## THE JOURNAL OF PHILOSOPHY

## ON THE NATURE AND THE OBSERVABILITY OF THE CAUSAL RELATION <sup>1</sup>

THE aim of this paper is to set forth two related theses. The first is that the correct definition of the causal relation is to be framed in terms of one single case of sequence, and that constancy of conjunction is therefore no part of it, but merely, under certain conditions, a corollary of the presence of the causal relation. The second thesis is that the causal relation, when correctly defined, is as directly observable as many other facts, and that the alleged mysteriousness of the causal tie is therefore a myth due only to a mistaken notion of what a tie is.

I. Meaning of "a Correct Definition."—The problem of giving a "correct" definition of the causal relation is that of making analytically explicit the meaning which the term "cause" has in actual concrete phrases that our language intuition acknowledges as proper and typical cases of its use. For obviously it is one thing to "know what cause means" in the cheap sense of being able to understand intuitively such an assertion as that the Santa Barbara earthquake caused the collapse of numberless chimneys; and it is another and a much more difficult and rarer thing to "know what cause means" in the sense of being able to give a correct definition of it. To say that a definition of it is correct means that that definition can be substituted for the word "cause" in any such assertion as the above, in which the word occurs, without in the least changing the meaning which the assertion is felt to have. Any ventured definition of such a philosophical term as cause is thus capable of being correct or incorrect in strictly the same sense as that in which a scientific hypothesis is so, viz., either it fits the facts or it does not. The only difference is that in the case of scientific hypotheses the facts are perceptual objects and their relations, while in the case of philosophical hypotheses the facts are the intuited meanings of actual phrases in which the word to be defined occurs. The great inductive method of hypothesis-deduction-verification is thus no less that of philosophy than that of science.

<sup>&</sup>lt;sup>1</sup> Read before the Meeting of the Eastern Division of the American Philosophical Association, at Northampton, Mass., December, 1925.

- II. Two Preliminary Remarks.—Before attempting to formulate a definition of the term "cause," attention must briefly be called to two essential preliminary points.<sup>2</sup>
- 1. The first is that nothing can, in strict propriety, ever be spoken of as a cause or an effect, except an event. And by an event is to be understood either a change or an absence of change (whether qualitative or relational) of an object.<sup>3</sup> On the other hand, objects themselves (in the sense of substances, e.g., gold; or things, e.g., a tree) never can properly be spoken of as causes or effects,<sup>4</sup> but only as agents or patients, as components or compounds, as parts or wholes. These relations, although closely allied to the causal relation, are nevertheless distinct from it, and can not be discussed here.
- 2. The second point to be borne in mind is that when the term "causal connection" is used, any one of four distinct objective relations may actually be meant, namely, objectively sufficient to. necessary to, necessitated by, contingent upon. And to these four relations correspond respectively the four functional terms, cause, condition, effect, resultant. So that, more explicitly, if a given particular event is regarded as having been sufficient to the occurrence of another, it is said to have been its cause; if regarded as having been necessary to the occurrence of another, it is said to have been a condition of it; if regarded as having been necessitated by the occurrence of another, it is said to have been its effect; and if regarded as having been contingent upon the occurrence of another, it is said to have been a resultant of that other. Much confusion has resulted in discussions of causality from the failure to keep these four relations at all times clearly distinguished, Mill, indeed, pushing perversity to the point of convincing himself and some of his readers that there was no sound basis for a distinction between cause and condition. But it is, on the contrary, essential to remember that to be sufficient is one thing, to be necessary another thing, and to be both sufficient and necessary (which is what Mill's definition would make cause mean) yet a third thing.

Of the four relations, cause, condition, effect, resultant, which a given particular event may have to another with which it is connected, we shall have space here to discuss only the first, namely, cause. And we shall, moreover, confine ourselves to cases—much the

<sup>&</sup>lt;sup>2</sup> In a monograph on causation published by the writer some time ago, these two points are argued at some length. See *Causation and the Types of Necessity*, Univ. of Washington Press, 1924, pp. 52 ff.

<sup>&</sup>lt;sup>3</sup> More technically, an event can be defined as either a change or an absence of change in the relation of an object to either an intensive or an extensive standard of reference, during a specified time interval.

<sup>4</sup> Cf. Schopenhauer, The Fourfold Root of the Principle of Sufficient Reason, Trans. Hillebrand, pp. 38 ff.; and Wundt, Logik, Third Ed., Vol. I., p. 586.

more frequent—where the events contemplated are changes, rather than absences of change.

III. Definition of Cause.—Taking it as an admitted fact of the language that if the occurrence of a particular change sufficed to the occurrence of a given other it is then said to have caused that other, the all-important question now arises how such sufficing is to be defined. I suggest that the correct definition of it, framed in terms of a hypothetical situation, is as follows:

Considering two changes, C and K (which may be either of the same or of different objects), the change C is said to have been sufficient to, *i.e.*, to have caused, the change K, if:

- 1. The change C occurred during a time and through a space terminating at the instant I at the surface S.<sup>5</sup>
- 2. The change K occurred during a time and through a space beginning at the instant I at the surface S.
- 3. No change other than C occurred during the time and through the space of C, and no change other than K during the time and through the space of K.

More roughly, but in briefer and more easily intuited terms, we may say that the cause of the particular change K was such particular change C as alone occurred in the immediate environment of K immediately before.

- IV. Some Bearings of the Definition.—A number of important points may be noted in connection with the above definition of cause.
- 1. The first is that it presents the causal relation as involving not two terms only, but essentially three terms, namely, (a) the environment of an object, (b) some change in that environment, (c) the resulting change in the object. As soon as it is clearly realized that the expression "the cause of an event" thus has any meaning at all only in terms of some definite environment, either concretely given or abstractly specified, Mill's contention that the distinction between cause and conditions is arbitrary and capricious, is seen to be absurd. To take up the environment into the "cause," as Mill's definition of cause tries to do, is impossible because the cause consists of a change in that environment. No event can be spoken of as the cause of anything, except relatively to certain conditions; and vice versa, as regards conditions.
  - 2. The second remark for which the definition of cause above gives
  - <sup>5</sup> The limit of a change of a solid is obviously a surface, not a point.
- 6 "The cause . . . is the sum total of the conditions, positive and negative taken together . . . which being realized, the consequent invariably follows" (Syst. of Logic, Bk. III., Ch. V., No. 3). This definition is obviously in flagrant contradiction with Mill's characterization of the cause as the single difference in the circumstances, in the canon of the "Method of Difference."

occasion concerns the immediate spatial and temporal contiguity of cause and effect. The alleged impossibility of such immediate contiguity is the chief ground upon which Mr. Russell has advocated the extrusion of the term "cause" from the philosophical vocabulary. The difficulties raised by him, however, are easily disposed of if two things are kept in mind. The first is that the terms "a time" and "a place" are ambiguous. It is essential to distinguish clearly "a time" in the sense of an instant, i.e., a cut of the time series, from "a time" in the sense of a segment of the time series, limited by two cuts. And similarly with regard to the space order, the cuts of it (viz., points, lines, or surfaces according as one, two, or three dimensional space is considered) are to be carefully distinguished from the parts of space, which have such cuts as limits. The second thing to bear in mind is that an event (whether a change or an "unchange''s) can not be said to occur at a time (cut), but only during a time (segment); nor at a point (or other cut of space), but only through a space (between cuts). Thus, a change is essentially a process which has extent both in time and in space, and is therefore divisible; any division yielding segments of the process that are themselves extended in time and space and therefore further divisible, ad infinitum. The immediate contiguity of cause and effect in space and time, specified in our definition, then means only that one identical space-time cut marks both the end of the cause process and the beginning of the effect process; the one extending up to, and the other from, that cut; the cut itself, however (by the very nature of a cut as distinguished from a segment), having no space-time dimension at all. 10 With cause and effect and their space-time relation 11 so conceived, there is no possibility that, as Mr. Russell contended, some other event should creep in between the cause and the effect

- 7 Proc. Arist. Soc., Vol. XIII., "On the Notion of Cause."
- <sup>8</sup> The apt term "unchange" is borrowed from Dr. Charles Mercier's book, Causation and Belief.
- <sup>9</sup> A stage might, however, conceivably be reached, at which the parts obtained by the division of a change, would, in terms of the particular test of changing used at the previous stages of division, be themselves not changes, but unchanges (though, of course, none the less extended in time and space and therefore divisible). That is, the assertion that something changes, or, equally, does not change, remains ambiguous so long as some definite test of such change has not been specified as standard. Thus the assertion might be true in terms of one test and false in terms of another. Cf. "A Liberalistic View of Truth," by the writer, in the Philos. Review for November, 1925.
- <sup>10</sup> In practice, no space-time dimension of a relevant order of magnitude. Clock ticks and graduation lines as used are never perfectly dimensionless.
- <sup>11</sup> This view of the space-time relation of cause and effect, I was gratified to find, is also that set forth by Mr. Johnson in Vol. III. of his *Logic* (p. 74), which appeared at virtually the same time as the monograph on causation referred to above.

and thwart the production of the effect. Nor are we compelled, as he also contended, to trim down indefinitely the beginning part of the cause (and, mutatis mutandis, the end part of the effect) on the ground that the early part of the cause is not necessary to the effect so long as the end part of the cause occurs. For, once more, the cause means something which was sufficient, and not as the objection assumes something which was both sufficient and necessary, to the effect. Thus the space-time limit of the cause process at the outer end is as elastic as we please, and varies with the space-time scope of the particular description of the cause that we give in each concrete case. And the same is true of the outer end of the effect process.<sup>12</sup>

3. The third observation to be made on the definition of cause proposed is that it defines the cause of a particular event in terms of but a single occurrence of it, and thus in no way involves the supposition that it, or one like it, ever has occurred before or ever will again. The supposition of recurrence is thus wholly irrelevant to the meaning of cause; that supposition is relevant only to the meaning of law. And recurrence becomes related at all to causation only when a law is considered which happens to be a generalization of facts themselves individually causal to begin with. A general proposition concerning such facts is, indeed, a causal law, but it is not causal because general. It is general, i.e., a law, only because it is about a class of resembling facts; and it is causal only because each of them already happens to be a causal fact individually and in its own right (instead of, as Hume would have it, by right of its co-membership with others in a class of pairs of successive events). The causal relation is essentially a relation between concrete individual events; and it is only so far as these events exhibit likeness to others, and can therefore be grouped with them into kinds, that it is possible to pass from individual causal facts to causal laws. the other hand, in the case of laws obtained, not by experimentation and generalization of the result of it by abstraction, but in a purely statistical manner (the only manner directly relevant to Hume's notion of cause), it is only quite accidentally that the terms of such "constant conjunctions" as these laws describe stand one to the other

12 It is interesting to note that the analysis of the space-time relation of cause and effect given above reveals an essential connection between the two notions of Change and of Causation. For, taking any given change process, by specifying a space-time cut of it, one splits it into a cause and an effect; and, on the other hand, taking any given cause and its effect, by abstracting from the particular space-time cut in terms of which as common limit the cause process is distinguished from the effect process, one obtains a process describable as one change. This calls to mind Kant's very inadequately argued contention in the Second Analogy, that (objective) change involves the category of causation.

as cause and effect. Much more frequently they are not such and are not regarded as such; and uniformity of succession thus constitutes not at all the meaning of the cause-effect relation, but at the most only evidence of the existence of some causal connection, perhaps very remote and indirect, and yet to be discovered, between the terms of the succession. A causal connection explains the regularity of the succession, but is not constituted by such regularity, which is but a corollary of the causal connection whenever the cause or the chain of causes happens to occur again. Hume himself, indeed, on the very page of the *Inquiry* where he gives his definition of cause (in terms of regularity of succession), says that that definition is "drawn from circumstances foreign to the cause"; "from something extraneous and foreign to it." And it was to avoid having to say, as Hume's definition would require, that day was the cause of night and night the cause of day, that Mill added, in his own definition, the requirement of "unconditionality" to that of invariability of sequence—without perceiving, however, that as soon as "unconditionality" was introduced, invariability became superfluous. the effect "unconditionally" follows from the cause, i.e., is necessitated by the cause, then, obviously, as often as the cause recurs the effect must recur also. But this so-called unconditionality of an effect, upon a cause, i.e., the necessitation of the effect by the cause, was the very thing which Mill had declared was not revealed by mere observed regularity of sequence. It must then be ascertained by the experimental "method of difference," i.e., by the analytical observation of an individual case. But Mill never sees that this amounts to defining cause in terms of single difference in one experiment. refers to single difference as a "Rule" by which to judge of causes and effects. 18 and Mill, borrowing the blunder, throughout persists in regarding single difference as a "method" for the roundabout ascertainment of something other than itself, viz., of invariable sequence; instead of, and properly, regarding it as the very definition of cause. This is perhaps in part explicable by the fact that Mill never clearly perceived the difference between experimentation and generalization 14 by abstraction; he never was adequately conscious that it is one thing to introduce a single difference, i.e., make a single change, in a given concrete set of circumstances, and note what happens; and a very different thing to compare two such experiments, one of which yielded a certain effect and the other failed to, and note what single difference there was between the single antecedent changes introduced in the two cases into the (same) set of circumstances.

<sup>13</sup> Treatise, Bk. I., Part III., No. 15.

<sup>&</sup>lt;sup>14</sup> This has been noted by Jevons, Pure Logic and Other Minor Works, p. 251.

4. As a last remark upon the definition of cause in terms of a single case given above, it may be noted that it is the only one which is faithful to the manner in which the word "cause" is actually used by every person whose English has not been contaminated by Hume. As Mr. Russell himself notes, we can not without "intolerable circumlocution" 15 avoid speaking of one particular event as causing another particular event. And, I ask, why seek to avoid it, when just that is so plainly what we do mean? When any philosophically pure-minded person sees a brick strike a window and the window break, he judges that the impact of the brick was the cause of the breaking, because he believes that impact to have been the only change which took place then in the immediate environment of the window. He may, indeed, have been mistaken, and acknowledge that he was mistaken, in believing that impact to have been the only change in the environment. But if so he will nevertheless maintain that if it had been the only change, it would have been the cause. That is, he will stand by the definition of cause, and admit merely that what he perceived was not a true case of what he meant and still means by cause.

The Observability of the Causal Relation.—This now brings us to the second of the two theses mentioned at the beginning of this paper, namely, that concerning the observability of the causal relation. Hume's view that no connection between a cause and its effect is objectively observable would be correct only under the assumption that a "connection" is an entity of the same sort as the terms themselves between which it holds, that is, for Hume and his followers, a sense impression. For it is true that neither a color, nor an odor, nor a sound, nor a taste, nor any other sense impression, "connecting" the cause and the effect, is observable between them. Indeed, we must even add that if a sense impression were present between those said to constitute the cause and the effect, it would, from its very nature as a sense impression, be quite incapable of doing any connecting and would itself but constitute one more of the entities to be connected. This is true in particular of the feeling of expectation which Hume would have us believe is what the words "necessary connection" ultimately denote.

But there is fortunately no need for us to attempt to persuade ourselves that whenever people during the past centuries have talked of objective connection they thus have not really meant it at all. For the fact is that a causal connection is not a sensation at all, but a relation. The nature of that relation has already been minutely described above. It is, as we have seen, a relation which has individual concrete events for its terms; and, as analyzed by us, its pres-

<sup>15</sup> Scientific Method in Philosophy, p. 220.

ence among such events is to be observed every day. We observe it whenever we perceive that a certain change is the *only* one to have taken place immediately before, in the immediate environment of another.

But at this point it becomes necessary for us to consider two apparently weighty objections, which can be urged against the observability of what we have defined as constituting the causal relation. One of them is that we are never theoretically certain that we have observed as much as the definition demands; and the other is that, on the other hand, we are often certain that the cause is less than the definition would permit us so to call. Each of these difficulties in turn must be carefully examined.

1. The first of them, more explicitly stated, is this: We never can be certain that the change which we have observed in any given case was, as the definition requires, the only change that occurred then and there, and therefore it is always possible that a part of the cause has escaped us. In considering this objection, it is, of course, well to bear in mind that our definition specifies contiguity in space as well as in time of the cause to the effect, and in addition permits us to set the outer space-time limit of the environment to be observed as near to the effect as we find convenient; so that the definition relieves us of the sometimes alleged obligation to observe the antecedent change of the entire universe. But even confining our observation to as externally limited a region of the contiguous space-time as we please, the possibility still always remains that we have not in a given case observed the whole of the change in that environment.

This predicament, it must frankly be admitted, is inescapable. But we must state also, and with all possible emphasis, that it is not peculiar to the definition of causation proposed. Nor, indeed, is it, in its essence, peculiar even to definitions of cause. Rather it is a predicament involved in every attempt to observe a universal negative. Thus, even such an assertion as that "this man is Mr. So-andso" is theoretically always precarious in exactly the same manner, for there is no theoretically absolute guarantee that the man before us is not some one else, who merely happens to be exactly like Mr. So-andso in the particular respects to which our observation has turned. The predicament mentioned, thus, does not constitute the

16 The corresponding difficulty with the Humian definition of cause as regular sequence is that experience never can guarantee that exceptions to the regularity of the sequence have not escaped our observation; or, more generally, that the sample of the character of the sequence, which we have observed, is a "fair sample."

17 This difficulty becomes particularly acute when the opportunity for observation is limited, as, e.g., in establishing one's identity over the telephone; or,

least evidence against the correctness of our definition of cause, for the very same difficulty would arise no matter what other definition were proposed.

All that we are then called upon to do in connection with that predicament is, first, to call attention to its existence and nature. and sagely class it as a fact illustrating the platitude that life is a precarious business in many ways; and, second, to state explicitly the proviso subject to which cases of causation as defined are observable. This proviso is obviously that the change which we observed in the antecedently contiguous space-time was really the only change which That is not something which we know to be true, occurred in it. but only something which we hope is true, and which for practical purposes we must suppose true; i.e., it is a postulate—the first of those underlying the present theory of causation. There is, however, no doubt that when, as in the laboratory, we have a high degree of control over the environment, and good opportunity to observe what occurs in it at a given moment, we do make the assumption just stated.

2. The second of the difficulties which we have to examine is of a logical rather than of a practical nature. It arises from the fact that in the face of the definition of cause given, we can not without a contradiction refuse to take into the cause any part of the total change observed in the contiguous space-time environment of the effect: while, on the contrary, we very frequently in fact seem so to use the word "cause" as to do just that. Thus, at the instant a brick strikes a window pane, the pane is struck, perhaps, by the air waves due to the song of a canary near by. Yet we usually would say that the cause of the breakage was the impact of the brick, and that the impact of the air waves, although it was part of the prior total change in the contiguous space-time, was no part of the cause. This being the way in which the word "cause" actually is used, how, then, can a definition which forbids us to call the cause anything less than the whole of the prior change in the contiguous space-time be regarded as a correct analysis of the meaning which the term "cause" actually possesses?

The contradiction, however, is only apparent, and depends upon a confusion between two different questions, due in turn to a certain ambiguity in the expression "the cause of an event." The first of the two questions is, what did cause, i.e., what did then and there suffice to, the occurrence of that concrete individual event? The second question, on the other hand, is really a double question, for it assumes the answer to the first as already possessed, and goes on again, in the endeavor of psychical researchers to check up the alleged identity of the "controls" of their mediums.

to ask, which part of what did suffice would be left if we subtracted from what did suffice such portions of it as were unnecessary to such an effect? This is a perfectly significant question, for to say "sufficient to" is one thing; and to say "no more than sufficient to" is another thing: a hundred-pound rock may well have been that which sufficed to the crushing of a worm, but it can not be said to have been no more than what would have sufficed, since the tenth part of it would also have been enough. The second and double question, moreover, is usually that which we mean to ask when we inquire concerning the cause of an event; but, as will appear directly, it is not, like the first, really an inquiry after the cause of one individual concrete event strictly as such. It is, on the contrary, an inquiry concerning what is common to it and to the causes of certain other events of the same kind. This much of a generalization, indeed, is indissolubly involved in the mere assigning of a name to the cause and to the effect perceived; although it is not involved in the merely perceiving them. This is an extremely important point, which constitutes the very key to the whole matter. That this is so will become fully evident if the significance of the second of the two questions above is more explicitly analyzed.

If we inquire what exactly is required to define the meaning of that (double) question, we find that at least two hypothetical cases are needed. For to say that in a given case a certain change sufficed to the occurrence of a given event, means, as we have seen, that no other change than it did occur in the prior contiguous space-time; and to say that a certain portion of that change was unnecessary means that in a case where that portion of the change did not occur which case therefore can not be the very identical case, but only a case that is otherwise similar—an (other) event of the same sort as the effect considered nevertheless did result. But now the fact that at least two hypothetical cases are thus necessary to define the meaning of our second question above, implies that that question is wholly meaningless with regard to one single concrete event. It is a question not, like the first, concerning the cause of one single concrete event, but concerning what was, or would be, common to the causes of at least two such.

The apparent contradiction which we faced is therefore now disposed of, for if, by "the cause of an event," we really mean the cause of one individual concrete event, and not merely of some case of a sort of event, then we must include in our answer the whole of the antecedent change in the contiguous space-time. And if, on the other hand, our answer leaves out any part of that change (as it often does), then the only question to which it can be a correct an-

swer is one as to what was common to the individual causes of two or more individual events of a given sort. Thus, if we say that the impact of a brick was the cause of the breaking of the window, and that the song of the canary had no part in it, then the words "the breaking of the window" do not refer to an individual event considered in its full concreteness, but only to a case-of-a-kind, uniquely placed and dated indeed, but not qualitatively specified otherwise than by the characters that define its kind, viz., "breaking of window." And it is solely owing to this that we can truly say that the song of the canary had nothing to do with it, for that means, then, nothing to do with what occurred in so far as what occurred is viewed merely as a case of breakage of a window. As already explained, to say that the song of the canary was unnecessary is not to say that it was not part of what did then and there suffice: it is to say only that in another case, otherwise similar, where the song did not occur, an effect of the same sort, viz., breaking, nevertheless did occur.

The whole of our answer to the objection we have been discussing may, after all this detail, be summarized by saying that the expression "the cause of the breaking of this window" has two senses, one strict, and the other elliptical. In the strict sense, it means "the fully concrete individual event which caused all the concrete detail of this breaking of this window." In the elliptical (and indeed more practically interesting) sense, it means "that which the cause of this breaking of this window has in common with the individual causes of certain other individual events of the same sort."

VI. The Generalization of Observed Causal Facts.—It is, of course, to be acknowledged that, as the parenthesis in the last sentence suggests, we are interested in causes and effects primarily for practical purposes, and that for such purposes causal knowledge is of direct value only so far as it has been generalized. This means that the interest of strictly concrete individual facts of causation to us is chiefly the indirect one of constituting raw material for generalization. And this explains why we so naturally and so persistently confuse the question, what did cause one given concrete event, with the very different question, in what respects does that cause resemble the causes of certain other events of the same sort previously observed in similar environments. For it is from the answer to this second question that we learn, what in such environments is the most we must do to cause the occurrence of another event of the given sort. And evidently just that is the very practically valuable information that we desire ultimately to obtain. But although it is true that, as practical beings, we are not directly interested in concrete individual facts of causation, it is not true that there are no such facts; nor, as we have seen, is it true that generality or recurrence is any part of the meaning of cause.

To round out the outline of the theory of the causal relation which this paper sets forth, there remains only to state the two postulates which condition, respectively, the validity of the descriptions by names which we formulate to fit sets of individual causal facts, and the validity of the applications we make of such generalizing descriptions to new cases.

The postulate which conditions the correctness of any answer we venture to give to the problem of description, viz., the problem in what respects the cause of a given concrete event resembles the causes of certain others of the same sort previously observed in similar environments, 18 is that the respects of resemblance which we include in our answer (through the name by which we describe the cause) are really the only ones that there were. This postulate, which may be called that of the descriptibility of our causal observations, is then the second postulate of our theory. The first, which it will be recalled was that no change that was not observed occurred in the prior contiguous space-time environment, may be called that of the observability of causal facts. And the third postulate, which we may term that of the applicability of our descriptions of our observations of causal facts to new cases, is that the new case (or cases) differs from those on the basis of which the description was formulated not otherwise nor more widely than they differed among themselves.

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## THE REFORM OF THE LEIBNIZIAN MONADOLOGY 1

## INTRODUCTION

EIBNIZ'S Monadology is a metaphysical treatise intended as a logical deduction of the nature of the reality which is the basis of physics. The treatise itself was practically unknown to Leibniz's contemporaries. It was written by him in 1714, two years before his death, in response to a request from one of his princely patrons, Prince Eugene of Savoy, and it was published in 1720, four years after his death. No classical work of the modern period approaches it in the fascination it has exercised on philosophers and in its in-

<sup>&</sup>lt;sup>18</sup> Mill correctly states that "It is inherent in a description to be the statement of a resemblance, or resemblances," Logic, p. 452.

<sup>&</sup>lt;sup>1</sup> Read before the meeting of the Pacific Division of the American Philosophical Association, Stanford University, November 27, 1925.