Announcements & Overview

- Administrative Stuff
  - ☞ **Everything has been pushed back by a week** (including mid-term).
    - ∗ **HW #2 due on Friday (2/19).**
      - · Consult *Homework Guidelines & Tips* handout re HW #2
  - – I have posted a bunch of additional LSL symbolization problems, with solutions. See new handout on our course website.
  - – **Our final exam schedule has been announced**
    - ∗ Morning Section: 8–10am, April 28 (location TBA)
    - ∗ Afternoon Section: 8–10am, April 29 (location TBA)
- Today: Unit #2, Continued
  - – Numerical expressions like “at least one of” or “exactly two of”
  - – Symbolizing English *Arguments* into LSL
  - – Next: Unit #3 — The (Truth-Functional) Semantics of LSL

Symbolizing Numerical Expressions

- Let $A \overset{\text{def}}{=}$ Alice will attend the party, $B \overset{\text{def}}{=}$ Bill will attend the party, and $C \overset{\text{def}}{=}$ Carol will attend the party. And consider these

  (1) *At least two of* Alice, Bill, and Carol will attend the party.

  (2) *Exactly one of* Alice, Bill, and Carol will attend the party.

  (3) *At most two of* Alice, Bill, and Carol will attend the party.

  (4) *Exactly two of* Alice, Bill, and Carol will attend the party.

- Some hints:
  - "Exactly 2 of $\{A, B, C\}$ are true" is equivalent to a *disjunction of conjunctions*, in which each conjunction expresses *one specific way* in which exactly two of $\{A, B, C\}$ could be true.
  - "At least 2 of $\{A, B, C\}$ are true" is equivalent to: *either exactly* 2 of $\{A, B, C\}$ are true *or all three* of $\{A, B, C\}$ are true.

Symbolizing/*Reconstructing* Entire English Arguments

- Naïvely, an argument is "just a collection of sentences". So, naïvely, one might think that symbolizing arguments should just boil down to symbolizing a bunch of individual sentences. It's not so simple.
- An argumentative passage has more structure than an individual sentence. This makes argument *reconstruction* more subtle.
- We must now make sure we capture the inter-relations of content across the various sentences of the argument.
- To a large extent, these interrelations are captured by a judicious choice of atomic sentences for the reconstruction.
- It is also crucial to keep in mind the overall intent of the argumentative passage — the intended argumentative strategy.
- Forbes glosses over the art of (charitable!) argument reconstruction. I will be a bit more explicit about this today in some examples.

Symbolizing Entire Arguments: An Example

- 'If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient. But, if God exists then He is none of these, and there is evil in the world. So, we must conclude that God does not exist.'
- Step 0: Decide on atomic sentences and letters.

  G: God exists. E: There is evil in the world.

  J: God is just. O: God is omnipotent.

  K: God is omniscient.
- Step 1: Identify (and symbolize) the *conclusion* of the argument:
  - 'God does not exist.' (which is just '$\sim G$' in LSL)
- Step 2: Symbolize the premises (in this case, there are two):
  - Premise #1: 'If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient.'

Symbolizing Arguments: Example #2

- Premise #1: 'If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient.'

  If G, then ($\sim E$ unless ($\sim J$ or ($\sim O$ or $\sim K$)))

  $G \rightarrow (\sim E \vee (\sim J \vee (\sim O \vee \sim K)))$

- Premise #2: 'If God exists then He is none of these (*i.e.*, He is *neither* unjust *nor*...), and there is evil in the world.'

  If G, then not not-J and not not-O and not not-K, and E.

  $[G \rightarrow (\sim\sim J \,\&\, (\sim\sim O \,\&\, \sim\sim K))] \,\&\, E$

- This yields the following (valid!) sentential form:

$$G \rightarrow (\sim E \vee (\sim J \vee (\sim O \vee \sim K)))$$

$$[G \rightarrow (\sim\sim J \,\&\, (\sim\sim O \,\&\, \sim\sim K))] \,\&\, E$$

$$\therefore \sim G$$

Symbolizing Arguments: Example #2 Notes

- The sentential form:

  $G \rightarrow (\sim E \vee (\sim J \vee (\sim O \vee \sim K)))$

  $[G \rightarrow (\sim\sim J \,\&\, (\sim\sim O \,\&\, \sim\sim K))]$

  E

  $\therefore \sim G$

  with *three* premises is *equivalent* to the *two*-premise sentential form we wrote down originally (why?).

- Alternative for premise #1: '$G \rightarrow \{\sim[\sim J \vee (\sim O \vee \sim K)] \rightarrow \sim E\}$'.

- Moreover, if we had written '$(\sim\sim K \,\&\, (\sim\sim J \,\&\, \sim\sim O))$' rather than '$(\sim\sim J \,\&\, (\sim\sim O \,\&\, \sim\sim K))$' in premise #2, we would have ended-up with yet another *equivalent* sentential form (why?).

- All of these forms capture the meaning of the premises and conclusion, and all are close to the given form. So, all are OK.

Symbolizing Arguments: Example #2 More Notes

- Premise #1: If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient.
- Two Questions: ① Why render this as (*i*) $\ulcorner p \to (q \text{ unless } r)\urcorner$, as opposed to (*ii*) $\ulcorner (p \to q) \text{ unless } r\urcorner$? ② *Does it matter* (*semantically*)?

① First, there's no comma after 'world'. Second, (*i*) is probably *intended*. The second answer assumes (*i*) and (*ii*) are *not* equivalent *in English*.

- That *may* be right, but it's not clear. It presupposes two things:
  (1) *In English*, $\ulcorner q \text{ unless } r\urcorner$ is equivalent to $\ulcorner$If not r, then $q\urcorner$.
  (2) *In English*, $\ulcorner$If p, then (if q then r)$\urcorner$ [*i.e.*, $\ulcorner p \to (q \to r)\urcorner$] is *not* equivalent to $\ulcorner$If (p and q), then $r\urcorner$ [*i.e.*, $\ulcorner (p \,\&\, q) \to r\urcorner$].
- We're *assuming* (1) in this class. (2) is controversial (but defensible).

② In LSL, (*i*) and (*ii*) *are equivalent*, *i.e.*, *in LSL* (2) is *false*. Thus, it seems to me that both readings are probably OK. This is a subtle case.

Symbolizing Arguments: Example #3

If Yossarian flies his missions then he is putting himself in danger, and it is irrational to put oneself in danger. If Yossarian is rational he will ask to be grounded, and he will be grounded only if he asks. But only irrational people are grounded, and a request to be grounded is proof of rationality. Consequently, Yossarian will fly his missions whether he is rational or irrational.

- Basic Sentences: Yossarian flies his missions (F), Yossarian puts himself in danger (D), Yossarian is rational (R), Yosarian asks to be grounded (A).
- Premise #1: If F then D, and if D then not R. [$(F \to D) \,\&\, (D \to \sim R)$]
- Premise #2: If R then A, and not F only if A. [$(R \to A) \,\&\, (\sim F \to A)$]
- Premise #3: But not F only if not R, and A implies R. [$(\sim F \to \sim R) \,\&\, (A \to R)$]
- Conclusion: Consequently, F whether R or not R. [$(R \to F) \,\&\, (\sim R \to F)$]. [Alternatively, the conclusion could be symbolized as: '$(R \vee \sim R) \to F$']
- Note: this is a valid form (we'll be able to prove this pretty soon).

Symbolizing Arguments: Example #4

Suppose no two contestants enter; then there will be no contest. No contest means no winner. Suppose all contestants perform equally well. Still no winner. There won't be a winner unless there's a loser. And conversely. Therefore, there will be a loser only if at least two contestants enter and not all contestants perform equally well.

- Step 0: Decide on atomic sentences and letters.

  T: At least two contestants enter. C: There is a contest.
  E: All contestants perform equally well.
  W: There is a winner. L: There is a loser.

- Step 1: Identify (and symbolize) the *conclusion* of the argument:
  - Conclusion: There will be a loser only if at least two contestants enter and not all contestants perform equally well.
    * "Logish": L only if T and not E.
    * LSL: $L \to (T \,\&\, {\sim}E)$.

– Step 2: Symbolize the premises (here, there are as many as five):

(1) Suppose no two contestants enter; then there will be no contest.

- "Logish": Suppose that not T; then it is not the case that C.
- LSL: '$\sim T \rightarrow \sim C$'.

(2) No contest means no winner.

- "Logish": Not C means not W. [*i.e.*, not C *implies* not W.]
- LSL: '$\sim C \rightarrow \sim W$'.

(3) Suppose all contestants perform equally well. Still no winner.

- "Logish": Suppose E. Still not W. [*i.e.*, E *also* implies not W.]
- LSL: '$E \rightarrow \sim W$'.

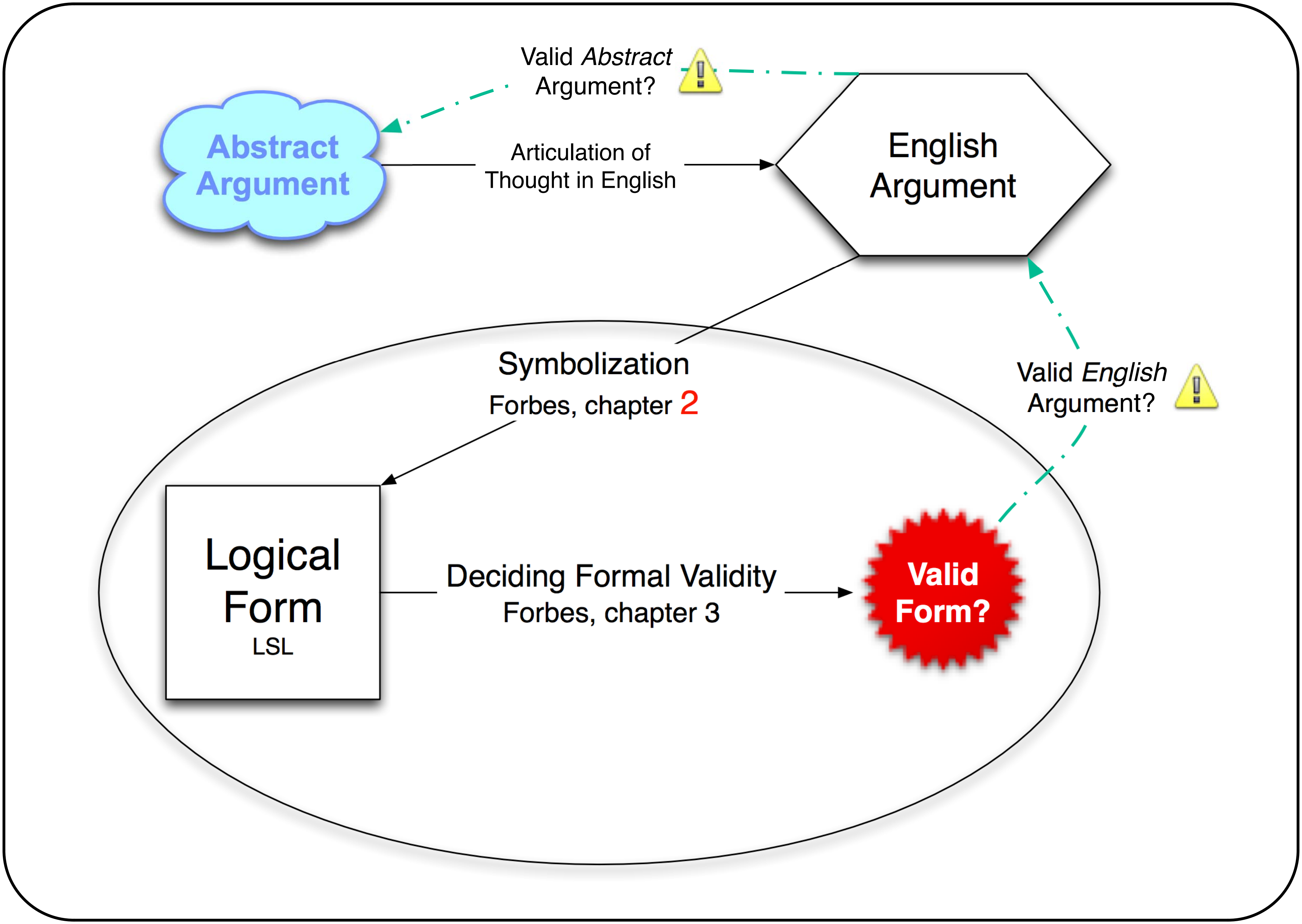
(4) There won't be a winner unless there's a loser. And conversely.

- "Logish": Not W unless L, *and conversely*.
- LSL: '$(\sim L \rightarrow \sim W)\,\&\,(\sim W \rightarrow \sim L)$'. [*i.e.*, not W *iff* not L.]

∗ The final product is the following *valid* sentential form:
$\sim T \rightarrow \sim C$. $\sim C \rightarrow \sim W$. $E \rightarrow \sim W$. $\sim L \leftrightarrow \sim W$. Therefore,
$L \rightarrow (T \,\&\, \sim E)$.

A Few Final Remarks on Symbolizing Arguments

- We saw the following premise our last argument: 'There won't be a winner unless there's a loser. And conversely.' I symbolized it as:
  - "Logish": If not L, then not W, *and conversely*. [*i.e.*, not L *iff* not W.]
  - LSL: '$\sim L \leftrightarrow \sim W$', *equivalently*: '$(\sim L \rightarrow \sim W)\ \&\ (\sim W \rightarrow \sim L)$'.
- One might wonder why I didn't interpret the "and conversely" to be operating on the *unless* operator itself, rather than the *conditional* operator. This would yield the following *different* symbolization:
  - "Logish": not W unless L, and L unless not W.
  - LSL: '$(\sim L \rightarrow \sim W)\ \&\ (\sim\sim W \rightarrow L)$', *equivalently*: '$(\sim L \rightarrow \sim W)\ \&\ (W \rightarrow L)$'.
- Answer: This is a *redundant* symbolization in LSL, since '$\sim L \rightarrow \sim W$' *is equivalent to* '$W \rightarrow L$'. Moreover, the resulting argument *isn't* valid.
- **Principle of Charity**. If an argument $\mathscr{A}$ has two *plausible but semantically distinct* LSL symbolizations (where neither is *obviously* preferable) — and *only one of them is valid — choose the valid one*.


Valid *Abstract* Argument?
Abstract Argument
Articulation of Thought in English
English Argument
Symbolization
Forbes, chapter 2
Valid *English* Argument?
Logical Form
LSL
Deciding Formal Validity
Forbes, chapter 3
Valid Form?

Chapter 3 — Semantics of LSL: Truth Functions I

- The semantics of LSL is *truth-functional* — the truth value of a compound statement is a *function* of the truth values of its parts.
- Truth-conditions for each of the five LSL statement forms are given by *truth tables*, which show how the truth value of each type of complex sentence depends on the truth values of its constituent parts.
- Truth-tables provide a very precise way of thinking about *logical possibility*. Each row of a truth-table can be thought of as a *way the world might be*. The actual world falls into *exactly one* of these rows.
- In this sense, truth-tables provide a way to "see" *logical space*.
- Truth-tables will also provide us with a rigorous way to establish whether an argument form in LSL is valid (*i.e.*, sentential validity).
- We just look for rows of a salient truth-table in which all the premises are true and the conclusion is false. That's where we're headed.

Chapter 3 — Semantics of LSL: Truth Functions II

- We begin with negations, which have the simplest truth functions. The truth table for negation is as follows (we use $\top$ and $\bot$ for true and false):

p	$\sim p$
$\top$	$\bot$
$\bot$	$\top$

- In words, this table says that if p is true than $\sim p$ is false, and if p is false, then $\sim p$ is true. This is quite intuitive, and corresponds well to the English meaning of 'not'. Thus, LSL negation is like English negation.
- Examples:
  - It is not the case that Wagner wrote operas. ($\sim W$)
  - It is not the case that Picasso wrote operas. ($\sim P$)
- '$\sim W$' is false, since 'W' is true, and '$\sim P$' is true, since 'P' is false (like English).

Chapter 3 — Semantics of LSL: Truth Functions III

p	q	$p \,\&\, q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$

- Notice how we have four (4) rows in our truth table this time (not 2), since there are four possible ways of assigning truth values to p and q.
- The truth-functional definition of & is very close to the English 'and'. A LSL conjunction is true if *both* conjuncts are true; it's false otherwise.
  - Monet and van Gogh were painters. ($M \,\&\, V$)
  - Monet and Beethoven were painters. ($M \,\&\, B$)
  - Beethoven and Einstein were painters. ($B \,\&\, E$)
- '$M \,\&\, V$' is true, since both 'M' and 'V' are true. '$M \,\&\, B$' is false, since 'B' is false. And, '$B \,\&\, E$' is false, since 'B' and 'E' are both false (like English).

Chapter 3 — Semantics of LSL: Truth Functions IV

p	q	$p \vee q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$

- Our truth-functional $\vee$ is not as close to the English 'or'. An LSL disjunction is true if *at least one* disjunct is true (false otherwise).
- In English, 'A or B' often implies that 'A' and 'B' are *not both true*. That is called *exclusive* or. In LSL, '$A \vee B$' is *not* exclusive; it is *inclusive* (true if both disjuncts are true). But, we *can* express exclusive or in LSL. How?
  - Either Jane austen or René Descartes was novelist. ($J \vee R$)
  - Either Jane Austen or Charlotte Bronte was a novelist. ($J \vee C$)
  - Either René Descartes or David Hume was a novelist. ($R \vee D$)
- The first two disjunctions are true because at least one their disjuncts is true, but the third is false, since both of its disjuncts are false.