

# **Declare Your Language**

## **Chapter 9: Dynamic Semantics**

**Eelco Visser**

**IN4303 Compiler Construction**

**TU Delft**

**November 2017**

# Outline

**The meaning of programs**

**Operational semantics**

**DynSem: A DSL for dynamic semantics specification**

**Interpreter generation**

**Scopes describe frames**

# Reading Material



# Executable specification of dynamic semantics

RTA 2015

<http://dx.doi.org/10.4230/LIPICs.RTA.2015.365>

## DynSem: A DSL for Dynamic Semantics Specification

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### Abstract

The formal semantics of a programming language and its implementation are typically separately defined, with the risk of divergence such that properties of the formal semantics are not properties of the implementation. In this paper, we present DynSem, a domain-specific language for the specification of the dynamic semantics of programming languages that aims at supporting both formal reasoning and efficient interpretation. DynSem supports the specification of the *operational semantics* of a language by means of *statically typed conditional term reduction rules*. DynSem supports *concise* specification of reduction rules by providing *implicit build and match coercions* based on reduction arrows and implicit term constructors. DynSem supports *modular* specification by adopting implicit propagation of semantic components from I-MSOS, which allows omitting propagation of components such as environments and stores from rules that do not affect those. DynSem supports the declaration of *native operators* for delegation of aspects of the semantics to an external definition or implementation. DynSem supports the definition of auxiliary *meta-functions*, which can be expressed using regular reduction rules and are subject to semantic component propagation. DynSem specifications are *executable* through automatic generation of a Java-based AST interpreter.

**1998 ACM Subject Classification** F.3.2. Semantics of Programming Languages (D.3.1.)

**Keywords and phrases** programming languages, dynamic semantics, reduction semantics, semantics engineering, IDE, interpreters, modularity

**Digital Object Identifier** 10.4230/LIPICs.RTA.2015.365

### 1 Introduction

The specification of the dynamic semantics is the core of a programming language design as it describes the runtime behavior of programs. In practice, the implementation of a compiler or an interpreter for the language often stands as the *only* definition of the semantics of a language. Such implementations, in a traditional programming language, often lack the clarity and the conciseness that a specification in a formal semantics framework provides. Therefore, they are a poor source of *documentation* about the semantics. On the other hand, formal definitions are not executable to the point that they can be used as implementations to run programs. Even when both a formal specification and an implementation co-exist, they typically diverge. As a result, important properties of a language as established based on its formal semantics may not hold for its implementation. Our goal is to unify the semantics engineering and language engineering of programming language designs [22] by providing a notation for the specification of the dynamic semantics that can serve both as a readable formalization as well as the source of an execution engine.

In this paper, we present DynSem, a DSL for the concise, modular, statically typed, and executable specification of the dynamic semantics of programming languages. DynSem



# Alignment between static binding/type system and dynamic memory layout

For the interested

EC00P 2016

<http://dx.doi.org/10.4230/LIPIcs.EC00P.2016.20>

## Scopes Describe Frames: A Uniform Model for Memory Layout in Dynamic Semantics (Artifact)\*

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### Abstract

Our paper introduces a systematic approach to the alignment of names in the static structure of a program, and memory layout and access during its execution. We develop a uniform memory model consisting of frames that instantiate the scopes in the scope graph of a program. This provides a language-independent correspondence between static scopes and run-time memory layout, and between static resolution paths and run-time memory access paths. The approach scales to a range of binding features, supports straightforward type soundness proofs,

and provides the basis for a language-independent specification of sound reachability-based garbage collection.

This Coq artifact showcases how our uniform model for memory layout in dynamic semantics provides structure to type soundness proofs. The artifact contains type soundness proofs mechanized in Coq for (supersets of) all languages in the paper. The type soundness proofs rely on a language-independent framework formalizing scope graphs and frame heaps.

1998 ACM Subject Classification F.3.1 Specifying and Verifying and Reasoning about Programs  
Keywords and phrases Dynamic semantics, scope graphs, memory layout, type soundness, operational semantics  
Digital Object Identifier 10.4230/DARTS.2.1.10  
Related Article Casper Bach Poulsen, Pierre Néron, Andrew Tolmach, and Eelco Visser, “Scopes Describe Frames: A Uniform Model for Memory Layout in Dynamic Semantics”, in Proceedings of the 30th European Conference on Object-Oriented Programming (ECOOP 2016), LIPIcs, Vol. 56, pp. 20:1–20:26, 2016.  
<http://dx.doi.org/10.4230/LIPIcs.EC00P.2016.20>  
Related Conference 30th European Conference on Object-Oriented Programming (ECOOP 2016), July 18–22, 2016, Rome, Italy

### 1 Scope

The artifact is designed to document and support repeatability of the type soundness proofs in the companion paper [2], using the Coq proof assistant.<sup>1</sup> In particular, the artifact provides a

\* This work was partially funded by the NWO VICI *Language Designer’s Workbench* project (639.023.206). Andrew Tolmach was partly supported by a Digiteo Chair at Laboratoire de Recherche en Informatique, Université Paris-Sud.

<sup>1</sup> <https://coq.inria.fr/>



Automatically checking type safety of interpreters.

Just out of the oven

POPL 2018

<http://casperbp.net/store/intrinsicallytyped.pdf>

<https://doi.org/10.1145/3158104>

## Intrinsically-Typed Definitional Interpreters for Imperative Languages

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A definitional interpreter defines the semantics of an object language in terms of the (well-known) semantics of a host language, enabling understanding and validation of the semantics through execution. Combining a definitional interpreter with a separate type system requires a separate type safety proof. An alternative approach, at least for pure object languages, is to use a dependently-typed language to encode the object language type system in the definition of the abstract syntax. Using such intrinsically-typed abstract syntax definitions allows the host language type checker to verify automatically that the interpreter satisfies type safety. Does this approach scale to larger and more realistic object languages, and in particular to languages with mutable state and objects?

In this paper, we describe and demonstrate techniques and libraries in Agda that successfully scale up intrinsically-typed definitional interpreters to handle rich object languages with non-trivial binding structures and mutable state. While the resulting interpreters are certainly more complex than the simply-typed  $\lambda$ -calculus interpreter we start with, we claim that they still meet the goals of being concise, comprehensible, and executable, while guaranteeing type safety for more elaborate object languages. We make the following contributions: (1) A *dependent-passing style* technique for hiding the weakening of indexed values as they propagate through monadic code. (2) An Agda library for programming with *scope graphs* and *frames*, which provides a uniform approach to dealing with name binding in intrinsically-typed interpreters. (3) Case studies of intrinsically-typed definitional interpreters for the simply-typed  $\lambda$ -calculus with references (STLC+Ref) and for a large subset of Middleweight Java (MJ).

CCS Concepts: • **Theory of computation** → **Program verification**; *Type theory*; • **Software and its engineering** → **Formal language definitions**;

Additional Key Words and Phrases: definitional interpreters, dependent types, scope graphs, mechanized semantics, Agda, type safety, Java

### ACM Reference Format:

Casper Bach Poulsen, Arjen Rouvoet, Andrew Tolmach, Robbert Krebbers, and Eelco Visser. 2018. Intrinsically-Typed Definitional Interpreters for Imperative Languages. *Proc. ACM Program. Lang.* 2, POPL, Article 16 (January 2018), 34 pages. <https://doi.org/10.1145/3158104>

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2475-1421/2018/1-ART16

<https://doi.org/10.1145/3158104>

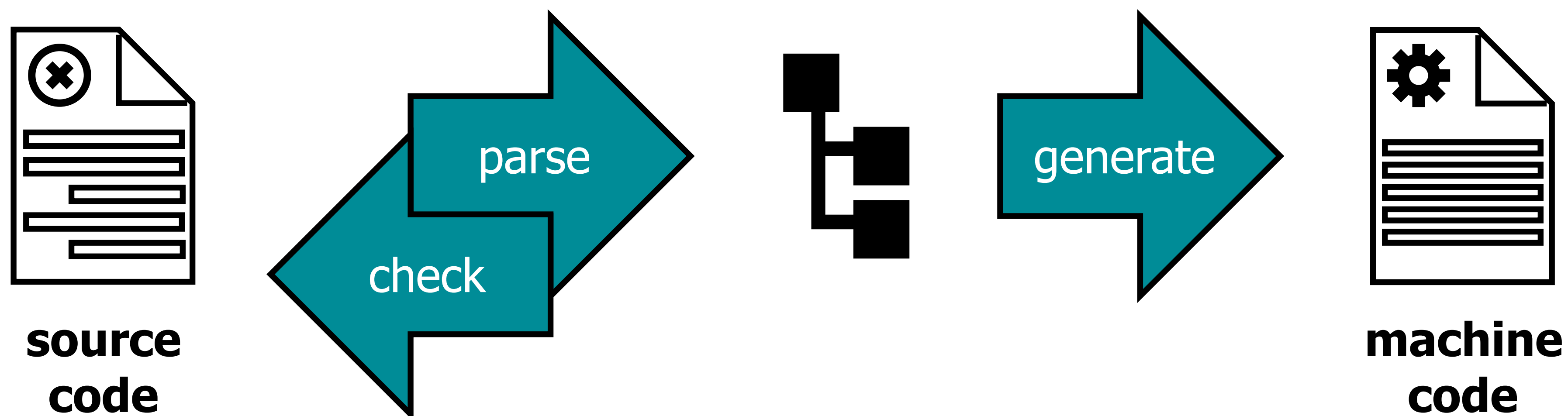
Proceedings of the ACM on Programming Languages, Vol. 2, No. POPL, Article 16. Publication date: January 2018.

# Semantics



# What is the meaning of a program?

$$\text{meaning}(p) = \text{behavior}(p)$$



$\text{meaning}(p)$  = what happens when executing the generated (byte) code to which  $p$  is compiled



# What is the meaning of a program?

$$\text{meaning}(p) = \text{behavior}(p)$$

What *is* behavior?

How can we *observe* behavior?

Mapping input to output

Changes to state of the system

Which behavior is essential, which accidental?

# How can we define the semantics of a program?

Compiler defines *translational* semantics

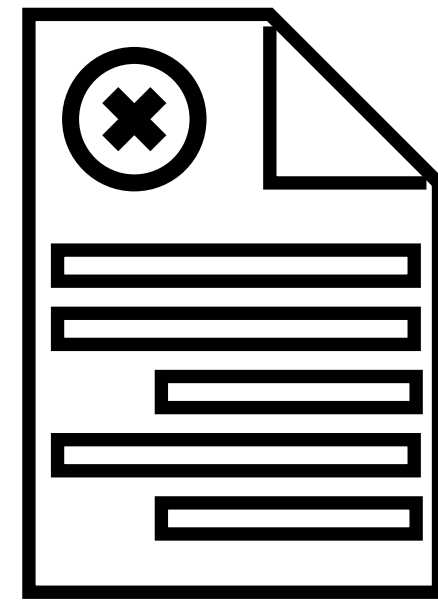
$$\text{semanticsL1}(p) = \text{semanticsL2}(\text{translate}(p))$$

Requires understanding `translate` and `semanticsL2`

How do we know that `translate` is correct?

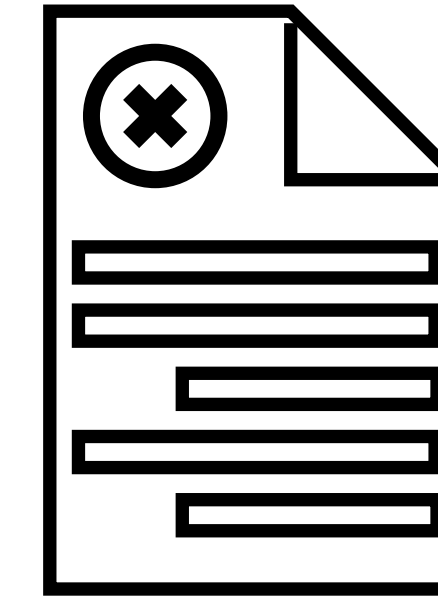
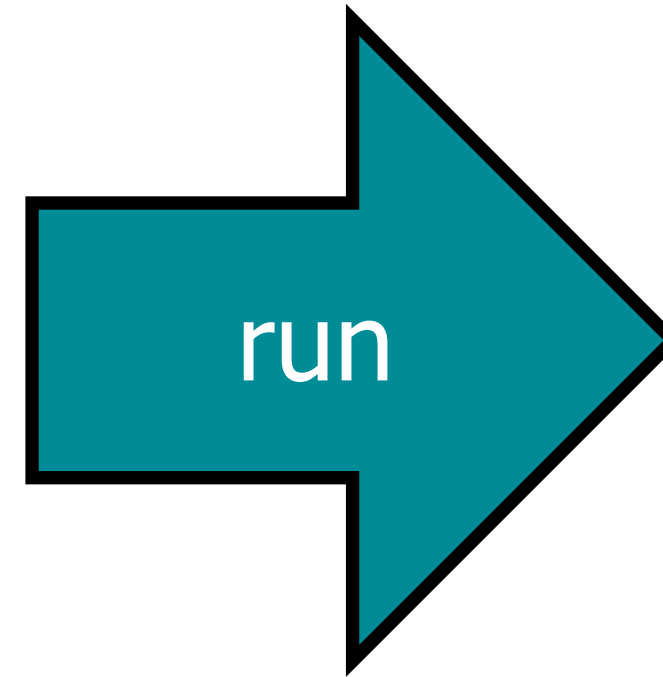
Is there a more ***direct description*** of `semanticsL1`?



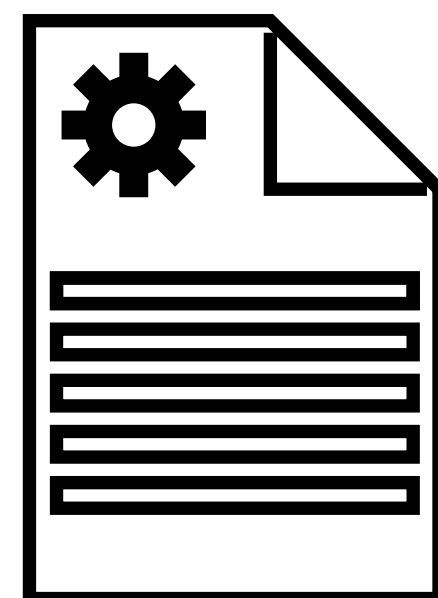
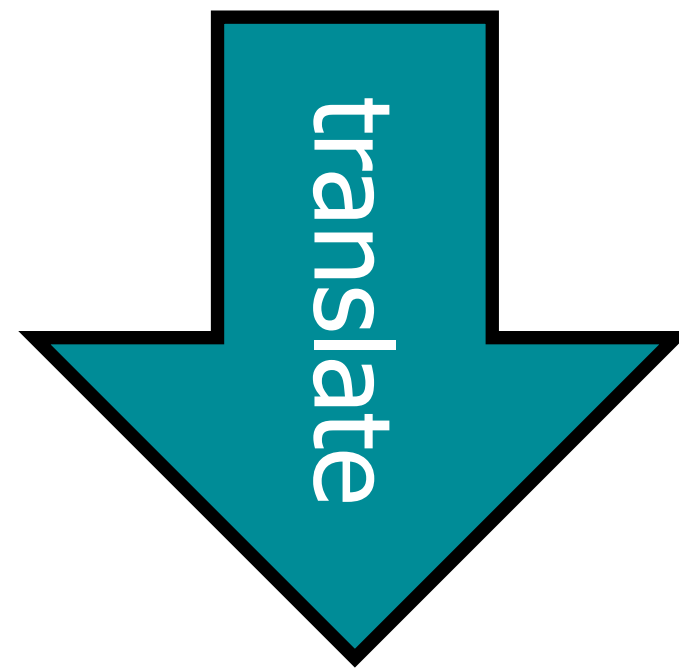


**source  
code**

semanticsL1

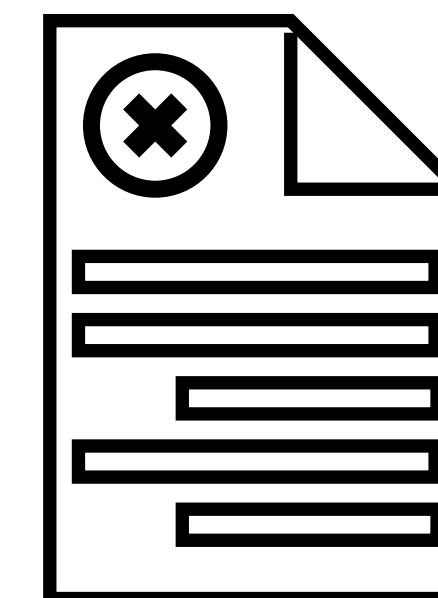
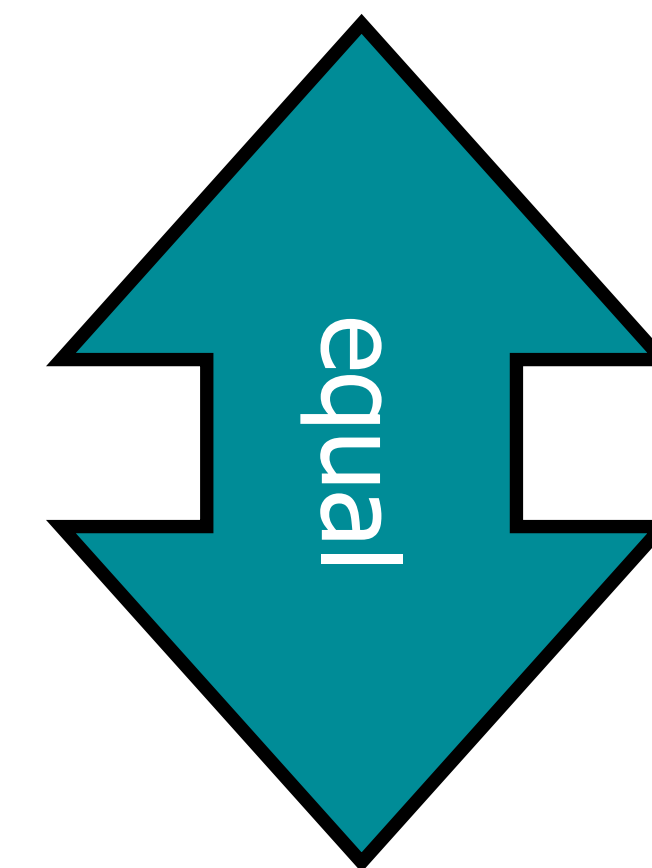
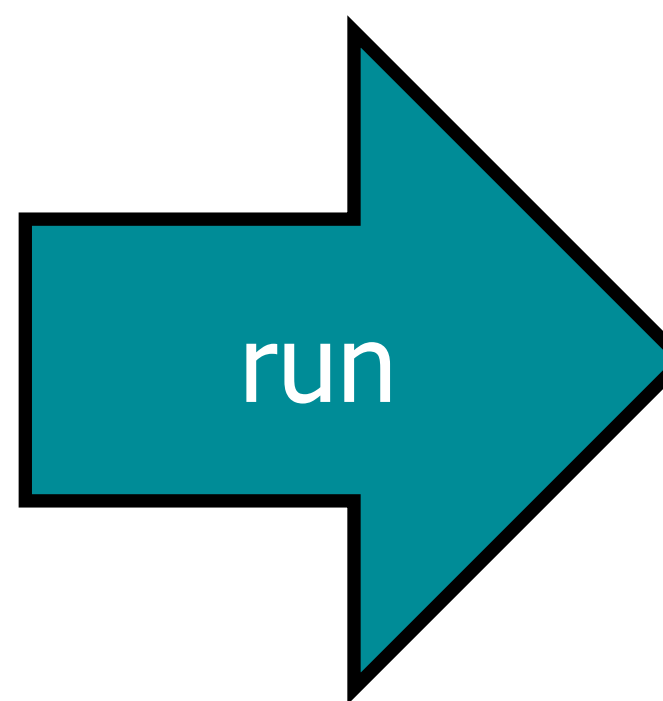


**value**



**machine  
code**

semanticsL2



**value**

# Verifying Compiler Correctness

Direct semantics of source language provides a specification

How to check correctness?

**Testing:** for *many* programs  $p$  (and inputs  $i$ ) **test** that

$$\text{run}(p)(i) == \text{run}(\text{translate}(p))(i)$$

**Verification:** for *all* programs  $p$  (and inputs  $i$ ) **prove** that

$$\text{run}(p)(i) == \text{run}(\text{translate}(p))(i)$$



# Validating Semantics

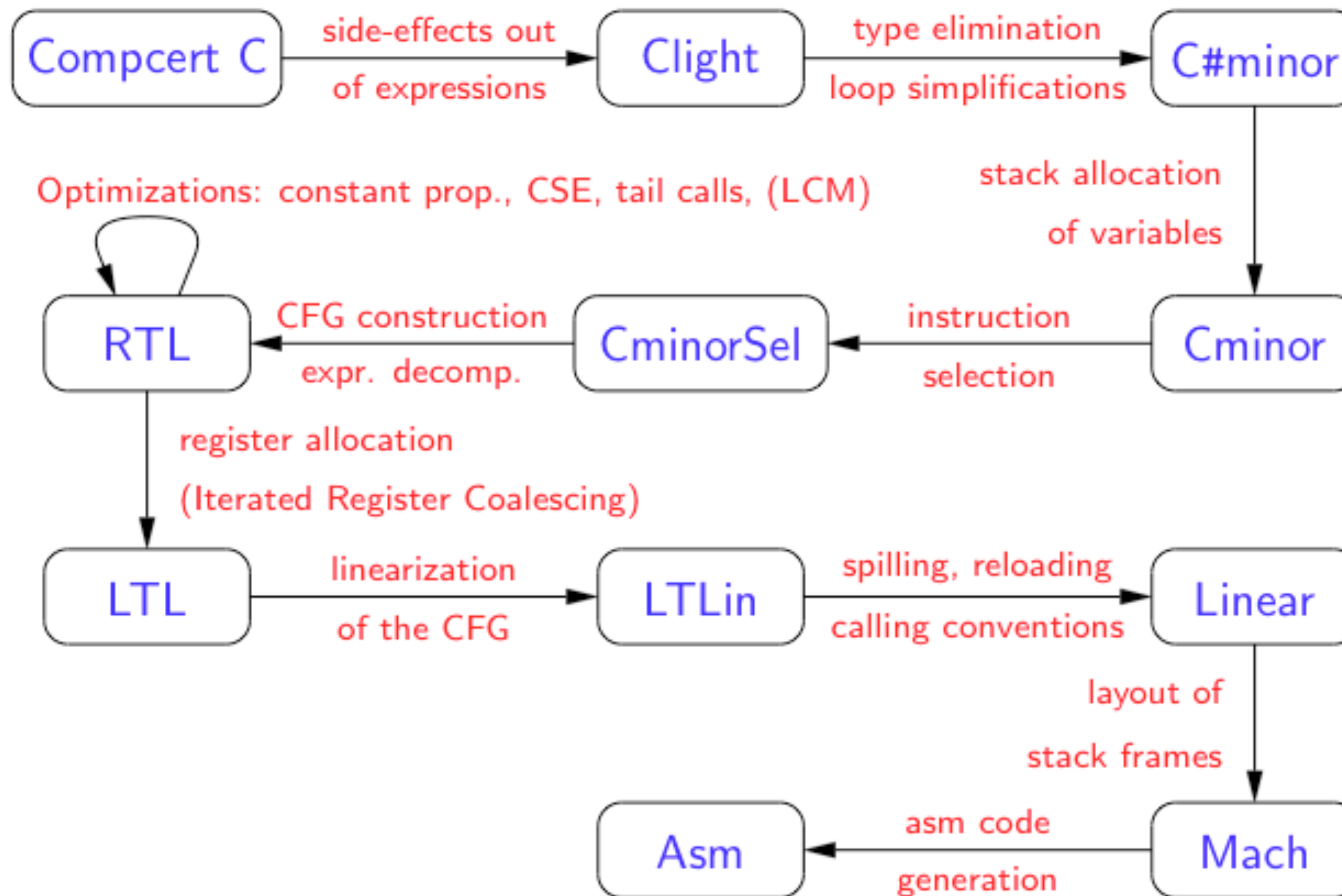
Is this the right semantics?

**Testing:** for *many* programs  $p$  (and inputs  $i$ ) *test* that

$$\text{run}(p)(i) == v$$

Requires specifying desired  $\langle p, i, v \rangle$  combinations  
(aka unit testing)

# The CompCert C Compiler





# Compiler Construction Courses of the Future

## Language Specification

syntax definition  
name binding  
type system  
dynamic semantics  
translation  
transformation  
  
safety properties

## Language Implementation

generating implementations  
from specifications  
  
parser generation  
constraint resolution  
partial evaluation  
  
...

## Language Testing

test generation

## Language Verification

proving correctness

# Operational Semantics



# Operational Semantics

**What** is the **result** of execution of a program  
and **how** is that result achieved?

**Structural Operational Semantics:** What  
are the individual steps of an execution?

**Natural Semantics:** How is overall result of  
execution obtained?

Defined using a **transition system**

# Transition System

rule 
$$\frac{e_1 \rightarrow e_1' \dots e_n \rightarrow e_n'}{e \rightarrow e'}$$
 premises  
conclusion

axiom  $e \rightarrow e'$

reduction  $p \rightarrow v$  derivation tree to prove  
 $v$  is value of program  $p$

# Structural Operational (Small-Step) Semantics

$$e = x \mid i \mid e + e \mid \backslash x.e \mid e \ e \mid \text{ifz}(e) \ e \ \text{else} \ e$$

$$\frac{e_1 \rightarrow e_1'}{e_1 + e_2 \rightarrow e_1' + e_2}$$

$$\frac{e_2 \rightarrow e_2'}{e_1 + e_2 \rightarrow e_1 + e_2'}$$

$$i + j \rightarrow i \pm j$$

$$\frac{e_1 \rightarrow e_1'}{\text{ifz}(e_1) \ e_2 \ \text{else} \ e_3 \rightarrow \text{ifz}(e_1') \ e_2 \ \text{else} \ e_3}$$

$$\text{ifz}(0) \ e_2 \ \text{else} \ e_3 \rightarrow e_2$$

$$\frac{i \neq 0}{\text{ifz}(i) \ e_2 \ \text{else} \ e_3 \rightarrow e_3}$$

$$e \rightarrow e$$

reducing expressions

$$\frac{e_1 \rightarrow e_1'}{e_1 \ e_2 \rightarrow e_1' \ e_2}$$

$$\frac{e_2 \rightarrow e_2'}{v \ e_2 \rightarrow v \ e_2'}$$

$$(\backslash x.e) \ v_1 \rightarrow e[x := v_1]$$

order of evaluation?



# Structural Operational (Small-Step) Semantics

$$e = x \mid i \mid e + e \mid \backslash x.e \mid e \ e \mid \text{ifz}(e) \ e \ \text{else} \ e$$

$$\frac{e_1 \rightarrow e_1'}{e_1 + e_2 \rightarrow e_1' + e_2}$$

$$\frac{e_2 \rightarrow e_2'}{v_1 + e_2 \rightarrow v_1 + e_2'}$$

$$i + j \rightarrow i \pm j$$

$$\frac{e_1 \rightarrow e_1'}{\text{ifz}(e_1) \ e_2 \ \text{else} \ e_3 \rightarrow \text{ifz}(e_1') \ e_2 \ \text{else} \ e_3}$$

$$\text{ifz}(0) \ e_2 \ \text{else} \ e_3 \rightarrow e_2$$

$$\frac{i \neq 0}{\text{ifz}(i) \ e_2 \ \text{else} \ e_3 \rightarrow e_3}$$

$$e \rightarrow e$$

reducing expressions

$$\frac{e_1 \rightarrow e_1'}{e_1 \ e_2 \rightarrow e_1' \ e_2}$$

$$\frac{e_2 \rightarrow e_2'}{v \ e_2 \rightarrow v \ e_2'}$$

$$(\backslash x.e) \ v_1 \rightarrow e[x := v_1]$$

order of evaluation?

# Natural (Big-Step) Semantics

$e = x \mid i \mid e + e \mid \backslash x.e \mid e \ e \mid \text{ifz}(e) \ e \ \text{else} \ e$

$$E \vdash i \Rightarrow \text{NumV}(i)$$
$$E \vdash e_1 \Rightarrow \text{NumV}(i)$$
$$E \vdash e_2 \Rightarrow \text{NumV}(j)$$

---

$$E \vdash e_1 + e_2 \Rightarrow \text{NumV}(i + j)$$
$$E \vdash e_1 \Rightarrow \text{NumV}(0)$$
$$E \vdash e_2 \Rightarrow v$$

---

$$E \vdash \text{if}(e_1) \ e_2 \ \text{else} \ e_3 \Rightarrow v$$
$$E \vdash e_1 \Rightarrow \text{NumV}(i), \ i \neq 0$$
$$E \vdash e_3 \Rightarrow v$$

---

$$E \vdash \text{if}(e_1) \ e_2 \ \text{else} \ e_3 \Rightarrow v$$
$$E \vdash e \Rightarrow v$$

reducing expressions to values

$$E[x] = v$$

---

$$E \vdash x \Rightarrow v$$
$$E \vdash \backslash x.e \Rightarrow \text{ClosV}(x, e, E)$$
$$E_1 \vdash e_1 \Rightarrow \text{ClosV}(x, e, E_2)$$
$$E_1 \vdash e_2 \Rightarrow v_1$$
$$\{x \mapsto v_1, E_2\} \vdash e \Rightarrow v_2$$

---

$$E_1 \vdash e_1 \ e_2 \Rightarrow v_2$$

# **DynSem: A DSL for Dynamic Semantics Specification**

Vlad Vergu, Pierre Neron, Eelco Visser

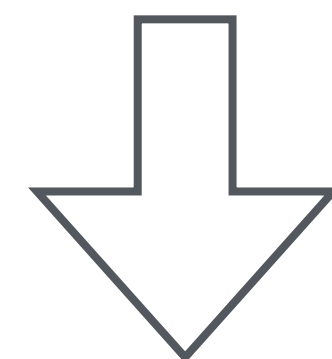
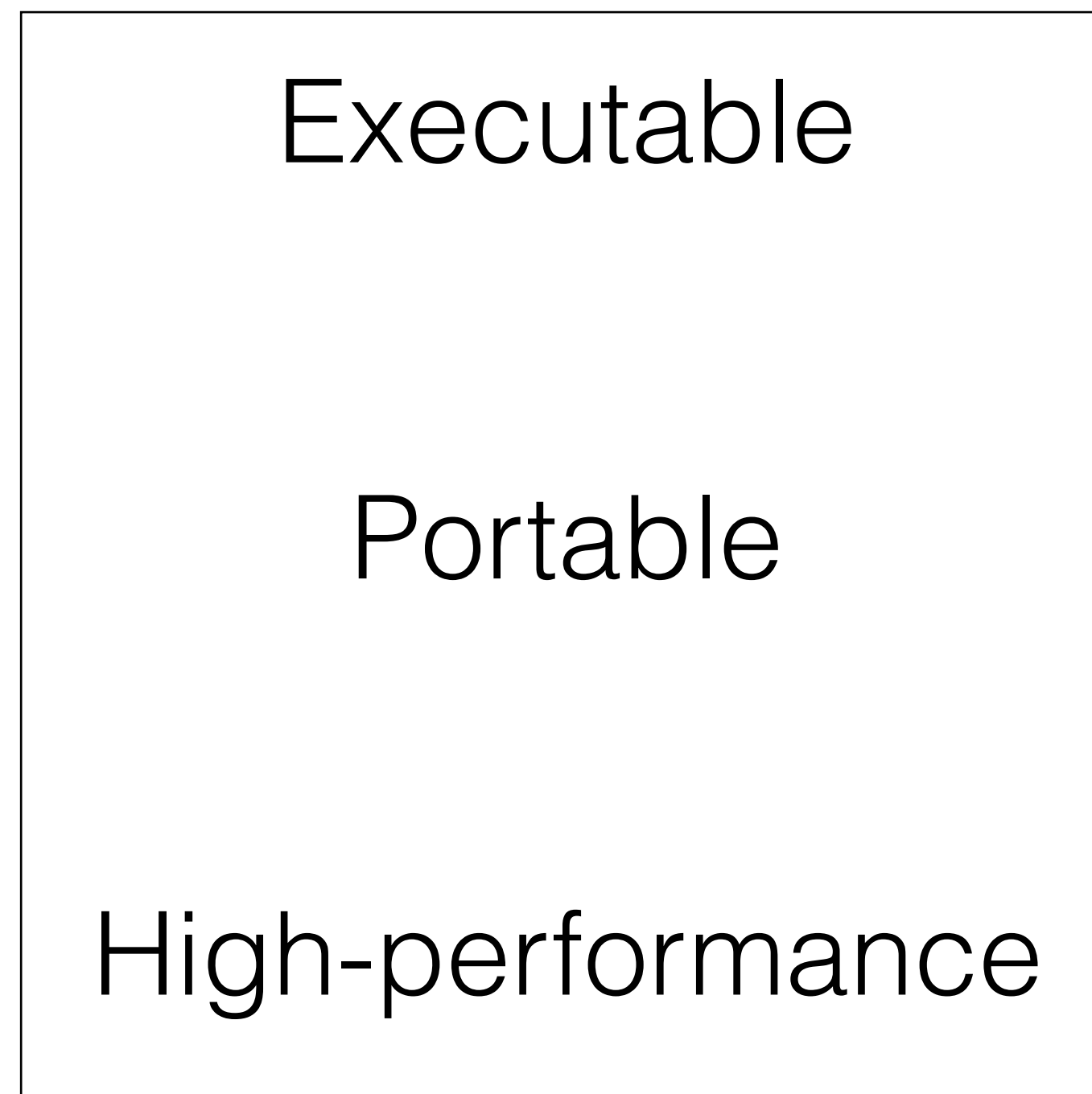
RTA 2015

# Interpreters for Spoofox Languages

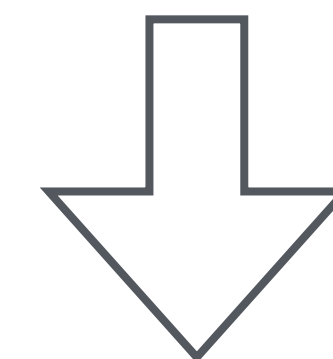
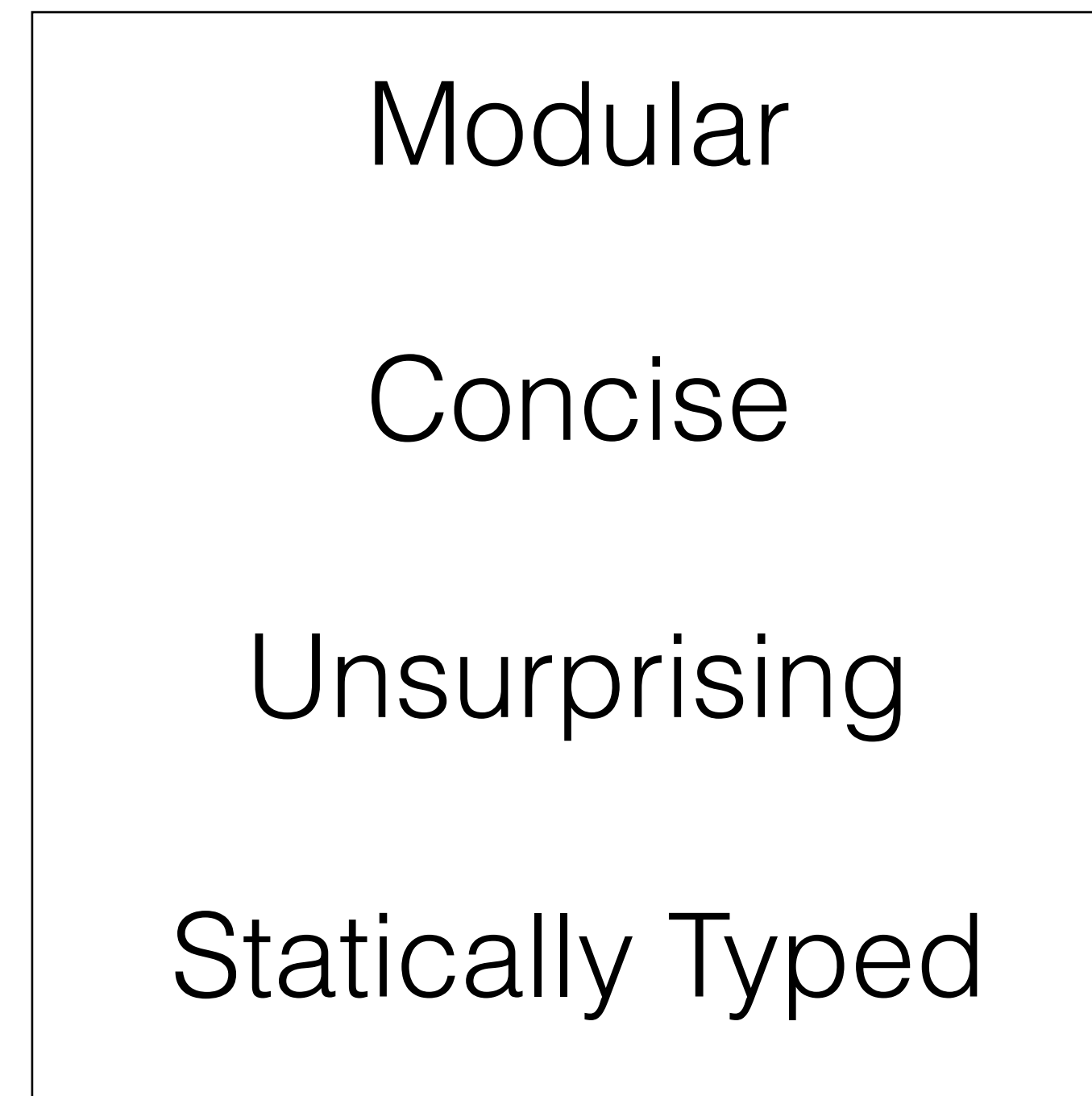
gcdAB.pbox	gcdAB.aterm	gcdAB.evaluated.aterm
<pre>1 let 2   gcd = box(0) 3 in 4   let f = 5     fun (a, b) { 6       if (b == 0) 7         a 8       else 9         unbox(gcd)(b, a % b) 10    end 11  } 12 in 13   setbox(gcd, f); 14   unbox(gcd)(1134903170, 1836) 15 end 16 end 17</pre>	<pre>1 Let( 2   [Bind("gcd", Box(Num("0")))] 3 , Let( 4   [ Bind( 5     "f" 6     , Fun( 7       ["a", "b"] 8       , If( 9         Eq(Var("b"), Num("0")) 10        , Var("a") 11        , App( 12          Unbox(Var("gcd")) 13          , [Var("b"), Mod(Var("a"), Var("b"))] 14        ) 15      ) 16    ) 17  ) 18 ] 19 , Seq( 20   SetBox(Var("gcd"), Var("f")) 21   , App( 22     Unbox(Var("gcd")) 23     , [Num("1134903170"), Num("1836")] 24   ) 25 ) 26 ) 27 )</pre>	<pre>1 R_default_VC 2   NumV(34) 3 , Map( 4   "Store" 5   , Bind(923, RefV(925)) 6   , Bind( 7     925 8     , ObjV( 9       Map( 10        "Env" 11        , Bind("outer", 922) 12        , Bind("self", 923) 13        , Bind("super", 924) 14      ) 15    ) 16  ) 17 , Bind( 18   926 19   , ClosV( 20     Fun( 21       ["a", "b"] 22       , If( 23         Eq(Var("b"), Num("0")) 24         , Var("a") 25         , App( 26           Unbox(Var("gcd")) 27           , [Var("b"), Mod(Var("a"), V 28         )</pre>



# Design Goals



Big-Step



I-MSOS

M. Churchill, P. D. Mosses, and P. Torrini.  
Reusable components of semantic  
specifications. In MODULARITY, April 2014.

# Example: DynSem Semantics of PAPL-Box

```
let
  fac = box(0)
in
  let f = fun (n) {
    if (n == 0)
      1
    else
      n * (unbox(fac) (n - 1))
    end
  }
  in
    setbox(fac, f);
    unbox(fac)(10)
  end
end
```

## Features

- Arithmetic
- Booleans
- Comparisons
- Mutable variables
- Functions
- Boxes

## Components

- Syntax in SDF3
- Dynamic Semantics in DynSem

# Abstract Syntax from Concrete Syntax

```
module Arithmetic
```

```
imports Expressions
```

```
imports Common
```

```
context-free syntax
```

```
Expr.Num = INT
```

```
Expr.Plus = [[Expr] + [Expr]] {left}
```

```
Expr.Minus = [[Expr] - [Expr]] {left}
```

```
Expr.Times = [[Expr] * [Expr]] {left}
```

```
Expr.Mod = [[Expr] % [Expr]] {left}
```

```
context-free priorities
```

```
{left: Expr.Times Expr.Mod }
```

```
> {left: Expr.Minus Expr.Plus }
```

src-gen/ds-signatures/Arithmetic-sig

```
module Arithmetic-sig
```

```
imports Expressions-sig
```

```
imports Common-sig
```

```
signature
```

```
sorts
```

```
Expr
```

```
constructors
```

```
Num : INT -> Expr
```

```
Plus : Expr * Expr -> Expr
```

```
Minus : Expr * Expr -> Expr
```

```
Times : Expr * Expr -> Expr
```

```
Mod : Expr * Expr -> Expr
```

# Values, Meta-Variables, and Arrows

```
module values

signature
  sorts V Unit
  constructors
    U : Unit
  variables
    v : V
```

```
module expressions

imports values
imports Expressions-sig

signature
  arrows
    Expr --> V
  variables
    e : Expr
    x : String
```



# Term Reduction Rules

```
module arithmetic-explicit
```

```
imports expressions primitives Arithmetic-sig
```

```
signature
```

```
  constructors
```

```
    NumV: Int -> V
```

```
rules
```

```
  Num(__String2INT__(n)) --> NumV(str2int(n)).
```

```
  Plus(e1, e2) --> NumV(plusI(i1, i2))
```

```
  where
```

```
    e1 --> NumV(i1); e2 --> NumV(i2).
```

```
  Minus(e1, e2) --> NumV(minusI(i1, i2))
```

```
  where
```

```
    e1 --> NumV(i1); e2 --> NumV(i2).
```

```
module primitives
```

```
signature
```

```
  native operators
```

```
    str2int : String -> Int
```

```
    plusI   : Int * Int -> Int
```

```
    minusI  : Int * Int -> Int
```

# Native Operations

```
module primitives
signature
  native operators
    str2int : String -> Int
    plusI   : Int * Int -> Int
    minusI  : Int * Int -> Int
```

```
public class Natives {

  public static int plusI_2(int i1, int i2) {
    return i1 + i2;
  }

  public static int str2int_1(String s) {
    return Integer.parseInt(s);
  }
}
```

# Arrows as Coercions

rules

$\text{Plus}(\text{NumV}(i1), \text{NumV}(i2)) \dashrightarrow \text{NumV}(\text{plusI}(i1, i2)).$

signature

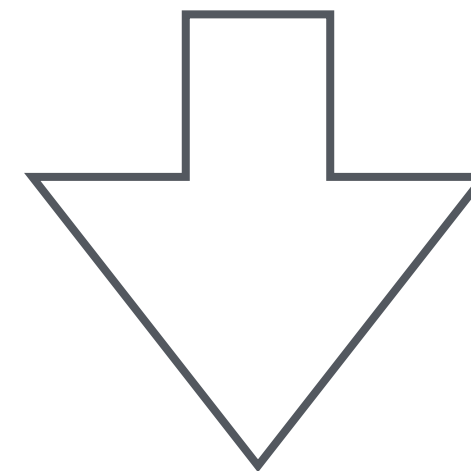
constructors

$\text{Plus} : \text{Expr} * \text{Expr} \rightarrow \text{Expr}$

$\text{NumV} : \text{Int} \rightarrow V$

arrows

$\text{Expr} \dashrightarrow V$



rules

$\text{Plus}(e1, e2) \dashrightarrow \text{NumV}(\text{plusI}(i1, i2))$

where

$e1 \dashrightarrow \text{NumV}(i1);$

$e2 \dashrightarrow \text{NumV}(i2).$

# Modular

```
module arithmetic

imports Arithmetic-sig
imports expressions
imports primitives

signature
  constructors
    NumV: Int -> V

rules

  Num(str) --> NumV(str2int(str)).

  Plus(NumV(i1), NumV(i2)) --> NumV(plusI(i1, i2)).

  Minus(NumV(i1), NumV(i2)) --> NumV(minusI(i1, i2)).

  Times(NumV(i1), NumV(i2)) --> NumV(timesI(i1, i2)).

  Mod(NumV(i1), NumV(i2)) --> NumV(modI(i1, i2)).
```

```
module boolean

imports Booleans-sig expressions

signature
  constructors
    BoolV: Bool -> V

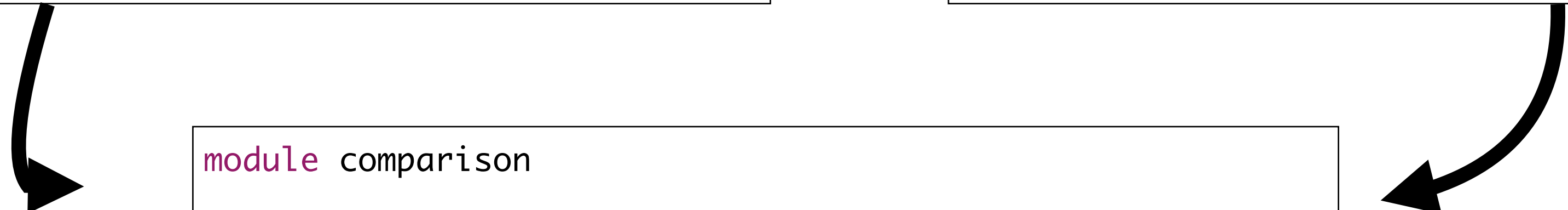
rules

  True() --> BoolV(true).
  False() --> BoolV(false).

  Not(BoolV(false)) --> BoolV(true).
  Not(BoolV(true)) --> BoolV(false).

  Or(BoolV(true), _) --> BoolV(true).
  Or(BoolV(false), e) --> e.

  And(BoolV(false), _) --> BoolV(false).
  And(BoolV(true), e) --> e.
```



```
module comparison

imports Comparisons-sig arithmetic boolean

rules

  Gt(NumV(i1), NumV(i2)) --> BoolV(gtI(i1, i2)).

  Eq(NumV(i1), NumV(i2)) --> BoolV(eqI(i1, i2)).

  Eq(BoolV(b1), BoolV(b2)) --> BoolV(eqB(b1, b2)).
```



# Control-Flow

```
module controlflow

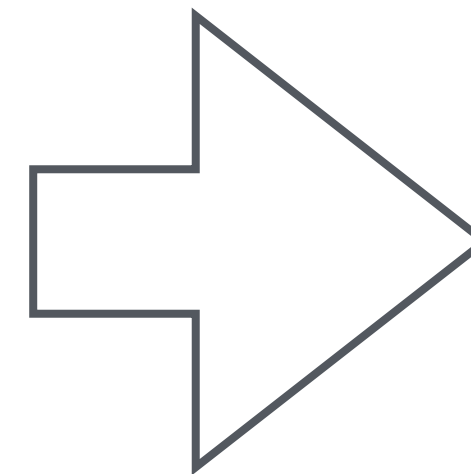
imports ControlFlow-sig
imports expressions
imports boolean

rules

  Seq(v, e2) --> e2.

  If(BoolV(true), e1, _) --> e1.

  If(BoolV(false), _, e2) --> e2.
```



```
module controlflow

imports ControlFlow-sig
imports expressions
imports boolean

rules

  Seq(e1, e2) --> v2
  where
    e1 --> v1;
    e2 --> v2.

  If(e1, e2, e3) --> v
  where
    e1 --> BoolV(true);
    e2 --> v.

  If(e1, e2, e3) --> v
  where
    e1 --> BoolV(false);
    e3 --> v.
```

# Immutable Variables: Environment Passing

## constructors

Let : ID \* Expr \* Expr -> Expr

Var : ID -> Expr

## module variables

imports Variables-sig environment

## rules

$E \vdash \text{Let}(x, v: V, e2) \dashrightarrow v2$

where

$\text{Env } \{x \dashrightarrow v, E\} \vdash e2 \dashrightarrow v2.$

$E \vdash \text{Var}(x) \dashrightarrow E[x].$

## module environment

imports values

## signature

sort aliases

Env = Map<String, V>

variables

E : Env

# First-Class Functions: Environment in Closure

constructors

Fun : ID \* Expr -> Expr

App : Expr \* Expr -> Expr

module unary-functions

imports expressions environment

signature

constructors

ClosV : String \* Expr \* Env -> V

rules

E |- Fun(x, e) --> ClosV(x, e, E).

E |- App(e1, e2) --> v

where

E |- e1 --> ClosV(x, e, E');

E |- e2 --> v2;

Env {x |--> v2, E'} |- e --> v.

module environment

imports values

signature

sort aliases

Env = Map<String, V>

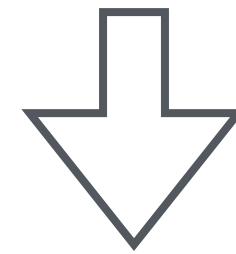
variables

E : Env

# Implicit Propagation

rules

$\text{Plus}(\text{NumV}(i1), \text{NumV}(i2)) \rightarrow \text{NumV}(\text{plusI}(i1, i2)).$



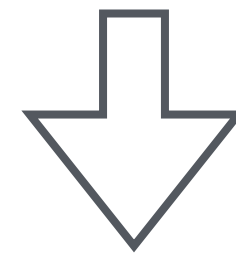
rules

$\text{Plus}(e1, e2) \rightarrow \text{NumV}(\text{plusI}(i1, i2))$

where

$e1 \rightarrow \text{NumV}(i1);$

$e2 \rightarrow \text{NumV}(i2).$



rules

$E \vdash \text{Plus}(e1, e2) \rightarrow \text{NumV}(\text{plusI}(i1, i2))$

where

$E \vdash e1 \rightarrow \text{NumV}(i1);$

$E \vdash e2 \rightarrow \text{NumV}(i2).$

# Mutable Boxes: Store

```
module box
imports store arithmetic
signature
  constructors
    Box      : Expr -> Expr
    Unbox    : Expr -> Expr
    SetBox   : Expr * Expr -> Expr
  constructors
    BoxV: Int -> V
```

rules

```
Box(e) :: S --> BoxV(loc) :: Store {loc |--> v, S'}
where e :: S --> v :: S';
      fresh => loc.
```

```
Unbox(BoxV(loc)) :: S --> S[loc].
```

```
SetBox(BoxV(loc), v) :: S --> v :: Store {loc |--> v, S}.
```

```
module store
```

```
imports values
```

```
signature
```

```
  sort aliases
```

```
    Store = Map<Int, V>
```

```
  variables
```

```
    S : Store
```



# Mutable Variables: Environment + Store

## constructors

```
Let : ID * Expr * Expr -> Expr
Var : ID -> Expr
Set : String * Expr -> Expr
```

```
module variables-mutable
imports Variables-sig store
rules
```

```
E |- Var(x) :: S --> v :: S
where E[x] => loc; S[loc] => v.
```

```
E |- Let(x, v, e2) :: S1 --> v2 :: S3
where
  fresh => loc;
  {loc |--> v, S1} => S2;
  Env {x |--> loc, E} |- e2 :: S2 --> v2 :: S3.
```

```
E |- Set(x, v) :: S --> v :: Store {loc |--> v, S}
where E[x] => loc.
```

```
module store
```

```
imports values
```

```
signature
```

```
sort aliases
```

```
Env = Map<ID, Int>
```

```
Store = Map<Int, V>
```

```
variables
```

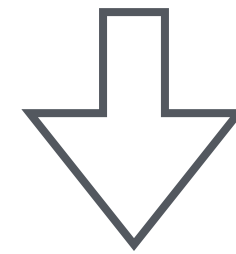
```
E : Env
```

```
S : Store
```

# Implicit Store Threading

rules

$\text{Plus}(\text{NumV}(i1), \text{NumV}(i2)) \rightarrow \text{NumV}(\text{plusI}(i1, i2)).$



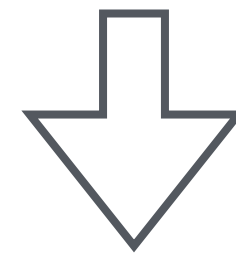
rules

$\text{Plus}(e1, e2) \rightarrow \text{NumV}(\text{plusI}(i1, i2))$

where

$e1 \rightarrow \text{NumV}(i1);$

$e2 \rightarrow \text{NumV}(i2).$



rules

$E \vdash \text{Plus}(e1, e2) :: S1 \rightarrow \text{NumV}(\text{plusI}(i1, i2)) :: S3$

where

$E \vdash e1 :: S1 \rightarrow \text{NumV}(i1) :: S2;$

$E \vdash e2 :: S2 \rightarrow \text{NumV}(i2) :: S3.$

# Abstraction: Env/Store Meta Functions

```
module store

imports values

signature
  sort aliases
    Env = Map<String, Int>
    Store = Map<Int, V>
  variables
    E : Env
    S : Store
  arrows
    readVar   : String --> V
    bindVar   : String * V --> Env
    writeVar  : String * V --> V

    allocate  : V --> Int
    write     : Int * V --> V
    read      : Int --> V
```

```
rules

allocate(v) --> loc
where
  fresh => loc;
  write(loc, v) --> _.

write(loc, v) :: S -->
  v :: Store {loc |--> v, S}.

read(loc) :: S --> S[loc] :: S.

rules

bindVar(x, v) --> {x |--> loc}
where allocate(v) --> loc.

E |- readVar(x) --> read(E[x]).

E |- writeVar(x, v) --> write(E[x], v).
```

# Boxes with Env/Store Meta Functions

```
module boxes
```

```
signature
```

```
constructors
```

```
Box      : Expr -> Expr
```

```
Unbox    : Expr -> Expr
```

```
SetBox   : Expr * Expr -> Expr
```

```
constructors
```

```
BoxV: V -> V
```

```
rules
```

```
Box(v) --> BoxV(NumV(allocate(v))).
```

```
Unbox(BoxV(NumV(loc))) --> read(loc).
```

```
SetBox(BoxV(NumV(loc)), v) --> write(loc, v).
```

# Mutable Variables with Env/Store Meta Functions

## constructors

```
Let  : String * Expr * Expr -> Expr  
Var  : String -> Expr  
Set  : String * Expr -> Expr
```

## module variables

```
imports expressions store
```

## rules

```
Var(x) --> readVar(x).
```

```
E |- Let(x, v1, e) --> v2
```

```
where
```

```
    bindVar(x, v1) --> E';
```

```
    Env {E', E} |- e --> v2.
```

```
Set(x, v) --> v
```

```
where
```

```
    writeVar(x, v) --> _.
```



# Functions with Multiple Arguments

```
module functions

imports Functions-sig
imports variables

signature
  constructors
    ClosV      : List(ID) * Expr * Env -> V
    bindArgs   : List(ID) * List(Expr) --> Env

rules

  E |- Fun(xs, e) --> ClosV(xs, e, E).

  App(ClosV(xs, e_body, E_clos), es) --> v'
  where
    bindArgs(xs, es) --> E_params;
    Env {E_params, E_clos} |- e_body --> v'.

  bindArgs([], []) --> {}.

  bindArgs([x | xs], [e | es]) --> {E, E'}
  where
    bindVar(x, e) --> E;
    bindArgs(xs, es) --> E'.
```

# Tiger in DynSem

```
store.ds
1
2
3 imports dynamics/values
4
5 signature // lvalue
6 sorts LValue
7 arrows
8   LValue -lval-> Int
9 variables
10  lv : LValue
11
12 signature // environment
13 sorts Id
14 sort aliases
15   // Address = Int
16   Env = Map(Id, Int)
17 variables
18   a : Int
19 components
20   E : Env
21 arrows
22   lookup(Id) --> Int
23   bind(Id, Int) --> Env
24
25
26 rules
27
28 E |- lookup(x) --> E[x].
29
30 E |- bind(x, a) --> {x l--> a, E}.
31
32 signature // heap
33 sort aliases
34   Heap = Map(Int, V)
35 components
36   H : Heap
37 arrows
38   read(Int) --> V
39   allocate(V) --> Int
40   write(Int, V) --> V
41
42 rules
43
44 read(a) :: H --> H[a].
45
46 write(a, v) :: H --> v :: H {a l--> v, H}
47
48 allocate(v) --> a
49 where
50   fresh => a;
51   write(a, v) --> ..
52

numbers.ds
1 module functions
2
3 imports ds-signatures/Functions-sig
4 imports dynamics/base
5 imports dynamics/store
6 imports dynamics/bindings
7
8 signature
9   constructors
10    ClosureV : List(FArg) * Exp * Env -> V
11   arrows
12    E |- funEnv(List(FunDec)) :: H --> Env :: H
13    E |- evalFuns(List(FunDec)) :: H --> Env :: H
14    E |- evalArgs(List(FArg), List(Exp)) :: H --> Env ::
15
16 rules // function definition
17
18 FunDecs(fds) --> E
19 where
20   funEnv(fds) --> E;
21   E |- evalFuns(fds) --> ..
22
23 E |- funEnv([]) --> E.
24
25 funEnv([FunDec(f, _, _, _) | fds]) --> E
26 where
27   E bindVar(f, UndefV()) |- funEnv(fds) --> E.
28
29 E |- evalFuns([]) --> E.
30
31 E |- evalFuns([FunDec(f, args, _, e) | fds]) --> evalFu
32 where
33   writeVar(f, ClosureV(args, e, E)) --> ..
34
35 rules // function call
36
37 Call(f, es) --> v
38 where
39   readVar(f) --> ClosureV(args, e, E);
40   evalArgs(args, es) --> E';
41   E {E', E} |- e --> v.
42
43 evalArgs([], []) --> {}.
44
45 evalArgs([FArg(x, _) | args], [v | es]) --> {x l--> a,
46 where
47   allocate(v) --> a;
48   evalArgs(args, es) --> E.
49
50 rules // procedure definition
51

control-flow.ds
12 signature
13 sort aliases
14   Idx = Map(Int, Int)
15 variables
16   I : Idx
17 constructors
18   ArrayV : Idx -> V
19 arrows
20   initArray(Int, Int, V, Idx) --> Idx
21
22 rules
23
24 Array(_, IntV(i), v) --> ArrayV(I)
25 where
26   initArray(0, i, v, {}) --> I.
27
28 initArray(i, j, v, I) --> I'
29 where
30   case ltI(i, j) of {
31     1 =>
32       allocate(v) --> a;
33       initArray(addI(i, 1), j, v, {i l--> a, I})
34     0 =>
35       I => I'
36   }.
37
38 Subscript(a, IntV(i)) -lval-> I[i]
39 where
40   read(a) --> ArrayV(I).
41

arrays.ds
12 signature
13 sort aliases
14   Idx = Map(Int, Int)
15 variables
16   I : Idx
17 constructors
18   ArrayV : Idx -> V
19 arrows
20   initArray(Int, Int, V, Idx) --> Idx
21
22 rules
23
24 Array(_, IntV(i), v) --> ArrayV(I)
25 where
26   initArray(0, i, v, {}) --> I.
27
28 initArray(i, j, v, I) --> I'
29 where
30   case ltI(i, j) of {
31     1 =>
32       allocate(v) --> a;
33       initArray(addI(i, 1), j, v, {i l--> a, I})
34     0 =>
35       I => I'
36   }.
37
38 Subscript(a, IntV(i)) -lval-> I[i]
39 where
40   read(a) --> ArrayV(I).
41

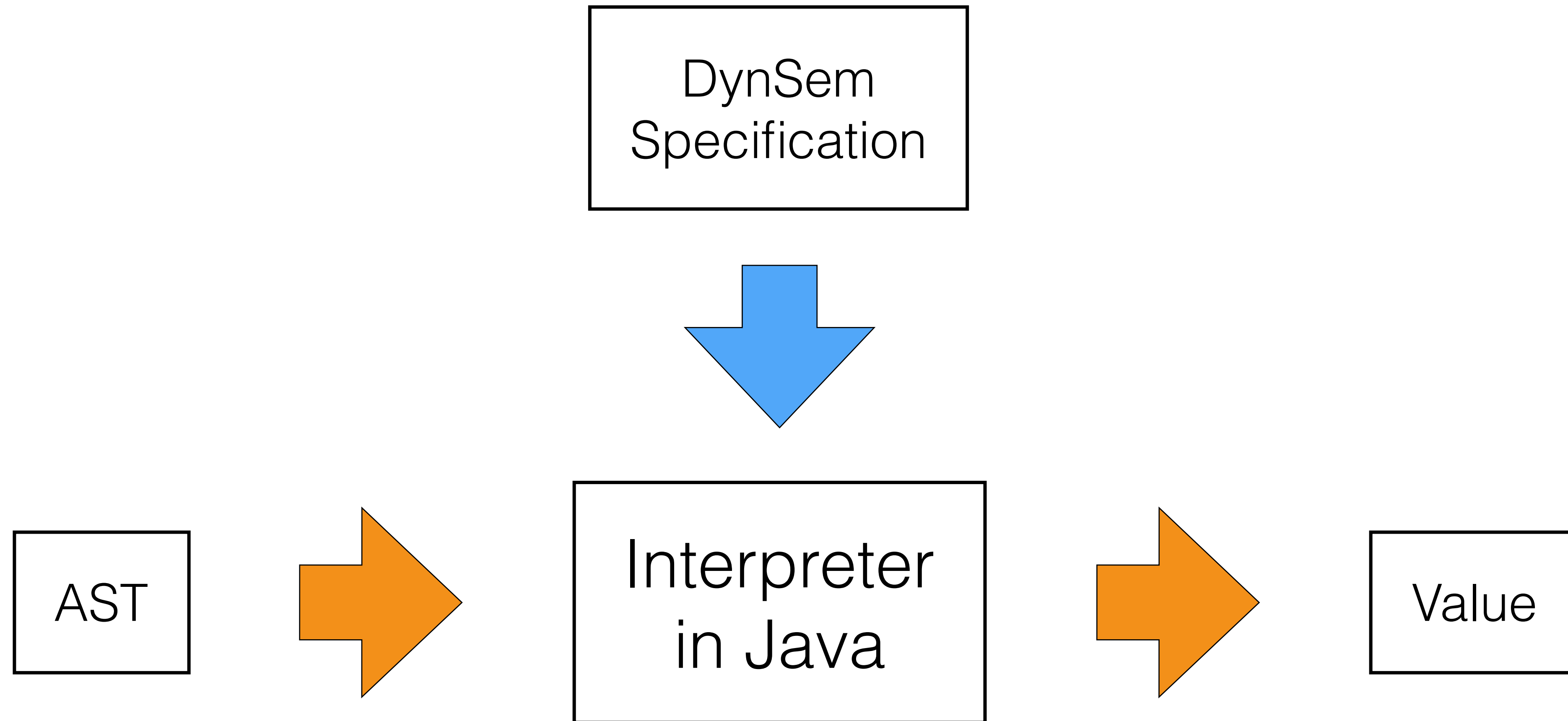
prettyprint.tig
1 let
2
3 type tree = {key: string, left : tree, right: tree}
4
5 function prettyprint(tree: tree) : string =
6 let
7
8   var output := ""
9
10  function write(s: string) =
11    output := concat(output, s)
12
13  function show(n: int, t: tree) =
14    let function indent(s: string) =
15      (write("\n");
16       for i := 1 to n
17         do write(" "))
18      output := concat(output, s))
19    in if t = nil then indent(".")
20       else (indent(t.key);
21            show(n+1, t.left);
22            show(n+1, t.right))
23  end
24
25 in show(0, tree);

prettyprint.aterm
1 Mod(
2   Let(
3     [ TypeDecs(
4       [ TypeDec(
5         "tree"
6         , RecordTy(
7           [ Field("key", Tid("string"))
8             , Field("left", Tid("tree"))
9             , Field("right", Tid("tree"))
10          ]
11        )
12      ]
13    )
14  ]
15  , FunDecs(
16    [ FunDec(
17      "prettyprint"
18      , [FArg("tree", Tid("tree"))]
19      , Tid("string")
20      , Let(
21        [ VarDecNoType("output", String("\n"
22          , FunDecs(
23            [ ProcDec(
24              "write"
25              , [FArg("s", Tid("string"))]
```

<https://github.com/MetaBorgCube/metaborg-tiger>

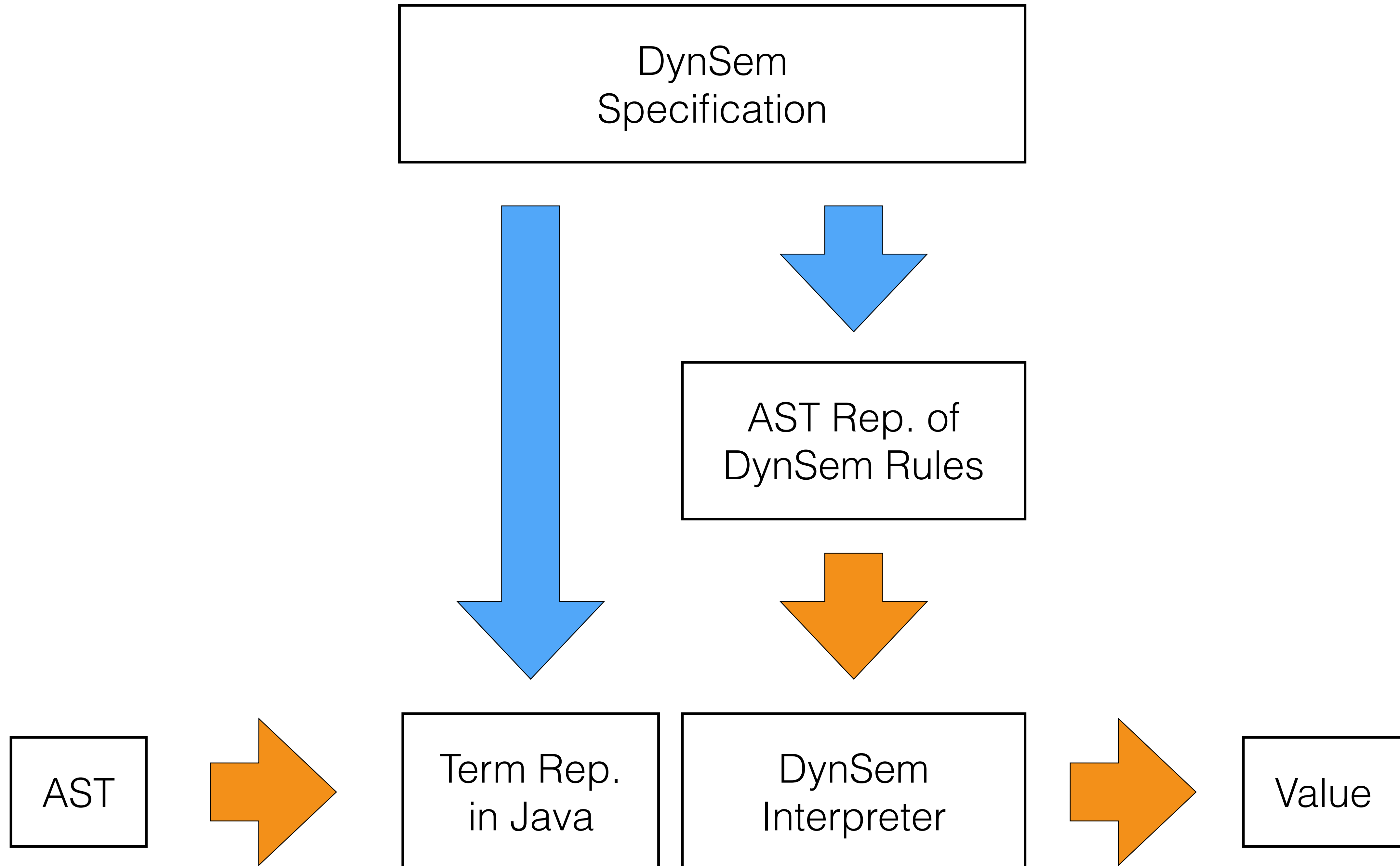
# Interpreter Generation

# Generating AST Interpreter from Dynamic Semantics



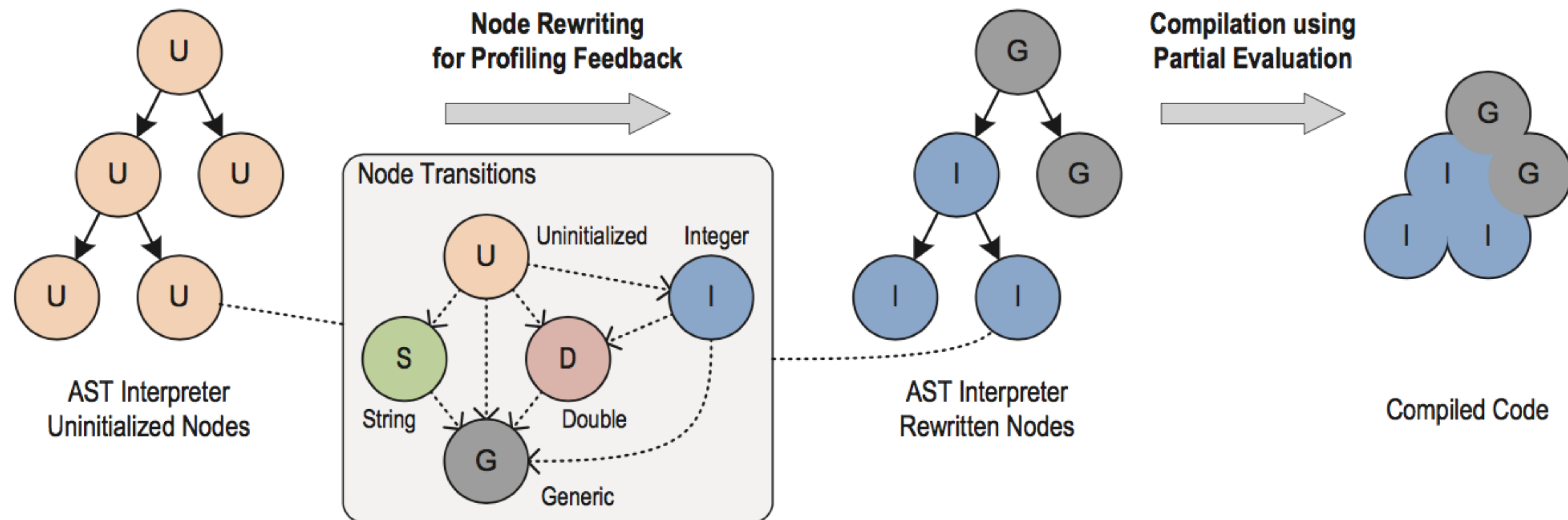


# DynSem Meta-Interpreter





# Truffle: Partial Evaluation of AST Interpreters

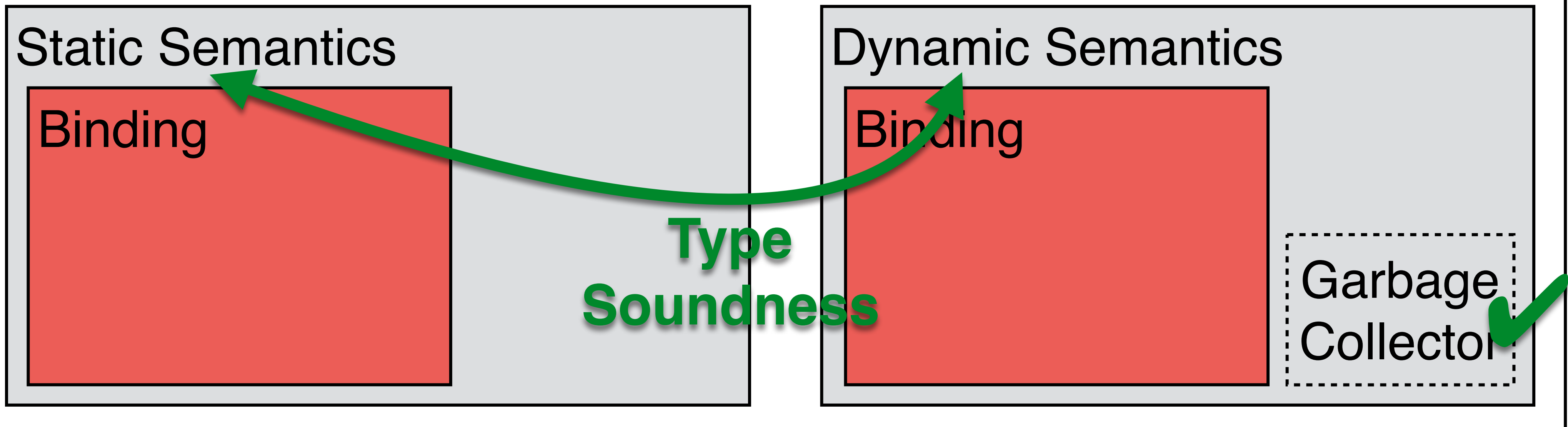


# Scopes Describe Frames: A uniform model for memory layout in dynamic semantics

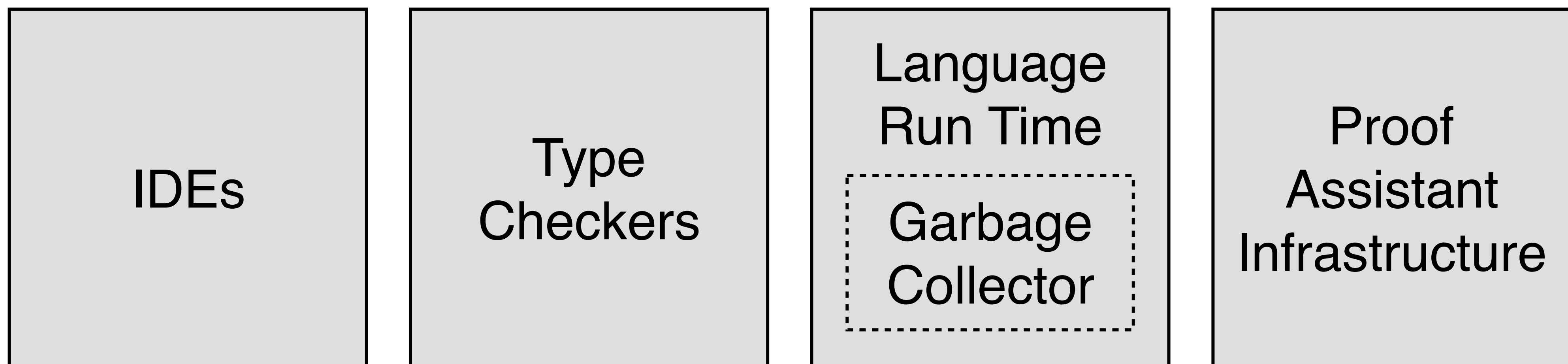
Casper Bach Poulsen, Pierre Néron, Andrew Tolmach, Eelco Visser

ECOOP 2016

# Semantic Specification

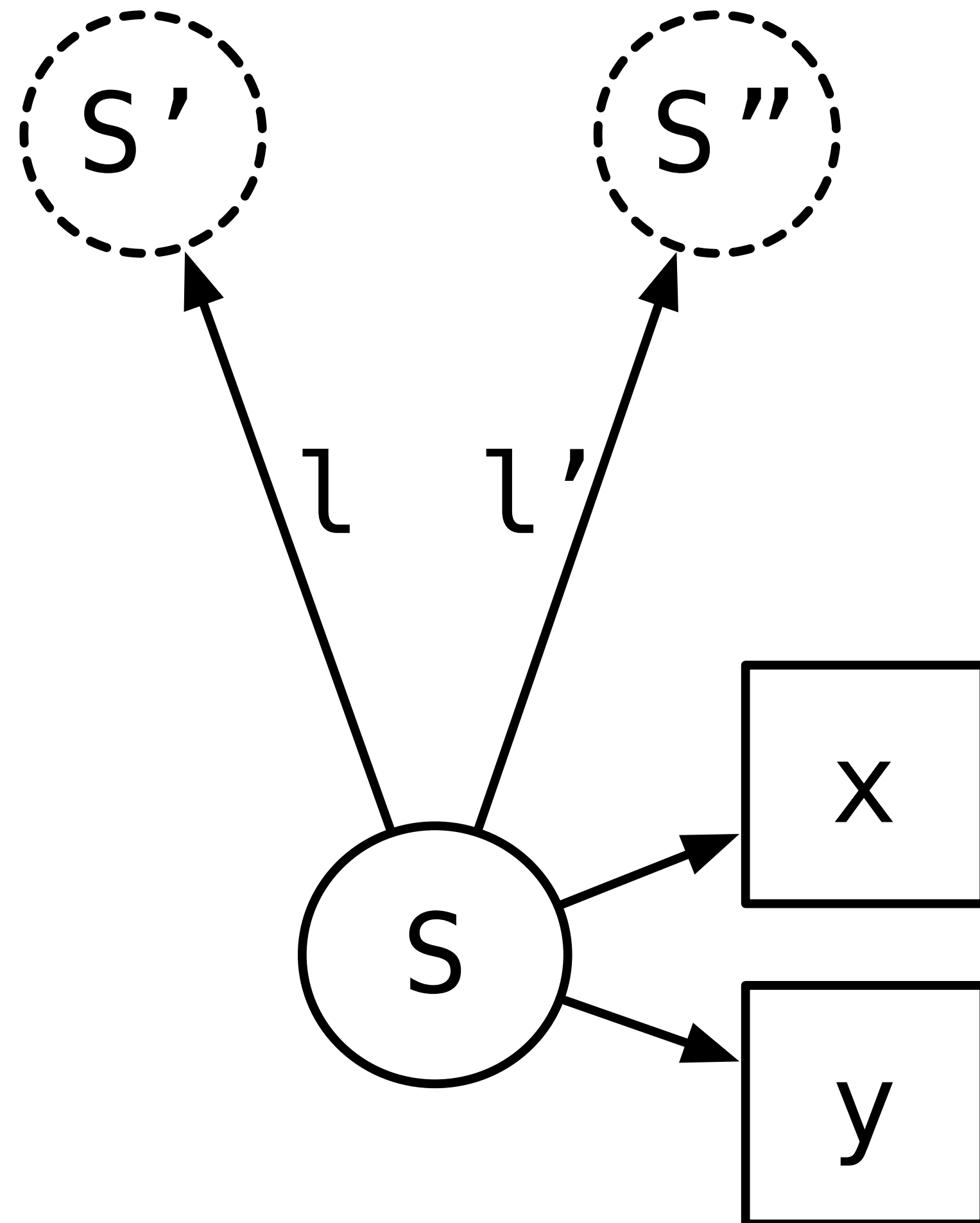


## Tools



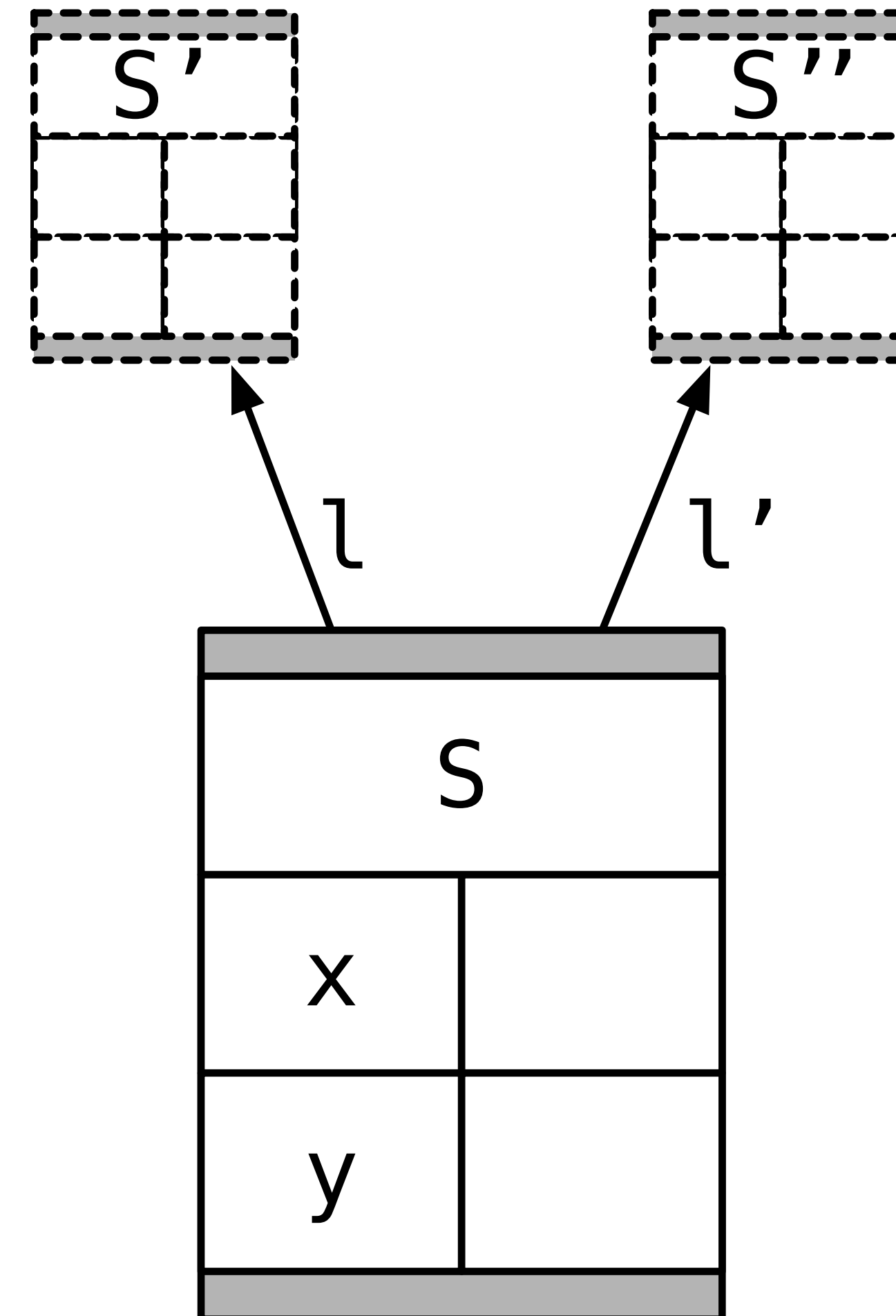
		Static	Dynamic
Lexical Mutable Objects	<pre> val x = 31; val y = x + 11; </pre>	Typing Contexts Type Substitution	Substitution Environments De Bruijn Indices HOAS
	<pre> var x = 31; x = x + 11; </pre>	Typing Contexts Store Typing	Stores/Heaps
	<pre> class A {   var x = 0;   var y = 42; } var r = new A(); </pre>	Class Tables	Mutable Objects Stores/Heaps

# Scope



[ESOP'15]

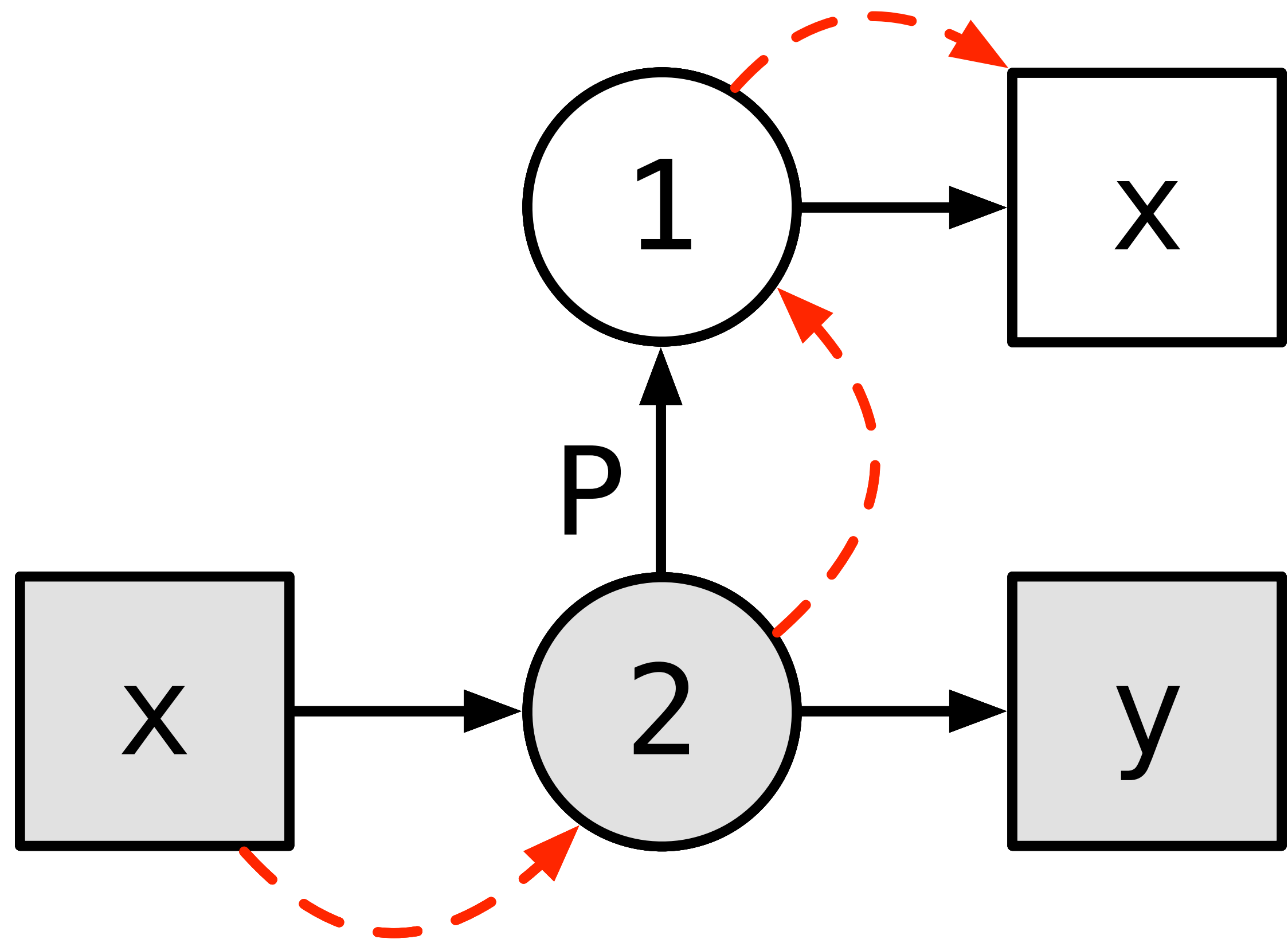
# Frame



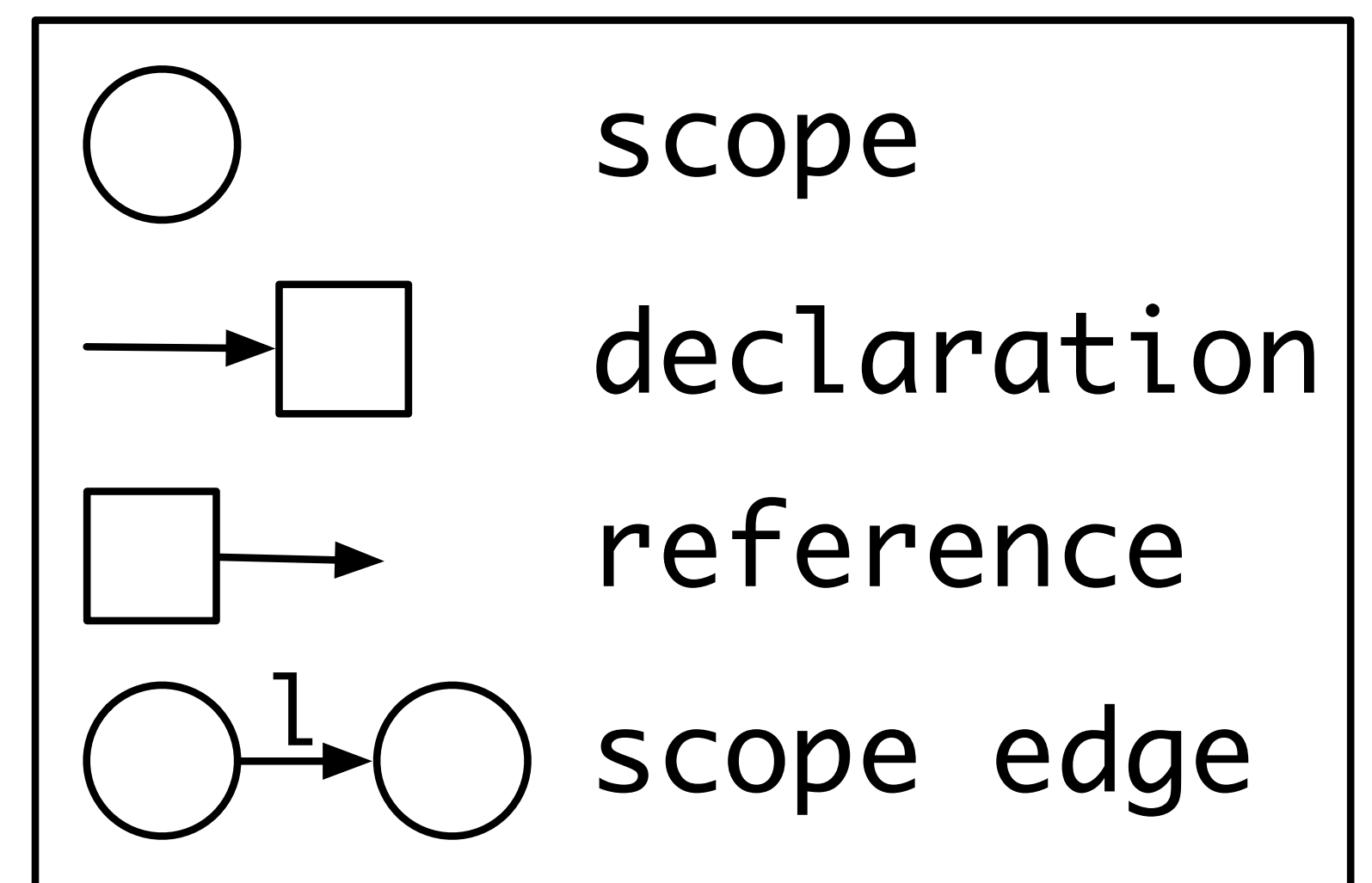
[ECOOP'16]

# Lexical Scoping

```
val x = 31;  
val y = x + 11;
```



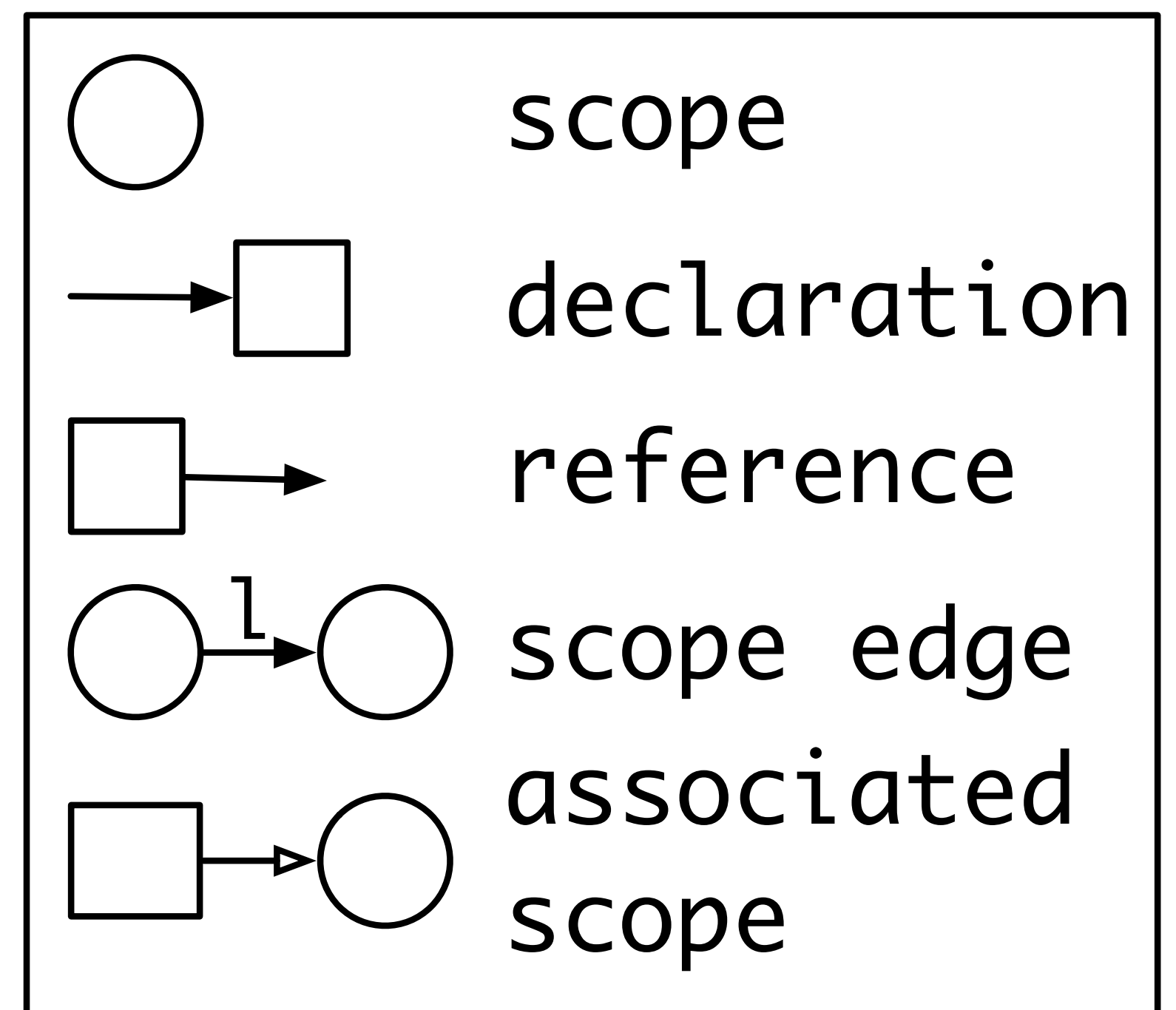
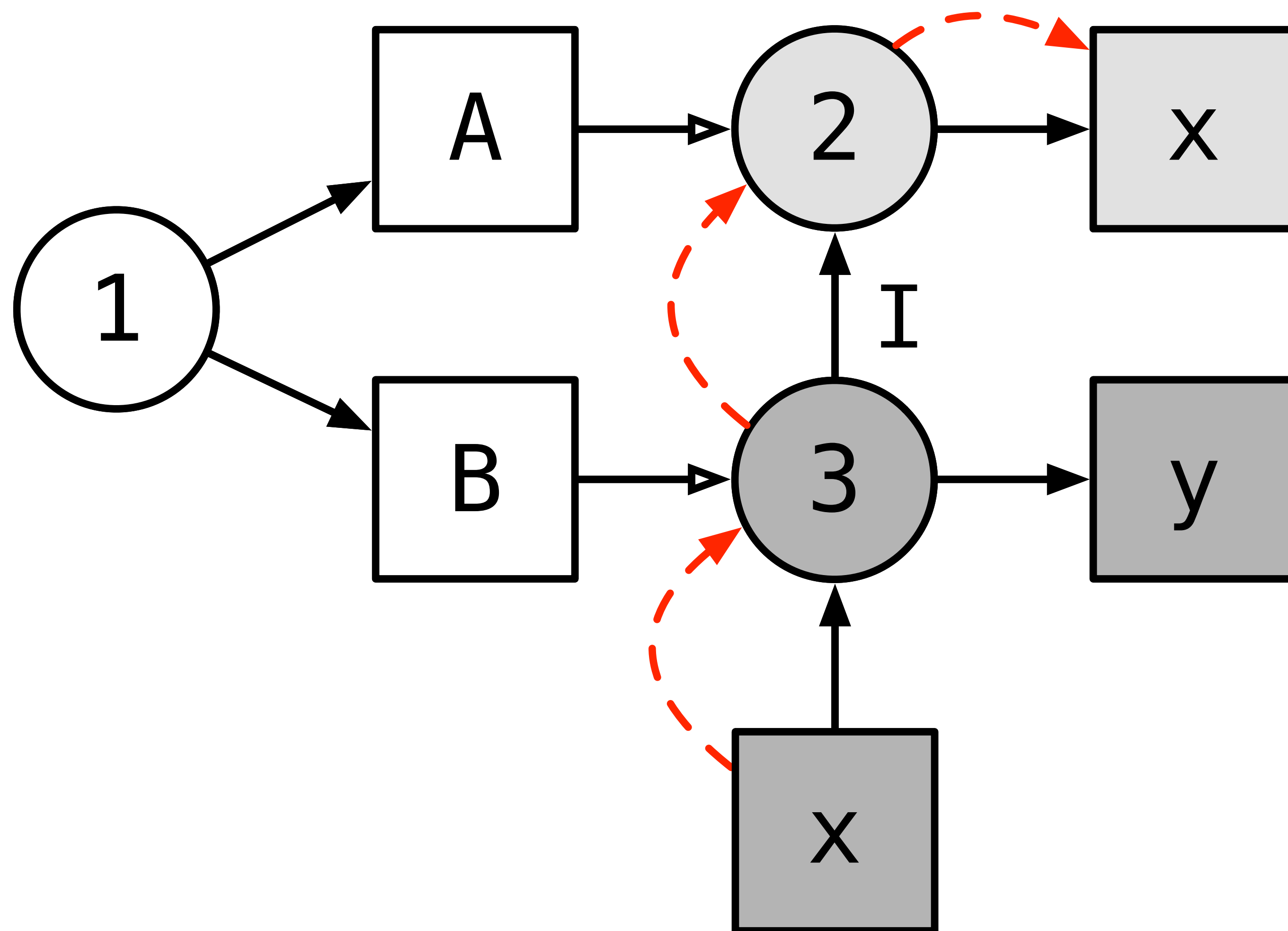
[ESOP'15; PEPM'16]





# Inheritance

```
class A { var x = 42; }  
class B extends A { var y = x; }
```



# More Binding Patterns

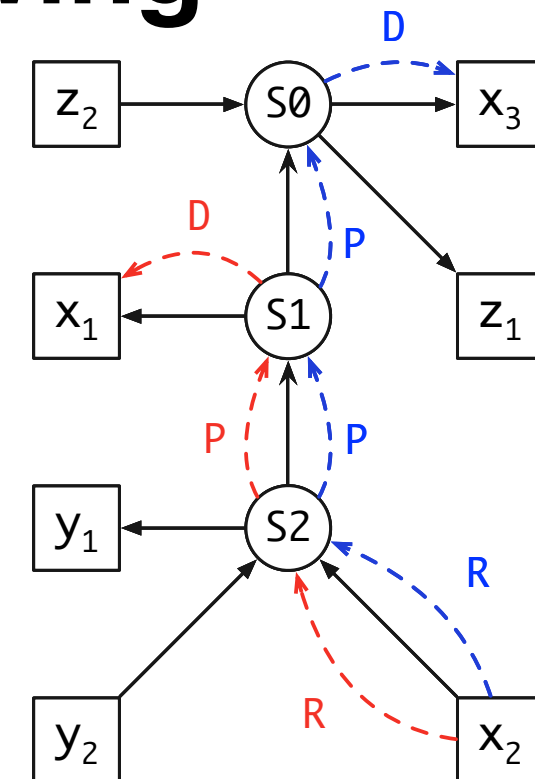
## Shadowing

```
def x3 = z2 5 7 S0
def z1 =
  fun x1 { S1
    fun y1 { S2
      x2 + y2
    }
  }
```

$D < P.p$

$p < p'$

$s.p < s.p'$

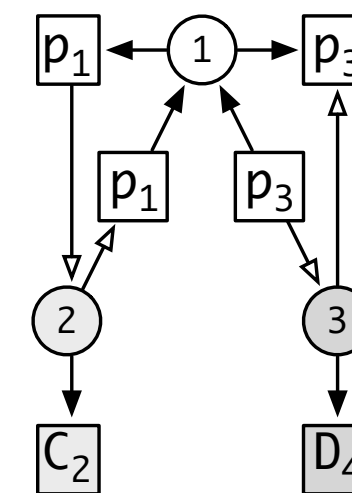


$R.P.D < R.P.P.D$

## Java Packages

```
package p1;
class C2 {}

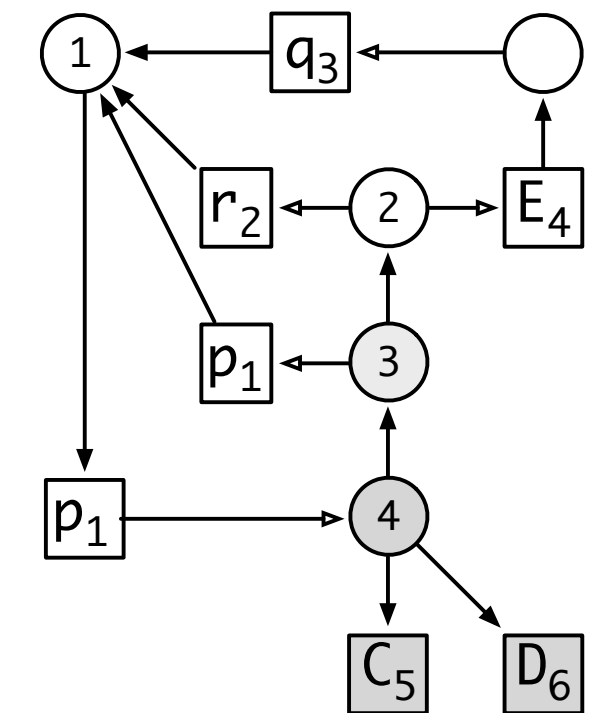
package p3;
class D4 {}
```



## Java Import

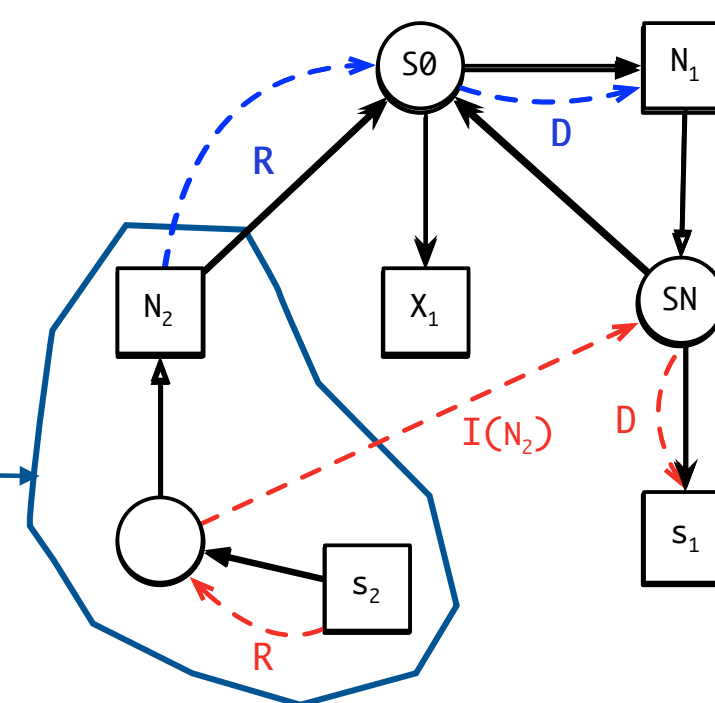
```
package p1;
imports r2.*;
imports q3.E4;

public class C5 {}
class D6 {}
```



## Qualified Names

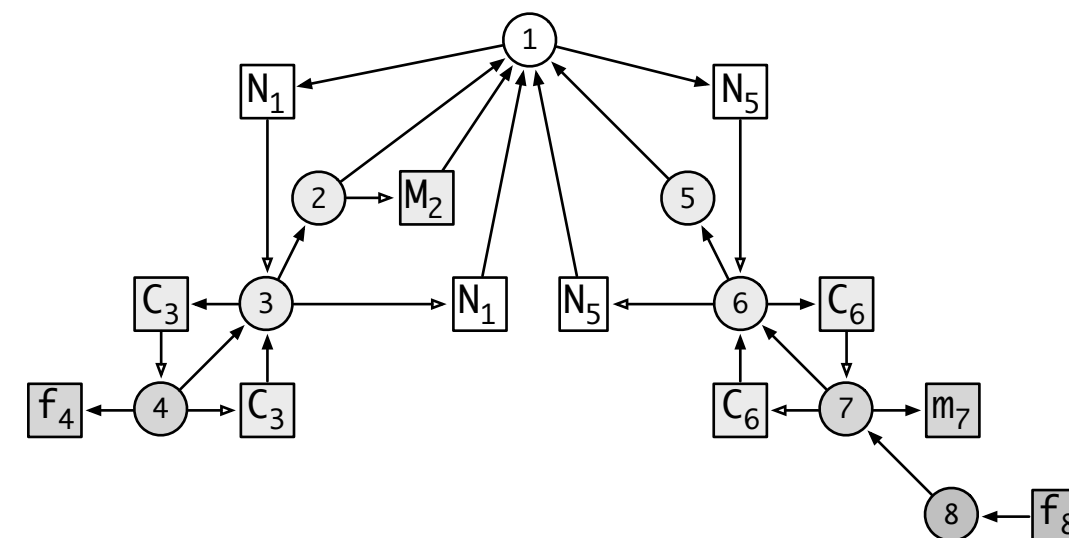
```
module N1 { S0
  def s1 = 5
}
module M1 {
  def x1 = 1 + N2.s2
}
```



## C# Namespaces and Partial Classes

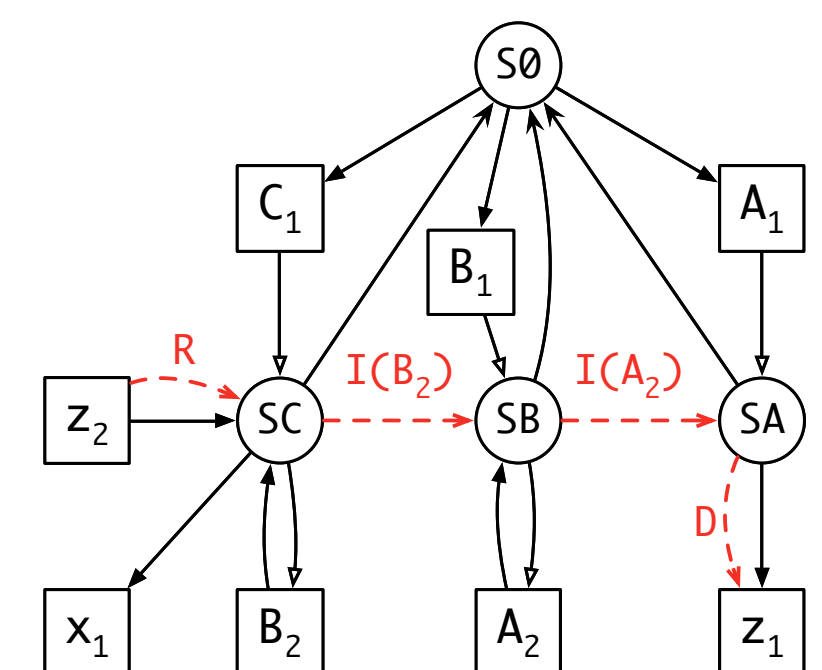
```
namespace N1 {
  using M2;
  partial class C3 {
    int f4;
  }
}
```

```
namespace N5 {
  partial class C6 {
    int m7 {
      return f8;
    }
  }
}
```



## Transitive vs. Non-Transitive

```
module A1 {
  def z1 = 5 SA
}
module B1 {
  import A2 SB
}
module C1 {
  import B2 SC
  def x1 = 1 + z2
}
```



With transitive imports, a well formed path is  $R.P*.I(_)*.D$

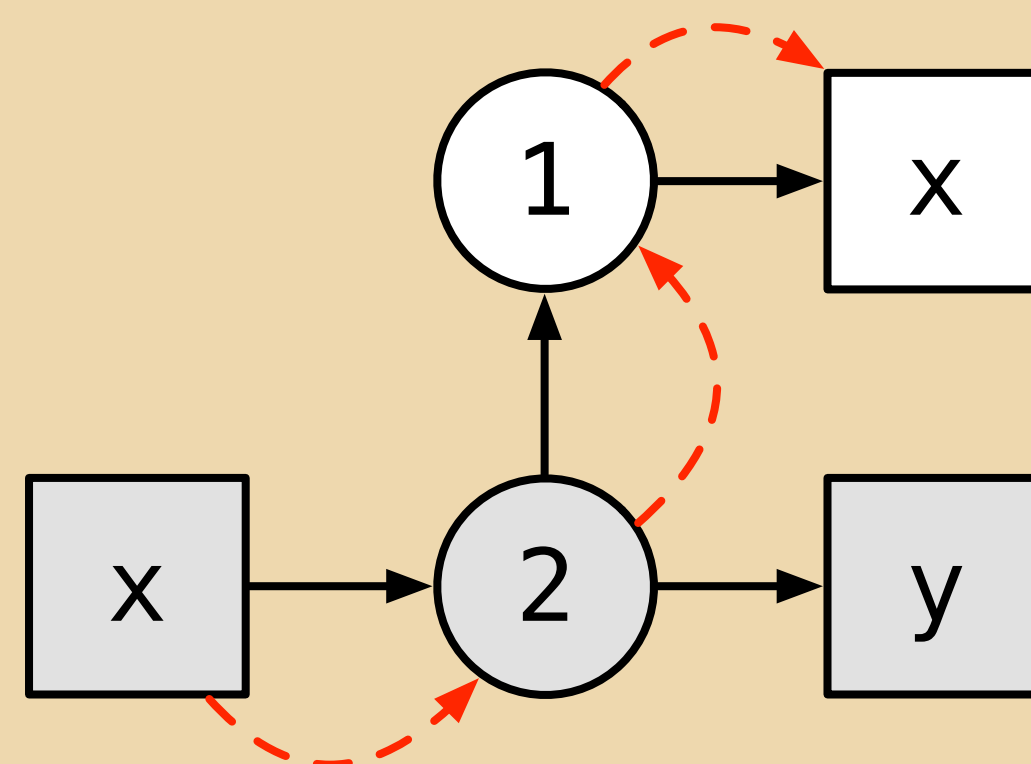
With non-transitive imports, a well formed path is  $R.P*.I(_)?*.D$

		Static	Dynamic
Lexical Mutable Objects	<pre> val x = 31; val y = x + 11; </pre>	Typing Contexts Type Substitution	Substitution Environments De Bruijn Indices HOAS
	<pre> var x = 31; x = x + 11; </pre>	Typing Contexts Store Typing	Stores/Heaps
	<pre> class A {   var x = 0;   var y = 42; } var r = new A(); </pre>	Class Tables	Mutable Objects Stores/Heaps

## Lexical

```
val x = 31;  
val y = x + 11;
```

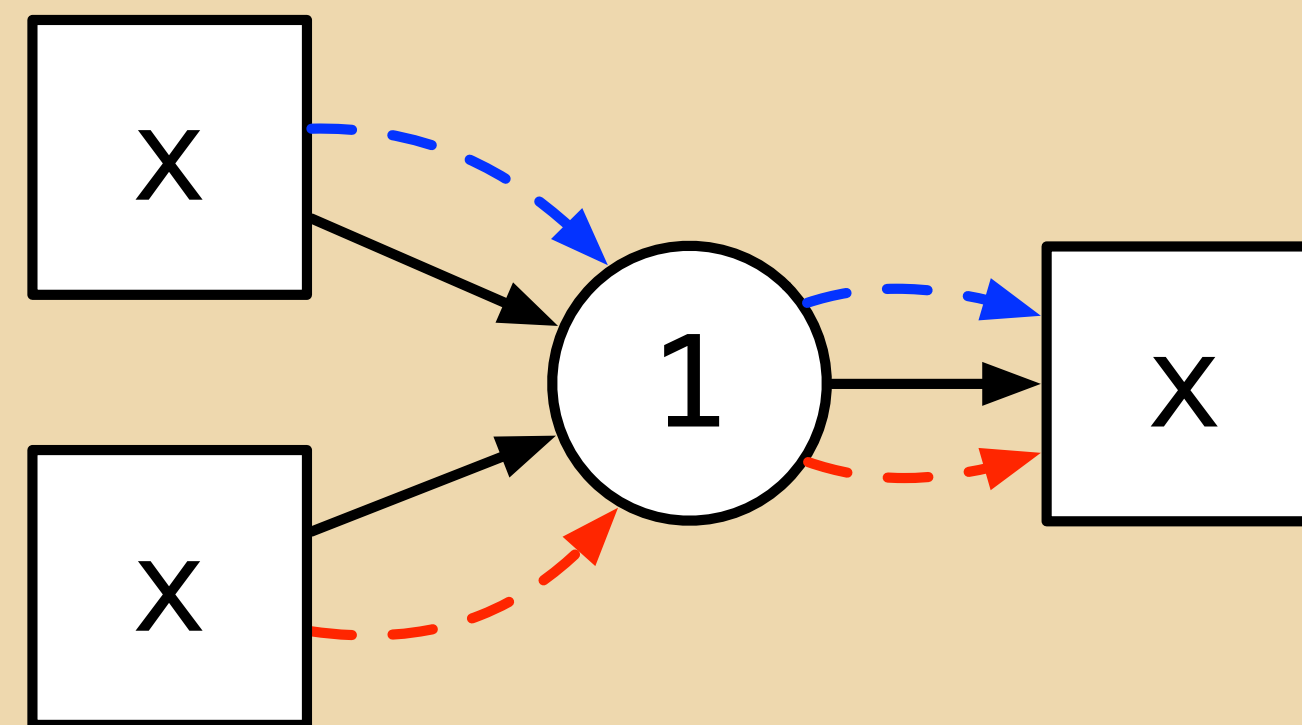
## Static



Substitution  
Environments  
De Bruijn Indices  
HOAS

## Mutable

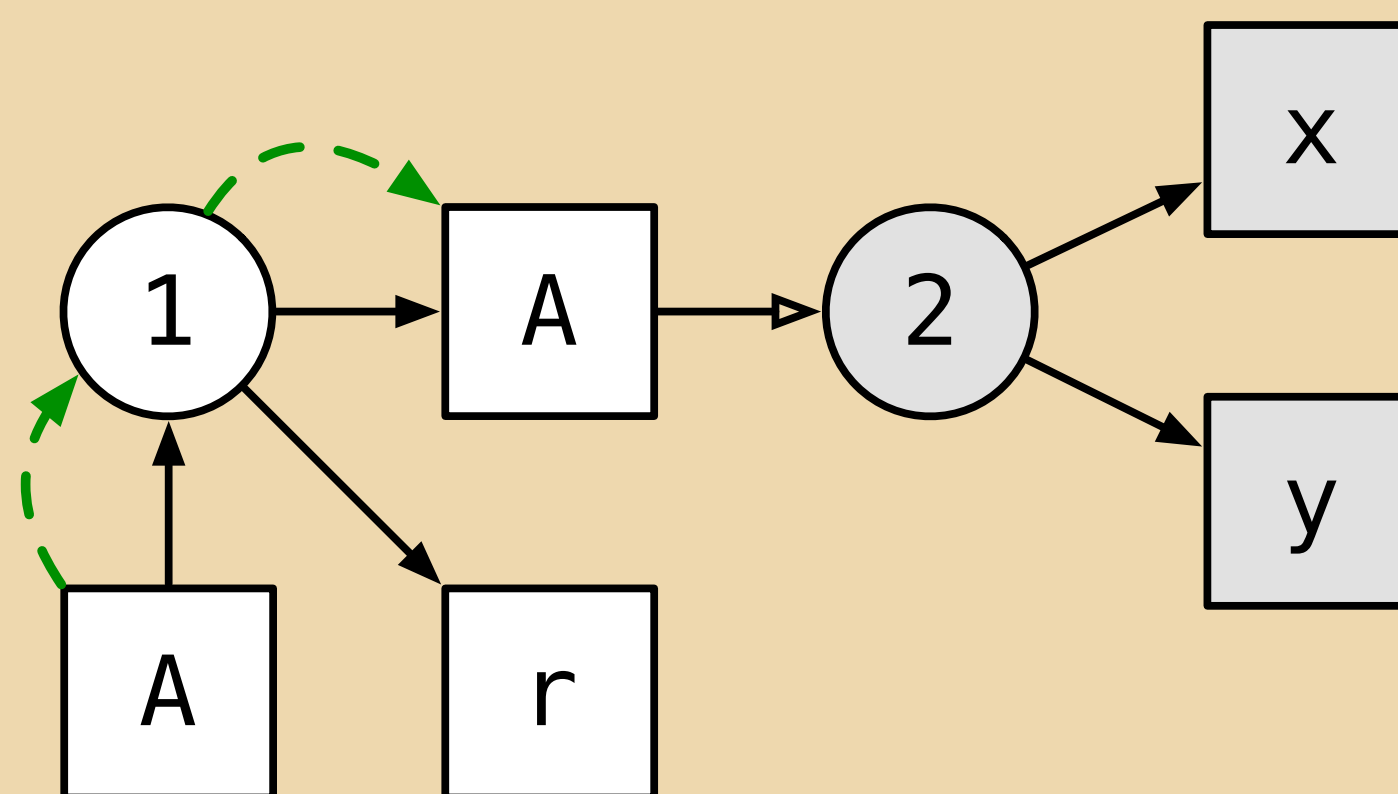
```
var x = 31;  
x = x + 11;
```



Stores/Heaps

## Objects

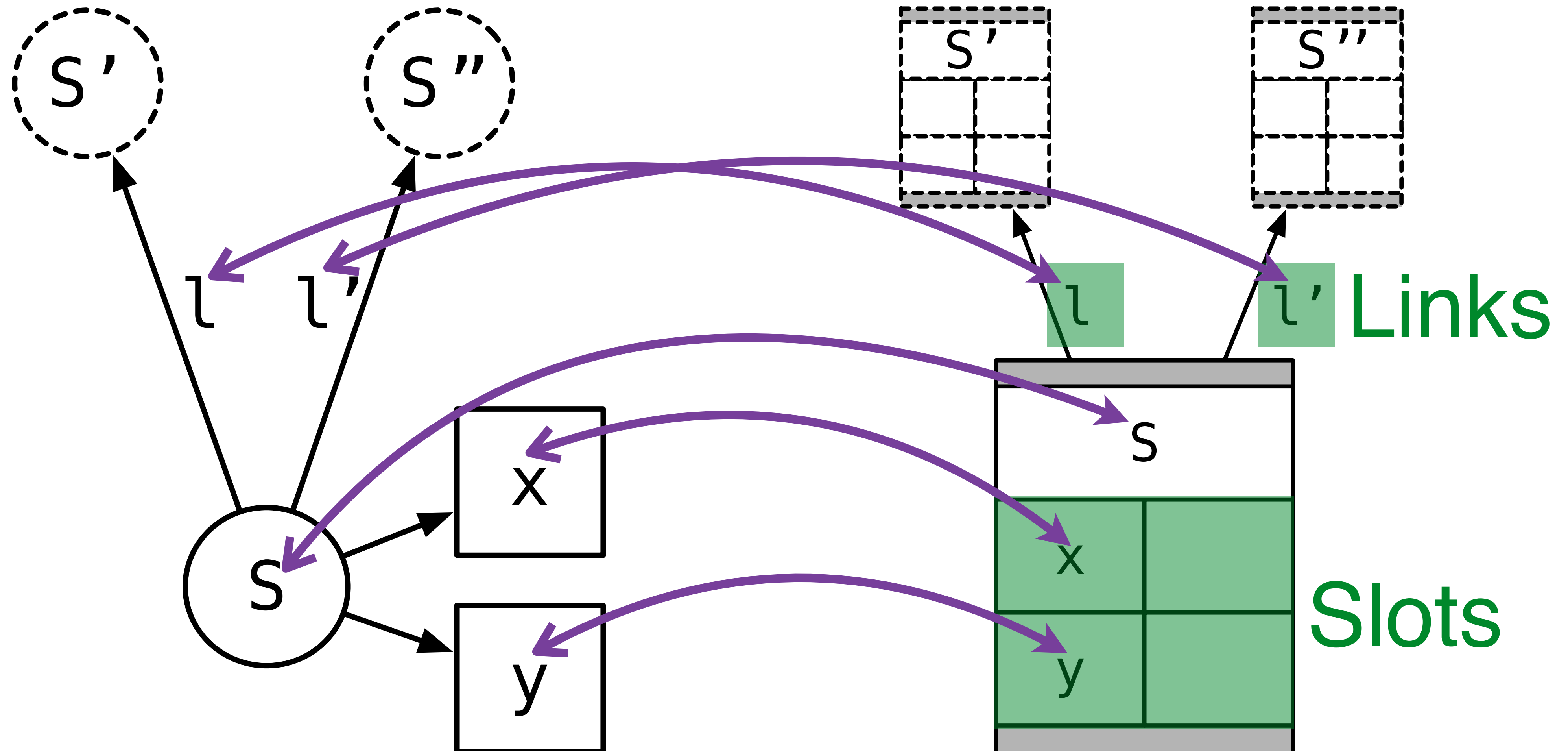
```
class A {  
  var x = 0;  
  var y = 42;  
}  
var r = new A();
```



Mutable Objects  
Stores/Heaps

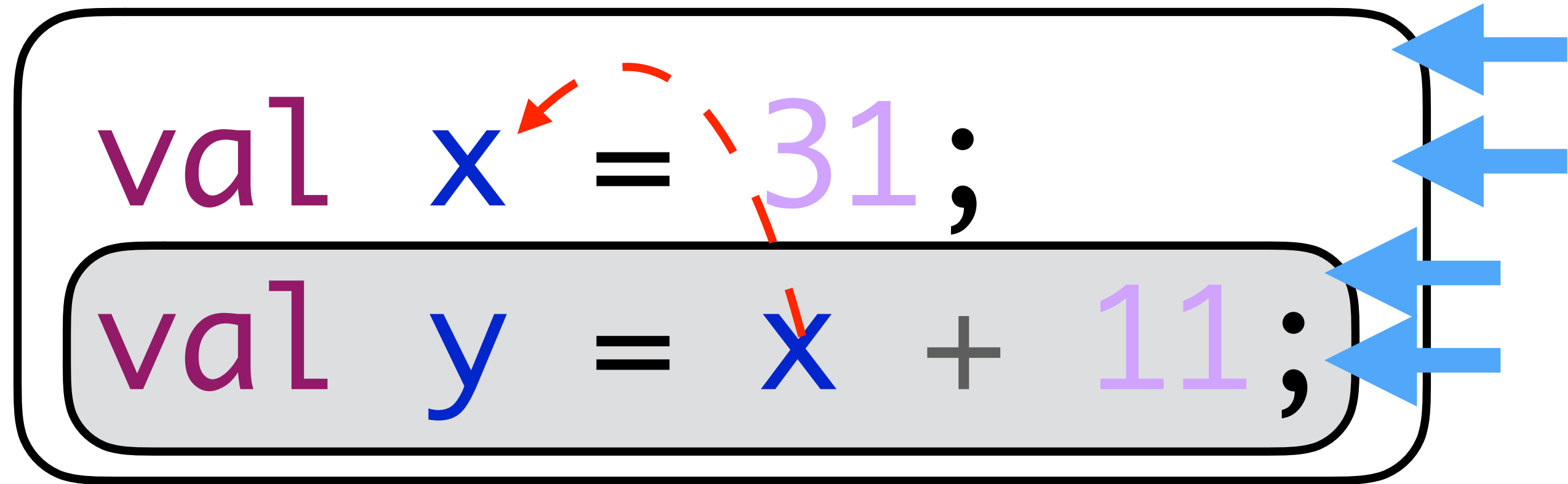
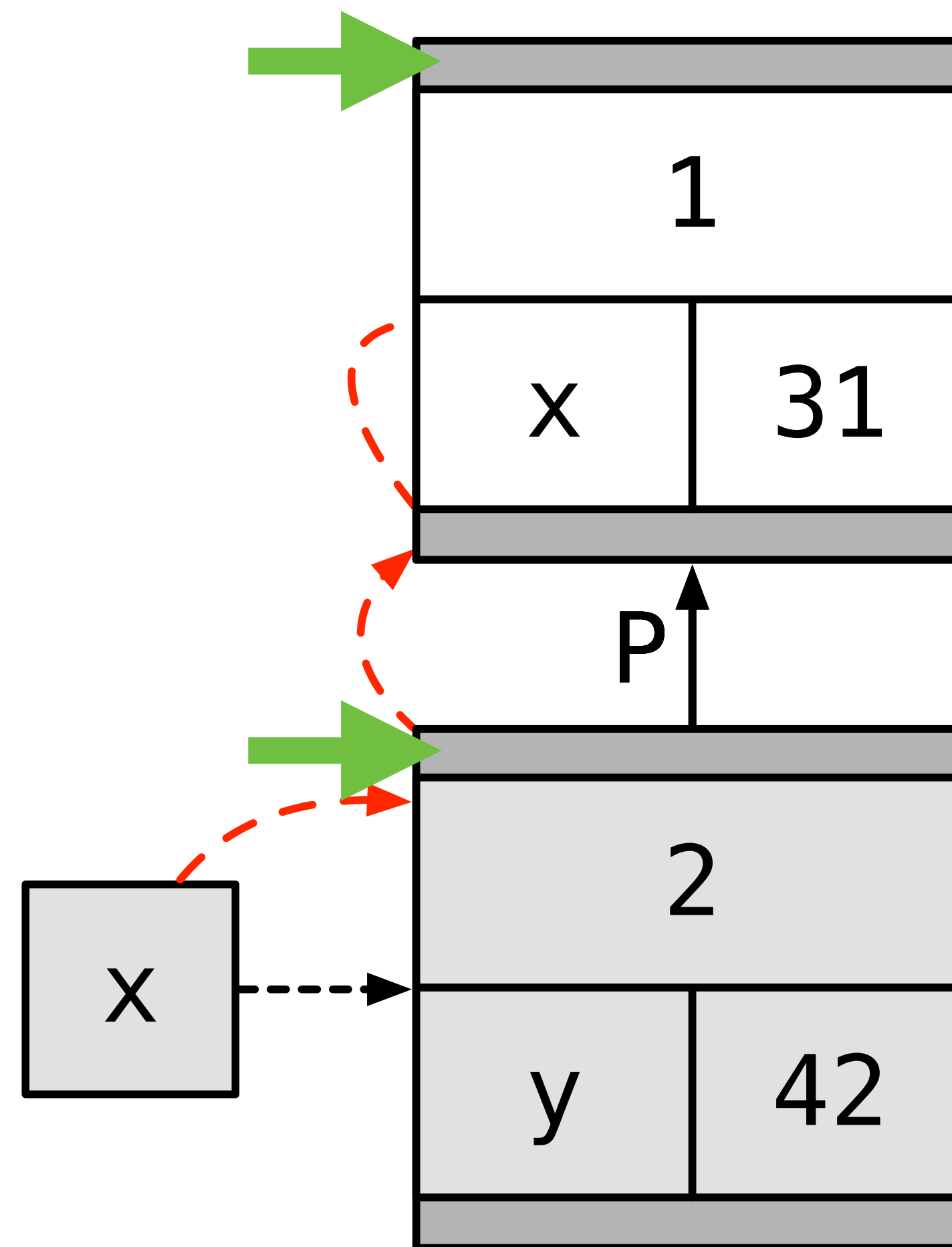
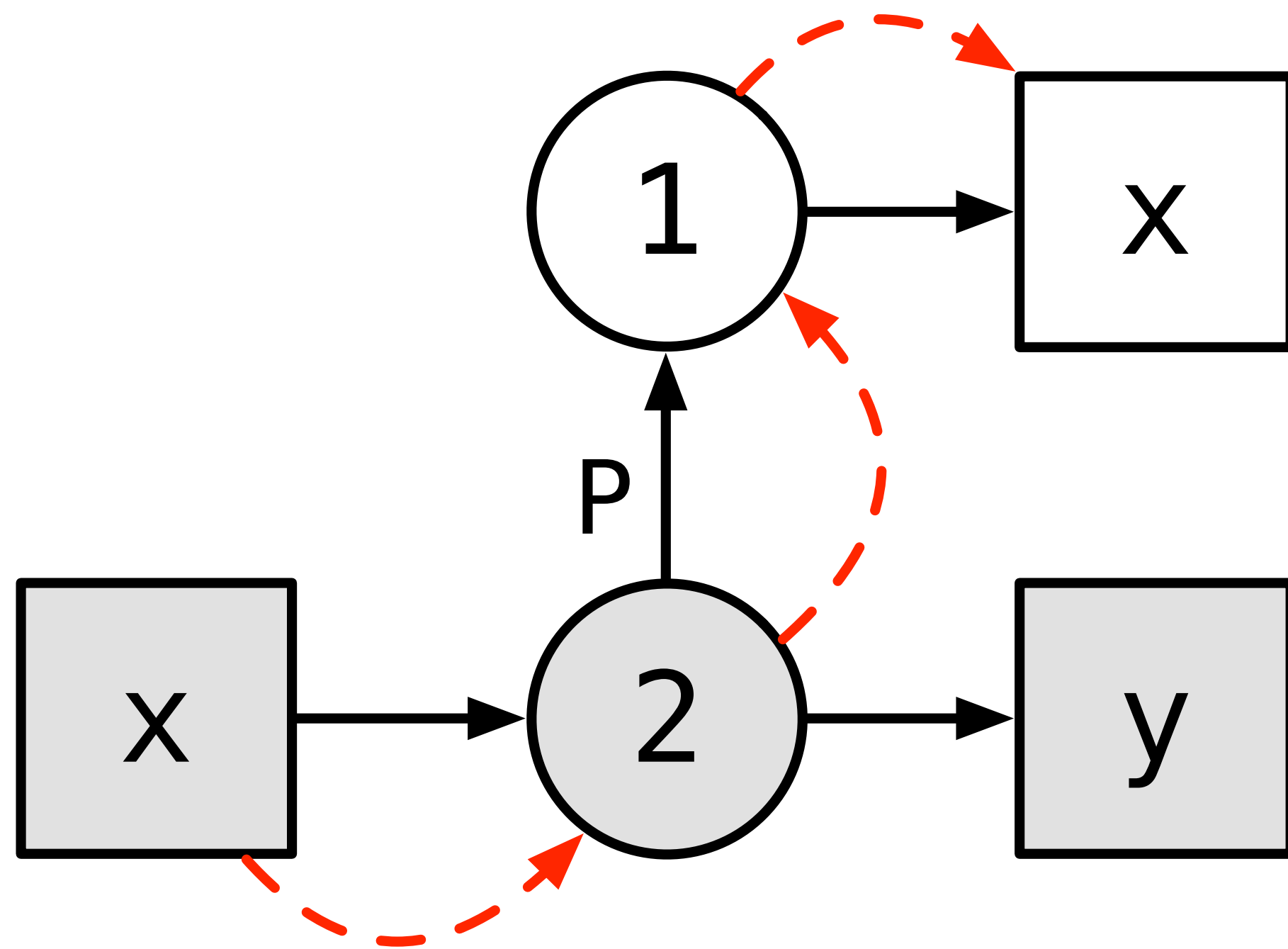
# Scope

# Frame



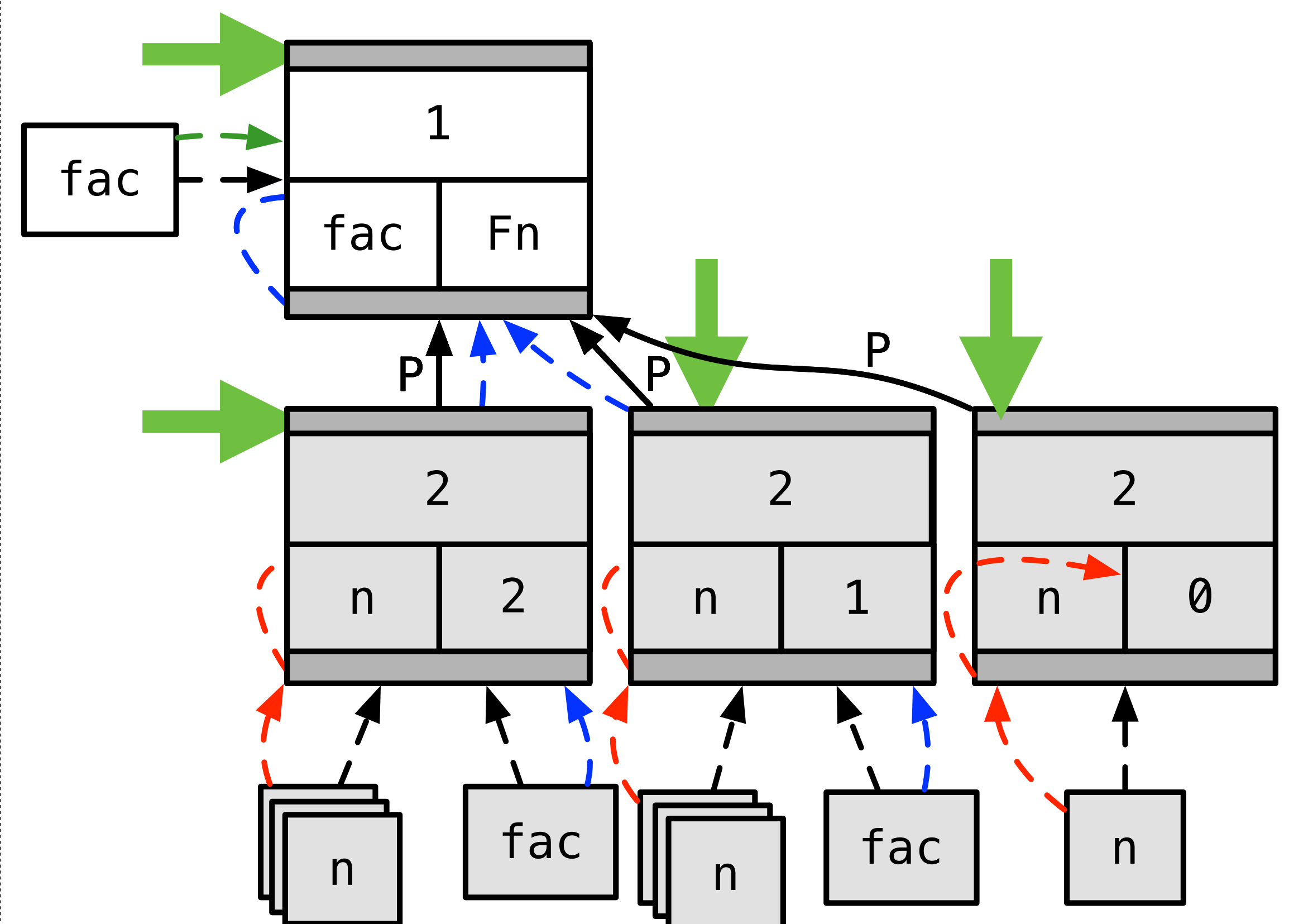
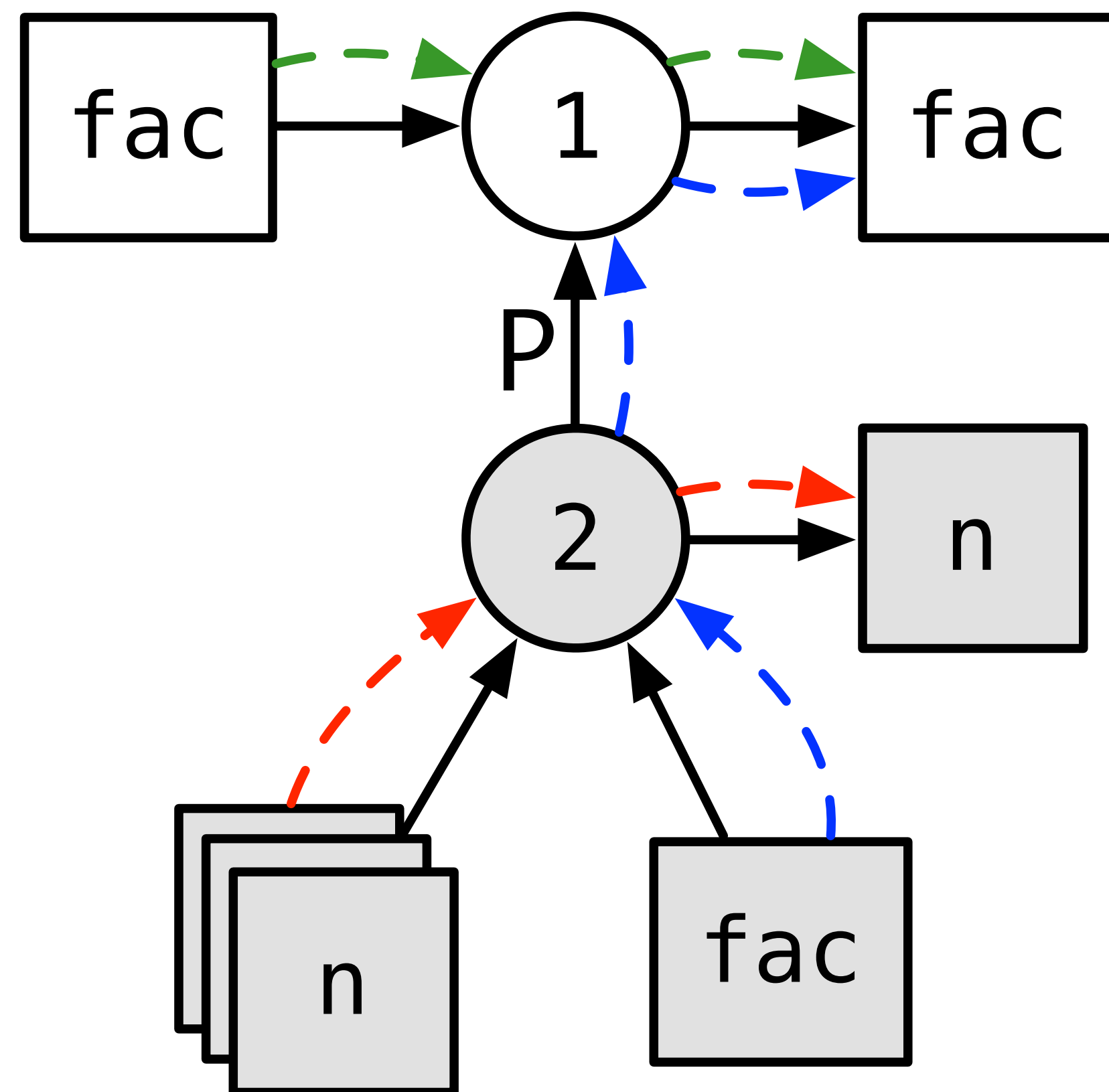
[ECOOP'16]

```
val x = 31;  
val y = x + 11;
```

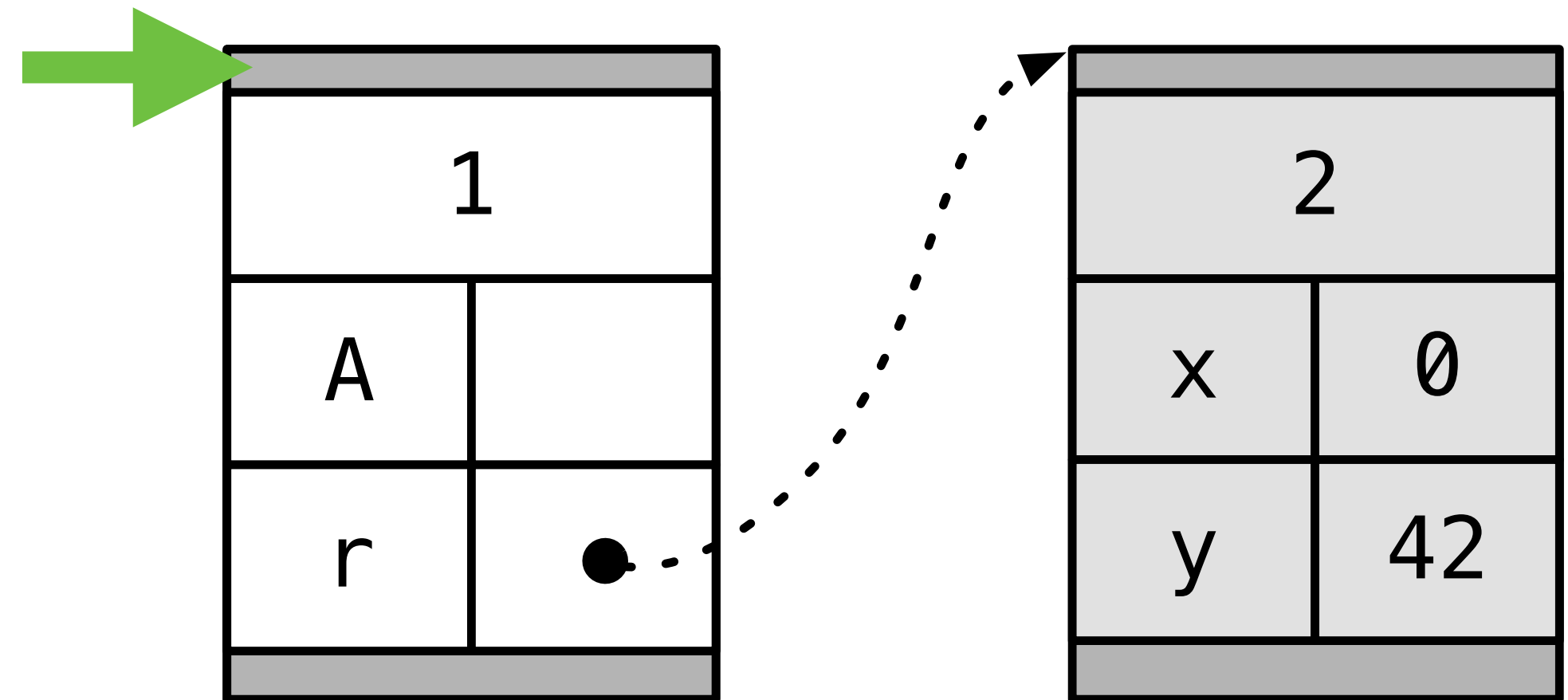
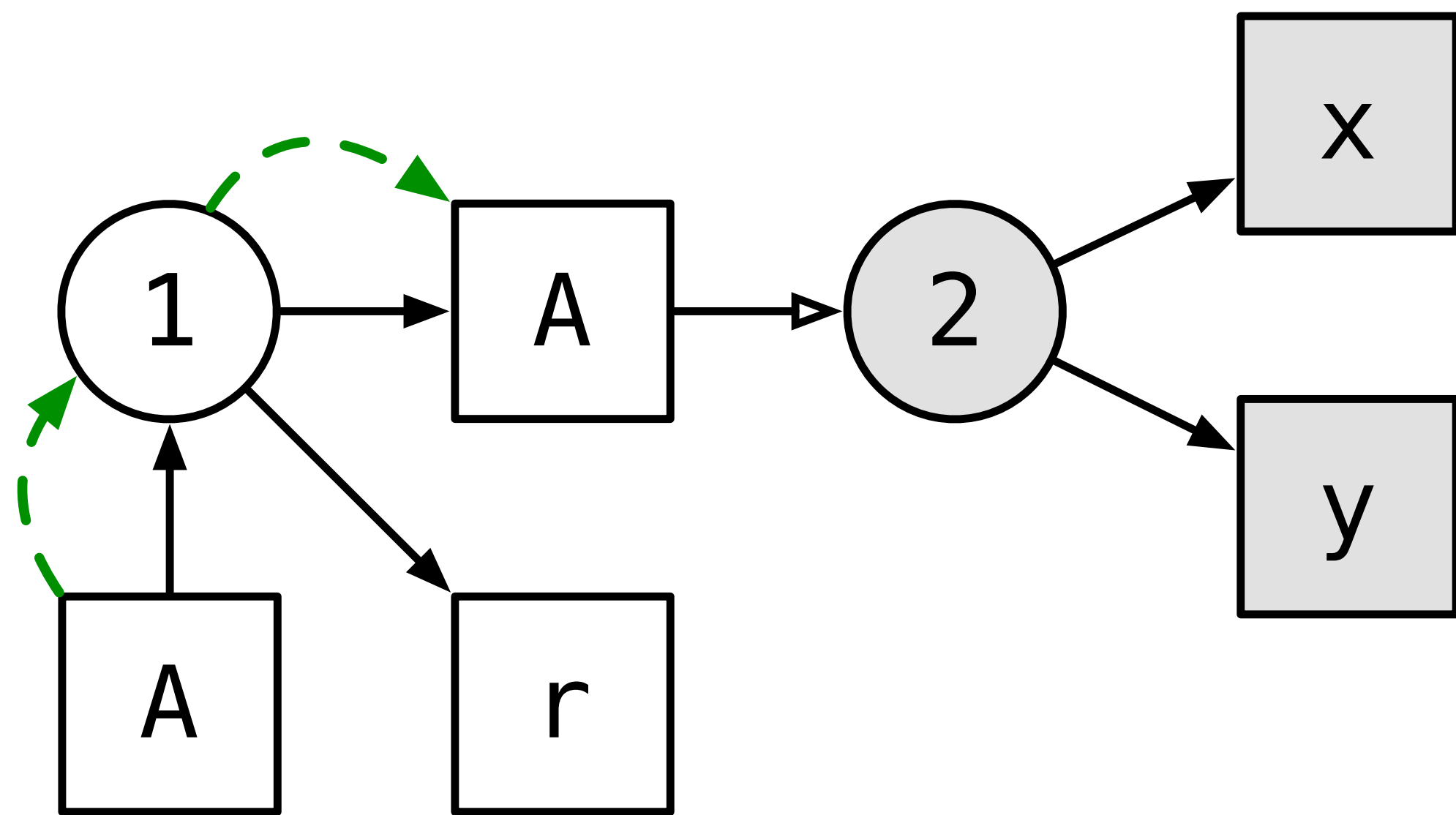
A diagram showing two code blocks. The top block contains 'val x = 31;' and the bottom block contains 'val y = x + 11;'. Blue arrows point from the right towards each line of code. A red dashed arrow points from the '31' in the first line to the 'x' in the second line.



```
def fac(n : Int) : Int = {
  if (n == 0) 1
  else n * fac(n - 1)
};
fac(2);
```



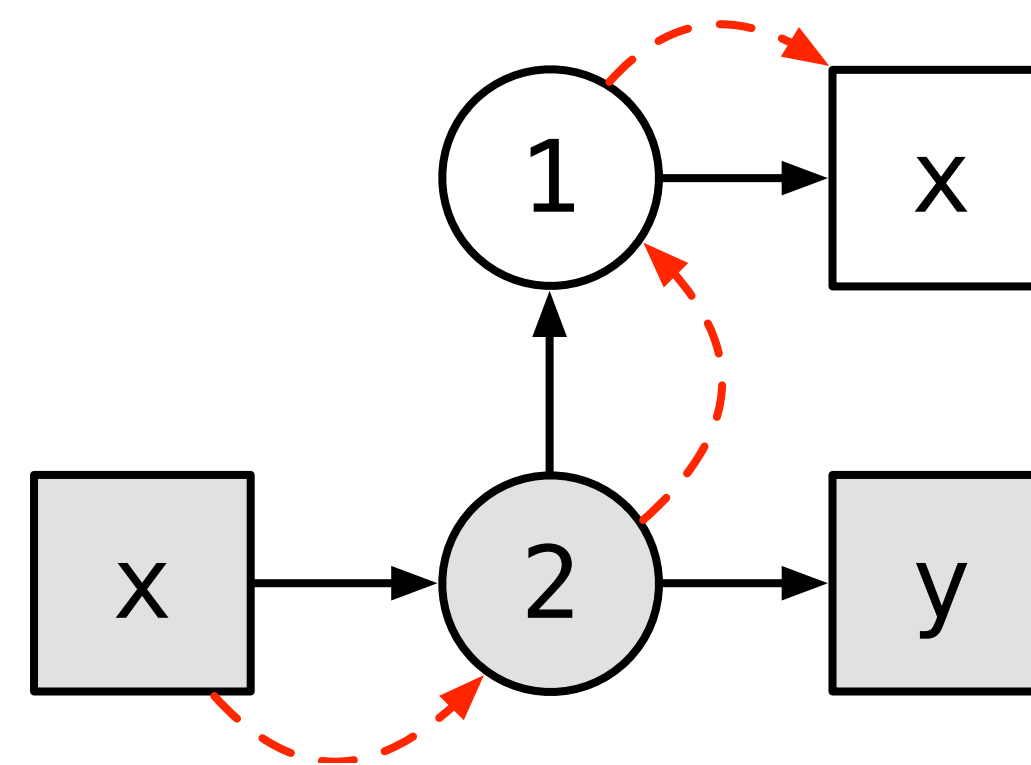
```
class A {  
    var x = 0;  
    var y = 42;  
}  
var r = new A();
```



## Lexical

```
val x = 31;  
val y = x + 11;
```

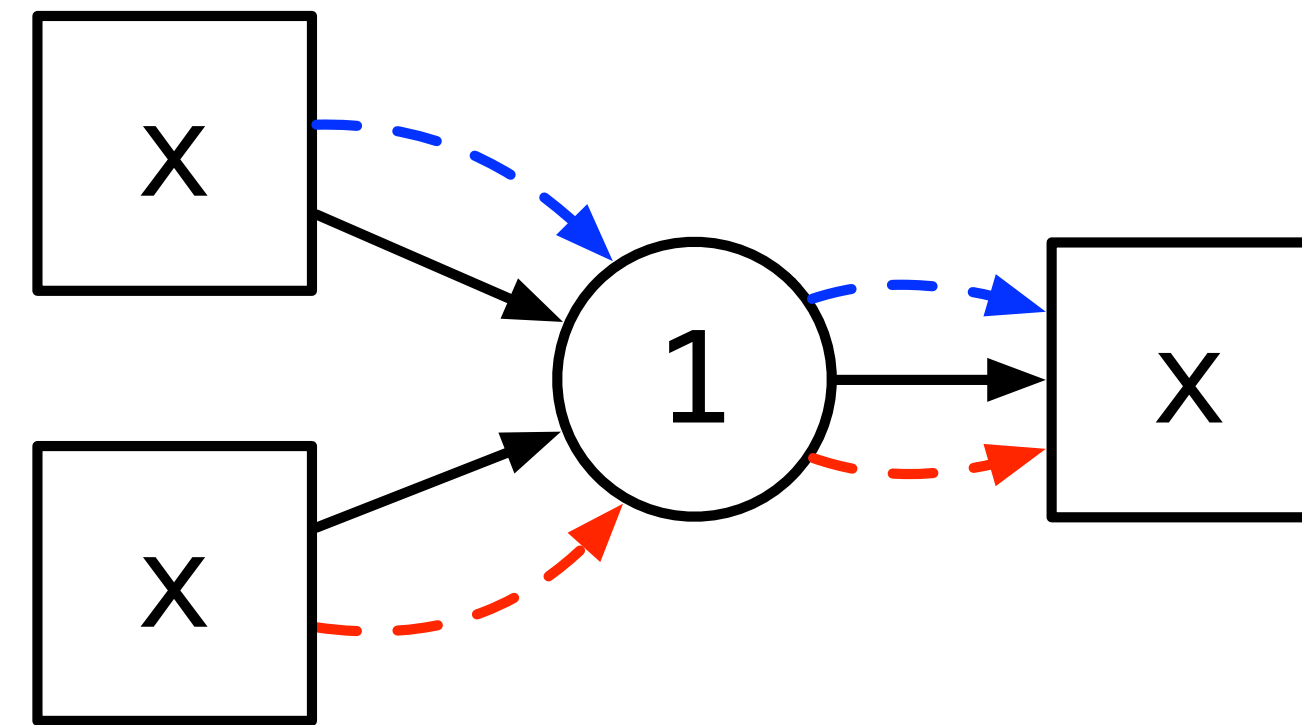
## Static



Substitution  
Environments  
De Bruijn Indices  
HOAS

## Mutable

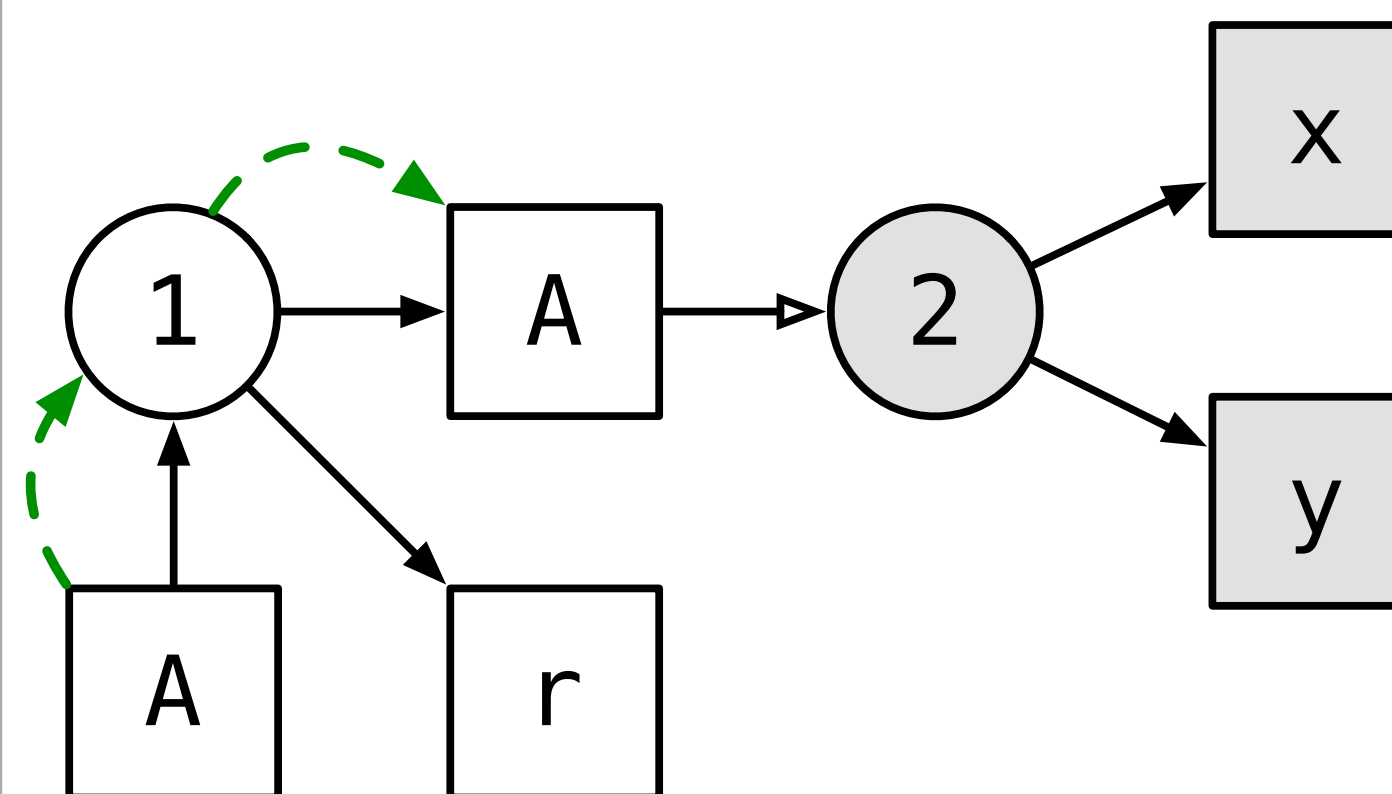
```
var x = 31;  
x = x + 11;
```



Stores/Heaps

## Objects

```
class A {  
  var x = 0;  
  var y = 42;  
}  
var r = new A();
```

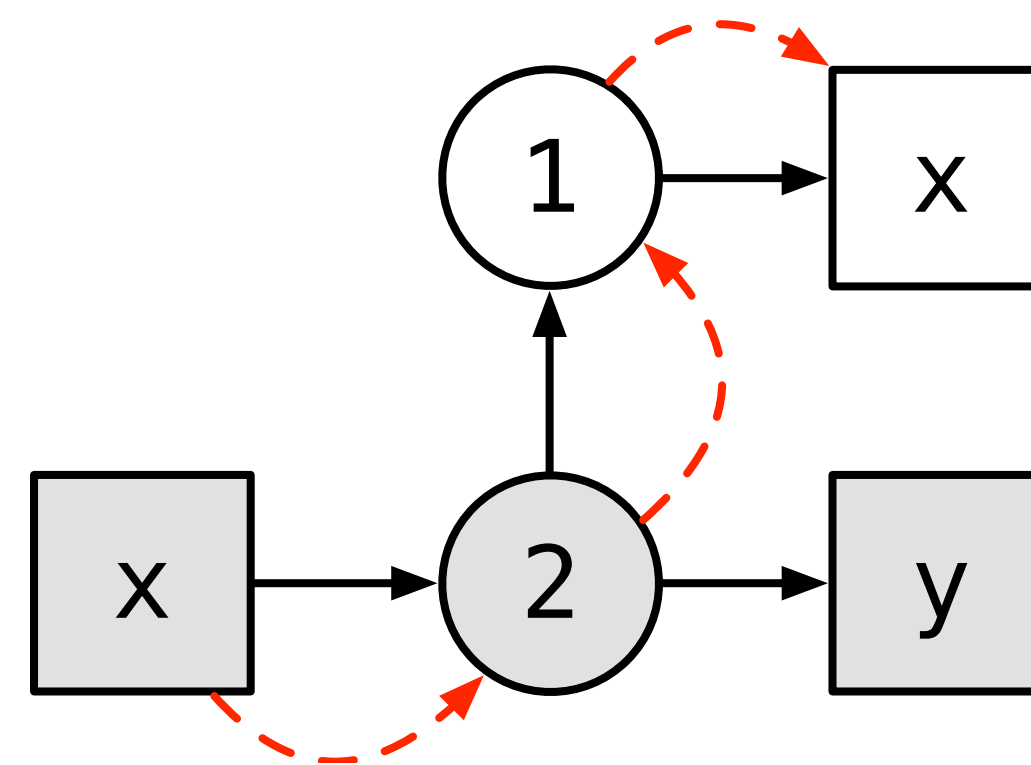


Mutable Objects  
Stores/Heaps

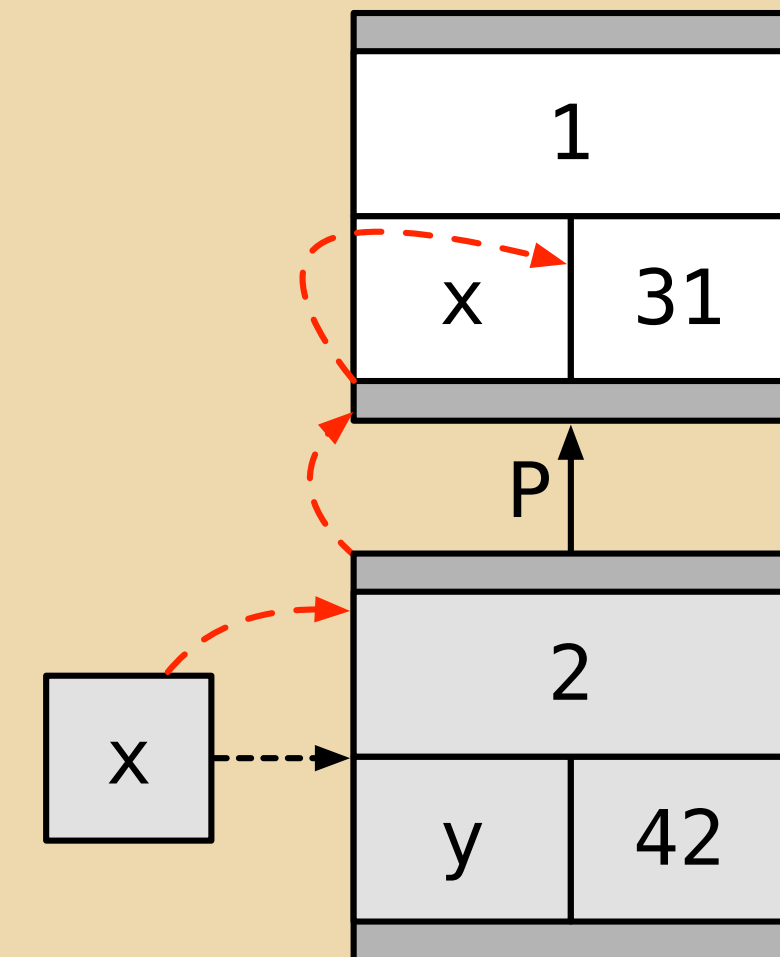
## Lexical

```
val x = 31;  
val y = x + 11;
```

## Static

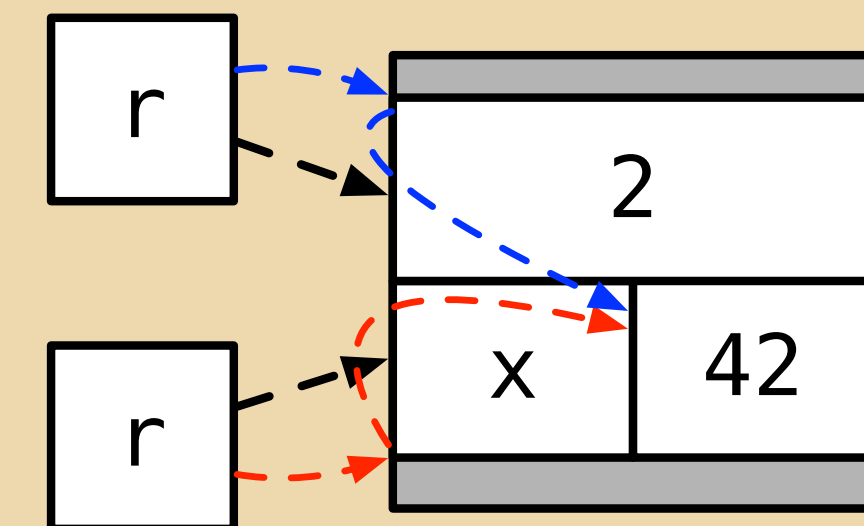
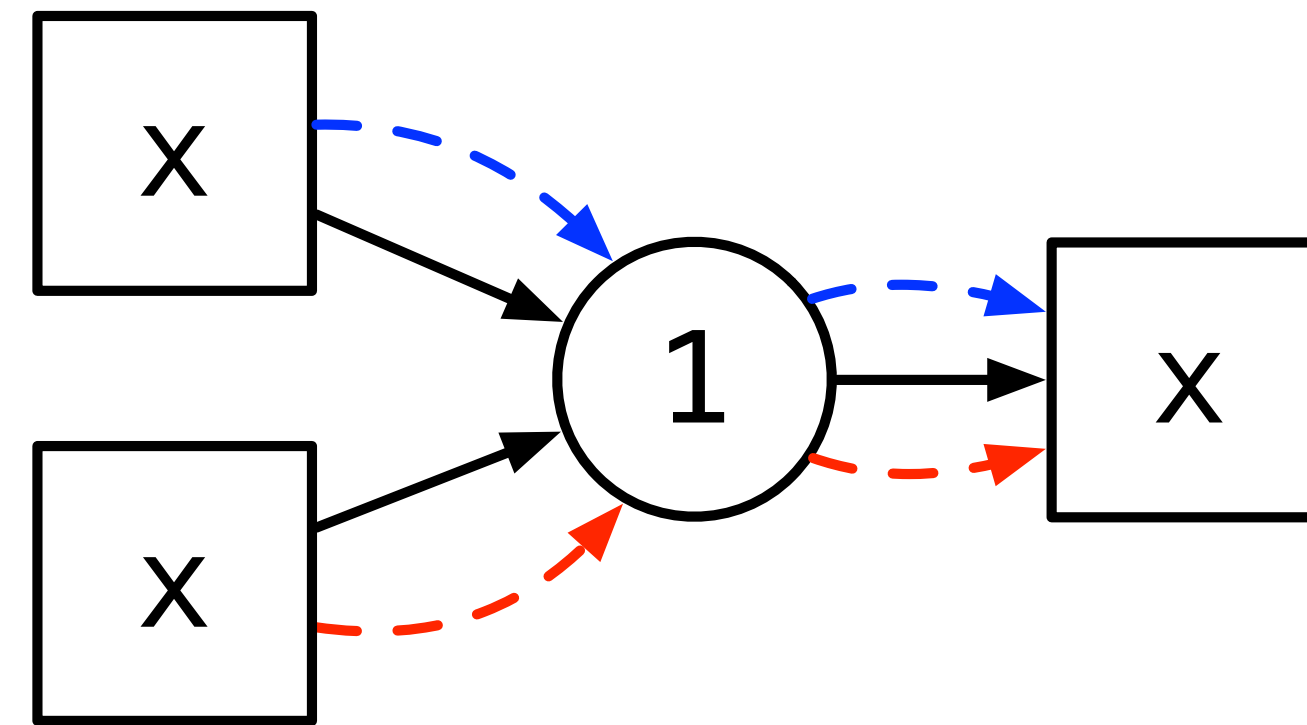


## Dynamic



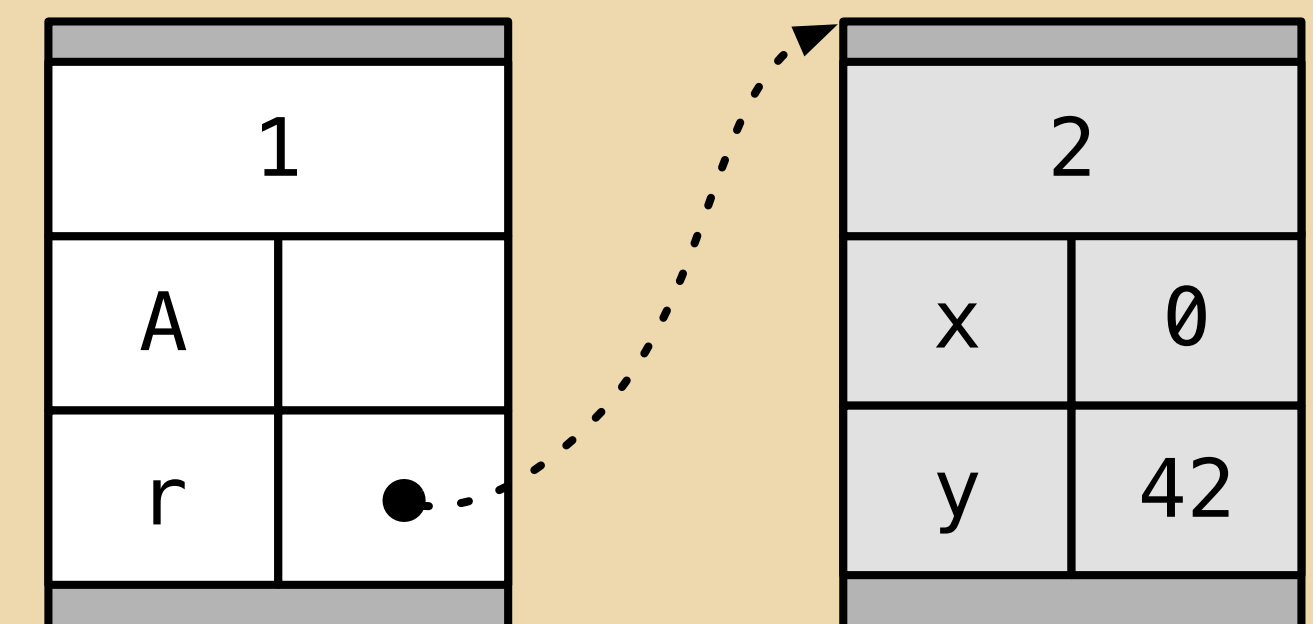
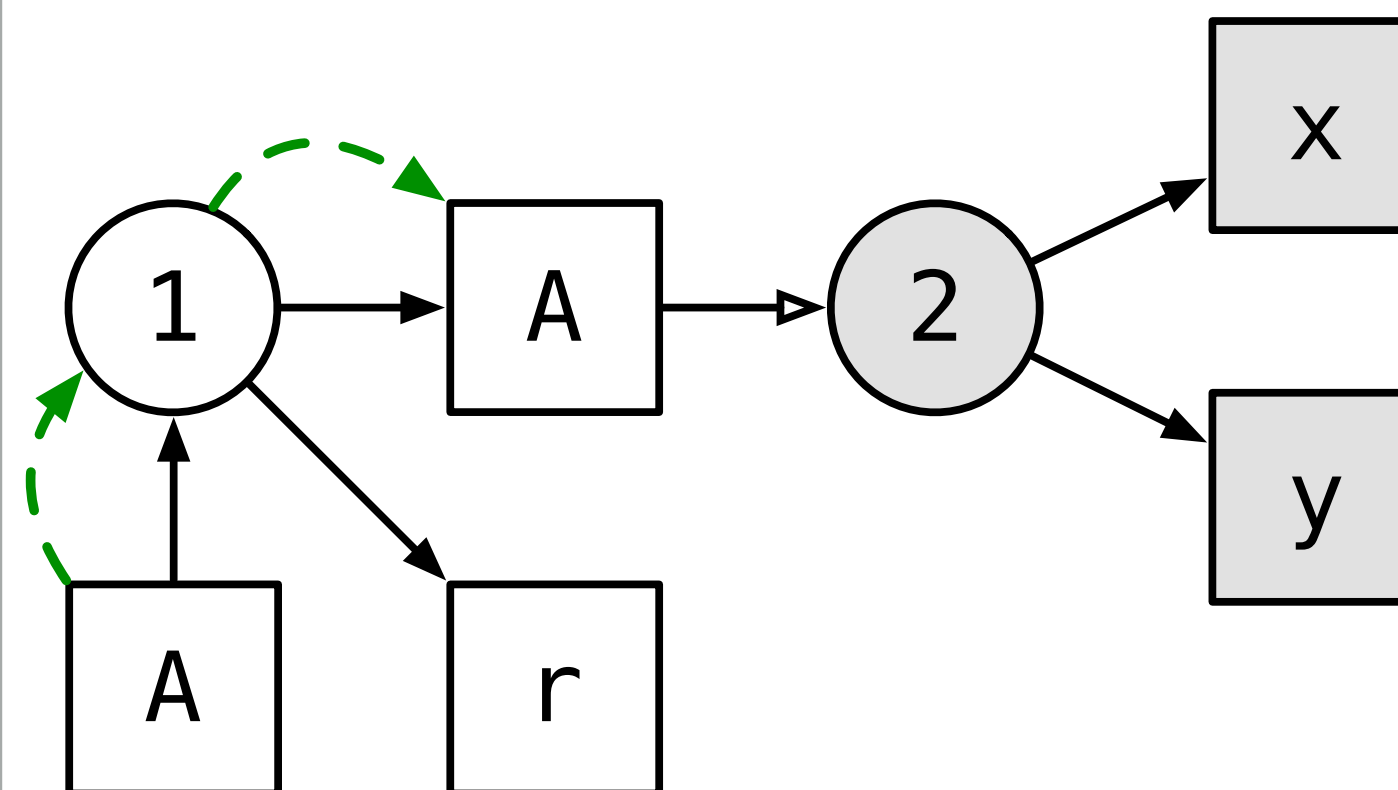
## Mutable

```
var x = 31;  
x = x + 11;
```



## Objects

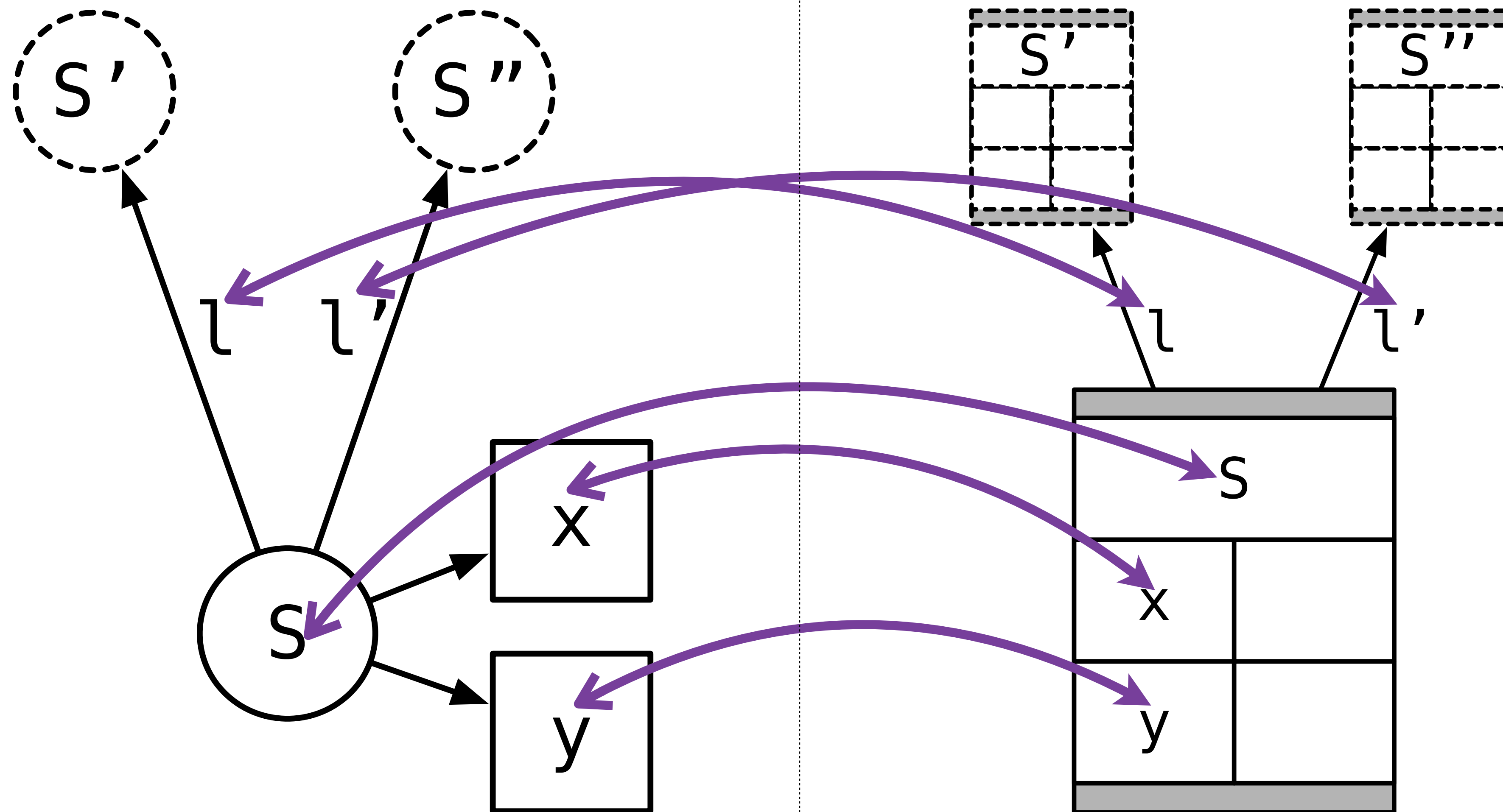
```
class A {  
  var x = 0;  
  var y = 42;  
}  
var r = new A();
```



# Well-Bound Frame

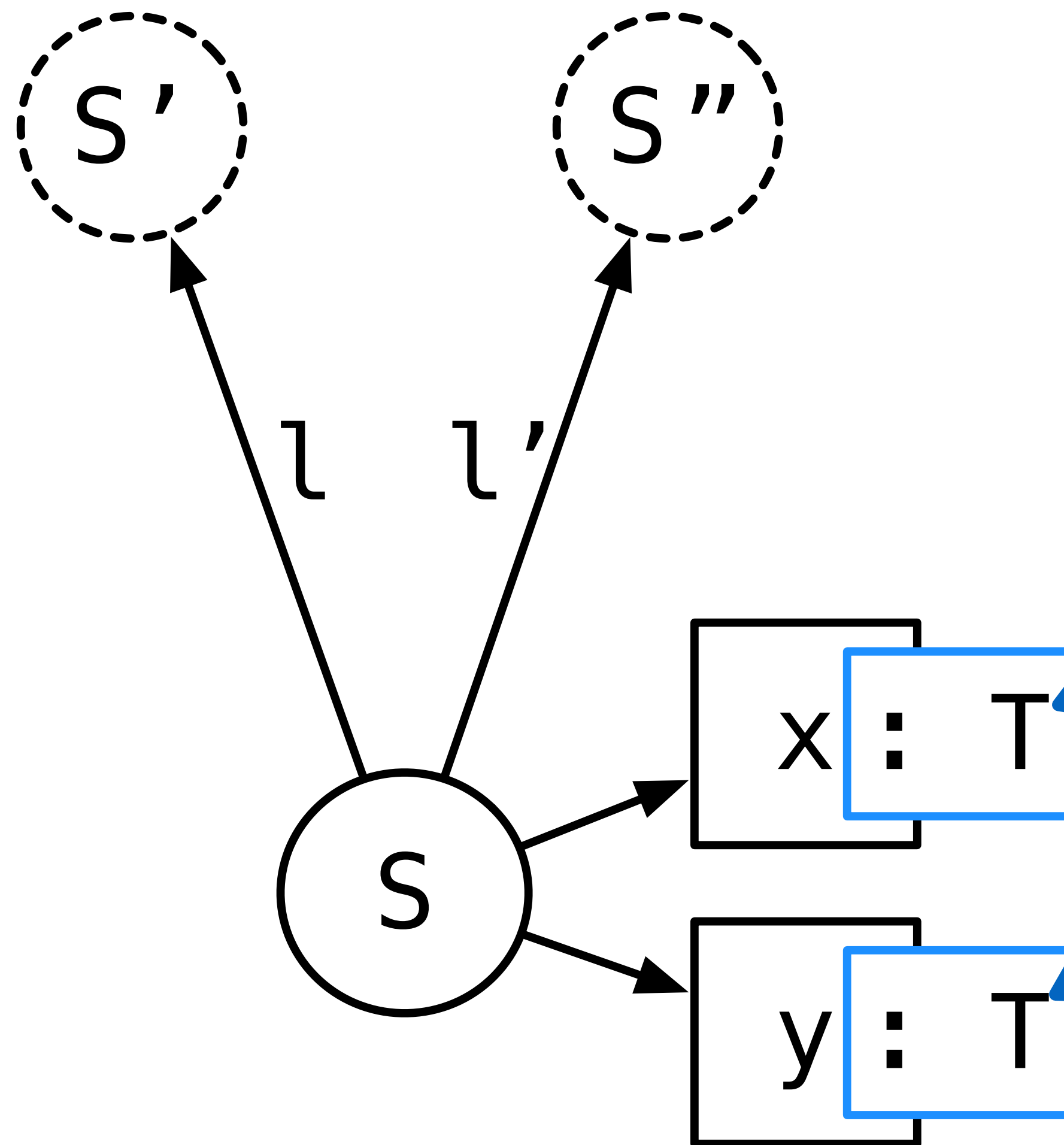
**Scope**

**Frame**

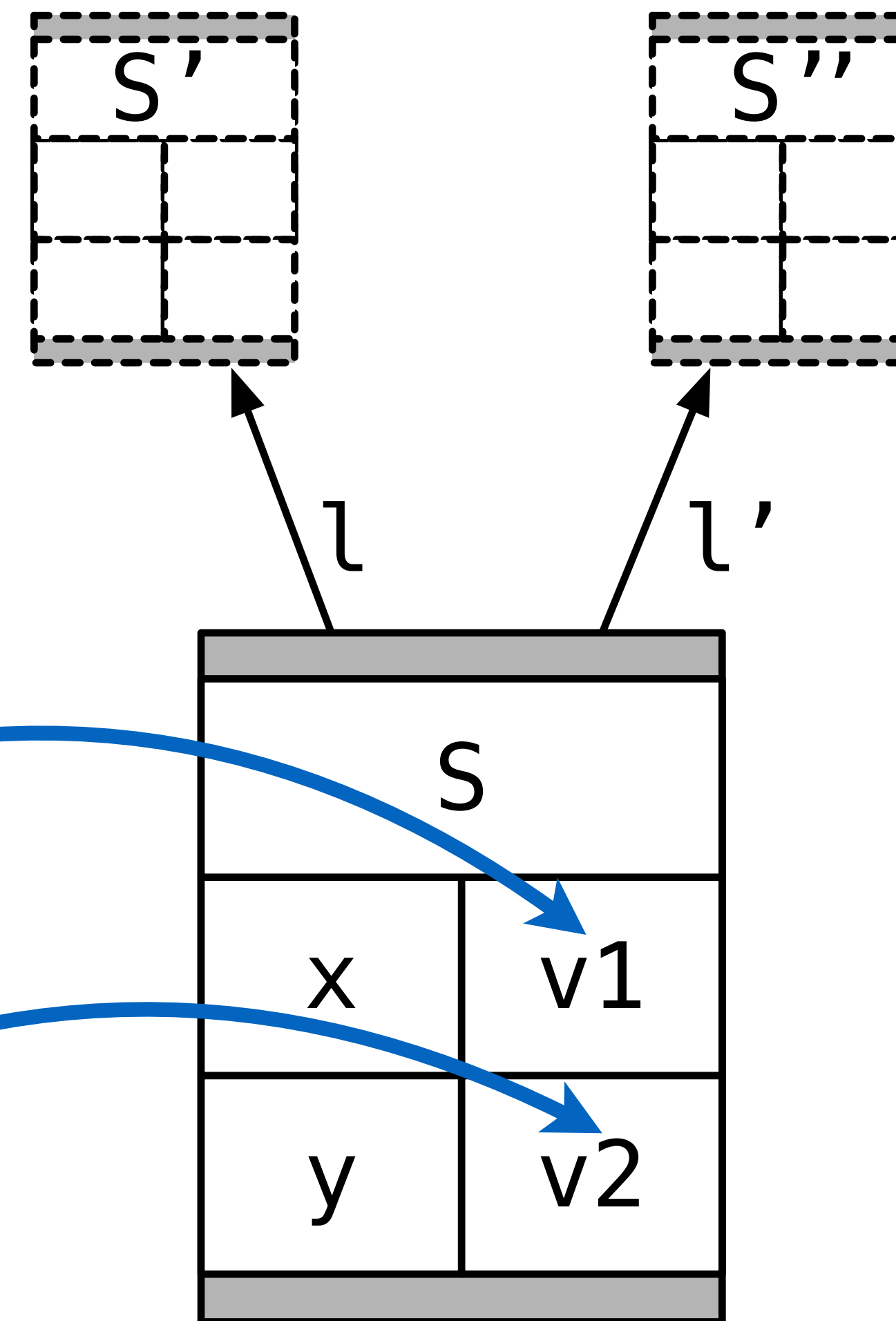


# Well-Typed Frame

**Scope**



**Frame**



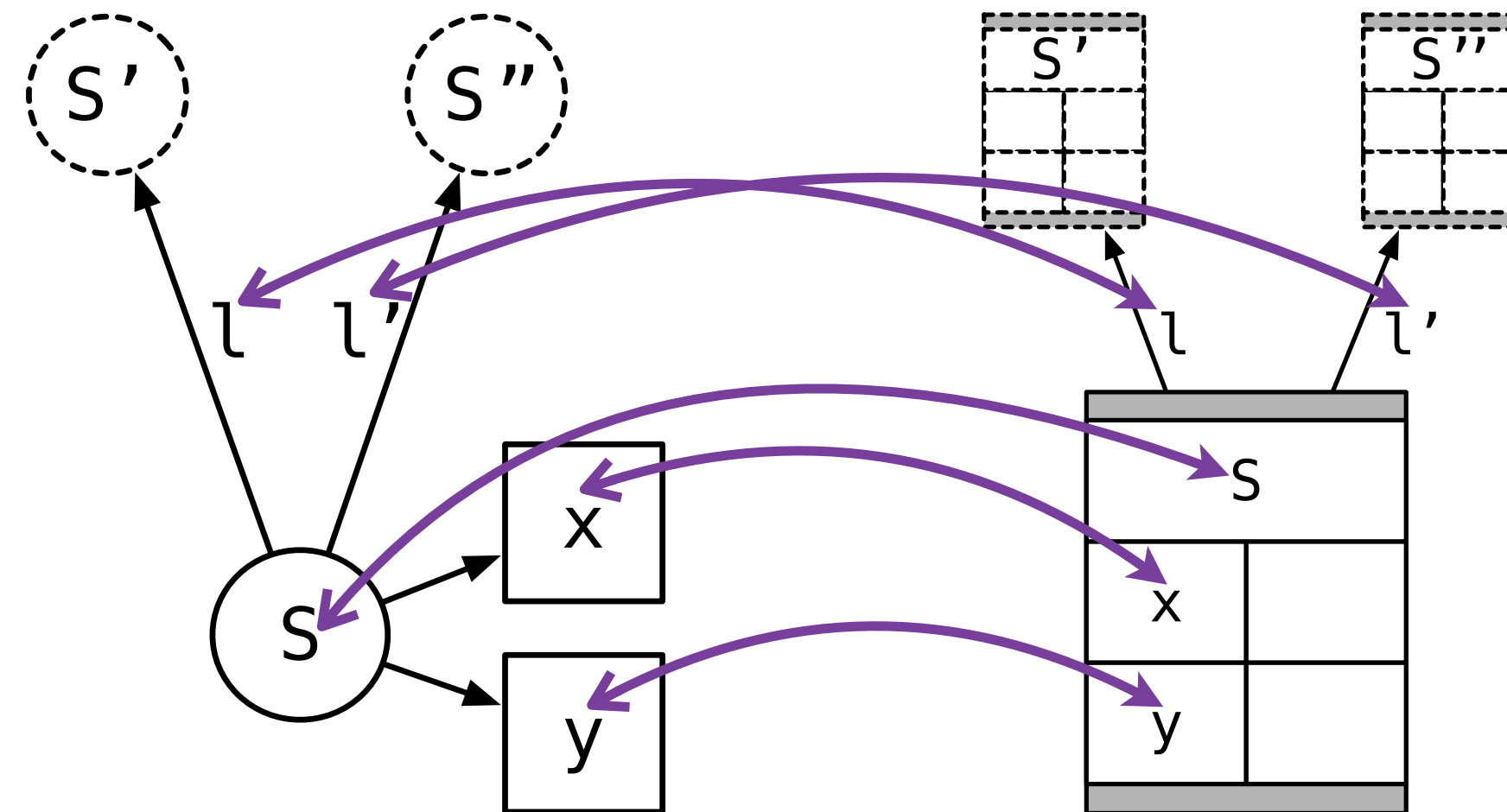


# Good Frame Invariant

## Well-Bound Frame

Scope

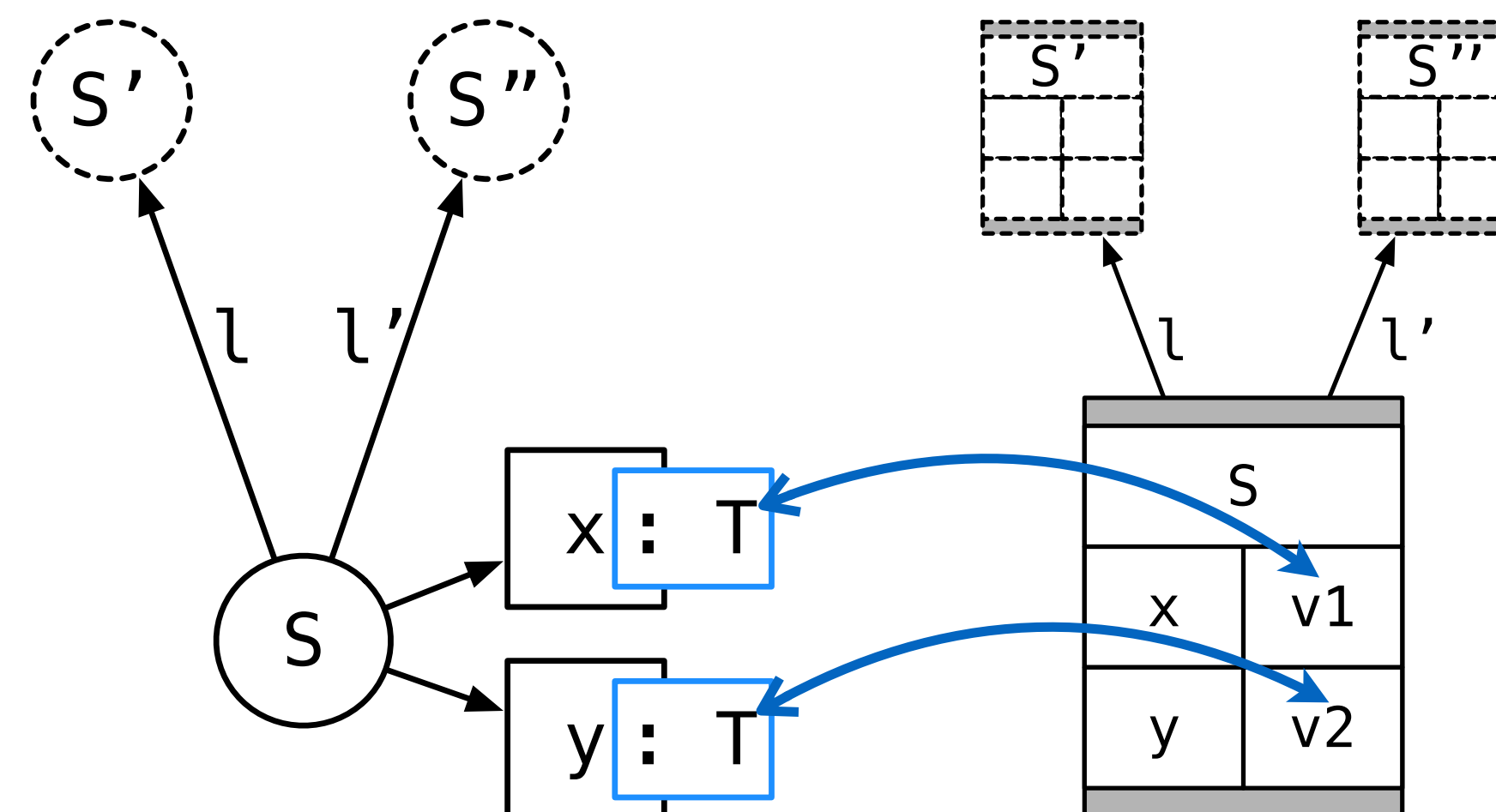
Frame



## Well-Typed Frame

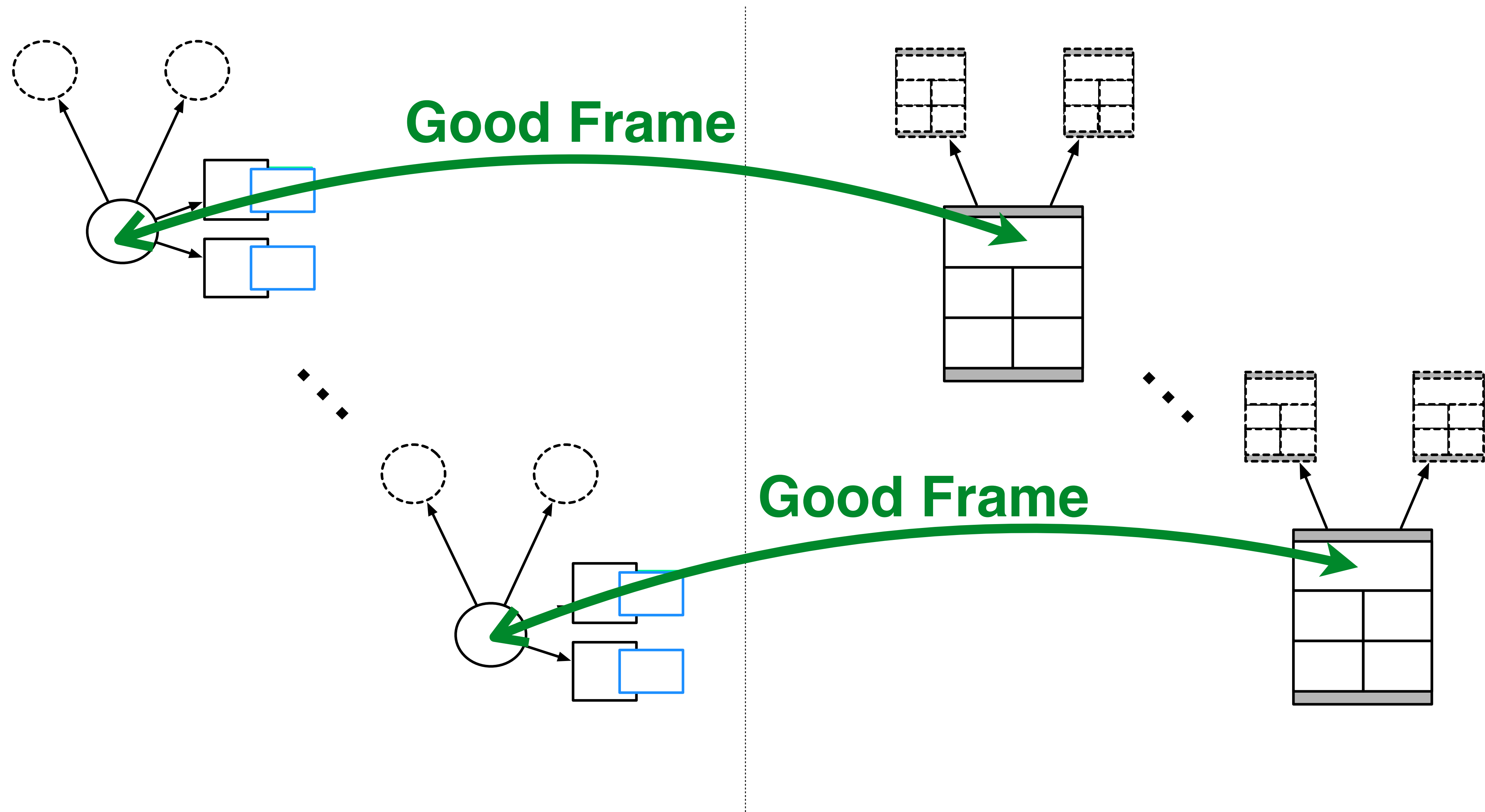
Scope

Frame

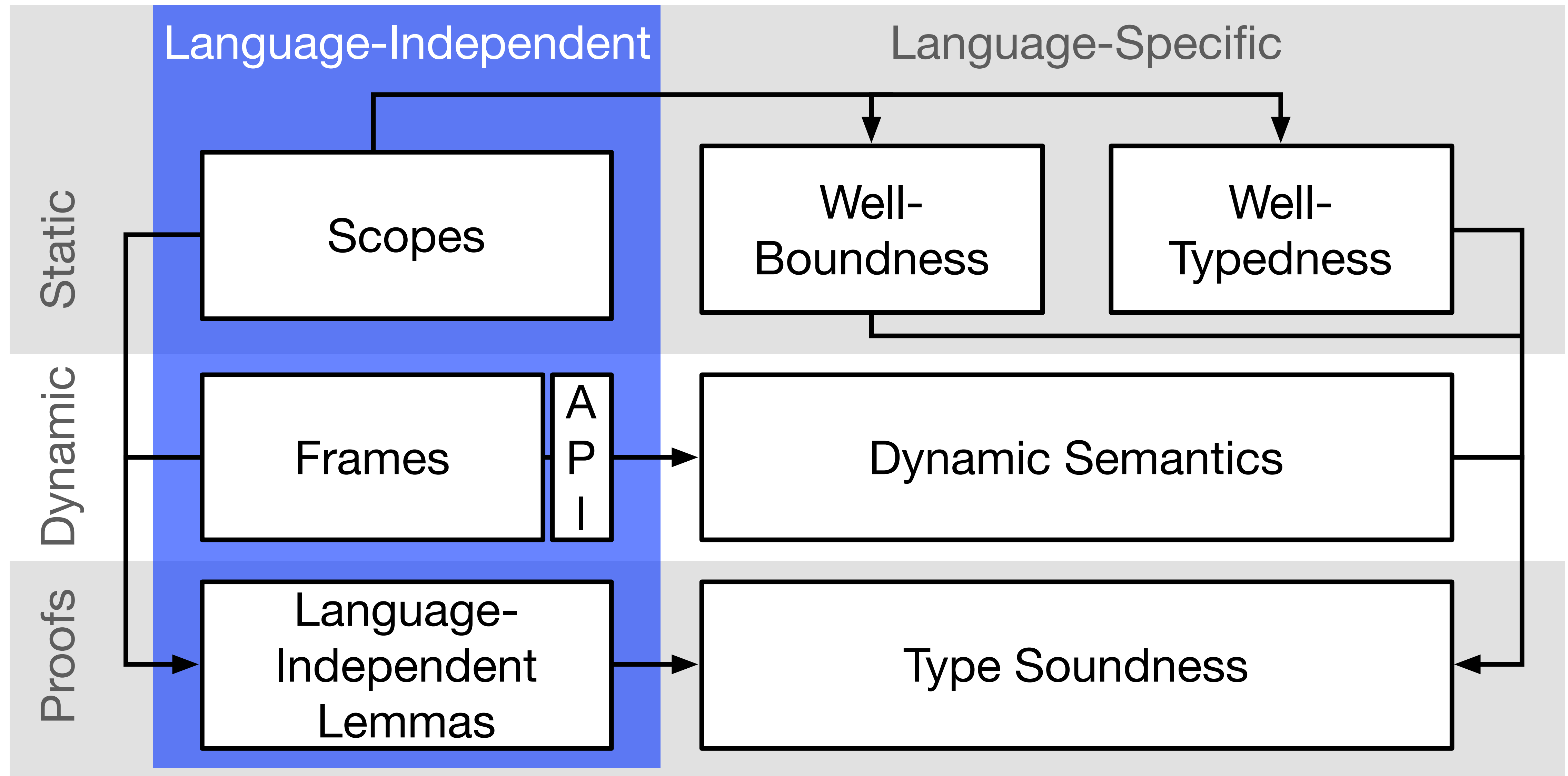


# Good Heap Invariant

## Every Frame is Well-Bound and Well-Typed

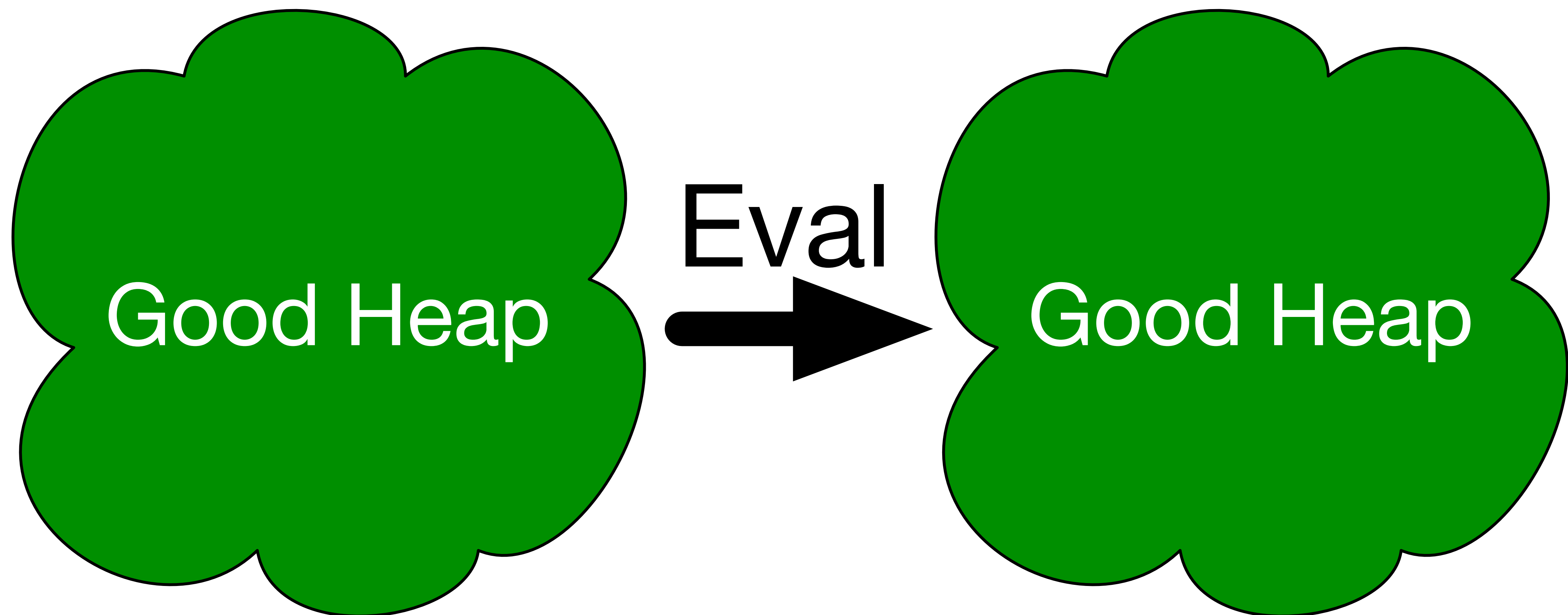


# Architecture of a Specification



# Type Soundness Principle

Evaluation Preserves  
Good Heap Invariant



# Summary

# Summary

Compilers provide de-facto semantics to programming languages

=> often unclear

Formal specification of source language

is essential to pin down design

Hard requirement for future programming languages

Formal semantics should be live (connected to implementation)

and understandable (through readable meta-DSL)



# Research Agenda

Abrupt termination?

Concurrency?

More case studies

Interpreter Generation

Optimization

Targeting (Graal+Truffle)/PyPy?

Type Soundness Verification