Homework #3 Solutions

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1. Truth-Table for '$A \to (B \to (A \,\&\, B))$' (main connective in red):

A	B	A	$\to$	$(B$	$\to$	$(A$	$\&$	$B))$
⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥
⊥	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥

$\therefore$ '$A \to (B \to (A \,\&\, B))$' is *tautological* (it is true on all interpretations).

5. Truth-Table for '$((F \,\&\, G) \to H) \to ((F \vee G) \to H)$' (main connective in red):

F	G	H	$((F$	$\&$	$G)$	$\to$	$H)$	$\to$	$((F$	$\vee$	$G)$	$\to$	$H)$
⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤
⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥
⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤	⊤
⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥

$\therefore$ '$((F \,\&\, G) \to H) \to ((F \vee G) \to H)$' is *contingent* (it is true on some interpretations, false on others).

7. Truth-Table for '$(A \leftrightarrow B) \,\&\, ((C \to {\sim}A) \,\&\, (B \to C))$' (main connective in red):

A	B	C	$(A$	$\leftrightarrow$	$B)$	$\&$	$((C$	$\to$	${\sim}A)$	$\&$	$(B$	$\to$	$C)$
⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥
⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤
⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊥
⊥	⊤	⊤	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤
⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊥

$\therefore$ '$(A \leftrightarrow B) \,\&\, ((C \to {\sim}A) \,\&\, (B \to C))$' is *contingent* (it is true on some interpretations, false on others).

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II. Truth-Tables for the sentences in question (main connectives in red):

(1)

A	B	A	$\vee$	B
⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊤	⊤	⊥
⊥	⊤	⊥	⊤	⊤
⊥	⊥	⊥	⊥	⊥

(2)

A	B	A	$\to$	B
⊤	⊤	⊤	⊤	⊤
⊤	⊥	⊤	⊥	⊥
⊥	⊤	⊥	⊤	⊤
⊥	⊥	⊥	⊤	⊥

(3)

A	B	${\sim}$	$(A$	$\&$	${\sim}$	$B)$
⊤	⊤	⊤	⊤	⊥	⊥	⊤
⊤	⊥	⊥	⊤	⊤	⊤	⊥
⊥	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊤	⊥	⊥	⊤	⊥

	A	B	$\sim$	$(\sim$	A	$\&$	$\sim$	$B)$
	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊤
(4)	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊥
	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊥

	A	B	$\sim$	A	$\vee$	B
	⊤	⊤	⊥	⊤	⊤	⊤
(5)	⊤	⊥	⊥	⊤	⊥	⊥
	⊥	⊤	⊤	⊥	⊤	⊤
	⊥	⊥	⊤	⊥	⊤	⊥

	A	$\sim$	A	$\vee$	A
(6)	⊤	⊥	⊤	⊤	⊤
	⊥	⊤	⊥	⊤	⊥

	A	$(A$	$\rightarrow$	$(A$	$\&$	$\sim$	$A))$	$\rightarrow$	$\sim$	A
(7)	⊤	⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤
	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊤	⊤	⊥

Therefore, we have the following equivalences:

- (6) and (7) are equivalent.
- (1) and (4) are equivalent.
- (2), (3), and (5) are equivalent.

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III. No, if p is not a tautology, it does *not* follow that $\ulcorner \sim p \urcorner$ is a tautology. I proved this in lecture (it's the metatheoretic question: If $\nvDash p$, then does it follow that $\vDash \sim p$?). There are LSL sentences such that *both* $\nvDash p$ *and* $\nvDash \sim p$. Any atomic wff (*e.g.*, 'A') will do. More generally, any *contingent* sentence p will, by definition, be such that $\nvDash p$ *and* $\nvDash \sim p$.