IE 251 - LINEAR PROGRAMMING

Fall 2024

Case Study 2

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1. INTRODUCTION

We offer a complete solution to the production and sales optimization issue IE-Tech Manufacturing Company faces in this study. IE-Tech specializes in manufacturing pre-assembled robotic kits and premium robotics components. To maximize profit, the manufacture of five essential components—aluminum frames, carbon fiber frames, manual modules, advanced control modules, and advanced sensor modules—must be optimized. The assembly and sale of robotic kits must also be considered. These kits require a certain number of different components to be assembled, requiring precise planning to Align production with demand and resource limits.

Production and assembly operations are carried out in plants in 3 cities: İstanbul, Ankara, and İzmir. Each plant is capable of producing all of the components and assembling the robotic kits, but certain restrictions such as production costs ***(Table 6)***, required labor and packing times ***(Table 1,2,3)***, monthly available minutes for labor and packing ***(Table 1,2,3)***, assembly times ***(Table 4)***, minimum and maximum product demands ***(Table 5)*** and selling prices ***(Table 7)*** varies between plants. Resources such as labor and packing time as well as assembly times, impose strict constraints on monthly production capacity. Additionally, the company also manages carbon fiber resources. Each carbon fiber frame requires 0.25 pounds of carbon fiber, and 1,000 pounds are available per month, limiting the production of carbon fiber frames. Since there is no beginning inventory and the planning period is two months, it is crucial to control production efficiency and inventory expenses, which add up to 8% of production costs at the end of each month. The 12% increase in manufacturing costs in the second month makes matters much more complicated and requires careful decisions to strike a balance between cost reduction and meeting demand.

The optimization model aims to balance production and sales while adhering to several constraints, including the minimum and maximum monthly demand for each product, the availability of materials, and plant-specific production capacities. The scheduling method is made more difficult by the time limits that each factory has for robotic kit assembly. The goal is to identify the best strategy that optimizes profitability while considering the different production costs and selling prices at each plant considering the %12 increase in the second month. Our report examines production planning strategies, examines the relationship between resource utilization and profit, and analyzes the effects of price and resource changes. After explaining the mathematical model and conducting a sensitivity analysis of the results, recommendations are presented for IE-Tech to implement the best manufacturing and sales strategy. The appendix contains all relevant tables and computations for your reference. This solution gives IE-Tech a solid plan to regulate its business processes, ensuring the company meets customer needs while maximizing profits.

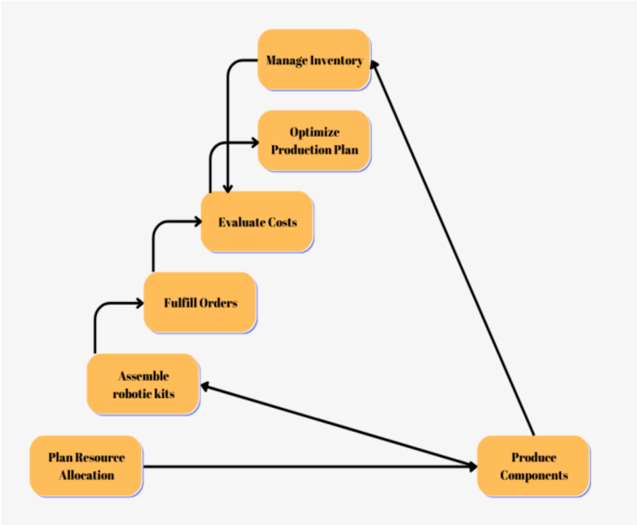

Figure 1:Precedence Diagram

2. MAIN BODY

Formulation of the Problem

Mathematical Model

Indices and Sets

i: Index of components $\boldsymbol{i} \in \boldsymbol{I}$ = {aluminum frames, carbon fiber frames, manual modules, advanced control modules, advanced sensor modules}

j: Index of plants $\boldsymbol{j} \in \boldsymbol{J}$ = {İstanbul, Ankara, İzmir}

t: Index of periods(months) $\boldsymbol{t} \in \boldsymbol{T}$ = {1, 2}

Decision Variables

X: Component ***i*** produced in plant ***j*** in month ***t***

R: Robotic kits assembled in plant ***j*** in month ***t***

XS: Component ***i*** sold in plant ***j*** in month ***t***

RS: Robotic kit sold in plant ***j*** in month ***t***

INVX: Inventory of component ***i*** in plant ***j*** at the end of month ***t***

INVR: Inventory of robotic kit in plant ***j*** at the end of month ***t***

Parameters

LABORDATA: Required labor minutes in plant ***j***

LABAV: Available monthly labor minutes in plant ***j***

PACKDATA: Required packing minutes in plant ***j***

PACKAV: Available monthly packing minutes in plant ***j***

ASSDATA: Required assembly minutes in plant ***j***

ASSAV: Available monthly assembly minutes in plant ***j***

MINDEMCOMP: Minimum demand for component ***i*** in plan*t* ***j***

MAXDEMCOMP: Maximum demand for component ***i*** in plant ***j***

MINDEMROBO: Minimum demand for robotic kit in plant ***j***

MAXDEMROBO: Maximum demand for robotic kit in plant ***j***

COSTCOMP: Production cost of component ***i*** in plant ***j***

COSTROBO: Production cost of a robotic kit in plant ***j***

SELLCOMP: Selling price of a component ***i*** in plant ***j***

SELLROBO: Selling price of a robotic kit in plant ***j***

REQ: Required component ***i*** for assembling a robotic kit

Model

Max $z = \sum_{i\in I}\sum_{j\in J}$ $[\ (XS_{ij1} * SELLCOMP_{ij} + RS_{j1} * SELLROBO_j) + (XS_{ij2} * SELLCOMP_{ij} + RS_{j2} * SELLROBO_j) - (COSTCOMP_{ij} * X_{ij1} + INVX_{ij1} *$

$COSTCOMP_{ij} * 0.08 + COSTCOMP_{ij} * X_{ij2} * 1.12 + INVX_{ij2} * COSTCOMP_{ij} * 0.08 * 1.12 + INVR_{j1} * COSTROBO_j * 0.08 + INVR_{j2} * COSTROBO_j * 1.12 * 0.08 + R_{j1} * COSTROBO_j + R_{j2} * COSTROBO_j * 1.12)$]

subject to

(1) $(i \in I)\sum X_{ijt} * LABORDATA_{ij} \leq LABAV_j$ $\forall j \in J, t \in T$

(2) $(i \in I)\sum X_{ijt} * PACKDATA_{ij} \leq PACKAV_j$ $\forall j \in J, t \in T$

(3) $R_{jt} * ASSDATA_j \leq ASSAV_j$ $\forall j \in J, t \in T$

(4) $(j \in J)\sum X_{2jt} \leq 4000$ $\forall t \in T$

(5) $X_{ijt} \geq MINDEMCOMP_{ij}$ $\forall i \in I, j \in J, t \in T$

(6) $X_{ijt} \leq MAXDEMCOMP_{ij}$ $\forall i \in I, j \in J, t \in T$

(7) $R_{jt} \geq MINDEMROBO_j$ $\forall j \in J, t \in T$

(8) $R_{jt} \leq MAXDEMROBO_j$ $\forall j \in J, t \in T$

(9) $X_{ij1} - XS_{ij1} - INVX_{ij1} - REQ_i * R_{j1} = 0$ $\forall i \in I, j \in J$

(10) $R_{j1} - RS_{j1} - INVR_{j1} = 0$ $\forall j \in J$

(11) $X_{ij2} - XS_{ij2} - INVX_{ij2} - REQ_i * R_{j2} + INVX_{ij1} = 0$ $\forall i \in I, j \in J$

(12) $R_{j2} - RS_{j2} - INVR_{j2} + INVR_{j1} = 0$ $\forall j \in J$

(13) $REQ_i * R_{jt} \leq X_{ijt}$ $\forall i \in I, j \in J, t \in T$

(14) $X_{ijt}, R_{jt}, XS_{ijt}, XR_{jt}, INVX_{ijt}, INVR_{ijt} >= 0$ $\forall i \in I, j \in J, t \in T$

Our objective function maximizes the total profit generated from the production and sale of components and robotic kits while deducting production and inventory costs. The expressions $XS_{ij1} * SELLCOMP_{ij} + RS_{j1} * SELLROBO_j$ and $XS_{ij2} * SELLCOMP_{ij} + RS_{j2} * SELLROBO_j$ represent the total revenue earned from selling components and robotic kits in ***periods 1 and 2***, respectively. The deductions include:

- **Production costs**: $COSTCOMP_{ij} * X_{ij}$ and $COSTCOMP_{ij} * X_{ij2} * 1.12$ (*reflecting a 12% increase in period 2*).

- **Inventory holding costs**: $INVX_{ij1} * COSTCOMP_{ij} * 0.08$ and $INVX_{ij2} * COSTCOMP_{ij} * 0.08 * 1.12$ for components, and $INVR_{j1} * COSTROBO_j * 0.08$ and $INVR_{j2} * COSTROBO_j * 0.08 * 1.12$ for robotic kits.

The objective ensures the company maximizes its profitability by optimizing production, sales, and inventory allocation across plants and periods.

Constraints

1. **Constraints (1) and (2): Labor and Packing Time**
   These constraints ensure that the total labor $(\sum_{i \in I} X_{ijt} * LABORDATA_{ij})$ and packing time $(\sum_{i \in I} X_{ijt} * PACKDATA_{ij})$ required for production in plant j do not exceed the available labor $(LABAV_j)$ and packing time $(PACKAV_j)$ for each period t.
2. **Constraint (3): Assembly Time**
   This constraint ensures that the total assembly time required for robotic kits $(R_{jt} * ASSDATA_j)$ in plant j does not exceed the available assembly time $(ASSAV_j)$ for each period t.
3. **Constraint (4): Carbon Fiber Constraint**
   This constraint limits the total production of carbon fiber frames $(\sum_{j \in J} X_{1jt})$ across ***all plants*** to ***4000 units*** per period t.
4. **Constraints (5) and (6): Component Demand**
   These constraints ensure that the production of components X_{ijt} in plant j for each period t meets the minimum demand $(MINDEMCOMP_{ij})$ and does not exceed the maximum demand $(MAXDEMCOMP_{ij})$.
5. **Constraints (7) and (8): Robotic Kit Demand**
   These constraints ensure that the production of robotic kits R_{jt} in plant j for each period t meets the minimum demand $(MINDEMROBO_j)$ and does not exceed the maximum demand $(MAXDEMROBO_j)$.
6. **Constraints (9) and (10): Normalization for Period 1**
   - Constraint (9): Ensures that the total production of components X_{ij1} equals the sum of sold components XS_{ij1}, leftover inventory $INVX_{ij1}$, and components used in robotic kit assembly $REQ_i * R_{j1}$.

- Constraint (10): Ensures that the total production of robotic kits $\boldsymbol{R_{j1}}$ equals the sum of sold kits $\boldsymbol{RS_{j1}}$ and leftover inventory $\boldsymbol{INVR_{j1}}$.

7. **Constraints (11) and (12): Normalization for Period 2**
   - Constraint (11): Accounts for components produced $\boldsymbol{X_{ij2}}$, sold $\boldsymbol{XS_{ij2}}$, leftover inventory $\boldsymbol{INVX_{ij2}}$, and those used in robotic kit assembly $\boldsymbol{REQ_i * R_{j2}}$, while including leftover inventory from period 1 $\boldsymbol{INVX_{ij1}}$.
   - Constraint (12): Ensures that the total production of robotic kits $\boldsymbol{R_{j2}}$ accounts for sold kits $\boldsymbol{RS_{j2}}$, leftover inventory $\boldsymbol{INVR_{j2}}$, and leftover inventory from period 1 $\boldsymbol{INVR_{j1}}$.
8. **Constraint (13): Component Requirement for Robotic Kits**
   This constraint ensures that the total components $\boldsymbol{X_{ijt}}$ available in plant $\boldsymbol{j}$ for period $\boldsymbol{t}$ are sufficient to assemble the robotic kits $\boldsymbol{REQ_i * R_{jt}}$.

Sign Restriction

All decision variables (Xijt,Rjt,XSijt,RSjt,INVXijt,INVRjt) are restricted to be non-negative real numbers to ensure feasible and realistic solutions.

Assumptions

Despite their usefulness, linear programming (LP) models frequently oversimplify difficult real-world situations. To make sure the LP model built for IE-Tech Manufacturing is correctly formulated, a few assumptions must be established. The model's compliance with the four primary LP model assumptions—additivity, proportionality, divisibility, and certainty—is examined in this section.

Additivity: Additivity presupposes that each decision variable makes an independent, additive contribution to the objective function and constraints. In this model: Sales of components ***(XS_{ijt})*** and robotic kits ***(RS_{jt})*** generate revenue contributions that are additive and independent of one another. The costs of ***labor***, ***production***, and ***inventory*** are all seen as distinct and cumulative contributions to the constraints and goal function. As long as the revenue contributions and costs are unrelated to one another, there do not seem to be any additivity violations.

Proportionality: A linear relationship between decision factors and their contributions to the goal function or restrictions is assumed by proportionality. Regarding this model:

It is expected that the cost of producing robotic kits ***($COSTROBO_j$)*** and components ***($COSTCOMP_{ij}$)*** will rise proportionately with the quantity produced. It is believed that the labor and packing times ***($LABORDATA_{ij}$ and $PACKDATA_{ij}$)*** would rise in proportion to the number of components produced. The amount of units stored determines the inventory holding cost, which is ***8%*** of manufacturing expenses. However, because of economies of scale or inefficiencies at greater production levels, proportionality may not hold true in practice. For instance, worker availability or weariness may have an impact on labor efficiency.

Divisibility: Divisibility allows for continuous solutions by assuming that choice variables can have any fractional value. Regarding this model: The model permits fractional production quantities since it is expected that the manufacturing of robotic kits ***(R_{jt})*** and components ***(X_{ijt})*** is divisible. Since manufacturing and sales quantities must be full numbers, divisibility may not be realistic even though it is true technically *(e.g., it is not viable to create 0.5 robotic kits)*. Rounding or an integer programming technique may be needed for practical implementation to guarantee realistic outcomes.

Certainty: Certainty presupposes that every model parameter is fixed and known. Regarding this model: Parameters such as production costs **($COSTCOMP_{ij}$,$COSTROBO_j$)**, selling prices ***($SELLCOMP_{ij}$,$SELLROBO_j$)*** , labor availability ***($LABAV_j$)*** , and packing availability ***($PACKAV_j$)*** are assumed to be constant and certain. In practice, assurance is rarely maintained. Unexpected events, supply chain interruptions, and market conditions can all affect costs, prices, and resource availability. For example: Supplier problems may result in changes to the price of raw materials (such as carbon fiber). Demand projections ***($MINDEMCOMP_{ij}$,$MAXDEMCOMP_{ij}$,$MINDEMROBO_j$,$MAXDEMROBO_j$)*** may fluctuate based on customer preferences or economic developments.

Conclusion

Although the LP model offers a structured approach to optimize IE-Tech's production and sales, the assumptions of additivity, proportionality, divisibility, and certainty highlight some limitations:

- In general, ***additivity*** and ***proportionality*** hold true within the model's parameters, but they might need to be adjusted for complexity in the real world.

- Although ***divisibility*** is mathematically satisfied, it needs to be modified for practical feasibility.
- ***Certainty*** is the most challenging assumption, as real-world parameters are often uncertain and subject to change.

The model's applicability and dependability in a dynamic manufacturing environment can be increased by addressing these limitations through scenario analysis, sensitivity analysis, or optimization.

3. DISCUSSIONS ABOUT FINDINGS

Results

By balancing production, inventory keeping, and sales decisions across several plants and periods, our model has successfully optimized total profit while considering real-world limitations including labor and packing availability and assembly capacity, material limits, and demand requirements. The specific results are listed below:

Resource Constraints

Labor (eLab): In a number of plants, labor resources are completely used, especially during times when output demand is high. The sensitivity analysis shows that labor limitations are binding. According to shadow prices, increased labor capacity at these times might greatly increase profitability; for example, some factories (such c_u_x112_) could yield returns of $3.62 per additional unit.

Shadow prices:
c_u_x112_: $3.61905 | **c_u_x113_:** $2.76571 | **c_u_x116_:** $2.66667

Implication: Expanding labor availability in plants where these constraints are binding could yield substantial profit improvements. For instance, adding one unit of labor for **c_u_x112_** increases profit by $3.62.

Packing (ePack): Just like labor, packing resources are used up completely during critical periods. The model contributes to a strictly optimized production schedule by guaranteeing that packing availability is not exceeded.

Shadow prices:
c_u_x114_: $4.44444 | **c_u_x115_:** $3.80444

> **Implication:** Increasing packing capacity in critical plants can provide higher production flexibility and profitability.
>
> **Assembly (eAss):** In some cases, especially when it comes to the production of robotic kits, assembly capacity constraint is binding. The substantial significance of assembly as a narrowing resource is indicated by the marginal value of easing these limitations (shadow prices).

Shadow prices:
c_u_x120_: $2.35294 | **c_u_x121_:** $2.01412

> **Implication:** Expanding assembly capacity increases robot production, which significantly impacts profitability.

Material Constraints

Carbon Fiber (eCar): The sensitivity analysis emphasizes the binding nature of this constraint, making it one of the most important restrictions. The shadow price highlights the potential profitability of expanding material availability, while the production of components needing carbon fiber is limited to 4000 units.

Shadow price:
c_l_x148_: $1.52941

> **Implication:** Increasing the carbon fiber availability directly impacts the profit by enabling more component production.

Numerical Analysis of Demand Constraints

Minimum Demand Constraints: In order to satisfy baseline client expectations, these limits mandate the manufacturing of a minimum quantity of robots or components.

Binding minimum demand restrictions frequently leads to:

c_l_x137_: $1.62667 | **c_l_x148_:** $1.52941 (shadow prices)

The opportunity cost of failing to meet baseline demand is represented by these shadow prices. For instance, profit would drop by $1.53 if production of **c_l_x148_** were to remain at least one unit above the minimal demand.

Observations

Constraints on binding minimum demand imply that production must continue even when resources are few, which could raise operating expenses. Reducing these restrictions may increase adaptability, particularly for goods with slight profit margins.

Maximum Demand Constraints: In order to prevent overproduction and match market capacity, these limitations set a production cap.

In the analysis of sensitivity:

Shadow Prices: The majority of maximum demand constraints have zero shadow prices, which means that the current optimal solution does not bind to these constraints.

This implies that some capacity is being unused because the model is not hitting maximum demand limitations.

Observations

If market capacity permits, non-binding maximum demand limits draw attention to the possibility of higher production and sales. Raising the upper limits for high-margin items could be a good move to make use of the resources at hand.

Inventory and Carrying Costs

Inventory Levels: The model's inventory choices take into account the trade-off between reducing holding costs and satisfying future demand. For instance, in order to offset higher second-period costs—which rise by 12% as a result of operational issues or inflation—first-period production frequently surpasses immediate sales.

Carrying Costs: Overproduction is penalized by inventory carrying costs, especially for robots and components. To reduce these expenses, the model carefully weighs inventory against anticipated demand.

Shadow Prices: Shadow prices for inventory constraints are primarily zero, reflecting no direct impact on profit unless capacity limits are reached.

Objective Function Value

The results highlight the total profit achieved under different scenarios:

Total Profit: $501,357.95

This represents the optimal solution under the current constraints without additional resource adjustments.

4. CONCLUSION & RECOMMENDATIONS

The company operates under severe resource constraints, with labor, packing, assembly, and carbon fiber availability serving as crucial restrictions that have a direct impact on profit, according to the optimization model and sensitivity analysis. The shadow prices linked to these constraints show significant room for development, including expanding material availability or resource capacity to open up new production possibilities and increase profit. Furthermore, non-binding maximum demand limitations draw attention to the unrealized potential to raise production in the event that demand rises, whereas minimum demand constraints enforce market requirements while imposing strict production limits that may put a strain on resources. The organization will be able to increase efficiency, flexibility, and profitability while staying in line with the market through strategic investments in resource expansion, demand alignment, and cost optimization, especially with regard to inventory and production scheduling.

Recommendations

IE-Tech should think about taking the following steps to improve operational effectiveness and profitability:

1) **Expand Critical Resources**

Workforce and Packing Ability: Invest in more packing materials and labor, especially in plants that have high shadow pricing (e.g., $4.44 for c_u_x114_ and $3.62 for c_u_x112_). This will facilitate increased output by easing production restrictions.

Capacity for Assembly: Increase the availability of assembly time in factories with high demand. Shadow prices, such as $2.35 for c_u_x120_, provide a significant opportunity to increase profits.

2) **Increase Carbon Fiber Availability**

To loosen the restriction, try negotiating larger carbon fiber quotas or looking into other suppliers (eCar). Even small increases in availability can result in measurable profit gains with a shadow price of $1.53.

3) **Optimize Demand Management**

Reevaluate the minimum demand restrictions: Assess the strategic alignment of standard minimum demand levels, especially for low-margin products such as some of the components. Reducing these restrictions may allow resources to be allocated to opportunities with better profit margins.

Leverage Non-Binding Maximum Demand: To make the most of production capacities for high-margin products which are robotic kits, increase market capacity or customer base.

4) **Enhance Inventory Strategies**

Reduce the expense of carrying inventory by switching to a just-in-time production approach. Adjust production schedules to better reflect anticipated demand. To better predict changes in demand and make the best inventory choices, use predictive analytics.

5) **Conduct a Scenario Analysis**

To comprehend the effects of particular changes, such as doubling labor availability, raising the carbon fiber limit, or alleviating particular demand limitations, do focused analysis. This can guarantee the most return on increased capacity and assist in prioritizing expenditures.

6) **Examine Cost-Cutting Strategies**

Examine the second-period production cost escalation factor (12%). To lessen this rise, try to renegotiate contracts or streamline procedures.

By applying these recommendations, IE-Tech can increase its potential for profit, lessen operational limitations, and provide flexibility to its demand planning and production procedures. In a market with limited resources, these strategic changes will guarantee competitive advantage and growth.

APPENDIX

Table 1: Resources for İstanbul

	Resources	
Components	**Labor (minutes/unit)**	**Packing (minutes/unit)**
Aluminum frames	1	4
Carbon fiber frames	1.5	4
Manual modules	1.5	5
Advanced control modules	3	6
Advanced sensor modules	4	6
Monthly Availability (minutes)	**12,000**	**20,000**

Table 2: Resources for Ankara

	Resources	
Components	**Labor (minutes/unit)**	**Packing (minutes/unit)**
Aluminum frames	3.5	7
Carbon fiber frames	3.5	7
Manual modules	4.5	8
Advanced control modules	4.5	9
Advanced sensor modules	5	7
Monthly Availability (minutes)	**15,000**	**40,000**

Table 3: Resources for İzmir

	Resources	
Components	**Labor (minutes/unit)**	**Packing (minutes/unit)**
Aluminum frames	3	7.5
Carbon fiber frames	3.5	7.5
Manual modules	4	8.5
Advanced control modules	4.5	8.5
Advanced sensor modules	5.5	8
Monthly Availability (minutes)	**22,000**	**35,000**

Table 4: Assembly Times

Plant	**Time (minutes per set)**	**Total Time Available (minutes)**
İstanbul	65	5500
Ankara	60	5000
İzmir	65	6000

Table 5: Minimum and Maximum Product Demand per Month

	Plants					
	İstanbul		**Ankara**		**İzmir**	
Products	**min**	**max**	**min**	**max**	**min**	**max**
Aluminum frames	0	2000	0	2000	0	2000
Carbon fiber frames	100	2000	100	2000	50	2000
Manual modules	200	2000	200	2000	100	2000
Advanced control modules	30	2000	30	2000	15	2000
Advanced sensor modules	100	2000	100	2000	100	2000
Robotic kit	0	200	0	200	0	200

Table 6: Production costs ($)

	Plants		
Products	**İstanbul**	**Ankara**	**İzmir**
Aluminum frames	6	5	7
Carbon fiber frames	19	18	20
Manual modules	4	5	5
Advanced control modules	10	11	12
Advanced sensor modules	26	24	27
Robotic kit	178	175	180

Table 7: Selling prices ($)

	Plants		
Products	**İstanbul**	**Ankara**	**İzmir**
Aluminum frames	10	10	12
Carbon fiber frames	25	25	30
Manual modules	8	8	10
Advanced control modules	18	18	22
Advanced sensor modules	40	40	45
Robotic kit	290	290	310