Estimates of the summary AUC under three settings of  $(c_1, c_2)$   $(\tau_1^2, \tau_2^2) = (0.5, 0.5)$ 

Yi

2022-01-21

Table 1: Summary of the SAUC estimates under the true selective publication mechanism of  $(c_1, c_2) = (1/\sqrt{2}, 1/\sqrt{2})$ 

			S = 15	S=25	S = 50	
No.	Methods	True	Median (Q1, Q3)	Median (Q1, Q3)	Median (Q1, Q3)	
1	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = c_2)$ Proposed $(c_1 = 0)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	62.0	66.1 (45.2, 77.2) 65.8 (42.5, 76.6) 69.3 (51.1, 78.3) 67.5 (52.1, 76.7) 69.8 (53.6, 78.7) 63.3 (48.5, 73.0)	64.3 (50.7, 74.1) 64.4 (51.5, 74.3) 69.5 (59.8, 76.3) 68.4 (57.9, 74.4) 69.7 (61.1, 76.6) 62.2 (53.7, 69.6)	62.5 (52.0, 70.4) 63.3 (53.9, 70.6) 69.2 (63.5, 73.9) 66.8 (60.3, 71.9) 69.4 (64.1, 74.3) 62.0 (56.5, 67.1)	
2	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = c_2)$ Proposed $(c_1 = 0)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	70.2	72.4 (61.2, 78.9) 72.0 (59.7, 78.6) 74.1 (65.9, 79.8) 72.5 (63.6, 78.0) 74.4 (66.5, 79.9) 71.5 (63.5, 76.9)	71.0 (62.5, 76.3) 71.0 (62.7, 76.6) 73.4 (67.8, 77.9) 71.8 (64.6, 76.3) 73.7 (68.8, 78.1) 70.5 (64.7, 74.7)	71.1 (65.3, 75.1) 71.0 (65.3, 75.1) 73.6 (70.4, 76.7) 71.9 (67.2, 75.2) 74.1 (70.7, 77.0) 70.3 (66.8, 73.4)	
3	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = c_2)$ Proposed $(c_1 = 0)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	84.6	84.1 (77.9, 87.5) 83.8 (76.7, 87.3) 84.4 (78.6, 87.7) 84.8 (79.4, 87.9) 85.5 (80.3, 88.4) 83.6 (78.1, 86.4)	84.6 (79.7, 87.2) 84.6 (79.1, 87.1) 85.1 (81.0, 87.4) 85.5 (82.1, 87.5) 86.2 (83.2, 88.2) 84.2 (80.7, 86.5)	84.6 (81.5, 86.6) 84.7 (81.4, 86.7) 85.5 (83.3, 87.1) 86.1 (83.9, 87.5) 86.7 (84.8, 88.1) 84.5 (82.5, 86.0)	
4	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = c_2)$ Proposed $(c_1 = 0)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	86.4	85.9 (82.8, 87.9) 85.9 (82.3, 87.8) 86.2 (83.3, 88.0) 86.3 (83.2, 88.0) 87.0 (84.4, 88.6) 85.8 (83.6, 87.5)	86.3 (84.2, 87.8) 86.3 (84.2, 87.8) 86.5 (84.6, 88.0) 86.5 (84.8, 88.0) 87.3 (85.8, 88.5) 86.1 (84.5, 87.5)	86.4 (85.1, 87.4) 86.4 (85.1, 87.5) 86.8 (85.6, 87.7) 86.8 (85.6, 87.7) 87.6 (86.6, 88.4) 86.3 (85.2, 87.2)	
5	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = c_2)$ Proposed $(c_1 = 0)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	87.7	85.7 (79.5, 89.4) 85.4 (78.8, 89.3) 84.5 (77.2, 88.9) 86.0 (80.0, 89.6) 86.0 (79.1, 89.8) 86.1 (80.9, 89.2)	86.7 (82.5, 89.4) 86.5 (82.6, 89.4) 85.6 (80.8, 89.1) 86.6 (82.5, 89.6) 87.1 (83.0, 89.9) 87.2 (83.8, 89.3)	87.6 (84.9, 89.4) 87.4 (85.1, 89.3) 86.3 (83.5, 88.7) 87.2 (84.8, 89.5) 87.7 (85.4, 89.8) 87.5 (85.5, 89.1)	
6	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = c_2)$ Proposed $(c_1 = 0)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	83.5	82.6 (76.0, 87.2) 82.3 (76.0, 86.9) 81.0 (74.6, 86.3) 82.5 (76.7, 87.1) 82.5 (76.9, 87.1) 82.5 (76.8, 86.7)	83.8 (79.0, 87.4) 83.6 (79.3, 87.1) 82.2 (77.7, 86.1) 83.8 (79.5, 86.7) 83.9 (80.1, 87.1) 83.3 (79.6, 86.5)	84.0 (81.1, 86.8) 83.7 (80.8, 86.5) 82.7 (79.8, 85.5) 83.5 (80.6, 86.3) 84.1 (81.5, 86.6) 83.3 (80.8, 85.7)	

Median with 25th empirical quartile (Q1) and 75th empirical quartile (Q3) and convergence rate (CR) are reported. No. correspon

Table 2: Summary of the SAUC estimates under the true selective publication mechanism of  $(c_1, c_2) = (1, 0)$ 

			S = 15	S = 25	S = 50	S = 200
No.		True	Median (Q1, Q3)	Median (Q1, Q3)	Median (Q1, Q3)	Median (Q1, Q3)
1	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = 1)$ Proposed $(c_1 = c_2)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	62.0	63.2 (47.5, 74.7) 64.2 (47.1, 75.2) 62.2 (48.0, 74.5) 61.3 (48.4, 71.9) 66.3 (51.9, 75.6) 63.3 (48.5, 73.0)	63.1 (50.9, 71.5) 64.8 (51.3, 73.1) 62.8 (51.3, 72.0) 62.4 (51.7, 69.9) 66.6 (56.4, 73.9) 62.2 (53.7, 69.6)	62.9 (54.5, 69.2) 62.7 (54.9, 69.9) 63.9 (56.4, 70.1) 60.8 (53.8, 66.8) 66.1 (60.8, 71.4) 62.0 (56.5, 67.1)	63.7 (59.7, 67.0) 62.3 (57.9, 65.5) 65.2 (62.1, 68.0) 59.9 (55.4, 63.4) 66.0 (63.0, 68.6) 61.8 (59.1, 64.6)
2	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = 1)$ Proposed $(c_1 = c_2)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	70.2	69.4 (57.5, 77.1) 71.3 (59.0, 77.9) 69.3 (57.3, 77.1) 67.3 (55.9, 74.8) 71.6 (61.6, 78.0) 71.5 (63.5, 76.9)	69.9 (60.4, 75.3) 71.1 (63.0, 75.9) 69.5 (60.3, 75.0) 66.9 (58.0, 72.8) 71.7 (65.5, 76.0) 70.5 (64.7, 74.7)	70.1 (64.3, 74.0) 71.1 (65.9, 74.8) 70.4 (64.7, 74.5) 67.2 (59.7, 71.7) 71.8 (67.7, 75.2) 70.3 (66.8, 73.4)	70.5 (68.4, 72.3) 70.3 (68.3, 72.1) 71.2 (69.1, 72.9) 65.9 (61.7, 69.1) 71.7 (70.0, 73.3) 70.1 (68.5, 71.7)
3	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = 1)$ Proposed $(c_1 = c_2)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	84.6	82.9 (75.0, 86.8) 83.0 (75.4, 86.9) 82.9 (74.1, 87.0) 83.3 (77.2, 87.2) 84.3 (77.8, 87.7) 83.6 (78.1, 86.4)	84.1 (78.9, 86.7) 84.4 (80.2, 86.9) 84.1 (79.2, 86.8) 84.8 (80.9, 87.0) 85.8 (82.2, 87.7) 84.2 (80.7, 86.5)	84.2 (80.6, 86.4) 84.5 (81.2, 86.5) 84.5 (81.0, 86.6) 85.1 (82.3, 86.8) 86.0 (83.6, 87.7) 84.5 (82.5, 86.0)	84.5 (83.0, 85.5) 84.6 (83.4, 85.7) 84.9 (83.5, 86.0) 85.3 (84.1, 86.3) 86.3 (85.3, 87.1) 84.6 (83.7, 85.5)
4	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = 1)$ Proposed $(c_1 = c_2)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	86.4	85.5 (81.9, 87.7) 85.5 (82.2, 87.8) 85.5 (81.7, 87.6) 85.5 (81.6, 87.5) 86.4 (83.6, 88.3) 85.8 (83.6, 87.5)	85.9 (83.7, 87.6) 86.1 (83.9, 87.7) 86.1 (83.8, 87.8) 85.9 (83.7, 87.6) 87.0 (85.1, 88.4) 86.1 (84.5, 87.5)	86.2 (84.9, 87.4) 86.4 (85.0, 87.4) 86.5 (85.2, 87.7) 86.4 (84.8, 87.4) 87.4 (86.4, 88.3) 86.3 (85.2, 87.2)	86.4 (85.8, 86.9) 86.4 (85.9, 87.0) 86.7 (86.1, 87.3) 86.4 (85.6, 87.0) 87.6 (87.1, 88.0) 86.4 (85.9, 86.8)
5	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = 1)$ Proposed $(c_1 = c_2)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	87.7	86.7 (80.0, 90.1) 85.5 (77.8, 89.3) 86.7 (79.8, 90.0) 87.4 (81.5, 90.6) 86.9 (79.4, 90.3) 86.1 (80.9, 89.2)	88.0 (84.2, 90.3) 86.8 (82.3, 89.7) 88.4 (85.0, 90.5) 88.8 (85.1, 90.9) 88.5 (84.7, 90.8) 87.2 (83.8, 89.3)	88.6 (86.3, 90.0) 87.8 (85.1, 89.5) 89.0 (87.1, 90.3) 89.3 (87.0, 90.6) 89.2 (86.9, 90.7) 87.5 (85.5, 89.1)	88.2 (86.9, 89.3) 87.7 (86.5, 88.8) 89.2 (88.3, 89.8) 89.3 (88.3, 90.2) 89.3 (88.4, 90.1) 87.7 (86.7, 88.5)
6	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = 1)$ Proposed $(c_1 = c_2)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	83.5	83.2 (75.7, 88.2) 81.7 (74.6, 86.9) 83.7 (76.5, 88.3) 84.5 (78.0, 88.8) 83.0 (76.2, 88.1) 82.5 (76.8, 86.7)	84.9 (79.4, 88.4) 83.6 (78.4, 87.3) 85.1 (80.5, 88.4) 85.3 (80.5, 88.6) 84.9 (80.3, 88.2) 83.3 (79.6, 86.5)	84.7 (81.2, 87.6) 83.7 (80.5, 86.3) 85.5 (82.4, 87.9) 85.8 (82.8, 88.3) 85.4 (82.3, 87.7) 83.3 (80.8, 85.7)	84.0 (82.2, 85.6) 83.5 (82.0, 84.8) 85.2 (83.8, 86.4) 85.7 (84.3, 87.1) 85.3 (84.0, 86.5) 83.4 (82.2, 84.5)

Median with 25th empirical quartile (Q1) and 75th empirical quartile (Q3) and convergence rate (CR) are reported. No. corresponds to the scenario number. S denotes the number of population studies. True denotes the true value of the SAUC. Proposed  $(hatc_1, hatc_2)$ , Proposed  $(c_1 = 1)$ , and Proposed  $(c_1 = c_2)$  denote the proposed method that estimates  $(c_1, c_2)$ , correctly specifies  $(c_1, c_2) = (1, 0)$ , and misspecifies  $(c_1, c_2) = (1/sqrt2, 1/sqrt2)$ , respectively; Heckman-type denotes the method of Piao et al.; Reitsma<sub>O</sub> and Reitsma<sub>P</sub> denote the Reitsma model based on N published studies and S population studies, respectively. All the entries are multiplied by 100.

Table 3: Summary of the SAUC estimates under the true selective publication mechanism of  $(c_1, c_2) = (0, 1)$ 

			S = 15	S = 25	S = 50	S = 200
No.		True	Median (Q1, Q3)	Median (Q1, Q3)	Median (Q1, Q3)	Median (Q1, Q3)
1	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = 0)$ Proposed $(c_1 = c_2)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	62.0	56.3 (33.3, 72.5) 62.2 (41.4, 74.4) 54.8 (32.1, 72.3) 59.5 (39.4, 73.0) 62.7 (40.8, 75.2) 63.3 (48.5, 73.0)	56.0 (39.8, 68.9) 62.7 (51.5, 72.3) 54.3 (38.3, 68.6) 60.2 (46.0, 70.6) 63.6 (51.2, 73.1) 62.2 (53.7, 69.6)	55.9 (42.6, 65.3) 62.4 (54.7, 68.7) 55.4 (42.2, 64.1) 60.1 (49.7, 67.4) 62.7 (55.2, 69.3) 62.0 (56.5, 67.1)	58.2 (52.8, 63.2) 61.9 (57.8, 65.2) 54.7 (49.1, 60.1) 59.2 (54.5, 63.5) 62.3 (58.3, 65.8) 61.8 (59.1, 64.6)
2	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = 0)$ Proposed $(c_1 = c_2)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	70.2	69.6 (51.8, 78.2) 71.2 (59.2, 78.9) 69.1 (48.7, 78.0) 69.9 (55.7, 77.7) 72.2 (59.3, 79.3) 71.5 (63.5, 76.9)	67.3 (53.8, 75.1) 69.9 (61.2, 75.8) 66.7 (51.2, 75.2) 68.0 (57.8, 74.4) 70.6 (61.5, 76.5) 70.5 (64.7, 74.7)	68.0 (60.3, 73.2) 70.6 (65.8, 74.4) 67.9 (60.1, 73.2) 68.7 (62.3, 73.1) 71.3 (66.6, 75.1) 70.3 (66.8, 73.4)	68.7 (65.8, 71.1) 70.1 (68.1, 72.1) 67.1 (63.9, 70.1) 68.6 (65.3, 71.0) 70.8 (68.8, 72.8) 70.1 (68.5, 71.7)
3	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = 0)$ Proposed $(c_1 = c_2)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	84.6	82.1 (69.4, 86.6) 83.3 (74.0, 87.2) 81.6 (68.0, 86.4) 82.2 (71.2, 86.4) 83.5 (73.8, 87.6) 83.6 (78.1, 86.4)	82.9 (74.6, 86.6) 84.0 (78.3, 87.1) 82.5 (74.1, 86.5) 82.9 (75.3, 86.2) 84.5 (78.3, 87.4) 84.2 (80.7, 86.5)	83.5 (78.6, 86.0) 84.6 (81.4, 86.7) 82.9 (77.9, 85.8) 83.3 (78.8, 85.9) 85.0 (81.6, 87.0) 84.5 (82.5, 86.0)	84.1 (82.3, 85.4) 84.6 (83.3, 85.7) 83.1 (80.8, 84.8) 83.2 (81.1, 84.8) 84.9 (83.6, 86.1) 84.6 (83.7, 85.5)
4	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = 0)$ Proposed $(c_1 = c_2)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	86.4	85.0 (80.6, 87.5) 85.5 (82.0, 87.9) 85.0 (79.6, 87.5) 85.0 (80.1, 87.1) 85.9 (82.3, 88.1) 85.8 (83.6, 87.5)	85.5 (82.5, 87.3) 86.1 (83.7, 87.7) 85.3 (82.2, 87.3) 85.3 (81.7, 87.1) 86.4 (84.0, 88.0) 86.1 (84.5, 87.5)	85.7 (83.9, 87.1) 86.3 (84.8, 87.4) 85.5 (83.5, 87.1) 85.4 (83.1, 86.8) 86.7 (85.2, 87.8) 86.3 (85.2, 87.2)	86.1 (85.4, 86.8) 86.4 (85.8, 87.0) 85.8 (84.9, 86.5) 85.4 (84.5, 86.2) 86.8 (86.1, 87.3) 86.4 (85.9, 86.8)
5	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = 0)$ Proposed $(c_1 = c_2)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	87.7	82.4 (72.6, 87.4) 84.3 (76.4, 88.4) 82.3 (72.6, 87.2) 83.5 (75.7, 87.9) 83.2 (73.1, 87.7) 86.1 (80.9, 89.2)	83.8 (76.9, 87.7) 86.0 (80.1, 88.7) 84.3 (77.6, 87.6) 85.1 (80.3, 88.1) 84.8 (77.0, 88.2) 87.2 (83.8, 89.3)	86.1 (82.4, 88.3) 87.2 (84.5, 88.9) 86.3 (82.9, 88.3) 86.2 (83.6, 88.1) 86.7 (83.2, 88.7) 87.5 (85.5, 89.1)	87.1 (85.5, 88.3) 87.6 (86.4, 88.6) 87.5 (86.1, 88.5) 87.1 (85.9, 88.1) 87.6 (86.2, 88.6) 87.7 (86.7, 88.5)
6	Proposed $(\hat{c}_1, \hat{c}_2)$ Proposed $(c_1 = 0)$ Proposed $(c_1 = c_2)$ Heckman-type Reistma <sub>O</sub> Reistma <sub>P</sub>	83.5	78.7 (69.4, 84.9) 80.9 (73.1, 85.9) 78.7 (69.2, 84.9) 80.0 (72.4, 85.4) 79.0 (70.1, 85.1) 82.5 (76.8, 86.7)	80.9 (75.1, 85.4) 82.8 (77.7, 86.3) 81.1 (75.3, 85.3) 82.0 (76.9, 85.9) 81.4 (75.5, 85.7) 83.3 (79.6, 86.5)	82.2 (78.3, 85.4) 83.4 (80.3, 86.0) 82.6 (78.8, 85.4) 83.0 (79.7, 85.7) 82.8 (79.2, 85.6) 83.3 (80.8, 85.7)	82.4 (80.7, 84.1) 83.4 (82.0, 84.8) 82.8 (81.2, 84.3) 83.5 (81.6, 85.3) 82.8 (81.4, 84.3) 83.4 (82.2, 84.5)

Median with 25th empirical quartile (Q1) and 75th empirical quartile (Q3) and convergence rate (CR) are reported. No. corresponds to the scenario number. S denotes the number of the population studies. True denotes the true value of the SAUC. Proposed  $(hatc_1, hatc_2)$ , Proposed  $(c_1 = 1)$ , and Proposed  $(c_1 = c_2)$  denote the proposed method that estimates  $(c_1, c_2)$ , correctly specifies  $(c_1, c_2) = (0, 1)$ , and misspecifies  $(c_1, c_2) = (1/sqrt2, 1/sqrt2)$ , respectively; Heckman-type denotes the method of Piao et al.; Reitsma<sub>O</sub> and Reitsma<sub>P</sub> denote the Reitsma model based on N published studies and S population studies, respectively. All the entries are multiplied by 100.