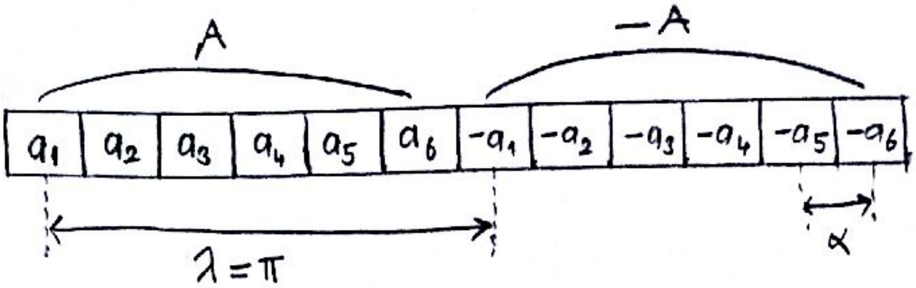
EE362 – HW2 – Solution

Q.1)

a)



b)

$$\alpha = \frac{180}{6} = 30^\circ \text{ (slot angle)}$$

$\lambda = \pi$ (pitch angle)

$q = 6$ (# of slots per pole per phase)

$$\Rightarrow k_{pr} = \sin(\lambda/2) = 1$$

$$\Rightarrow k_{dr} = \frac{\sin\left(\frac{q\alpha}{2}\right)}{q\sin\left(\frac{\alpha}{2}\right)} = 0.644$$

$$\left.\right\} k_{wr} = k_{pr}\cdot k_{dr} = \boxed{0.644}$$

c) $$F_r = \left(\frac{4}{\pi}\right)\left(N_{ph}\frac{I}{2}\right)k_{wr} = \left(\frac{4}{\pi}\right)\left(6\times 10\times \frac{10}{2}\right)\times 0.644 = \boxed{246 \text{ Amps}}$$

d) $B_{gr} = F_r \dfrac{\mu_0}{g}$ where $g = 0.002\,m$ $\longrightarrow$ $B_{gr} = 0.1545$ Tesla

$$\phi_{pr} = \frac{4rl}{p}B_{gr} = \frac{4\times 0.1\times 0.3}{2}\times 0.1514 = \boxed{9.27 \text{ mWb}}$$

e) $E = 4.44\, N_{ph,s} f \phi_p k_{w,s} = 4.44\times(6\times 30)\times 50\times 9.27\times 10^{-3}\times k_{w,s} = 350$ Volts.

⊛ f is calculated by : $f = \dfrac{p\times n_r}{120} = \dfrac{2\times 3000}{120} = 50$ Hz.

$$\Rightarrow k_{ws} = 0.945 = k_{ds}\times k_{ps}$$

$$k_{ds} = \frac{\sin\left(q\frac{\alpha}{2}\right)}{q\sin\left(\frac{\alpha}{2}\right)} = 0.9598 \Rightarrow k_{ps} = \frac{k_{ws}}{k_{ds}} = 0.9848 = \sin\left(\frac{\lambda}{2}\right)$$

$\Rightarrow \lambda_1 = 160^\circ$ (underpitched)

$\lambda_2 = 200^\circ$ (overpitched)

f) $\lambda_1 = 160^\circ$

A			−C			B			−A			C			−B		
a_1	a_2	a_3	$-c_4$	$-c_5$	$-c_6$	b_1	b_2	b_3	$-a_4$	$-a_5$	$-a_6$	c_1	c_2	c_3	$-b_4$	$-b_5$	$-b_6$
a_5	a_6	$-c_1$	$-c_2$	$-c_3$	b_4	b_5	b_6	$-a_1$	$-a_2$	$-a_3$	c_4	c_5	c_6	$-b_1$	$-b_2$	$-b_3$	a_4

λ_1 : 8 slots

$\lambda_2 = 200^\circ$

A			−C			B			−A			C			B		
a_1	a_2	a_3	$-c_4$	$-c_5$	$-c_6$	b_1	b_2	b_3	$-a_4$	$-a_5$	$-a_6$	c_1	c_2	c_3	$-b_4$	$-b_5$	$-b_6$
$-b_3$	a_4	a_5	a_6	$-c_1$	$-c_2$	$-c_3$	b_4	b_5	b_6	$-a_1$	$-a_2$	$-a_3$	c_4	c_5	c_6	$-b_1$	$-b_2$

$\lambda_2 = 10$ slots

g) n : harmonic order

$$k_{p\text{-}n} = \sin\left(n\frac{\lambda}{2}\right) = 0 \Rightarrow \sin\left(n\frac{4\pi}{9}\right) = 0 \Rightarrow n\frac{4\pi}{9} = k\pi \quad (k \text{ is integer})$$

$$\Rightarrow \boxed{n=9}$$

$\lambda_1 = 160^\circ$ is more advantageous since the winding resistance and copper mass (hence copper loss) is smaller.

h) Balanced 3ϕ currents

$i_a(t) = 5\cos(2\pi f t)$

$i_b(t) = 5\cos(2\pi f t - 2\pi/3)$

$i_c(t) = 5\cos(2\pi f t + 2\pi/3)$ where $f = 50$ Hz and phase sequence is A-B-C.

$$\mathcal{F}_s = \left(\frac{3}{2}\right)\left(\frac{4}{\pi}\right)\left(N_{ph\text{-}s} \times \frac{I}{2}\right)k_{ws} = \boxed{812.16 \text{ Amps}}$$

$$B_{gs} = \mathcal{F}_s \frac{\mu_0}{g} = \boxed{0.51 \text{ Tesla}}$$

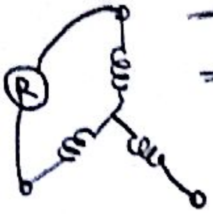
$$\phi_{ps} = \frac{4rl}{p} B_{gs} = \boxed{30.6 \text{ mWb}}$$

$$E = 4.44\, N_{phr}\, f\, \phi_p\, k_{wr} = 4.44 \times (6\times 10) \times 50 \times 30.6 \times 10^{-3} \times 0.644$$

$$\boxed{E = 262.5 \text{ Volts}}$$

Q.2)

a)



$\rightarrow R_{dc} = 0.06\,\Omega$

$\rightarrow R_{1dc} = 0.03\,\Omega$

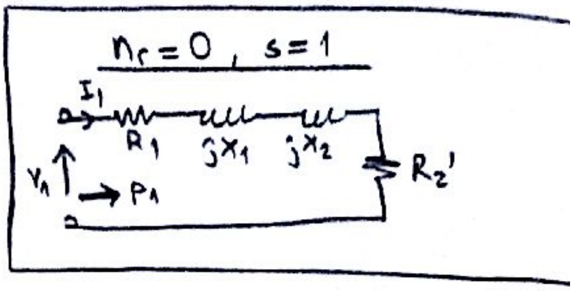
$R_1 = R_{1dc} \times 1.1 = 0.033\,\Omega$ //

$\downarrow$ skin effect @ 50 Hz

Locked Rotor Test

$P_1 = (R_1 + R_2')I_1^2 = 4\,kW \Rightarrow R_2' = 0.031\,\Omega$ /

$X_1 = X_2' = \frac{1}{2}\sqrt{\left(\frac{V_1}{I_1}\right)^2 - (R_1+R_2')^2} = 0.06\,\Omega$ //



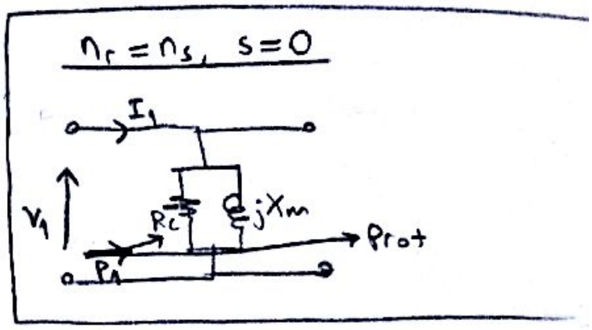
No Load Test

$P_1 = P_c + P_{rot} \rightarrow P_c = 3\,kW$

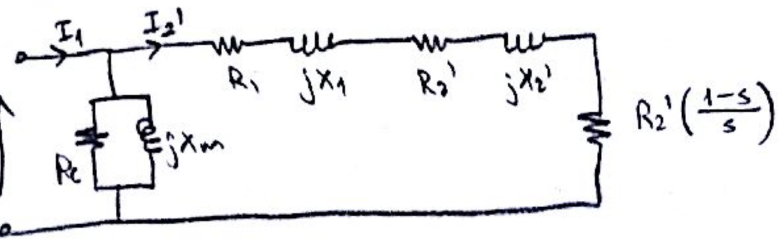
$P_1 \downarrow 4.5\,kW$ $\quad P_{rot} \downarrow 4.5/3 = 1.5\,kW$

$R_c = \frac{V_1^2}{P_c} = 53.3\,\Omega$ //

$X_m = \frac{1}{\sqrt{\left(\frac{I_1}{V_1}\right)^2 - \frac{1}{R_c^2}}} \approx 100\,\Omega$ //



Stator referred equivalent circuit (shunt branch moved to stator terminals)



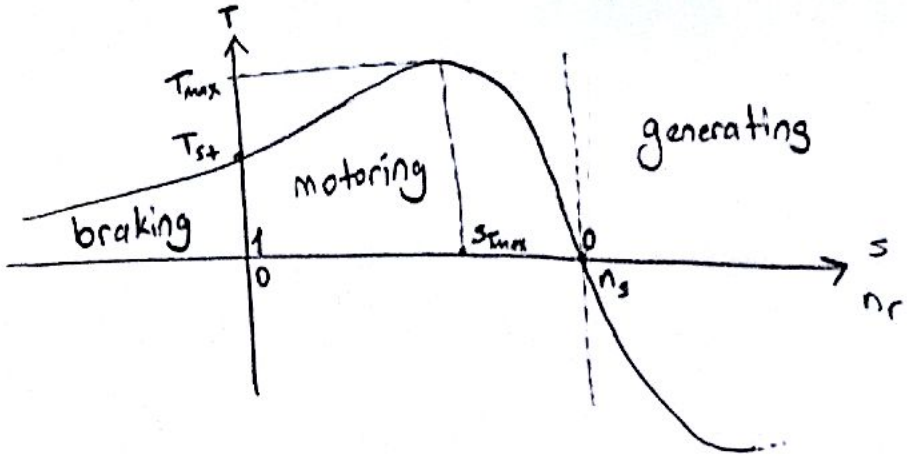
b) $n_s = \frac{120f}{P} = 1500\,rpm$

c) $s_{T_{max}} = \pm\frac{R_2'}{\sqrt{R_1^2 + (X_1+X_2')^2}} = 0.25$

d) $T_{max} = \frac{3V_1^2}{2\omega_s}\,\frac{1}{R_1 + \sqrt{R_1^2 + (X_1+X_2')^2}} = 9750\,Nm$

e) $T_{st} = \frac{3V_1^2}{\omega_s}\,\frac{R_2'}{(R_1+R_2')^2 + (X_1+X_2')^2} = 5122\,Nm$

f)

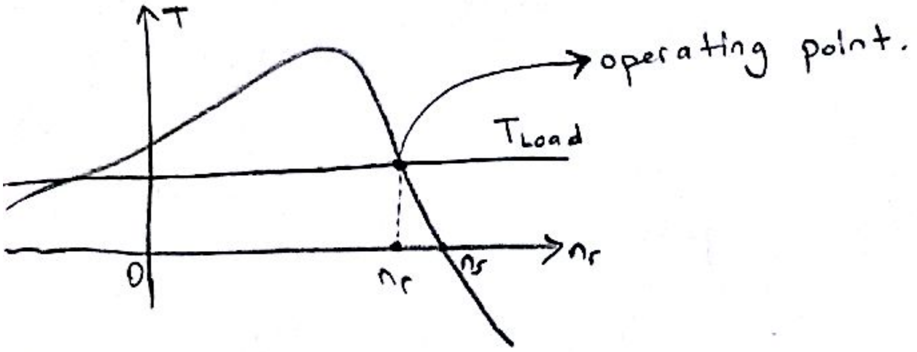


g)
- Increases
- No change
- Increases
- Increases

h) $\underline{s = 0.05}$

$$T = \frac{3V_1^2}{w_s} \cdot \frac{1}{\left(R_1 + \frac{R_2'}{s}\right)^2 + \left(x_1 + x_2'\right)^2} \cdot \frac{R_2'}{s} \approx 4300\,\text{Nm}$$

$$n_r = n_s(1-s) = 1425 \text{ rpm.}$$



i) $\omega_r = n_r \times \left(\frac{2\pi}{60}\right) = 149.2\ \text{rad/sec} \Rightarrow P_m = T_m \times \omega_r = 641.6\ \text{kW}$ /

$P_{fw} = 4.5\ \text{kW}$ / (given) $\Rightarrow P_m = P_{out} + P_{fw} \Rightarrow P_{out} = 637.1\ \text{kW}$ //

$P_m = 3(I_2')^2 R_2' \left(\frac{1-s}{s}\right) \Rightarrow (I_2')^2 = \frac{P_m}{3R_2'} \cdot \left(\frac{s}{1-s}\right) \Rightarrow I_2' = 602.6\ A$

$P_{cur} = 3(I_2')^2 R_2' = 33.77\ \text{kW}$ // $\quad P_{cus} = 3(I_2')^2 R_1 = 35.95\ \text{kW}$ //

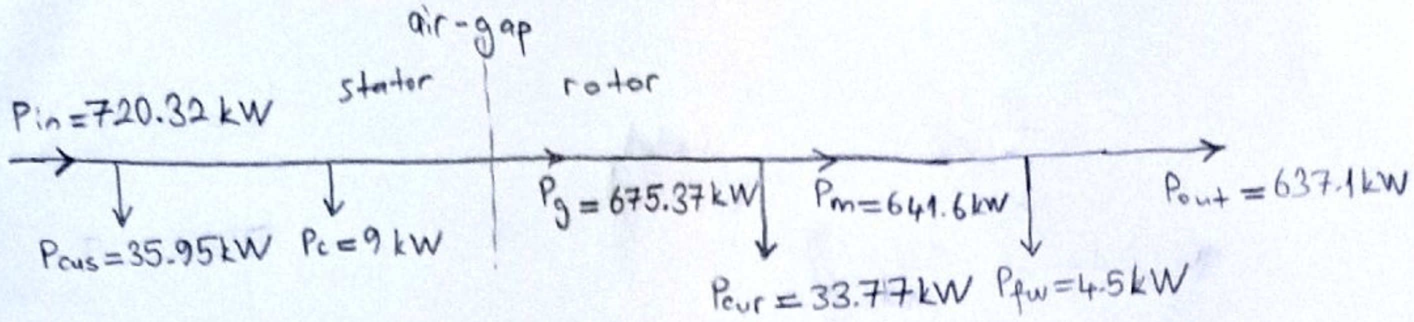
$P_g = P_m + P_{cur} = 675.37\ \text{kW}$

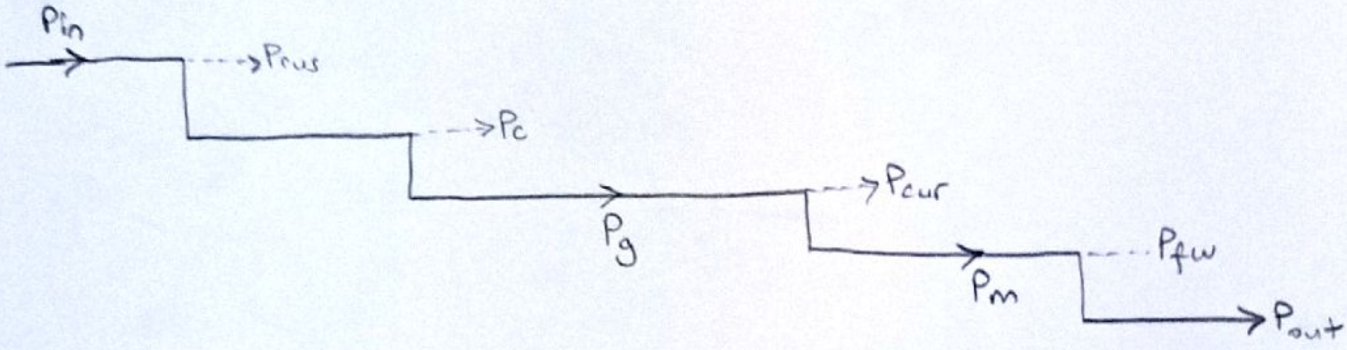
$P_c = 3\frac{V_1^2}{R_c} = 9\ \text{kW}$ //

$\Rightarrow P_{in} = P_c + P_{cus} + P_g = 720.32\ \text{kW}$

$\eta = \frac{P_{out}}{P_{in}} \times 100 = 88.5\%$ //

i)


air-gap
stator
rotor
Pin = 720.32 kW
Pcus = 35.95 kW
Pc = 9 kW
Pg = 675.37 kW
Pm = 641.6 kW
Pcur = 33.77 kW
Pfw = 4.5 kW
Pout = 637.1 kW


Pin
Pcus
Pc
Pg
Pcur
Pm
Pfw
Pout