

Overview of Today's Lecture

- Music: Robin Trower, "Daydream" (*King Biscuit Flower Hour* concert, 1977)
- Administrative Stuff (lots of it)
 - Course Website/Syllabus [*i.e.*, syllabus handout]
 - * Textbook & Supplemental Materials
 - * What, When, Where, Why?
 - * Grades, Assignments, Exams, and all that...
 - * Group Work and Individual Work
 - * Tentative Course Schedule [+ Home Page, bspace site, Email]
 - * MacLogic Software (more on this later in the course)
 - Please fill-out an index card with the following information:
 - * Name, SID, email, year, major, section prefs — rank these 6 pairs:
 - (1) 10–11 MW, (2) 12–1 MW, (3) 10–11 TR
 - (4) 11–12 TR (5) 1–2 MW, (6) 2–3 MW
- Introduction to the Course & Chapter 1 of Forbes

What Logic is *Not*

- Often, people will say: "That person is logical" or "That decision is logical", etc. What they *mean* is that the person/decision/etc is *reasonable* or *rational*. Logic (in our sense) has little to do with this.
- Logic is not about people or how they think or how they ought to think. How people *actually* think is a *psychological* question. How people *ought* to think is an *epistemological* (or perhaps *ethical*) question.
- Logic is abstract. It is not about concrete entities. In this sense, it is like mathematics. But, it is more basic and fundamental than mathematics.
- Logic is not about debating or arguing. It is also not about persuading or convincing people of things (or any human activities, for that matter).
- Logic is not empirical (like physics). Nor is it subjective (like, perhaps, matters of taste). It isn't mysterious or unclear either. So, what *is* it?

Background 1: Propositions and Sentences

- *Propositions* are the basic units of logical analysis. They are expressed by declarative sentences like "Snow is white."
- Not all sentences express propositions (*e.g.*, "What time is it?").
- Propositions are not identical to declarative sentences that express them. Consider: "Snow is white" and "Schnee ist weiß."
- Propositions are either true or false (not both). *True* and *False* are called *truth-values*. Propositions have exactly one truth-value. The truth-value of a proposition is *objective*.
- That is, whether a proposition is true or false (in a given situation) does not depend on what anyone thinks about *that* proposition or on how that proposition happens to be expressed.
- Even if a proposition is *about* something subjective, its truth-value remains objective (*e.g.*, Branden believes that the Yankees will win.)

Background 2: Actual, Possible, and Necessary Truth

- Some propositions are actually true (Snow is white), and some are not (Al Gore is President of the United States in 2007).
- Other propositions are not *actually* true, but still *possibly* true. Al Gore is not *actually* our President in 2007, but he *might have been*. As such, it is *possibly* true that Al Gore is President in 2007.
- Some propositions are not even *possibly* true. For instance:
 1. My car has traveled faster than the speed of light.
 2. $2 + 2 = 5$.
 3. Branden weighs 200 lbs and Branden does not weigh 200 lbs.
- (1) violates the laws of physics: it is *physically impossible*. (2) violates the laws of arithmetic: it is *arithmetically impossible*.
- (3) violates the laws of *logic*: it is *logically impossible*.

- This is the kind of impossibility that interests the logician. In slogan form, we might call this “the strongest possible kind of impossibility.”
- Some propositions are not only *actually* true, but (logically) *necessarily* true. These *must* be true, on pain of *self-contradiction*:
 - Either Branden weighs 200lbs or he does not weigh 200lbs.
 - If Branden is a good man, then Branden is a man.
- Logical possibility and logical necessity are central concepts in this course. We will make extensive use of them.
- We will look at two precise, formal logical theories in which the notion of logical necessity will have a more precise meaning.
- But, before we get into our formal theorizing, we will look informally at the *following-from* relation between propositions.
- As we will see, understanding the following-from relation will require a grasp of the notions of logical necessity (and logical truth).

Bakckground 3: Arguments, Following-From, and Validity

- An *argument* is a collection of propositions, one of which (the *conclusion*) is supposed to *follow from* the rest (the *premises*).

All men are mortal. [premise]
Socrates is a man. [premise]
Therefore, Socrates is mortal. [conclusion]
- If the conclusion of an argument *follows from* its premises, then the argument is said to be *valid* (otherwise, it's *invalid*).

☞ **Definition.** An argument \mathcal{A} is *valid* if and only if:

Rendition #1. It is (logically!) *necessary* that *if* all of the premises of \mathcal{A} are true, *then* the conclusion of \mathcal{A} is also true.

Rendition #2. It is (logically!) *impossible* for both of the following to be true simultaneously: (1) all of the premises of \mathcal{A} are true, *and* (2) the conclusion of \mathcal{A} is false. [For us, this will be *equivalent* to #1.]

Background 4: Validity, Soundness, and “Good” Arguments

- A “good” argument is one in which the conclusion follows from the premises. But, intuitively, there is more to a “good” argument (all things considered) than mere validity.
 - Ideally, arguments should also have (actually) *true premises*. If the premises of an argument are (actually) false, then (intuitively) the argument isn't very “good” — even if it is valid. *Why not?*
- ☞ **Definition.** An argument \mathcal{A} is *sound* if and only if *both*:
- (i) \mathcal{A} is valid, *and* (ii) all of \mathcal{A} 's premises are (actually) true.
- So, there are two components or aspects of “good” arguments:
 - Logical Component: Is the argument valid?
 - Non-Logical Component: Are the premises (actually) true?
 - This course is only concerned with the *logical* component.

Is it *possible* that all of \mathcal{A} 's premises are true, *but* \mathcal{A} 's conclusion is false?

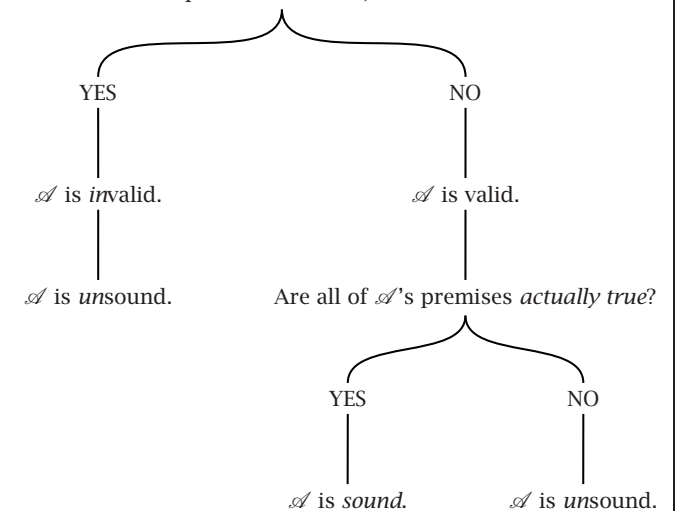


Figure 1: Testing an argument \mathcal{A} for validity and soundness.

Why study logic *formally* or *symbolically*?

- Ultimately, we want to decide whether arguments expressible in *natural* languages are valid. But, in this course, we will only study arguments expressible in *formal* languages. And, we will use *formal* tools. *Why?*
- Analogous question: What we want from natural science is explanations and predictions about *natural* systems. But, our theories (strictly) apply only to systems faithfully describable in *formal, mathematical* terms.
- Although formal models are *idealizations* which abstract away some aspects of natural systems, they are *useful idealizations* that help us understand *many* natural relationships and regularities.
- Similarly, studying arguments expressible in formal languages allows us to develop powerful tools for testing validity. We won't be able to capture *all* valid arguments this way. But, we can grasp many.

A Subtle Argument, and the Notion of Logical Form

- (i) John is a bachelor.
 \therefore John is unmarried.
- Is (i) valid? Well, this is tricky. Intuitively, being unmarried is part of the *meaning* of "bachelor". So, it *seems* like it is (intuitively) logically impossible for the premise of (i) to be true while its conclusion is false
- This suggests that (i) is (intuitively/absolutely) valid.
- On the other hand, consider the following argument:
 If John is a bachelor, then John is unmarried.
- (ii) John is a bachelor.
 \therefore John is unmarried.
- The correct judgment about (ii) seems *clearly* to be that it is valid – *even if we don't know the meaning of "bachelor" (or "unmarried")*.
- This is clear because the logical form of (ii) is *obvious* [(i)'s form is not].

Logical Form II

- This suggests the following additional "conservative" heuristic:
 \Rightarrow We should conclude that an argument \mathcal{A} is valid only if we can see that \mathcal{A} 's conclusion follows from \mathcal{A} 's premises *without appealing to the meanings of the predicates involved in \mathcal{A}* .
- But, if validity does not depend on the meanings of predicates, then what *does* it depend on? This is a deep question about logic. We will not answer it here. That's for more advanced philosophical logic courses.
- What we will do instead is adopt a conservative methodology that only classifies *some* "intuitively/absolutely valid" arguments as valid.
- The strategy will be to develop some *formal* methods for *modeling* intuitive/absolute validity of arguments expressed in English.
- We won't be able to capture *all* intuitively/absolutely valid arguments with our methods, but this is OK. [Analogy: mathematical physics.]

Logical Form III

- We will begin with *sentential logic*. This will involve providing a characterization of valid *sentential forms*. Here's a paradigm example:
 Dr. Ruth is a man.
 (1) If Dr. Ruth is a man, then Dr. Ruth is 10 feet tall.
 \therefore Dr. Ruth is 10 feet tall.
- (1) is a set of sentences with a valid sentential form. So, whatever argument it expresses is a valid argument. What's its *form*?
 p .
 (1_f) If p , then q .
 $\therefore q$.
- (1)'s valid *sentential form* (1_f) is so famous it has a name: *Modus Ponens*. [Usually, latin names are used for the *valid* forms.]

☞ **Definition.** The *sentential form* of an argument (or, the sentences faithfully expressing an argument) is obtained by replacing each basic (or, atomic) sentence in the argument with a single (lower-case) letter.

- What's a "basic" sentence? A basic sentence is a sentence that doesn't contain any sentence as a proper part. How about these?
 - (a) Branden is a philosopher and Branden is a man.
 - (b) It is not the case that Branden is 6 feet tall.
 - (c) Snow is white.
 - (d) Either it will rain today or it will be sunny today.
- Sentences (a), (b), and (d) are *not* basic (we'll call them "complex" or "compound"). Only (c) is basic. We'll also use "atomic" for basic.
- What's the sentential form of the following argument (is it valid?):

If Tom is at his Fremont home, then he's in California.
 Tom is in California.
 ∴ Tom is at his Fremont home.

Two "Strange" Valid Sentential Forms

(+) p . Therefore, either q or not q .

- (+) is valid because it is (logically) *impossible* that *both*:
 - (i) p is true, *and*
 - (ii) "either q or not q " is false.

This is impossible because (ii) *alone* is impossible.

(±) p and not p . Therefore, q .

- (±) is valid because it is (logically) *impossible* that *both*:
 - (iii) " p and not p " is true, *and*
 - (iv) q is false.

This is impossible because (iii) *alone* is impossible.

- We'll soon see why we have these "oddities". They stem from our semantics for "If ... then" statements (and our first def. of validity).

Some Valid and Invalid Sentential Forms

Sentential Argument Form	Name	Valid/Invalid
$\frac{p}{\text{If } p, \text{ then } q}$ $\therefore q$	<i>Modus Ponens</i>	Valid
$\frac{q}{\text{If } p, \text{ then } q}$ $\therefore p$	Affirming the Consequent	Invalid
$\frac{\text{It is not the case that } q}{\text{If } p, \text{ then } q}$ $\therefore \text{It is not the case that } p$	<i>Modus Tollens</i>	Valid
$\frac{\text{It is not the case that } p}{\text{If } p, \text{ then } q}$ $\therefore \text{It is not the case that } q$	Denying the Antecedent	Invalid
$\frac{\text{If } p, \text{ then } q}{\text{If } q, \text{ then } r}$ $\therefore \text{If } p, \text{ then } r$	Hypothetical Syllogism	Valid
$\frac{\text{It is not the case that } p}{\text{Either } p \text{ or } q}$ $\therefore q$	Disjunctive Syllogism	Valid

Logical Form IV — Beyond Sentential Form

- The first half of the course involves developing a precise *theory* of *sentential* validity, and several rigorous techniques for *deciding* whether a sentential form is (or is not) valid. This only takes us so far.
- Not all (absolutely) valid arguments have valid *sentential* forms, *e.g.*:

All men are mortal.

(2) Socrates is a man.

∴ Socrates is mortal.
- The argument expressed by (2) seems clearly valid. But, the sentential form of (2) is not a valid form. Its sentential form is:

p .

(2_f) q .

∴ r .

- In the second half of the course, we'll see a more general theory of logical forms which will encompass both (2) and (1) as valid forms.
- In this more general theory, we will be able to see that (2) has something like the following (non-sentential!) logical form:

All Xs are Ys.

(2_f*) *a* is an X.

∴ *a* is a Y.

- But, we won't need to worry about such non-sentential forms until chapter 7. Meanwhile, we will focus on *sentential logic*.
- This will involve learning a (simple) purely formal language for talking about sentential forms, and then developing rigorous methods for determining whether sentential forms are valid.
- As we will see, the fit between our simple formal sentential language and English (or other natural languages) is not perfect.

Validity and Soundness of Arguments — Some Non-Sentential Examples

- Can we classify the following according to validity/soundness?

1) All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage.	5) All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine.
2) All wines are whiskeys. Chardonnay is a wine. Therefore, chardonnay is a whiskey.	6) All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine.
3) All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink.	7) All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine.
4) All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey.	8) All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine.

	Valid	Invalid
True premises True conclusion	All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage. [sound]	All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine. [unsound]
True premises False conclusion	Impossible None exist	All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine. [unsound]
False premises True conclusion	All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink. [unsound]	All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine. [unsound]
False premises False conclusion	All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey. [unsound]	All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine. [unsound]

- See, also, our validity and soundness handout ...

Some Brain Teasers Involving Validity and Soundness

- Here are two very puzzling arguments:

(\mathcal{A}_1) Either \mathcal{A}_1 is valid or \mathcal{A}_1 is invalid.

∴ \mathcal{A}_1 is invalid.

(\mathcal{A}_2) \mathcal{A}_2 is valid.

∴ \mathcal{A}_2 is invalid.

- I'll discuss \mathcal{A}_2 (\mathcal{A}_1 is left as an exercise).
 - If \mathcal{A}_2 is valid, then it has a true premise and a false conclusion. But, this means that if \mathcal{A}_2 is valid, then \mathcal{A}_2 invalid!
 - If \mathcal{A}_2 is invalid, then its conclusion must be true (as a matter of logic). But, this means that if \mathcal{A}_2 is invalid then \mathcal{A}_2 is valid!
 - This *seems* to imply that \mathcal{A}_2 is *both valid and invalid*. But, remember our conservative validity-principle. What is the *logical form* of \mathcal{A}_2 ?