Announcements & Overview

- Administrative Stuff
  - ☞ **Everything has been pushed back by one week** (due to mourning).
    - ∗ HW #1 grades posted (with detailed solutions). See histogram.
    - ∗ **HW #2 due *next* Friday (2/19).**
      - · Consult *Homework Guidelines & Tips* handout re HW #2
  - – HWs are now worth 25% of your final grade, and Participation is worth 8% (*via* ungraded TurningPoint Cloud quizzes).
  - – Consult the latest revision of the Syllabus & Website for details.
- Today: Introduction to Unit #2 (**L**anguage of **S**entential **L**ogic)
  - – 5 sentential (*truth-functional*) connectives (logical constants of LSL)
  - – First Steps in Symbolization: English $\mapsto$ LSL
  - – Symbolizing English Sentences
  - – Then: Symbolizing English Arguments

Introduction to the Syntax of the LSL: The Lexicon

- The syntax of LSL is quite simple. Its lexicon has the following symbols:
  - Upper-case letters 'A', 'B', ... which stand for *basic sentences*.
  - Five *sentential connectives/operators* (one *unary*, four *binary*):

Operator	**Name**	**Logical Function**	**Used to symbolize**
'$\sim$'	tilde	negation	not, it is not the case that
'&'	ampersand	conjunction	and, also, moreover, but
'$\vee$'	vee	disjunction	or, either ... or ...
'$\rightarrow$'	arrow	conditional	if ... then ..., only if
'$\leftrightarrow$'	double arrow	biconditional	if and only if

  - Parentheses '(', ')', brackets '['. ']', and braces '{', '}' for grouping.
- If a string of symbols contains anything else, then it's not a sentence of LSL. And, only *certain* strings of these symbols are LSL sentences.
- Some LSL symbol strings aren't *well-formed*: '$(A \,\&\, B$', '$A \,\&\, B \vee C$', *etc*.

The Five Kinds (Forms) of *Non-Basic* LSL Sentences

- Sentences of the form $\ulcorner p \,\&\, q \urcorner$ are called *conjunctions*, and their constituents (p, q) are called *conjuncts*.
- Sentences of the form $\ulcorner p \vee q \urcorner$ are called *disjunctions*, and their constituents (p, q) are called *disjuncts*.
- Sentences of the form $\ulcorner p \to q \urcorner$ are called *conditionals*. p is called the *antecedent* of $\ulcorner p \to q \urcorner$, and q is called its *consequent*.
- Sentences of the form $\ulcorner p \leftrightarrow q \urcorner$ are called *biconditionals*. p is called the *left-hand side* of $\ulcorner p \leftrightarrow q \urcorner$, and q is its *right-hand side*.
- Sentences of the form $\ulcorner {\sim} p \urcorner$ are called *negations*. The sentence p is called the *negated sentence*.
- These 5 kinds of sentences (+ *atoms*) are the *only* kinds in LSL.
- Next, we begin to think about "translation" from English into LSL.

English $\mapsto$ LSL I: Basic Steps Toward Symbolization

- Sentences with *no* connectives are *trivial* to symbolize:
  - 'It is cold.' $\mapsto$ 'C'.
  - 'It is rainy.' $\mapsto$ 'R'.
  - 'It is sunny.' $\mapsto$ 'S'.
- Sentences with just one sentential connective are also pretty easy:
  - 'It is cold and rainy.' $\mapsto$ '$C \,\&\, R$'. [why two atomic letters?]

☞ Try to give the most *precise* (fine-grained) LSL rendition you can, and try to come as close as possible to capturing the meaning of the original.

- Sentences with two connectives can be trickier:
  - 'Either it is sunny or it is cold and rainy.' $\mapsto$ '$S \vee (C \,\&\, R)$'.
- Q: Why is '$(S \vee C) \,\&\, R$' incorrect? A: The English is *not* a conjunction.

English ↦ LSL II: Symbolizing in Two Stages

☞ When symbolizing English sentences in LSL (especially complex ones), it is useful to perform the symbolization in (at least) *two stages*.

Stage 1: Replace all basic sentences (explicit or implicit) with atomic letters. This yields a sentence in "Logish" (neither English nor LSL).

Stage 2: Eliminate remaining English by replacing English connectives with LSL connectives, and properly grouping the resulting symbolic expression (w/parens, *etc.*) to yield pure LSL.

- Here are some simple examples involving only single connectives:

English:	**"Logish":**	**LSL:**
Either it's raining or it's snowing.	Either R or S.	$R \vee S$
If Dell introduces a new line, then Apple will also.	If D, then A.	$D \to A$
Snow is white and the sky is blue.	W and B.	$W \,\&\, B$
It is not the case that Emily Bronte wrote *Jane Eyre*.	It is not the case that E.	$\sim E$
John is a bachelor if and only if he is unmarried.	J if and only if not M.	$J \leftrightarrow \sim M$

English $\mapsto$ LSL III: Symbolizations involving '&' and '$\vee$'

- We use '&' to symbolize a variety of English connectives, including:
  - 'and', 'yet', 'but', 'however', 'moreover', 'nevertheless', 'still', 'also', 'although', 'both', 'additionally', 'furthermore' (and others)
- There is often more to the meaning of 'but', 'nevertheless', 'still', 'although', 'however' (and other such English connectives) than merely 'and'. But, in LSL, the closest we can get to these connectives is '&'.
- On the other hand, there are fewer English expressions that we will symbolize using '$\vee$'. Typically, these involve either 'or' or 'either … or'.
- But, less typically and more controversially, there is one other English connective we will symbolize as '$\vee$', and that is 'unless'. Seem strange?
- Intuitively, $\ulcorner p$ unless $q\urcorner$ means something like $\ulcorner$if not q, then $p\urcorner$. But, in LSL, $\ulcorner{\sim}q \to p\urcorner$ is *equivalent* to (*means the same as*) $\ulcorner p \vee q\urcorner$. [Ch. 3.]

English $\mapsto$ LSL IV: Symbolizations involving '$\rightarrow$' (and '$\leftrightarrow$')

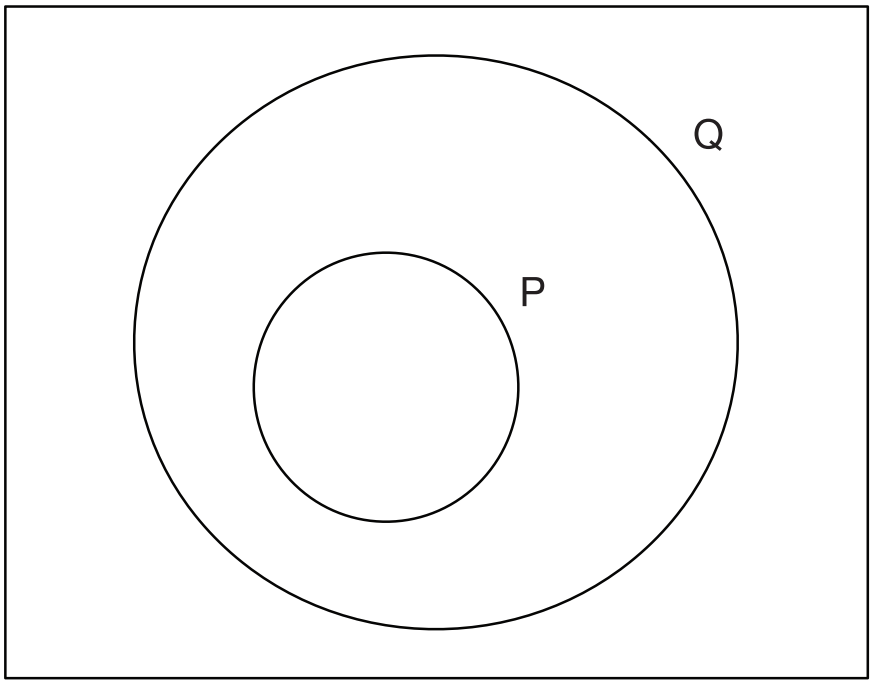
☞ We will use '$\rightarrow$' to symbolize *many* different English expressions. These will be the most controversial and tricky of our LSL symbolizations. *E.g.*:

- $\ulcorner$if p then $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ implies $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ only if $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner q$ if $p\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ is a sufficient condition for $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner q$ is a necessary condition for $p\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner q$ provided $p\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner q$ whenever $p\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ is contingent upon $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$

- $\ulcorner p \leftrightarrow q\urcorner$ is equivalent to $\ulcorner (p \rightarrow q) \,\&\, (q \rightarrow p)\urcorner$ (so mastering '$\rightarrow$' is key)

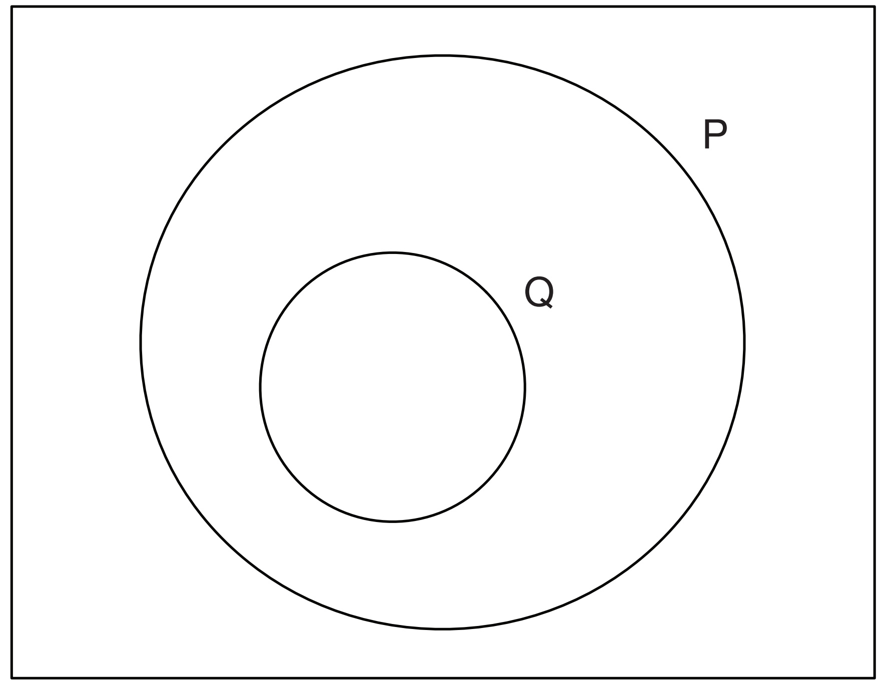
Picturing *If* vs. *Only if*

All possible worlds.



If P, then Q.
Q if P.
P only if Q.
P is sufficient for Q.
Q is necessary for P.

All possible worlds.



If Q, then P.
P if Q.
Q only if P.
Q is sufficient for P.
P is necessary for Q.

English $\mapsto$ LSL V: Grouping Two or More Binary Connectives

- Whenever three or more LSL sentence letters appear in an LSL sentence, parentheses (or brackets or braces) must be used (carefully!) to indicate the intended *scope* of the connectives. Otherwise, problems ensue ...
- *E.g.*, '$A \,\&\, B \vee C$' is *not* an LSL sentence. It is *ambiguous* between '$(A \,\&\, B) \vee C$' and '$A \,\&\, (B \vee C)$', which are *distinct* LSL sentences.
- The term "*well-formed formula of LSL*" ("LSL WFF") is synonymous with "*LSL sentence*." Non-well-formed strings of symbols aren't sentences.
- In English, the string of English words 'Porch on the is cat a there' is ungrammatical — it is *not well-formed*. All of its *constituent parts* are English words/letters, but (*as a whole*) it's not an English sentence.
- Similarly, in LSL, the following strings of symbols are not WFFs:

'$A \to \vee B$' '$A \,\&\, B \vee C$' '$A \to B \to C$' '$\sim \vee B(\vee C)$' '$A \,\&\, B \,\&\, C$'

English $\mapsto$ LSL VI: Negation, Conjunction, and Disjunction

- The tilde '$\sim$' operates *only* on the unit that *immediately* follows it. In '$\sim K \vee M$,' $\sim$ affects only 'K'; in '$\sim(K \vee M)$,' $\sim$ affects the entire '$K \vee M$'.
- 'It is not the case that K or M' is *ambiguous* between '$\sim K \vee M$,' and '$\sim(K \vee M)$.' **Convention**: 'It is not the case that K or M' $\mapsto$ '$\sim K \vee M$'.
- 'Not both S and T' $\mapsto$ '$\sim(S \,\&\, T)$'. [Chapter 3: '$\sim(S \,\&\, T)$' *means the same as* '$\sim S \vee \sim T$'. But, '$\sim(S \,\&\, T)$' does *not* mean the same as '$\sim S \,\&\, \sim T$'.]
- 'Not either S or T' $\mapsto$ '$\sim(S \vee T)$'. [Chapter 3: '$\sim(S \vee T)$' *means the same as* '$\sim S \,\&\, \sim T$', but '$\sim(S \vee T)$' does *not* mean the same as '$\sim S \vee \sim T$'.]
- Here are some examples involving $\sim$, &, and $\vee$ (not, and, or):
  1. Shell is not a polluter, but Exxon is. $\mapsto$??
  2. Not both Shell and Exxon are polluters. $\mapsto$??
  3. Both Shell and Exxon are not polluters. $\mapsto$??

4. Not either Shell or Exxon is a polluter. $\mapsto$??

5. Neither Shell nor Exxon is a polluter. $\mapsto$??

6. Either Shell or Exxon is not a polluter. $\mapsto$??

- Summary of translations involving $\sim$, &, and $\vee$ (not, and, or):

"Logish"	**LSL**
Not either A or B.	$\sim(A \vee B)$
Either not A or not B	$\sim A \vee \sim B$
Not both A and B.	$\sim(A \,\&\, B)$
Both not A and not B. (Neither A nor B.)	$\sim A \,\&\, \sim B$

- DeMorgan Laws (we will *prove* these laws is Chapters 3 & 4):

  $\ulcorner \sim(p \vee q) \urcorner$ is equivalent to (means the same as) $\ulcorner \sim p \,\&\, \sim q \urcorner$

  $\ulcorner \sim(p \,\&\, q) \urcorner$ is equivalent to (means the same as) $\ulcorner \sim p \vee \sim q \urcorner$

- But, $\ulcorner \sim(p \vee q) \urcorner$ is *not* equivalent to $\ulcorner \sim p \vee \sim q \urcorner$.

- And, $\ulcorner \sim(p \,\&\, q) \urcorner$ is *not* equivalent to $\ulcorner \sim p \,\&\, \sim q \urcorner$.

English $\mapsto$ LSL VII: Summary of the LSL Connectives

English Expression	LSL Connective
not, it is not the case that, it is false that	$\sim$
and, yet, but, however, moreover, nevertheless, still, also, although, both, additionally, furthermore	&
or, unless, either ... or ...	$\vee$
if ... then ..., only if, given that, in case, provided that, on condition that, sufficient condition, necessary condition, unless (**Note**: don't confuse antecedents/consequents!)	$\rightarrow$
if and only if (iff), is equivalent to, sufficient and necessary condition for, necessary and sufficient condition for	$\leftrightarrow$

English $\mapsto$ LSL X (&, $\rightarrow$): Example #1

- 'John will study hard and also bribe the instructor, and if he does both then he'll get an "A", provided the instructor likes him.'
  - Step 0: Decide on atomic sentences and letters.

    S: John will study hard. A: John will get an "A".

    B: John will bribe the instructor. L: The instructor likes John.
  - Step 1: Substitute into English, yielding "Logish":

    S and B, and if S and B then A, provided L.
  - Step 2: Make the transition into LSL (in stages as well, perhaps):

    S and B, and if L, then if S and B then A.

    $(S \,\&\, B) \,\&\, (L \rightarrow$ (if S and B then A)).

    Final Product: $(S \,\&\, B) \,\&\, (L \rightarrow ((S \,\&\, B) \rightarrow A))$

English ↦ LSL II ($\sim$, &, $\vee$, $\rightarrow$, $\leftrightarrow$): Example #2

- 'Sara is going unless either Richard or Pam is going, and Sara is not going if, and only if, neither Pam nor Quincy are going.'
  - Step 0: Decide on atomic sentences and letters.

    P: Pam is going. Q: Quincy is going.

    R: Richard is going. S: Sara is going.
  - Step 1: Substitute into English, yielding "Logish":

    S unless either R or P, and not S iff neither P nor Q.
  - Step 2: Make the transition into LSL (in stages again):

    S unless $(R \vee P)$, and $\sim S$ iff $(\sim P \,\&\, \sim Q)$

    $(\sim(R \vee P) \rightarrow S) \,\&\, (\sim S \leftrightarrow (\sim P \,\&\, \sim Q))$
- It is also acceptable to replace the 'unless' with '$\vee$', yielding:

  $(S \vee (R \vee P)) \,\&\, (\sim S \leftrightarrow (\sim P \,\&\, \sim Q))$

English $\mapsto$ LSL II ($\sim$, &, $\vee$, $\rightarrow$, $\leftrightarrow$): Example #3

- 'If you do not concentrate well unless you are alert, then provided that you are not a maniac, you will fly an airplane only if you are sober.'
  - Step 0: Decide on atomic sentences and letters.

    C: You concentrate well. M: You are a maniac.

    A: You are alert. F: You will fly an airplane.

    S: You are sober.
  - Step 1: Substitute into English, yielding "Logish":

    If not C unless A, then provided that not M, F only if S.
  - Step 2: Make the transition into LSL (in stages again):

    If $\sim C$ unless A, then if $\sim M$, then F only if S.

    Final Product: $(\sim A \rightarrow \sim C) \rightarrow (\sim M \rightarrow (F \rightarrow S))$.
- It is also acceptable to replace the 'unless' with '$\vee$', yielding:

  Alternative Final Product: $(\sim C \vee A) \rightarrow (\sim M \rightarrow (F \rightarrow S))$

English $\mapsto$ LSL II ($\sim$, &, $\leftrightarrow$): Example #4

- 'If, but only if, they have made no commitment to the contrary, may reporters reveal their sources, but they always make such a commitment and they ought to respect it.'
  - Step 0: Decide on atomic sentences and letters.

  S: Reporters may reveal their sources.

  C: Reporters have made a commitment to protect their sources.

  R: Reporters ought to respect their commitment to protect sources.

  - Step 1: Substitute into English, yielding "Logish":

    If, but only if, it is not the case that C, then S, but C and R.

  - Step 2: make the transition into LSL (in stages as well, perhaps):

    S iff not C, but C and R.

    Final Product: $(S \leftrightarrow \sim C)\ \&\ (C\ \&\ R)$

Symbolizing/*Reconstructing* Entire English Arguments

- Naïvely, an argument is "just a collection of sentences". So, naïvely, one might think that symbolizing arguments should just boil down to symbolizing a bunch of individual sentences. It's not so simple.
- An argumentative passage has more structure than an individual sentence. This makes argument *reconstruction* more subtle.
- We must now make sure we capture the inter-relations of content across the various sentences of the argument.
- To a large extent, these interrelations are captured by a judicious choice of atomic sentences for the reconstruction.
- It is also crucial to keep in mind the overall intent of the argumentative passage — the intended argumentative strategy.
- Forbes glosses over the art of (charitable!) argument reconstruction. I will be a bit more explicit about this today in some examples.

Symbolizing Entire Arguments: An Example

- 'If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient. But, if God exists then He is none of these, and there is evil in the world. So, we must conclude that God does not exist.'
- Step 0: Decide on atomic sentences and letters.

  G: God exists. E: There is evil in the world.

  J: God is just. O: God is omnipotent.

  K: God is omniscient.
- Step 1: Identify (and symbolize) the *conclusion* of the argument:
  - 'God does not exist.' (which is just '$\sim G$' in LSL)
- Step 2: Symbolize the premises (in this case, there are two):
  - Premise #1: 'If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient.'

Symbolizing Arguments: Example #2

- Premise #1: 'If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient.'

  If G, then ($\sim E$ unless ($\sim J$ or ($\sim O$ or $\sim K$)))

  $G \rightarrow (\sim E \vee (\sim J \vee (\sim O \vee \sim K)))$

- Premise #2: 'If God exists then He is none of these (*i.e.*, He is *neither* unjust *nor*...), and there is evil in the world.'

  If G, then not not-J and not not-O and not not-K, and E.

  $[G \rightarrow (\sim\sim J \;\&\; (\sim\sim O \;\&\; \sim\sim K))] \;\&\; E$

- This yields the following (valid!) sentential form:

  $G \rightarrow (\sim E \vee (\sim J \vee (\sim O \vee \sim K)))$

  $[G \rightarrow (\sim\sim J \;\&\; (\sim\sim O \;\&\; \sim\sim K))] \;\&\; E$

  $\therefore \sim G$

Symbolizing Arguments: Example #2 Notes

- The sentential form:

  $G \to (\sim E \vee (\sim J \vee (\sim O \vee \sim K)))$

  $[G \to (\sim\sim J \,\&\, (\sim\sim O \,\&\, \sim\sim K))]$

  E

  $\therefore \sim G$

  with *three* premises is *equivalent* to the *two*-premise sentential form we wrote down originally (why?).

- Alternative for premise #1: '$G \to \{\sim[\sim J \vee (\sim O \vee \sim K)] \to \sim E\}$'.

- Moreover, if we had written '$(\sim\sim K \,\&\, (\sim\sim J \,\&\, \sim\sim O))$' rather than '$(\sim\sim J \,\&\, (\sim\sim O \,\&\, \sim\sim K))$' in premise #2, we would have ended-up with yet another *equivalent* sentential form (why?).

- All of these forms capture the meaning of the premises and conclusion, and all are close to the given form. So, all are OK.

Symbolizing Arguments: Example #2 More Notes

- Premise #1: If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient.
- Two Questions: ① Why render this as (*i*) $\ulcorner p \to (q \text{ unless } r)\urcorner$, as opposed to (*ii*) $\ulcorner(p \to q) \text{ unless } r\urcorner$? ② *Does it matter* (*semantically*)?

① First, there's no comma after 'world'. Second, (*i*) is probably *intended*. The second answer assumes (*i*) and (*ii*) are *not* equivalent *in English*.

- That *may* be right, but it's not clear. It presupposes two things:
  (1) *In English*, $\ulcorner q \text{ unless } r\urcorner$ is equivalent to ⌜If not r, then q⌝.
  (2) *In English*, ⌜If p, then (if q then r)⌝ [*i.e.*, $\ulcorner p \to (q \to r)\urcorner$] is *not* equivalent to ⌜If (p and q), then r⌝ [*i.e.*, $\ulcorner(p \,\&\, q) \to r\urcorner$].
- We're *assuming* (1) in this class. (2) is controversial (but defensible).

② In LSL, (*i*) and (*ii*) *are equivalent*, *i.e.*, *in LSL* (2) is *false*. Thus, it seems to me that both readings are probably OK. This is a subtle case.

Symbolizing Arguments: Example #3

If Yossarian flies his missions then he is putting himself in danger, and it is irrational to put oneself in danger. If Yossarian is rational he will ask to be grounded, and he will be grounded only if he asks. But only irrational people are grounded, and a request to be grounded is proof of rationality. Consequently, Yossarian will fly his missions whether he is rational or irrational.

- Basic Sentences: Yossarian flies his missions (F), Yossarian puts himself in danger (D), Yossarian is rational (R), Yosarian asks to be grounded (A).
- Premise #1: If F then D, and if D then not R. [$(F \to D) \,\&\, (D \to \sim R)$]
- Premise #2: If R then A, and not F only if A. [$(R \to A) \,\&\, (\sim F \to A)$]
- Premise #3: But not F only if not R, and A implies R. [$(\sim F \to \sim R) \,\&\, (A \to R)$]
- Conclusion: Consequently, F whether R or not R. [$(R \to F) \,\&\, (\sim R \to F)$]. [Alternatively, the conclusion could be symbolized as: '$(R \vee \sim R) \to F$']
- Note: this is a valid form (we'll be able to prove this pretty soon).

Symbolizing Arguments: Example #4

Suppose no two contestants enter; then there will be no contest. No contest means no winner. Suppose all contestants perform equally well. Still no winner. There won't be a winner unless there's a loser. And conversely. Therefore, there will be a loser only if at least two contestants enter and not all contestants perform equally well.

- Step 0: Decide on atomic sentences and letters.

  T: At least two contestants enter. C: There is a contest.
  E: All contestants perform equally well.
  W: There is a winner. L: There is a loser.

- Step 1: Identify (and symbolize) the *conclusion* of the argument:
  - Conclusion: There will be a loser only if at least two contestants enter and not all contestants perform equally well.
    * "Logish": L only if T and not E.
    * LSL: $L \to (T \,\&\, {\sim}E)$.

– Step 2: Symbolize the premises (here, there are as many as five):

(1) Suppose no two contestants enter; then there will be no contest.

· "Logish": Suppose that not T; then it is not the case that C.

· LSL: '$\sim T \rightarrow \sim C$'.

(2) No contest means no winner.

· "Logish": Not C means not W. [*i.e.*, not C *implies* not W.]

· LSL: '$\sim C \rightarrow \sim W$'.

(3) Suppose all contestants perform equally well. Still no winner.

· "Logish": Suppose E. Still not W. [*i.e.*, E *also* implies not W.]

· LSL: '$E \rightarrow \sim W$'.

(4) There won't be a winner unless there's a loser. And conversely.

· "Logish": Not W unless L, *and conversely*.

· LSL: '$(\sim L \rightarrow \sim W)\,\&\,(\sim W \rightarrow \sim L)$'. [*i.e.*, not W *iff* not L.]

∗ The final product is the following *valid* sentential form:

$\sim T \rightarrow \sim C$. $\sim C \rightarrow \sim W$. $E \rightarrow \sim W$. $\sim L \leftrightarrow \sim W$. Therefore,

$L \rightarrow (T\,\&\sim E)$.