

# Simulation Result for Manuscript

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$$\text{Biomarker1 : } \tilde{X} = \begin{cases} 0.7 + 0.1e & \text{if } \tilde{T} \leq 2 \\ 0.3 + 0.3e & \text{if } \tilde{T} > 2 \end{cases}$$

$$\text{Biomarker2 : } \tilde{X} = \begin{cases} 0.6 + 0.2e & \text{if } \tilde{T} \leq 2 \\ 0.4 + 0.3e & \text{if } \tilde{T} > 2 \end{cases}$$

Table 1: Scenarios of simulation

Censoring	Biomarker	Subjects	$\beta$	$\alpha_{0.7}$	$\alpha_{0.5}$	$\alpha_{0.3}$
$Exp(0.2)$	1	$U(50, 150)$	1	-2.431	-3.199	-3.961
		$U(50, 300)$	1	-3.277	-4.178	-5.083
	2	$U(50, 150)$	1	-0.518	-1.270	-2.020
		$U(50, 300)$	1	-0.878	-1.649	-2.427
$U(1, 4)$	1	$U(50, 150)$	1	-2.968	-3.713	-4.452
		$U(50, 300)$	1	-3.945	-4.899	-5.818
	2	$U(50, 150)$	1	-0.795	-1.547	-2.286
		$U(50, 300)$	1	-1.233	-2.020	-2.802

Table 2: Summary of the estimated SAUC when the true censoring is distributed as  $Exp(0.2)$ .

Patients	S	p	Methods	Biomarker1		Biomarker2	
				Median (Q1, Q3)	CR	Median (Q1, Q3)	CR
50-150	70	0.7	HZ <sub>P</sub>	0.00 (73.70, 75.65)	100	0.00 (61.55, 62.84)	100
			HZ <sub>O</sub>	1.09 (74.59, 76.84)	100	1.78 (63.33, 64.68)	100
			Prop <sub>(0.7)</sub>	0.64 (73.90, 76.84)	77.3	0.02 (61.34, 63.09)	64
			Prop <sub>(0.7)</sub> <sup>*</sup>	0.49 (73.77, 76.77)	82	0.05 (61.42, 63.06)	77
		0.5	HZ <sub>O</sub>	1.64 (74.80, 77.76)	100	2.80 (64.07, 65.82)	100
			Prop <sub>(0.5)</sub>	0.70 (73.51, 77.38)	76.3	0.01 (60.92, 64.01)	72
			Prop <sub>(0.5)</sub> <sup>*</sup>	0.21 (72.99, 76.77)	74.8	-0.45 (60.63, 62.97)	73.4
		0.3	HZ <sub>O</sub>	2.63 (75.29, 79.32)	100	4.18 (65.14, 67.40)	99.9
			Prop <sub>(0.3)</sub>	0.88 (72.86, 78.31)	67.5	2.51 (60.89, 66.78)	79.1
			Prop <sub>(0.3)</sub> <sup>*</sup>	0.08 (72.01, 77.14)	63.6	-0.99 (59.70, 63.19)	64.2
	200	0.7	HZ <sub>P</sub>	0.00 (73.92, 75.13)	100	0.00 (61.86, 62.58)	100
			HZ <sub>O</sub>	1.03 (74.83, 76.19)	100	1.82 (63.59, 64.43)	100
			Prop <sub>(0.7)</sub>	0.49 (73.98, 75.94)	72.2	-0.14 (61.54, 62.56)	63
			Prop <sub>(0.7)</sub> <sup>*</sup>	0.45 (73.95, 75.99)	86	-0.10 (61.57, 62.62)	75.4
		0.5	HZ <sub>O</sub>	1.64 (75.25, 76.93)	100	2.85 (64.59, 65.57)	100
			Prop <sub>(0.5)</sub>	0.50 (73.68, 76.26)	76.3	-0.46 (61.03, 62.50)	67.2
			Prop <sub>(0.5)</sub> <sup>*</sup>	0.33 (73.48, 75.99)	85.4	-0.55 (60.90, 62.27)	77.8
		0.3	HZ <sub>O</sub>	2.29 (75.79, 77.85)	100	4.08 (65.67, 66.96)	100
			Prop <sub>(0.3)</sub>	0.30 (72.95, 76.73)	76.8	-0.17 (60.31, 66.21)	73.1
			Prop <sub>(0.3)</sub> <sup>*</sup>	-0.31 (72.53, 75.87)	83.1	-1.20 (59.88, 62.04)	73.1

Patients denote the range of the number of patients.  $S$  denotes the number of the population studies.  $p$  denotes the approximate proportion of the published studies among the population. Median with 25th and 75th empirical quartiles (Q1, Q3) of the SAUC at  $t = 2$  are reported. CR denotes the proportion of successfully converged estimates among 1000 repetition. HZ<sub>P</sub> denotes the HZ model using the population studies; HZ<sub>O</sub> denotes the HZ model using only the corresponding numbers of published studies; Prop<sub>(p)</sub> denotes the proposed sensitivity analysis method given  $p$  with initial values equal to the estimates from the trivariate normal model based on the published studies; Prop<sub>(p)</sub><sup>\*</sup> denotes the proposed method given  $p$  with initial values equal to the estimates from the trivariate normal model based on the population studies. The medians are multiplied by 100.

Table 3: Summary of the estimated SAUC when the true censoring is distributed as  $Exp(0.2)$ .

Patients	S	p	Methods	Biomarker1		Biomarker2	
				Median (Q1, Q3)	CR	Median (Q1, Q3)	CR
50-300	70	0.7	HZ <sub>P</sub>	0.00 (75.08, 76.48)	100	0.00 (62.05, 62.96)	100
			HZ <sub>O</sub>	0.77 (75.56, 77.47)	100	1.28 (63.35, 64.37)	100
			Prop <sub>(0.7)</sub>	0.48 (75.02, 77.27)	77.4	0.62 (62.39, 63.86)	73.9
			Prop <sub>(0.7)</sub> <sup>*</sup>	0.49 (75.08, 77.23)	82.3	0.13 (62.08, 63.34)	69.4
		0.5	HZ <sub>O</sub>	1.23 (75.69, 78.17)	100	2.06 (63.95, 65.26)	100
			Prop <sub>(0.5)</sub>	0.43 (74.59, 77.54)	74.8	0.92 (62.17, 64.50)	66.2
			Prop <sub>(0.5)</sub> <sup>*</sup>	0.35 (74.73, 77.43)	76.3	0.05 (61.60, 63.57)	68.7
		0.3	HZ <sub>O</sub>	1.57 (75.07, 79.13)	100	2.97 (64.68, 66.31)	99.9
			Prop <sub>(0.3)</sub>	-0.08 (72.84, 78.46)	75.7	1.90 (62.10, 65.79)	70.1
			Prop <sub>(0.3)</sub> <sup>*</sup>	0.00 (74.10, 77.87)	62.2	-0.06 (61.16, 64.11)	58.5
	200	0.7	HZ <sub>P</sub>	0.00 (75.07, 76.00)	100	0.00 (62.25, 62.82)	100
			HZ <sub>O</sub>	0.68 (75.62, 76.73)	100	1.31 (63.55, 64.18)	100
			Prop <sub>(0.7)</sub>	0.39 (75.23, 76.56)	78.9	0.57 (62.51, 63.77)	70.2
			Prop <sub>(0.7)</sub> <sup>*</sup>	0.39 (75.21, 76.54)	84.2	-0.01 (62.21, 62.92)	67.7
		0.5	HZ <sub>O</sub>	1.13 (75.87, 77.24)	100	2.05 (64.25, 64.98)	100
			Prop <sub>(0.5)</sub>	0.34 (74.99, 76.63)	75.3	-0.03 (61.97, 63.40)	59.9
			Prop <sub>(0.5)</sub> <sup>*</sup>	0.29 (74.98, 76.51)	84	-0.24 (61.80, 62.87)	65.8
		0.3	HZ <sub>O</sub>	1.55 (76.01, 78.06)	100	2.91 (65.04, 66.03)	100
			Prop <sub>(0.3)</sub>	-0.03 (74.19, 76.81)	79.6	0.09 (61.58, 65.08)	69.3
			Prop <sub>(0.3)</sub> <sup>*</sup>	-0.06 (74.42, 76.54)	85.3	-0.54 (61.23, 62.75)	68.7

Patients denote the range of the number of patients.  $S$  denotes the number of the population studies.  $p$  denotes the approximate proportion of the published studies among the population. Median with 25th and 75th empirical quartiles (Q1, Q3) of the SAUC at  $t = 2$  are reported. CR denotes the proportion of successfully converged estimates among 1000 repetition. HZ<sub>P</sub> denotes the HZ model using the population studies; HZ<sub>O</sub> denotes the HZ model using only the corresponding numbers of published studies; Prop<sub>(p)</sub> denotes the proposed sensitivity analysis method given  $p$  with initial values equal to the estimates from the trivariate normal model based on the published studies; Prop<sub>(p)</sub><sup>\*</sup> denotes the proposed method given  $p$  with initial values equal to the estimates from the trivariate normal model based on the population studies. The medians are multiplied by 100.

Table 4: Summary of the estimated SAUC when the true censoring is distributed as  $U(1, 4)$ , but a misspecified exponential distribution is fitted.

Patients	S	$p$	Methods	Biomarker1		Biomarker2	
				Median (Q1, Q3)	CR	Median (Q1, Q3)	CR
50-150	70	0.7	HZ <sub>P</sub>	0.00 (74.12, 75.80)	100	0.00 (61.71, 62.82)	100
			HZ <sub>O</sub>	0.92 (74.88, 77.02)	100	1.98 (63.61, 64.92)	100
			Prop <sub>(0.7)</sub>	0.58 (74.23, 77.04)	63.9	0.29 (61.79, 63.41)	44
			Prop <sub>(0.7)</sub> <sup>*</sup>	0.60 (74.18, 77.06)	64.8	0.41 (61.87, 63.51)	49.6
		0.5	HZ <sub>O</sub>	1.58 (75.20, 77.97)	100	3.08 (64.61, 66.19)	100
			Prop <sub>(0.5)</sub>	0.04 (73.40, 76.78)	56.8	0.67 (61.31, 65.20)	51.5
			Prop <sub>(0.5)</sub> <sup>*</sup>	0.15 (73.46, 76.81)	58.1	0.25 (61.35, 64.03)	51.3
		0.3	HZ <sub>O</sub>	2.32 (75.46, 79.32)	100	4.42 (65.71, 67.75)	100
			Prop <sub>(0.3)</sub>	-0.34 (71.97, 77.27)	45.4	3.98 (62.86, 67.51)	64.4
			Prop <sub>(0.3)</sub> <sup>*</sup>	-0.52 (72.00, 76.63)	43.1	0.39 (60.41, 65.50)	43.3
	200	0.7	HZ <sub>P</sub>	0.00 (74.17, 75.26)	100	0.00 (61.92, 62.61)	100
			HZ <sub>O</sub>	0.99 (75.13, 76.39)	100	1.97 (63.88, 64.62)	100
			Prop <sub>(0.7)</sub>	0.66 (74.50, 76.38)	79.6	-0.03 (61.72, 62.73)	39.2
			Prop <sub>(0.7)</sub> <sup>*</sup>	0.63 (74.46, 76.34)	81	0.03 (61.86, 62.74)	46.1
		0.5	HZ <sub>O</sub>	1.52 (75.48, 77.05)	100	3.15 (64.98, 65.82)	100
			Prop <sub>(0.5)</sub>	-0.03 (73.57, 75.94)	72.9	-0.32 (61.30, 62.84)	45.8
			Prop <sub>(0.5)</sub> <sup>*</sup>	-0.06 (73.54, 75.94)	77.8	-0.43 (61.25, 62.47)	49.6
		0.3	HZ <sub>O</sub>	2.17 (75.85, 77.98)	100	4.45 (66.07, 67.28)	100
			Prop <sub>(0.3)</sub>	-0.79 (72.54, 75.53)	67.9	3.68 (61.18, 66.96)	59.6
			Prop <sub>(0.3)</sub> <sup>*</sup>	-0.79 (72.61, 75.58)	70.1	-1.19 (60.13, 62.01)	49.7

Patients denote the range of the number of patients.  $S$  denotes the number of the population studies.  $p$  denotes the approximate proportion of the published studies among the population. Median with 25th and 75th empirical quartiles (Q1, Q3) of the SAUC at  $t = 2$  are reported. CR denotes the proportion of successfully converged estimates among 1000 repetition. HZ<sub>P</sub> denotes the HZ model using the population studies; HZ<sub>O</sub> denotes the HZ model using only the corresponding numbers of published studies; Prop<sub>( $p$ )</sub> denotes the proposed sensitivity analysis method given  $p$  with initial values equal to the estimates from the trivariate normal model based on the published studies; Prop<sub>( $p$ )</sub><sup>\*</sup> denotes the proposed method given  $p$  with initial values equal to the estimates from the trivariate normal model based on the population studies. The medians are multiplied by 100.

Table 5: Summary of the estimated SAUC when the true censoring is distributed as  $U(1, 4)$ , but a misspecified exponential distribution is fitted.

Patients	S	p	Methods	Biomarker1		Biomarker2	
				Median (Q1, Q3)	CR	Median (Q1, Q3)	CR
50-300	70	0.7	HZ <sub>P</sub>	0.00 (75.28, 76.70)	100	0.00 (62.11, 62.93)	100
			HZ <sub>O</sub>	0.74 (75.78, 77.68)	100	1.41 (63.50, 64.43)	100
			Prop <sub>(0.7)</sub>	0.23 (75.13, 77.37)	62.3	0.18 (62.08, 63.38)	40.6
			Prop <sub>(0.7)</sub> <sup>*</sup>	0.31 (75.19, 77.41)	62.3	0.12 (62.03, 63.21)	41.2
		0.5	HZ <sub>O</sub>	1.14 (75.76, 78.42)	100	2.29 (64.23, 65.37)	100
			Prop <sub>(0.5)</sub>	-0.22 (74.31, 76.98)	58.1	0.55 (61.88, 64.49)	41.5
			Prop <sub>(0.5)</sub> <sup>*</sup>	-0.03 (74.73, 77.17)	57.5	-0.16 (61.51, 63.15)	35
		0.3	HZ <sub>O</sub>	1.38 (74.34, 79.47)	100	3.25 (64.98, 66.45)	100
			Prop <sub>(0.3)</sub>	-2.46 (34.02, 76.50)	54.7	2.41 (62.12, 66.19)	50.6
			Prop <sub>(0.3)</sub> <sup>*</sup>	-0.32 (73.60, 77.48)	42.3	-0.49 (60.90, 63.35)	31.7
	200	0.7	HZ <sub>P</sub>	0.00 (75.33, 76.18)	100	0.00 (62.33, 62.86)	100
			HZ <sub>O</sub>	0.62 (75.88, 76.93)	100	1.38 (63.69, 64.29)	100
			Prop <sub>(0.7)</sub>	0.40 (75.56, 76.77)	72.5	0.01 (62.24, 62.95)	34.6
			Prop <sub>(0.7)</sub> <sup>*</sup>	0.38 (75.56, 76.79)	72.1	0.03 (62.28, 62.98)	37.8
		0.5	HZ <sub>O</sub>	1.05 (76.10, 77.52)	100	2.21 (64.46, 65.15)	100
			Prop <sub>(0.5)</sub>	0.10 (75.02, 76.71)	74	-0.32 (61.82, 62.84)	36.8
			Prop <sub>(0.5)</sub> <sup>*</sup>	0.10 (75.04, 76.70)	72.3	-0.23 (61.91, 62.84)	40.2
		0.3	HZ <sub>O</sub>	1.42 (76.11, 78.26)	100	3.15 (65.28, 66.17)	100
			Prop <sub>(0.3)</sub>	-0.83 (73.69, 76.01)	69.4	-0.42 (61.54, 64.68)	42.3
			Prop <sub>(0.3)</sub> <sup>*</sup>	-0.48 (74.32, 76.41)	68	-0.74 (61.17, 62.42)	40.7

Patients denote the range of the number of patients.  $S$  denotes the number of the population studies.  $p$  denotes the approximate proportion of the published studies among the population. Median with 25th and 75th empirical quartiles (Q1, Q3) of the SAUC at  $t = 2$  are reported. CR denotes the proportion of successfully converged estimates among 1000 repetition. HZ<sub>P</sub> denotes the HZ model using the population studies; HZ<sub>O</sub> denotes the HZ model using only the corresponding numbers of published studies; Prop<sub>(p)</sub> denotes the proposed sensitivity analysis method given  $p$  with initial values equal to the estimates from the trivariate normal model based on the published studies; Prop<sub>(p)</sub><sup>\*</sup> denotes the proposed method given  $p$  with initial values equal to the estimates from the trivariate normal model based on the population studies. The medians are multiplied by 100.