# Probabilistic Model for Infinite Memory  LLM: Variational Markov Transformer

(theory part complete, experiment in progress, paper not finished yet)

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## Abstract

Current Long Language Models (LLMs) are constrained by limited window sizes and require substantial resources for training and inference. Addressing these limitations, we propose a novel approach that establishes a foundational probabilistic framework for modeling the distribution of sequences of arbitrary lengths, including the comprehensive corpus used to train LLMs.

Our methodology innovatively integrates Deep Hidden Markov Models (DHMMs) as the core probabilistic framework for representing sequence collections. Moving beyond the traditional HMM algorithm's reliance on discrete hidden states, we employ a novel combination of the Evidence Lower Bound (ELBO) objective from Variational Autoencoders (VAEs) with the Truncated Backpropagation Through Time (TBTT) algorithm in Recurrent Neural Networks (RNNs) for learning DHMMs. This integration leverages the strengths of VAEs in mitigating gradient explosion and the insignificance of initial hidden states, common challenges in RNNs. Simultaneously, the TBTT algorithm facilitates the learning of long-term dependencies by sharing hidden states across sequence segments. This synergistic approach enables our model to effectively capture complex sequence patterns, achieving theoretically infinite dependencies and advancing the capabilities of sequence modeling.

In addition, for language tasks, we use LLMs as pre-trained text encoders, we also invent stochastic transformers to model the posterior distribution of VAEs used in DHMM, overcoming the sequential sampling of the markov property. Our method significantly reduces the required input window size for LLMs during both training and inference phases, thereby decreasing the overall computational demands and resource consumption of LLM.

## Introduction

The theoretical memory window of an RNN  cannot be said to be infinite. If it were infinite, then the hidden state would have to contain the entire memory of the RNN agent. However, during training, the starting hidden state of each training sequence is either zero or noise. It's important to note that if the RNN is supposed to output a long thesis, how could the starting memory possibly be zero? Or how could it be the same to the RNN  that outputs a sequence  of a child's writing?   Perhaps, when training each sequence, calculating the most suitable starting hidden state value first is the most consistent approach, but RNNs lack a strict probabilistic model to achieve this. Moreover, RNNs also have problems with gradients and the inability to train parallel sequence data.

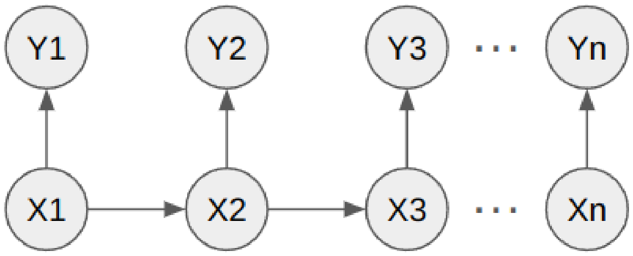
As for transformers, all current attempts are engineering efforts to expand the window or to extrapolate longer lengths than trained using attention and relative position encoding. Some works are aimed to combine the above RNN with the transformer architecture thus still suffering from the RNN's drawbacks.

## Probabilistic modeling of the training sequence

The memory should be a compression of what we have seen before but not all the raw data we have seen, when we want to recall something, we don't need to reread all what we read before.

What I currently want to try is a method similar to deep HMM. It's based on deriving the maximum likelihood training target from VAE, striving to solve the aforementioned problems, equipping the RNN with a strict probabilistic model to find the most suitable initial hidden state given a list of observations.
For training,  sample a continue subsequence of any length(n>=2) from the training sequence:
**Y1    Y2    Y3  …   Yn**

We could suppose the following probabilistic model:



X1, X2 … Xn are hidden variables, during the inference time, Xk contains the key to activate all memory before time step k,  including the memory before X1. We suppose the dimension of Xk does not need to be bigger than the dimension of Yk, as said above, Xk is not a storage of all memory  before, the true memory is stored inside the entire neural network, Xk just acts as the key to activate the memory belonging to  it's time step.

For each continue subsequence, we aim to maximize **P(Y1,  Y2,  Y3…Yn):**

$$P(Y) = \frac{P(Y,X)}{P(X|Y)} = \frac{P(Y,X)Q(X|Y)}{P(X|Y)Q(X|Y)}$$
$$\Rightarrow logP(Y) = logP(Y,X) + log\frac{Q(X|Y)}{P(X|Y)} - logQ(X|Y)$$
$$\Rightarrow logP(Y) = \int Q(X|Y)logP(Y)dX$$
$$= \int Q(X|Y)(logP(Y,X) - logQ(X|Y))dX + \int Q(X|Y)log\frac{Q(X|Y)}{P(X|Y)}dX$$
$$= \int Q(X|Y)(logP(Y,X) - logQ(X|Y))dX + KL(Q(X|Y)||P(X|Y))$$
$$\Rightarrow logP(Y) \geq \int Q(X|Y)(logP(Y,X) - logQ(X|Y))dX = ELBO$$

## Training VMT with pretrained GPT

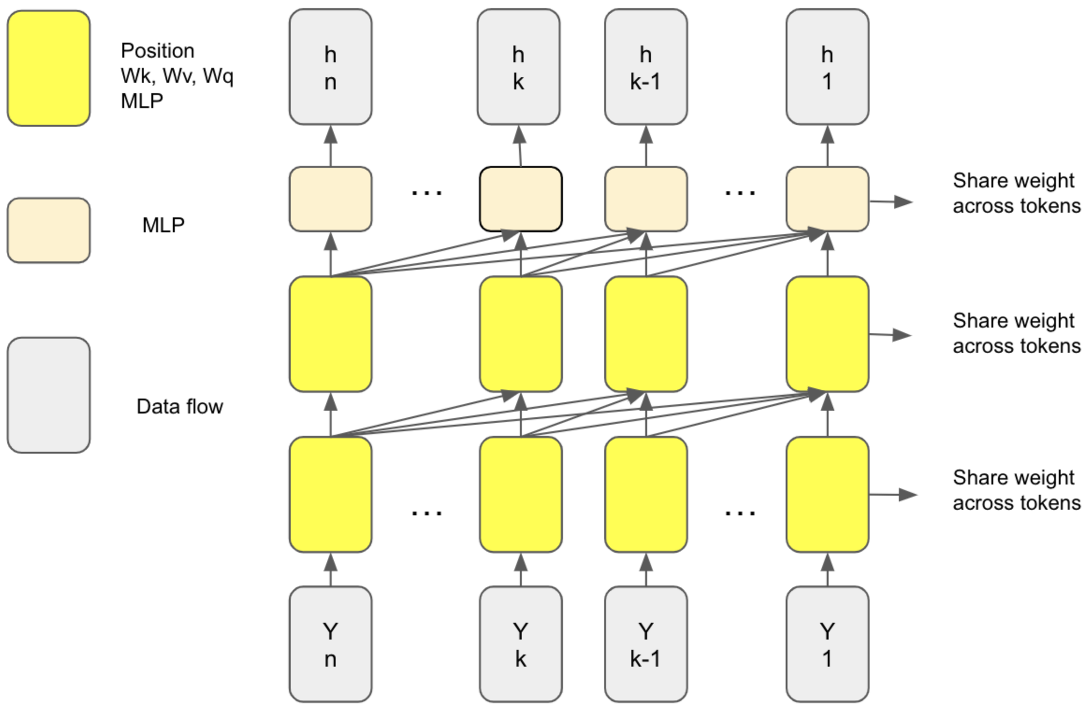
$$P(Y,X) = P(y_1, y_2 ... y_n, x_1, x_2 ... x_n)$$
$$= P(x_1)P(y_1|x_1)\prod_{k=1}^{n-1}(P(x_{k+1}|x_k)P(y_{k+1}|x_{k+1}))$$
$$Q(X|Y) = P(x_1, x_2 ... x_n|y_1, y_2 ... y_n)$$
$$= Q(x_1|y_1, y_2 ... y_n)\prod_{k=1}^{n-1}Q(x_{k+1}|x_k, y_{k+1} ... y_n)$$
$$\Rightarrow$$
$$ELBO = \mathop{\mathbb{E}}_{x_1 \sim Q(x_1|y_1 ... y_n)}(logP(x_1) + logP(y_1|x_1) - logQ(x_1|y_1 ... y_n) + ELBO_1)$$
$$k = 1..n-1$$
$$ELBO_k = \mathop{\mathbb{E}}_{x_{k+1} \sim Q(x_{k+1}|x_k, y_{k+1} ... y_n)}(logP(x_{k+1}|x_k) + logP(y_{k+1}|x_{k+1}) - logQ(x_{k+1}|x_k, y_{k+1} ... y_n) + ELBO_{k+1})$$
$$ELBO_n = 0$$

By the property of MarKov chain we could parametrize

1. P(x_k)
2. P(y_k|x_k)
3. P(x_k+1|x_k)
4. Q(x1|y1…yn)
5. Q(x_k+1|x_k, y_k+1…y_n)

For **any k** and **any subsequence** from the training document set using the same 5 neural networks above.
Free to choose adaptive architecture for 1, 2, 3 according to the nature of the problem.
I personally prefer to use my BreezeForest and conditional BreezeForest, as they are universal density estimators.
Here we focus on how to choose architecture for 4. and 5.
By leveraging a pre-trained GPT like LLM, we could compress the sequence of yn, y_n-1…y_k into a single hidden state input for 4. And 5. as the following:

Take an autoregressive transformer of window size n as the figure below:


Position
Wk, Wv, Wq
MLP
MLP
Data flow
h n
h k
h k-1
h 1
…
Share weight across tokens
Share weight across tokens
Share weight across tokens
Y n
Y k
Y k-1
Y 1

The weight are shared across tokens, let's say the weight is W_tf, for both training and inference, given y1, y2 … yn a continue sequence sampled from the ground truth sequence, as shown above we inverse the order of the sequence then compute the feature set used in posterior distribution:
hn, h_n-1 … h1 = f(W_tf, yn, y_n-1…y1), where hk gathers all the information from yn down to yk.

Q(x1|y1…yn) = Q(x1|h1, T) -> T is number of read states, here is n, choose a conditional generative model as you want
Q(x_k+t|xk, y_k+1…yn)= Q(x_k+t|xk, h_k+1, t, T) ->T is the number of states read from yn to y_k+1, t is 1 for the training objective, t may be bigger than one for accelerating the reading on the later stage. choose a conditional generative model as you

If W_tf is a GPT like transformer which has not been trained on sequence of inverse order, the weight of W_tf may need to be finetuned during the training.

**Algorithm 1: Training VMT using Batched TBTT starting from a pre-trained GPT**

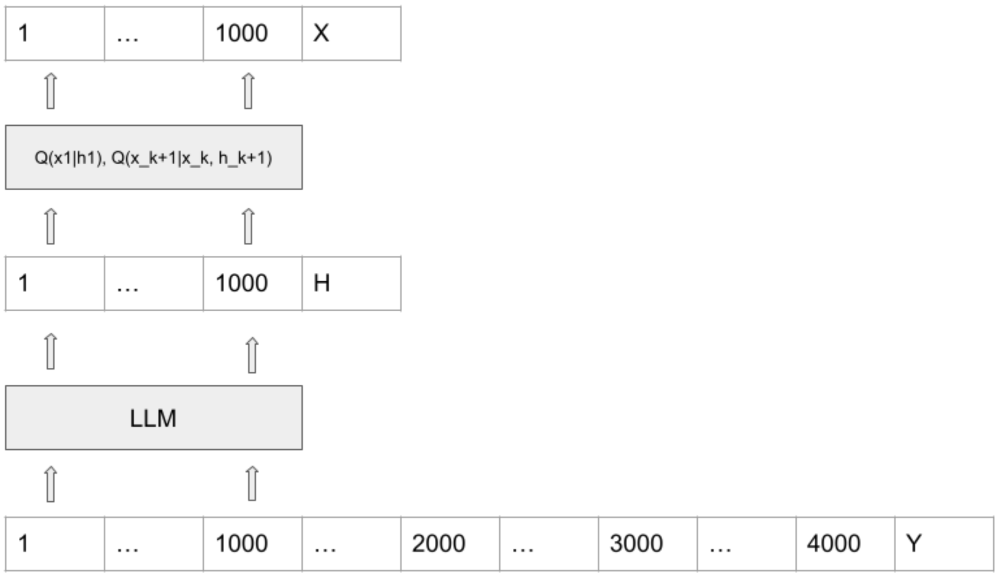
```
Init GPT: f(W_tf, …)
Init Theta for Markov part of weight
```

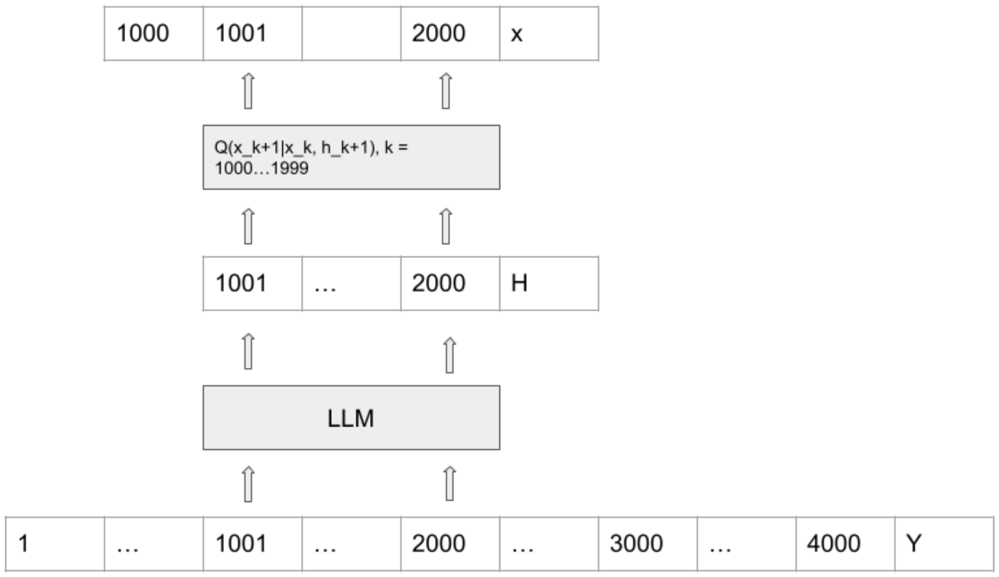
```
    1. P(x_k)
    2. P(y_k|x_k)
    3. P(x_k+1|x_k)
    4. Q(x1|..., T)
    5. Q(x_k+t|x_k, …, t, T)
Init optimizer
1. Organize sequences into batches, begin from longest to shortest
2. Set reference window size n, can be variable during the training, to simplify, we fix it except
encountering the end of a sequence in a batch

Init x0 to None
Loop batch  from beginning of the sequences:
   If a sequence in the batch ends within the current window:
      reduce the window size to the nearest ending position among all sequences
   For sequences without initial state X0(at the beginning of newly added sequences):
        Use Q(x1|..., T) to sample x1 for these sequences
   For sequences having initial state x0(at the beginning of newly added sequences):
        Use Q(x1| x0, ...,1, T) to sample x1 for these sequences
   For all sequences in the batch sample x2, x3, x4… sequentially using:
   Q(x_2|x_1, …, 1, T-1), Q(x_3|x_2, …, 1, T-2), Q(x_4|x_3, …, 1, T-3)...
   Until the end of the current window size, all sequences are processed simultaneously.
   Given X1…Xend, we can update the ELBO:
      Loss_elbo = -ELBO(h_end …  h2, h1,y_end, y_L-1…y1, x_end…x_2, x_1, Theta )

      # here we add more training steps than the pure objective
      For sequences where x1 is sampled from Q(x1| x0, ...,1, T):
            Loss_init =  -log Q(x1| …,T) # we use x1 to train the initialization posterior
      For all sequences:
           Loss_prior = -(log P(x1) + logP(x2) … logP(xn))
      # this will train the posterior to skip, in order to accelerate reading
      For all sequences, sample a t from 2 to T,
           Loss_skip =  -log(Q(x_(t+1)|x_1, …, t, T-1))

      Loss_ttl = Loss_elbo + (Loss_init + Loss_prior + Loss_skip)*ponderation(<1)
      grad1 = dLoss/dTheta
      grad2  = dLoss/dW_tf
      Theta, W_tf <- optimizer(grad1, grad2,  Theta, W_tf)
   Replace sequences that gets end by new sequences, for sequences not terminated yet:
      x0=x_end
```

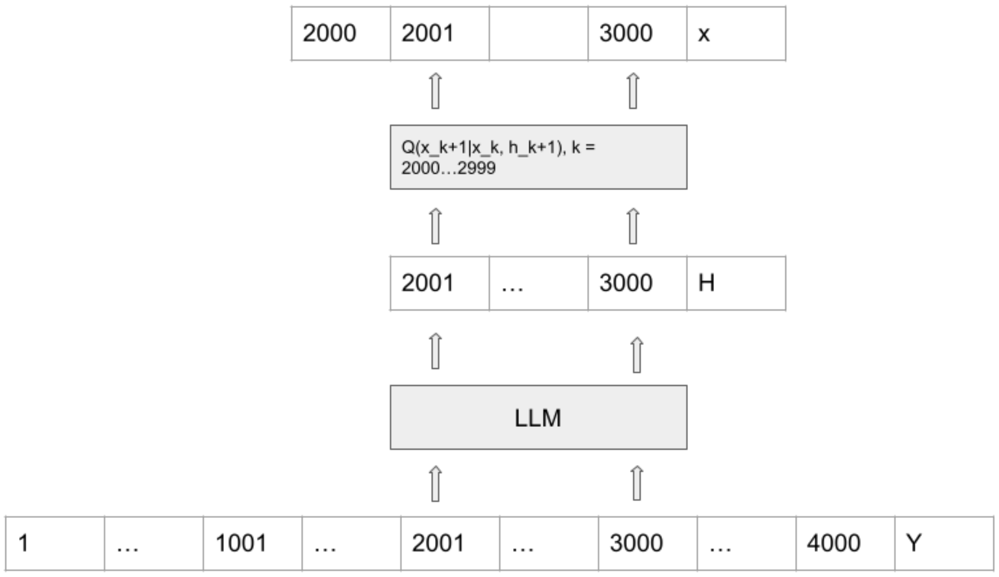
```
Repeat the above process for several epoch over the entire training dataset
```

```
Return 1, 2, 3, 4, 5 and W_tf
```

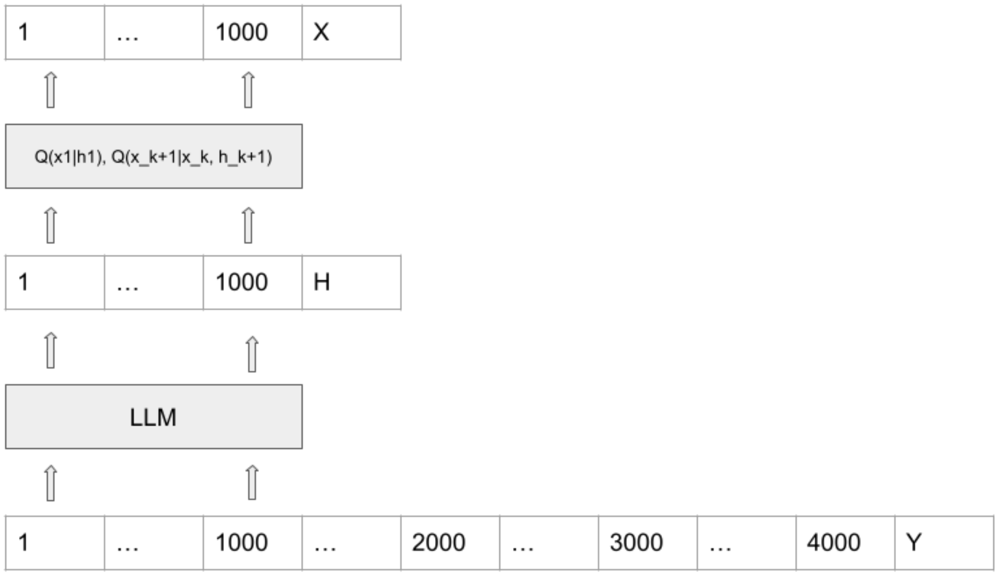
Illustration of a simple case with  batch size one:

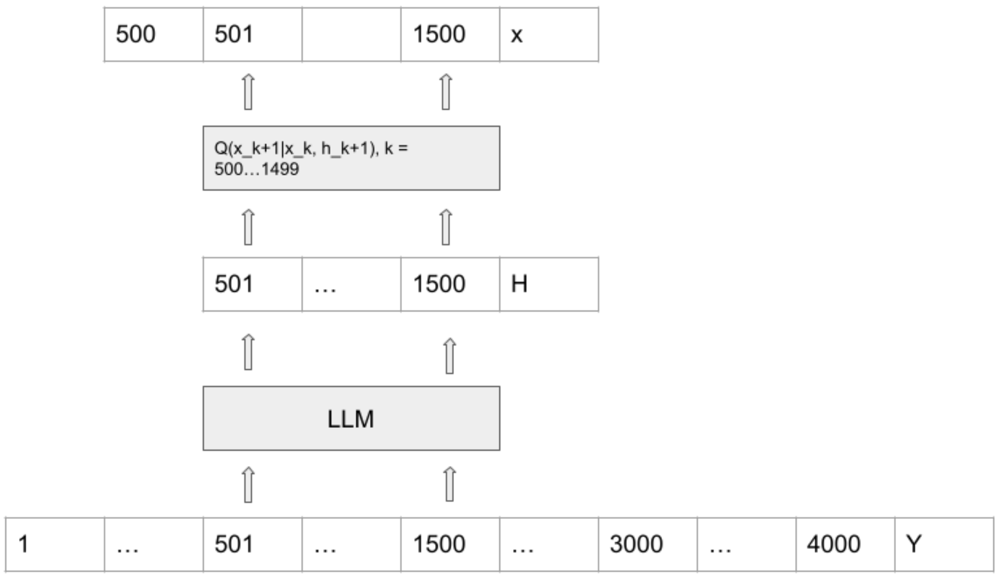


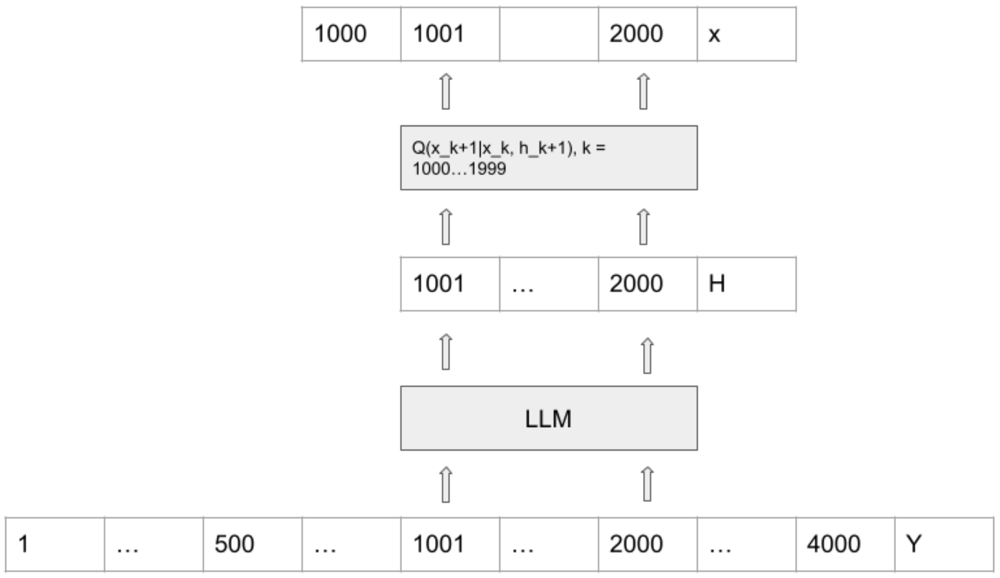




We can see that the last state X_end which will be passed to the next trunk is predicted at the end, without knowing the later Ys, which may lead to imprecision of the estimation of the value of X_end, however, as the state passed across trunk play a crucial role in long term memory, an improvement would be to overlap half of each trunk:







Thus, the state X_end passed across window will be predicted by knowing half of window size of Ys after it’s position.

## Training reverse VMT with fixed weight pretrained GPT

$$P(Y,X) = P(y_1, y_2 ... y_n, x_1, x_2 ... x_n)$$
$$= P(x_n)P(y_n|x_n)\prod_{k=1}^{n-1}(P(x_{n-k}|x_{n-k+1})P(y_{n-k}|x_{n-k}))$$
$$Q(X|Y) = P(x_1, x_2 ... x_n|y_1, y_2 ... y_n)$$
$$= Q(x_n|y_1, y_2 ... y_n)\prod_{k=1}^{n-1}Q(x_{n-k}|x_{n-k+1}, y_1 ... y_{n-k})$$
$$\Rightarrow$$
$$ELBO = \mathop{\mathbb{E}}_{x_n \sim Q(x_n|y_1...y_n)}(logP(x_n) + logP(y_n|x_n) - logQ(x_n|y_1 ... y_n) + ELBO_1)$$
$$k = 1..n-1$$
$$ELBO_k = \mathop{\mathbb{E}}_{x_{n-k} \sim Q(x_{n-k}|x_{n-k+1}, y_1 ... y_{n-k})}(logP(x_{n-k}|x_{n-k+1}) + logP(y_{n-k}|x_{n-k}) -$$
$$logQ(x_{n-k}|x_{n-k+1}, y_1 ... y_{n-k}) + ELBO_{k+1})$$
$$ELBO_n = 0$$

By the property of MarKov chain we could parametrize

1. P(x_k)
2. P(y_k|x_k)
6. Pr(x_k-1 |xk)
7. Qr(xn|y1…yn)
8. Qr(x_k-1|xk, y1…y_k-1)

for any k and any subsequence using the same 5 neural networks.
Free to choose adaptive architecture for 1, 2, 3 according to the nature of the problem.
Here we focus on how to choose architecture for 4. and 5.
By leveraging a pre-trained GPT like LLM, we could compress the sequence of y1…y_k-1 into a single hidden state input for 4. And 5 as the following:

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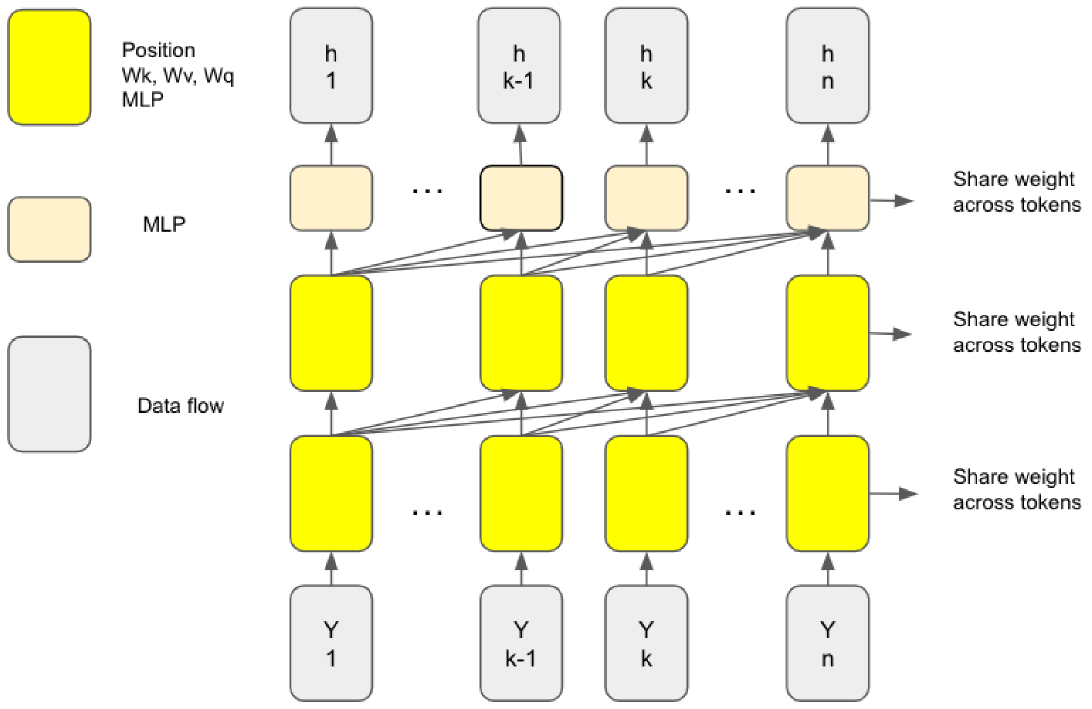
Position
Wk, Wv, Wq
MLP
MLP
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h 1
h k-1
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h n
Share weight across tokens
Share weight across tokens
Share weight across tokens
Y 1
Y k-1
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Y n

The weight are share across tokens, let's say the weight is W_tf, for both training and inference, given y1, y2 … yn a continue sequence sampled from the ground truth sequence, as shown above we first compute the feature set used in posterior distribution:
f(W_tf, y1, y2…yn) = h1, h2 … hn, where hk gathers all the information from y1 to yk.

Qr(xn|y1…yn) =  Qr(xn|hn)  -> choose a conditional generative model as you want
Qr(x_k-1|xk, y1…y_k-1)= Qr(x_k-1|xk, h_k-1) -> choose a conditional generative model as you want.

## Inference

Now we can obtain 5 neural networks from normal order training

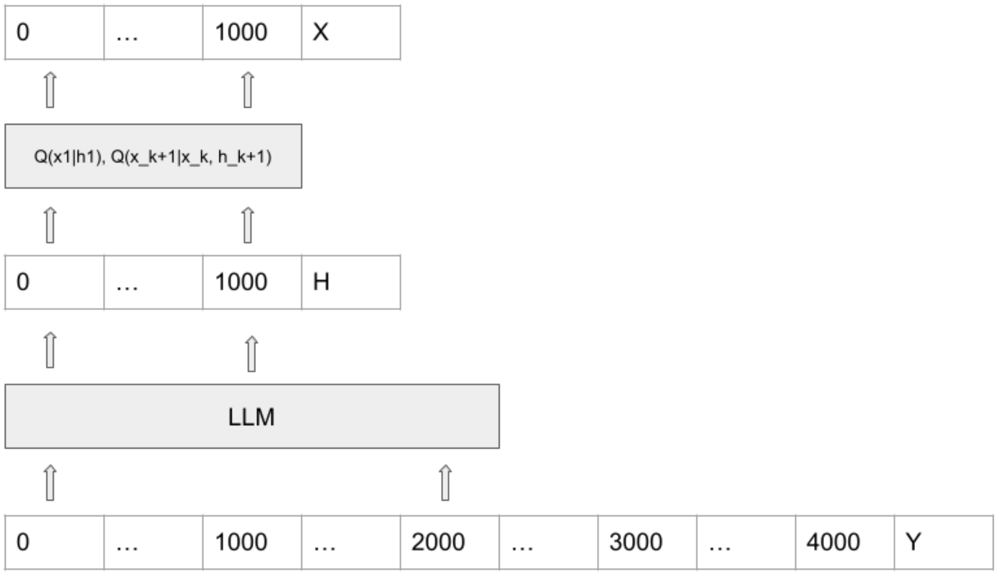
- P(x_k)
- P(y_k|x_k)
- P(x_k+1|x_k)
- Q(x1|y1…yn)
- Q(x_k+1|x_k, y_k+1…y_n)
- W_tf_reverse

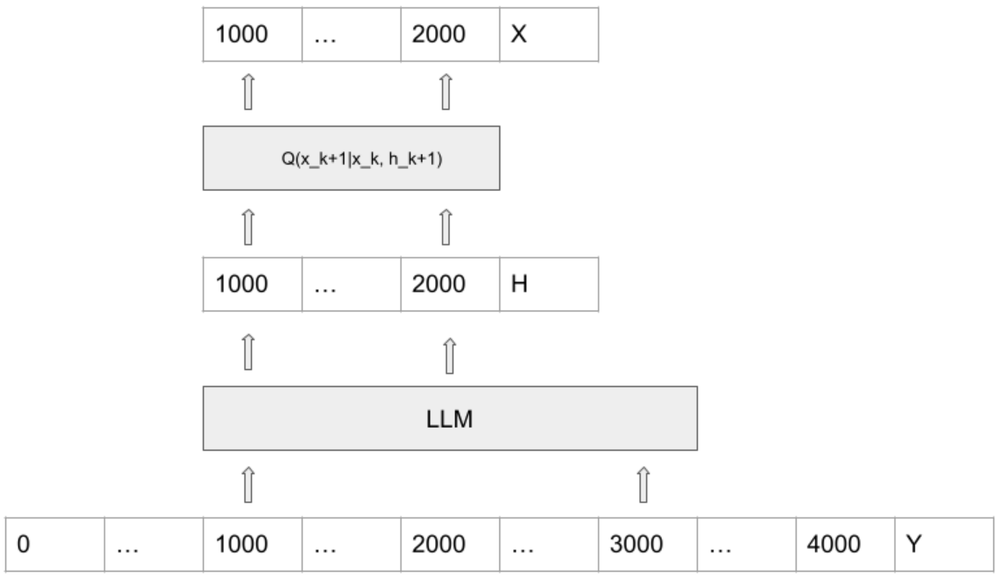
And 3 additional neural network from reverse order training
- Pr(x_k-1 |xk)

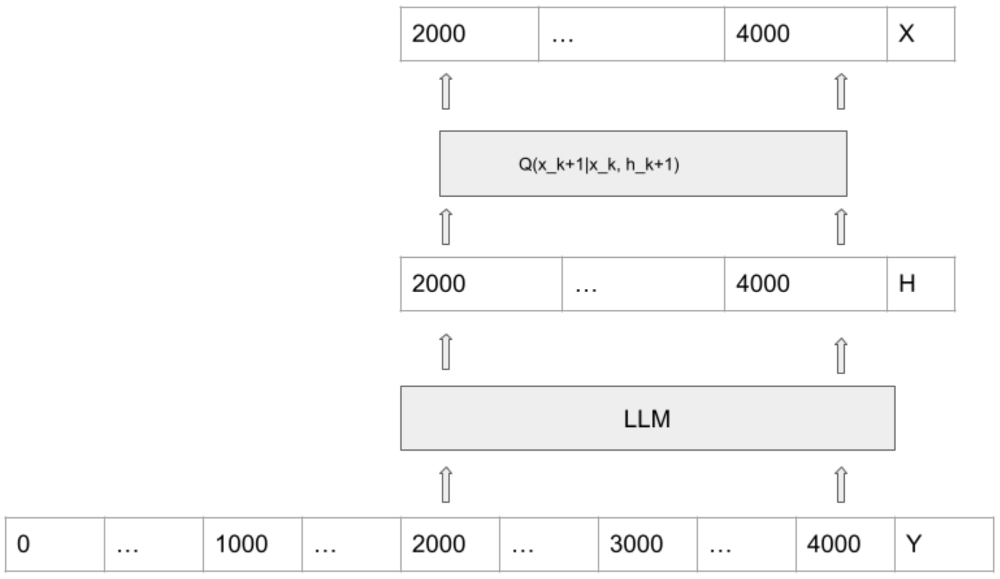
- Qr(xn|y1…yn)
- Qr(x_k-1|xk, y1…y_k-1)
- W_tf_gpt

How to use them during inference? Or concretely for example, how to use them for a chatbot application?
The user uploads a document then asks to summarize, the total length of the user's first input is of length Y1 …Y4000, the chatbot needs to read it first, then give its summary. Then the user will ask another question…

**Reading:**







As the training sequence length is 2000, we begin with the first 2000 tokens(even  it's possible to exceed it), reading it with W_tf_reverse to get h1…h2000 then use Q(x1|h1)  to sample the first X1 at the beginning of the input sequence.
Once we have X1, we could you Q(x_k+1|X_k, h_k+1) to sample the next Xs until X_1000, Now we could do another forward pass of  W_tf_reverse to get h1000…h3000, thus we can adjust the Xs after X_1000 using more information. We repeat this process until we reach X_4000.

An acceleration trick is to use Qr(xn|y1…yn) to read immediately the first n tokens without updating X iteratively from x1 to xn.

**Reading acceleration**
One may notice that the reading during the inference might be slow, even with the acceleration of Qr(xn|yn) at the very beginning.  One improvement to this is to change Q(x_k+1|xk, h_k+1) into Q(x_k+t|xk, h_k+1, T, t), where T is the total number of items between yn and x_k+1. The Q(x_k+t|xk, h_k+1, T, t) can be trained at the same time of the initial training or be distilled later by Q(x_k+1|xk, h_k+1). Thus, when t >> 1, the model can read by skipping several steps.

**Writing:**
The goal of the reading is to find the fittest hidden state X according to the input content, Now, we could use P(x_k-1|x_k)  and P(yk|xk) to sample X_4000+ and Y_4000+. Then reuse Q(x_k+1|X_k, h_k+1)  to read new content from users.

Other possible usages include:

1. Reverse generation of text such as writing a prequel for a novel using Pr(x_k-1 |xk)
2. Sample randomly a text using P(x_k)

## The distribution of P(X)

In this section, we discuss the nature of the hidden state X and its distribution P(X).

Every intellectual agent, its sequence of mental states from its birth until now can be seen as a sequence of Xs. If we put all Xs from the sequences of all  intellectual agents who have generated the training data together, we get P(X).

We could imagine that P(X) may be a multimodal distribution as different types of text (Romantic novel Vs scientific paper) may correspond to very different sequences.

If X1 and X2 are close to each other, then the sequences they belong to may also share common memory. This makes it useful to store special Xs of a running sequence using a vector database with certain ponderation as long term memories. Each time the sequence encounters a x which is close to another x’ that the database has stored long before, The model can use x’ to generate recall memory sequence y’ then read it. This may enhance the long term memory which is related to the agent’s current state.

The X’ stored in the vector database may vanish if never visited again.

## Architecture choice

**P(x_k)**

Should be possible to evaluate LLH easily, can effectively model multimodal distribution, example of choice: BreezeForest

**P(y_k|x_k)**
Similar to the last layer in GPT that transforms a transformer's output into token's softmax.

**P(x_k+1|x_k), Pr(x_k-1 |xk)**
Conditional BreezeForest or simple gaussian

**Q(x1|h1), Qr(xn|hn)**
Conditional BreezeForest, conditional Affine coupling flow, when the condition sequence is short, Q(x1|h1), Qr(xn|hn) will resemble the property of P(X_k)

**W_tf_reverse**
a fine tuned GPT like LLM

**W_tf_gpt**
A pretrained GPT like LLM with fixed weight

**Qr(x_k-1|xk, h_k-1), Q(x_k|x_k-1, h_k)**
Conditional BreezeForest, conditional Affine coupling flow or gaussian

## Answer to the most asked questions:

**1. What's the difference with RNN, how to handle gradient exploding?**
RNN predicts the next Y only by knowing the current hidden state, this gives huge uncertainty, especially, it should do the same thing recurrently for several steps. If one step goes wrong so that the hidden X does not match the corresponding Y, even with teacher forcing of correct Y labels, the mismatch between Y and X would not be corrected, this may leads to exploding error.
Unlike RNN, the training process of DHMM is an autoencoder instead of a predictor, when you predict x_k with Q(x_k|x_k-1, y_k, y_k+1…yn), you already know all Ys later, this could help predicting the adequate X at each step.

Another difference with RNN is said at the introduction, the initial state of RNN is not significant while DHMM could predict the fittest initial hidden state for a given observation sequence.

Finally, DHMM is a rigorous probabilistic model modeling long sequences distribution, whereas the RNN is a simple recurrent predictor.

**2. During the training, VMT needs to sample sequentially each Xs, this may slow down the training.**

Although I have 2 methods in mind which may completely break the markov property to generate all hidden states at once during the training, these 2 methods are not ready to present yet.
Let’s first admit this is true, what’s the consequences?
Assume that you have a max GPU memory limit M, with traditional GPT training of a batch of sequence of length L, you can train (M/L^2)*L tokens by unitary time, where the memory complexity of the transformer is L^2, the number of sequences in a batch is (M/L^2).
With the same GPU to train a VMT of window size l, you can train (M/l^2)*l tokens by l unitary time.
So the training speed is M/l^2 instead of M/L, but note that with VMT you don’t need to have a huge window size to ensure long term memory, so the actual window size during the training can be much smaller than L. In fact, we only need to choose a l < sqrt(L) in order to be more efficient than traditional GPT training.

**3. Do you need to retrain the LLM?**
We may need to fine tune the LLM or only distill from a fixed weight LLM, this depends on the LLM’s capability.

**4. Would you face the same problem of RNN by forgetting the information seen a long time ago?**
I don’t have fixed answer yet without experimentation, but since DHMM is a probabilistic model modeling the entire corpus of sequences, the hidden state of each trunk are computed during the training, if during the training VMT can correctly predict each trunk given the beginning hidden state, it would also do so during the inference time.