The Probability of the Evidence

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Received View: P(e) should be low

- P(e) should not be 1 because then P(h/e) = P(h), so e not prob. relevant to h, therefore can't be evidence for h.

- It's good if P(e) is low because that corresponds to surprisingness of the evidence, and surprising evidence is more confirming.

- Conformably, P(e) is in denominator of Bayes equation. So, low P(e) makes P(h/e) high, right? (Actually, wrong.)

To Show: It's a good thing if P(e) is high

- Mathematical argument: Lower bounds on P(e) combined with favorable likelihood ratio yield lower bounds on P(h/e).

- Intuitive argument:

  P(e) needs to be high to justify
  Bayesian conditionalization.

  High P(e) and high likelihood ratio
  correspond to eliminative reasoning

Mathematical argument

- Call P(e/h)/P(e/-h) "LR" for likelihood ratio.
- Assume it's good if LR > 1.

  (Note LR>1 implies positive relevance.)
- Assume the higher the LR, the better.
- Note:

$$P(h/e) = (LR - P(e/h)/P(e))/(LR - 1),$$

by a rearrangement of the Bayes equation.

Mathematical Argument

- Note from this equation,

$$P(h/e) = (LR - P(e/h)/P(e))/(LR - 1),$$

  that values for P(e/h), P(e/-h) and P(e) are sufficient to determine P(h/e).

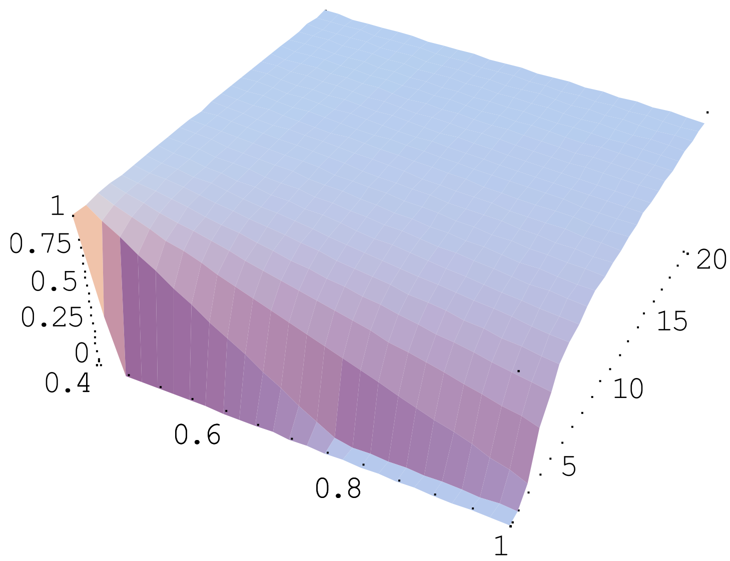
- y-axis → LR

  x-axis → P(e/h)

  z-axis → P(h/e)

We inspect graph for increasing constant values of P(e).

- All graphed values of LR are >1.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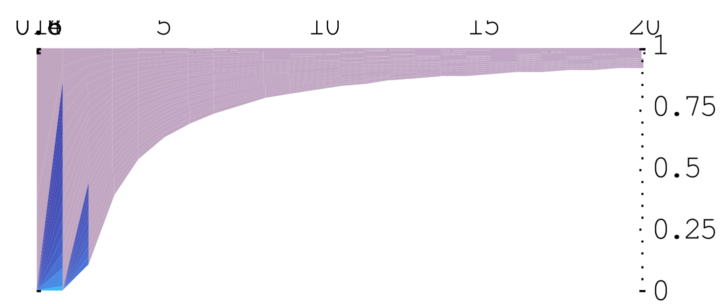
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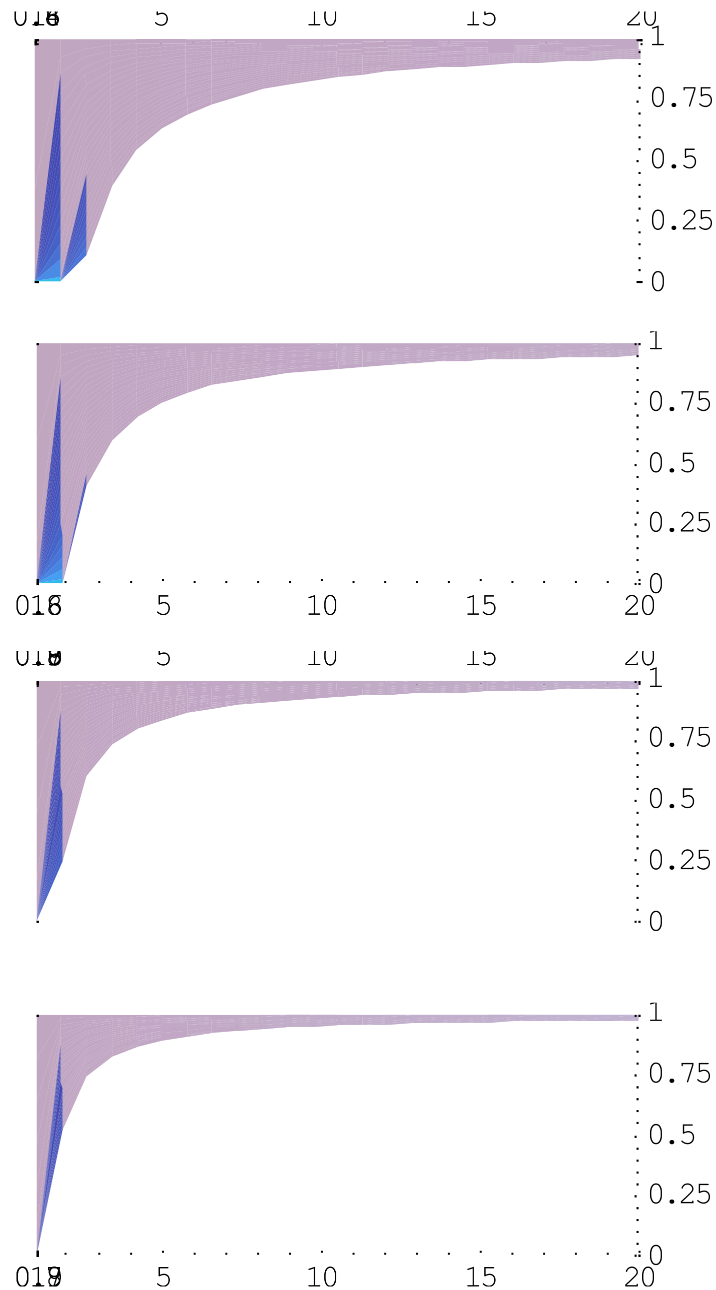

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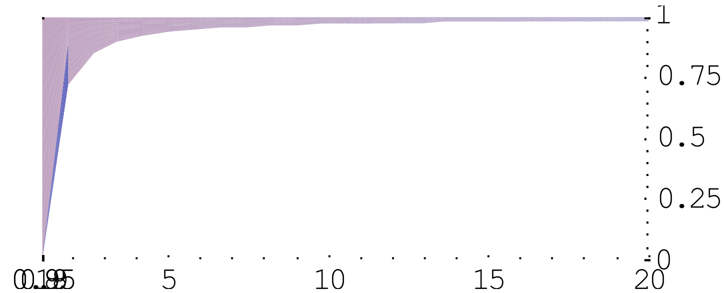

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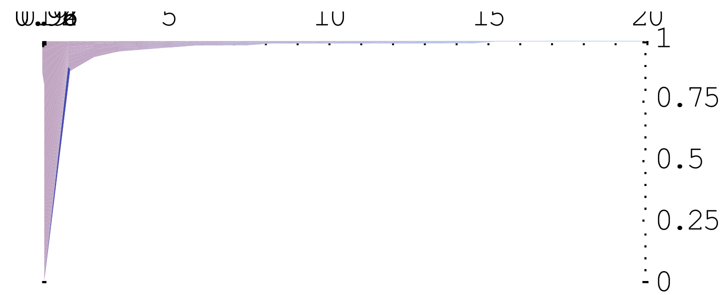

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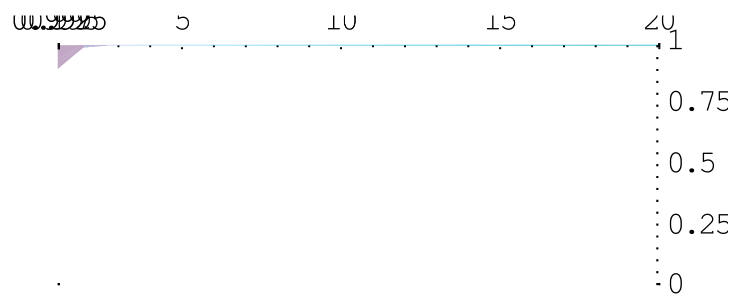

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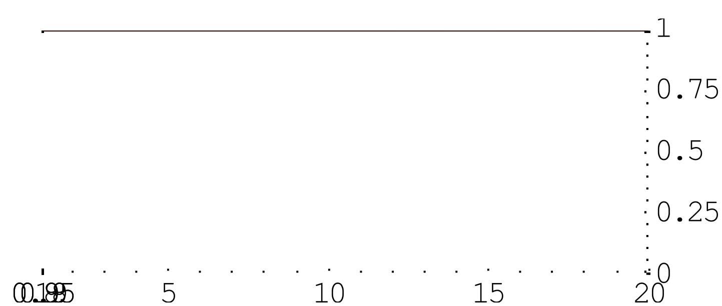

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Mathematical Argument

- We graphed P(e/h), P(e), and the LR, but have a result independent of P(e/h), depending only on P(e) and the LR:

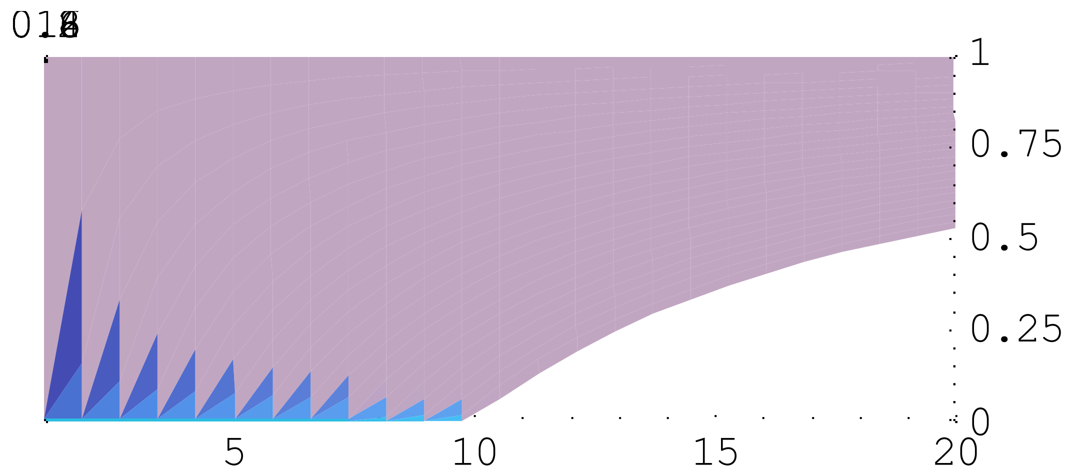
Fixed P(e) with increasing LR yields increased minimum values for P(h/e).

Fixed LR with increasing P(e) yields increased minimum values for P(h/e).

→ A lower bound on the LR with a lower bound on P(e) yields a lower bound on P(h/e).

Claims not true if we replace LR with P(e/h)/P(e).

Do get lower bound on P(h/e) with lower bounds on P(h) and P(e/h)/P(e).



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Mathematical Argument

Evaluations of $P(e)$ and $P(e/h)$, $P(e/-h)$ are sufficient to determine $P(h/e)$:

$$P(e) = P(e/h)\,\mathbf{P(h)} + P(e/-h)\,\mathbf{P(-h)}$$

$$1 = \mathbf{P(h)} + \mathbf{P(-h)}$$

Mathematical Argument

It's good to have a lower bound on $P(h/e)$:

good reason to believe h true.

Neither positive relevance nor high LR alone give this.

Some thresholds of interest:

$P(e) > .5$, $LR > 3 \rightarrow P(h/e) > .5$

$P(e) > .75$, $LR > 3 \rightarrow P(h/e) > .82$

$P(e) > .75$, $LR > 7 \rightarrow P(h/e) > .95$

Intuitive Argument

Part I: Conditionalization

Part II: Eliminative Reasoning

Intuitive Argument, Part I

- $P'(h) = P(h/e) = P(e/h)P(h)/P(e)$
- $P(e)$ is your degree of belief in e before you conditionalize, the degree of belief in e that *justifies* conditionalization on e.
- $P(e)$ is also standardly assumed to be degree of belief in e before observing e, but that can’t be right—conditionalization wouldn’t be justified.

Intuitive Argument, Part I

Howson and Urbach (1993, 99):

> When your degree of belief in e goes to 1, but no stronger proposition also acquires probability 1, set P'(a) = P(a/e) for all a in the domain of P, where P is your probability function immediately prior to the change.

Intuitive Argument, Part II
Eliminative Reasoning

High P(e) ……………….. It occurred that e.

High P(e/h)/P(e/-h) …. It is more likely that h is responsible for e than that –h is responsible for e.

Surprising Evidence

Standard argument:

$$P(h/e_1)/P(h/e_2) = (P(h)P(e_1/h))/P(e_1) \text{ x } P(e_2)/(P(h)P(e_2/h))$$

$$= P(e_2)/P(e_1)$$

Surprising Evidence

Standard argument:

$$P(h/e_1)/P(h/e_2) = (P(h)P(e_1/h))/P(e_1) \text{ x } P(e_2)/(P(h)P(e_2/h))$$

$$= P(e_2)/P(e_1)$$

But…

$$= (P(e_2/h)P(h) + P(e_2/-h)P(-h))/(P(e_1/h)P(h) + P(e_1/-h)P(-h))$$