What might be the case after a change in view

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The Problem

Fuhrmann Triviality Result

Belief revision cannot be "preservative" for reflective agents

This is usually put AGM-wise:

- Epistemic states are belief sets—sets of sentences of our favorite language
- ullet An agent in state K believes arphi iff $arphi\in K$
- Rationality constraints on revision are constraints on the K's

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- Rationality constraints are only as good as the relations of "epistemic commitment"—consequence relations!—they are built on
- Os the consequence relations should be an explicit part of our modeling, not hidden in the background. It's prettier to do that model-theoretically.
- When we do this for modals, it will be a dynamic consequence relation that I will push for

A Not-Very-Diplomatic Subtitle



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Outline

- First Pass
 - Ideology
 - Doxastic Conservatism
 - Reflective Modality: 'Might'
- 2 Triviality
 - One Way
 - And Another Way
- Preservation vs. Persistence
 - Two Ways Out
 - Does LI Plus Vacuity Really Entail Preservation?
- 4 The Positive Bit
 - Updates
 - Back To Revision Models



Preservation

The Conservative's Credo

Information is not gratuitous! Belief change should minimize information loss

We are dealing here with coarse-grained qualitative models of belief change, so this is naturally codified as

Preservation

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Two truisms about might

You have two marbles (red, yellow) and a box. You put one of the marbles in the box without showing me which one. Then I ought to believe

(1) The yellow marble might (in view of what else I believe) be in the box.

Conversely: if I believe something like (1), then I **ought not** believe the yellow marble **isn't** in the box.

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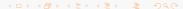
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Well, they're truisms if we assume ...

- This might is epistemic and solpisistic
 - Intuitively: a consistency check on I believe
- 2 That 'belief'-talk is suitably permissive
 - Maybe things like might p aren't truth-bearing, and so maybe strictly speaking belief isn't quite the attitude we have toward them

In Other Words: might is a reflective modal

- My epistemic state commits me to *might* p iff it doesn't commit me to $\neg p$.
- Dually for must p.



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Basic AGM Revision

- States: $K, K', \dots \in \mathbf{K}$
- (More about K in a minute)

Two Constraints Not Up For Grabs

$$S_{AGM} \varphi \in K \star \varphi$$

 $\mathsf{C}_{\mathsf{AGM}}$ If $\neg \varphi \notin \mathsf{Cn}(\emptyset)$ then $K \star \varphi$ is consistent

A belief set K is **consistent** (w.r.t. L^+) iff for no $\varphi \in L^+$ is it the case that $\varphi, \neg \varphi \in K$

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Our Two Players

AGM Preservation

$$P_{AGM}$$
 If $\neg \varphi \notin K$, then $K \subseteq K \star \varphi$

A model is basic iff it satisfies the two non-negotiable constraints plus $P_{\mbox{\scriptsize AGM}}$

Take a $\varphi \in \mathsf{CPL}$ and belief set K. $\mathsf{Poss}(K)$ is the smallest set s.t.

- if $\varphi \in K$, then $\Box \varphi \in K$
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Closure Under Poss

All belief sets $K \in \mathbf{K}$ are closed under Poss—i.e., Poss $(K) \subseteq K$



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The Fuhrmann Result

An (AGM-wise) model is **non-trivial** iff:

For Some $\varphi \in \mathsf{CPL}$ and $K \colon \varphi \not\in K \& \neg \varphi \not\in K$

Proposition (Fuhrmann, Levi)
If a model $\langle K, \star \rangle$ is basic, it is trivial

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Revision Models

Worlds, states Fix a set W of worlds. States s, s', \ldots are subsets of W. I is the set of such s's.

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Consequence relation \models \subseteq I \times L^+
Revision model M = \langle I, \circ, \models \rangle
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C If
$$\llbracket \neg \varphi \rrbracket \neq W$$
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Preservation

P If
$$s \not\models \neg \varphi$$
, then $\{\psi : s \models \psi\} \subseteq \{\psi : s \circ \varphi \models \psi\}$

A model $M = \langle I, \circ, \models \rangle$ is basic iff it satisfies S, C, and P

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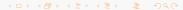
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 $\models \subseteq I \times L^+$ is basically reflective iff:

- $s \models \varphi$ iff $s \subseteq \llbracket \varphi \rrbracket$, for $\varphi \in \mathsf{CPL}$
- if $s \models \varphi$, then $s \models \Box \varphi$
- if $s \not\models \neg \varphi$, then $s \models \Diamond \varphi$
- (truth-functionally equivalent subformulas can be swapped inside the scope of the modals)
- s is consistent w.r.t. \models iff for no $\varphi \in L^+$ is it the case that $s \models \varphi$ and $s \models \neg \varphi$ —i.e., not both are in $\{\psi : s \models \psi\}$
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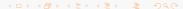


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A Contrived Example

Fix a state s, and form the autoepistemic closure (in L^+) of it. Then you've got yourself a basically reflective consequence relation.

Let K_s be the smallest set s.t.

- $\varphi \in K_s$ iff $s \subseteq \llbracket \varphi \rrbracket$ (for $\varphi \in CPL$)
- if $\varphi \in K_s$, then $\Box \varphi \in K_s$;
- if $\neg \varphi \notin K_s$, then $\Diamond \varphi \in K_s$;
- if $[\alpha] = [\beta]$, then $\psi \in K_s$ iff $\psi[\alpha/\beta] \in K_s$.

Then define:

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$$s \models^+ \varphi$$
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Basic Commitment Is Well-Behaved

Observation

Where \models is a basically reflective relation, and s, s' any states:

- If $s \neq \bot$, then $s \models \Box \varphi$ iff $s \models \varphi$
- For any $\varphi \in \mathsf{CPL}$, either $s \models \Diamond \varphi$ or $s \models \neg \Diamond \varphi$
- If $\{\varphi \in \mathsf{CPL} : s \models \varphi\} = \{\varphi \in \mathsf{CPL} : s' \models \varphi\}$, then $\{\varphi \in L^+ : s \models \varphi\} = \{\varphi \in L^+ : s' \models \varphi\}$ (if s, s' are consistent)

The Fuhrmann Result, Again

- M is the class of revision models with a basically reflective consequence relation
- $\langle I, \circ, \models \rangle$ is **non-trivial** iff there $s \in I$, φ such that $s \not\models \varphi$ and $s \not\models \neg \varphi$

Proposition

If $M \in \mathbf{M}$ is basic, it is trivial.

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Give Up On Reflective Agents

- Things like $\Diamond \varphi$ don't express propositions of the normal sort, and so aren't really the kinds of things that can be the object of belief
- And so they don't really enter into our constraints on revision models at all

- Suppose our revision operator is governed by the Levi Identity—revising by φ decomposes into a contraction/downdate/weakening w.r.t. $\neg \varphi$ followed by an expansion/update w.r.t. φ
- Suppose contraction/downdate/weakening idles on non-belief (Easy Contraction)
- These entail Preservation
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Levi Identity

downdate: $s \subseteq s \downarrow \varphi$

M satisfies the Levi Identity (LI) iff: $s \circ \varphi = (s \downarrow \neg \varphi) \cap \llbracket \varphi \rrbracket$

EW If
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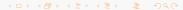
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LI + EW amounts to

$$\mathsf{ER} \ \text{if} \ s \not\models \neg \varphi \ \mathsf{then} \ s \circ \varphi = s \cap \llbracket \varphi \rrbracket$$

- Assume LI and EW for a model $M = \langle I, \circ, \models^+ \rangle$
 - = is our contrived example of a basically reflective consequence relation
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- Consider $s \circ p$. By LI + EW (= ER) $s \circ p = s \cap \llbracket p \rrbracket = \{w_1\}$
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So What's The Deal?

To get the entailment to go through, we also need to assume something about the consequence relation

Persistence

 $\models \subseteq I \times L$ is **persistent** iff for all $\varphi \in L$: if $s \models \varphi$ and $s' \subseteq s$, then $s' \models \varphi$

Observation

Consider a model $M = \langle I, \circ, \models \rangle$. If \models is persistent, then if M satisfies ER it satisfies P

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In the context of these modals persistence is just a bad idea

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Let L^{\diamond} be the smallest set including CPL closed under \neg, \wedge, \Diamond

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- Take any old "broadly conditional" revision model off the shelf
- Swap out the consequence relation in it, and put I⊢ in its place
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