## Announcements & Such

- *The Budos Band*
- Administrative Stuff
  - **HW #2 1st-submissions are due on Thursday (4pm, drop box).**
    * Note: This involves problems from chapters 2 and 3.
    * Consult the *HW Tips Handout* for helpful tips on HW #2.
  - ☞ **Homework formatting. Please put the following information:**
    * **Name, GSI, section time, and date.**
    **on all assignments and exams (upper-right corner of first page).**
- Chapter 2 — LSL Symbolizations of Entire English Arguments
  - The art of charitable argument reconstruction.
- Next: Chapter 3 — *Truth-Functional Semantics* for LSL
- First: a leftover from HW #1…[Note: the HW #1 solutions are posted.]

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## Rewind: The Last Problem on HW #1

- The last problem on HW #1 is about the following argument ($\mathscr{A}$):
  (1) If Prince William is unmarried, then Prince William is a bachelor.
  (2) Prince William is a bachelor.
  (3) Therefore, Prince William is unmarried.
- Is $\mathscr{A}$ is *absolutely sound*? Since both premises (1) and (2) of $\mathscr{A}$ are *actually* true, this question reduces to "Is $\mathscr{A}$ is *absolutely valid*?".
- $\mathscr{A}$ is clearly *not sententially* valid. Its sentential form is the fallacious form that I called *affirming the consequent* in a previous lecture.
- Nonetheless, one might be tempted to argue that $\mathscr{A}$ is *absolutely* valid on the grounds that $\mathscr{A}$'s conclusion (3) follows from premise (2) *alone.*
- We are conservative about such cases. We only call arguments valid *if we have a formal theory according to which they have a valid form*. As it turns out, we have no such theory. So, our answer will be: NO.
- Our course in philosophical logic (142) delves into this subtle issue.

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## Symbolizing Arguments: Example #2

If Yossarian flies his missions then he is putting himself in danger, and it is irrational to put oneself in danger. If Yossarian is rational he will ask to be grounded, and he will be grounded only if he asks. But only irrational people are grounded, and a request to be grounded is proof of rationality. Consequently, Yossarian will fly his missions whether he is rational or irrational.

- Basic Sentences: Yossarian flies his missions ($F$), Yossarian puts himself in danger ($D$), Yossarian is rational ($R$), Yosarian asks to be grounded ($A$).
- Premise #1: If $F$ then $D$, and if $D$ then not $R$. [$(F \to D)\,\&\,(D \to \sim R)$]
- Premise #2: If $R$ then $A$, and not $F$ only if $A$. [$(R \to A)\,\&\,(\sim F \to A)$]
- Premise #3: But not $F$ only if not $R$, and $A$ implies $R$. [$(\sim F \to \sim R)\,\&\,(A \to R)$]
- Conclusion: Consequently, $F$ whether $R$ or not $R$. [$(R \to F)\,\&\,(\sim R \to F)$]. [Alternatively, the conclusion could be symbolized as: '$(R \lor \sim R) \to F$']
- Note: this is a valid form (we'll be able to prove this pretty soon).

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## Symbolizing Arguments: Example #3

Suppose no two contestants enter; then there will be no contest. No contest means no winner. Suppose all contestants perform equally well. Still no winner. There won't be a winner unless there's a loser. And conversely. Therefore, there will be a loser only if at least two contestants enter and not all contestants perform equally well.

- Step 0: Decide on atomic sentences and letters.

  $T$: At least two contestants enter.          $C$: There is a contest.

  $E$: All contestants perform equally well.

  $W$: There is a winner.                    $L$: There is a loser.
- Step 1: Identify (and symbolize) the *conclusion* of the argument:
  - Conclusion: There will be a loser only if at least two contestants enter and not all contestants perform equally well.
    * "Logish": $L$ only if $T$ and not $E$.
    * LSL: '$L \to (T\,\&\,\sim E)$'. [Why not '$(L \to T)\,\&\,\sim E$'?]

- Step 2: Symbolize the premises (here, there are as many as five):

  (1) Suppose no two contestants enter; then there will be no contest.
    * "Logish": Suppose that not $T$; then it is not the case that $C$.
    * LSL: '$\sim T \to \sim C$'.

  (2) No contest means no winner.
    * "Logish": Not $C$ means not $W$. [*i.e.*, not $C$ *implies* not $W$.]
    * LSL: '$\sim C \to \sim W$'.

  (3) Suppose all contestants perform equally well. Still no winner.
    * "Logish": Suppose $E$. Still not $W$. [*i.e.*, $E$ *also* implies not $W$.]
    * LSL: '$E \to \sim W$'.

  (4) There won't be a winner unless there's a loser. And conversely.
    * "Logish": Not $W$ unless $L$, *and conversely*.
    * LSL: '$(\sim L \to \sim W) \& (\sim W \to \sim L)$'. [*i.e.*, not $W$ *iff* not $L$.]

- The final product is the following *valid* sentential form:

  $\sim T \to \sim C$. $\sim C \to \sim W$. $E \to \sim W$. $\sim L \leftrightarrow \sim W$. Therefore, $L \to (T \& \sim E)$.

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## A Few Final Remarks on Symbolizing Arguments

- We saw the following premise our last argument: 'There won't be a winner unless there's a loser. And conversely.' I symbolized it as:
  - "Logish": If not $L$, then not $W$, *and conversely*. [*i.e.*, not $L$ iff not $W$.]
  - LSL: '$\sim L \leftrightarrow \sim W$', *equivalently*: '$(\sim L \to \sim W) \& (\sim W \to \sim L)$'.

- One might wonder why I didn't interpret the "and conversely" to be operating on the *unless* operator itself, rather than the *conditional* operator. This would yield the following *different* symbolization:
  - "Logish": not $W$ unless $L$, and $L$ unless not $W$.
  - LSL: '$(\sim L \to \sim W) \& (\sim\sim W \to L)$', *equivalently*: '$(\sim L \to \sim W) \& (W \to L)$'.

- Answer: This is a *redundant* symbolization in LSL, since '$\sim L \to \sim W$' *is equivalent to* '$W \to L$'. Moreover, the resulting argument *isn't* valid.

- **Principle of Charity**. If an argument $\mathscr{A}$ has two *plausible but semantically distinct* LSL symbolizations (where neither is *obviously* preferable) — and *only one of them is valid* — *choose the valid one*.

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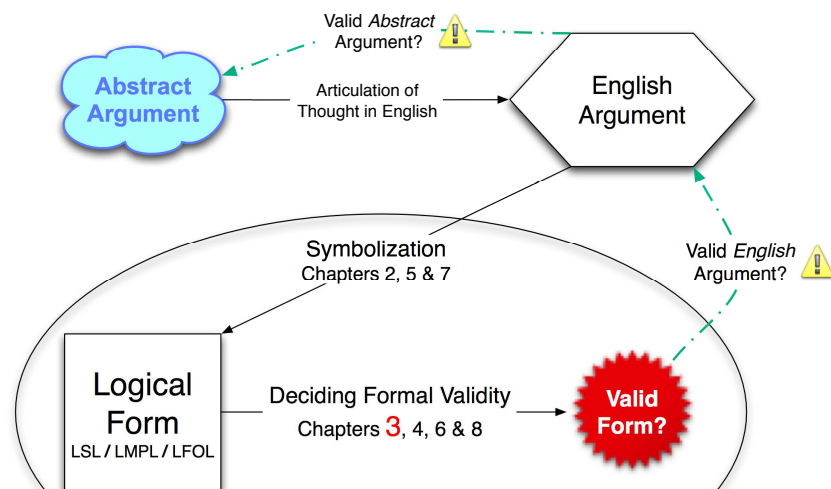
## Chapter 3 — Semantics of LSL: Truth Functions I

- The semantics of LSL is *truth-functional* — the truth value of a compound statement is a function of the truth values of its parts.

- Truth-conditions for each of the five LSL statement forms are given by *truth tables*, which show how the truth value of each type of complex sentence depends on the truth values of its constituent parts.

- Truth-tables provide a very precise way of thinking about *logical possibility*. Each row of a truth-table can be thought of as a *way the world might be*. The actual world falls into *exactly one* of these rows.

- In this sense, truth-tables provide a way to "see" "logical space."

- Truth-tables will also provide us with a rigorous way to establish whether an argument form in LSL is valid (*i.e.*, sentential validity).

- We just look for rows of a salient truth-table in which all the premises are true and the conclusion is false. That's where we're headed.

## Chapter 3 — Semantics of LSL: Truth Functions II

- We begin with negations, which have the simplest truth functions. The truth table for negation is as follows (we use $\top$ and $\bot$ for true and false):

| $p$ | $\sim p$ |
|---|---|
| $\top$ | $\bot$ |
| $\bot$ | $\top$ |

- In words, this table says that if $p$ is true than $\sim p$ is false, and if $p$ is false, then $\sim p$ is true. This is quite intuitive, and corresponds well to the English meaning of 'not'. Thus, LSL negation is like English negation.

- Examples:
  - It is not the case that Wagner wrote operas. ($\sim W$)
  - It is not the case that Picasso wrote operas. ($\sim P$)

- '$\sim W$' is false, since '$W$' is true, and '$\sim P$' is true, since '$P$' is false (like English).

## Chapter 3 — Semantics of LSL: Truth Functions III

| $p$ | $q$ | $p\ \&\ q$ |
|---|---|---|
| $\top$ | $\top$ | $\top$ |
| $\top$ | $\bot$ | $\bot$ |
| $\bot$ | $\top$ | $\bot$ |
| $\bot$ | $\bot$ | $\bot$ |

- Notice how we have four (4) rows in our truth table this time (not 2), since there are four possible ways of assigning truth values to $p$ and $q$.

- The truth-functional definition of & is very close to the English 'and'. A LSL conjunction is true if *both* conjuncts are true; it's false otherwise.
  - Monet and van Gogh were painters. ($M\ \&\ V$)
  - Monet and Beethoven were painters. ($M\ \&\ B$)
  - Beethoven and Einstein were painters. ($B\ \&\ E$)

- '$M\ \&\ V$' is true, since both '$M$' and '$V$' are true. '$M\ \&\ B$' is false, since '$B$' is false. And, '$B\ \&\ E$' is false, since '$B$' and '$E$' are both false (like English).

## Chapter 3 — Semantics of LSL: Truth Functions IV

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| $\top$ | $\top$ | $\top$ |
| $\top$ | $\bot$ | $\top$ |
| $\bot$ | $\top$ | $\top$ |
| $\bot$ | $\bot$ | $\bot$ |

- Our truth-functional $\vee$ is not as close to the English 'or'. An LSL disjunction is true if *at least one* disjunct is true (false otherwise).

- In English, '$A$ or $B$' often implies that '$A$' and '$B$' are *not both true*. That is called *exclusive* or. In LSL, '$A \vee B$' is *not* exclusive; it is *inclusive* (true if both disjuncts are true). But, we *can* express exclusive or in LSL. How?
  - Either Jane austen or René Descartes was novelist. ($J \vee R$)
  - Either Jane Austen or Charlotte Bronte was a novelist. ($J \vee C$)
  - Either René Descartes or David Hume was a novelist. ($R \vee D$)

- The first two disjunctions are true because at least one their disjuncts is true, but the third is false, since both of its disjuncts are false.

## Chapter 3 — Semantics of LSL: Truth Functions V

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| $\top$ | $\top$ | $\top$ |
| $\top$ | $\bot$ | $\bot$ |
| $\bot$ | $\top$ | $\top$ |
| $\bot$ | $\bot$ | $\top$ |

- Our truth-functional $\to$ is farther from the English 'only if'. An LSL conditional is false iff its antecedent is true and its consequent is false.

- Consider the following English conditionals. [$M$ = 'the moon is made of green cheese', $O$ = 'life exists on other planets', and $E$ = 'life exists on Earth']
  - If the moon is made of green cheese, then life exists on other planets.
  - If life exists on other planets, then life exists on earth.

- The LSL translations of these sentences are both true. '$M \to O$' is true because its antecedent '$M$' is false. '$O \to E$' is true because its consequent '$E$' is true. This seems to deviate from the English 'if'.

  [Soon, we'll *prove* the following *equivalence*: $\ulcorner p \to q \urcorner \dashv\vDash \ulcorner \sim p \vee q \urcorner$.]

## Chapter 3 — Semantics of LSL: Truth Functions VI

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| ⊤ | ⊤ | ⊤ |
| ⊤ | ⊥ | ⊥ |
| ⊥ | ⊤ | ⊥ |
| ⊥ | ⊥ | ⊤ |

- Our truth-functional ↔ is also farther from the English 'if and only if'. An LSL biconditional is true iff both sides have the same truth value.

- Consider these two biconditionals. [$M$ = 'the moon's made of green cheese', $U$ = 'there are unicorns', $E$ = 'life exists on Earth', and $S$ = 'the sky is blue']
  - The moon is made of green cheese if and only if there are unicorns.
  - Life exists on earth if and only if the sky is blue.

- The LSL translations of these sentences are true. $M \leftrightarrow U$ is true because $M$ and $U$ are false. $E \leftrightarrow S$ is true because $E$ and $S$ are true. This seems to deviate from the English 'iff'. Soon, we'll *prove* the following:

$$\ulcorner p \leftrightarrow q \urcorner \dashv\vDash \ulcorner (p \,\&\, q) \vee (\sim p \,\&\, \sim q) \urcorner$$

## Chapter 3 — Semantics of LSL: Truth Functions VII

- If our truth-functional semantics for '→' doesn't perfectly capture the English meaning of 'if ... then ...', then why do we define it this way?

- The answer has two parts. First, our semantics is *truth-functional*. This is an *idealization* — it yields the *simplest* ("Newtonian") semantics.

- And, there are only $2^4 = 16$ possible binary truth-functions. Why?

- So, unless one of the *other* 15 binary truth-functions is *closer* to the English conditional than '→' is, it's *the best we can do*, *truth-functionally*.

- More importantly, there are certain *logical properties* that the conditional *must* have. It can be shown that our definition of '→' is the *only* binary truth-function which satisfies all three of the following:

(1) *Modus Ponens* [$p$ and $\ulcorner p \to q \urcorner \therefore q$] is a valid sentential form.

(2) Affirming the consequent [$q$ and $\ulcorner p \to q \urcorner \therefore p$] is *not* a valid form.

(3) All sentences of the form $\ulcorner p \to p \urcorner$ are logical truths.

## Chapter 3 — Semantics of LSL: Truth Functions VIII

- Here are all of the 16 possible binary truth-functions. I've given them all names or descriptions. [Only a few of these names were made up by me.]

| $p$ | $q$ | ⊤ | NAND | → | $\sim p$ | FI (←) | $\sim q$ | ↔ | NOR | ∨ | NIFF | $q$ | NFI | $p$ | NIF | & | ⊥ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ⊤ | ⊤ | ⊤ | ⊥ | ⊤ | ⊥ | ⊤ | ⊥ | ⊤ | ⊥ | ⊤ | ⊥ | ⊤ | ⊥ | ⊤ | ⊥ | ⊤ | ⊥ |
| ⊤ | ⊥ | ⊤ | ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | ⊥ | ⊥ | ⊤ | ⊤ | ⊥ | ⊥ |
| ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊥ | ⊥ | ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊥ | ⊥ | ⊥ | ⊥ |
| ⊥ | ⊥ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊤ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ | ⊥ |
| (1)? | | | | Yes | | | | | | | | | | | | | |
| (2)? | | | | Yes | | | | | | | | | | | | | |
| (3)? | | | | Yes | | | | | | | | | | | | | |

- Exercise: fill-in the three rows at the bottom (except for →, which I have done for you already) concerning (1), (2), and (3) from the previous slide.

- You should be able to do this pretty soon (within the next week) ...

## Chapter 3 — Semantics of LSL: Additional Remarks on →

- Above, I explained *why* our conditional → behaves "like a disjunction":

1. We want a *truth-functional* semantics for →. This is a simplifying *idealization*. Truth-functional semantics are the simplest compositional semantics for sentential logic. [A "Newtonian" semantic model.]

2. Given (1), the *only* way to define → is *our* way, since it's the *only* binary truth-function that has the following three essential *logical* properties:
   (i) *Modus Ponens* [$p$ and $\ulcorner p \to q \urcorner \therefore q$] is a valid sentential form.
   (ii) Affirming the consequent [$q$ and $\ulcorner p \to q \urcorner \therefore p$] is *not* a valid form.
   (iii) All sentences of the form $\ulcorner p \to p \urcorner$ are logical truths.

- There are *non*-truth-functional semantics for the English conditional.

- These may be "closer" to the English *meaning* of "if". But, they agree with our semantics for →, when it comes to the crucial *logical* properties (i)–(iii). Indeed, our → captures *most* of the (intuitive) *logical* properties of "if".

## Interpretations and the Relation of Logical Consequence

- An *interpretation* of an LSL formula $p$ is an assignment of truth-values to all of the sentence letters in $p$ — *i.e.*, a row in $p$'s truth-table.

- A formula $p$ is a *logical consequence* of a set of formulae $S$ [written $S \vDash p$] just in case there is no interpretation (*i.e.*, no row in the joint truth-table of $S$ and $p$) on which all the members of $S$ are $\top$ but $p$ is $\bot$.

- $S \vDash p$ is another way of saying that the argument from $S$ to $p$ is *valid*.

- Two LSL sentences $p$ and $q$ are said to be *logically equivalent* [written $p \dashv\vDash q$] iff they have the same truth-value on all (joint) interpretations.

- That is, $p$ and $q$ are logically equivalent iff *both $p \vDash q$ and $q \vDash p$*.

- I will often express $\ulcorner p \vDash q \urcorner$ by saying that $\ulcorner p$ entails $q \urcorner$. This is easier than saying that $\ulcorner q$ is a logical consequence of $p \urcorner$.

- The logical consequence relation $\vDash$ is our central theoretical relation.

## Logical Truth, Logical Falsity, and Contingency: Definitions

- A statement is said to be logically true (or tautologous) if it is $\top$ on all interpretations. *E.g.*, any statement of the form $p \leftrightarrow p$ is tautological.

| $p$ | $p$ | $\leftrightarrow$ | $p$ |
|-----|-----|-----|-----|
| $\top$ | $\top$ | $\top$ | $\top$ |
| $\bot$ | $\bot$ | $\top$ | $\bot$ |

- A statement is logically false (or self-contradictory) if it is $\bot$ on all interpretations. *E.g.*, any statement of the form $p \,\&\, {\sim}p$ is logically false:

| $p$ | $p$ | $\&$ | $\sim$ | $p$ |
|-----|-----|-----|-----|-----|
| $\top$ | $\top$ | $\bot$ | $\bot$ | $\top$ |
| $\bot$ | $\bot$ | $\bot$ | $\top$ | $\bot$ |

- A statement is contingent if it is *neither* tautological *nor* self-contradictory. Example: '$A$' (or *any* basic sentence) is contingent.

| $A$ | $A$ |
|-----|-----|
| $\top$ | $\top$ |
| $\bot$ | $\bot$ |

## Equivalence, Contradictoriness, Consistency, and Inconsistency

- Statements $p$ and $q$ are equivalent [$p \dashv\vDash q$] if they have the same truth-value on all interpretations. For instance, '$A \rightarrow B$' and '$\sim A \vee B$'.

| $A$ | $B$ | $A$ | $\rightarrow$ | $B$ | $\sim$ | $A$ | $\vee$ | $B$ |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\bot$ | $\top$ | $\top$ | $\top$ |
| $\top$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\bot$ | $\top$ | $\bot$ | $\bot$ |
| $\bot$ | $\top$ | $\bot$ | $\top$ | $\top$ | $\top$ | $\bot$ | $\top$ | $\top$ |
| $\bot$ | $\bot$ | $\bot$ | $\top$ | $\bot$ | $\top$ | $\bot$ | $\top$ | $\bot$ |

- Statements $p$ and $q$ are contradictory [$p \dashv\vDash \sim q$] if they have opposite truth-values on all interpretations. For instance, '$A \rightarrow B$' and '$A \,\&\, {\sim}B$'.

| $A$ | $B$ | $A$ | $\rightarrow$ | $B$ | $A$ | $\&$ | $\sim$ | $B$ |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\bot$ | $\bot$ | $\top$ |
| $\top$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\top$ | $\top$ | $\top$ | $\bot$ |
| $\bot$ | $\top$ | $\bot$ | $\top$ | $\top$ | $\bot$ | $\bot$ | $\bot$ | $\top$ |
| $\bot$ | $\bot$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\bot$ | $\top$ | $\bot$ |

- Statements $p$ and $q$ are inconsistent [$p \vDash \sim q$] if there is no interpretation on which they are both true. For instance, '$A \leftrightarrow B$' and '$A \,\&\, {\sim}B$' are inconsistent [Note: they are *not* contradictory!].

| $A$ | $B$ | $A$ | $\leftrightarrow$ | $B$ | $A$ | $\&$ | $\sim$ | $B$ |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\bot$ | $\bot$ | $\top$ |
| $\top$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\top$ | $\top$ | $\top$ | $\bot$ |
| $\bot$ | $\top$ | $\bot$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\bot$ | $\top$ |
| $\bot$ | $\bot$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\bot$ | $\top$ | $\bot$ |

- Statements $p$ and $q$ are consistent [$p \nvDash \sim q$] if there's an interpretation on which they are both true. *E.g.*, '$A \,\&\, B$' and '$A \vee B$' are consistent:

| $A$ | $B$ | $A$ | $\&$ | $B$ | $A$ | $\vee$ | $B$ |
|-----|-----|-----|-----|-----|-----|-----|-----|
| $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\top$ | $\top$ |
| $\top$ | $\bot$ | $\top$ | $\bot$ | $\bot$ | $\top$ | $\top$ | $\bot$ |
| $\bot$ | $\top$ | $\bot$ | $\bot$ | $\top$ | $\bot$ | $\top$ | $\top$ |
| $\bot$ | $\bot$ | $\bot$ | $\bot$ | $\bot$ | $\bot$ | $\bot$ | $\bot$ |