Simulation Result for Manuscript

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$$\begin{split} \text{Biomarker1}: \tilde{X} &= \begin{cases} 0.7 + 0.1e & \text{if } \tilde{T} \leq 2\\ 0.3 + 0.3e & \text{if } \tilde{T} > 2 \end{cases} \\ \text{Biomarker2}: \tilde{X} &= \begin{cases} 0.6 + 0.2e & \text{if } \tilde{T} \leq 2\\ 0.4 + 0.3e & \text{if } \tilde{T} > 2 \end{cases} \end{split}$$

Table 1: Scenarios of simulation

Censoring	Biomarker	Subjects	β	$\alpha_{0.7}$	$\alpha_{0.5}$	$\alpha_{0.3}$
Exp(0.2)	1	U(50, 150)	1	-2.431	-3.199	-3.961
		U(50, 300)	1	-3.277	-4.178	-5.083
	2	U(50, 150)	1	-0.518	-1.270	-2.020
		U(50, 300)	1	-0.878	-1.649	-2.427
U(1, 4)	1	U(50, 150)	1	-2.968	-3.713	-4.452
		U(50, 300)	1	-3.945	-4.899	-5.818
	2	U(50, 150)	1	-0.795	-1.547	-2.286
		U(50, 300)	1	-1.233	-2.020	-2.802

Table 2: Summary of the estimated SAUC when the true censoring is distributed as Exp(0.2).

				Biomarker1		Biomarker2	
Patients	\mathbf{S}	p	Methods	Median (Q1, Q3)	CR	Median (Q1, Q3)	CR
50-150	70		HZ_P	0.00 (73.70, 75.65)	100	0.00 (61.55, 62.84)	100
		0.7	HZ_O $Prop_{(0.7)}^*$ $Prop_{(0.7)}^*$ $Prop_{(0.7)}^{**}$	1.09 (74.59, 76.84) 0.64 (73.90, 76.84) 0.49 (73.77, 76.77) 0.52 (73.80, 76.64)	100 77.3 82 83.9	1.78 (63.33, 64.68) 0.02 (61.34, 63.09) 0.05 (61.42, 63.06) 1.08 (62.17, 64.16)	100 64 77 88.1
		0.5	HZ_O $Prop_{(0.5)}^*$ $Prop_{(0.5)}^*$ $Prop_{(0.5)}^{**}$	1.64 (74.80, 77.76) 0.70 (73.51, 77.38) 0.21 (72.99, 76.77) 0.16 (72.97, 76.76)	100 76.3 74.8 74.5	2.80 (64.07, 65.82) 0.01 (60.92, 64.01) -0.45 (60.63, 62.97) 1.95 (62.18, 65.47)	100 72 73.4 85.4
		0.3	HZ_O $Prop_{(0.3)}^*$ $Prop_{(0.3)}^{**}$ $Prop_{(0.3)}^{**}$	2.63 (75.29, 79.32) 0.88 (72.86, 78.31) 0.08 (72.01, 77.14) -0.16 (71.63, 76.99)	100 67.5 63.6 60.6	4.18 (65.14, 67.40) 2.51 (60.89, 66.78) -0.99 (59.70, 63.19) 3.17 (62.19, 66.94)	99.9 79.1 64.2 82.2
	200	0.7	HZ_P HZ_O $\mathrm{Prop}_{(0.7)}$	0.00 (73.92, 75.13) 1.03 (74.83, 76.19) 0.49 (73.98, 75.94)	100 100 72.2	0.00 (61.86, 62.58) 1.82 (63.59, 64.43) -0.14 (61.54, 62.56)	100 100 63
			$Prop^*_{(0.7)}$ $Prop^*_{(0.7)}$	0.45 (73.95, 75.99) 0.54 (73.97, 76.00)	86 90	-0.10 (61.57, 62.62) 0.58 (62.00, 63.88)	75.4 86.5
		0.5	HZ_O $Prop_{(0.5)}^*$ $Prop_{(0.5)}^*$ $Prop_{(0.5)}^{**}$	1.64 (75.25, 76.93) 0.50 (73.68, 76.26) 0.33 (73.48, 75.99) 0.25 (73.48, 75.94)	100 76.3 85.4 87.6	2.85 (64.59, 65.57) -0.46 (61.03, 62.50) -0.55 (60.90, 62.27) 2.52 (61.89, 65.40)	100 67.2 77.8 91.5
		0.3	HZ_O $Prop_{(0.3)}^*$ $Prop_{(0.3)}^*$ $Prop_{(0.3)}^{**}$	2.29 (75.79, 77.85) 0.30 (72.95, 76.73) -0.31 (72.53, 75.87) -0.34 (72.49, 75.76)	100 76.8 83.1 79.4	4.08 (65.67, 66.96) -0.17 (60.31, 66.21) -1.20 (59.88, 62.04) 3.32 (61.32, 66.58)	100 73.1 73.1 90.7

Patients denote the range of the number of patients. S denotes the number of the population studies. p denotes the approximate proportion of the published studies among the population. Median with 25th and 75th empirical quartiles (Q1, Q3) of the SAUC at t=2 are reported. CR denotes the proportion of successfully convergenced estimates among 1000 repetition. HZ_P denotes the HZ model using the population studies; HZ_O denotes the HZ model using only the corresponding numbers of published studies; $Prop_{(p)}$ denotes the proposed sensitivity analysis method given p with initial values equal to the etimates from the trivariate normal model based on the published studies; $Prop_{(p)}^*$ denotes the proposed method given p with initial values equal to the etimates from the trivariate normal model based on the population studies; $Prop_{(p)}^{**}$ denotes the proposed method given p with vague initial values (0,0,0,0.1,0.1,-0.1,-0.1,-0.1). The medians are multiplied by 100.

Table 3: Summary of the estimated SAUC when the true censoring is distributed as Exp(0.2).

				Biomarker1		Biomarker2	
Patients	\mathbf{S}	p	Methods	Median (Q1, Q3)	CR	Median (Q1, Q3)	CR
50-150	70		HZ_P	0.00 (75.08, 76.48)	100	0.00 (62.05, 62.96)	100
		0.7	HZ_O	$0.77 \ (75.56, 77.47)$	100	$1.28 \ (63.35, \ 64.37)$	100
			$\text{Prop}_{(0.7)}$	$0.48 \ (75.02, 77.27)$	77.4	$0.62 \ (62.39, \ 63.86)$	73.9
			$\text{Prop}_{(0.7)}^*$	$0.49 \ (75.08, 77.23)$	82.3	$0.13 \ (62.08, \ 63.34)$	69.4
			$\text{Prop}_{(0.7)}^{**}$	$0.50 \ (75.08, 77.27)$	80.3	$1.01\ (62.72,\ 64.19)$	88
		0.5	HZ_O	$1.23\ (75.69,\ 78.17)$	100	$2.06 \ (63.95, \ 65.26)$	100
			$Prop_{(0.5)}$	$0.43 \ (74.59, 77.54)$	74.8	0.92 (62.17, 64.50)	66.2
			$\text{Prop}_{(0.5)}^*$	0.35 (74.73, 77.43)	76.3	$0.05 \ (61.60, \ 63.57)$	68.7
			$\operatorname{Prop}^{**}_{(0.5)}$	$0.35 \ (74.70, 77.43)$	68.9	1.69 (63.14, 65.03)	85.5
		0.3	HZ_O	1.57 (75.07, 79.13)	100	2.97 (64.68, 66.31)	99.9
			$\text{Prop}_{(0.3)}$	-0.08 (72.84, 78.46)	75.7	1.90 (62.10, 65.79)	70.1
			$\text{Prop}_{(0.3)}^{*}$	$0.00\ (74.10,\ 77.87)$	62.2	-0.06 (61.16, 64.11)	58.5
			$\text{Prop}_{(0.3)}^{**}$	$-0.17 \ (73.81, \ 77.74)$	55.9	$2.43 \ (63.02, \ 66.01)$	78.7
	200		HZ_P	$0.00\ (75.07,\ 76.00)$	100	$0.00 \ (62.25, \ 62.82)$	100
		0.7	HZ_O	$0.68\ (75.62,\ 76.73)$	100	1.31 (63.55, 64.18)	100
			$\text{Prop}_{(0.7)}$	$0.39 \ (75.23, 76.56)$	78.9	0.57 (62.51, 63.77)	70.2
			$\text{Prop}_{(0.7)}^{*}$	$0.39\ (75.21,\ 76.54)$	84.2	-0.01 (62.21, 62.92)	67.7
			$\text{Prop}_{(0.7)}^{**}$	$0.40\ (75.23,\ 76.56)$	90.9	$1.16 \ (63.10, \ 64.10)$	89.7
		0.5	HZ_O	1.13 (75.87, 77.24)	100	2.05 (64.25, 64.98)	100
			$\text{Prop}_{(0.5)}$	$0.34\ (74.99,\ 76.63)$	75.3	-0.03 (61.97, 63.40)	59.9
			$\text{Prop}_{(0.5)}^{\hat{*}}$	$0.29\ (74.98,\ 76.51)$	84	-0.24 (61.80, 62.87)	65.8
			$\text{Prop}_{(0.5)}^{**}$	$0.29\ (74.99,\ 76.51)$	77.4	$1.97 \ (63.98, \ 64.90)$	92.4
		0.3	HZ_O	$1.55\ (76.01,\ 78.06)$	100	$2.91\ (65.04,\ 66.03)$	100
			$\text{Prop}_{(0.3)}$	-0.03 (74.19, 76.81)	79.6	$0.09 \ (61.58, 65.08)$	69.3
			$\operatorname{Prop}_{(0.3)}^*$	-0.06 (74.42, 76.54)	85.3	-0.54 (61.23, 62.75)	68.7
			$\operatorname{Prop}_{(0.3)}^{**}$	-0.07 (74.43, 76.52)	69.9	2.57 (62.60, 65.79)	86.7

Patients denote the range of the number of patients. S denotes the number of the population studies. p denotes the approximate proportion of the published studies among the population. Median with 25th and 75th empirical quartiles (Q1, Q3) of the SAUC at t=2 are reported. CR denotes the proportion of successfully convergenced estimates among 1000 repetition. HZ_P denotes the HZ model using the population studies; HZ_O denotes the HZ model using only the corresponding numbers of published studies; $Prop_{(p)}$ denotes the proposed sensitivity analysis method given p with initial values equal to the etimates from the trivariate normal model based on the published studies; $Prop_{(p)}^*$ denotes the proposed method given p with initial values equal to the etimates from the trivariate normal model based on the population studies; $Prop_{(p)}^{**}$ denotes the proposed method given p with vague initial values (0,0,0,0.1,0.1,0.1,-0.1,-0.1,0.1). The medians are multiplied by 100.

Table 4: Summary of the estimated SAUC when the true censoring is distributed as U(1,4), but a misspecified exponential distribution is fitted.

				Biomarker1		Biomarker2	
Patients	\mathbf{S}	p	Methods	Median (Q1, Q3)	CR	Median (Q1, Q3)	CR
50-150	70		HZ_P	0.00 (74.12, 75.80)	100	0.00 (61.71, 62.82)	100
		0.7	HZ_O	0.92 (74.88, 77.02)	100	1.98 (63.61, 64.92)	100
			$Prop_{(0.7)}$	$0.58 \ (74.23, 77.04)$	63.9	0.29 (61.79, 63.41)	44
			$\operatorname{Prop}^*_{(0.7)}$	$0.60\ (74.18,\ 77.06)$	64.8	0.41 (61.87, 63.51)	49.6
			$\operatorname{Prop}_{(0.7)}^{**}$	$0.61\ (74.26,\ 77.11)$	59.1	$0.87\ (62.02,\ 64.32)$	57.9
		0.5	HZ_O	1.58 (75.20, 77.97)	100	3.08 (64.61, 66.19)	100
			$Prop_{(0.5)}$	$0.04 \ (73.40, 76.78)$	56.8	$0.67 \ (61.31, \ 65.20)$	51.5
			$\text{Prop}_{(0.5)}^*$	$0.15 \ (73.46, 76.81)$	58.1	$0.25 \ (61.35, 64.03)$	51.3
			$\text{Prop}_{(0.5)}^{**}$	$-0.02 \ (73.15, 76.73)$	49.7	$1.78 \ (61.71, 65.66)$	60.2
		0.3	HZ_O	2.32 (75.46, 79.32)	100	4.42 (65.71, 67.75)	100
			$Prop_{(0.3)}$	-0.34 (71.97, 77.27)	45.4	3.98 (62.86, 67.51)	64.4
			$\text{Prop}_{(0.3)}^{*}$	-0.52 (72.00, 76.63)	43.1	$0.39 \ (60.41, 65.50)$	43.3
			$\text{Prop}_{(0.3)}^{**}$	-1.27 (71.12, 76.50)	37.9	$3.28 \ (61.72,\ 67.15)$	58
	200		HZ_P	$0.00\ (74.17,\ 75.26)$	100	$0.00 \ (61.92, \ 62.61)$	100
		0.7	HZ_O	$0.99\ (75.13,\ 76.39)$	100	1.97 (63.88, 64.62)	100
			$Prop_{(0.7)}$	$0.66 \ (74.50, 76.38)$	79.6	-0.03 (61.72, 62.73)	39.2
			$\text{Prop}^{*}_{(0.7)}$	$0.63\ (74.46,\ 76.34)$	81	$0.03 \ (61.86, \ 62.74)$	46.1
			$\text{Prop}_{(0.7)}^{**}$	$0.64\ (74.44,\ 76.38)$	76.7	$0.27 \ (61.98, \ 63.65)$	51.7
		0.5	HZ_O	1.52 (75.48, 77.05)	100	3.15 (64.98, 65.82)	100
			$\text{Prop}_{(0.5)}$	-0.03 (73.57, 75.94)	72.9	-0.32 (61.30, 62.84)	45.8
			$\operatorname{Prop}^*_{(0.5)}$	-0.06 (73.54, 75.94)	77.8	-0.43 (61.25, 62.47)	49.6
			$\operatorname{Prop}_{(0.5)}^{**}$	-0.03 (73.59, 76.00)	66	$0.37 \ (61.59, 65.22)$	60.9
		0.3	HZ_O	2.17 (75.85, 77.98)	100	$4.45 \ (66.07, \ 67.28)$	100
			$Prop_{(0.3)}$	-0.79 (72.54, 75.53)	67.9	$3.68 \ (61.18, 66.96)$	59.6
			$\operatorname{Prop}_{(0.3)}^*$	-0.79 (72.61, 75.58)	70.1	-1.19 (60.13, 62.01)	49.7
			$\operatorname{Prop}_{(0.3)}^{**}$	-0.77 (72.35, 75.53)	55.6	-0.13 (60.69, 66.25)	60

Patients denote the range of the number of patients. S denotes the number of the population studies. p denotes the approximate proportion of the published studies among the population. Median with 25th and 75th empirical quartiles (Q1, Q3) of the SAUC at t=2 are reported. CR denotes the proportion of successfully convergenced estimates among 1000 repetition. HZ_P denotes the HZ model using the population studies; HZ_O denotes the HZ model using only the corresponding numbers of published studies; $\mathrm{Prop}_{(p)}^*$ denotes the proposed sensitivity analysis method given p with initial values equal to the etimates from the trivariate normal model based on the published studies; $\mathrm{Prop}_{(p)}^*$ denotes the proposed method given p with initial values equal to the etimates from the trivariate normal model based on the population studies; $\mathrm{Prop}_{(p)}^*$ denotes the proposed method given p with vague initial values (0,0,0,0.1,0.1,0.1,-0.1,-0.1,-0.1). The medians are multiplied by 100.

Table 5: Summary of the estimated SAUC when the true censoring is distributed as U(1,4), but a misspecified exponential distribution is fitted.

				Biomarker1		Biomarker2	
Patients	\mathbf{S}	p	Methods	Median (Q1, Q3)	CR	Median (Q1, Q3)	CR
50-150	70		HZ_P	0.00 (75.28, 76.70)	100	0.00 (62.11, 62.93)	100
		0.7	HZ_O $Prop_{(0.7)}^*$ $Prop_{(0.7)}^*$ $Prop_{(0.7)}^{**}$	0.74 (75.78, 77.68) 0.23 (75.13, 77.37) 0.31 (75.19, 77.41) 0.26 (75.18, 77.42)	100 62.3 62.3 42.1	1.41 (63.50, 64.43) 0.18 (62.08, 63.38) 0.12 (62.03, 63.21) 1.00 (62.63, 64.27)	100 40.6 41.2 60.1
		0.5	HZ_O $Prop_{(0.5)}^*$ $Prop_{(0.5)}^*$ $Prop_{(0.5)}^{**}$	1.14 (75.76, 78.42) -0.22 (74.31, 76.98) -0.03 (74.73, 77.17) -0.06 (74.46, 77.16)	100 58.1 57.5 36.3	2.29 (64.23, 65.37) 0.55 (61.88, 64.49) -0.16 (61.51, 63.15) 1.83 (62.93, 65.20)	100 41.5 35 58.4
		0.3	HZ_O $Prop_{(0.3)}^*$ $Prop_{(0.3)}^{**}$ $Prop_{(0.3)}^{**}$	1.38 (74.34, 79.47) -2.46 (34.02, 76.50) -0.32 (73.60, 77.48) -1.19 (72.76, 76.84)	100 54.7 42.3 26.2	3.25 (64.98, 66.45) 2.41 (62.12, 66.19) -0.49 (60.90, 63.35) 2.39 (62.21, 66.16)	100 50.6 31.7 53.6
	200		HZ_P	0.00 (75.33, 76.18)	100	0.00 (62.33, 62.86)	100
		0.7	HZ_O $Prop_{(0.7)}^*$ $Prop_{(0.7)}^*$ $Prop_{(0.7)}^{**}$	0.62 (75.88, 76.93) 0.40 (75.56, 76.77) 0.38 (75.56, 76.79) 0.37 (75.52, 76.73)	100 72.5 72.1 54.3	1.38 (63.69, 64.29) 0.01 (62.24, 62.95) 0.03 (62.28, 62.98) 1.12 (62.64, 64.27)	100 34.6 37.8 52.7
		0.5	HZ_O $Prop_{(0.5)}^*$ $Prop_{(0.5)}^*$ $Prop_{(0.5)}^{**}$	1.05 (76.10, 77.52) 0.10 (75.02, 76.71) 0.10 (75.04, 76.70) 0.10 (75.04, 76.77)	100 74 72.3 41.1	2.21 (64.46, 65.15) -0.32 (61.82, 62.84) -0.23 (61.91, 62.84) 1.90 (62.63, 65.00)	100 36.8 40.2 58
		0.3	HZ_O $Prop_{(0.3)}^*$ $Prop_{(0.3)}^*$ $Prop_{(0.3)}^{**}$	1.42 (76.11, 78.26) -0.83 (73.69, 76.01) -0.48 (74.32, 76.41) -0.43 (74.34, 76.49)	100 69.4 68 37.7	3.15 (65.28, 66.17) -0.42 (61.54, 64.68) -0.74 (61.17, 62.42) 2.38 (61.83, 65.91)	100 42.3 40.7 56.9

Patients denote the range of the number of patients. S denotes the number of the population studies. p denotes the approximate proportion of the published studies among the population. Median with 25th and 75th empirical quartiles (Q1, Q3) of the SAUC at t=2 are reported. CR denotes the proportion of successfully convergenced estimates among 1000 repetition. HZ_P denotes the HZ model using the population studies; HZ_O denotes the HZ model using only the corresponding numbers of published studies; $Prop_{(p)}$ denotes the proposed sensitivity analysis method given p with initial values equal to the etimates from the trivariate normal model based on the published studies; $Prop_{(p)}^*$ denotes the proposed method given p with initial values equal to the etimates from the trivariate normal model based on the population studies; $Prop_{(p)}^{**}$ denotes the proposed method given p with vague initial values (0,0,0,0.1,0.1,0.1,-0.1,-0.1). The medians are multiplied by 100.