Branden Fitelson

Philosophy 12A Notes

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Announcements & Such

- Administrative Stuff
 - Take-Home Mid-Term re-subs are due today.
 - When you turn in resubmissions, make sure that you staple them to your original homework submission.
 - We will be discussing the grade curve for the course as soon as all of the mid-term grades are in (both take-home and in-class).
 - Branden will not be holding office hours this week.
 - HW #4 posted but, not due (1st sub) until after spring break.
- Today: Chapter 4 Natural Deduction Proofs for LSL
 - Today: *lots of proofs* using the basic natural-deduction rules.
 - After spring break: finishing-up Chapter 4 & moving on to Chap. 5.
 - **MacLogic** a useful computer program for natural deduction.
 - * See http://fitelson.org/maclogic.htm.
- Make sure you do lots of proofs practice is the key here.

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The Introduction and Elimination Rules for \lor

Rule of \vee -**Introduction**: For any formula p, if p has been inferred at line j, then, for any formula q, *either* $^{\mathsf{r}}p \vee q^{\mathsf{r}}$ *or* $^{\mathsf{r}}q \vee p^{\mathsf{r}}$ may be inferred at line k, labeling the line ' $j \vee l$ ' and writing on its left the same premise and assumption numbers as appear on the left of j.

$$a_1, \dots, a_n$$
 (j) p a_1, \dots, a_n (j) q
$$\vdots$$
 OR
$$\vdots$$

$$a_1, \dots, a_n$$
 (k) $p \lor q$ $j \lor I$
$$a_1, \dots, a_n$$
 (k) $p \lor q$ $j \lor I$

- The VI rule is very simple an intuitive. Basically, it says that you may infer a disjunction from *either* of its disjuncts.
- The *elimination* rule (\vee E) for \vee , on the other hand, is considerably more complex to state and apply. It's the hardest of our rules.

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Rule of \vee -**Elimination**: If a disjunction $p \vee q$ occurs at line g of a proof, p is assumed at line h, r is derived at line i, q is assumed at line j, and r is derived at line k, then at line m we may infer r, labeling the line 'g, h, i, j, k \vee E' and writing on its left every number on the left at line g, and at line i (except h), and at line k (except j).

$$a_1, \dots, a_n$$
 (g) $p \lor q$
 \vdots
 h (h) p Assumption
 \vdots
 b_1, \dots, b_u (i) r
 \vdots
 j (j) q Assumption
 \vdots
 c_1, \dots, c_w (k) r
 \vdots
 \mathscr{A} (m) r g, h, i, j, k \lor E

where \mathscr{A} is the set: $\{a_1, ..., a_n\} \cup \{b_1, ..., b_u\}/h \cup \{c_1, ..., c_w\}/j$.

CHAPTER 4

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General Tips on Proof Strategy and Planning

- As a first line of attack, always try to prove your conclusion by using the introduction rule for its main connective as the main strategy.
- This will indicate what assumptions, if any, need to be made and what other formulae will need to be derived. This is "working backward".
- If these other formulae also contain connectives, then try to prove them by introducing their main connectives. Work backward, as far as possible.
- When this technique can no longer be applied, inspect your current stock of premises and assumptions to see if they have any *obvious* consequences.
- If your current premises and assumption contain a disjunction $\lceil r \vee s \rceil$, see if you can prove your current goal formula p from each of its disjuncts r and s (using your current premises and assumptions). If you think you can, then try using $\vee E$ to prove p. If no disjunction appears anywhere in your current of premises/assumptions, then $\vee E$ is probably not a good strategy.
- If you have tried everything you can think of to prove your current goal p, try assuming $\lceil \sim p \rceil$ and aim for $\lceil \sim \sim p \rceil$ by $\sim E$, $\sim I$; then use DN.

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When to Make Assumptions, and When *Not* to

- In constructing a proof, any assumptions you make must eventually be discharged, so you should only make assumptions in connection with the three rules which discharge assumptions.
- In other words, if you make an assumption p in a proof, you must be able to give one of the following three reasons:
 - 1. p is the antecedent of a conditional $p \rightarrow q$ you are trying to derive using the \neg **I** rule (then, try to prove *q*).
 - 2. You are trying to derive $\lceil \sim p \rceil$, so you assume p with an eye toward using the \sim I rule (then, try to prove \curlywedge).
 - 3. p is one of the disjuncts of a disjunction $p \vee q$ (somewhere in your current stock of premises and assumptions!) to which you will be applying $\vee E$ (then, try to prove some r from each).
- Remember, only the three rules $\neg I$, $\sim I$, and $\vee E$ involve making assumptions. No other rules can discharge assumptions.

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10 More Examples Involving VI and VE

1. $(A \& B) \lor (A \& C) \vdash A$

2. $(A \rightarrow \land) \lor (B \rightarrow \land) . B \vdash \sim A$ [p. 116, §4.5, ex. 11]

[p. 111, ex. 2]

[not in text]

3. $(A \lor B) \lor C \vdash A \lor (B \lor C)$ [p. 116, ex. 19]

 $A. A \lor B \vdash (A \rightarrow B) \rightarrow B$ [p. 116, ex. 10]

5. $A \& B \vdash \sim (\sim A \lor \sim B)$ [p. 116, ex. 14 (\vdash)]

6. $A \vee B \vdash \sim (\sim A \& \sim B)$ [p. 116, ex. 13]

7. $\sim (A \& B) \vdash \sim A \lor \sim B$ [p. 116, ex. 16 (\dashv)]

8. $\sim C \vee (A \rightarrow B) \vdash (C \& A) \rightarrow B$ [not in text]

9. $\vdash (A \rightarrow B) \lor (B \rightarrow A)$

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10. $\sim (A \vee B) \vdash \sim A \& \sim B$ [not in text]

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Proof of Example #1

Problem is: $(A\&B)_{\vee}(A\&C) + A$

4

1

1 (1) $(A&B)_{\vee}(A&C)$ Premise 2 (2) A&B Assumption (VE) 2

(3) A 2 &E

4 (4) A&C Assumption (VE) (5) A 4 &E

(6) A 1,2,3,4,5 VE

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Proof of Example #2

Problem is: $(A \rightarrow \Lambda) \vee (B \rightarrow \Lambda)$, B + $\sim A$

2

3

4

1,2

(1) $(A \rightarrow \Lambda) \vee (B \rightarrow \Lambda)$ Premise (2) B Premise

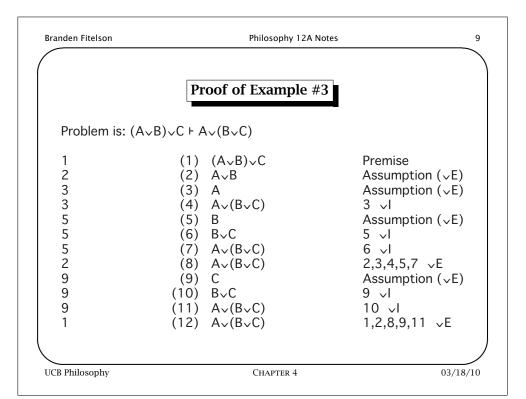
(3) A Assumption (~I)

(4) A→Λ Assumption (\vee E) 3,4 (5) Λ 4.3 →E

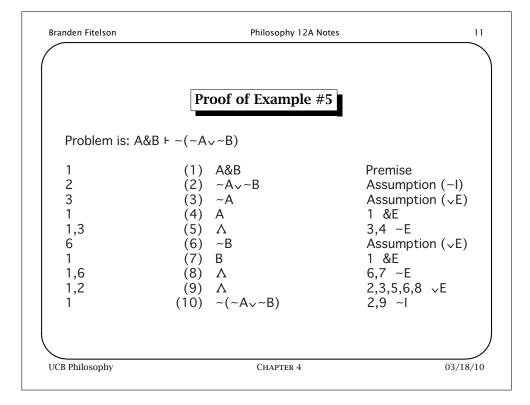
6 (6) B→Λ Assumption (\sqrt{E}) 2.6 6.2 →E (7) Λ

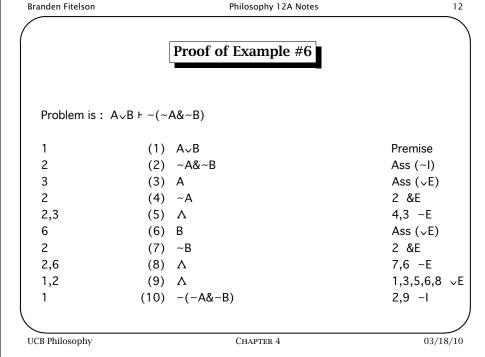
1,4,5,6,7 VE 1,2,3 Λ (8) (9) ~A 3,8 ~1

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	Proo	f of Example #4	4
Problem is	: A∨B ⊦ (A→	B)→B	
1	(1)	A√B	Premise
2	(2)	A→B	Ass (→I)
3	(3)	Α	Ass (√E)
2,3	(4)	В	2,3 →E
5	(5)	В	Ass (√E)
1,2	(6)	В	1,3,4,5,5 √E
1	(7)	(A→B)→B	2,6 →I





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	Proof of Example #	[‡] 7
Problem is:	~(A&B) + ~A~~B	
1 2 3 3 2,3 2 2 8 8 2,8 2 2 2 2 1,2 1	(1) ~(A&B) (2) ~(~A\~B) (3) ~A (4) ~A\~B (5) \(\Lambda\) (6) ~~A (7) \(\Lambda\) (8) ~B (9) ~A\~B (10) \(\Lambda\) (11) ~~B (12) \(\Bar{1}\) (13) \(\Lambda\) (14) \(\Lambda\) (15) ~~(~A\~B) (16) ~A\~B	Premise Assumption (~I) Assumption (~I) 3 ✓I 2,4 ~E 3,5 ~I 6 DN Assumption (~I) 8 ✓I 2,9 ~E 8,10 ~I 11 DN 7,12 &I 1,13 ~E 2,14 ~I 15 DN
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