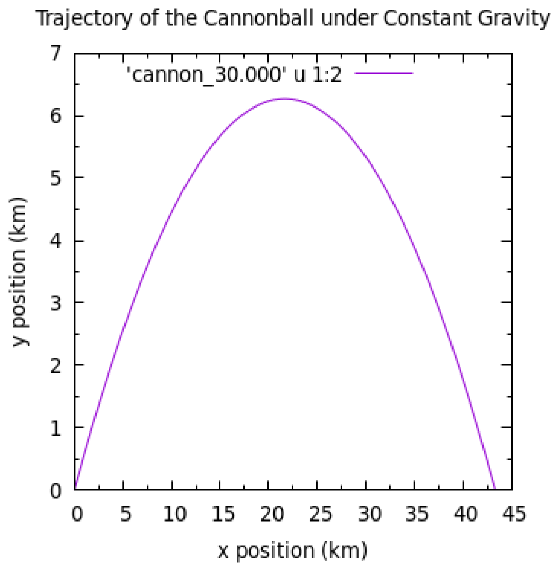


(b) 45° under constant gravity

Figure 3: 45° Test for varying and constant gravity



(a) 30° under varying gravity



(b) 30° under constant gravity

Figure 4: 30° Test for varying and constant gravity

Note that, while the varying gravity allows for the cannonball to be shot further, the effects under the conditions tested are near negligible. The final distances are nearly the same for varying and constant gravity. For varying gravity, the optimized angle for the furthest distance is 50232.02 meters when the cannonball is launched at 45.4°. This was calculated using the same method in section 1.2 with the height set to zero meters.

1.5 Variation in Gravity and Height-Dependent Air Drag

Lastly, we will investigate the effects of using both the variation in gravity and the height-dependent air drag as described in Eq. 2.24. The goal of this section is to calculate the angle, within $0.01°$, that shoots the cannonball to 19.500 kilometers. To do this, a new parameter is introduced in the code to determine if optimization should be done to maximize distance or to acquire the angle desired in this section. When the parameter optimization_input is set to anything besides zero, the angle to reach 19.5 kilometers is acquired. As expected there are two optimal angles found. The higher optimal angle found is $64.53°$, with a final distance of 19.499 kilometers. The smaller optimal angle found is $21.54°$, with a final distance of 19.509 kilometers.

1.6 Conclusion

We examined the trajectory of a cannonball under linear air drag, two different altitude-dependent air drag models, and varying and constant gravity. One notable conclusion about the air drag models is that under Eq. 2.24 (air drag as calculated under the adiabatic approximation), the distance traveled is far less than that of the other models. Another conclusion is that the variation of Earth's gravitational force is near negligible in this model. This is to be expected as the altitude the cannonball reaches is several orders of magnitude less than the radius of the Earth, meaning the gravitational force changes very little.