Focused Correlation and Confirmation

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coherence theory and the 'truth connection'

- Bonjour's 'cognitively spontaneous' beliefs (1985)
- Klein and Warfield's observation (1994-96)
  - *One can increase coherence of a set without increasing likelihood of truth.*
- March of the Probabilists (late '90s)
  - Shogenji, Fitelson, Olsson . . .
  - *Under what conditions is coherence 'truth-conducive'?*
- Impossibility results (early '00s)
  - Bovens & Hartmann: *There is no coherence measure that induces an ordering on sets of variables.*
  - Olsson: *There is no informative link between coherence and likelihood of truth*
- 'Irrelevancy' arguments (lately)
  - Schupbach, *ceteris paribus* conditions (FEW 2007)
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  Schupbach, *ceteris paribus* conditions (FEW 2007)

  ⋆ —, artifact of testimonial systems.

coherence measures

the mini-series:

- *Some good news:* Focused Correlation, coherence and the 'truth connection'
- *Bad news:* Coherence measures and the dynamics of belief change.
- *More bad news:* Bayesian coherentism and anti-realism (with a surprise guest).
- *Finale:* An alternative to Bayesian coherentism (with a surprise guest).

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what this talk is about

Good news

- There is an informative relationship between ‘coherence’ and ‘truth’, when
  - *coherence* $=_{df}$ association (deviation from independence), and
  - *truth conduciveness* $=_{df}$ boost of incremental confirmation.

‘coherence’

- is a *distance from independence* property of sets of binary variables, interpreted as sets of propositions.

‘truth-conduciveness’

- is an increase in posterior probability;
- an increase in incremental confirmation.

some machinery

incremental convergence (Cohen 1977)

- Evidence A and B *converge upon* h if $\Pr(h|A \cap B) > \Pr(h|A)$.
- $i_c(\cdot) =_{df} \Pr(h|A \cap B) \;/\; \Pr(h|A)$ measures *incremental convergence* from A to $\{A, B\}$ on h:

$$i_c(h, A, \{A, B\}) = r \text{ is } \begin{cases} \text{positive}, & \text{if } r > 1 \\ \text{neutral}, & \text{if } r = 1 \\ \text{negative}, & \text{if } r < 1 \end{cases}$$

the Wayne-Shogenji measure

$$S(A, B) = \frac{\Pr(A \cap B)}{\Pr(A)\Pr(B)} = s \text{ is } \begin{cases} \text{positive}, & \text{if } s > 1 \\ \text{neutral}, & \text{if } s = 1 \\ \text{negative}, & \text{if } s < 1 \end{cases}$$

Is coherence truth conducive?

- *In order for coherence to be truth conducive, more coherence must imply higher posterior probability* (Olsson 2005, 136).

Answers

Is coherence truth conducive?

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Answers

- Klein and Warfield: (not always).
- Olsson: NO.

example 1 (− cor, + conf)

h_1: The drawn ball is the 2 ball.	$\Pr(h_1) = 0.2$
A_1: The drawn ball is solid.	$\Pr(A_1) = 0.6$
A_2: The drawn ball is even.	$\Pr(A_2) = 0.4$

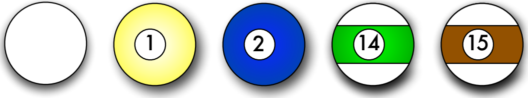
$$S(A_1, A_2) = \frac{\Pr(A \cap B)}{\Pr(A)\Pr(B)} \approx 0.833$$

$$\frac{\Pr(h_1|A_1)}{\Pr(h_1)} \approx 1.667, \quad \frac{\Pr(h_1|A_2)}{\Pr(h_1)} = 2.5$$

but

$$\frac{\Pr(h_1|A_1 \cap A_2)}{\Pr(h_1|A_1)} = 3, \quad \frac{\Pr(h_1|A_1 \cap A_2)}{\Pr(h_1|A_2)} = 2.$$

example 2 (+ cor, + conf)



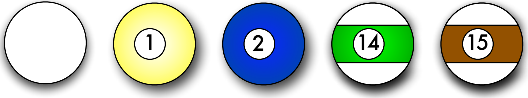
h_2: The drawn ball is striped.
A_3: The drawn ball is odd.
A_4: The drawn ball is an even or an odd stripe.

$S(A_3, A_4) = 1.25 \quad \Pr(h_2) = \Pr(A_3) = \Pr(A_4) = 0.4$

$$\frac{\Pr(h_2|A_3)}{\Pr(h_2)} = 1.25 = \frac{\Pr(h_2|A_4)}{\Pr(h_2)}$$

$$\frac{\Pr(h_2|A_3 \cap A_4)}{\Pr(h_2|A_3)} = 2 = \frac{\Pr(h_2|A_3 \cap A_4)}{\Pr(h_2|A_4)}$$

example 3 (− cor, − conf)



h_3: The drawn ball is solid.
A_5: The drawn ball is odd.
A_6: The drawn ball is even.

$S(A_5, A_6) = 0 \qquad \Pr(h_3) = \Pr(A_5) = \Pr(A_6) = 0.4$

$$\frac{\Pr(h_3|A_5)}{\Pr(h_3)} = 1.25 = \frac{\Pr(h_3|A_6)}{\Pr(h_3)}$$

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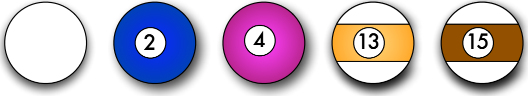
example 4 (+ cor, − conf)



h_4: The drawn ball is even.
A_7: The drawn ball is the 2 ball.
A_8: The drawn ball is solid.

$S(A_7, A_8) = 2.5$

$\Pr(h_4) = \Pr(A_8) = 0.4,$
$\Pr(A_7) = 0.2$

$$\frac{\Pr(h_4|A_7)}{\Pr(h_4)} = 2.5 = \frac{\Pr(h_4|A_8)}{\Pr(h_4)}$$

$$\frac{\Pr(h_4|A_7 \cap A_8)}{\Pr(h_4|A_7)} = 1 = \frac{\Pr(h_4|A_7 \cap A_8)}{\Pr(h_4|A_8)}.$$

Formal question

- Is there an informative relationship between a *dfi*-measure and confirmation?

Implementation question

- What is the mechanism for ensuring independent ‘cognitively spontaneous beliefs’?

Formal question

- Is there an informative relationship between a *dfi*-measure and confirmation? (YES)

Implementation question

What is the mechanism for ensuring independent 'cognitively spontaneous beliefs'?

the Wayne-Shogenji measure

$$S(A,B) = \frac{\Pr(A \cap B)}{\Pr(A)\Pr(B)} = s \text{ is } \begin{cases} \text{positive}, & \text{if } s > 1 \\ \text{neutral}, & \text{if } s = 1 \\ \text{negative}, & \text{if } s < 1 \end{cases}$$

Conditional W-S measure (Wayne 1995)

$$S(A,B|h) = \frac{\Pr(A \cap B|h)}{\Pr(A|h)\Pr(B|h)} = t \text{ is } \begin{cases} \text{positive}, & \text{if } t > 1 \\ \text{neutral}, & \text{if } t = 1 \\ \text{negative}, & \text{if } t < 1 \end{cases}$$

Focused Correlation

Suppose

$S(A, B) = \frac{\Pr(A \cap B)}{\Pr(A)\Pr(B)}$, and

$S(A, B|h) = \frac{\Pr(A \cap B|h)}{\Pr(A|h)\Pr(B|h)}$. Then,

$$For_h(A, B) := \frac{S(A, B|h)}{S(A, B)} = \frac{\Pr(h|A \cap B)}{\Pr(h)} \times \frac{\Pr(h)}{\Pr(h|A)} \times \frac{\Pr(h)}{\Pr(h|B)}.$$

Interpreting Focused Correlation

$$\frac{\Pr(h|A \cap B)}{\Pr(h)} = For_h(A,B) \times \frac{\Pr(h|A)^{*}}{\Pr(h)} \times \frac{\Pr(h|B)^{*}}{\Pr(h)}.$$

if $For_h(A,B) > 1$, then combining *increases* $\Pr_{post}(h)$.
if $For_h(A,B) < 1$, combining does **not** *increase* $\Pr_{post}(h)$.
if $For_h(A,B) = 1$, then combining yields no difference.
if $For_h(A,B) = 0$, $\{A,B\}$ gives no information on $\Pr_{post}(h)$.

* *only if* $\Pr(h|\cdot) > \Pr(h)$.

Interpreting Focused Correlation

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if $For_h(A, B) < 1$, then combining *decreases* $\Pr_{post}(h)$.
if $For_h(A, B) > 1$, combining does **not** *decrease* $\Pr_{post}(h)$.
if $For_h(A, B) = 1$, then combining yields no difference.
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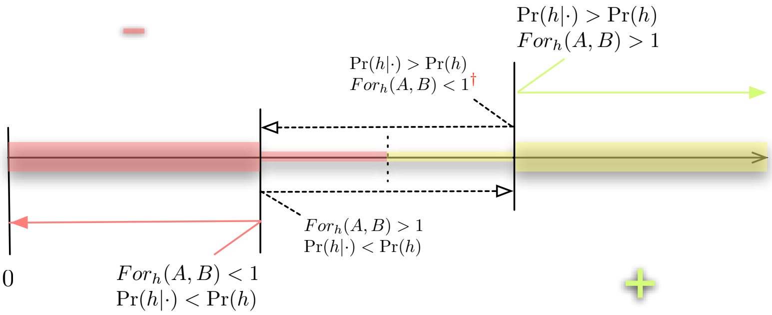
interpreting focused correlation



†when $For_h(A,B) \neq 0$.

example 1 (− cor, + conf)

h_1: The drawn ball is the 2 ball.
A_1: The drawn ball is solid.
A_2: The drawn ball is even.

$$S(A_1, A_2) \approx 0.833 \; For_h(A_1, A_2) = \frac{S(A_1, A_2|h)}{S(A_1, A_2)} \approx 1.20$$

$$\frac{\Pr(h_1|A_1)}{\Pr(h_1)} \approx 1.667, \;\; \frac{\Pr(h_1|A_2)}{\Pr(h_1)} = 2.5$$

$$\frac{\Pr(h_1|A_1 \cap A_2)}{\Pr(h_1|A_1)} = 3, \;\; \frac{\Pr(h_1|A_1 \cap A_2)}{\Pr(h_1|A_2)} = 2.$$

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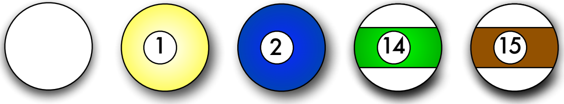

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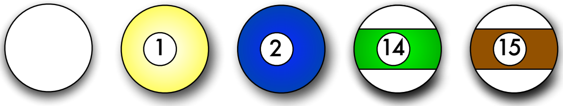

h_3: The drawn ball is solid.
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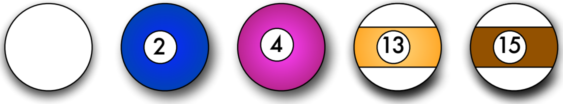

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* *Within a testimonial system.*

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- Wheeler: YES†

* *Within a testimonial system.*

† *if you don't mind talking this way.*

features of focused correlation

- The results do not depend upon conditional independence assumptions (Cf. Bovens, Hartmann, Olsson, Shogenji).
- No restrictions on logical relationships between evidence and hypothesis variables.
- Highlights that linking *dfi* of n evidence variables to confirmation of h is an $n+1$ relation.
- Reveal important asymmetries in incremental confirmation; ordering of evidence variables.

some limitations of focused correlation

- How do you model ‘cognitively spontaneous’ belief?
- Do we really want to drag h into this?
- Does the For_h answer to Olsson obstruct giving an answer to Bovens & Hartmann?
- For_h ignores caveats and precautions that attend drawing inferences from correlation measures (just like everybody else.)

summary

For$_h$ offers some good news:

- Focused Correlation resolves a purely formal question raised by the 'truth connection' problem for coherence measures.
- Locates the action in Olsson's impossibility result within his testimonial model and its conditional independence assumptions.
- Identifying the parameters necessary for *For$_h$* can locate the most informative evidence in a distribution for a given hypothesis variable. Evidence variables can then be ordered by coarse to fine-grained impact upon a hypothesis.
- Values for *For$_h$* can reveal whether evidence is (positively or negatively) monotone.

future work

For$_h$ in context:

- ⋆ Some good news and many open questions . . .
- ◦ *Bad news:* Coherence measures and the dynamics of belief change.
- ◦ *More bad news:* Bayesian coherentism and anti-realism (with a surprise guest).
- ◦ *Finale:* An alternative to Bayesian coherentism (with a surprise guest).

thank you!