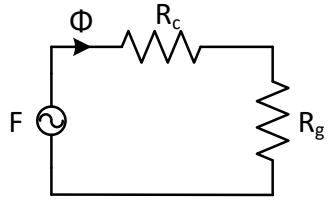
**Homework 1 – Solution**

**Q.1.**
F: Magnetomotive force
$R_c$: Core reluctance
$R_g$: Air gap reluctance
Φ: Flux (same everywhere)




**a)** $R_c$ is zero since the core is infinitely permeable.
Loop equation: $N\ I = H\ l$

$$H_g = \frac{N\ I}{g}$$

$$B_g = \frac{N\ I}{g}\ \mu_0$$

$$I = \frac{B_g\ g}{N\ \mu_0} = 2.5\ Amps$$

**b)** $R_c$ is constant since the core is linear.
Loop equation: $N\ I = H_c\ l_c + H_g\ g$

$$l_c = 2l_1 + 2l_2$$

$$B_g = H_g\ \mu_0$$

$$B_c = H_c\ \mu_0\ \mu_r$$

Since flux is same everywhere and area is same everywhere, $B_g = B_c = B$

$$I = \frac{B\ l_c}{N\ \mu_r\ \mu_0} + \frac{B\ g}{N\ \mu_0} = 2.95\ Amps$$

**c)** The core is non-linear with the following B-H characteristics:

Since flux is same everywhere and area is same everywhere, $B_g = B_c = B$

$$H_g = {B_g}/{\mu_0}$$

$H_c$ is determined by the B-H characteristics (as nearly 60 H/m) and marked on the figure.

$$I = \frac{H_c\ l_c}{N} + \frac{H_g\ g}{N} = 2.92\ Amps$$

**d)** B = 1.25 Tesla.
Infinitely permeable core:

$$I = \frac{B_g\ g}{N\ \mu_0} = 4.97\ Amps$$

Linear core:

$$I = \frac{B\ l_c}{N\ \mu_r\ \mu_0} + \frac{B\ g}{N\ \mu_0} = 5.84\ Amps$$

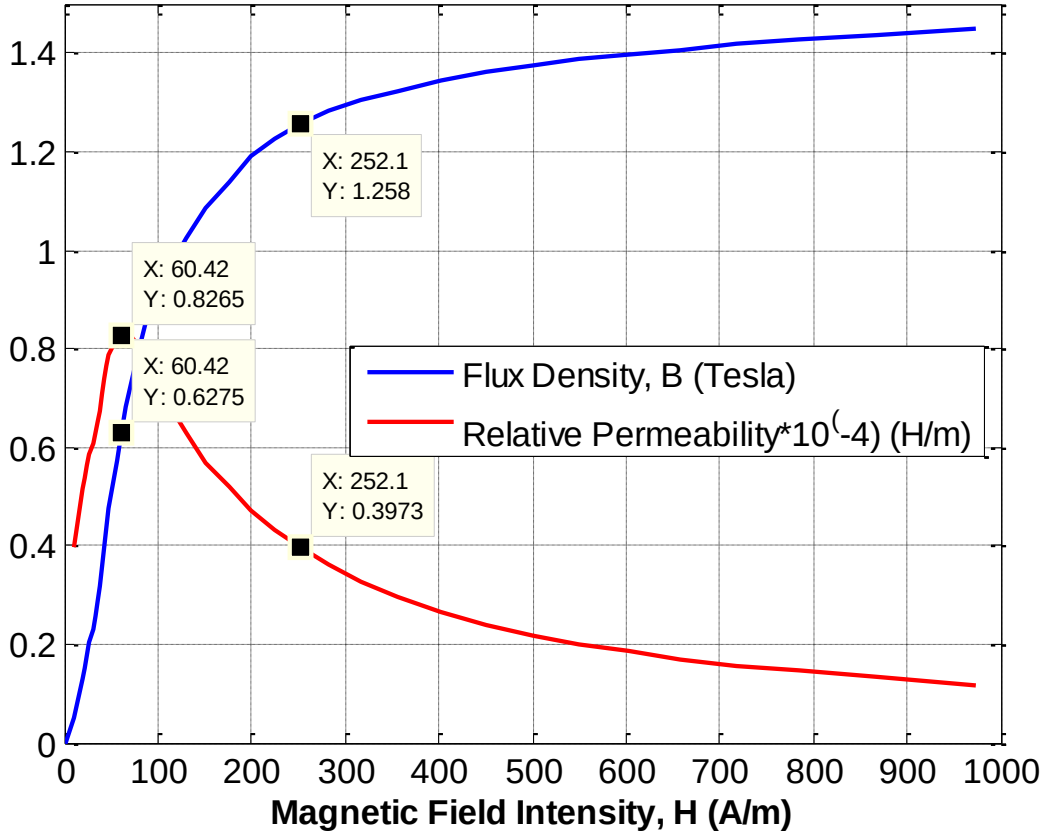
Core with realistic B-H char:

$H_g = {}^{B_g}/_{\mu_0}$

$H_c$ is determined by the B-H characteristics (as nearly 252 H/m) and marked on the figure.

$I = \frac{H_c\, l_c}{N} + \frac{H_g\, g}{N} = 6.72\ Amps$

**e)**




**f)** (a-c comparison) When the reluctances of the steel core and air gap are comparable, it would not be practical to treat the core as infinitely permeable. In this case, neglecting the core reluctance yielded us a loss of 0.5A calculation error (16 %).

(b-c comparison) When the operating point is known, linearization of the core around that point may yield accurate results. In our case (0.63 Tesla), the relative permeability holds and the results are nearly same.

The utilization of realistic characteristics yields most accurate results.

(part d) When the operating point is changed, the linearization does not hold any more. This is shown by the comparison of result with linear core and realistic core (13 % error). It may also be observed by the variation of relative permeability by the figure. The relative permeability is around 8000 for 0.63 Tesla, however it is half as much (around 4000) for 1.25 Tesla.

**Q.2.**

**a)**

$L = \frac{d\lambda}{di}$

where λ is the flux linkage.

$L = N\frac{d\Phi}{di}$

$L = NA\frac{dB}{di}$ .... (1)

$NI = Hl_c$

$NI = \frac{Bl_c}{\mu_r\,\mu_0}$

$B = \frac{NI\mu_r\,\mu_0}{l_c}$ .... (2)

Put (2) into the inductance equation (1):

$L = NA\frac{N\,\mu_r\,\mu_0}{l_c}$

$L = \frac{N^2 A\,\mu_r\,\mu_0}{2l_1 + 2l_2}$

**b)**

$N = \sqrt{\frac{L(2l_1 + 2l_2)}{A\,\mu_r\,\mu_0}}$

$N = 23.84$

24 turns should be wounded.

**c)**

$B_{sat} = 0.8\,Tesla$

$I_{max} = \frac{B_{sat}(2l_1 + 2l_2)}{N\,\mu_r\,\mu_0}$

$I_{max} = 9.5\,Amps$

**d)**

Method 1: Energy density:

$w = \frac{{B_{sat}}^2}{2\,\mu_r\,\mu_0}$

$w = 182\,J/m^3$

$W = v\,w$ where v is volume.

$v = (2l_1 + 2l_2)A = 5x10^{-4}\,m^3$

$W = 90\,mJ$

Method 2: Stored energy:

$W = \frac{1}{2}L{I_{max}}^2$

$W = 90\,mJ$

**e)**

$$L = \frac{d\lambda}{di}$$

where λ is the flux linkage.

$$L = N\frac{d\Phi}{di}$$

$$L = NA\frac{dB}{di} \quad \text{.... (1)}$$

$$NI = H_c l_c + H_g g$$

$$NI = \frac{B l_c}{\mu_r \, \mu_0}$$

$$B = \frac{NI\mu_r \, \mu_0}{l_c + g\mu_r} \quad \text{.... (2)}$$

Put (2) into the inductance equation (1):

$$L = \frac{N^2 A \, \mu_r \, \mu_0}{2l_1 + 2l_2 + g\mu_r}$$

where g = 1 mm.

**f)**

$$N = \sqrt{\frac{L(2l_1 + 2l_2 + g\mu_r)}{A \, \mu_r \, \mu_0}}$$

$$N = 73.09$$

73 turns should be wounded.

**g)**

$$B_{sat} = 0.8 \, Tesla$$

$$I_{max} = \frac{B_{sat}(2l_1 + 2l_2 + g\mu_r)}{N \, \mu_r \, \mu_0}$$

$$I_{max} = 30 \, Amps$$

**h)**

Stored energy:

$$W = \frac{1}{2} L {I_{max}}^2$$

$$W = 857 \, mJ$$

**i)** Introducing an air gap yielded a higher maximum current. Meaning of this is that the inductor can be operated for higher currents without saturating the core.
This situation can also be observed by the stored energy. Higher energies can be stored on the inductor with an air gap.
On the other hand, required turn number increased by gapping the core. This is a limitation for the design. The window area may not be sufficient to wound high number of turns. Moreover, cost (increased copper) and copper loss increase.