Philosophy 1115 Homework Assignment #6

April 8, 2016 (due on 04/22/16)

1 Problem #1

For this problem, please use the following stochastic truth table to determine all of your algebraic translations for probabilistic claims involving $\{X, Y, Z\}$.

State (s_i)	X	Y	Z	$\Pr(s_i)$
s_1	$\top$	$\top$	$\top$	$\Pr(s_1) = a_1$
s_2	$\top$	$\top$	$\bot$	$\Pr(s_2) = a_2$
s_3	$\top$	$\bot$	$\top$	$\Pr(s_3) = a_3$
s_4	$\top$	$\bot$	$\bot$	$\Pr(s_4) = a_4$
s_5	$\bot$	$\top$	$\top$	$\Pr(s_5) = a_5$
s_6	$\bot$	$\top$	$\bot$	$\Pr(s_6) = a_6$
s_7	$\bot$	$\bot$	$\top$	$\Pr(s_7) = a_7$
s_8	$\bot$	$\bot$	$\bot$	$\Pr(s_8) = a_8$

The goal is to prove the following general claim (the Unconditional Sure Thing Principle holds for Factor #2):

$$[\Pr(Z \mid X \,\&\, Y) > \Pr(Z) \text{ and } \Pr(Z \mid X \,\&\, {\sim}Y) > \Pr(Z)] \Longrightarrow \Pr(Z \mid X) > \Pr(Z).$$

That is, the goal is the prove that the following two assumptions:

(1) $\Pr(Z \mid X \,\&\, Y) > \Pr(Z)$

(2) $\Pr(Z \mid X \,\&\, {\sim}Y) > \Pr(Z)$

generally entail this third claim:

(3) $\Pr(Z \mid X) > \Pr(Z)$

In order to do this, you should follow these two steps:

Step 1. Translate claims (1)–(3) into their algebraic counterparts, using our definitions of unconditional and conditional probability (and the above table for the salient variables). That is, using:

$$\Pr(p) \overset{\text{def}}{=} \sum_{s_i \vDash p} \Pr(s_i) = \sum_{s_i \vDash p} a_i$$

$$\Pr(p \mid q) \overset{\text{def}}{=} \frac{\Pr(p \,\&\, q)}{\Pr(q)}, \text{ provided that } \Pr(q) > 0.$$

Step 2. Use our two general assumptions about the a_i's:

(i) Each of the a_i's are on $[0, 1]$. That is: $a_1, \ldots, a_8 \in [0, 1]$.

(ii) The a_i's must sum to 1. That is: $\sum_{i=1}^{8} a_i = 1$.

to show (in algebraic terms) that *whenever (1) and (2) are both true, (3) must also be true.*

2 Problem #2

Suppose we have an urn containing 320 objects. We are going to sample a single object o at random from the urn (each individual object is equally likely to be chosen). Consider the following three statements:

- B = o is black ($\sim B$ = o is white).
- M = o is metal ($\sim M$ = o is plastic).
- S = o is a sphere ($\sim S$ = o is a cube).

Assume that these three properties are distributed according to the following *probabilistic truth-table*:

State (s_i)	B	M	S	$\Pr(w_i)$
s_1	$\top$	$\top$	$\top$	$\Pr(s_1) = a_1 = \frac{24}{320}$
s_2	$\top$	$\top$	$\bot$	$\Pr(s_2) = a_2 = \frac{6}{320}$
s_3	$\top$	$\bot$	$\top$	$\Pr(s_3) = a_3 = \frac{24}{320}$
s_4	$\top$	$\bot$	$\bot$	$\Pr(s_4) = a_4 = \frac{42}{320}$
s_5	$\bot$	$\top$	$\top$	$\Pr(s_5) = a_5 = \frac{33}{320}$
s_6	$\bot$	$\top$	$\bot$	$\Pr(s_6) = a_6 = \frac{33}{320}$
s_7	$\bot$	$\bot$	$\top$	$\Pr(s_7) = a_7 = \frac{47}{320}$
s_8	$\bot$	$\bot$	$\bot$	$\Pr(s_8) = a_8 = \frac{111}{320}$

That is, 24 of the 320 objects are black metallic spheres; 47 of the 320 objects are white plastic spheres *etc.* With these basic probabilities in mind, we can use our definitions of unconditional and conditional probability (on page 1) to calculate *any* probability in this example.

The HW is to answer the following eleven (11) questions. [Note: once you've answered questions (1)–(5), you'll have everything you need to answer questions (6)–(11). See my 03/29/16 lecture for the 3 Proposals.]

1. What is $\Pr(S)$?
2. What is $\Pr(S \mid B)$? [That is, what is $\frac{\Pr(S\&B)}{\Pr(B)}$?]
3. What is $\Pr(S \mid B\,\&\,M)$? [That is, what is $\frac{\Pr(S\&(B\&M))}{\Pr(B\&M)}$?]
4. What is $\Pr(B \to S)$? [Hint: do the truth-table for $B \to S$ to see in which of the 8 worlds $B \to S$ is true.]
5. What is $\Pr((B\,\&\,M) \to S)$? [Hint: do the truth-table for $(B\,\&\,M) \to S$ to see in which worlds it is true.]
6. Is the argument $\ulcorner B \therefore S \urcorner$ inductively strong, according to Proposal #1? [Hint: use your answer to (4).]
7. Is $\ulcorner B \therefore S \urcorner$ inductively strong, according to Proposal #2 (Skyrms's proposal)? [Hint: use (2).]
8. Is $\ulcorner B \therefore S \urcorner$ inductively strong, according to Proposal #3 (my proposal)? [Hint: use (2) and (1).]
9. Is the argument $\ulcorner B\,\&\,M \therefore S \urcorner$ inductively strong, according to Proposal #1? [Hint: use (5).]
10. Is $\ulcorner B\,\&\,M \therefore S \urcorner$ inductively strong, according to Proposal #2 (Skyrms's proposal)? [Hint: use (3).]
11. Is $\ulcorner B\,\&\,M \therefore S \urcorner$ inductively strong, according to Proposal #3 (my proposal)? [Hint: use (3) and (1).]