25.11.2016

EE361 – HW#2
Solution

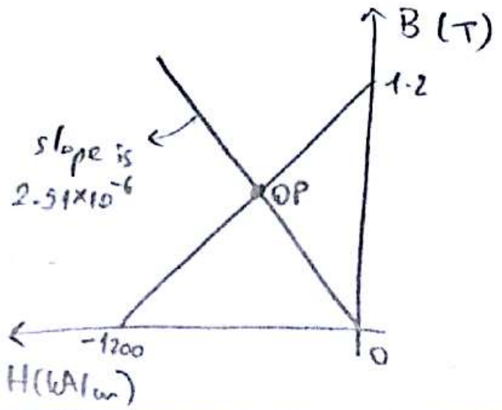
Q.1)
a) Ampere's Law: $H_m l_m + H_g l_g = 0$ .... (1)

Flux: $\phi_m = \phi_g \Rightarrow B_m A_m = B_g A_g$

$A_m = A_g \Rightarrow B_m = B_g$ .... (2)

Combine (1) and (2) $\Rightarrow$ $\boxed{B_m = -\frac{l_m}{l_g}\mu_0 H_m}$

Substitute: $\boxed{B_m = -2.51 \times 10^{-6} H_m}$ → Load line equation.



b) Magnet characteristics: $B_m = B_r + m H_m$ where m is the slope.

$B_r = 1.2$ T

Use $H_m = -1200$ kA/m point. $\Rightarrow 0 = 1.2 - m \times 1200 \times 10^3$

$\Rightarrow \boxed{m = 10^{-6}} \Rightarrow$ Magnet char: $\boxed{B_m = 1.2 + 10^{-6} H_m}$

To find the operating point (OP), use the intersection of the two lines:

$-2.51 \times 10^{-6} H_m = 1.2 + 10^{-6} H_m \Rightarrow H_m = -342$ kA/m

$\Rightarrow B_m = -2.51 \times 10^{-6} H_m = 0.858$ T.

$\boxed{OP = (0.858\ T, -342\ kA/m)}$

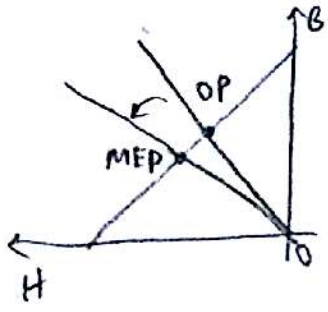
c) $EP = B_m \times H_m = (1.2 + 10^{-6} H_m) \times H_m = 1.2 H_m + 10^{-6} H_m^2$

$\frac{\partial (EP)}{\partial (H_m)} = 0$ @ MEP $\Rightarrow 1.2 + 2 \times 10^{-6} H_m = 0 \Rightarrow$ $\left.\begin{array}{l} H_m = -600 \text{ kA/m} \\ B_m = 0.6 \text{ T} \end{array}\right\}$ @ MEP operating point.

$MEP = (0.6T) \times (600\ kA/m) = \boxed{360\ kJ/m^3}$

d) OP : B = 0.858 T

MEP OP : B = 0.6 T



As shown on the figure, the load line should be shifted down such that it intersects with the magnet characteristics at MEP operating point.

$$B_m = -\frac{l_m}{l_g}\mu_0 H_m$$

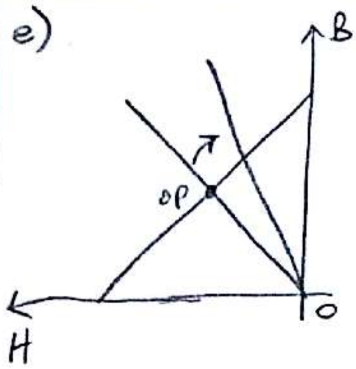
To do so, by looking at the equation above;

→ l_m can be decreased

→ l_g can be increased

---

e)



By looking at the equation in part (d), increasing l_m will shift the load line up as shown on the left. The operating point will move further away from MEP.

---

f) Since the core is infinitely permeable, the reluctance will be zero. The magnetic circuit will be as if it is short circuited.

$B_m = B_r = 1.2\ T$ //

$H_m = 0$ //

---

g) As the two magnets are in series and there is no air gap, the same phenomena will occur as in part (f).

$B_m = 1.2\ T$ //

$H_m = 0$ //

9.2) a) $N_1 = 60$ turns
$N_2 = 120$ turns

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{1}{2} \Rightarrow \text{step-up transformer.}$$

b) Since the secondary terminals are open circuited, the excitation current will only be composed of magnetizing current. Operating point is the saturation point: $H = 200\ A/m$.

$$N_1 I_1 = H\ell \rightarrow I_1 = \frac{200 \times 0.6}{60} = 2\ \text{Amps}$$

For AC excitation, if the transformer will not be saturated, the current found above should be the peak current.

$$I_{rms} = \frac{2}{\sqrt{2}} = \sqrt{2}\ \text{Amps}$$

c) At saturation point, $B = 1$ Tesla.

$$E_2 = 4.44\, N_2\, f\, B\, A$$

$$E_2 = 4.44 \times 120 \times 50 \times 1 \times 86 \times 10^{-4}$$

$$\boxed{E_2 = 230\ V_{rms}}$$

$$e_2(t) = 230\sqrt{2} \cdot \sin(2\pi 50 t)\ V$$

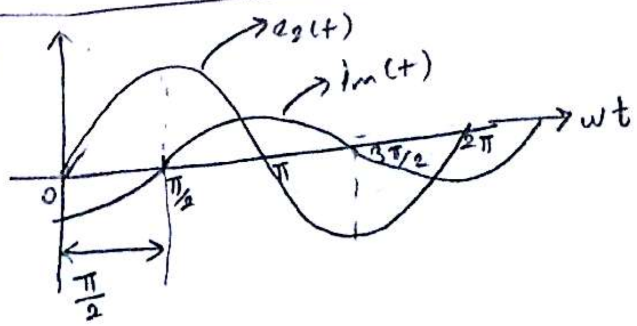
d) Alternatives

→ Increase number of turns (N_2) : More copper loss, more cost (copper)
→ Increase frequency (f) : Hard to implement since system frequency is fixed.
→ Increase core area : More cost (core), more core loss
→ Use another material with higher B_{sat} : Cost.

e) $I_m = \sqrt{2}\ A_{rms}$ (already found)

$$i_m(t) = 2 \cdot \sin(2\pi f t - \pi/2)$$

f)



The phase difference is 90°. It is due to Faraday's Law : $e = N\frac{d\phi}{dt}$

The magnetization demand of a transformer is purely inductive. Flux (also MMF) and voltage are 90° apart because of the derivative.

This situation can also be interpreted as the reactive power demand of the transformer.

9) No hysteresis loop $\Rightarrow$ No hysteresis loss.

It does not mean the core loss is zero. There may still be eddy current loss.

Q.3) Part I

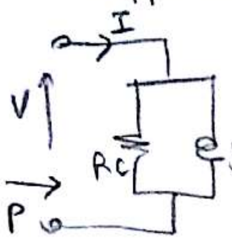
a) $I_{rated} = \frac{P_{rated}}{V_{2rated}} = \frac{10\ kVA}{440\ V} = 22.7\ A$ (secondary)

b) Open circuit test

→ There is no current on the secondary

→ The impedance of parallel branch is assumed to be much higher.

→ Copper loss resistances and leakage reactances can be neglected.



$I = 1.39\,A$

$V = 220\,V$

$P = 230\,W$

$R_c = \frac{V^2}{P} = 210\,\Omega$ //

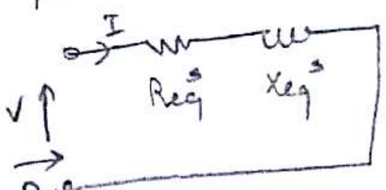
$S = VI = 305.8\ VA$

$Q = \sqrt{S^2 - P^2} = 201.5\ VAr$

$X_m = \frac{V^2}{Q} = 240\,\Omega$ //

Short circuit test

→ As the primary terminals are short circuited, the impedance of the parallel branch can be neglected.



The test is applied from secondary. The results will be "referred results".

$I = 22.7\,A$

$V = 17.7\,V$

$P = 257\,W$

$R_{eq}^{s} = \frac{P}{I^2} = 0.5\,\Omega$

$|Z_{eq}^{s}| = \frac{V}{I} = 0.78\,\Omega$

$X_{eq}^{s} = \sqrt{|Z_{eq}^{s}|^2 - R_{eq}^{s\,2}} = 0.6\,\Omega$

On the primary side:

$R_{eq} = (N_1/N_2)^2 R_{eq}^{s} = 0.125\,\Omega$

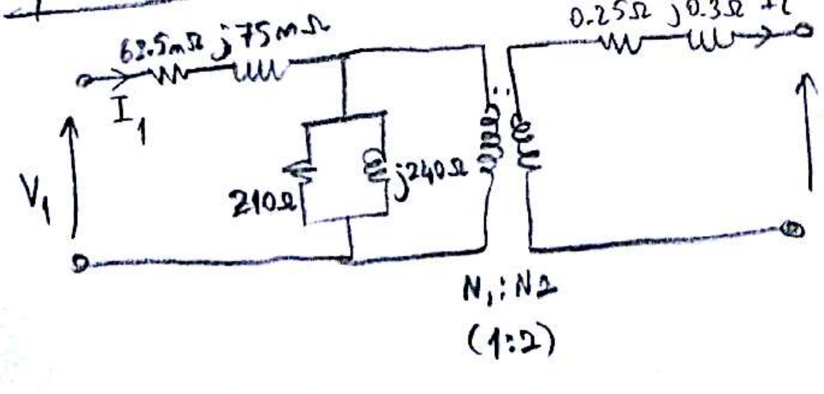
$X_{eq} = (N_1/N_2)^2 X_{eq}^{s} = 0.15\,\Omega$

Assume that $R_1 = R_2'$ and $X_1 = X_2'$

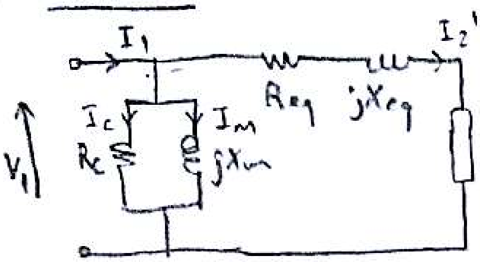
$\Rightarrow R_1 = 62.5\,m\Omega$, $R_2 = 0.25\,\Omega$

$X_1 = 75\,m\Omega$, $X_2 = 0.3\,\Omega$

Equivalent circuit:



Part II



Load: 10 kVA @ 1.0 pf $\Rightarrow$ 10 kW $= P_{out}$

$V_2 = 440\,V \Rightarrow V_2' = 220\,V$ (take as the reference phasor)

$I_2' = \dfrac{P_{out}}{V_2'} = 45.5\,A$ (in-phase with the secondary voltage, V_2')

$\vec{V}_1 = \vec{V}_2' + \vec{I}_2' \vec{Z}_{eq}$ where $\vec{Z}_{eq} = R_{eq} + jX_{eq} = 0.125 + j\,0.15$

$\Rightarrow \vec{V}_1 = 220 + 5.69 + j6.83 = 225.69 + j\,6.83 = 225.8 \angle 1.7^\circ$

$\Rightarrow \boxed{V_1 = 225.8\,V}$

$\vec{I}_c = \dfrac{\vec{V}_1}{R_c} = \dfrac{225.8 \angle 1.7^\circ}{210} = 1.08 \angle 1.7^\circ\,A$

$\vec{I}_m = \dfrac{\vec{V}_1}{jX_m} = \dfrac{225.8 \angle 1.7^\circ}{240 \angle 90^\circ} = 0.94 \angle -88.3^\circ\,A$

$\vec{I}_2' = I_2' = 45.5 \angle 0^\circ\,A$

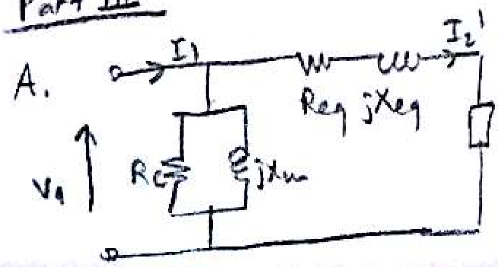
$\Rightarrow \vec{I}_1 = \vec{I}_2' + \vec{I}_c + \vec{I}_m = 45.5 + 1.08 + j0.03 + 0.03 - j0.94 = 46.61 - j0.97$

$\Rightarrow \vec{I}_1 = 46.62 \angle -1.2^\circ$

$\Rightarrow \boxed{I_1 = 46.62\,A}$

Part III

A.



Load: 8 kVA @ 1.0 pf $\Rightarrow$ 8 kW $= P_{out}$

$V_2 = 440V \Rightarrow V_2' = 220V$

$I_2' = \dfrac{P_{out}}{V_2'} = 36.4\,A$

$\vec{V}_1 = \vec{V}_2' + \vec{I}_2' \vec{Z}_{eq}$

$\vec{V}_1 = 220 + (36.4 \angle 0^\circ)(0.195 \angle 50^\circ) = 220 + 4.56 + j5.44 = 224.6 \angle 1.39^\circ$

$\Rightarrow \boxed{V_1 = 224.6\,V}$

$P_{in} = P_{out} + P_{loss}$, $P_{loss} = \dfrac{V_1^2}{R_c} + (I_2')^2 R_{eq} = \dfrac{(224.6)^2}{210} + (36.4)^2 \times 0.125 = 406\,W$

$\Rightarrow \boxed{P_{in} = 8406\,W}$

Part III

B. Load: 8 kW, 0.8 pf lagging. (Inductive Load)

$\vec{V_2}' = 220V$ (reference phasor)

$I_2' = \frac{S_2}{V_2'} = \frac{10 \text{ kVA}}{220V} = 45.45 A \Rightarrow \vec{I_2}' = 45.45 \angle -37^\circ$ → due to lagging pf

$\vec{V_1} = \vec{V_2}' + \vec{I_2}' \vec{Z_{eq}}$

$\vec{V_1} = 220 + (45.45 \angle -37^\circ)(0.195 \angle 50^\circ) = 220 + 8.19 \angle 13^\circ = 228.7 + j2 V$

$\Rightarrow \vec{V_1} = 228.7 \angle 0.5^\circ \Rightarrow \boxed{V_1 \cong 229 V}$

$P_{in} = P_{out} + P_{loss}, \quad P_{loss} = \frac{V_1^2}{R_c} + (I_2')^2 R_{eq} = \frac{(229)^2}{210} + (45.45)^2 \times 0.125$

$\Rightarrow P_{loss} = 508 W$

$\Rightarrow \boxed{P_{in} = 8508 W}$

C. Load: 8 kW, 0.8 pf leading (capacitive load)

$\vec{V_2}' = 220V$ (reference)

$I_2' = \frac{S_2}{V_2'} = 45.45A \rightarrow \vec{I_2}' = 45.45 \angle 37^\circ$ → due to leading pf.

$\vec{V_1} = \vec{V_2} + \vec{I_2}' \vec{Z_{eq}}$

$\vec{V_1} = 220 + (45.45 \angle 37^\circ)(0.195 \angle 50^\circ) = 220 + 8.9 \angle 87^\circ = 220.5 + j8.9 V$

$\Rightarrow \vec{V_1} = 220.7 \angle 2.3^\circ \Rightarrow \boxed{V_1 \cong 221 V}$

$P_{loss} = \frac{V_1^2}{R_c} + (I_2')^2 R_{eq} = \frac{(221)^2}{210} + (45.45)^2 \times 0.125$

$\Rightarrow P_{loss} = 491 W$

$\Rightarrow \boxed{P_{in} = 8491 W}$

D. Regulation

1) $reg = \frac{224.6 - 220}{220} \times 100 = 2\%$

2) $reg = \frac{229 - 220}{220} \times 100 = 4\%$

3) $reg = \frac{221 - 220}{220} \times 100 = 0.5\%$

Efficiency

1) $\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{8000}{8406} \times 100 = 95\%$

2) $\eta = \frac{8000}{8508} \times 100 = 94\%$

3) $\eta = \frac{8000}{8491} \times 100 = 94.2\%$

Comment:

→ With capacitive load, regulation is best. It does not have to be negative!
→ With inductive load, regulation is worst.

→ Efficiency is maximum for unity pf because the current required to supply 8 kW is minimum under unity pf condition. I^2R (copper) losses are minimum.
→ In the capacitive load, efficiency is better than inductive load since V_1 is smaller and so that core loss is smaller $\left(\frac{V_1^2}{R}\right)$.