PROPOSITIONAL REASONING THAT TRACKS PROBABILISTIC REASONING*

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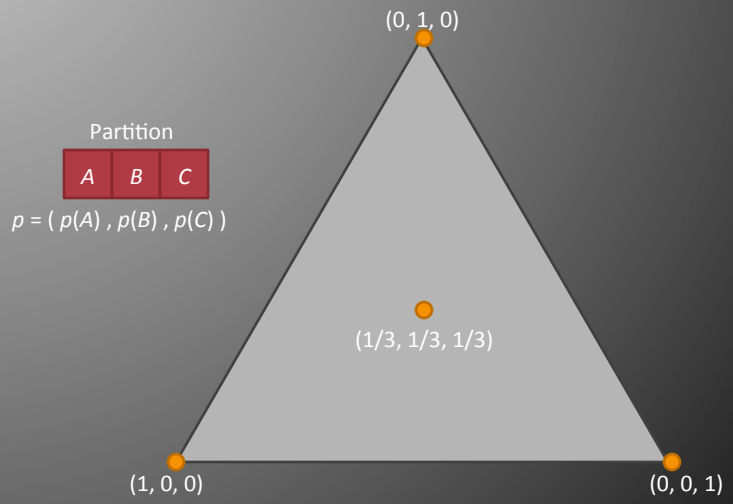
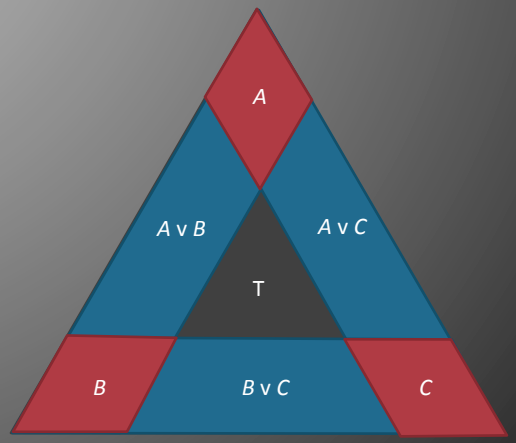
*This work originated from our project to develop a new justification for Ockham's razor, supported by NSF grant 0750681.

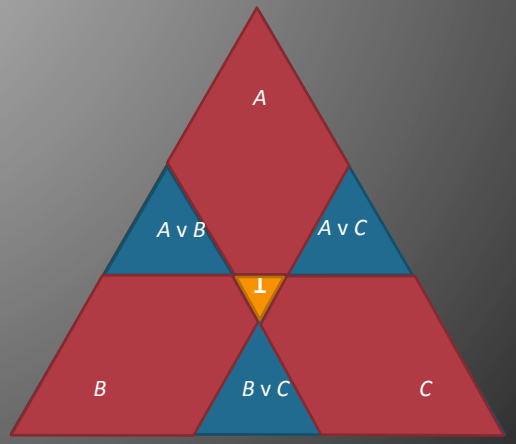
Space of Bayesian States



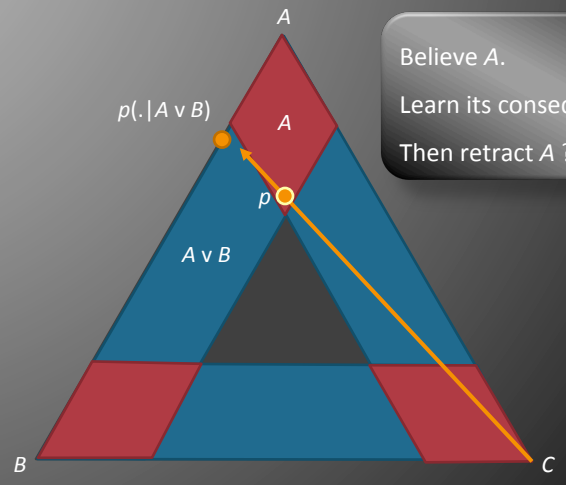
Probability Threshold Rule



The Lottery Paradox



New Riddle



Believe *A*.

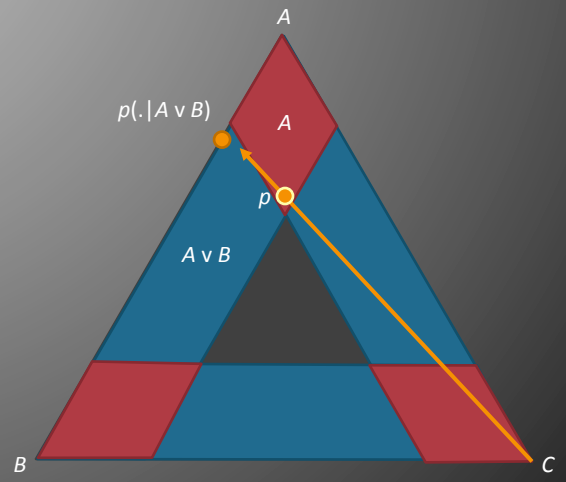
Learn its consequence *A* v *B*.

Then retract *A* ?!

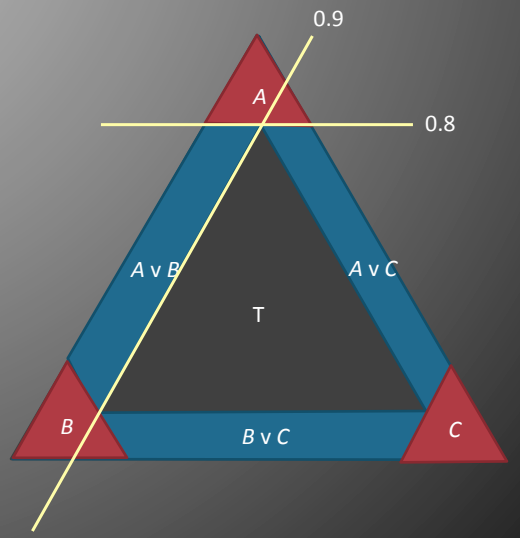
"Cautious" Monotonicity
= Hypothetico-Deductive Monotonicity

If you believe a hypothesis, don't *retract* it when you learn what it *entails* (i.e. *predicts*).

New Riddle



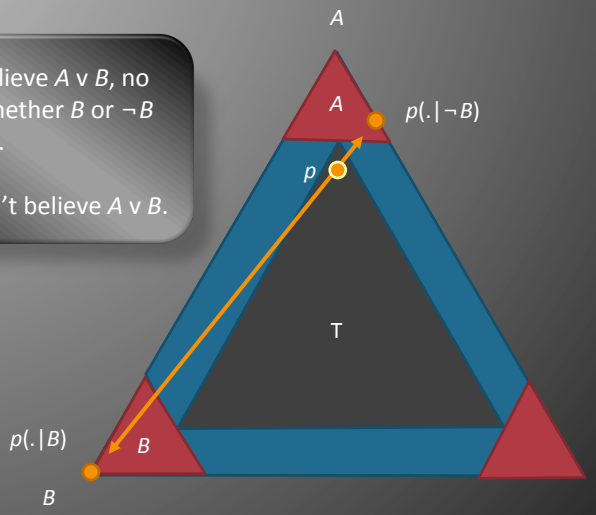
A Better Idea?



Another Riddle

Would believe $A \vee B$, no matter whether B or $\neg B$ is learned.

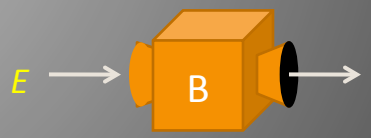
But doesn't believe $A \vee B$.



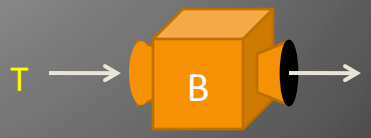
Case Reasoning

Believe a hypothesis, if you will believe it no matter whether E is learned or $\neg E$ is learned.

Belief Revision Rule

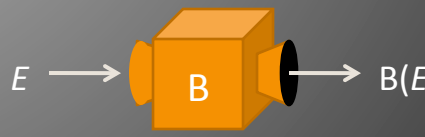


new propositional belief state in light of information E



initial belief state

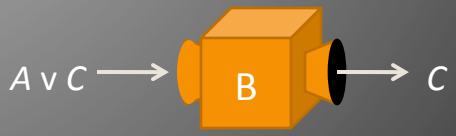
Another Notation

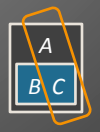


$B(T) * E = B(E)$

Example: AGM Revision

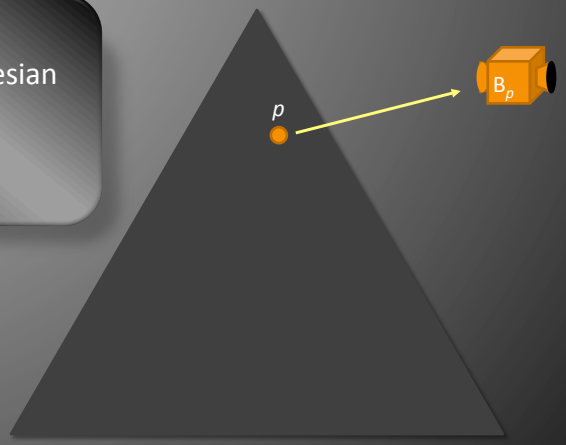
- Rank atoms.
- B(*E*) = the disjunction of all the minimal atoms compatible with *E*.





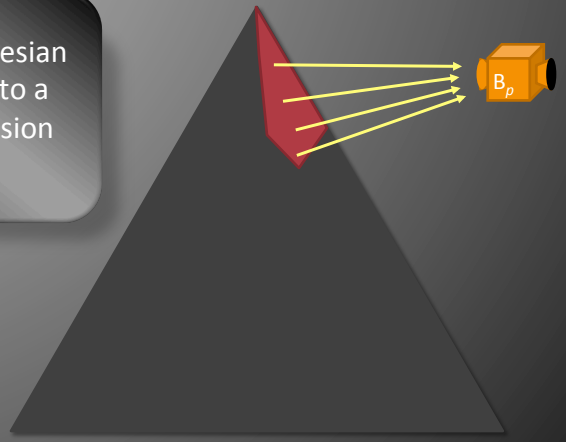
Belief Revision Policy

Maps each Bayesian state to a belief revision rule.



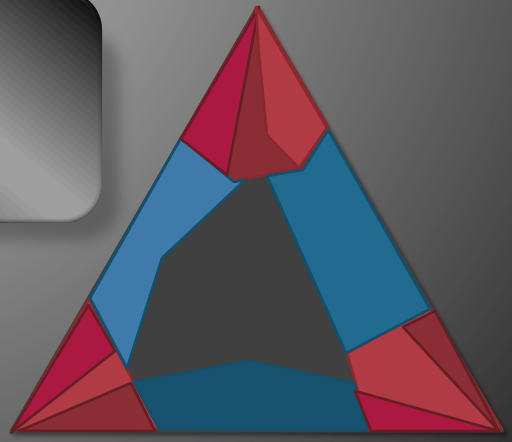
Rule Zone

Contains all Bayesian states assigned to a given belief revision rule.



Belief Revision Policy

Zones cover the simplex.

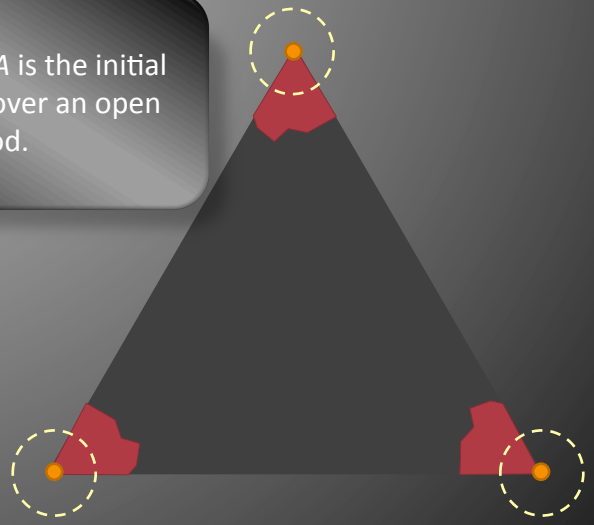


Consistent Policies

Inconsistency is believed at no Bayesian state

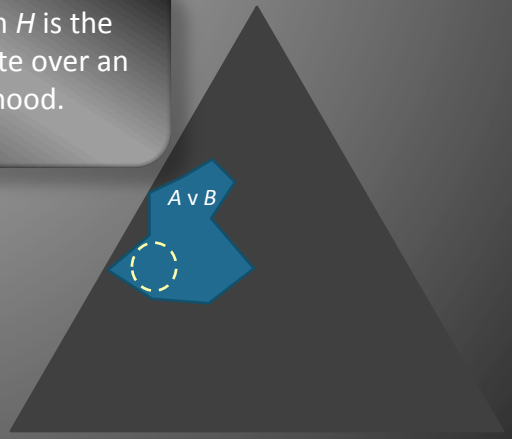
Non-Skeptical Policies

Every atom *A* is the initial belief state over an open neighborhood.



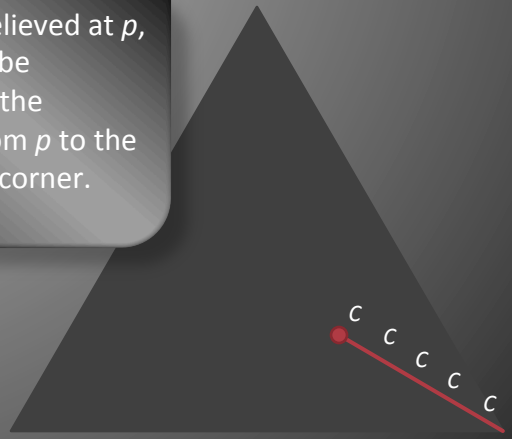
Non-Opinionated Policies

Some non-atom *H* is the initial belief state over an open neighborhood.



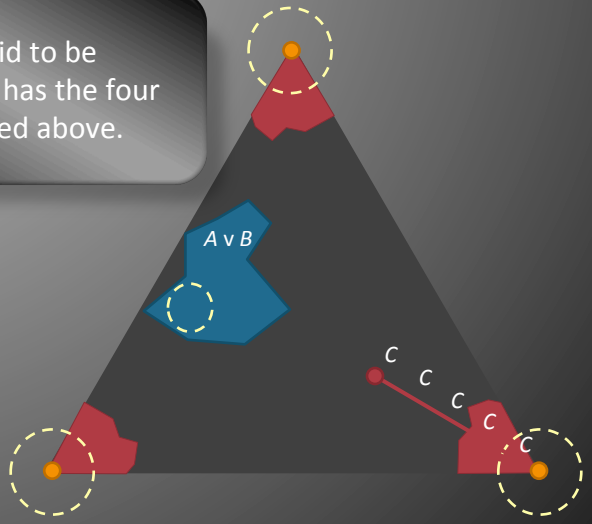
Corner-Monotone Policies

If an atom is believed at *p*, it continues to be believed along the straight line from *p* to the corresponding corner.

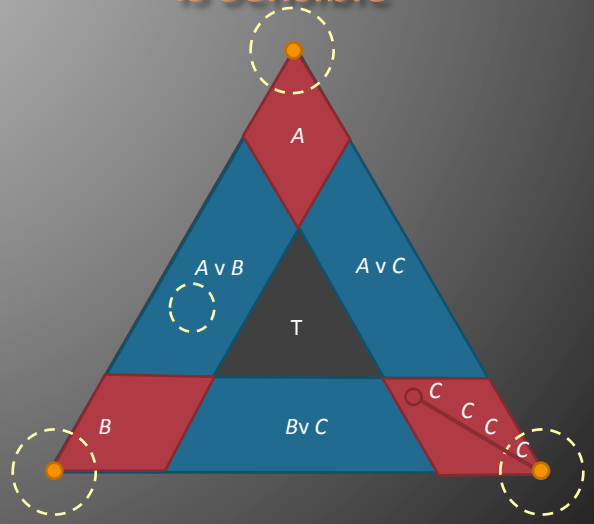


Sensible Policies

A policy is said to be *sensible* iff it has the four virtues defined above.

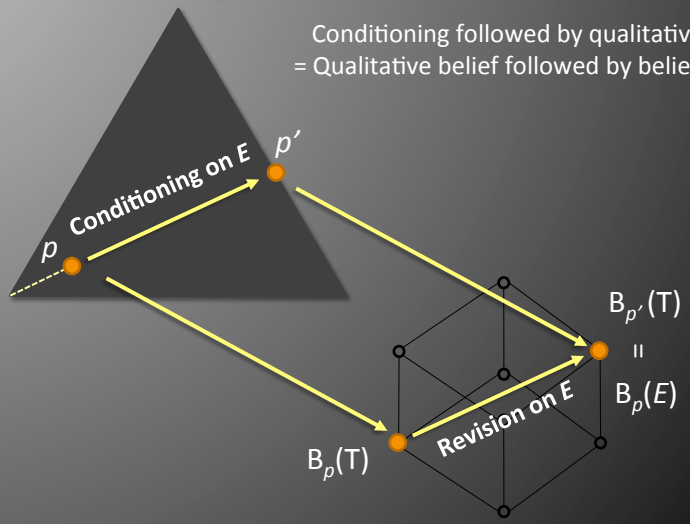


E.g., the Probability Threshold Policy is Sensible

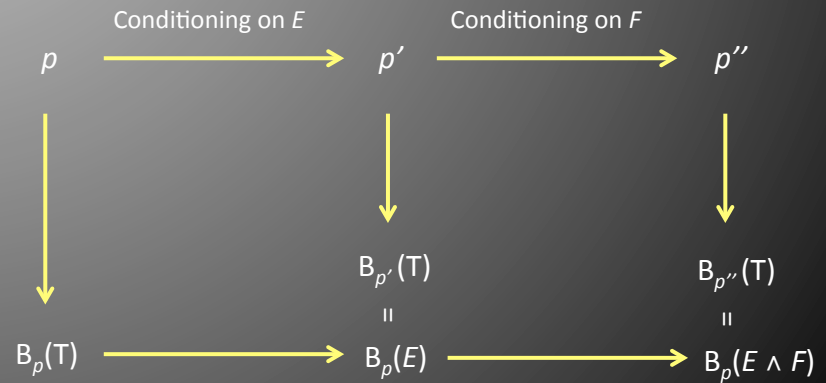


Revision Policy that *Tracks* Conditioning

Conditioning followed by qualitative belief
= Qualitative belief followed by belief revision.



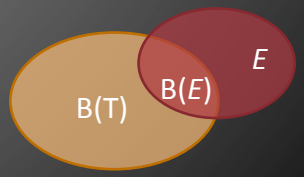
Pre-Established Harmony!



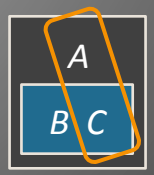
Accretive Belief Revision

If B(T) is logically compatible with *E*, then:

B(*E*) = B(T) ∧ *E* .



E.g., AGM is Accretive



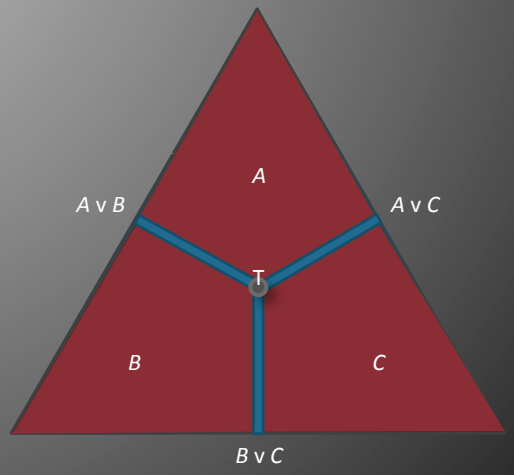
No-Go Theorem

No sensible belief revision policy that tracks conditioning is accretive .

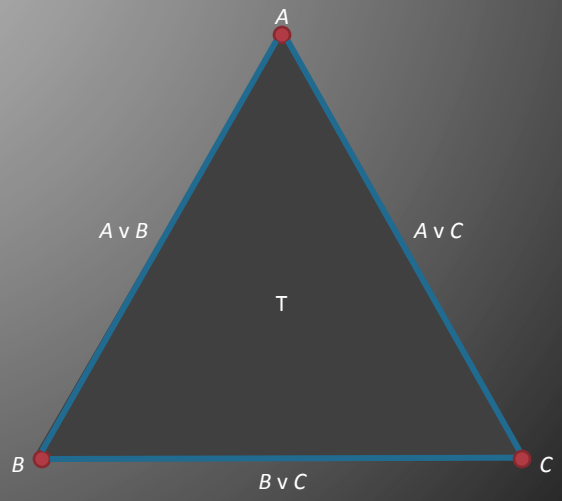
Corollary

No sensible belief revision policy that tracks conditioning is AGM.

Option: Opinionation

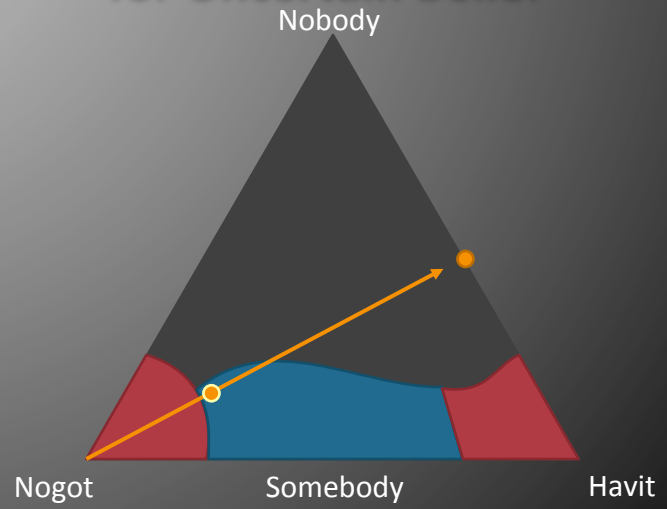


Option: Skepticism

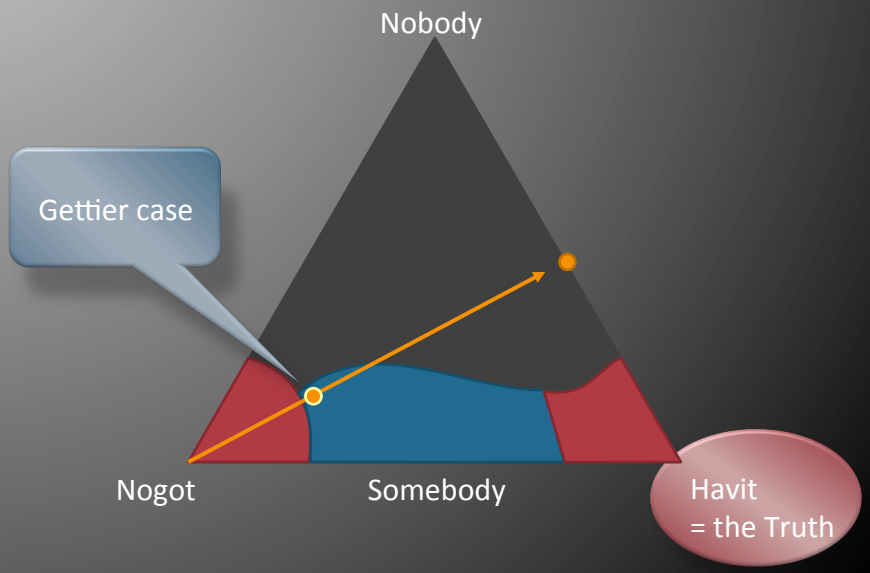


Option: *Dump* Accretive Belief Revision for Uncertain Belief

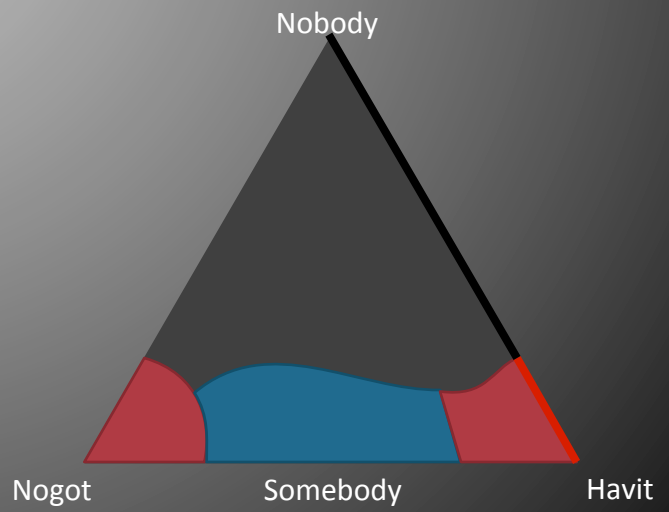
Anyway, Accretion is Dubious for Uncertain Belief



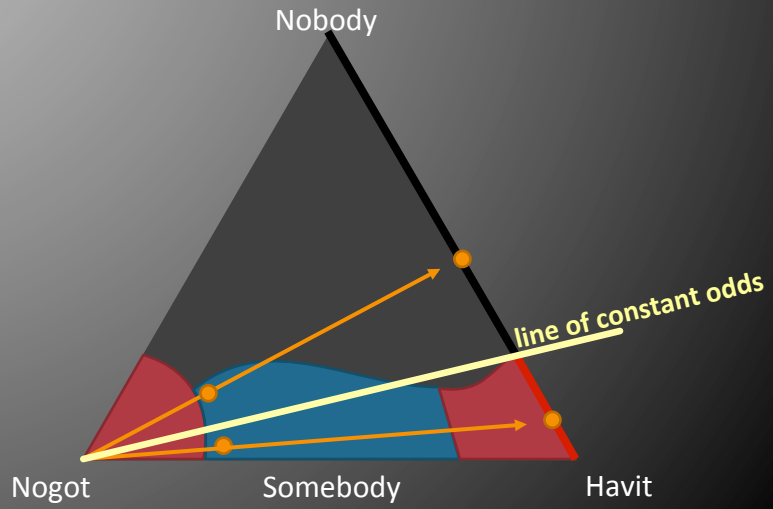
No-False-Lemma Gettier Case



The Target



Aiming at the Target

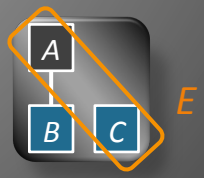


Lesson

A belief revision policy that tracks conditioning should be based on odds thresholds.

Shoham Revision (1987)

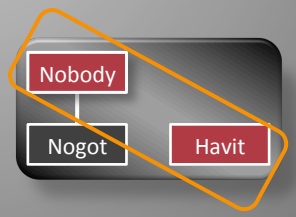
- Take a partial order of the atoms,* called a *plausibility order*.
- B(*E*) = the disjunction of the most plausible atoms compatible with *E*.



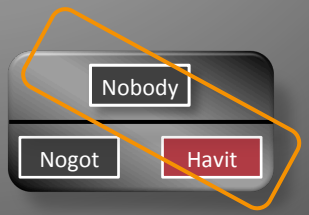
B(*E*) = *A* v *C*

Shoham Revision vs. AGM Revision

Shoham Revision vs. AGM Revision

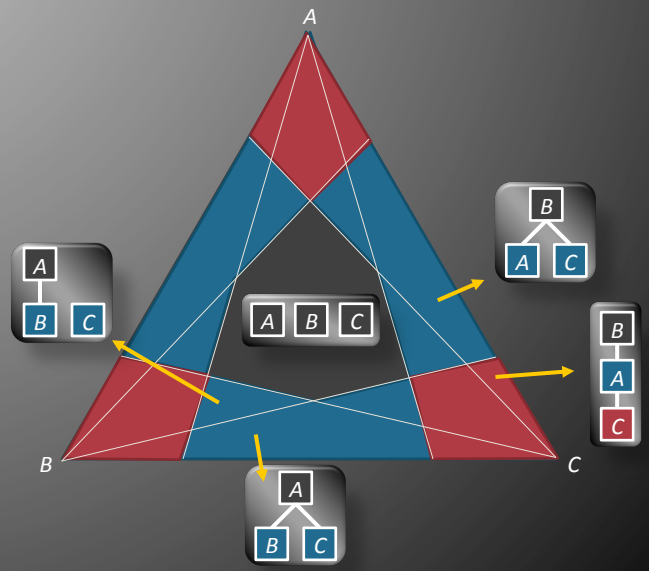


"Check your reasons"

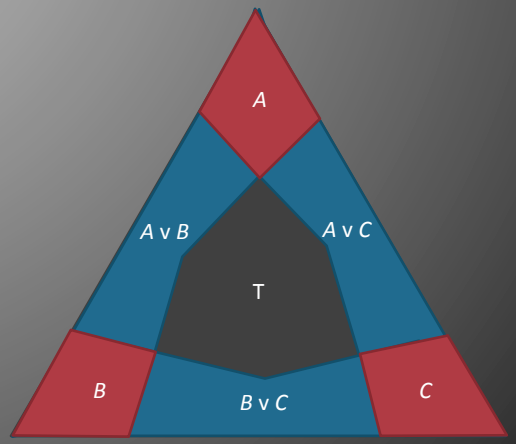


"Trust your belief state"

Odds Threshold Policy

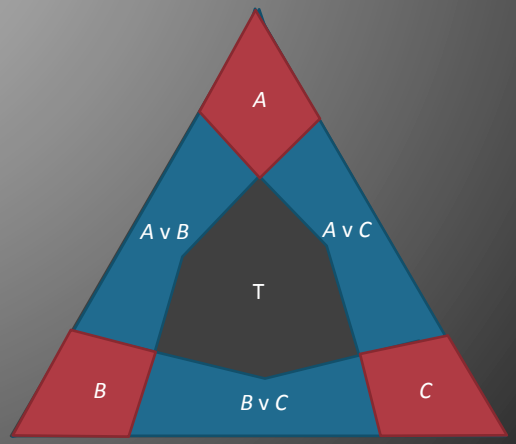


Odds Threshold Policy*

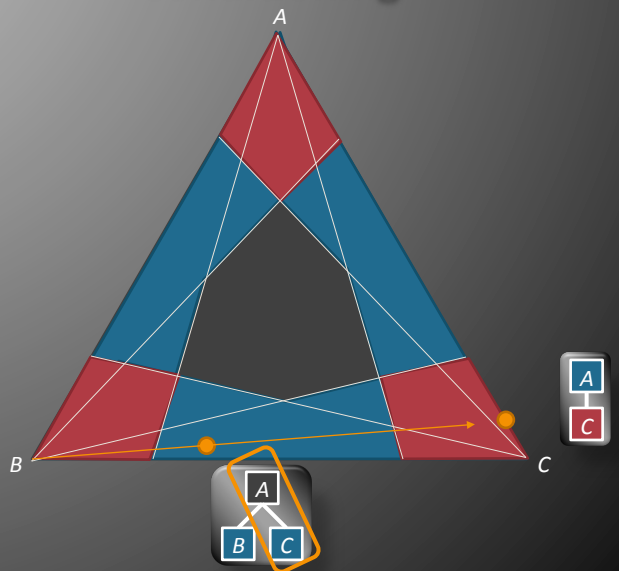


*Invented and rejected by Levi (1996)

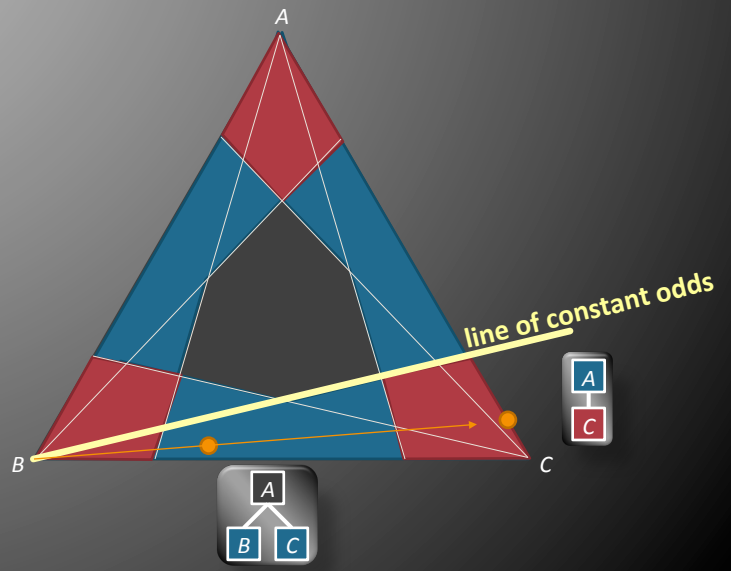
The Odds Threshold Policy is Sensible



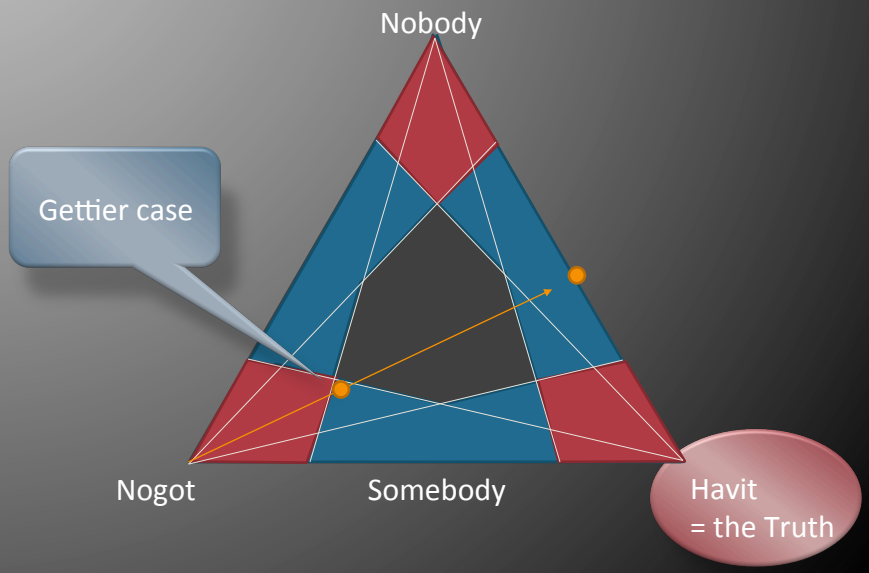
The Odds Threshold Policy Tracks Conditioning



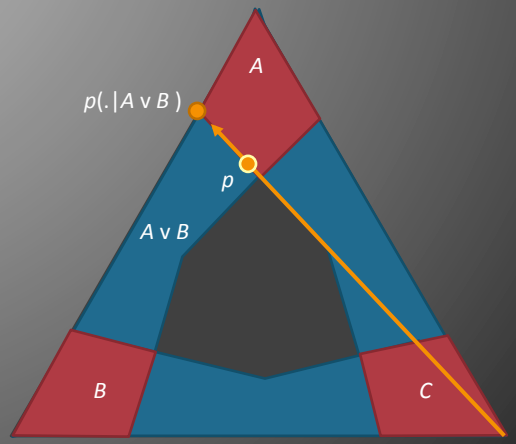
Accretion Depends on Odds



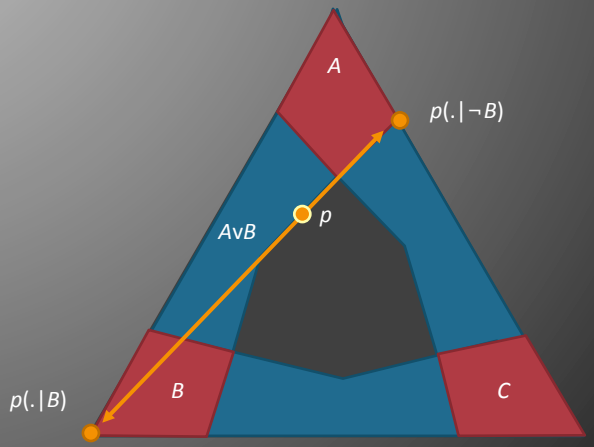
No-False-Lemma Gettier Case



Free from New Riddle 1



Free from New Riddle 2



Proposition*

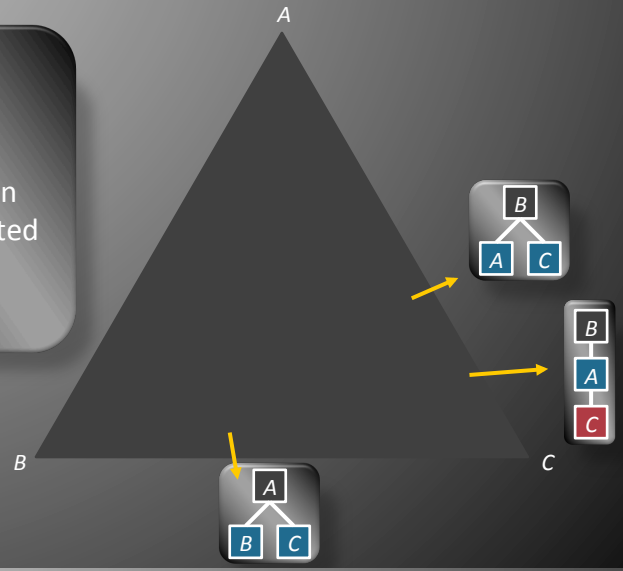
The odds threshold policy has the following virtues:

- ☑ sensible
- ☑ tracks conditioning
- ☑ avoids both riddles
- ☑ represents no-false-lemma Gettier cases

*Works for each countable dimension, and for each odds threshold t such that $1 < t < \infty$.

Shoham-driven Belief Revision Policy

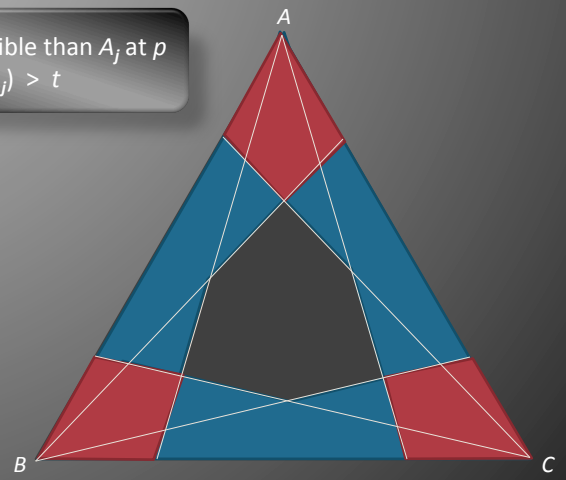
A *Shoham-driven* policy assigns only belief revision rules generated by Shoham models.



Does a Shoham-driven policy have to be based on odds thresholds in order to track conditioning?

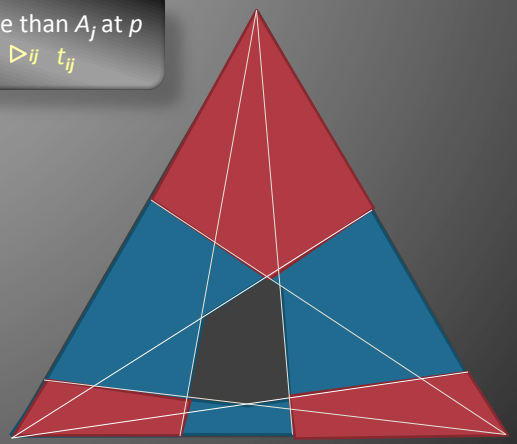
Constant Odds Thresholds

A_i is more plausible than A_j at p
$\Leftrightarrow$ $p(A_i) / p(A_j) > t$



Generalized Odds Thresholds

A_i is more plausible than A_j at p
$\Leftrightarrow$ $p(A_i) / p(A_j) \rhd_{ij} t_{ij}$



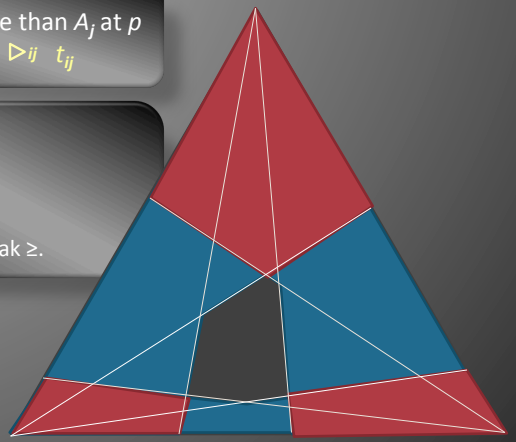
Odds-Based Shoham-Driven Policy

A_i is more plausible than A_j at p
$\Leftrightarrow$ $p(A_i) / p(A_j)$ $\rhd_{ij}$ t_{ij}

where:

$0 \leq t_{ij} \leq \infty$,

$\rhd_{ij}$ is strict $>$ or weak $\geq$.



Representation Theorem

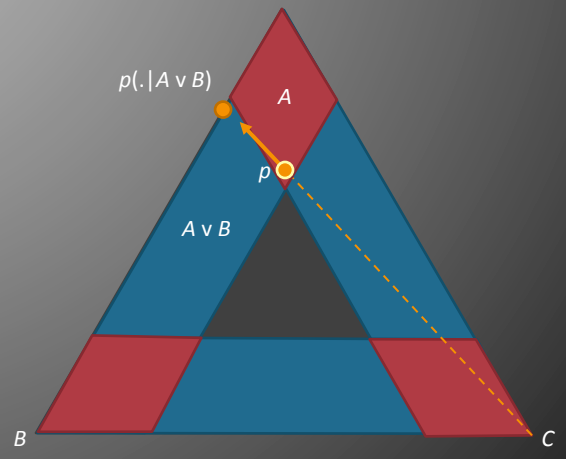
For each Shoham-driven belief revision policy B,

B is odds-based $\Leftrightarrow$ B is corner-monotone and tracks conditioning.

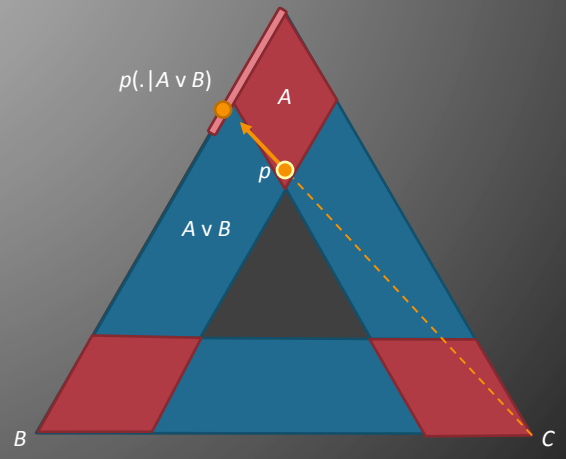
THANK YOU!

Blunt Diamond Theorem

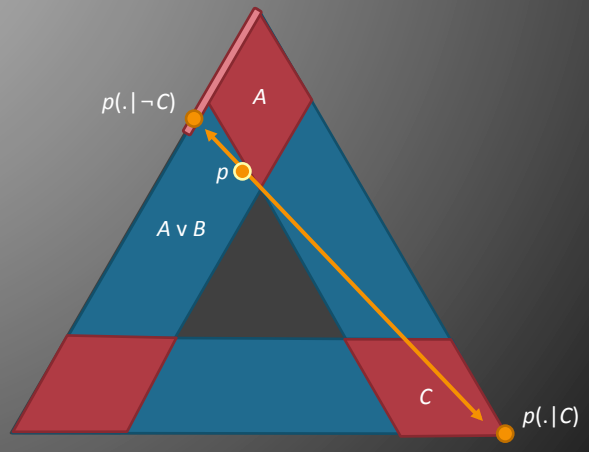
First Riddle



A Contextualist Objection



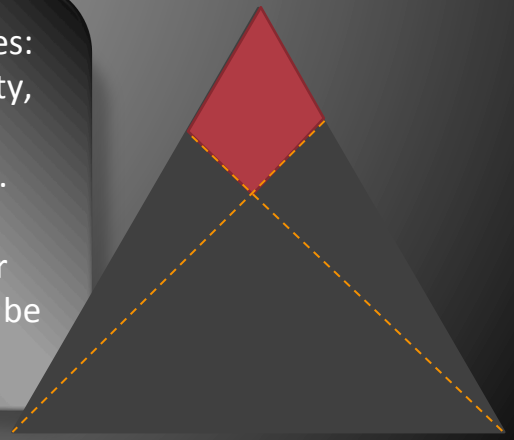
But That's Why We Have *Two* Riddles



Blunt Diamond Theorem*

Suppose policy B satisfies:
- cautious monotonicity,
- case reasoning,
- corner-monotonicity.

Then, for B, the zone for believing an atom must be a "blunt diamond".



*Lin and Kelly (forthcoming) "A Geo-Logical Solution to the Lottery Paradox"

No-Go Theorem for Cross-Question Policies

No-Go Theorem 1 for Cross-Question Policies*

No sensible cross-question policy is invariant under question refinement.

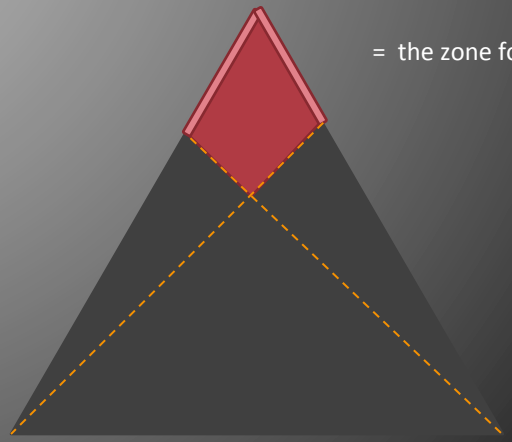
*Lin and Kelly (forthcoming) "A Geo-Logical Solution to the Lottery Paradox"

No-Go Theorem 2 for Cross-Question Policies*

No sensible cross-question policy that satisfies:
- cautious monotonicity
- case reasoning
- preservation under question refinement

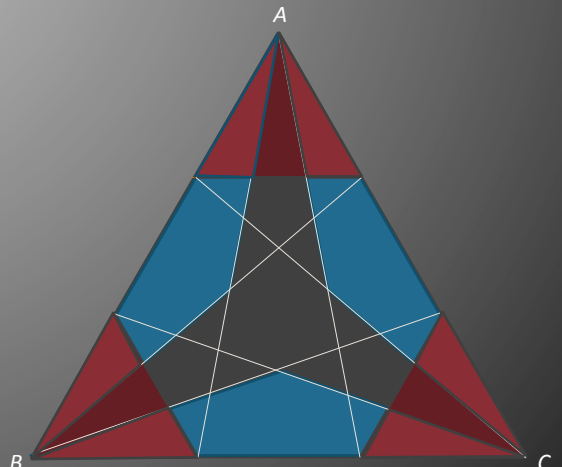
*Lin and Kelly (forthcoming) "A Geo-Logical Solution to the Lottery Paradox"

Blunt Diamond Theorem

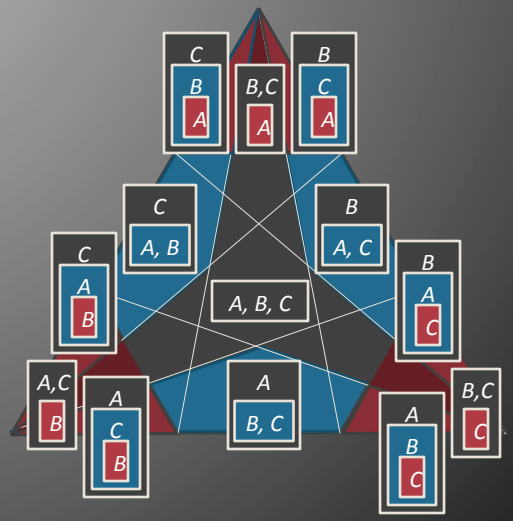


= the zone for believing *A*.

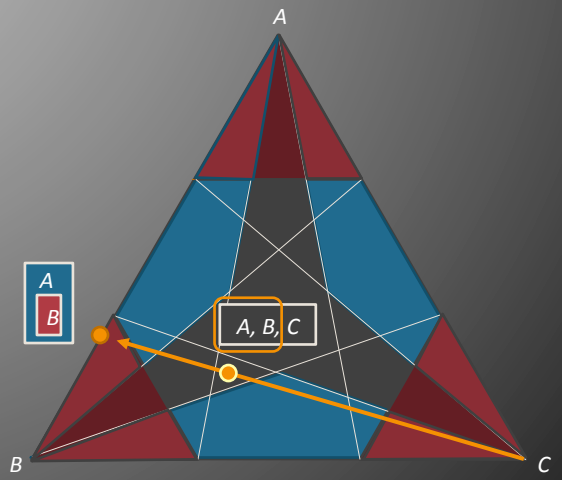
Example: Leitgeb's (2010) AGM Policy



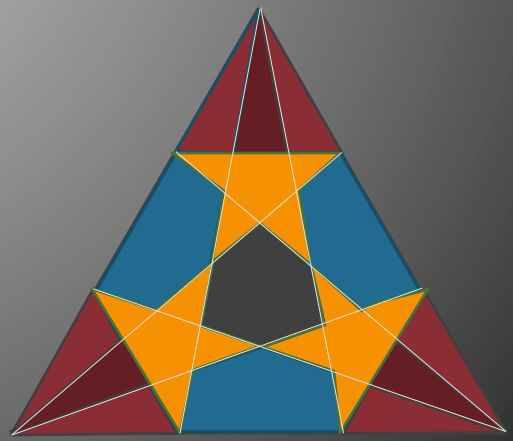
Example: Leitgeb's (2010) AGM Policy:



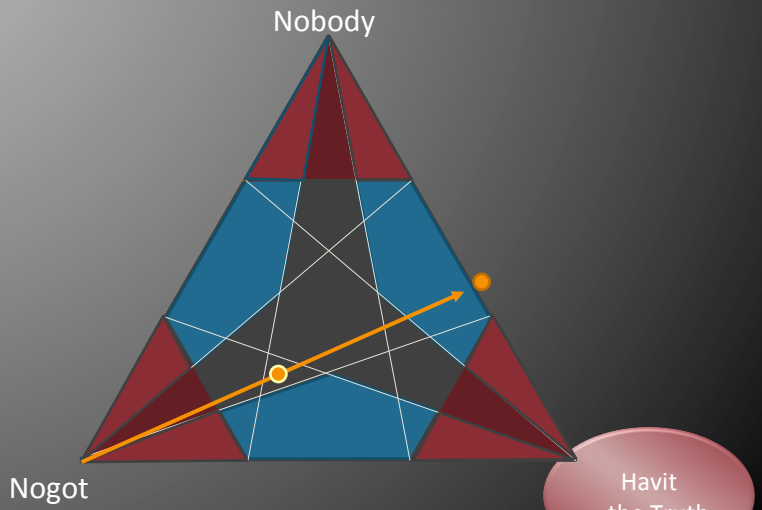
Tracking Failure



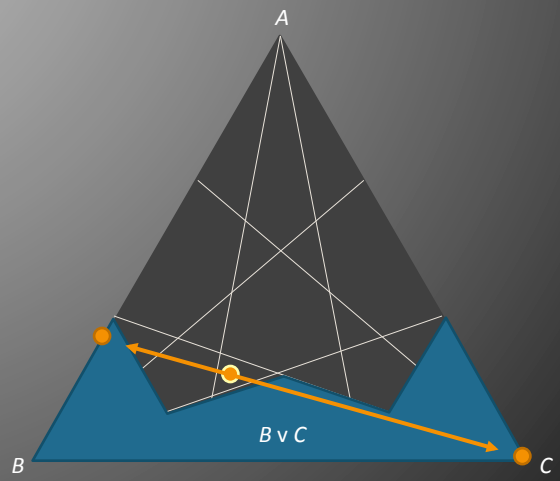
Tracking Failure Zones



No Representation of No-False Lemma Gettier Cases

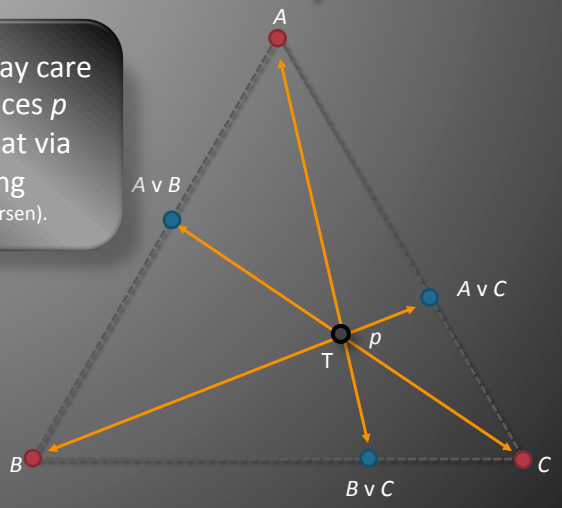


Disjunction Zone



Example: Restricted Belief Revision Policy

Individual *p* may care only about places *p* would end up at via *full* conditioning (Arlo-Costa and Pedersen).



Motive for General Policies: *Partial* Information