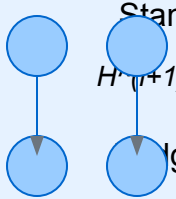
Mathematical Framework for Bridge Neural Networks

Modified Transformer Equations



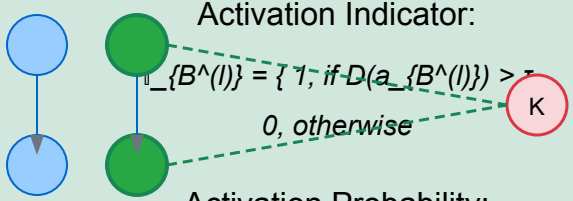
Standard Transformer Flow:

$$H^{(l+1)} = TransformerLayer_l(H^{(l)})$$

Bridge Neural Network Flow:

$$H^{(l+1)} = TransformerLayer_l(H^{(l)}) + \mathbb{1}_{B^{(l)}} \cdot I(K(Q(a_{B^{(l)}})))$$

Bridge Activation Function



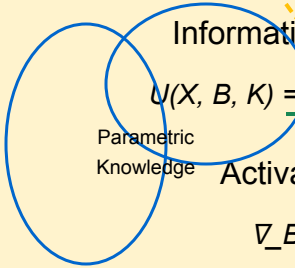
Activation Indicator:

$$\mathbb{1}_{B^{(l)}} = \begin{cases} 1, & \text{if } D(a_{B^{(l)}}) > \tau \\ 0, & \text{otherwise} \end{cases}$$

Activation Probability:

$$P(B^{(l)} \mid X) = \sigma(W_d \cdot a_{B^{(l)}} + b_d)$$

Information Theoretic Perspective



Information Utility Function:

$$U(X, B, K) = I(Y; X, B, K) - \lambda C(B, K)$$

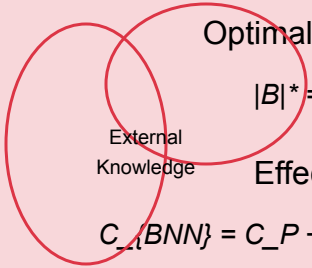
Activation Condition:

$$\nabla_B U(X, B, K) > 0$$

Decision Boundary

$$D(a_{B^{(l)}}) = \tau$$

Capacity Analysis



Optimal Bridge Allocation:

$$|B|^* = (1/\beta)((\alpha\beta/\lambda) - 1)$$

Effective Capacity:

$$C_{BNN} = C_P + \sum^{L}_{l=1} \mathbb{E}[\mathbb{1}_{B^{(l)}}] \cdot C_N$$