Declare Your Language

Chapter 10: Data-Flow Analysis

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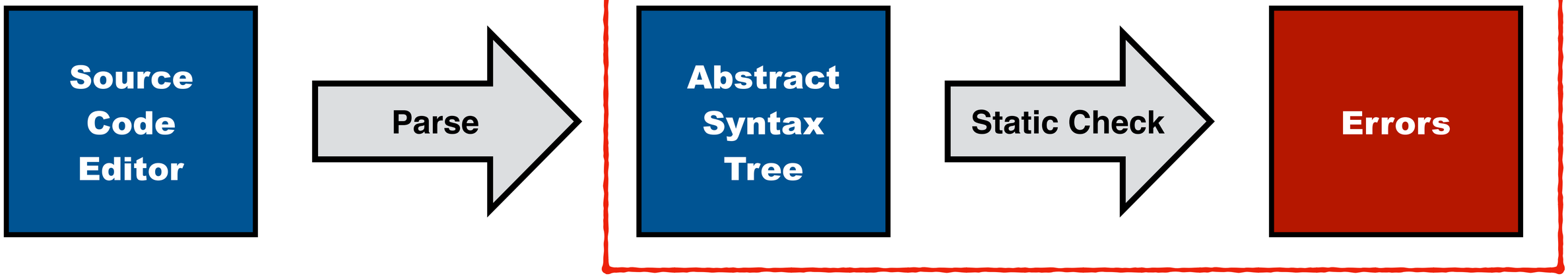
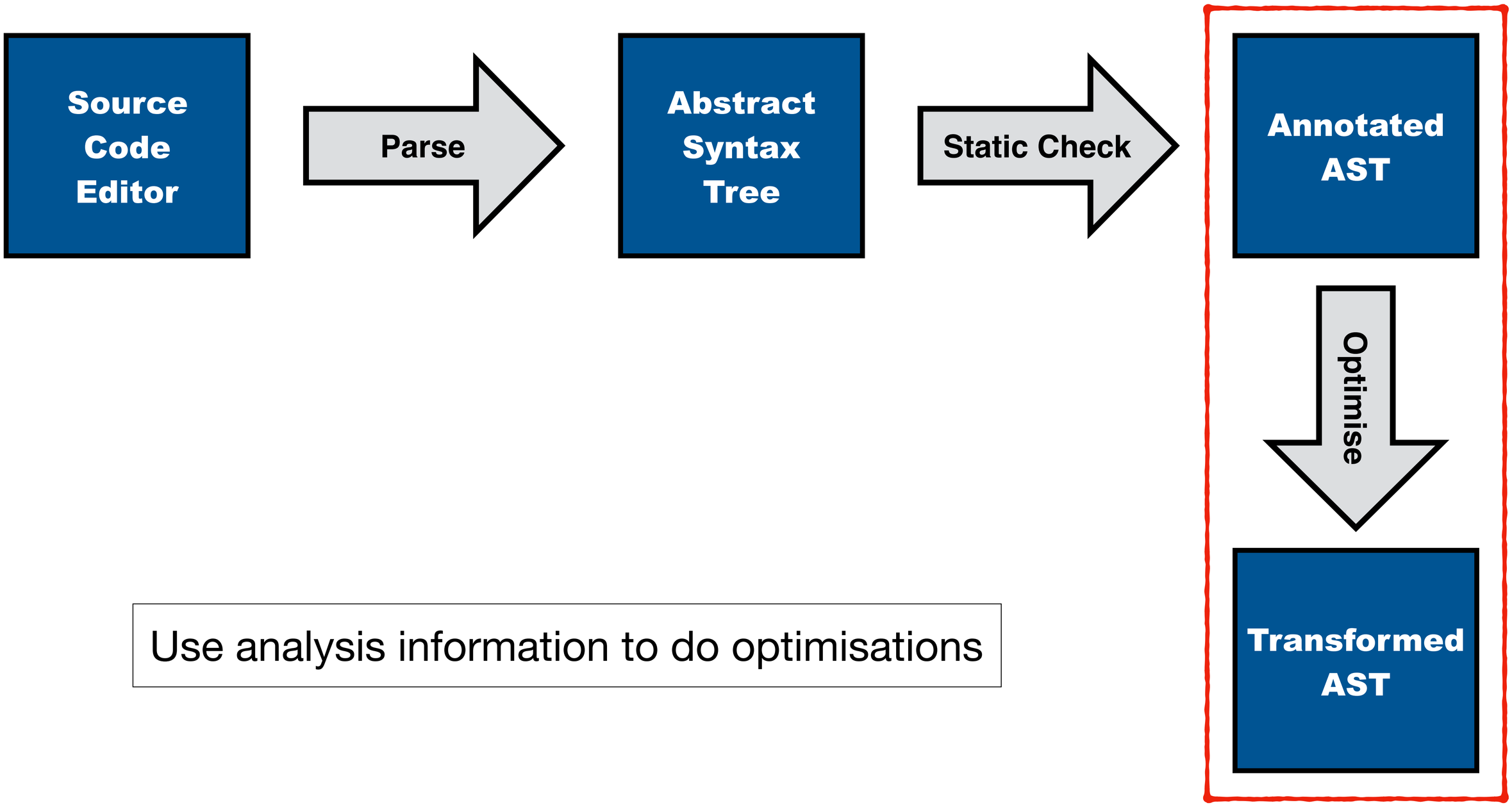
IN4303 Compiler Construction
TU Delft
November 2017

TU Delft



Check such things as final fields being initialised by a constructor



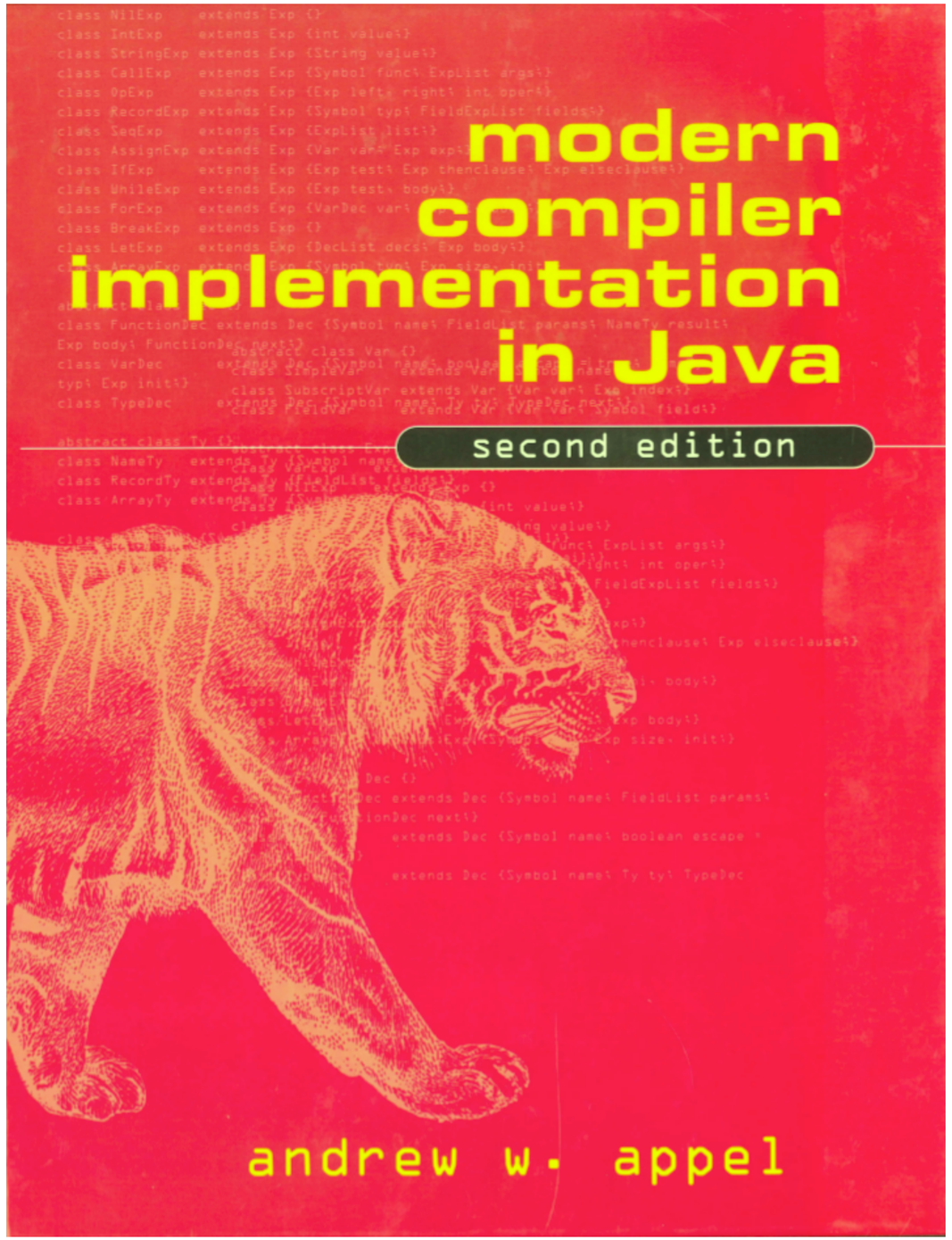
Use analysis information to do optimisations

Reading Material

Chapter 10: Liveness Analysis

Chapter 17: Dataflow Analysis

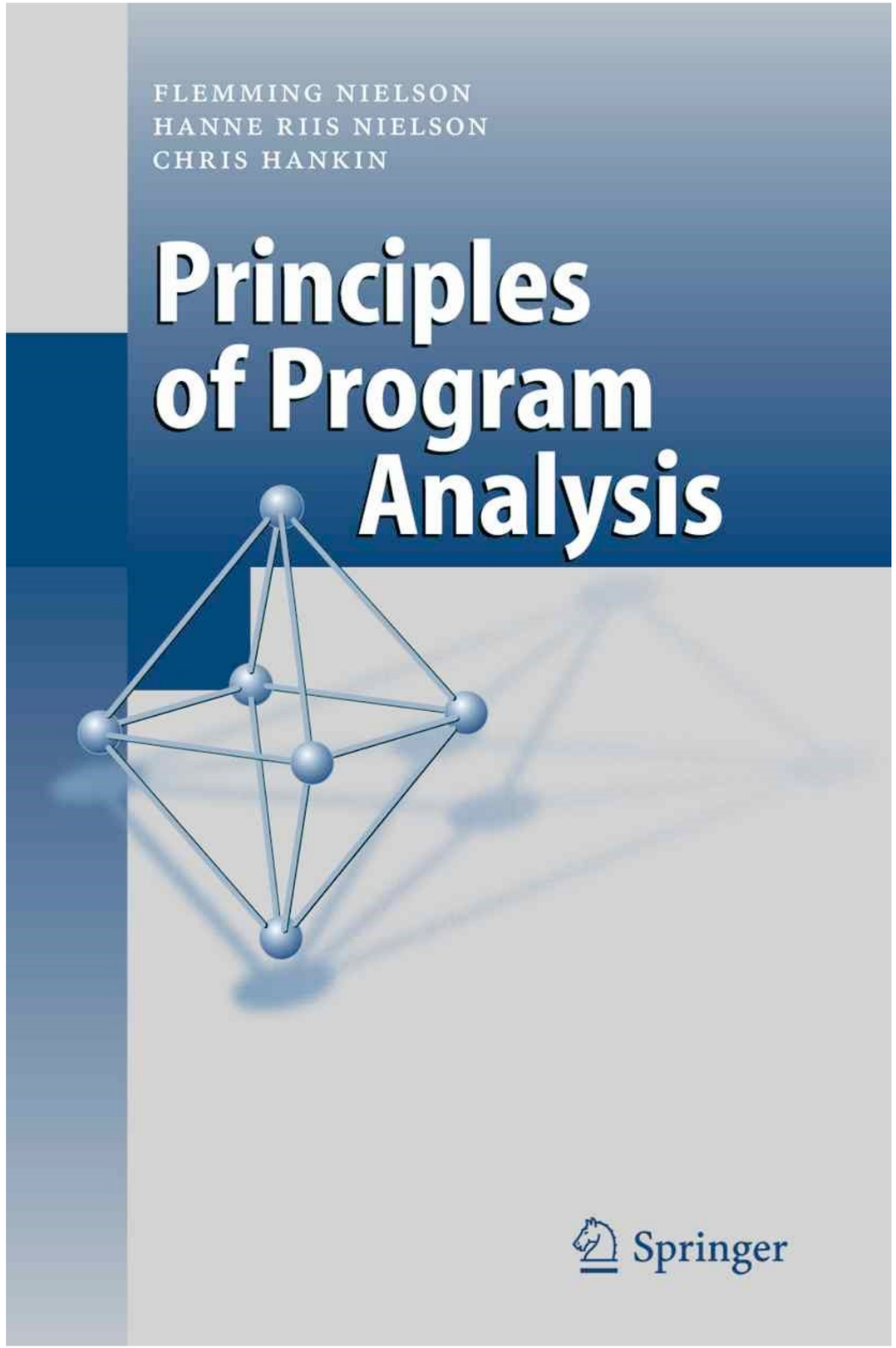
The *applied*/“cookbook” version



Chapter 1: Introduction

Chapter 2: Data Flow Analysis

The theoretical/“general” version



Data-Flow

Examples in Tiger

Available Expressions

```
let
  var x : int := a + b
  var y : int := a * b
 in
  while y > a + b do
    (
      a := a + 1;
      x := a + b
    )
end
```

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Available Expressions

Common subexpression elimination

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end
```

a + b
a + b, a * b
a + b

Live Variables

```
x := 2;
y := 4;
x := 1;
if y > x then
  z := y
else
  z := y * y;
x := z
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Live Variables

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x := 2;
y := 4;
x := 1;
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Live Variables

Dead code elimination

```
y := 4;
x := 1;
if y > x then
  z := y
else
  z := y * y;
x := z
```

What is Data-Flow Analysis?

Data-flow analysis is a technique for gathering information about the possible set of values calculated at various points in a computer program. A program's control flow graph (CFG) is used to determine those parts of a program to which a particular value assigned to a variable might propagate. The information gathered is often used by compilers when optimizing a program. A canonical example of a data-flow analysis is reaching definitions.

https://en.wikipedia.org/wiki/Data_flow_analysis

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- The *possible* set of values at various point in a program

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Gathering information about

- The *possible* set of values at various point in a program
- The program's *control flow graph* is used to …

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Control Flow Graphs

What is a Control Flow Graph?

A **control flow graph** (**CFG**) in computer science is a representation, using graph notation, of all paths that might be traversed through a program during its execution.

https://en.wikipedia.org/wiki/Control_flow_graph

Control Flow Graphs

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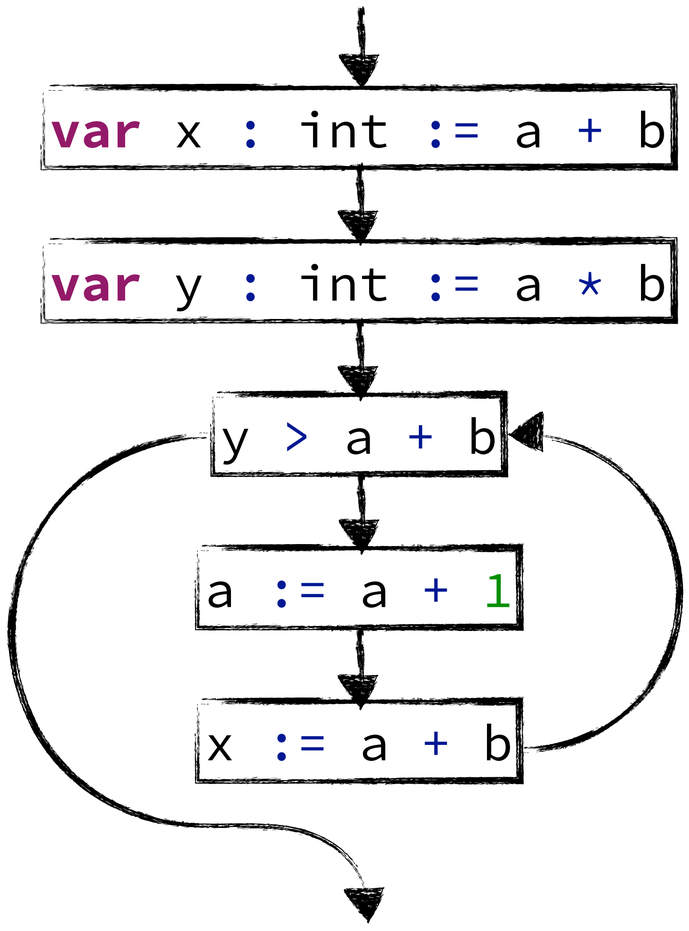
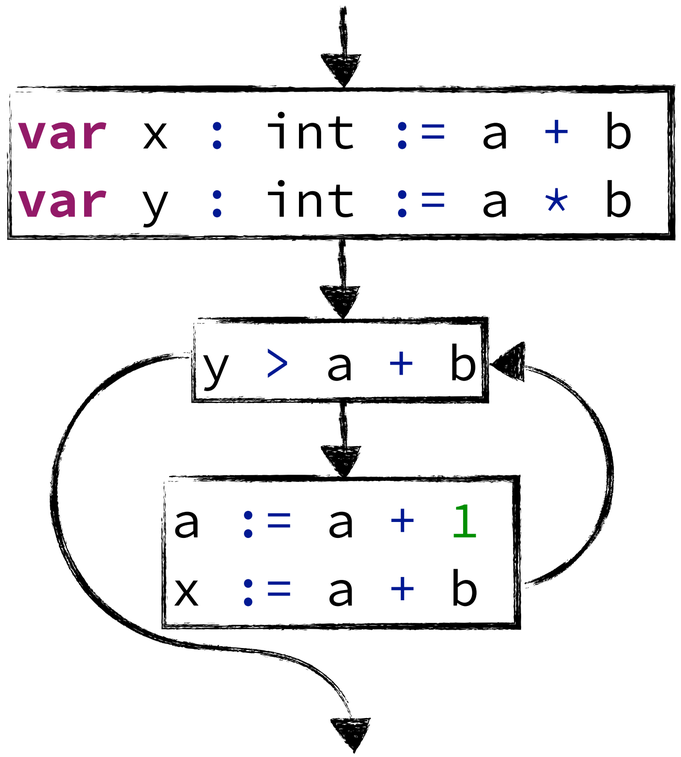
Control Flow Graphs

Full flow graph



Control Flow Graphs

Basic Blocks



Control Flow Graphs

Ingredients

Control Nodes

- Usually outermost statements and expressions
- Or blocks for consecutive statements

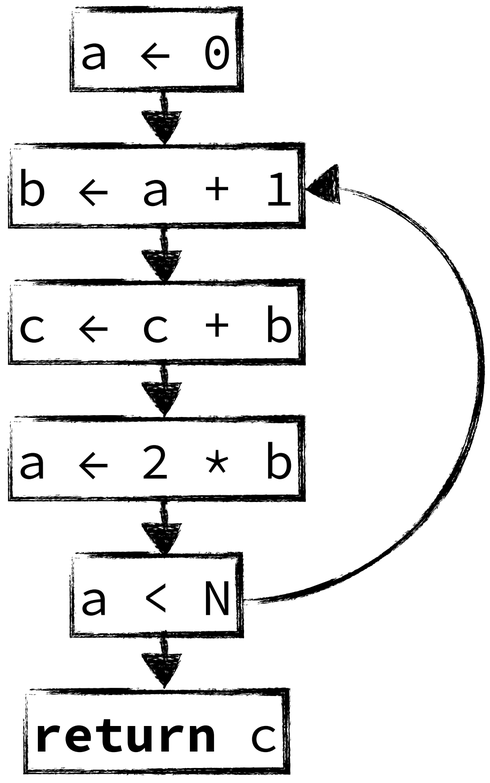
Control Edges

- Back edges: show loops
- Splits: conditionally split the control flow
- Merges: combine previously split control flow

Control Flow Graphs

Supports unstructured control flow

```
    a ← 0
L1: b ← a + 1
    c ← c + b
    a ← 2 * b
    if a < N goto L1
    return c
```



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A simple way to perform data-flow analysis of programs is to set up data-flow equations for each node of the control flow graph and solve them by repeatedly calculating the output from the input locally at each node until the whole system stabilizes, i.e., it reaches a fixpoint. This general approach was developed by Gary Kildall while teaching at the Naval Postgraduate School.[1]

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What is Data-Flow Analysis?

Possible set of values

- at various points in a computer program

Data-flow equations

- for each node of the control flow graph

Solve them by repetition

- i.e., it reaches a fixpoint

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- express the ‘effect’ of a control node

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Calculate a fixpoint

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- for each node of the control flow graph
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Calculate a fixpoint

- because loops

Traditional Gen/Kill Sets

Traditional set base analysis

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  - previousSet ∖ kill(currentNode) ∪ gen(currentNode)

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Depends on the direction of the analysis

Available Expressions

"An **expression** is **available** if it *must* have already been computed, and not later modified, on all paths to the program point"

```
kill(Assign(var, e1)) :=
  { e2 ∈ AllAE | var ∈ FV(e2) }
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gen(Assign(var, e1)) :=
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AllAE

- All Available Expressions in the program

FV

- Free Variables of the argument

SE

- Subexpressions of the argument

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This mistake can be prevented by making the control flow more explicit in the graph: First we execute the right-hand side, then we do the assignment.

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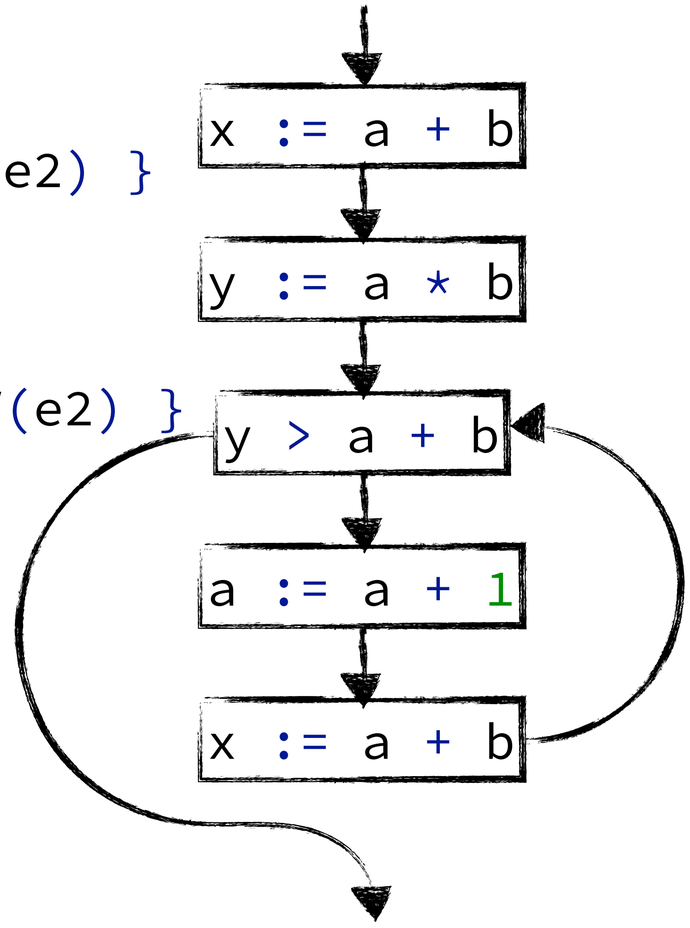

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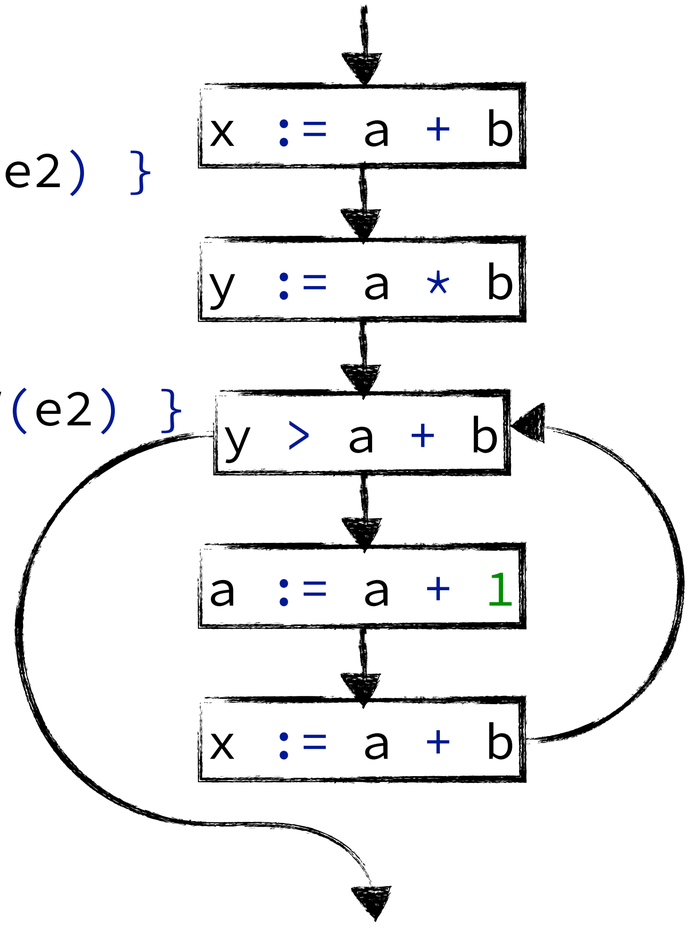

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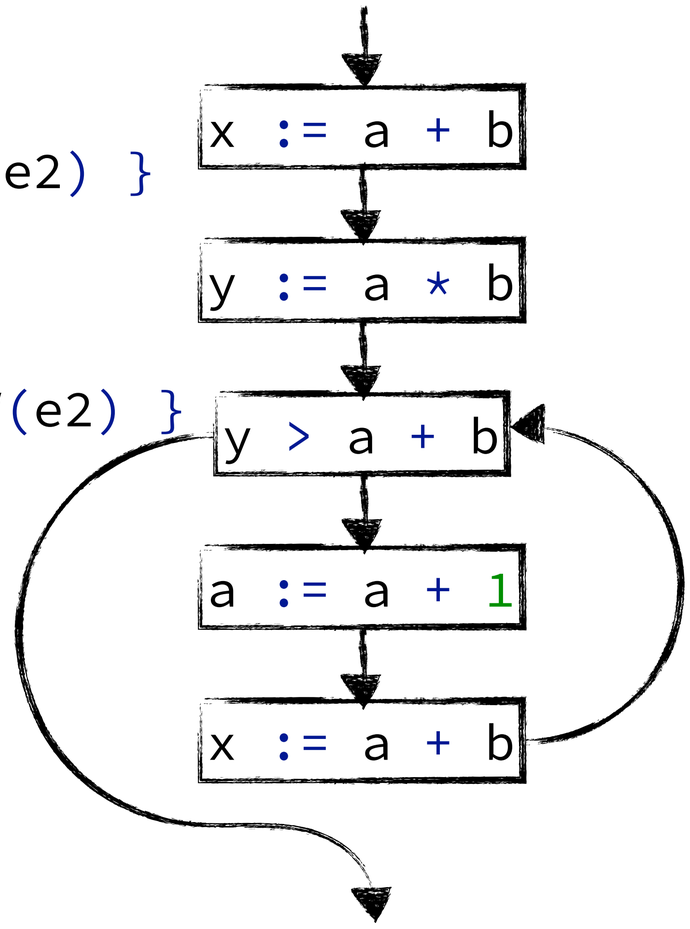

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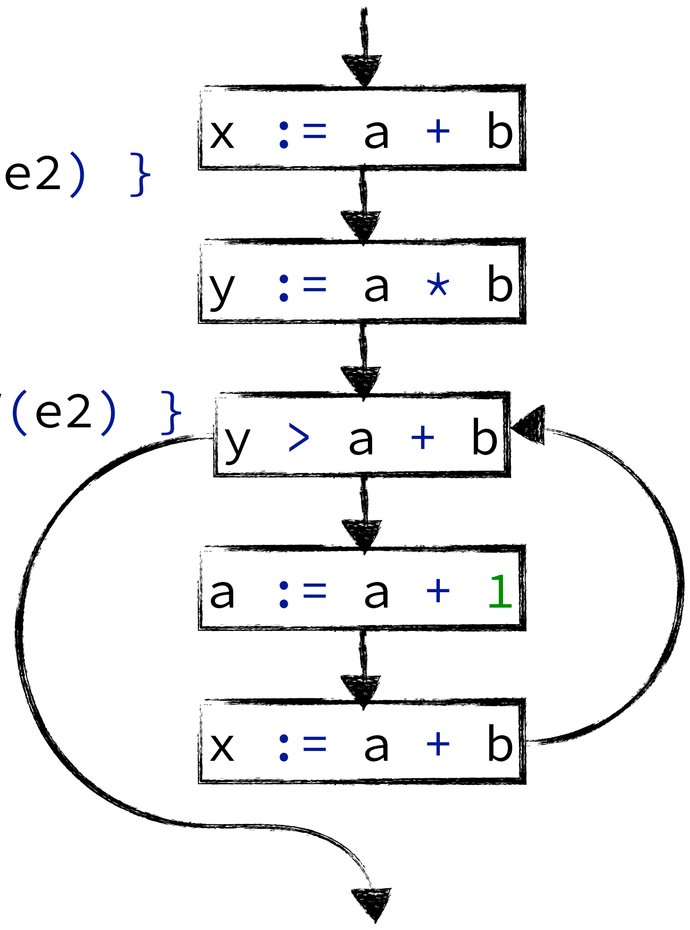


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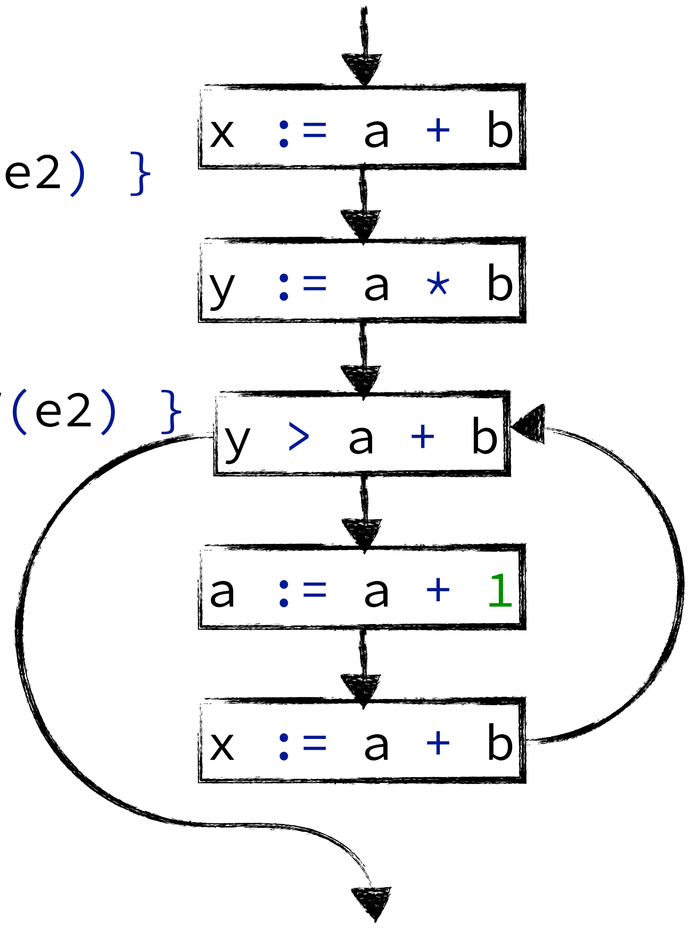


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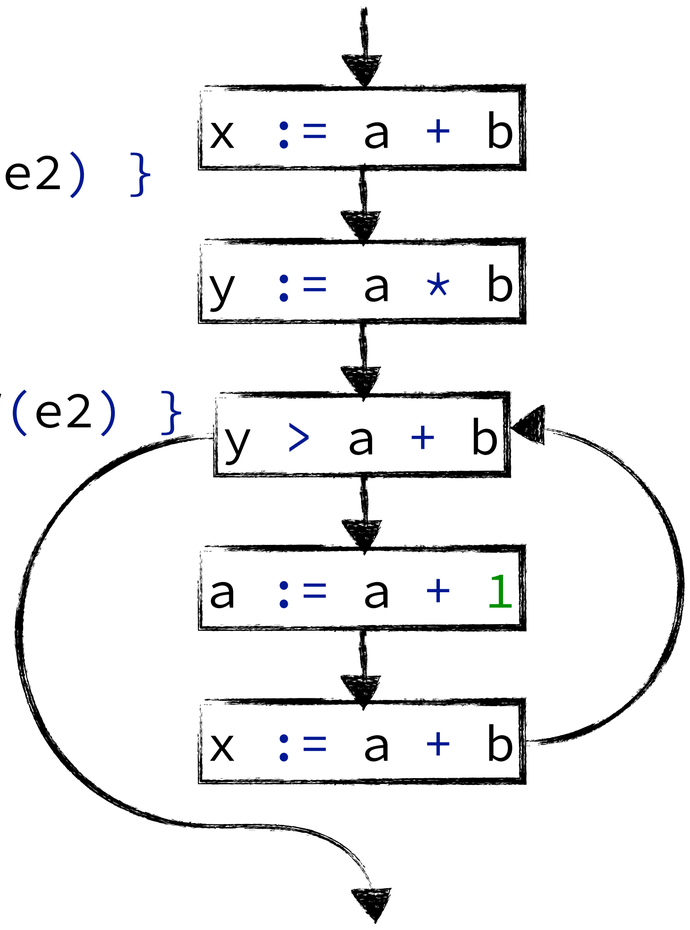


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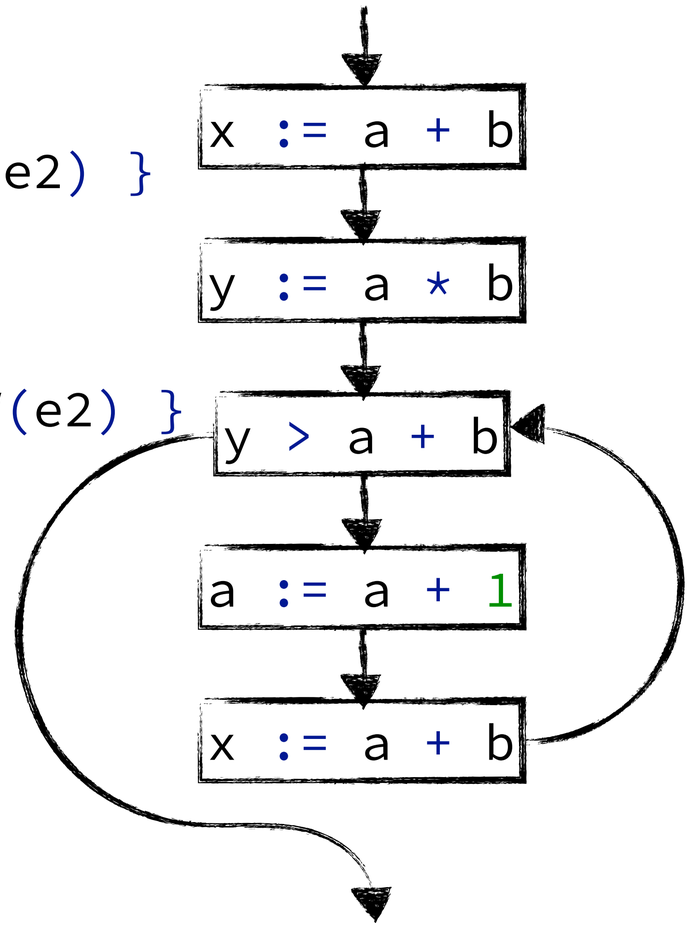

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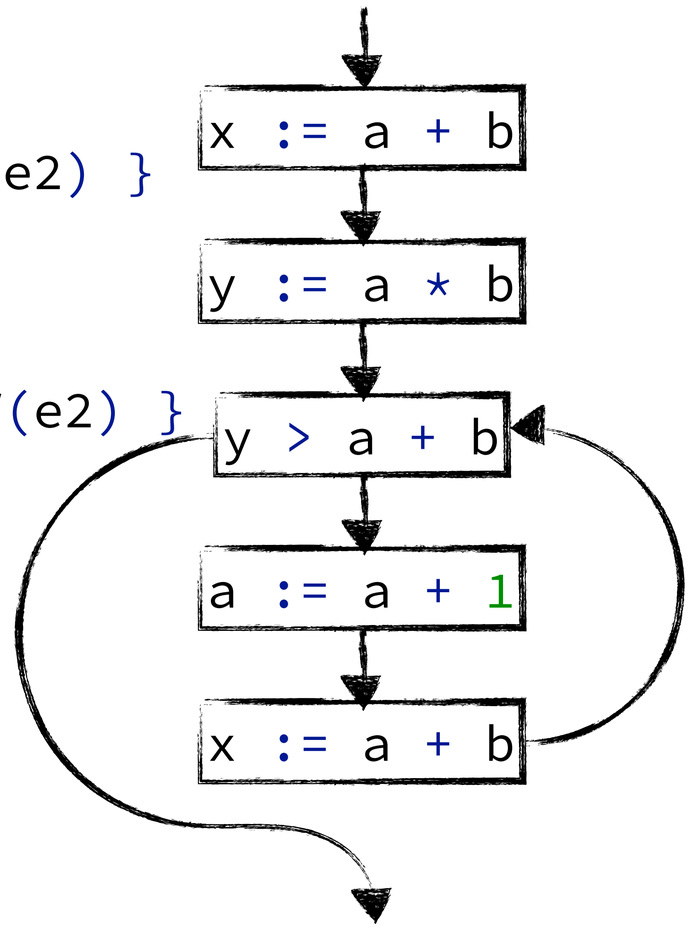

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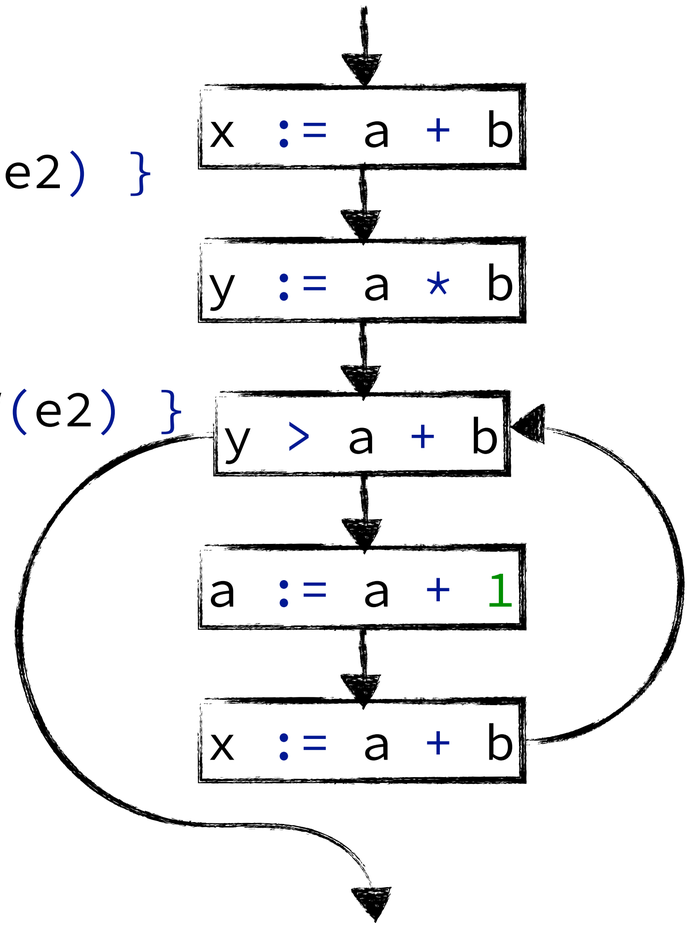

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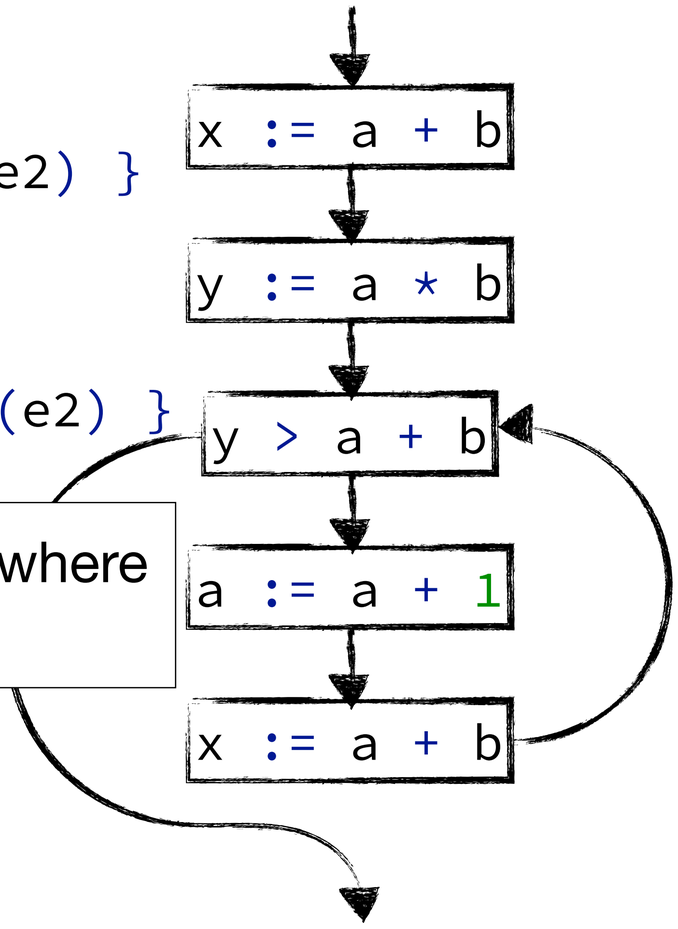
A *must* analysis intersects sets where control flow merges

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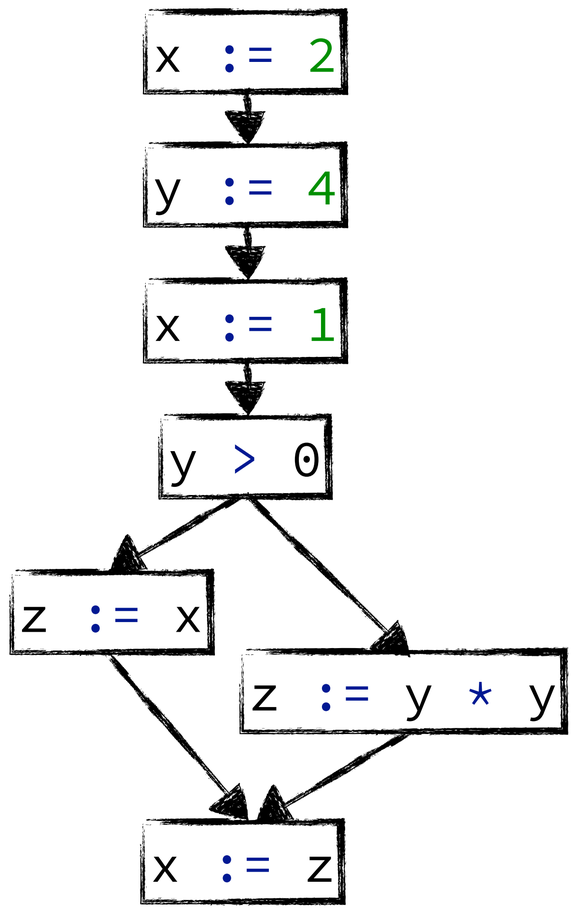

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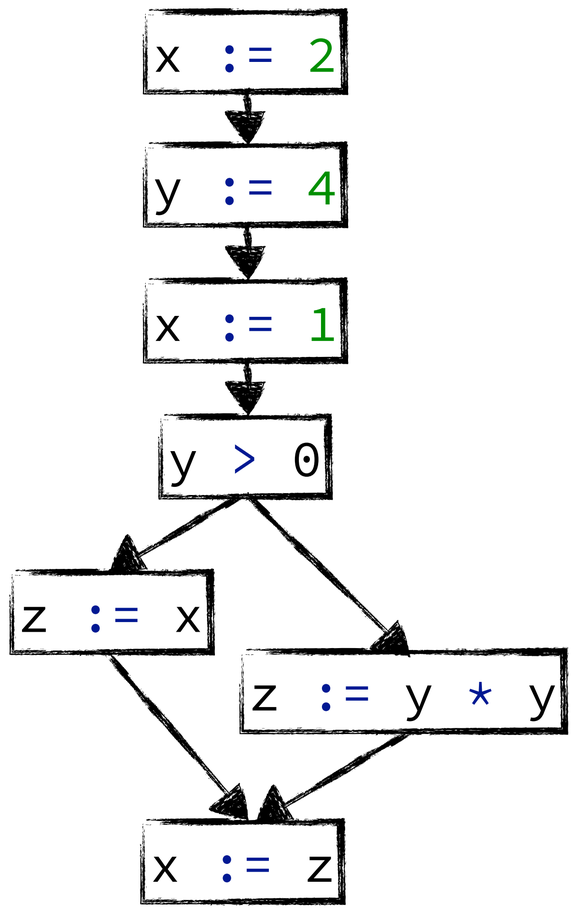

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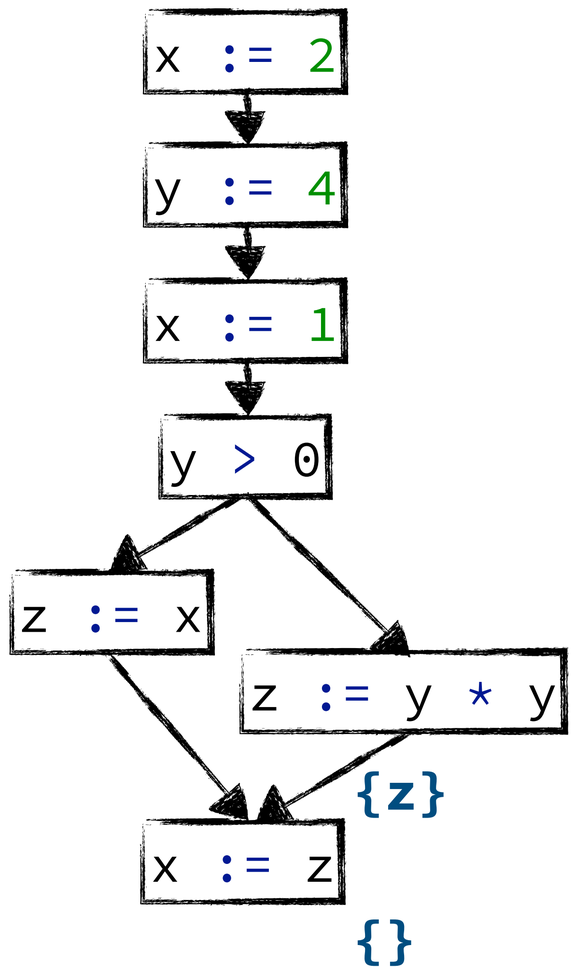

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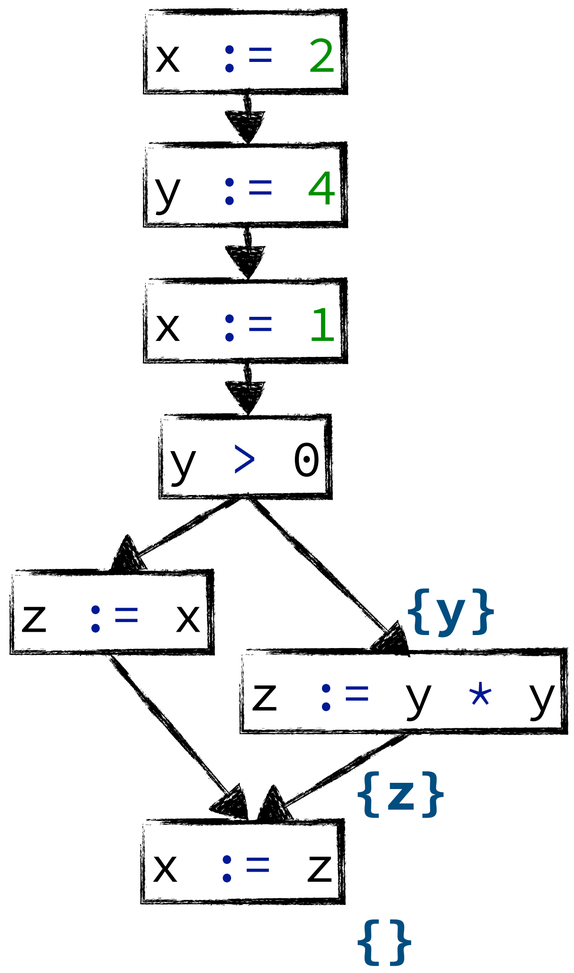

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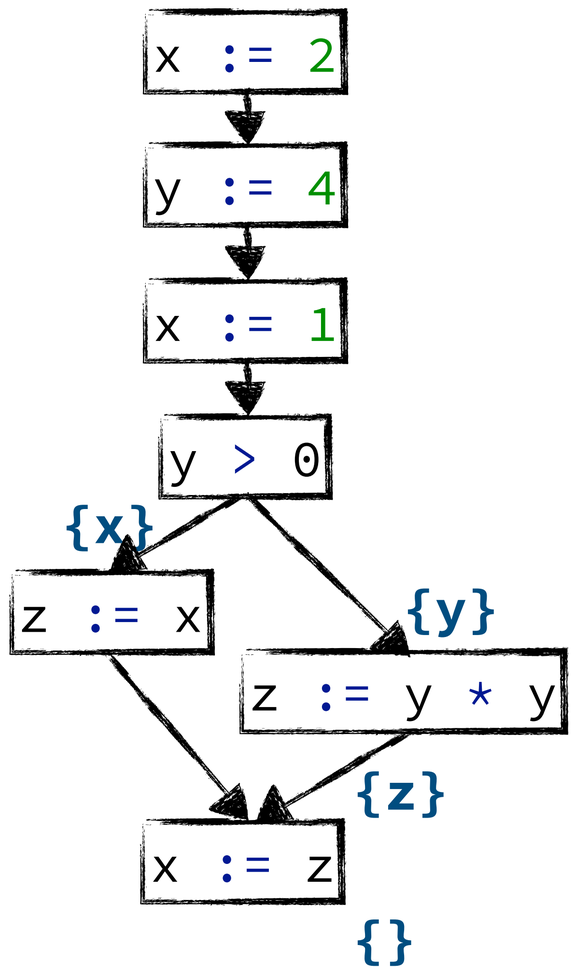

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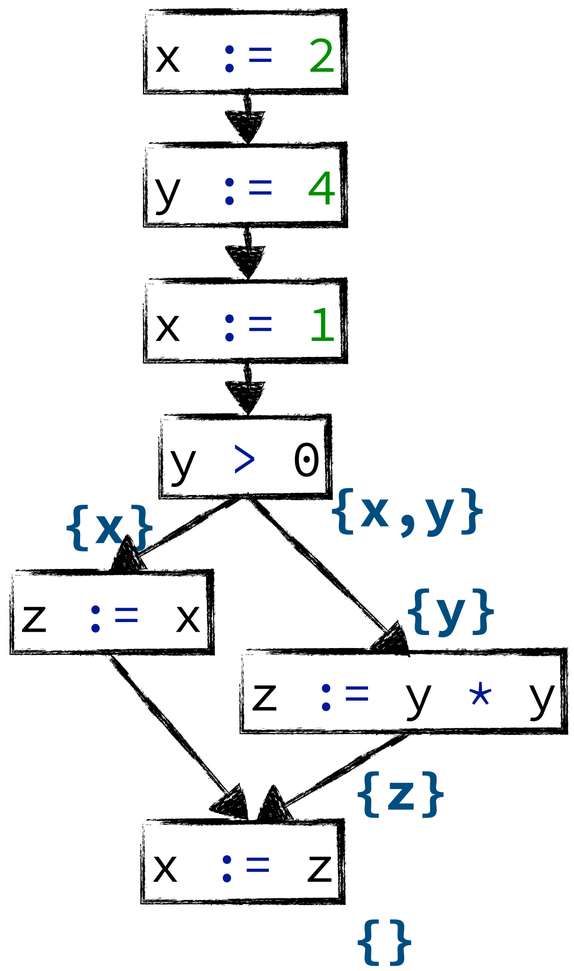

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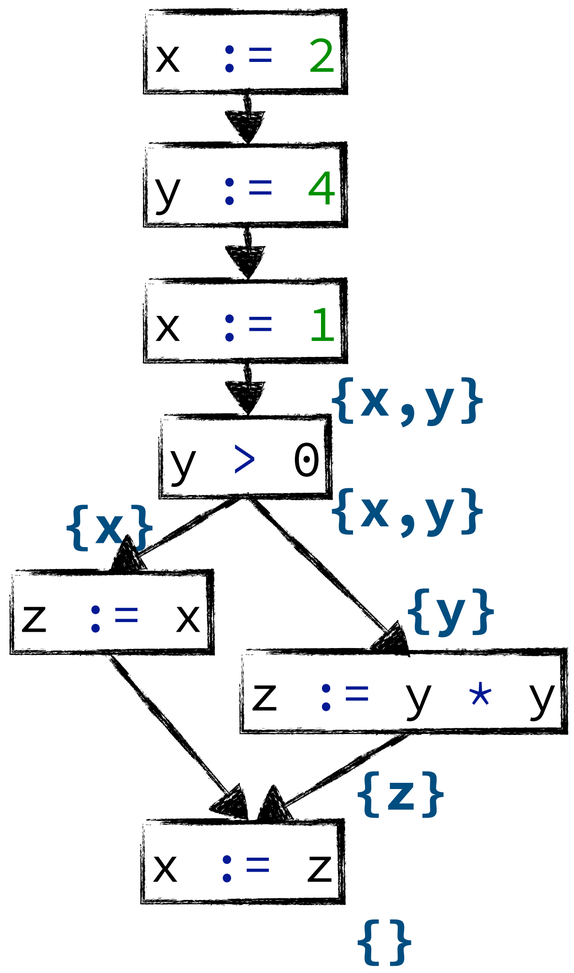

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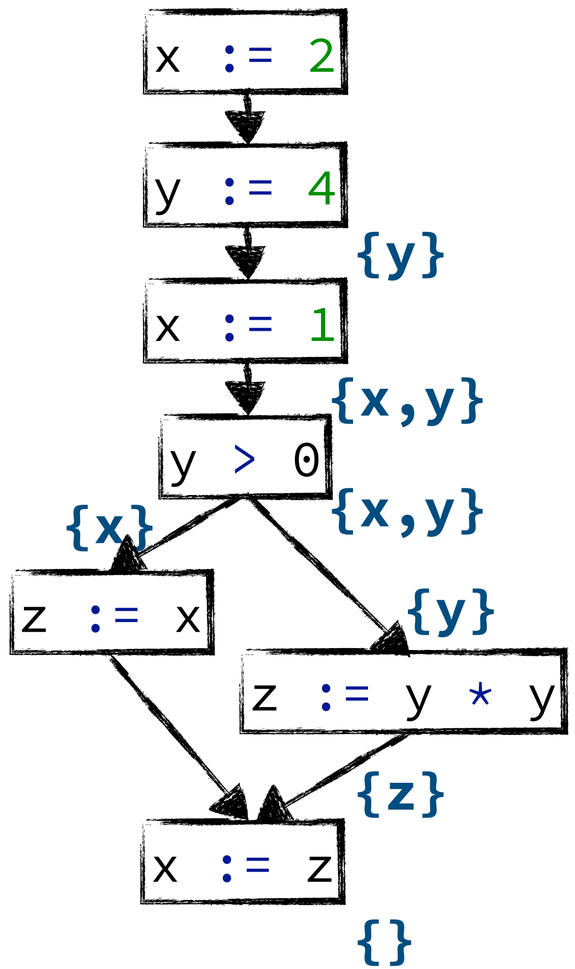

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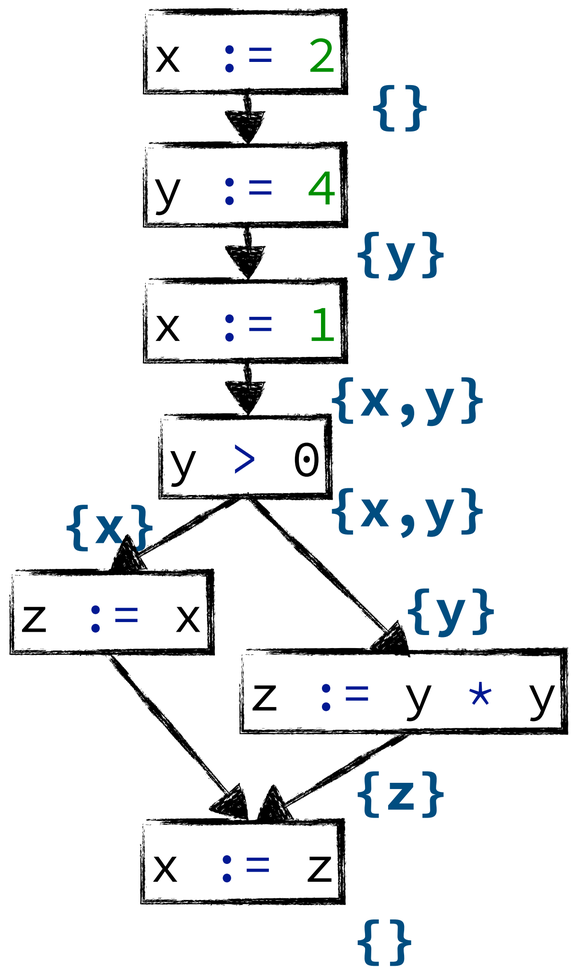

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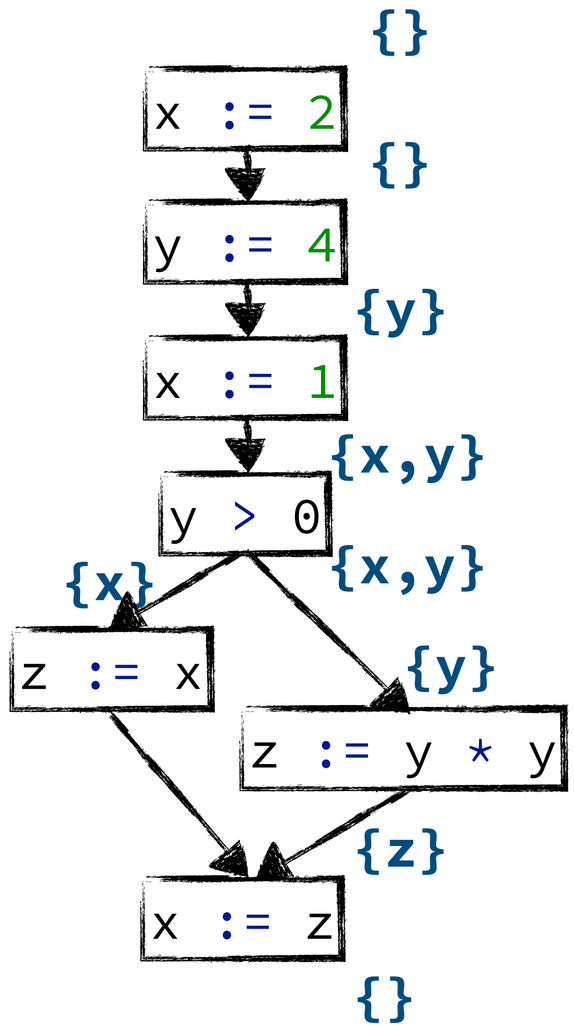

Traditional set based analysis

Sets as analysis information

Kill and **gen** sets per control node type

- previousSet ∖ **kill**(currentNode) ∪ **gen**(currentNode)

Can propagate either **forward** or **backward**

Can merge information with either **union** or **intersection**

- Respectively called **may** and **must** analyses

Beyond Sets

Constant propagation and folding

```
let
  var a : int := 0
  var b : int := a + 1
 in
    c := c + b;
    a := 2 * b
end
```

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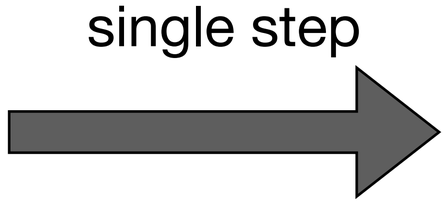

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full propagation

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end
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$a \mapsto 0$

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$a \mapsto 0, b \mapsto 1$

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$a \mapsto 0,\ b \mapsto 1$

$a \mapsto 0,\ b \mapsto 1,\ c \mapsto ?$

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Constant propagation and folding

The type of the analysis information

- Variables bound to either a particular *constant* or a *marker for non-constants*

The transfer functions per control node

- Basically an interpreter implementation for constants
- Needs to propagate markers when found

Monotone Frameworks

Termination

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Data-Flow Analysis needs fixpoint computation

- Because of loops

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Data-Flow Analysis needs fixpoint computation

- Because of loops

To terminate, there needs to *be* a fixpoint

- **And** we need to actually *get* to that fix point
- How can we check this?

Lattice Theory

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A set X is totally ordered under ≤ if for a, b, c ∈ X

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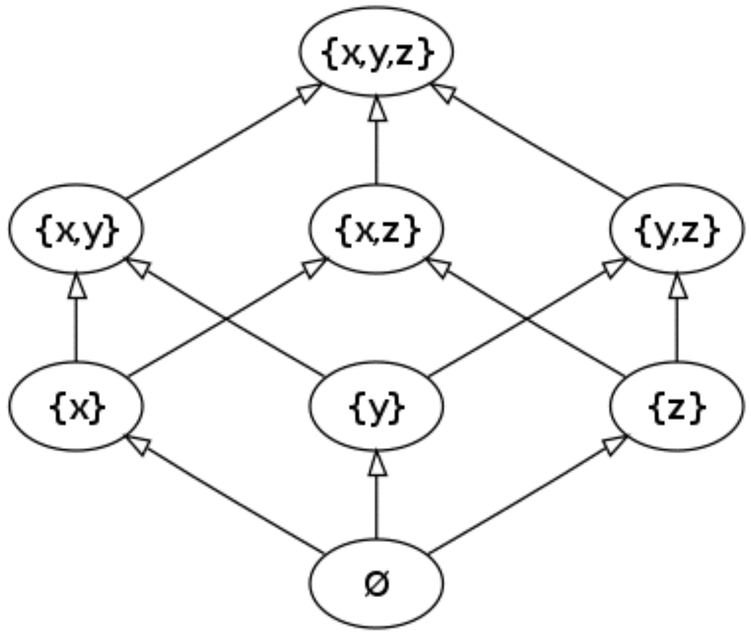

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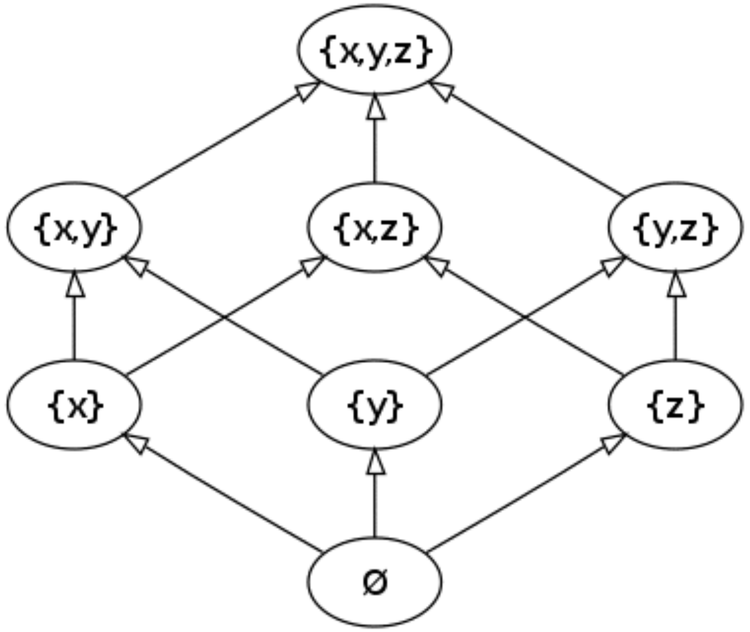

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A *Lattice* is a partially ordered set where

- every two elements have a *unique* least upper bound (or supremum or join)
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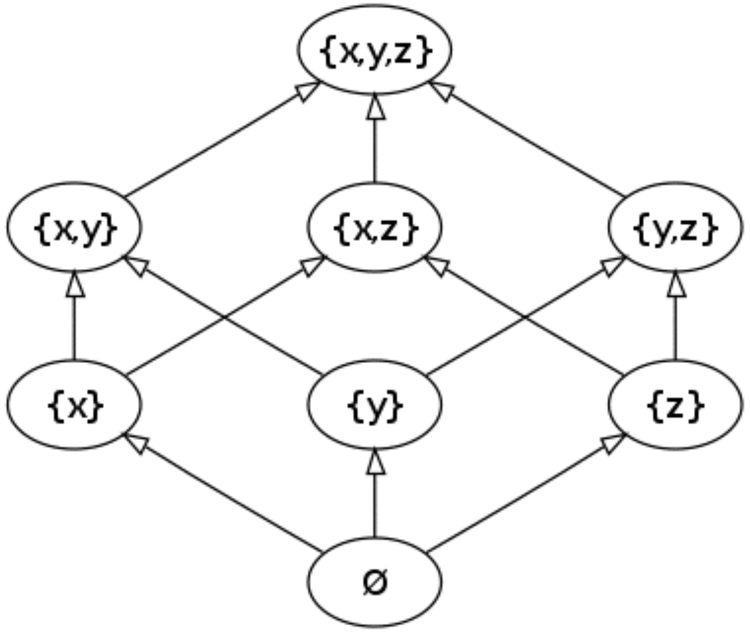

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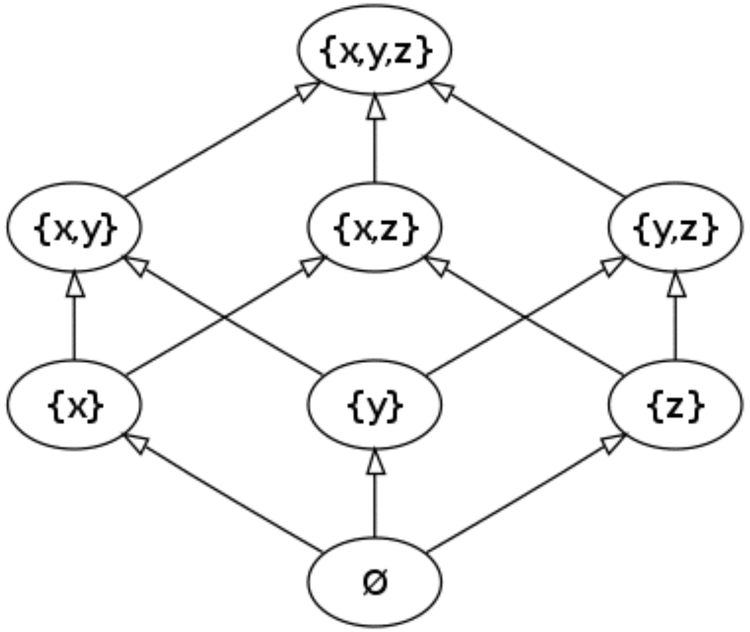

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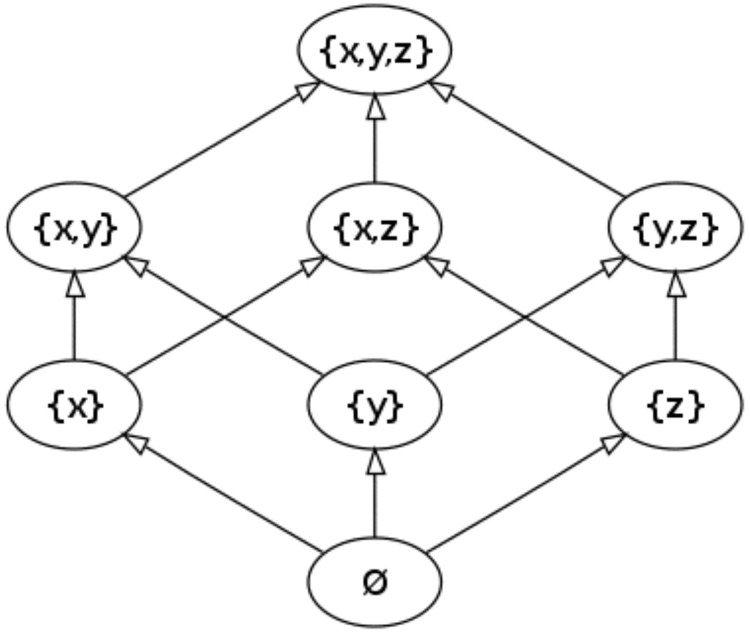

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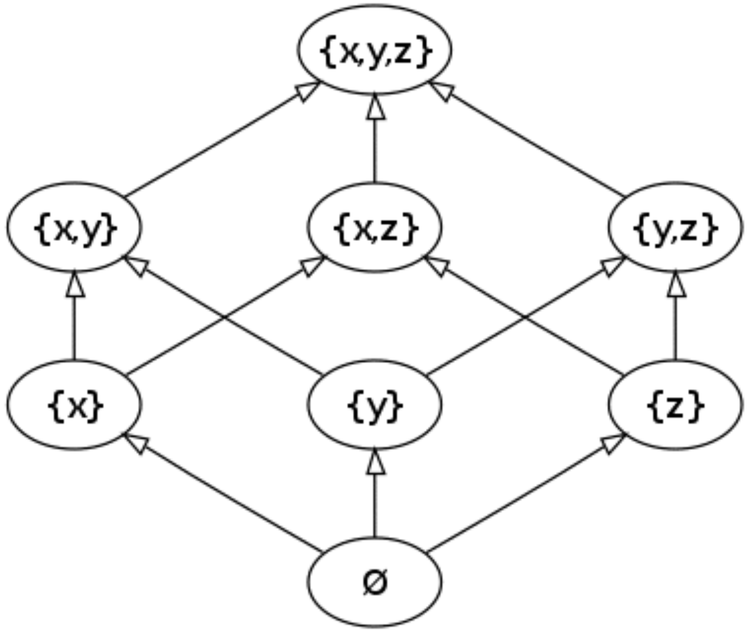

A *bounded* lattice has a top and bottom

- These are $\top$ and $\bot$ respectively

Lattices for data-flow analysis

Consider $\top$ as the coarsest approximation

- It's a safe approximation, because it says we're not sure of anything

Then we can combine data-flow information with $\sqcup$

- It is the most information preserving combination of information

Lattices for data-flow analysis

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General interval analysis has an infinite lattice

- $\top = [-\infty,\infty]$
- If a loop adds a finite number to a variable, you never get to ∞

Recap

An analysis consists of

- The type of the analysis information
- The *transfer functions* that express the ‘effect’ of a control node
- The initial analysis information

Recap

An analysis consists of

- The type of the analysis information, **and the lattice instance for that type**
- The *transfer functions* that express the ‘effect’ of a control node
  - **These should be monotone with respect to the lattice**
- The initial analysis information

Data-Flow Languages

How to define the data-flow rules of a language

?

Data-Flow specification approaches

Languages used for specifying data-flow analysis

- Attribute grammars: JastAdd [Söderberg'13]
- Datalog [Smaragdakis'15]: Doop [Kasterinis'13], Flix [Madsen'16]
- $INCA_L$ / MPS-DF [Szabó'18 / '16]
- Temporal Logic: Silver [VanWyk'10]

Separation of Concerns

Representation

- **Control Flow Graphs**
- **Before/After data-flow information on CFG nodes**

Declarative Rules

- To define control-flow rules of a language
- To define data-flow rules of a language

Language-Independent Tooling

- Analysis
- Code completion
- Refactoring
- Optimisation
- …

Control-flow graphs in FlowSpec

FlowSpec

Example program

```
x := 1;
if y > x then
  z := y;
else
  z := y * y;
y := a * b;
while y > a + b do
 (a := a + 1;
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Control-flow graphs in FlowSpec

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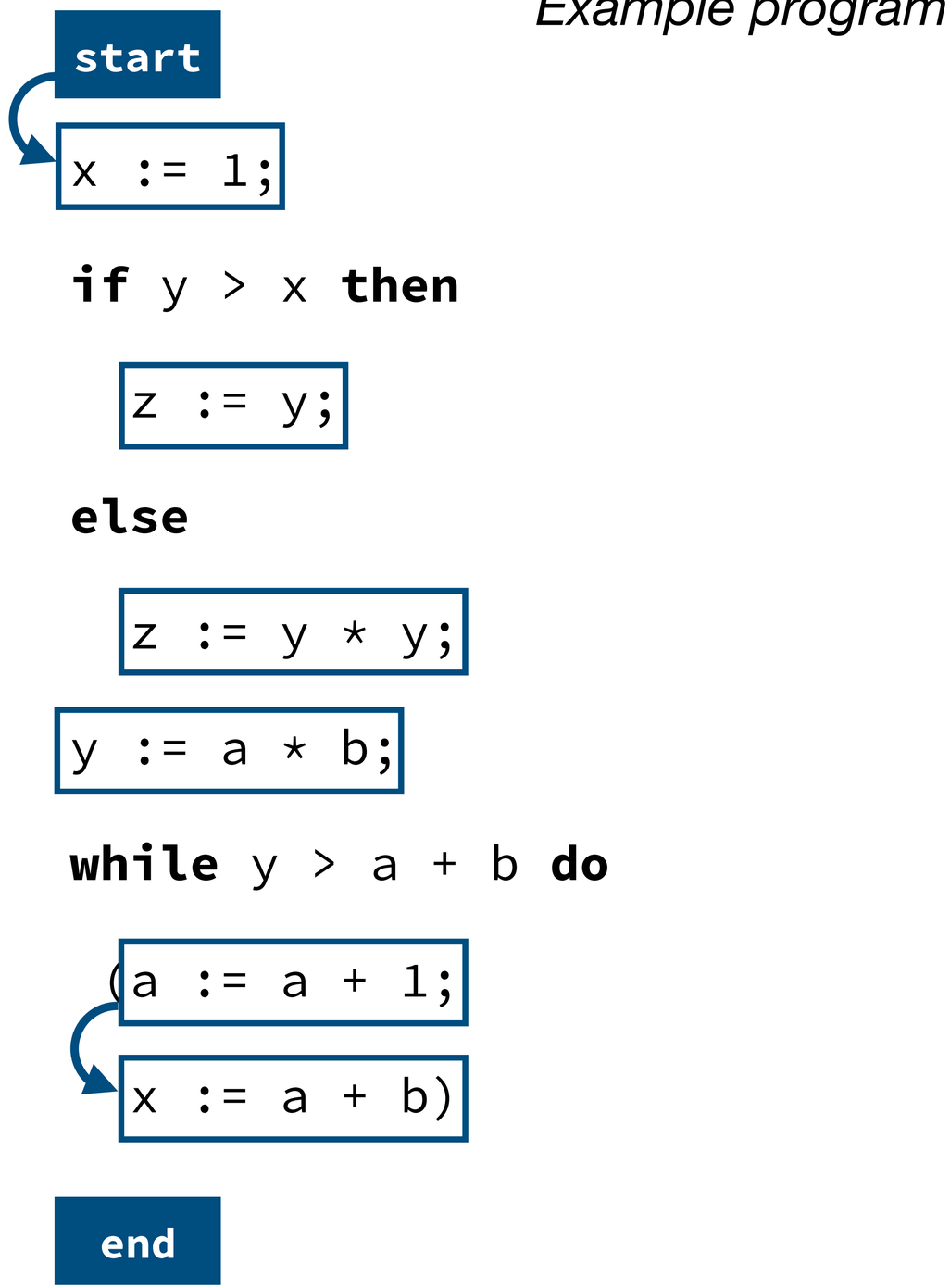

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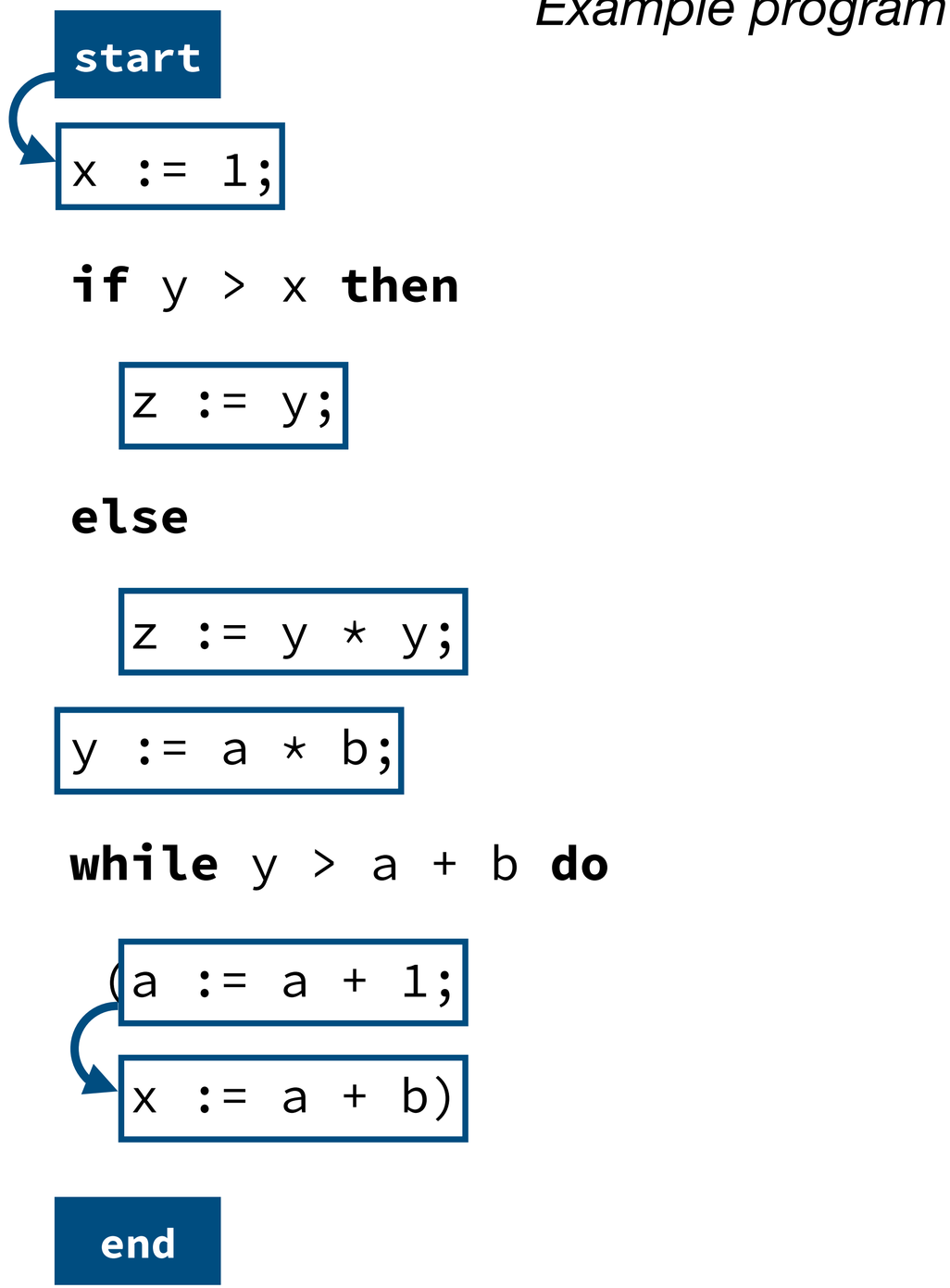

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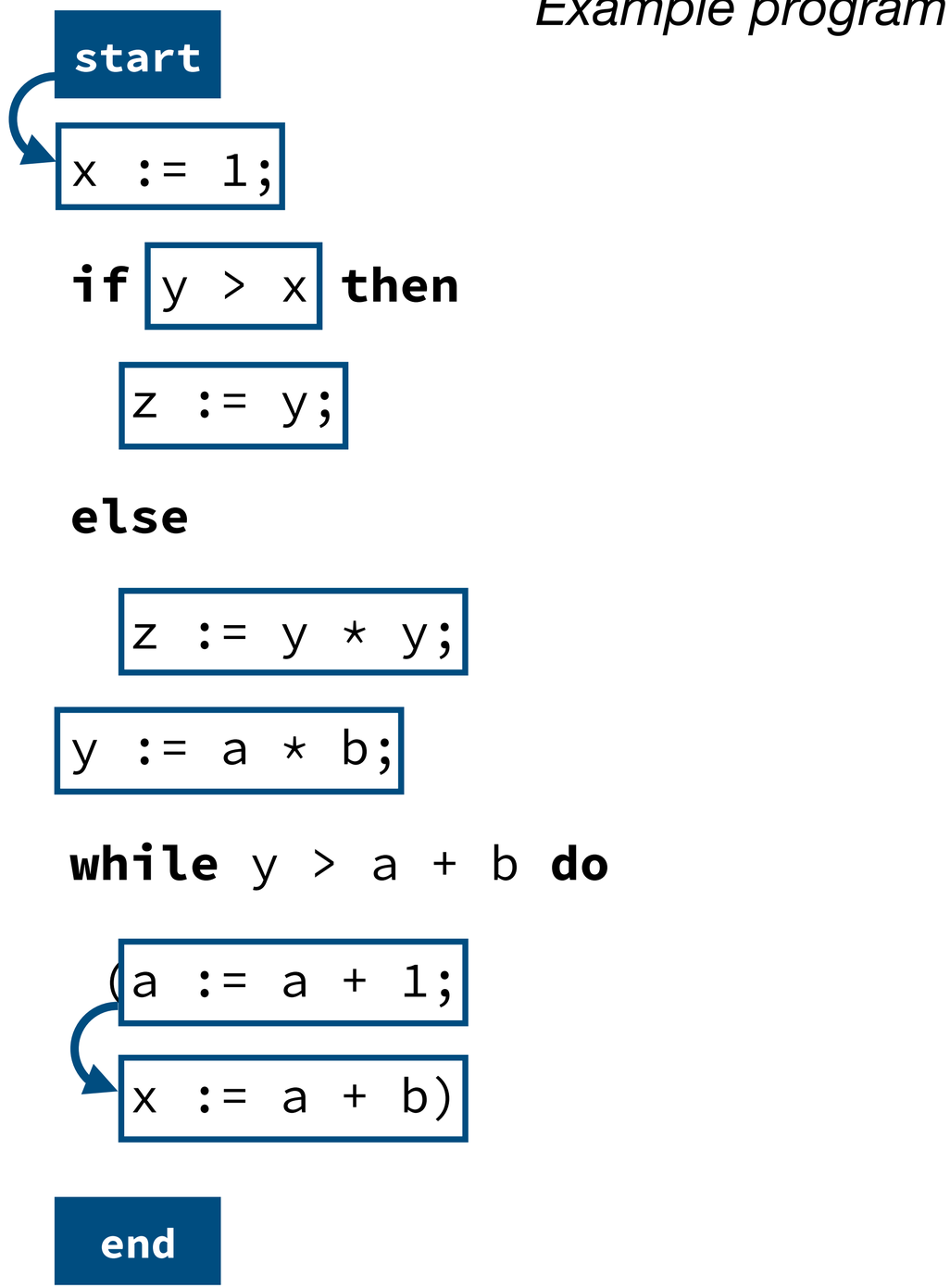

Control-flow graphs in FlowSpec

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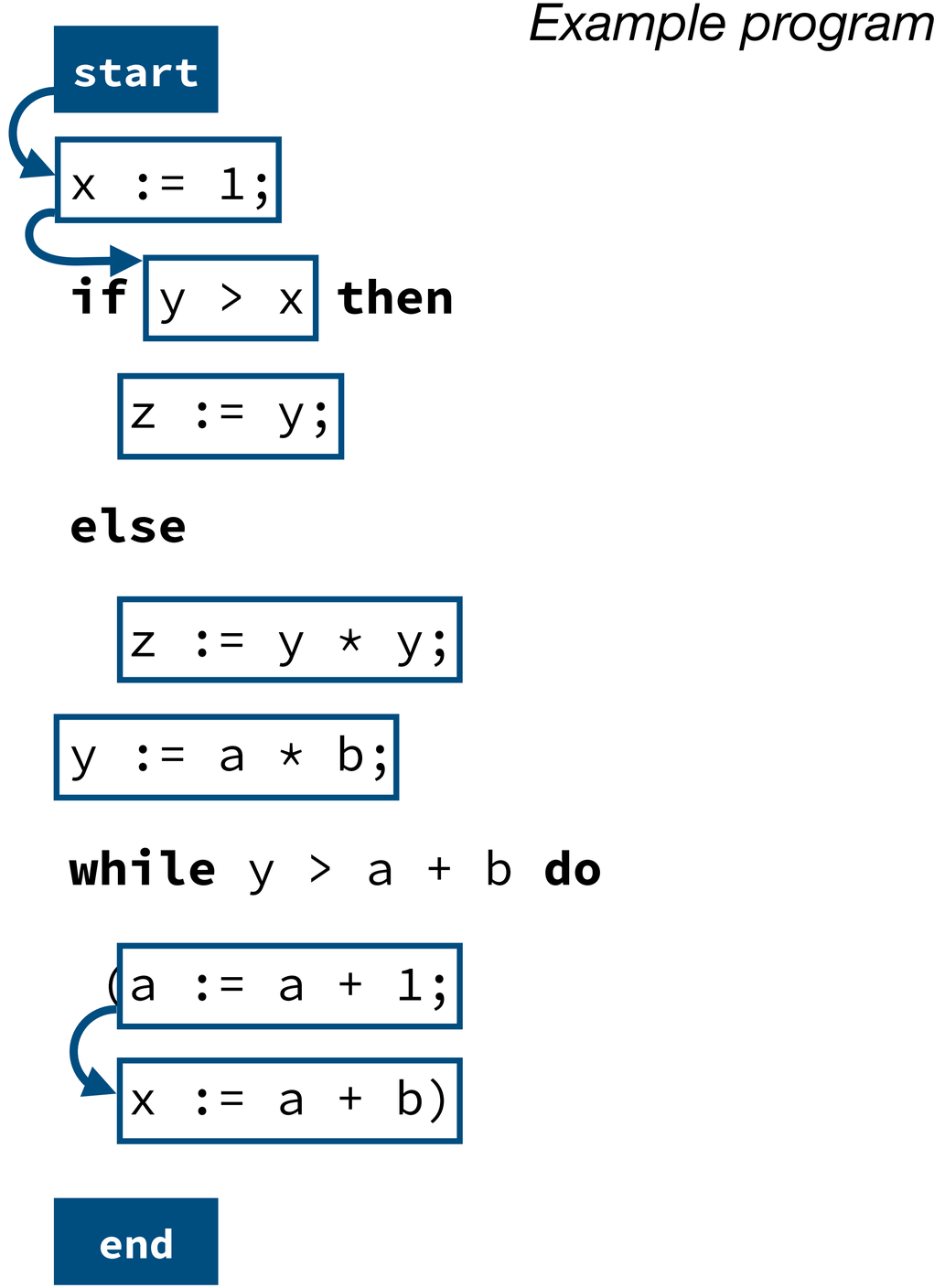

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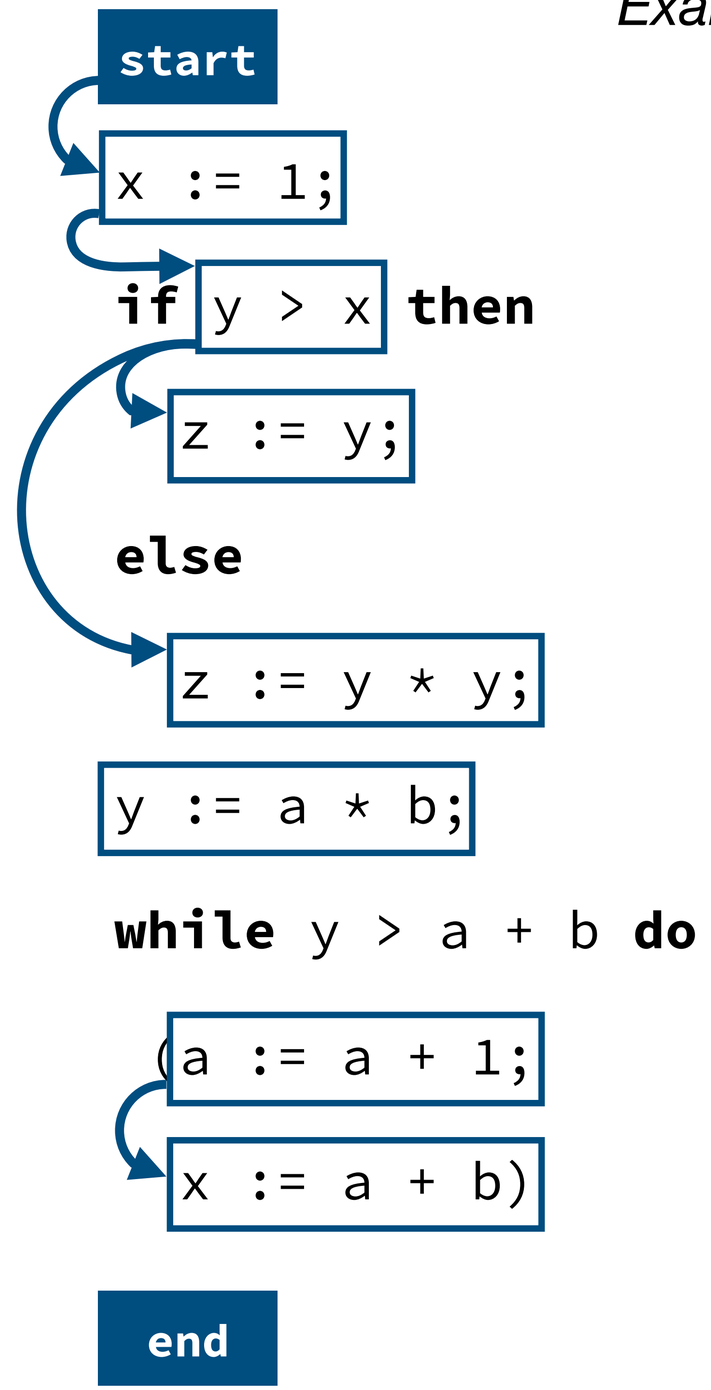

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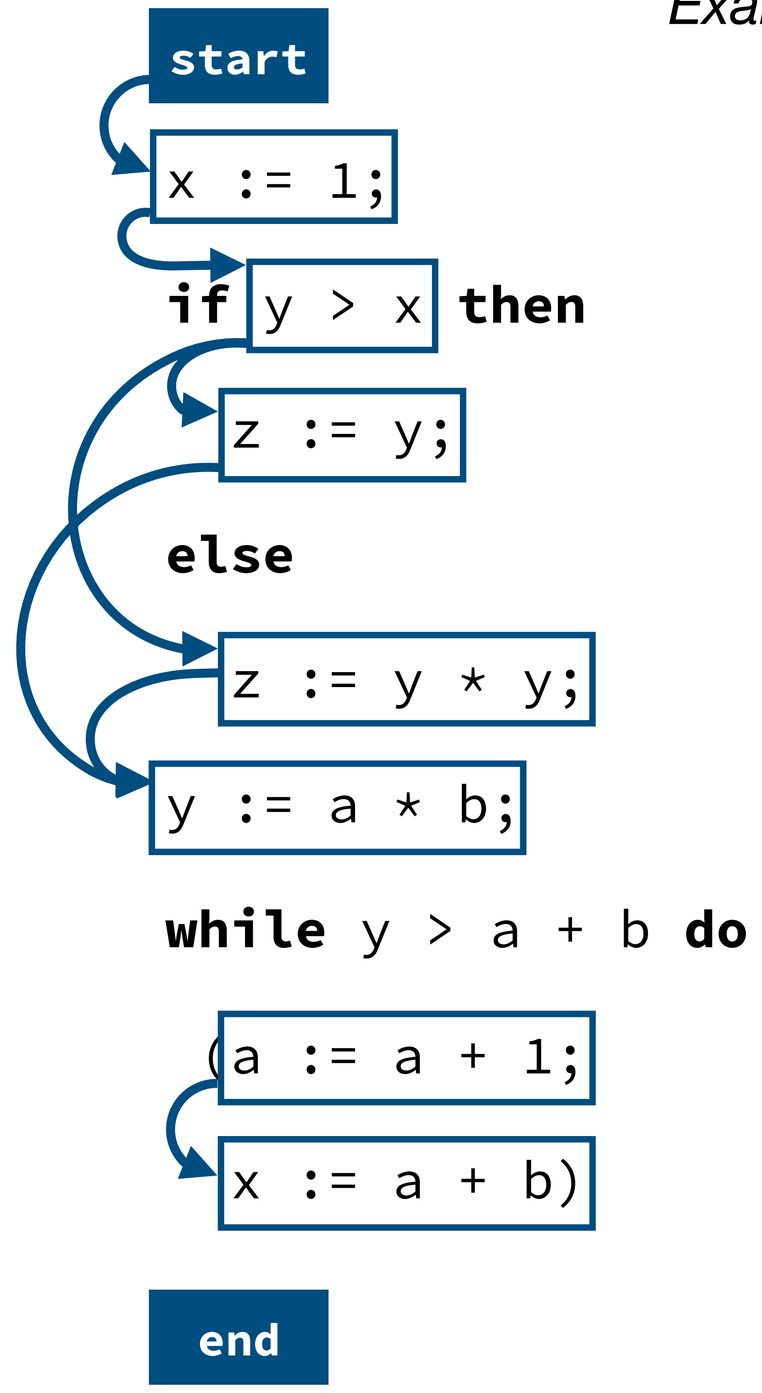

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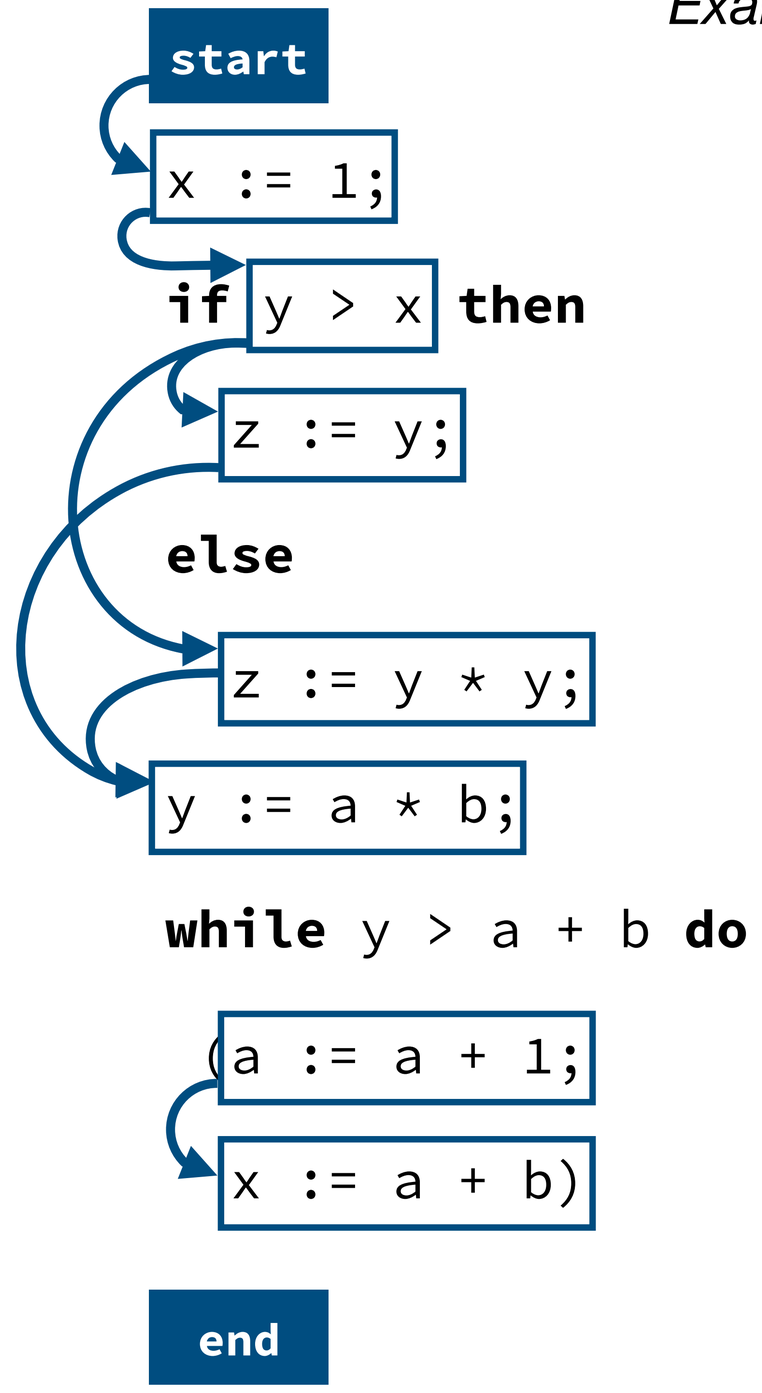

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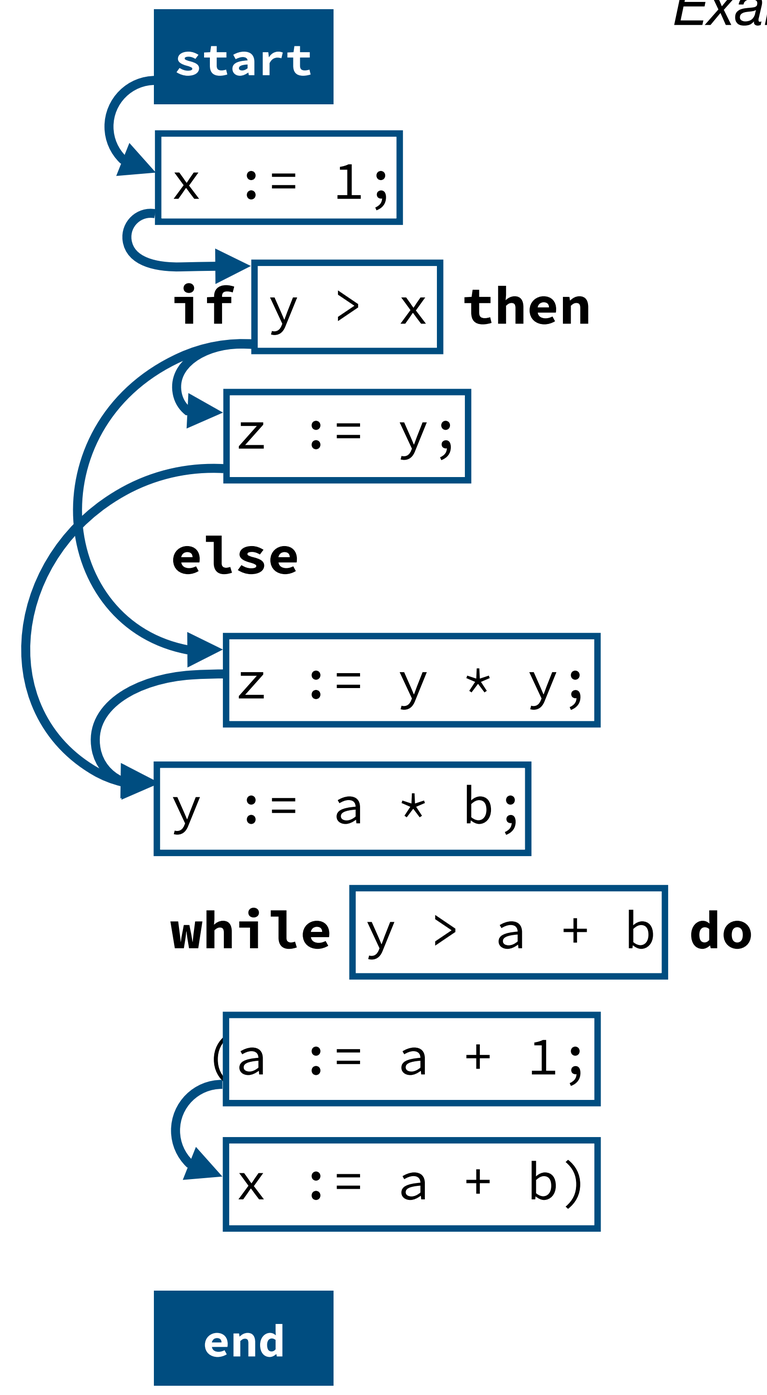

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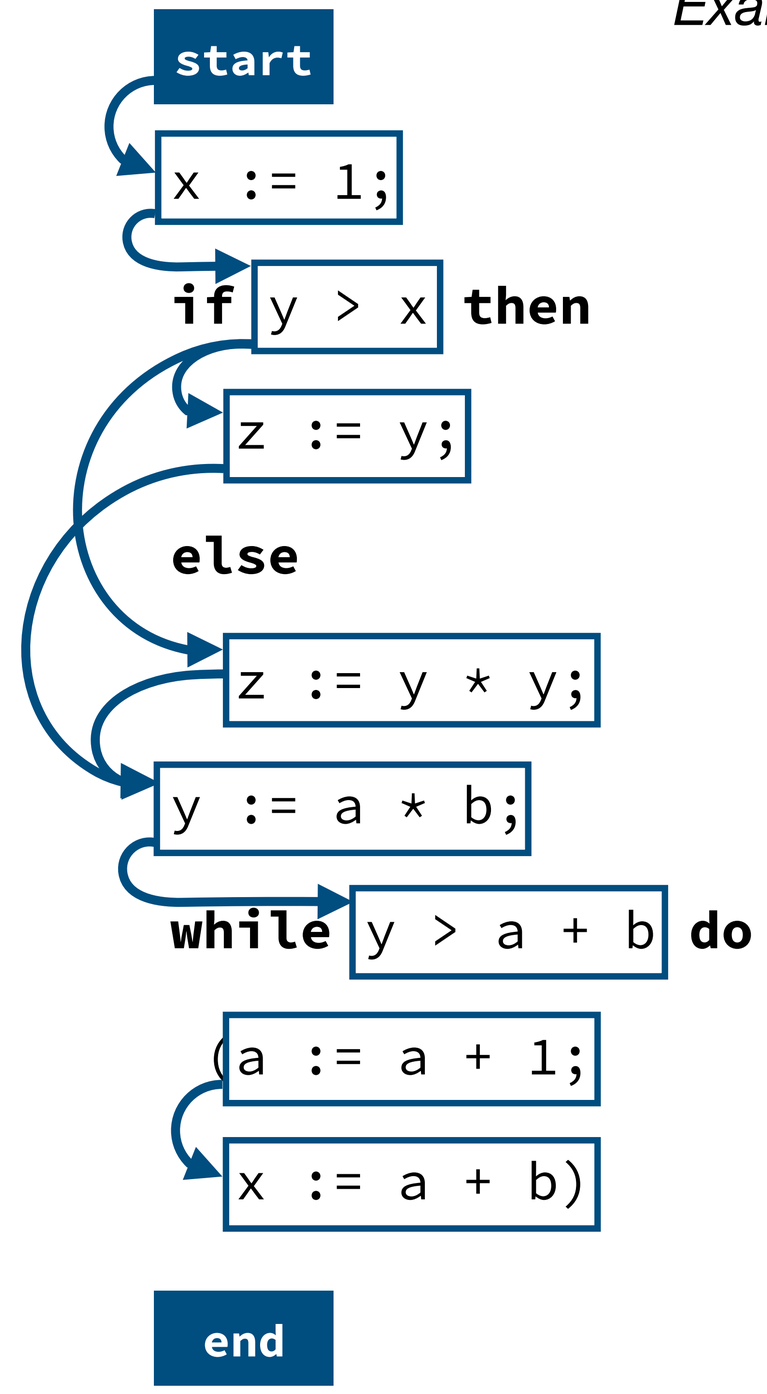

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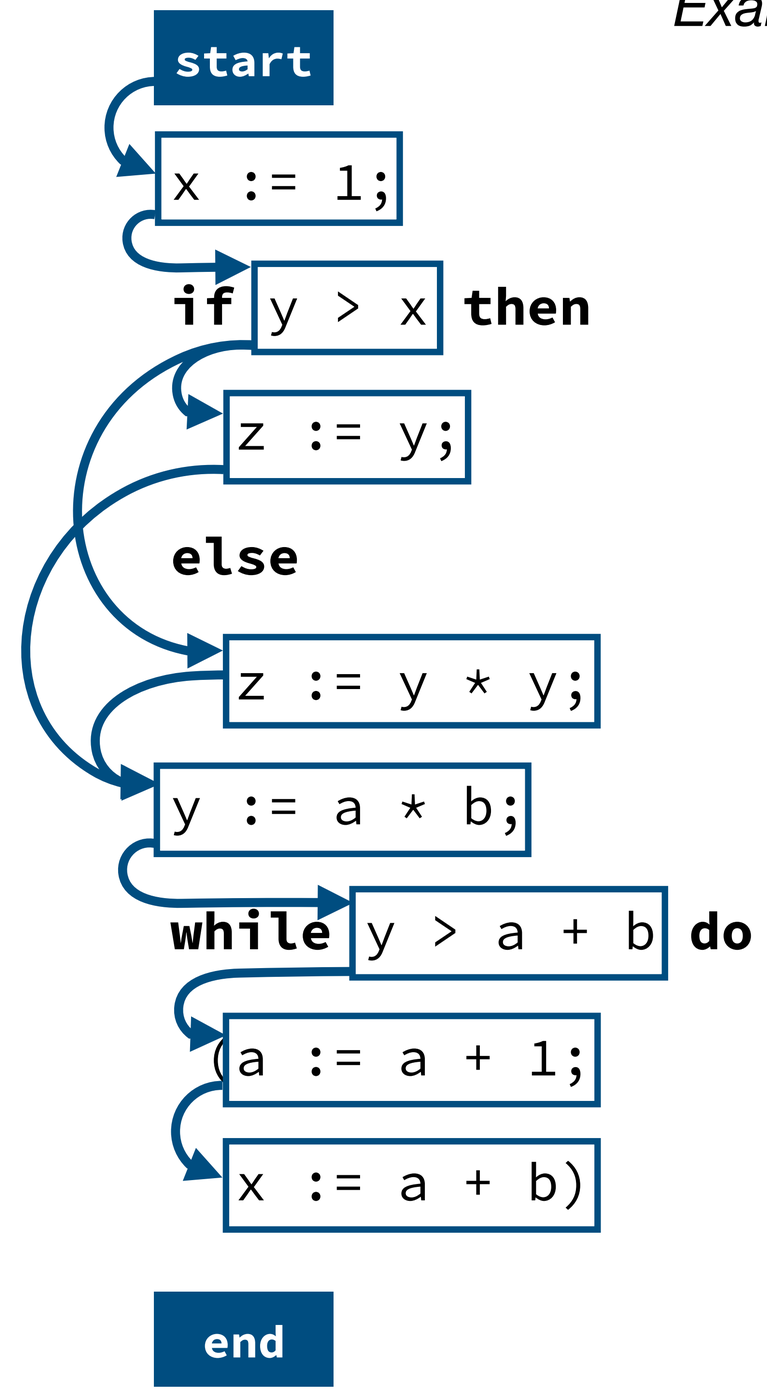

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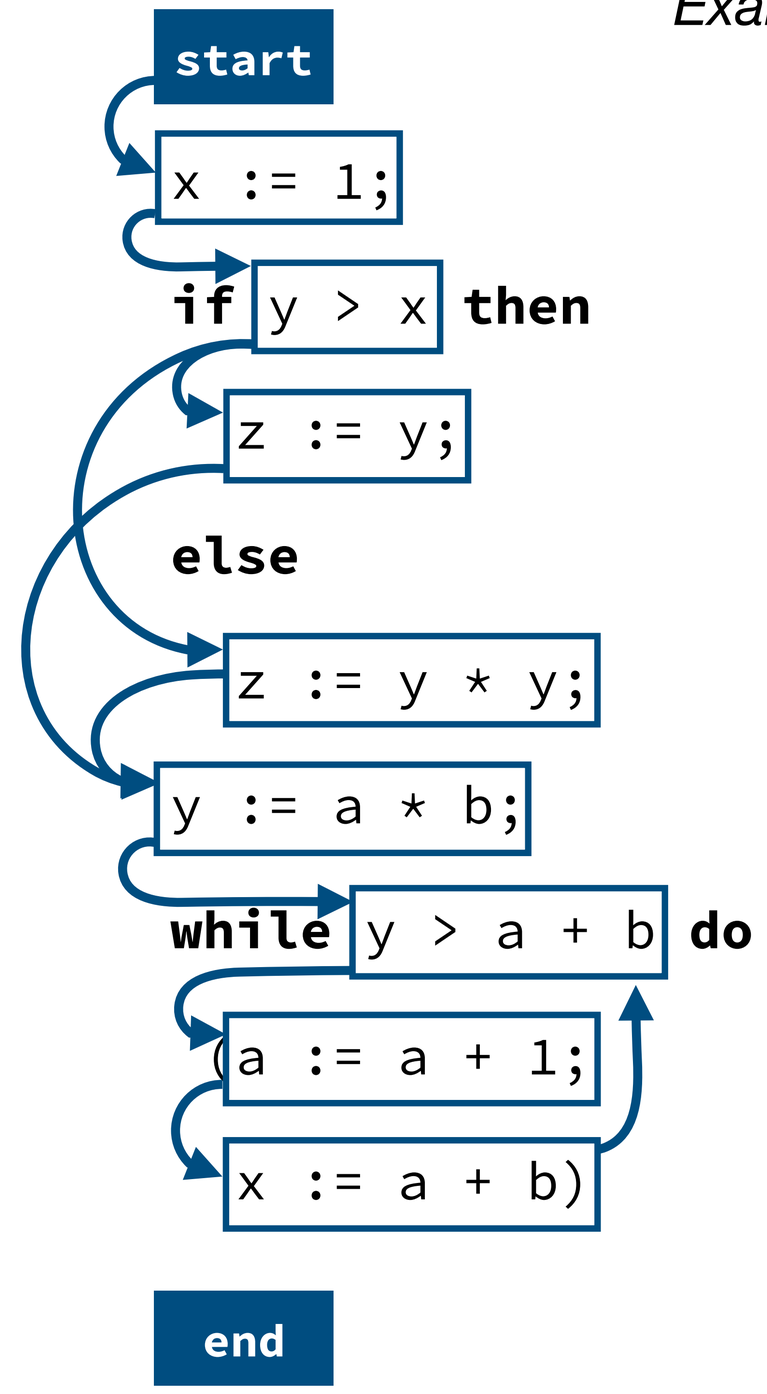

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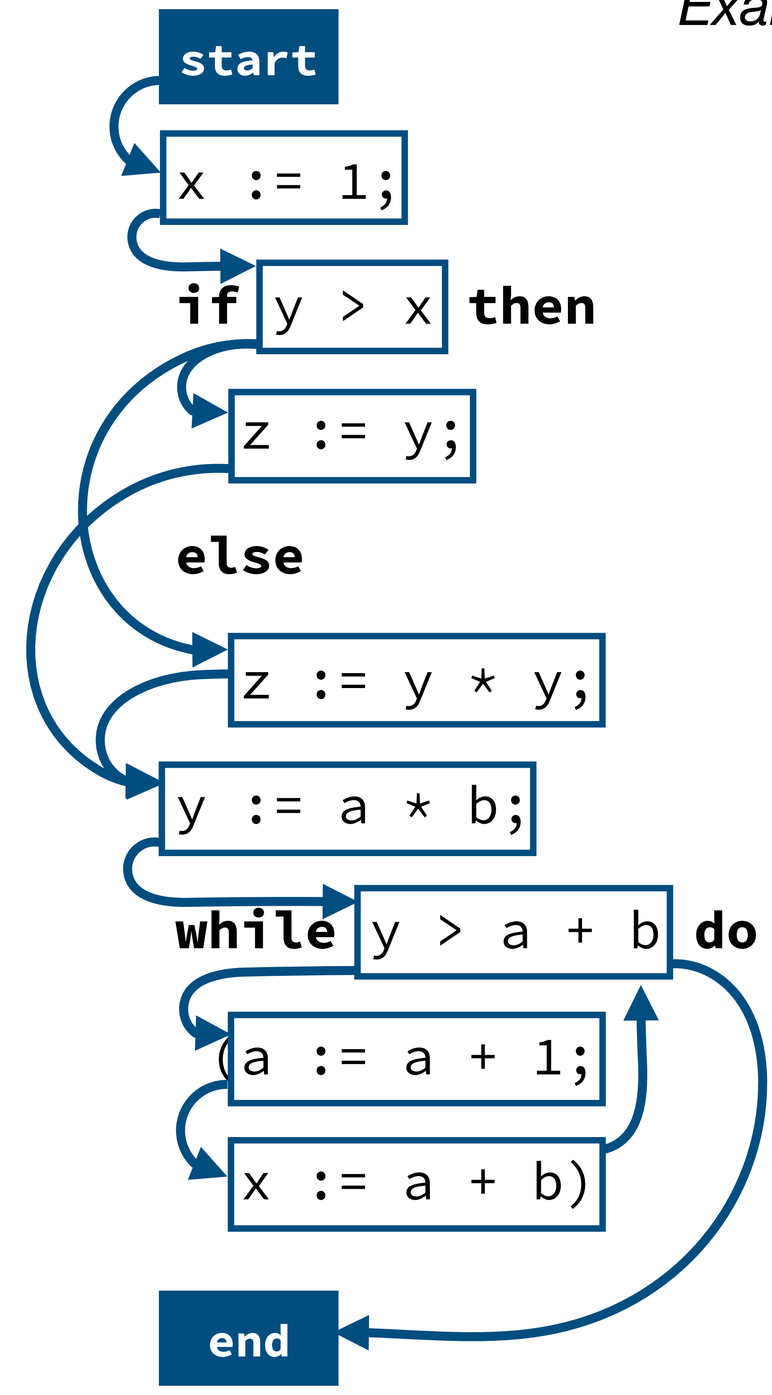

Live Variables in FlowSpec

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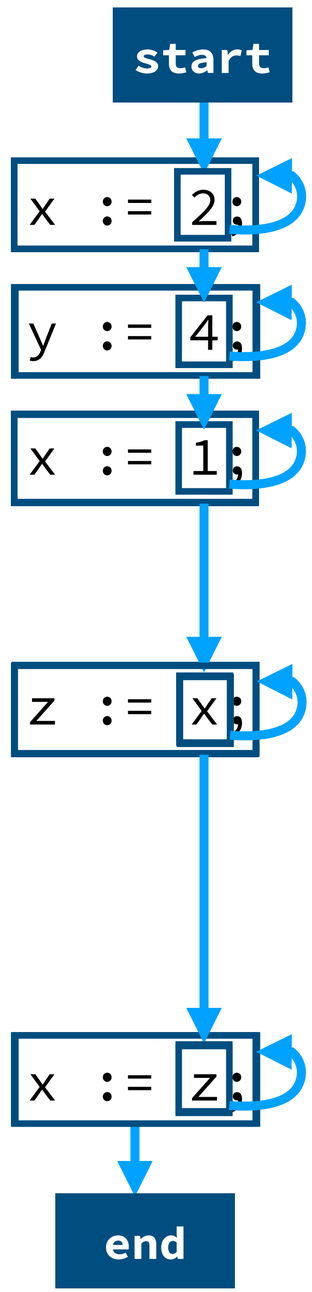

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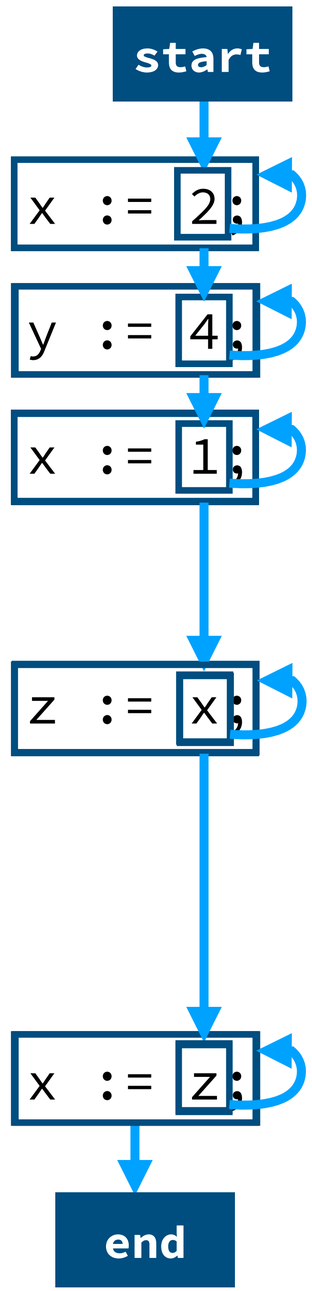
```
prop live:     Set(name)
```



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A variable is *live* if the current value of the variable *may* be read further along in the program

```
prop live:     Set(name)
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```
live(end) =
  {}

live(_ -> next) =
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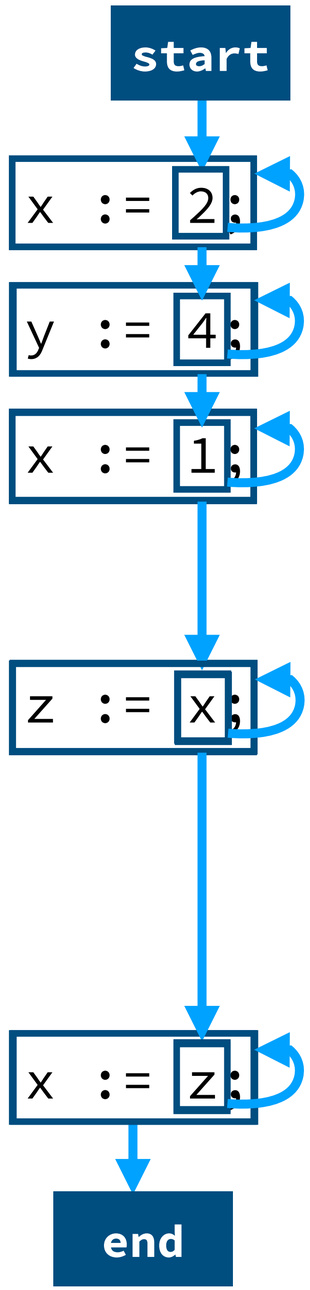

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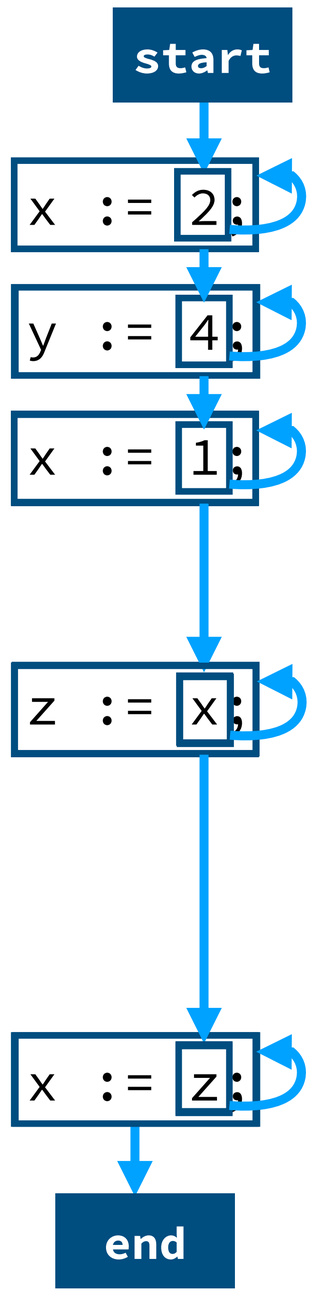

{}

{}

{}

{}

{}

{}

{}

{}

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```
prop live:     Set(name)

live(Ref(n) -> next) =
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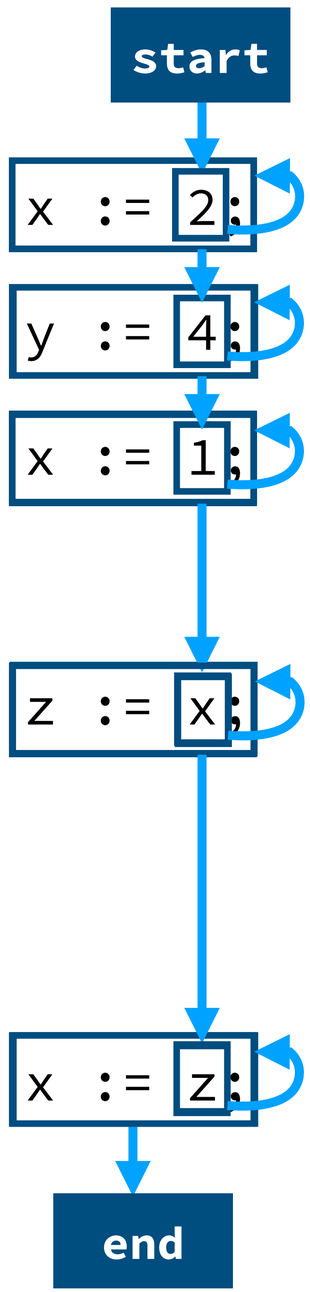
```



{}

{}

{}

{}

{}

{}

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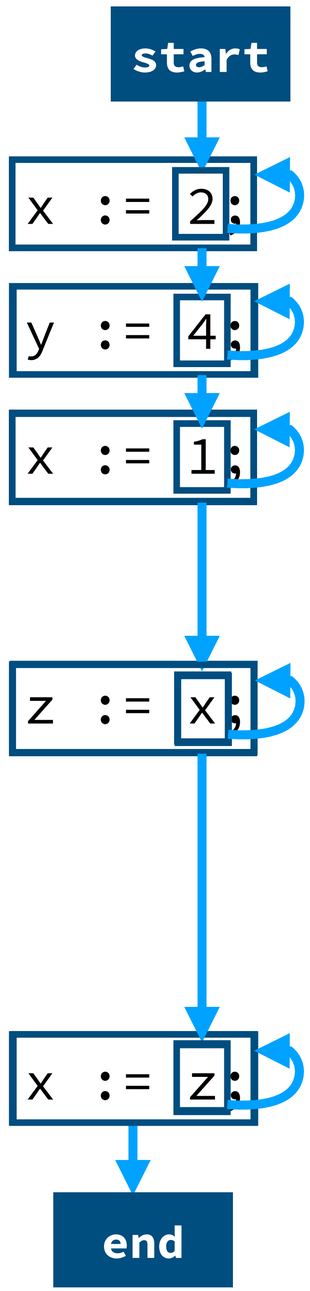

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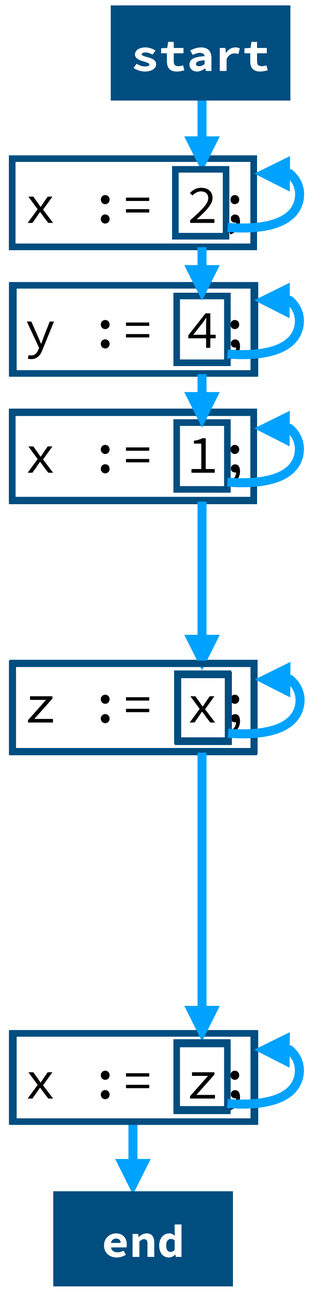


```
{z,x}
{z,x}
{z,x}
{z,x}
{z,x}
{z}
{}
{}
```

Live Variables in FlowSpec

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prop live:     Set(name)

live(Ref(n) -> next) =
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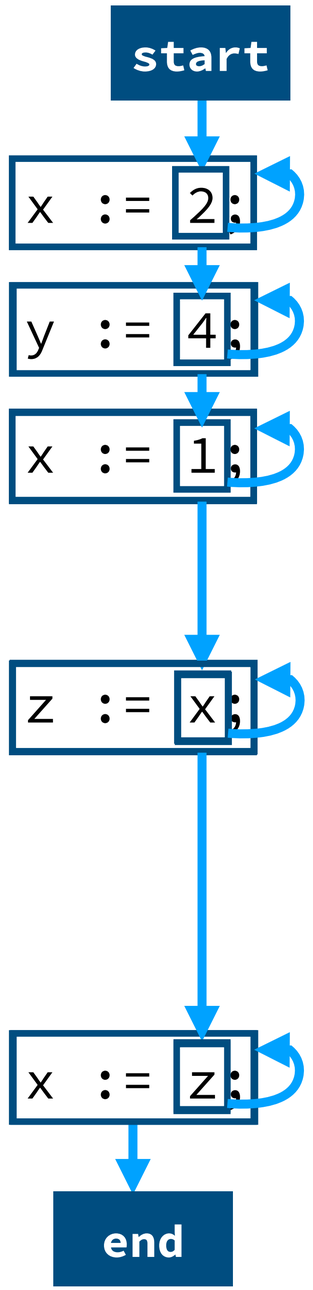

{z,x}

{z,x}

{z,x}

{z,x}

{z,x}

{z}

{}

{}

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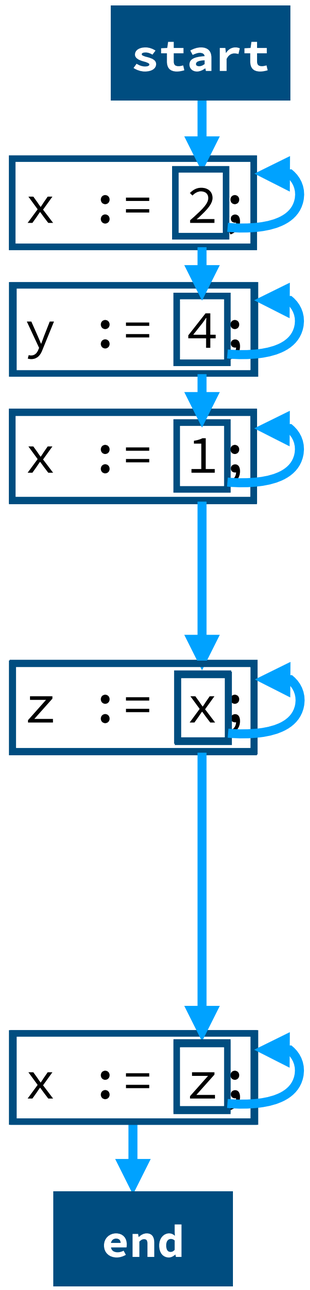

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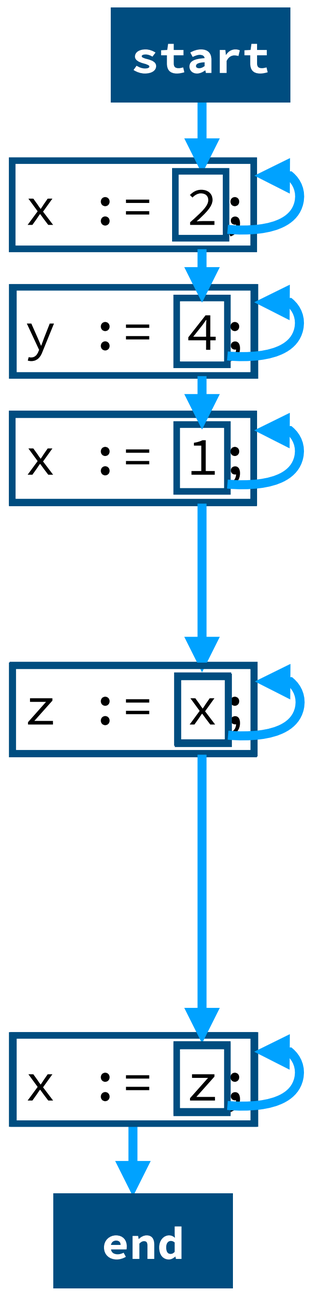

{}

{}

{}

{x}

{z}

{}

{}

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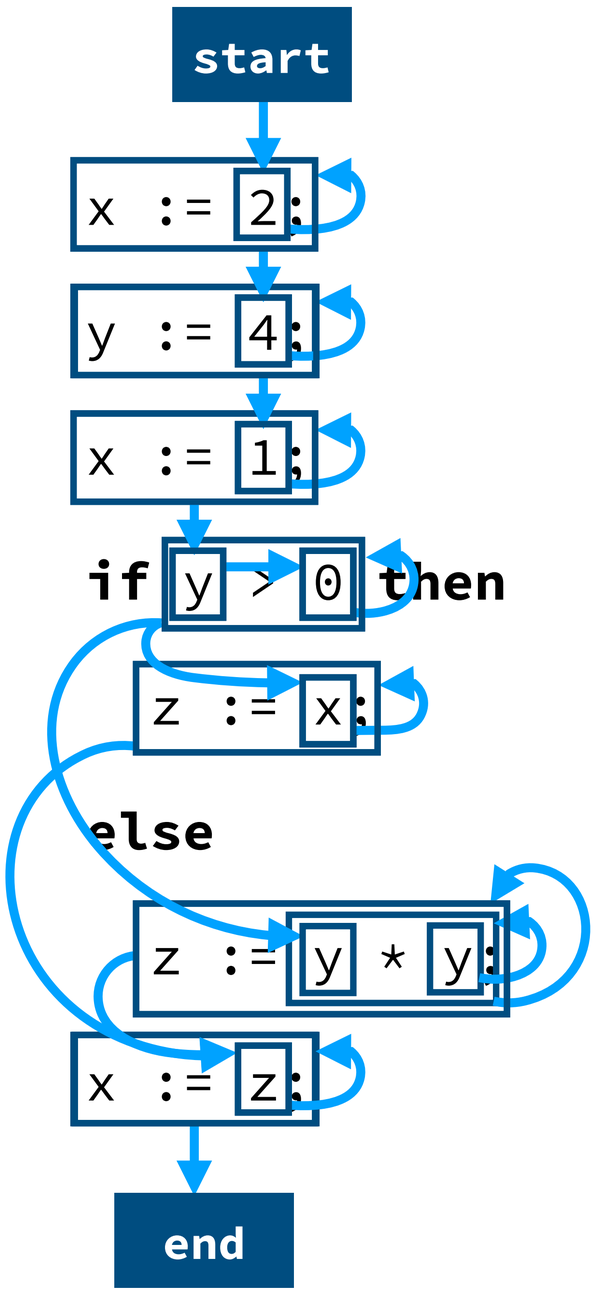

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```
x := 2;

y := 4;

x := 1;

if y > 0 then

  z := x;

else

  z := y * y;

x := z;
```

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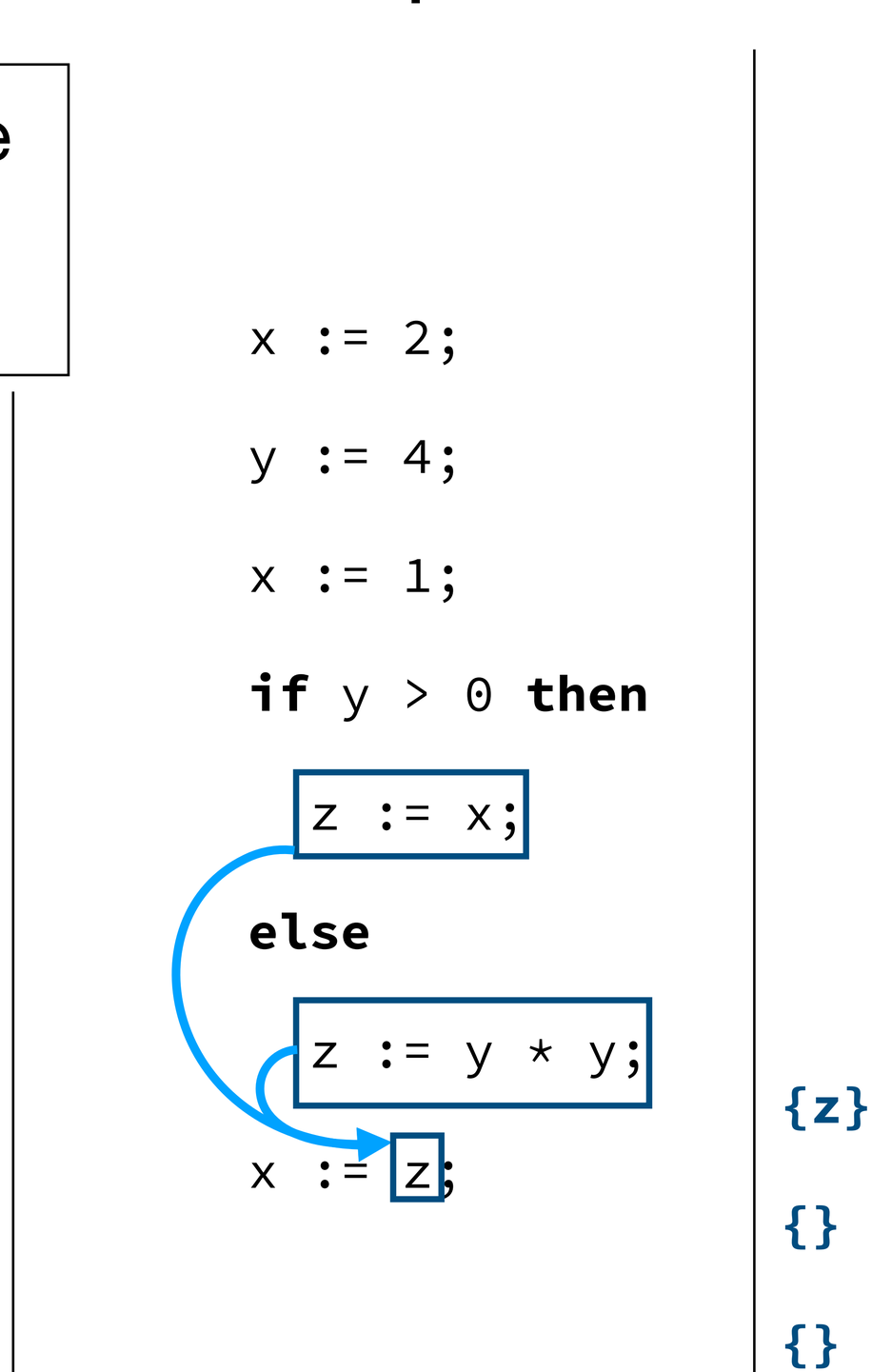

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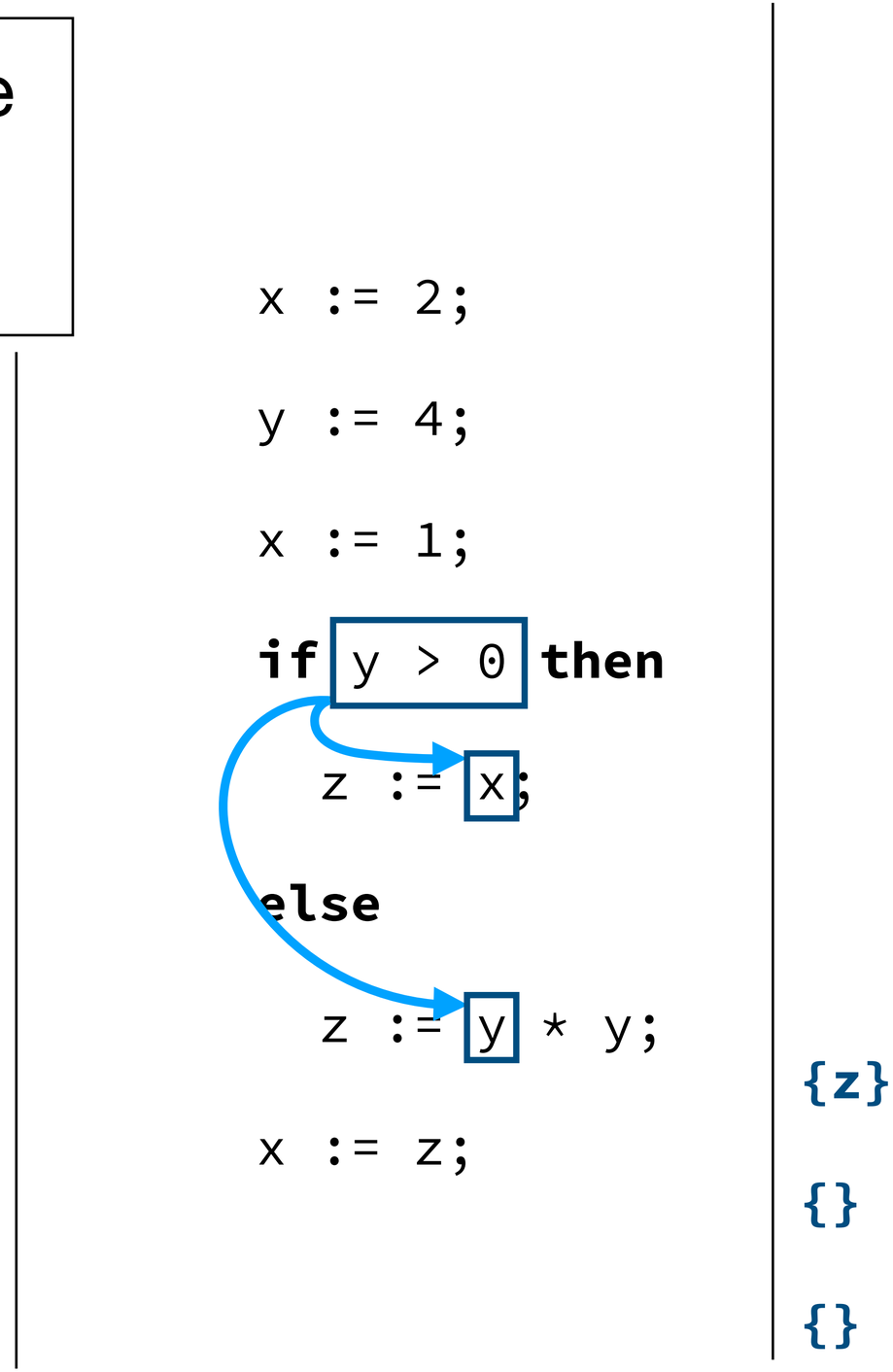

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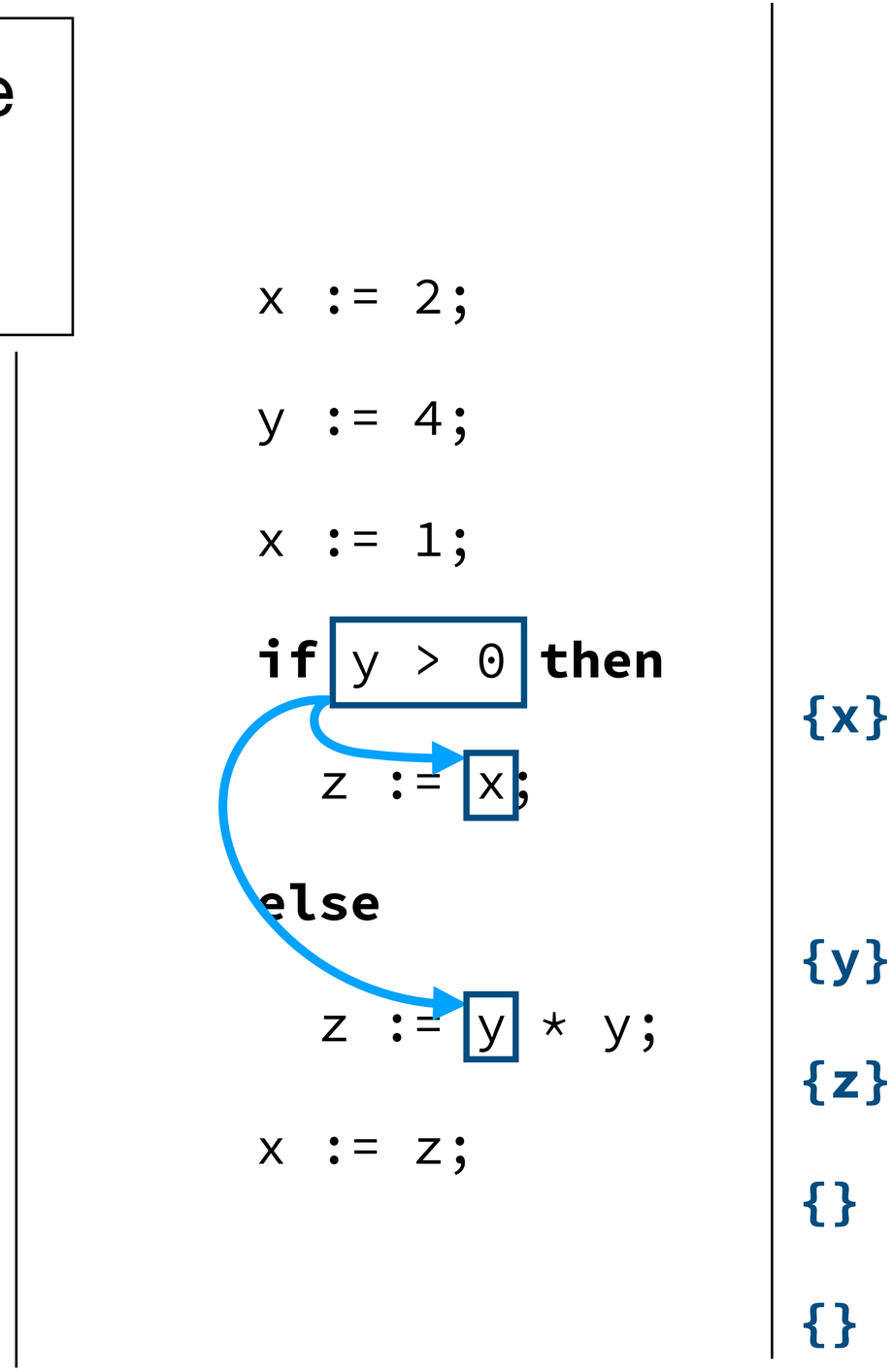

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```
prop live: MaySet(name)

live(Ref(n) -> next) =
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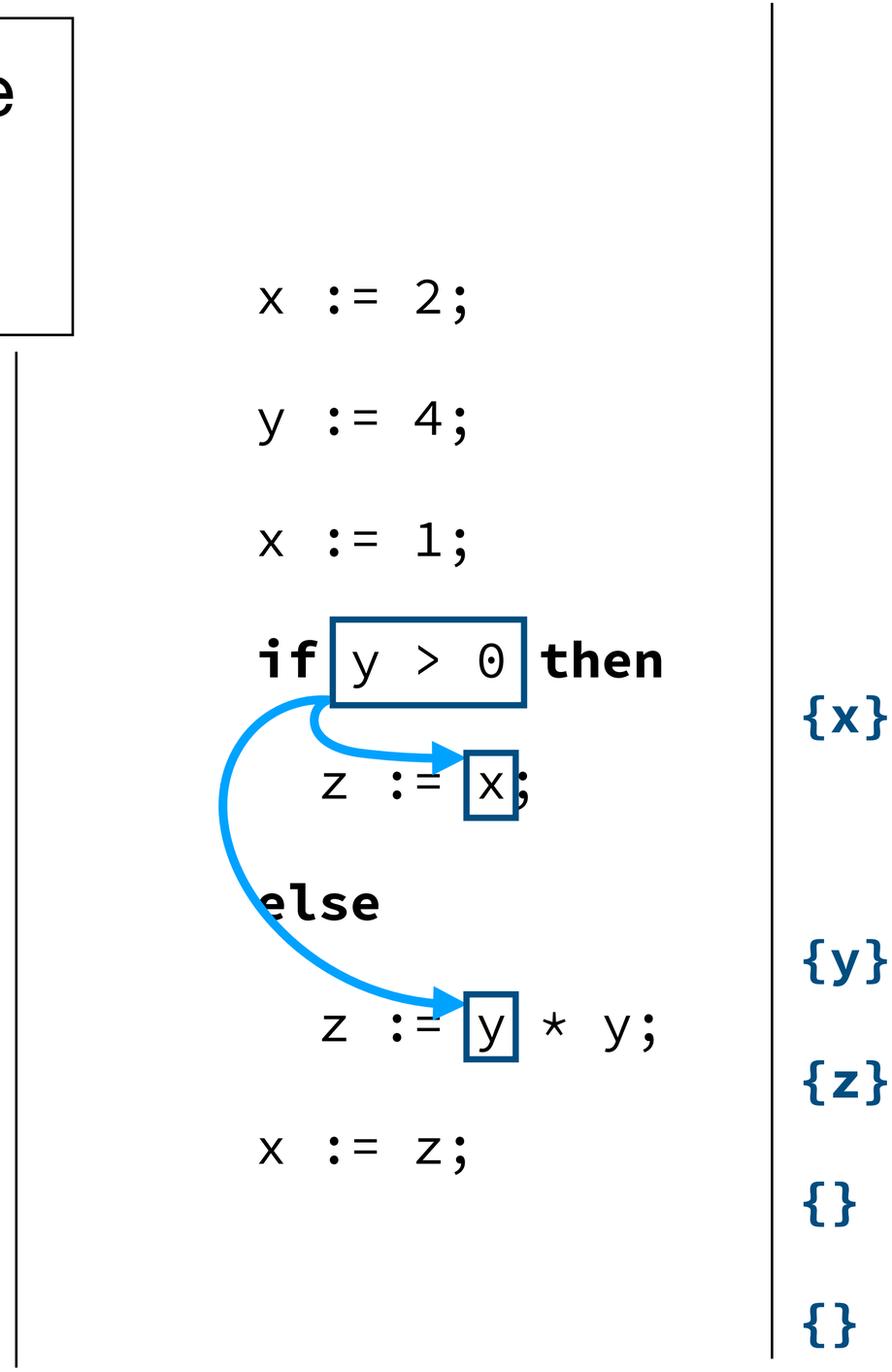

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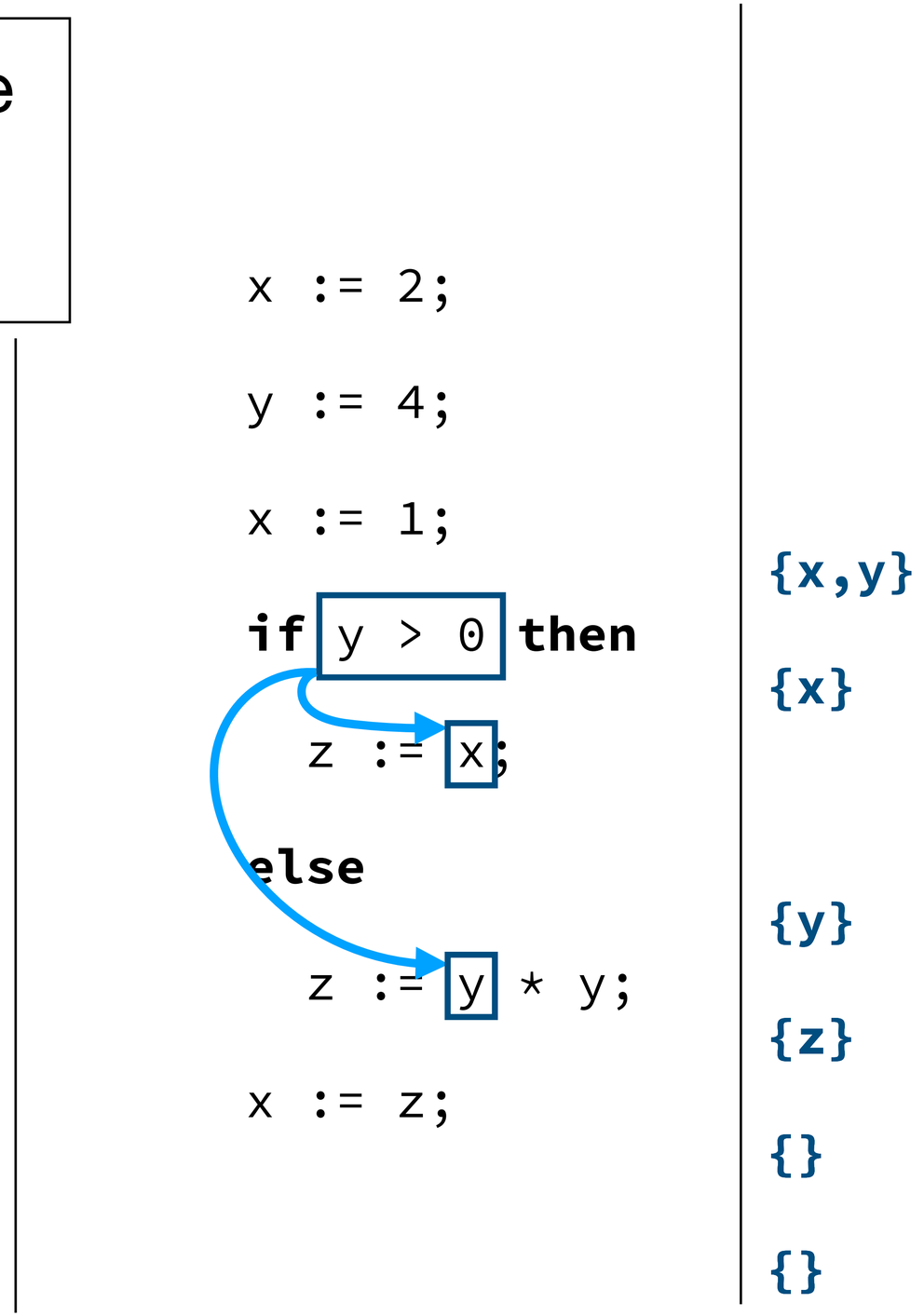

Closing

But wait! There’s more!

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- Follow function calls / where were we called from?

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Incrementality

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- Follow function calls / where were we called from?
- Dynamic dispatch: where did objects come from?
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Incrementality

- Optimisation: transform the program into a more efficient one

But wait! There's more!

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- User changes part of the program (typing, or refactoring)

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- Optimisation: transform the program into a more efficient one
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- Analysis results are *partially* invalid now

But wait! There's more!

Context-sensitivity

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Incrementality

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External PhD candidate Tamás Szabó researches incremental static analysis under supervision of Sebastian Erdweg

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PhD candidate Sven Keidel (HB08.260) researches abstract interpretation under supervision of Sebastian Erdweg

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- Can you generate a proof to show this property?
  - Perhaps using parameterised program equivalence
  - You might leverage scope graphs in your model!

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PhD candidate Arjen Rouvoet (HB08.260) researches optimisation correctness under supervision of Eelco Visser

Future Work

A running FlowSpec prototype

- So, no, you can't try out FlowSpec yet
- Almost done though, promise!

Evaluation of FlowSpec

- Is it powerful enough to capture everything we want?

Expand the control flow model

- Unordered control flow in expressions
- Parallel control flow for concurrent/parallel loops

Path-sensitive data-flow analysis

- Learning facts about conditions in conditional control flow

Supporting mechanised language meta-theory

- Relating static analysis and dynamic semantics

Summary: Data-Flow Analysis

Representation: Control Flow Graphs + Data Flow information

- Standardised representation for control flow
- Annotate with information from analysis

Formalism: Lattices and Transfer Functions

- Captures the type of information
- How to combine that information
- How to check that the analysis terminates

Limitations: Intra-procedural, context-insensitive

- So no function/method calls
- No tracking of objects or pointers for dynamic dispatch
- But at least we are *flow-sensitive*

Separation of Concerns

Representation

- **Control Flow Graphs**
- **Before/After data-flow information on CFG nodes**

Declarative Rules

- To define control-flow rules of a language
- To define data-flow rules of a language

Language-Independent Tooling

- Analysis
- Code completion
- Refactoring
- Optimisation
- …

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