Investigation of the Central Limit Theorem Using the Exponential Distribution

Alex Van Russelt

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Overview

This document investigates the central limit theorem (CLT) through use of an exponential distribution. Random variables are generated using such a distribution, and then averaged. This is repeated a number of times, and it is apparent that the resulting distribution can be well approximated by a normal distribution, as predicted by the CLT.

Simulations

```r
set.seed(123)
simulation_count <- 1000 # The number of times to repeat the experiment
exponential_count <- 40 # The number of exponentials to average
lambda <- 0.2

means <- numeric(length=simulation_count)

for (i in 1:simulation_count){
  data = rexp(exponential_count, lambda)
  means[i] <- mean(data)
}

df_means <- data.frame(means)
```

The `rexp` function is used to generate 40 random variables using the exponential distribution with rate paramater `lambda` set equal to `0.2`. The mean is recorded. This is repeated 1000 times.

Comparison of sample mean to the theoretical mean

```r
sample_mean <- mean(means)
message(paste("Theoretical mean:", format(1/lambda, nsmall=2, digits=2)))
```

```
## Theoretical mean: 5.00
```

```r
message(paste("Sample mean:", format(sample_mean, nsmall=2, digits=2)))
```

```
## Sample mean: 5.01
```

The theoretical mean of an exponential distribution with rate parameter λ is given by $\frac{1}{\lambda}$. In this case, $\lambda = 0.2$, so the expected mean is 5.00. The sample mean is 5.01, which is very close to the theoretical value.

Variance of the sample compared to theoretical variance

```
sample_variance <- var(means)
standard_error <- (1 / lambda) ^ 2 / exponential_count
message(paste("Theoretical variance:", format(standard_error, nsmall=2, digits=2)))
```

```
## Theoretical variance: 0.62
```

```
message(paste("Sample variance:", format(sample_variance, nsmall=2, digits=2)))
```

```
## Sample variance: 0.60
```

The theoretical variance is given by the squared standard error of the means:

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$

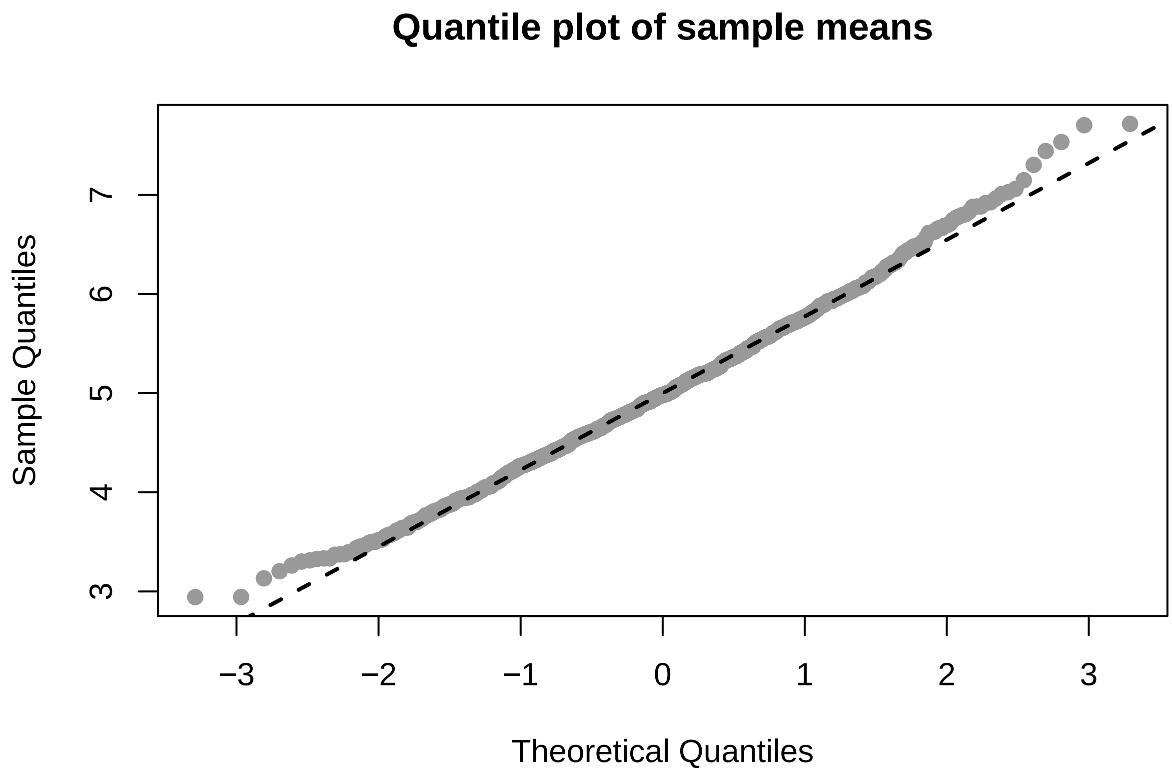
where:

1. $\sigma_{\bar{x}}$ is the standard error of the mean.
2. σ is the standard deviation of the population (which for an exponential distribution is equal to $\frac{1}{\lambda}$).
3. n is the number of observations (in this case, 40).

This produces a theoretical variance of 0.62. The sample mean is 0.60, which is very close to the theoretical value.

Comparing distribution of means to a normal distribution

```
qqnorm(means, col="#999999", pch=19,
       main='Quantile plot of sample means')
qqline(means, lwd=2, lty='dashed')
```



Here, a quantile plot is used to compare the quantiles of the means to that of a normal distribution. All data points lie close to the reference line, therefore, the distribution of the means is approximately normal.