Time Complexity - 1

- → Time Complexity & Space Complexity
- → Asymptotic Analysis
- → Big O notation
- → TLE - Time Limit Exceeded

} Not Today next class

Today: How to calculate number of iterations ?

Quiz 1: Sum of first N natural no. =

$N * (N+1)/2$

Quiz 2: How many numbers are there in range [3,10] ?

[ ] → closed bracket / inclusive

( ) → open " / exclusive

[3,10] → 3, 4, 5, 6, 7, 8, 9, 10 → 8#

[3,8) → 3, 4, 5, 6, 7, ~~8~~ → 5#

[a, b] → b - a + 1

[a, b) → b - a

(a, b) → b - a - 1

if a > b ? invalid input

$(3, 8) \rightarrow 4, 5, 6, 7 \rightarrow 4\#$

Quiz 3 : How many times do we need to divide N by 2 to reduce it to 1 ?

if $\Rightarrow N = 10$

$10/2 \rightarrow 5/2 \rightarrow 2/2 \rightarrow 1$

3 times

$N \xrightarrow{/2} N/2 \xrightarrow{/2} N/4 \xrightarrow{/2} N/8 \rightarrow \dots \rightarrow 1$

assume no. of times need to divide $= K$

$N \xrightarrow{1} N/2 \xrightarrow{2^{nd}} N/2^2 \xrightarrow{3^{rd}} N/2^3 \xrightarrow{4^{th}} N/2^4 \rightarrow \dots \xrightarrow{K^{th}} N/2^K$

after K times, N becomes 1.

$\log_a a^b = b$

$N/2^K = 1 \Rightarrow N = 2^K$

$\log_2 N = \log_2 2^K$

$= K$

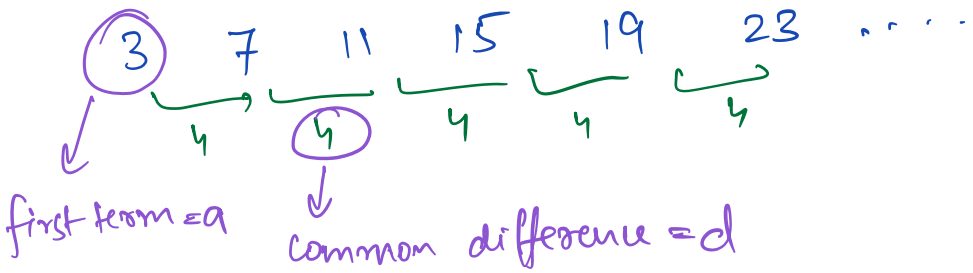
$K = \log_2 N$

Instead of 2 divide by 3 $\Rightarrow$

after K times $\Rightarrow$ $\boxed{N/3^K = 1}$

$\boxed{K = \log_3 N}$

Arithmetic Progressions



N^{th} term of this AP = ?

$a \quad a+d \quad a+2d \quad a+3d \quad \ldots\ldots \quad \boxed{a+(n-1)d}$

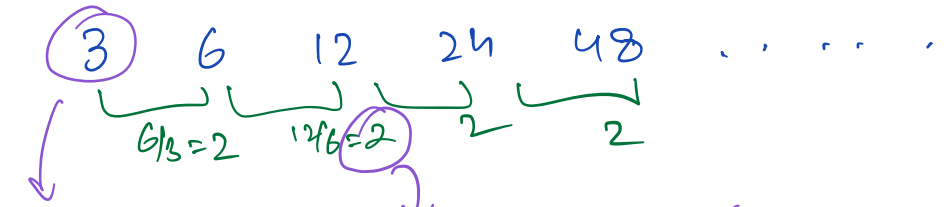
$1 \quad 2 \quad 3 \quad 4$ — n^{th} term

Sum of first N terms of an AP ?

$\boxed{\frac{n}{2}[2a+(n-1)d]}$

$\boxed{\frac{n}{2}(a+l) \qquad l = N^{th} \text{ term} = a+(n-1)d}$

Geometric Progressions



first term = a common ratio = r

N^{th} term of a GP?

$a, ar, ar^2, ar^3, \ldots\ldots$

$1 \quad 2 \quad 3 \quad 4$

ar^{n-1} — N^{th} term

Sum of first N terms of a GP?

$a + ar + ar^2 + \ldots + ar^{n-1}$

$$\boxed{a\left(\frac{r^n-1}{r-1}\right)} \quad r \neq 1$$

Quiz 4 : $\log_a a^x = ?$ → x

Quiz 5 :

```
for (i = 1; i <= N; ++i) {
    S = S + i;
}
```

$i = 1, 2, \ldots\ldots, N$

$i = [1, N]$

count = $N - 1 + 1 = N$

$O(N)$

```
void func(N, M) {
    for (i=1; i<=N; ++i) {
        if (i%2==0)
            print(i);
    }
    for (i=1; i<=M; ++i){
        if(i%2==1)
            print(i);
    }
}
```

N iterations

M iterations

$N/2 + M/2$

$N + M$ ✓

$N * M$

$O(N+M)$

$O(\max(N,M))$ ✓

Quiz 6:

```
for (i=0; i<=100; ++i) {
    s=s+i;
}
```

$i = 0, 1, 2, \ldots, 100$

$= [0, 100]$

count = $100-0+1 = 101$

$O(1)$

Quiz 7:

```
for (i=1; i*i<=N; ++i){
    s=s+i;
}
```

$\hookrightarrow i^2 <= N$

$i <= \sqrt{N}$

$i = [1, \sqrt{N}]$

count = $\sqrt{N}+1-1 = \sqrt{N}$

$O(\sqrt{N})$

Quiz 8

```
i=N;
while (i>1) {
    i=i/2;
}
```

$i = N, N/2, N/4, \ldots, 1$

count = # of times taken to divide N by 2 to reach 1.

$O(\log N)$

Based on above question:

$\boxed{count = \log_2 N}$

Quiz 9

```
for (i=0; i<=N; i=i*2) {
    s=s+i;
}
```

$i = 0 \xrightarrow{*2} 0 \xrightarrow{*2} 0 \rightarrow 0 \ldots \rightarrow 0$

```
for (i=1; i<N; i=i*2) {
    s=s+i;
}
```

$i = 1 \xrightarrow{*2} 2 \xrightarrow{*2} 4 \xrightarrow{*2} 8 \rightarrow \ldots \rightarrow N$

$1, 2, 4, 8, \ldots, N$

if this is k^{th} term

how many values = K

$a = 1, r = 2$

$ar^{K-1} = N$

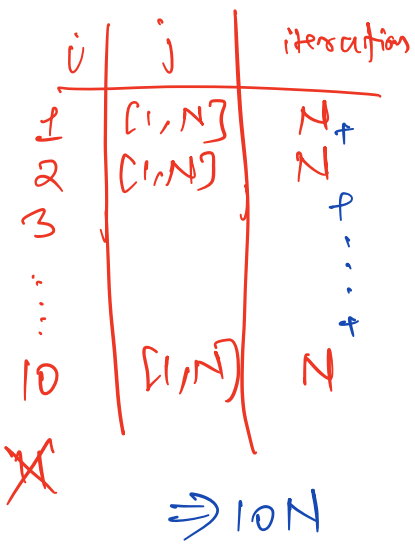
$2^{K-1} = N \Rightarrow K-1 = \log_2 N$

$O(\log N)$ $\quad K = \log_2 N + 1$

Quiz 10

```
for (i=1; i<=10; ++i) {
    for (j=1; j<=N; ++j) {
        s=s+i;
    }
}
```

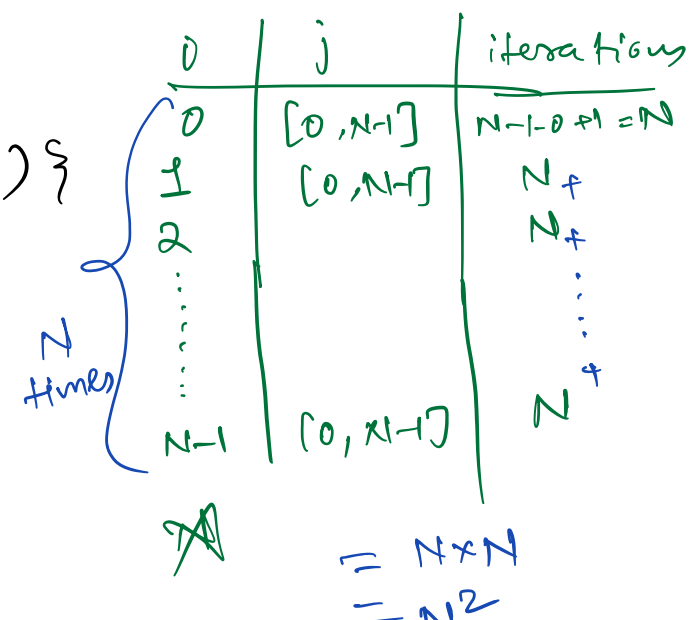
$O(N)$



Quiz 11

```
for (i=0; i<N; ++i) {
    for (j=0; j<N; ++j) {
        s=s+i;
    }
}
```

$O(N^2)$



Quiz 11

```
for (i=0 ; i<N ; ++i){
   for (j=0; j<=i ; ++j ) {
        s=s+i;
   }
}
```

i	j : [0, i]	iterations
0	[0,0]	0-0+1 = 1
1	[0,1]	2
2	[0,2]	3
⋮	⋮	⋮
N-1	[0,N-1]	N

$1 + 2 + 3 + \dots + N$

$\frac{N \times (N+1)}{2} \Rightarrow \frac{N^2 + N}{2}$

$O(N^2)$

Quiz 12

```
for ( i=1 ; i<=N; ++i){
   for (j=1 ; j<=N ; j=j*2) {
        s=s+i;
   }
}
```

↳ $j = 1, 2, 4, 8, \dots, N$

$= \log_2 N$

i	j	iterations
1	[1,N]	$\log_2 N$
2	[1,N]	$\log_2 N$
⋮	⋮	⋮
N		$\log_2 N$

$N \times \log_2 N$

$O(N \log N)$

Quiz 13

```
for ( i=1 ; i<=N; ++i ){
   for (j=1 ; j<=2^i ; ++j ){
        s=s+i;
   }
}
```

i	j : [1, 2^i]	iterations
1	[1, 2^1]	2^1
2	[1, 2^2]	2^2
3	[1, 2^3]	2^3
⋮		⋮

3

N	[1, 2^N]	2^N

$2^1 + 2^2 + 2^3 + \dots + 2^N$

terms = N

a = 2

r = 2

$sum = a\left(\frac{r^n - 1}{r-1}\right)$

$= 2\left(\frac{2^N - 1}{2-1}\right) = 2(2^N - 1)$

$O(2^N)$

Quiz 4

```
for (i=N; i>0; i=i/2) {
    for (j=1; j<=i; ++j) {
        S=S+i;
```

3 3

i	j : [1, i]	iteration
N	[1, N]	N
N/2	[1, N/2]	N/2
N/4	[1, N/4]	N/4
...	...	...
1	[1, 1]	1

$N + N/2 + N/4 + \dots + 1$

$N/2^0 + N/2^1 + N/2^2 + \dots + N/2^K$

1 2 3

$K+1^{th}$ term

$N/2^K = 1$

$K = \log_2 N$

total term = $\log_2 N + 1$

a = N

r = 1/2

$$a\left(\frac{r^n - 1}{r - 1}\right) \Rightarrow N\left(\frac{(1/2)^{\log_2 N + 1} - 1}{1/2 - 1}\right)$$

$$\Rightarrow N\left(\frac{1 - (1/2)^{\log_2 N + 1}}{1 - 1/2}\right)$$

$$(1/2)^{\log_2 N + 1}$$

$$1 / 2^{\log_2 N + 1}$$

$$1/N$$

$$\Rightarrow N\left(\frac{1 - 1/N}{1 - 1/2}\right)$$

$$\Rightarrow 2N\left(\frac{N - 1}{N}\right)$$

$$\Rightarrow 2N - 2 \approx N$$

$$O(N)$$

$$2^{\log_2 N} = N \; ?$$

$$\log_2 2^K = K$$

$$2^{\log_2 K} = K$$

$\log_2 N$ < sqrt(N)

$N = 2^{10}$

10 $\sqrt{2^{10}} = 2^5$

$10 < 32$

$N < N\log_2 N$

$N\log N < N\,\mathrm{sqrt}(N)$

$N \times 1 < (\log_2 N) N$

$N\,\mathrm{sqrt}(N) < N^2$

$N^2 < 2^N$

$$1 < \log_2 N < \sqrt{N} < N < N\log_2 N < N\sqrt{N} < N^2 < N^3 < 2^N$$

How to write Big O ?

what ? X

why ? X

1. Calculate iteration based on input
2. Neglect lower order terms.
3. Neglect constant coefficent term

$N^2 + N \Rightarrow O(N^2)$

$10N^2 + 2N\log N + 5 \Rightarrow O(N^2)$

Quizes

$f(N) = 4N^2 + 3N + 10^6$

$O(f(N)) = ?$

$O(N^2)$