# Testing Arguments for Validity and Soundness

Philosophy 1115 (Logic)
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## 1 Visualizing the Procedure for Validity/Soundness Testing

Figure 1 provides a series of questions (and their possible answers), which will help us to determine whether an argument is valid (or sound). In the next section, I will apply this method to several arguments from our introductory lectures.

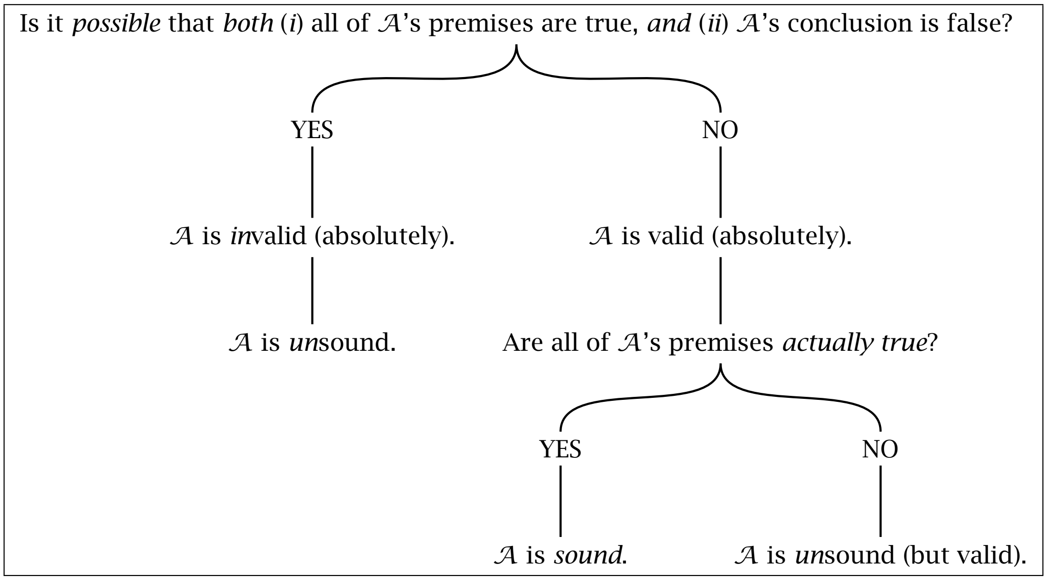


Figure 1: Testing an argument $\mathcal{A}$ for (absolute) validity and soundness.

## 2 Applying the Test to Some Examples

### 2.1 Example #1 — An "Easy" Valid Argument

Recall our first example from last time:

$\mathcal{A}_1$:
Dr. Ruth is a man.
If Dr. Ruth is a man, then Dr. Ruth is 10 feet tall.
∴ Dr. Ruth is 10 feet tall.

The method depicted visually in Figure 1 leads to the following sequence of questions (and answers) about argument $\mathcal{A}_1$.

$\mathsf{Q}_1$: Is it *possible* that *both* (*i*) all of the premises of $\mathcal{A}_1$ are true, *and* (*ii*) the conclusion of $\mathcal{A}_1$ is false?

$\mathsf{A}_1$: NO. Imagine a world in which it is true that Dr. Ruth is a man *and* it is true that *if* Dr. Ruth is a man, *then* Dr. Ruth is 10 feet tall. Any possible world of this kind will also be a possible world in which Dr. Ruth is 10 feet tall. So, there is no possible world in which (*i.e.*, it is *impossible* that) both (*i*) and (*ii*) obtain. Therefore, $\mathcal{A}_1$ is valid.

$\mathsf{Q}_2$: Are all of $\mathcal{A}_1$'s premises *actually true*?

$\mathsf{A}_2$: NO. In fact, *neither* of $\mathcal{A}_1$'s premises is true in the *actual* world. Therefore, $\mathcal{A}_1$ is *un*sound (but *valid*, nonetheless!).

### 2.2 Example #2 — A "Tricky" Valid Argument

$\mathcal{A}_2$:
Branden weighs 200 lbs and Branden does not weigh 200 lbs.
∴ The moon is made of green cheese.

This time, we have the following sequence of questions (and answers) about argument $\mathcal{A}_2$.

$\mathsf{Q}_1$: Is it *possible* that *both* (*i*) all of the premises of $\mathcal{A}_2$ are true, *and* (*ii*) the conclusion of $\mathcal{A}_2$ is false?

$A_1$: NO. Try to imagine a possible world in which the premise of $\mathcal{A}_2$ is true and the conclusion of $\mathcal{A}_2$ is false. This would have to be a world in which *all* of the following three propositions are true:

(1) Branden weighs 200 lbs.

(2) Branden does not weigh 200 lbs.

(3) The moon is not made of green cheese.

Of course, there is no problem imagining a world in which (3) is true (our very own actual world will do just fine!). But, there can be *no* possible world in which *both* (1) *and* (2) are true simultaneously, since (2) is just the *denial* of (1). So, there is no possible world in which (*i.e.*, it is *impossible* that) both (*i*) and (*ii*) obtain. Therefore, $\mathcal{A}_2$ is valid.

$Q_2$: Are all of $\mathcal{A}_2$'s premises *actually true*?

$A_2$: NO. In fact, $\mathcal{A}_2$'s premise is false in *all* possible worlds (not just ours!). Therefore, $\mathcal{A}_2$ *un*sound (but *valid*, nonetheless!).

## 2.3 Example #3 — Another "Tricky" Valid Argument

$\mathcal{A}_3$: Glass is a liquid.
∴ If Branden is 10 feet tall, then Branden is 10 feet tall.

$Q_1$: Is it *possible* that *both* (*i*) all of the premises of $\mathcal{A}_3$ are true, *and* (*ii*) the conclusion of $\mathcal{A}_3$ is false?

$A_1$: NO. Try to imagine a possible world in which the premise of $\mathcal{A}_3$ is true and the conclusion of $\mathcal{A}_3$ is false. This would have to be a world in which *both* of the following two propositions are true:

(1) Glass is a liquid.

(2) *It is not the case that* if Branden is 10 feet tall, then Branden is 10 feet tall.

Of course, there is no problem imagining a world in which (1) is true (our very own actual world will do just fine!). But, there is *no* possible world in which (2) is true. Statements of the form "If *p*, then *p*" are called *tautologies* (this term will be defined and discussed in chapter 3) — they are *necessarily true* (*i.e.*, it is *impossible* for them to be false). So, there is no possible world in which (*i.e.*, it is *impossible* that) both (*i*) and (*ii*) obtain. Therefore, $\mathcal{A}_3$ is valid.

$Q_2$: Are all of $\mathcal{A}_3$'s premises *actually true*?

$A_2$: YES. In the actual world, glass *is* a liquid. Therefore, $\mathcal{A}_3$ is *sound*!

## 2.4 Example #4 — An Invalid Argument

$\mathcal{A}_4$: Most professional basketball players are over 6 feet tall.
Joe is a professional basketball player.
∴ Joe is over 6 feet tall.

Q: Is it *possible* that *both* (*i*) all of the premises of $\mathcal{A}_4$ are true, *and* (*ii*) the conclusion of $\mathcal{A}_4$ is false?

A: YES. It is easy to imagine a world in which *most* professional basketball players are over 6 feet tall, but *some* (*e.g.*, Joe) are *not*.[1] So, it *is* possible that both (*i*) and (*ii*) obtain. Therefore, $\mathcal{A}_4$ is *in*valid (*i.e.*, *NOT* valid) and *un*sound.

[1]If this "most" were changed to "all," then argument $\mathcal{A}_4$ *would* be valid. *Why*?