Equilibrium among Coalitions

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Suppose that three people A, B, and C may divide \$6 by majority rule. The coalition $\{A, B\}$ may capture all the money and split it just two ways. Listing dollars allocated to people in alphabetical order, it may achieve the allocation $(3, 3, 0)$. However, the coalition $\{A, B\}$ is not stable. The excluded person C may offer B more to form the coalition $\{B, C\}$ than A offers B to form the coalition $\{A, B\}$. For example, C may propose the allocation $(0, 4, 2)$. In fact, there is no way to divide \$6 among A, B, and C so that each possible coalition receives at least as much as it can obtain by forming and acting alone. Every division leaves some coalition an opportunity to do better. The coalitional game lacks a core allocation.[1]

Coalitional games show that rationality does not require pursuit of all incentives. Agents in games with empty cores cannot pursue all incentives, but rationality is still attainable. Its demands adjust to circumstances. This essay introduces strategic equilibrium as a generalization of a core allocation's realization in a coalitional game. Strategic equilibrium is attainable. Achieving it replaces achieving a core allocation as a requirement of rationality.

To prepare for introduction of the standard of strategic equilibrium, the first section defines a coalition's incentives, and the second section defines a coalition's pursuit of incentives. The third section presents and supports the standard of strategic equilibrium for coalitional games. It verifies this standard of collective rationality by showing that individual rationality entails its satisfaction.[2]

1. A Coalition's Incentives

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[1] For a discussion of coalitional games and the problem of the empty core, see Binmore (2007: Chap. 18).
[2] This essay draws on Weirich (forthcoming).

Solutions to both cooperative and noncooperative games specify collective acts that meet standards of collective rationality. In an ideal game players are rational, informed, and in perfect conditions for playing their game. Because being an equilibrium is necessary for being a solution, and because every ideal game has a solution, every ideal game has an equilibrium. Coalitional games may lack core allocations. Yet rational players reach a type of equilibrium. This essay proposes weakening the standard of the core to obtain an equilibrium standard attainable in all ideal coalitional games. The weakening proposed has the support of attainable standards of rationality for agents. These standards require self-supporting strategies rather than strategies that nonconditionally maximize utility.

Weirich (1998) presents strategic equilibrium for noncooperative games. Strategic equilibrium's extension from noncooperative games to coalitional games shows that a single type of equilibrium, generated by a single standard of collective rationality, suits both types of game. Its extension resembles the extension of Nash equilibrium in noncooperative games to the core in coalitional games. The extension of Nash equilibrium identifies a coalition's incentives. The extension of strategic equilibrium identifies a coalition's sufficient incentives. Extending strategic equilibrium to coalitional games unifies the treatment of noncooperative and coalitional games. Because strategic equilibrium rests on rules for individuals making decisions, adopting it also unifies decision theory and game theory. It promotes unification of the branches of game theory and also unification of principles of individual and collective rationality.

In a coalitional game a strategy profile (an assignment of exactly one strategy to each coalition) is a strategic equilibrium if and only if no coalition has a sufficient incentive to change its strategy given the profile. Incentives come in two types. An opportunity for improvement generates an objective incentive, and knowledge of an opportunity for improvement generates a subjective incentive. In ideal coalitional games the two types of incentive agree. However, a theory of rationality addresses subjective incentives. Accordingly, as this essay understands a coalition's incentives, they depend on the coalition's options and its preferences among them.

This section defines a coalition's incentives, and the next section considers which incentives constitute sufficient reasons for options.

An account of a coalition's incentives must use technical definitions of a coalition's options and preferences because coalitions do not have minds, and so lack options and preferences in the ordinary sense. This section technically defines a coalition's options and preferences to suit principles of rationality extended from individuals to coalitions. The definitions use the options, beliefs, and desires of the coalition's members. They make standards of rationality for coalitions compatible with standards of rationality for individuals. In ideal cases the definitions make standards of collective rationality agree with intuitive goals of rational collective action. Fruitfulness for the theory of collective rationality justifies the definitions.

Because a coalition acts freely, it literally has options. It may form or not form and, if it forms, may divide its profits in many ways. In general, a coalition's options are possible collective acts constituted by options of the coalition's members. This characterization leaves some crucial points unsettled, however, so a coalition's set of options needs a technical specification.

In a coalitional game individuals collectively realize a strategy profile. A strategy profile specifies for every coalition (including unit-coalitions) whether the coalition forms and its strategy if it forms. A coalition's nonformation is a collective act, although not a joint act, and counts as its strategy. A strategy profile's coalition structure specifies the coalitions that form. Those coalitions constitute a partition of agents, that is, a collection of sets of agents in which the sets are nonempty, disjoint, and exhaustive. Inevitably, in a coalitional game the agents realize a unique coalition structure.

Because a game's coalition structure is open, each multimember coalition has the option to form or not to form. More fully, a multimember coalition's options are not to form, or to form and adopt a joint strategy. A joint strategy requires the members' collaboration, although not necessarily their efficient collaboration. If they do not collaborate, the coalition adopts no joint strategy and has not formed. A joint act requires the coalition's formation. For a multimember

coalition, failure to form is in the hands of its members. One member's recalcitrance is all it takes for the coalition not to form. Collaboration is unnecessary for the coalition's nonformation. The coalition's formation requires the action of all its members, but its failure to form requires the action of only one member.

A coalition's option by definition is an act that the coalition fully controls. A unit-coalition fails to form only if its unique member joins a larger coalition. This happens only given collaboration with others. A coalition fully controls an act just in case if the coalition were to realize the act, no outside agent would exercise any control over it. Hence a unit-coalition does not have full control over not forming. No individual can on his own not form his unit-coalition. A unit-coalition has the option of attempting not to form but lacks the option of not forming.

A unit-coalition may (1) attempt not to form or (2) not attempt not to form. The success of an attempt not to form depends on others; the unit-coalition may form despite the attempt not to form. The outcome of not attempting not to form is assured; the unit-coalition forms. Because the outcome is assured, I call the strategy of not attempting not to form "forming." But failing to exercise that strategy is attempting not to form; it is not failing to form. A unit-coalition cannot fail to form without nonmembers' assistance. A unit-coalition's nonformation is just a by-product of the formation of a larger coalition that includes the unit-coalition's unique member.

Formation of some coalitions prevents formation of other coalitions. Not every strategy of every coalition is compatible with every coalition structure. Hence not every profile of feasible strategies is feasible. A feasible profile has strategies that are compatible with one another. It assigns strategies to coalitions to form a coalition structure. In noncooperative games, where every profile of feasible strategies is itself feasible, the feasibility of a strategy profile goes without saying. In coalitional games, specifying that a strategy profile is feasible adds clarity, although for brevity this essay often takes feasibility as understood.

If two strategies are incompatible, then given the first, the second is not realized. Nonetheless, given the first, the second remains realizable. Although their combination is not realizable, each strategy is realizable. Modal logic similarly observes that the impossibility of a

conjunction entails that if one conjunct is true, then the other conjunct is false. The conjunction's impossibility does not entail that if one conjunct is true, then the other conjunct is impossible.

In a single-stage coalitional game, if a coalition with an individual forms, another coalition including that individual does not also form, because an individual does not belong to more than one coalition that forms. A coalition's realization of a strategy blocks the realization of strategies of other coalitions. For example, if $\{A, B\}$ forms, then $\{A\}$ does not form. Nonetheless, in a single-stage coalitional game, where choices are simultaneous, one coalition's strategy does not curtail other coalitions' strategies. Although given $\{A, B\}$'s forming, $\{A\}$ does not form, the unit-coalition retains the option of forming. All it takes for $\{A\}$ to form is A's choosing to form $\{A\}$. Agent A fully controls formation of his unit-coalition. He can form his unit-coalition until the coalitional game ends without its formation. Until that time, he can form $\{A\}$ by leaving $\{A, B\}$ even if he does not do that. An individual's adopting an option prevents his adopting other options, but it does not entail that he lacks other options. Similarly, a coalition's strategy may prevent another coalition's formation, but it does not eliminate the other coalition's option to form. The strategies available to coalitions do not change until the game ends.

The form of supposition the theory of rationality adopts for deliberations recognizes options' persistence. In deliberations, supposition of a strategy profile preserves the strategies available to agents. In a coalitional game with two individuals A and B, given $\{B\}$'s formation $\{A\}$ forms as well. Supposition of $\{B\}$'s formation does not preclude the possibility of $\{A, B\}$'s formation, however. If $\{B\}$ forms, then $\{A, B\}$ does not form. Nonetheless, if $\{B\}$ forms, $\{A, B\}$'s formation is still an option. It is still possible even if not actual. Taking supposition this way facilitates rejection of self-justifying mistakes. A mistake may be self-justifying if its realization disqualifies rivals as options. The theory of rationality prevents self-justifying mistakes by recognizing unrealized options.

In some cases, if a coalition were to act differently, other coalitions would act differently too. Such dependencies hold because of an incompatibility of coalitions. They do not signal a change in strategies available to coalitions. If a coalition were to act differently, then others would have

the same strategies. They just would not realize the same strategies. The difference in strategy realized would come from a difference in the acts of the relevant individuals, perhaps a switch from a rational to an irrational act. Because in a single-stage coalitional game all coalitions realize their strategies in the same stage, one coalition's strategy does not change other coalitions' options. If one coalition structure forms, others do not form. They can form, nonetheless, until the game ends without their formation.

A sequential game in which individuals propose coalitions realizes a single-stage coalitional game in which individuals achieve a coalitional structure. Although moves in a single-stage coalitional game do not cause other moves, moves in the underlying sequential game may cause other moves. A coalition's formation does not causally preclude formation of other coalitions comprising its members. However, in the underlying sequential game, members of coalitions with incentives to create a coalition structure may instigate it. Then their formation of coalitions causes the nonformation of coalitions outside the structure. In other cases, individuals' nonformation of some coalition may cause formation of another coalition. A representation of the underlying sequential game may display causal relations among moves that lead to formation of coalitions. Moves of a single-stage coalitional game are not causally related, however. Realization of a coalition structure occurs all at once.

A coalition may have an incentive to move from one option to another. A coalition's incentives depend on its members' incentives. A multimember coalition has an incentive to form when it has a joint strategy that benefits each member. It has an incentive not to form when some member does better on her own or in another multimember coalition under some joint strategy of that coalition. Even one member's profiting from a coalition's not forming creates a coalition's incentive not to form. A coalition has an incentive to disband if any member has an incentive to depart. Although in a single-stage coalitional game, coalitions do not form and then disband, deliberations may suppose a coalition's formation. Given its formation, it may have an incentive to disband prior to the game's conclusion.

Members' unanimous preferences are necessary and sufficient for a coalition's preference

between two joint acts. According to this definition, a coalition's preference ranking of joint acts is incomplete in many cases. For some pairs of joint acts, members' preferences between the acts are not unanimous, and no collective preference exists between them.

The standard to follow collective preferences among joint acts agrees with the standard to follow unanimous individual preferences. The standard of unanimity does not require a definition of collective preferences. The definition of collective preferences and the standard of collective preferences are dispensable in favor of the basic standard of unanimity. It governs joint action in ideal conditions. The standard to follow collective preferences generates a shortcut method of evaluating a joint act. In ideal conditions a coalition following its preferences meets the standard of unanimity.

According to the definition of a coalition's incentives, if a coalition's value exceeds the sum of its members' payoffs given some outcome, then the coalition has an incentive to form and realize an alternative outcome because its members have incentives to form the coalition and realize the alternative. A coalition's incentive holds on balance, factoring in others' response to its pursuit. A coalition's pursuing an incentive to switch strategy may not end a coalitional game because the switch may prompt another coalition's formation. The switch offers a chance that the coalition moves up its preference ranking, however, if there is a chance that the switch is final, that is, a chance that its outcome is a stopping point for coalitions' pursuit of incentives.

All theorists take a coalition's members' unanimous preferences as sufficient for the coalition's having an incentive to change its joint strategy. Not all take it as necessary, however. Some hold that preferences of some members and others' indifference also yield an incentive to change joint strategy. However, making unanimous preferences necessary for a coalition's incentive to switch joint strategies has a theoretical advantage. Implementation of a switch in joint strategies requires all members' participation. Members who are indifferent to the switch may not participate. Rational members may block a coalition's following its preference unless its preference requires their preferences, too. Therefore, I take a coalition to have an incentive to switch joint strategy if and only if all members prefer switching. It follows that a strategy profile

is efficient if and only if the grand coalition lacks an incentive to deviate.[3]

To make strategic reasoning vivid and to follow traditional terminology, I speak of coalitions responding to each other. In their coalitional game their responses are not causal, however. Coalitions do not observe the moves of others before making their own moves. All moves occur in the same stage. A response to a coalition's strategy is just the strategies other coalitions adopt if the coalition adopts the strategy. A response to a coalition's strategy, conjoined with the coalition's strategy, yields a strategy profile with a coalition structure and a joint strategy for each multimember coalition that forms. Agents may causally respond to each other's moves in the underlying sequential game only. There, individuals may accept or reject others' proposals of a strategy profile for the coalitional game. Many combinations of individuals' acts may realize the same joint acts of the coalitional game.

Information affects a coalition's preferences among its options. This section technically defines a coalition's knowledge so that a coalition knows a fact if and only if all its members do. In ideal conditions for joint action, communication is perfect among individuals and coalitions. Because conditions for communication are ideal, information that aids a coalition's joint action spreads among its members. Hence a coalition knows a relevant fact if and only if some member does. This essay treats a coalition's knowledge only in such ideal cases.

In an ideal single-stage coalitional game, each coalition knows others' strategies directly. It does not need to infer their strategies from its strategy or to replicate the reasoning of their members. Individuals have direct, nonstrategic knowledge of the profile realized because they participate in joint acts constituting the profile. Also, because of ideal communication, if a coalition changes its strategy, other coalitions' information changes to preserve its accuracy. Coalitions are prescient about others' acts. Each knows, for each of its strategies, others' response. Supposition of a profile's realization carries to all agents direct, nonstrategic

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[3] In coalitional games where utility is transferable, if a coalition's formation may yield gains for some members without loses for other members, then the coalition may also distribute its value so that each member gains. Consequently, under both accounts of collective incentives, a coalition has an incentive to form if and only if all members may gain from its formation.

knowledge of its realization. A coalition's incentives to switch strategy arise from its knowledge of others' response to its strategy. A strategy occurs in many profiles, of which only one gives others' response to the strategy. A coalition may have an incentive to switch away from the strategy relative to the profile containing the response to it, but lack an incentive to switch away from the strategy relative to some other profile. Incentives are relative to a profile's realization.

An incentive to deviate from a strategy profile not realized rests on knowledge of the profile if it is realized. Supposition of a profile's realization may counterfactually change incentives through changes in information about agents' acts. Supposing that an agent deviates from a strategy profile minimally revises the strategy profile. In a noncooperative game, supposition of an agent's change in strategy does not include other agents' changes in strategy. In a coalitional game, they may include such changes. In a two-person coalitional game, for example, $\{A, B\}$ does not switch from formation to nonformation unless $\{A\}$ switches from nonformation to formation. Calculation of the utilities of an agent's strategies given a strategy profile does not hold fixed the strategies of other agents if the agent's switching strategies entails that other agents also switch strategies.

A profile of strategies for agents indicates the coalitions that form and the joint strategies they adopt. Given a profile, a coalition has an incentive to switch from a strategy s to a strategy s' if and only if (1) s' is a joint strategy and given s each of the coalition's members prefers s' to s, or (2) s' is nonformation and given s at least one of the coalition's members prefers s' to s. This definition acknowledges that a coalition's joint strategy requires the assistance of all its members, whereas any member of a multimember coalition can by herself block the coalition's formation by forming her unit-coalition.

In a coalitional game, each coalition is an agent, and also the group of coalitions is an agent. The group of coalitions achieves the game's outcome. It differs from the grand coalition of all individuals. The grand coalition need not form in order for the group of coalitions to act. If many coalitions form and act, the group of coalitions acts although the grand coalition does not. The group of coalitions resembles a noncollaborating group of individuals in a noncooperative

game. The group of coalitions acts collectively, but not jointly, whatever coalition structure is realized. If two coalitions communicate and strike a bargain, then their members form and act jointly within the coalition of their combined members. The pair of coalitions does not act jointly. In cases where it appears that the group of coalitions acts jointly, coalitions smaller than the grand coalition do not form and their members form the grand coalition and act jointly within it. Strategy profiles are not joint strategies of the group of coalitions because the profiles require only the joint action of coalitions in the coalition structure the profile realizes, and not the joint action of all coalitions.

In some contexts it may be fruitful to treat the whole group of coalitions as an agent. Its acts are rational if the coalitions' acts are rational, so a standard of rationality for it may supply a shortcut method of evaluating coalitions.[4] This essay does not explore such shortcuts and works only with principles applying to coalitions of collaborating individuals. It does not assign options and incentives to the group of coalitions.

2. Paths of Incentives

The next section uses coalitions' paths of pursued incentives to define a strategic equilibrium of a coalitional game. As a preliminary, this section explains such paths. It treats a coalition's pursuit of incentives and a coalition's halting pursuit of incentives.

In a coalitional game a strategy profile specifies a coalition structure according to which some coalitions form and others do not form. The nodes of a path of incentives are strategy profiles. The utility profiles they generate explain incentives to change strategy profile. A profile may start a path of incentives that begins with a formed coalition's incentive to switch joint strategy or with an unformed coalition's incentive to switch from nonformation to formation. A path of incentives for a coalition involves relative incentives, that is, incentives to

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[4] Also, treating the group of coalitions as an agent may yield a shortcut method of identifying a coalitional game's equilibria using an extension of the equilibrium-search methods in Weirich (1998: Sec. 6.4).

switch strategy in the context of a profile. It terminates in a strategy in the context of a profile if, for instance, the coalition has no incentive to switch from the strategy given the profile.

A path of incentives for a coalition depends on other coalitions' pursuit of incentives because a coalition's incentives depend on other coalitions' responses to its strategies. Consider three-person majority-rule division of \$6. The coalition $\{A, B\}$ has an incentive to move from $(2, 2, 2)$ to $(3, 3, 0)$. Suppose that the coalition $\{A, C\}$ pursues its incentive to move from $(3, 3, 0)$ to $(4, 0, 2)$, and the coalition $\{B, C\}$ pursues its incentive to move from $(4, 0, 2)$ to $(0, 2, 4)$. Then, if the coalition $\{A, B\}$ pursues its incentive to move from $(2, 2, 2)$ to $(3, 3, 0)$, the result of the move is $(0, 2, 4)$, and the coalition has a subsequent incentive to move to $(2, 4, 0)$. A path of incentives for multiple coalitions implies a path of incentives for its initial coalition. Deleting other coalitions' responses yields a path of incentives for the coalition that initiates the multicoalition path. Their responses just explain the origin of the coalition's incentives, as the figure shows. Its derived path puts in brackets the response to the coalition $\{A, B\}$'s pursuing its initial incentive.

Multicoalition path: $(2, 2, 2) \rightarrow_{\{A, B\}} (3, 3, 0) \rightarrow_{\{A, C\}} (4, 0, 2) \rightarrow_{\{B, C\}} (0, 2, 4) \rightarrow_{\{A, B\}} (2, 4, 0)$

Derived path for $\{A, B\}$: $(2, 2, 2) \rightarrow_{\{A, B\}} (3, 3, 0) \ [(0, 2, 4)] \rightarrow_{\{A, B\}} (2, 4, 0)$

Figure. Multi- and Single-Coalition Paths of Incentives

A strategy profile specifies a feasible coalition structure and specifies each formed coalition's division of its profits. Several coalitions may have the power to change one strategy profile into another. In a game with exactly two players A and B, both $\{A\}$ and $\{B\}$ have the power to change the coalition structure from $\{\{A, B\}\}$ to $\{\{A\}, \{B\}\}$. I assume that exactly one coalition is the instigator of any change in strategy profile. The instigator is responsible for the change between successive nodes of a multicoalition path of incentives. The coalition instigating a change switches strategy without outsiders' assistance, although its switch may require other

coalitions to form or disband. Individuals' acts in the underlying sequential game make some coalition the instigator of a change. A coalition's pursuing an incentive in a coalitional game instigates a change if and only if in the underlying sequential game some member's pursuing an incentive instigates the coalition's pursuit of the incentive.

Given a profile's realization, an agent may have an incentive to switch strategy. This depends on the consequences of the agent's switching to an alternative strategy. If the agent were to switch to the alternative, what would happen? According to a standard interpretation of conditionals, his switch triggers a minimal departure from the profile. The agent's switch yields the causally nearest world in which he adopts the alternative strategy. The context, especially the type of game, influences the nearness of worlds and thus events in the nearest world with the agent's switch. In a noncooperative game a minimal departure from a profile does not include other agents' switches. In a coalitional game it may.[5]

A path of incentives represents dispositions to pursue incentives if any are pursued. A strategy profile in a path of incentives is the nearest alternative to the preceding profile. Imagining a profile's realization is an evidential supposition. The evidence it carries may bear on the results of a coalition's switching strategy. Imagining a switch is a causal supposition. An agent's switching strategy may prompt other agents' switches in the underlying sequential game. Nearest alternative profiles summarize agents' selection of incentives in a coalitional game. Paths of selected incentives may involve many agents, many profiles, and many incentives.

Some paths of incentives are endless. A path of pursued incentives specifies a halting place for pursuit of incentives. In an ideal game, it contains sufficient incentives to switch strategy. If a nonterminal node is realized, some agent fails to pursue a sufficient incentive to switch strategy. Rational ideal agents pursue all sufficient incentives and do not stop short of a path's terminal node. Of course, stopping short changes pursuit of incentives, but stopping's rationality depends on actual not hypothetical pursuit of incentives.

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[5] I assume the existence of a nearest alternative profile in the cases I treat. A more complex general theory handles cases in which no alternative is nearest.

If the previous section did not restrict coalitions' options, in some ideal coalitional games every profile would generate for some coalition a sufficient incentive to switch strategy. Consider an ideal version of three-person division of $6 by majority rule. Suppose that coalitions pursue optimal incentives. Profiles of the resulting game generate sufficient incentives to switch if, contrary to the account of options, unit-coalitions have nonformation as an option. In that case, for every profile either the grand coalition or a unit-coalition has a sufficient incentive to switch strategy, as the following two paragraphs show.

Take a profile where a unit-coalition forms. Imagine that the coalitional structure is $\{\{A, B\}, \{C\}\}$, and it yields $(3, 3, 0)$. Grant that $\{C\}$ has the option to disband in order to form the coalition $\{B, C\}$ and produce, say, $(0, 4, 2)$. Then it has a path away from formation of $\{C\}$. It has an incentive to switch from formation to nonformation. Others' responses to $\{C\}$'s nonformation may make $\{C\}$ form, but it has no incentive to switch from nonformation to formation. Its formation produces 0, and every other outcome yields at least 0 for $\{C\}$. Thus $\{C\}$ has a path away from the original profile, and that path terminates in nonformation. Given nonformation as a strategy for unit-coalitions, there is a similar terminating path away from every unit-coalition's part in every profile where some unit-coalition forms. A unit-coalition, if rational, pursues such incentives if it controls their pursuit.

In majority-rule division of $6, in a profile where no unit-coalition forms, the grand coalition forms. In every profile where the grand coalition forms, it has a terminating path of incentives away from formation to nonformation. No matter how the grand coalition proposes to divide the $6, two members of the grand coalition have an incentive to desert the grand coalition to form a two-person coalition that gains the $6. This constitutes an incentive of the grand coalition to disband, because a coalition has an incentive to disband if any subcoalition has an incentive to form. The grand coalition has no incentive to form again after it has disbanded. As long as two-person coalitions pursue optimal incentives, the grand coalition cannot give each individual more than she receives in a rival two-person coalition; the grand coalition is not more productive than is a two-person coalition. So the grand coalition has an incentive away from formation to

nonformation, and no incentive away from nonformation. Its path of incentives away from formation terminates in nonformation. Given its formation, it has a sufficient incentive not to form and so pursues that incentive. The grand coalition's formation starts a path of pursued incentives that terminates in a profile where the grand coalition does not form.

In an ideal version of the majority-rule game the grand coalition has an undeniable incentive to disband. Consequently, providing for rationality's attainability requires not recognizing an incentive of a unit-coalition to disband. Rejecting nonformation as an option for a unit-coalition discredits such an incentive. Moreover, as the previous section observes, independent reasons support not recognizing that option.

The previous section defines a coalition's incentives in terms of its members' unanimous preferences. Consequently, a coalition's incentives never proceed from one joint strategy to another in a circle back to the original joint strategy. However, a coalition's incentives to switch strategy, from formation to nonformation and the reverse, may lead the coalition around a circle. A coalition may have an incentive to switch from nonformation to a joint strategy. The joint strategy may then create an incentive for a member to defect, and thus an incentive for the coalition to disband. This cycle does not show that the coalition's incentives to switch strategy are irrational. Incentives to switch strategy are conditional preferences, and rational conditional preferences may generate a cycle. A restless but rational traveler may prefer being in Paris to being in Venice given that he is in Venice, and prefer being in Venice to being in Paris given that he is in Paris.

In a coalitional game, independence of a coalition's value from the acts of outsiders mitigates incentives' relativity to profiles. A coalition's incentives if it forms are constant. Despite entailment relationships among coalitions' strategies, a multimember coalition's payoff from its strategies is independent of others' strategies. If the coalition forms, it gains its value (using an efficient joint strategy). If it does not form, it gains nothing (although its members may gain as members of other coalitions). A change in others' strategies does not influence the coalition's payoff from its strategy. Whatever others do, its possible strategies are unchanged, and its

payoff from its strategy depends only on the strategy it realizes.[6]

A multimember coalition may have a self-undermining incentive. If it does not form, it may have an incentive to form, and yet realize that if it forms it has an incentive to disband. Its formation may lead to responses that it anticipates and that leave it with an incentive to disband. Then its incentive to switch from nonformation to formation is self-undermining. For example, in a three-person game where $\{A, B\}$ does not form, that coalition may have an incentive to form to obtain gains for A and B. But C's response if $\{A, B\}$ forms may give that coalition an incentive to disband. Agent C may offer B inducements to collaborate exclusively with him and so give B an incentive to leave the coalition $\{A, B\}$. Agent C's response does not dissolve the coalition $\{A, B\}$ but gives the coalition an incentive to disband. Because of that incentive, its incentive to switch to formation is self-undermining.

Because of self-undermining incentives, the grand coalition may rationally adopt a collective strategy that does not achieve efficiency. It may fail to form because members pursue incentives to form other coalitions. By halting pursuit of incentives at a profile in which it does not form, it may forgo pursuing an incentive to achieve an outcome better for each member. Rationality requires pursuit of sufficient incentives only, and self-undermining incentives are not sufficient.

Principles of rationality regulate pursuit of incentives, in particular, instigation of a change in strategy profile. A coalition's pursuit of incentives, to be rational, must conform to standards of rationality for selecting incentives to pursue when there are several, and for stopping pursuit of incentives when pursuit of selected incentives is endless. The standards leave agents some latitude. So paths of pursued incentives depend on agents' psychologies. The psychologies of a coalition's members settle its pursuit of incentives. Its pursuit of incentives then settles the behavior rational for it. Rational behavior for the coalition depends on its members' psychologies, as a person's rational choice between chocolate and vanilla ice cream depends on

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[6] Similar independence does not hold for a unit-coalition's payoffs from its strategies. Its payoff from forming is independent of others. It is its value. Its payoff from not forming is also independent of others. It is zero (although the unit-coalition's unique member may gain as a member of a multi-individual coalition that forms). However, as Section 1 explains, the unit-coalition has attempting not to form rather than not forming as an option. Its payoff from attempting not to form depends on others, because its nonformation depends on others.

her tastes.

A selection rule governs the incentive pursued if any is pursued. A stopping rule governs the place pursuit of incentive stops if it stops. The selection and stopping rules that Weirich (1998: Chap. 4) presents for individuals in noncooperative games extend to agents in coalitional games. This section presents only the extension's key points. It does not explore in detail the requirements of rational pursuit of incentives because for its purpose their main consequence is just that rational coalitions may stop pursuit of incentives. Some incentives are insufficient. The stopping rule says that when pursuit of selected incentives is endless, an agent may stop at an incentive that is not a sufficient reason to switch. Pursuit of incentives need not be relentless if that makes pursuit endless.

The details of rational pursuit of incentives, although they add flesh to examples, do not affect Section 3's theoretical points about collective rationality. That section establishes the existence of a strategic equilibrium in an ideal coalitional game without showing that a particular profile is a strategic equilibrium. It derives the collective standard of strategic equilibrium from individual standards using only the general assumption that individuals and coalitions pursue incentives rationally. The derivation does not require rules that explicate rational pursuit of incentives.

This section formulates simplified selection and stopping rules to illustrate such rules. Examples treating ideal games assume that pursuit of incentives is completely rational and complies with the simplified rules. Compliance with those rules may not be sufficient for rationality and may be merely consistent with, rather than necessary for, rationality.

Selection and stopping rules for coalitional games address the incentives of all agents together and through them the incentives of each agent separately. The selection rule permits selecting any coalition with an incentive when there are several and for that coalition selects an incentive optimal to pursue, if one exists. This rule puts aside any considerations favoring a coalition with an incentive that starts a terminating rather than an endless path. It also puts aside any global strategic considerations favoring pursuit of a suboptimal incentive. The stopping rule

permits any agent's halting at any node in a circular path of selected incentives and at any node in a noncircular but endless path of selected incentives, except a node with the agent's initial strategy. It puts aside, for instance, considerations favoring nodes in a cycle to which the path leads.

The selection and stopping rules govern ideal games in which individuals have opportunities for communication and, if necessary, binding agreement, to achieve efficient outcomes for coalitions to which they belong if those coalitions form. A coalition that forms adopts an efficient strategy if it pursues an optimal incentive. Its strategy is efficient for the coalition, given the behavior of other coalitions that form. For example, given the unit-coalition structure, if each unit-coalition maximizes utility among its independent strategies, then the resulting strategy profile achieves efficiency within coalitions that form. Because paths of selected incentives follow optimal incentives, if a coalition pursues incentives beyond its initial strategy, it does not halt pursuit of incentives at a profile in which it forms and adopts an inefficient joint strategy.

Paths of pursued incentives, that is, paths of nearest alternative profiles, depict a coalitional game's deliberational dynamics. They use dispositions to pursue incentives in hypothetical situations to explain how players' strategic reasoning leads them to a particular profile. The dynamics move through tentative decisions about coalition formation and joint action. Stages are argumentative, not temporal, although they have temporal counterparts in the underlying sequential game.

A coalitional game's dynamics depend on the incentives pursued at each strategy profile, given pursuit of some incentive, and the stopping point for pursuit of incentives along an endless path of selected incentives. The selection rule ensures that paths of pursued incentives do not fork, and the stopping rule governs their termination. A path moves from a profile generating an incentive along a single path to a terminal profile. Each starting point yields exactly one endpoint. A strategic equilibrium in the coalitional game depends on constraints on the dynamics of pursuit of incentives, as an equilibrium state of a ball in a basin depends on

momentum and gravity's constraints on the dynamics of the ball's motion in the basin. A strategic equilibrium is a steady state of the dynamics. Its realization responds to all sufficient incentives.

The players at stages of deliberation survey the whole game and deliberation's progress. They process all reasons. The players are strategic, look-ahead reasoners and so can find a global maximum, not just a local maximum. As one sees where a ball rolling in a basin will come to rest and may place the ball there immediately, players see where their tentative decisions will end and may go there immediately. Also, rational players can jump to the best basin of attraction in one step. They need not wait for chance events to create the jump, as in an evolutionary dynamics. The players' realization that they are headed toward an inferior profile may either make them swerve toward a superior profile, or restart their deliberations on a better trajectory.

The dynamics move from one stage of deliberation to another until deliberations reach a halt. When there are several halting places, the dynamics eventually settle on one. The final profile arises from the agents' rationality and their psychologies. The agents' preparation for a coalitional game yields their coordination to realize a particular profile. Deliberation for strategic reasoners may work backwards from a desired end point to a starting point that leads there. Players settling incentives to pursue may begin by observing where they want to end and then devise a way to arrive there. Although a rational player pursues her goals at each stage, the players may reach a particular profile by in advance forming suitable rational dispositions to pursue incentives. Those dispositions create their deliberational dynamics.

3. Strategic Equilibria in Coalitional Games

This section introduces the standard of strategic equilibrium for coalitional games. The standard formulates a requirement of collective rationality. It governs after-the-fact evaluation of a group's act. It does not explicitly direct a group's act although it implies some procedural

principles for favorable cases. After defining strategic equilibrium, the section shows that in every ideal coalitional game a strategic equilibrium exists, and individuals' rationality entails realization of a strategic equilibrium.

Equilibrium is a reasonable requirement for a solution to a coalitional game, but the requirement needs an account of equilibrium according to which every ideal coalitional game has at least one equilibrium. Equilibrium outcomes may exist more widely than do core allocations. The core overlooks types of equilibrium broader than joint utility maximization. The decision principle of self-support, a generalization of the principle of utility maximization, yields another type of equilibrium, namely, joint self-support. In coalitional games the principle of self-support applies to coalitions as well as to individuals.[7]

A strategy profile's being out of equilibrium is a matter of opposition to its strategies. A profile is out of equilibrium if an alternative profile opposes it. One profile opposes another profile if in deliberations it is rational to consider it instead of the other profile. An out-of-equilibrium profile may be decisively rejected in favor of an alternative. Decisive rejection entails that rational deliberation never returns to the profile. Suppose that some coalition has an incentive to switch away from a profile. A second coalition has an incentive to abandon the profile that the first coalition's switch would realize. A third coalition has an incentive to abandon the profile that the second coalition's switch would realize. This path of incentives continues until some coalition's incentive to switch leads it to the original profile. The return to the original profile allows it to be an equilibrium despite the first coalition's incentive to switch away from it.

A path of incentives leading away from a profile does not disqualify that profile as an equilibrium unless it settles on some alternative profile. If the path of incentives does not terminate, then it does not produce an alternative that decisively replaces the original profile in deliberations. For example, suppose that a group may select an annual income for itself. For

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[7] Weirich (1998: Chap. 4) presents the principle of self-support. It is a generalization of the principle of ratification stated in Jeffrey (1983: Chap. 1).

each figure it may select, some alternative is better. Although some path of incentives leads away from selection of any given income, no such path terminates. As a result, the group may select a figure despite the availability of higher figures. Selecting the figure may be an equilibrium of rational deliberations.

Rational pursuit of incentives does not require relentless pursuit of incentives. Pursuit of incentives may stop with a self-supporting strategy, one from which an agent has no sufficient incentive to deviate. Halting pursuit of incentives is not irrational in cases where not all can pursue incentives relentlessly. Although rational individuals seek gains, some may lose without being irrational, as a child playing musical chairs may lose without being irrational.

Applying selection and stopping rules to prune and truncate paths of incentives yields paths of pursued incentives. A path of pursued incentives terminates in a strategy profile. A profile meets opposition if it initiates a path of pursued incentives. Otherwise, it is an equilibrium. It is at the bottom of a basin of attraction in the dynamics pursuit of incentives creates. A strategic equilibrium is a feasible strategy profile such that given the profile no coalition has a path of pursued incentives away from the profile.

Strategic equilibria exist in concrete realizations of coalitional games. An outcome's being a strategic equilibrium depends on more features of a concrete coalitional game than a characteristic function represents (by specifying the value that each coalition can achieve on its own). In particular, it depends on the incentives that coalitions pursue when they cannot pursue all incentives. An adequate representation of a concrete coalitional game supplements a characteristic function with a representation of coalitions' pursuit of incentives, such as a directed graph of the type in Section 2's figure.

A characteristic function represents bargaining leverage that comes from coalition formation but not leverage that comes from, say, being more patient than others are. Pursuit of incentives registers bargaining power that a characteristic function omits. It represents players' psychologies. Concrete coalitional games are not characteristic-function-form games in the sense of being adequately represented by characteristic functions. Because being a strategic

equilibrium is a necessary condition of being a solution, pursuit of incentives is a relevant feature of a concrete coalitional game. An adequate abstract representation of the game includes it.

Using coalitions' paths of pursued incentives to define a strategic equilibrium is equivalent to using multicoalition paths for the same work. If a profile starts no path of pursued incentives, then it starts no path of a single coalition's pursued incentives. Also, any path of pursued incentives away from a profile entails that some coalition has a path of pursued incentives away from the profile. The path's first incentive belongs to some coalition and starts a path for that coalition. Hence a profile starts no path of pursued incentives just in case it starts no path of pursued incentives for any single coalition.

Strategic equilibria are more prevalent than profiles realizing core allocations but still narrow the set of possible solutions in most concrete coalitional games. In a coalitional game with the characteristic function $v(A) = v(B) = v(C) = 1, v(AB) = v(BC) = v(AC) = 4, v(ABC) = 12$, not all strategy profiles are strategic equilibria. Coalitions rationally pursuing incentives continue until the grand coalition forms and divides its value. Profiles without formation of the grand coalition are not strategic equilibria.

The strategic equilibria of a concrete version of three-person majority-rule division of $6 depend on the agents' psychologies. If A does not pursue incentives, then a strategic equilibrium yields $(0, 3, 3)$, and no strategic equilibrium yields $(2, 2, 2)$. The standard of strategic equilibrium narrows the field of candidates for a solution in light of the agents' pursuit of incentives. Other standards of collective rationality may narrow the field further. A solution requires joint rationality, and an equilibrium offers only joint self-support. Joint self-support attends to sufficient incentives but does not consider all reasons. It yields an equilibrium among reasons of a certain type. Some strategic equilibria may fail to be solutions.

In an ideal coalitional game every coalition has a self-supporting strategy. For a strategy fails to be self-supporting only if pursuit of incentives leads to a self-supporting strategy. A profile of self-supporting strategies may not be jointly self-supporting, however, and so may not be a strategic equilibrium. An agent's information about other agents' strategies depends on the

profile realized. A profile of self-supporting strategies' realization may affect information, incentives, and so self-support. In an ideal game, all agents know the coalition structure realized given the profile realized. They know which strategies are feasible given the coalition structure realized. One coalition's self-supporting strategy may be incompatible with another coalition's self-supporting strategy. A profile of self-supporting strategies may not be feasible.

For example, suppose that formation strategies for $\{A\}$ and $\{A, B\}$ are self-supporting taken by themselves. The coalition $\{A, B\}$ may have no incentive to deviate from its formation strategy because neither member does better by deviating. The coalition $\{A\}$ may also have no incentive to deviate from its formation strategy because it does not have the option to disband and form $\{A, B\}$. Although forming $\{A\}$ and forming $\{A, B\}$ are self-supporting, they are not self-supporting taken together because they are incompatible.

Rolling back idealizations may facilitate reaching a strategic equilibrium. If an agent has no information about others' responses to his strategies, problems of strategic reasoning do not arise. An agent's strategy does not furnish evidence about other agents' strategies and so create incentives to deviate. Removing idealizations may make equilibrium easier to achieve by eliminating incentives to deviate from a strategy in the context of a profile. Agents may more easily pursue all incentives, which is a way of achieving strategic equilibrium.

In an ideal (concrete) coalitional game, agents' full rationality includes their rational pursuit of incentives, and their complete information about their game includes information about their pursuit of incentives. Although existence of a strategic equilibrium does not follow from existence of self-supporting strategies, every ideal coalitional game has at least one strategic equilibrium.

The proof that a strategic equilibrium exists shows that for a profile to not be an equilibrium, some other profile must be an equilibrium. To begin, select any profile of an ideal coalitional game. Suppose that it is not an equilibrium. Then it starts a path of pursued incentives. The path terminates with a profile that is an equilibrium because a terminal profile initiates no path. If a coalition has a path of pursued incentives away from a profile, then the terminal strategy for

the coalition and the response to it form an equilibrium. These simple points establish the existence of a strategic equilibrium in every ideal coalitional game.

In a coalitional game, a strategic equilibrium has jointly self-supporting strategies for coalitions. Realization of a strategic equilibrium is equivalent to realization of jointly self-supporting strategies for individuals in the underlying sequential game. Joint self-support for coalitions agrees with joint self-support for individuals because of the relation between incentives for coalitions and for individuals.

Suppose that no coalition has an incentive to deviate from a profile in a coalitional game. Individuals form unit-coalitions, so none has an incentive to deviate unilaterally. None has an incentive to deviate jointly with others, unless that deviation is implementable and so profits each participant. Those conditions yield a coalition's incentive to deviate and so contravene the supposition. Joint acts of coalitions of which an individual is a potential member represent all the individual's opportunities to participate in joint acts. So no individual in the underlying sequential game has an incentive to deviate from the profile. For example, consider an element of the core in a coalitional game with a nonempty core. No coalition, and thus no individual, has an incentive to depart from its realization. Moreover, if some coalition has an incentive to deviate from a profile, then some individual has an incentive to deviate from its realization. The coalition's incentive requires the incentives of all members. Therefore, a coalition has an incentive to deviate from a profile if and only if some individual in the underlying sequential game has an incentive to deviate from its realization.

These points about incentives hold for sufficient incentives also. If no coalition has a sufficient incentive to deviate from a profile, then no individual has a sufficient incentive to deviate either unilaterally or jointly with others. An individual's having a sufficient incentive to deviate requires some multimember- or unit-coalition's having a sufficient incentive to deviate. If all coalitions have excuses for not deviating, then their members have excuses. Moreover, if some coalition has a sufficient incentive to deviate from a profile, then some individual does. Rationality does not require a coalition to act unless it requires at least one member to act.

Therefore, in a coalitional game a profile has jointly self-supporting strategies for coalitions if and only if its realization in the underlying sequential game has jointly self-supporting strategies for individuals.

Because coalitions' realizing a strategic equilibrium is equivalent to individuals' realizing jointly self-supporting strategies, three points follow. (1) Strategic equilibria among coalitions identify strategic equilibria among individuals. (2) Strategic equilibrium is a standard for solutions to coalitional games. (3) Strategic equilibrium is a standard of collective rationality in ideal coalitional games. The remainder of this section establishes these points.

A coalitional game's underlying dynamics indicate strategic reasons for a strategy profile. Many avenues of strategic reasoning may lead to the profile. A profile's status as a strategic equilibrium in a coalitional game may be independent of detailed moves in the underlying sequential game, however. General features of the sequential game's dynamics may ensure that the profile is a strategic equilibrium. They offer a shortcut to identification of strategic equilibria.

For simplicity, a game's analysis using coalitions' pursuit of incentives does not decompose it into individuals' pursuit of incentives. Individuals may pursue incentives in varied ways that do not matter, as long as a coalition's incentives are pursued. The analysis may concentrate on collective dynamics instead of individual dynamics. Economy justifies an analysis that uses collective incentives. The analysis discovers a strategic equilibrium more easily than does application of first principles. Take a coalitional game with a single strategic equilibrium (which may be a core element). Spotting it is quicker than working through the individual incentives that generate it.

Identifying strategic equilibria for coalitions is a shortcut method of identifying strategic equilibria for individuals. As shown, in ideal coalitional games a strategic equilibrium for coalitions results from individual's compliance with the principle of self-support, and individuals' compliance with the principle yields a strategic equilibrium for coalitions. The shortcut method, by identifying strategic equilibria for coalitions, identifies all strategic equilibria for individuals

in the underlying sequential game. The shortcut saves computation but sacrifices generality in identifying profiles in which individuals achieve joint self-support. It addresses ideal games only.

Next, this section shows that strategic equilibrium is a standard for solutions to ideal coalitional games. This requires showing that strategic equilibrium is necessary for a solution. To show that a solution is a strategic equilibrium, one must show that a strategic equilibrium emerges if all coalitions are rational. A demonstration that collective rationality within and among coalitions generates a strategic equilibrium in an ideal coalitional game may use principles of collective rationality, such as the standard of self-support for coalitions. However, such principles are derivative and have restricted range. Support using first principles is more reliable. Standards of rationality apply to a group by way of its members because a group acts through its members and not directly. A group is rational if all its members are rational.[8]

A solution is a profile of strategies that are jointly rational. A solution's realization entails the joint rationality of all agents, and so the rationality of all agents. The rationality of all agents entails the rationality of all individuals. The rationality of all individuals entails realization of a strategic equilibrium. Hence, a solution's realization entails a strategic equilibrium's realization. To back this argument that being a strategic equilibrium is necessary for being a solution, the following paragraph shows that individuals' universal rationality entails realization of a strategic equilibrium. It shows that a strategic equilibrium emerges if all individuals in a coalitional game are rational. Showing that individuals acting rationally in a coalitional game realize a strategic equilibrium confirms that a solution is a strategic equilibrium.

A coalition's paths of incentives are constructed of individuals' paths of incentives. If a coalition fails to pursue a sufficient incentive, then some member fails to pursue a sufficient incentive. If a coalition irrationally fails to pursue an incentive, some member irrationally fails

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[8] Sen's (2002: 212) claim that individual and collective rationality conflict in the Prisoner's Dilemma mistakes efficiency for a requirement of collective rationality. It is only a goal of collective rationality. Weirich (forthcoming: Chap. 4) defends the view that individual rationality entails collective rationality.

to pursue an incentive. If the individuals in a coalitional game fail to realize a strategic equilibrium, then at least one is irrational. If a strategic equilibrium is not realized in an ideal coalitional game, then some individual fails to pursue incentives rationally. Violating a selection or stopping rule entails a coalition's, and at least one member's, irrational response to incentives. For example, stopping pursuit of incentives despite having a sufficient incentive to switch strategy is irrational for a coalition. It results from at least one member's irrationality. In the case of an incentive to form a coalition, it is contrary to a sufficient incentive of every agent. In the case of an incentive to disband, it is contrary to a sufficient incentive of some agent.

As shown, universal rationality entails strategic equilibrium. Using the technical definition of a coalition's options and incentives, the standard of strategic equilibrium for solutions to coalitional games follows from the basic standard of composition by rational acts of individuals. The standard requires a coalition to follow sufficient collective preferences among its options. Rational acts by a coalition's members entail the coalition's compliance with the standard. The entailment, besides verifying the standard of strategic equilibrium for solutions to coalitional games, also establishes the fruitfulness of the technical definitions of a coalition's options and incentives. The definitions make the standard of strategic equilibrium a sound principle of joint rationality.

One may illustrate universal rationality's entailment of strategic equilibrium by analyzing coalitional games as noncooperative games, for example, by analyzing coalition formation as a combination of individuals' strategies. An analysis shows how the entailment proceeds in a coalitional game, first, from individual rationality in the underlying sequential game to strategic equilibrium in the sequential game and, then, to strategic equilibrium in the coalitional game.

Take an ideal version of majority-rule division of \$6 among A, B, and C. Suppose that in the game's realization, the coalition $\{B, C\}$ pursues its incentive from $(2, 2, 2)$ to $(0, 4, 2)$, and its agreement on the division $(0, 4, 2)$ yields the outcome of the underlying sequential game and the coalitional game. In the coalitional game's concrete realization, the coalition $\{A, C\}$ does not pursue its incentive to switch from $(0, 4, 2)$ to $(2, 0, 4)$. Halting pursuit of incentives is rational

for it and for its members. All comply with the selection and stopping rules.

In the game the individuals adopt self-supporting strategies. These strategies, create no sufficient incentive to switch strategies, given each individual's knowledge of the response to all her strategies. Individual A prefers many outcomes to $(0, 4, 2)$ but cannot realize them. Individual C rebuffs her efforts to realize $(2, 0, 4)$, for instance. Although C prefers $(2, 0, 4)$ to $(0, 4, 2)$, he does not pursue that incentive. Consequently, A cannot instigate a switch to $(2, 0, 4)$. No individual has a sufficient incentive to deviate from her part in $(0, 4, 2)$. Each individual's strategy in the profile realizing that division is self-supporting.

Because the individuals have foreknowledge of the strategy profile they realize in the underlying sequential game, each individual's strategy is self-supporting given that profile, and so the strategies in the profile are jointly self-supporting. No individual has a sufficient incentive to switch strategies given the profile. Even taking account of opportunities to instigate a coalition's switch, none has a sufficient incentive to switch. Because no individual has a sufficient incentive to switch, no coalition has a sufficient incentive to switch. The coalition $\{A, C\}$ has an incentive to switch to $(2, 0, 4)$, but that incentive is not sufficient because C does not have a sufficient incentive to do his part in the switch. The strategy profile in the underlying sequential game that realizes $(0, 4, 2)$ is a strategic equilibrium, and so is the strategy profile realizing that division in the coalitional game.

This section's last step is to show that strategic equilibrium is a standard of collective rationality in ideal coalitional games. Collective rationality is not equivalent to realizing a solution. Collective rationality may require more than realizing a profile of strategies that are jointly rational. It may require realizing such a profile for the right reasons. It may have a procedural component. Also, collective rationality may require less than does realizing a solution. Realizing a solution demands realizing a profile of rational strategies. Collective rationality may not demand universal rationality. It tolerates inconsequential mistakes. The players in a game may be collectively rational although one player is irrational in a nondamaging way. A coalition with an irrational member may nonetheless pursue sufficient incentives, for

instance.

A strategic equilibrium is a profile of strategies. Two strategy profiles may generate the same outcome. That is, their outcomes may be the same in relevant respects. Collective rationality demands only an outcome equivalent to the outcome of a profile of universally rational strategies. So collective rationality does not entail realization of a strategic equilibrium. Strategic equilibrium is a standard of collective rationality in the sense that collective rationality entails an outcome the same in relevant respects as the outcome of a strategic equilibrium. The relevant respects depend on what matters to the individuals in the concrete game. Their utility functions represent what matters to them. So their collective rationality entails realizing a utility profile that a strategic equilibrium realizes.

Imagine an ideal coalitional game in which collective rationality requires the rationality of all players. This may happen because collective rationality requires every player to pull his oar; no player's irrationality is inconsequential. In such a game, a single player's irrationality entails collective irrationality. That is, collective rationality entails universal rationality. Because universal rationality entails strategic equilibrium, strategic equilibrium is then necessary for collective rationality in this type of ideal coalitional game.

In an ideal coalitional game, collective rationality yields a utility profile that a solution yields. Realizing a solution entails realizing a strategic equilibrium, as shown. If a utility profile is not the outcome of a strategic equilibrium, then it is not the outcome of a solution. Hence, it is not a product of collective rationality. Collective rationality yields the utility profile a strategic equilibrium generates. Therefore in an ideal coalitional game, strategic equilibrium is a standard of collective rationality in the appropriate sense.

As this essay shows, collective rationality supports strategic equilibria rather than core allocations in ideal coalitional games. Strategic equilibrium makes an attainable standard of collective rationality.

References

Binmore, K. 2007. *Playing for Real: A Text on Game Theory*. New York: Oxford University Press.

Jeffrey, R. 1983. *The Logic of Decision*. Second edition. Chicago: Chicago University Press.

Sen, A. 2002. *Rationality and Freedom*. Cambridge, MA: Harvard University Press.

Weirich, P. 1998. *Equilibrium and Rationality: Game Theory Revised by Decision Rules*. Cambridge: Cambridge University Press.

Weirich, P. Forthcoming. *Collective Rationality: Equilibrium in Cooperative Games*. New York: Oxford University Press.