Announcements & Overview

- Administrative Stuff
  ☞ **HW #1 Assigned (see website) – due next Friday (*via* `Blackboard`).**
- Today: Basic Underlying Concepts of Logic (Chapter 1, Cont'd)
  - A Re-Cap of some of the key concepts and definitions so far
    * propositions (as expressed by declarative sentences)
    * actual *vs.* logical truth/falsity
    * sentential form and (sentential) logical truth
    * validity (in general) and sentential validity (in particular)
  - Validity and Soundness of Arguments – Some Examples
  - Two Brain Teasers Involving Validity and Self-Reference
  - A "Big Picture" View of Part I of the Course
  - Preamble to Chapter 2 — The Use/Mention Distinction
  - Introduction to Chapter 2 (the language of sentential logic)

Brief Re-Cap of Concepts and Definitions, So Far

- **Propositions** are assertions that are either **true** or **false**, but not both. They are expressed by declarative sentences in natural language.
- Some propositions are not just true (or false), but **logically true** (or false). A logical truth (falsehood) is true (false) in **all possible worlds**.
- Logical falsehoods are **self-contradictory**. They are false in a way that does not depend on the meanings of their (non-logical) terms, or on which objects they are about. [Similarly for logical truths.]
- **Logical constants** are terms in language which have meanings that do not depend on which objects or concepts the claims in which they occur are about. Examples: **truth-functuonal connectives** (**and**, **or**, *etc.*).
- A **basic/atomic sentence** is one which containts no logical constants. In **sentential logic** (the "Newtonian" formal theory we'll study in Part I of the course), the basic sentences have no truth-functional connectives.

- The **sentential form** (of a sentence or an argument) is determined by replacing (sententially) basic sentences with sentence letters.
  - Branden is 10 feet tall and Branden is not 10 feet tall.
    * Sentential Form: T and not T.
    * This sentence is *logically false*, since it is a *self-contradiction* — no matter which proposition the sentence letter "T" denotes.
  - Either it will snow today or it will not snow today.
    * Sentential Form: Either S or not S.
    * *Logically true* — its falsity would be self-contradictory.
- An **argument** is a set of propositions, one of which (its **conclusion**) is meant to be supported by the rest (its **premises**).
- An argument $\mathscr{A}$ is **valid** (*i.e.*, $\mathscr{A}$'s conclusion **follows logically from** $\mathscr{A}$'s premises) if it is *logically impossible* (*i.e.*, contradictory) that *both*:
  1. $\mathscr{A}$'s premises are true, *and* (at the same time/in the same situation)
  2. $\mathscr{A}$'s conclusion is false.

- Arguments with valid *sentential* forms are said to be ***sententially* valid**.
- Not all ("absolutely") valid arguments are *sententially* valid. E.g.,

  All men are mortal.

  Socrates is a man.

  ∴ Socrates is mortal.
- This argument is valid ("absolutely"), since the only way its premises could be true *while its conclusion is false*, is if Socrates were somehow *both* mortal *and* ***not*** mortal (and that would be a *self-contradiction*).
- However, the *sentential* form of this argument (p. q. $\therefore r$) is not (generally) valid, because it has some invalid instances.
- All instances of a valid argument form are valid. Some instances of an invalid argument form will be invalid. But, some instances of an invalid argument form will also be *valid*!
- $\mathscr{A}$ is **sound** if (i) $\mathscr{A}$ is *valid*, *and* (ii) $\mathscr{A}$'s premises are *all (actually) true*.

Validity and Soundness of Arguments — Some Non-Sentential Examples

- Can we classify the following according to validity/soundness?

1) All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage.	**5)** All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine.
2) All wines are whiskeys. Chardonnay is a wine. Therefore, chardonnay is a whiskey.	**6)** All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine.
3) All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink.	**7)** All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine.
4) All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey.	**8)** All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine.

	Valid	**Invalid**
True premises True conclusion	All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage. [sound]	All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine. [unsound]
True premises False conclusion	**Impossible** None exist	All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine. [unsound]
False premises True conclusion	All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink. [unsound]	All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine. [unsound]
False premises False conclusion	All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey. [unsound]	All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine. [unsound]

- See, also, our validity and soundness handout . . .

Some Brain Teasers Involving Validity and Soundness

- Here are two very puzzling arguments:

($\mathscr{A}_1$) Either $\mathscr{A}_1$ is valid or $\mathscr{A}_1$ is invalid.
$\therefore$ $\mathscr{A}_1$ is invalid.

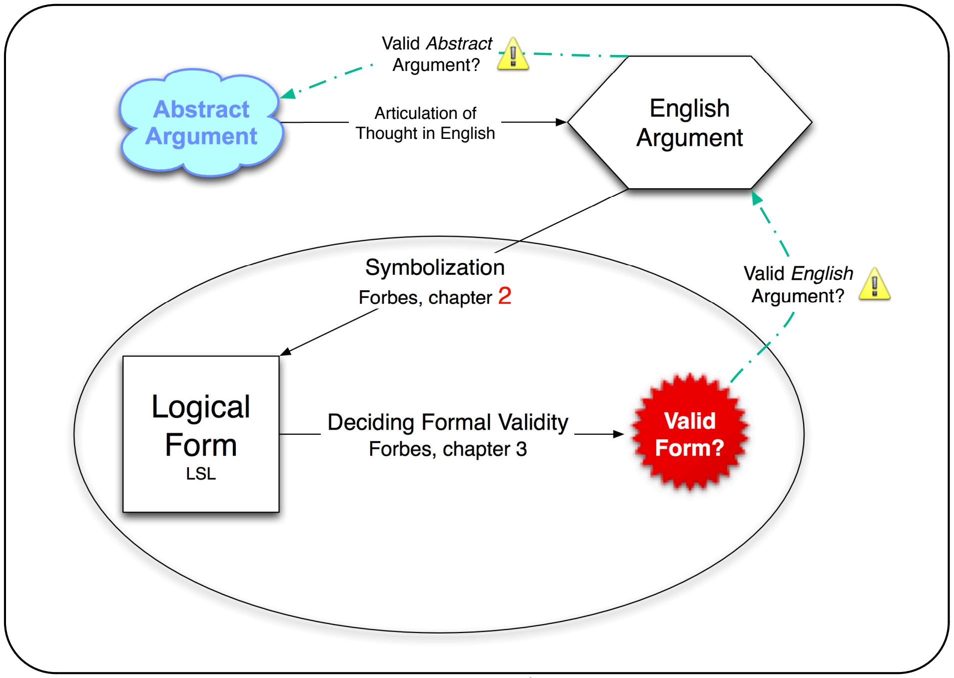
($\mathscr{A}_2$) $\mathscr{A}_2$ is valid.
$\therefore$ $\mathscr{A}_2$ is invalid.

- I'll discuss $\mathscr{A}_2$ ($\mathscr{A}_1$ is left as an exercise).
  - If $\mathscr{A}_2$ is valid, then it has a true premise and a false conclusion. But, this means that if $\mathscr{A}_2$ is valid, then $\mathscr{A}_2$ invalid!
  - If $\mathscr{A}_2$ is invalid, then its conclusion must be true (as a matter of logic). But, this means that if $\mathscr{A}_2$ is invalid then $\mathscr{A}_2$ is valid!
  - This *seems* to imply that $\mathscr{A}_2$ is *both valid and invalid*. But, remember our conservative validity-principle. What is the *logical form* of $\mathscr{A}_2$?

Absolute Validity *vs* Formal Validity

- Forbes calls the general, informal notion of validity "absolute validity".
- Our notion is a bit more conservative than his, since we'll only call an argument valid if one of our *formal theories* captures it as falling under a valid *form*. Our first formal theory (LSL) is about *sentential* validity.
- An argument is *sententially* valid if it has a valid *sentential form*.
- Sentential form is obtained by replacing each basic or atomic sentence in an argument with a corresponding lower-case letter.
- Once we know the sentential form of an argument (chapter 2), we will be able to apply purely formal, mechanical methods (chapters 3 and 4) for determining whether that sentential form is valid.

☞ Even if an argument fails to be *sententially* valid, it could still be valid according to a richer logical theory than LSL. I'll mention some other, more sophisticated theories of logical form later in the course.



Some True/False Questions about Unit #1

1. If the conclusion of an argument cannot (on pain of logical contradiction) be false, then said argument is valid.
2. An argument with premises that are all actually true and a conclusion that is actually false must be invalid.
3. Adding premises to a valid argument can never render it invalid.
4. If an argument is sententially invalid, then it is (absolutely) invalid.
5. All instances of an invalid argument form must be invalid.

Preamble for Chapter 2: The Use/Mention Distinction

- Consider the following two sentences:

  (1) California has more than nine residents.

  (2) ‘California’ has more than nine letters.

- In (1), we are *using* the word ‘California’ to talk about the State of California. But, in (2), we are merely *mentioning* the word ‘California’ (*i.e.*, we're talking about *the word itself*).
- If Jeremiah = ‘California’, which of these sentences are true?

  (3) Jeremiah has (exactly) eight letters [false].

  (4) Jeremiah has (exactly) ten letters [true].

  (5) ‘Jeremiah’ has eight letters [true].

  (6) ‘Jeremiah’ is the name of a state [false].

Preamble for Chapter 2: More on Use/Mention and ‘ ’ *versus* ⌜ ⌝

- Consider the following two statements about LSL sentences

  (*i*) If p and q are both sentences of LSL, then so is ⌜$(p \,\&\, q)$⌝.

  (*ii*) If p and q are both sentences of LSL, then so is ‘$(p \,\&\, q)$’.

- As it turns out, (*i*) is true, but (*ii*) is *false*. The string of symbols ‘$(p \,\&\, q)$’ *cannot* be a sentence of LSL, since ‘p’ and ‘q’ are *not* part of the lexicon of LSL. They allow us to talk about LSL *forms*.
- The trick is that ⌜$(p \,\&\, q)$⌝ abbreviates the long-winded phrase:
  - The symbol-string which results from writing ‘(’ followed by p followed by ‘&’ followed by q followed by ‘)’.
- In (*ii*), we are merely *mentioning* ‘p’ and ‘q’ (in ‘$(p \,\&\, q)$’). But, in (*i*), we are *using* ‘p’ and ‘q’ (in ⌜$(p \,\&\, q)$⌝) to talk about (forms of) sentences in LSL. In (*i*), ‘p’ and ‘q’ are *used* as *metavariables*.

Preamble for Chapter 2: Object language, Metalanguage, *etc.* ...

- LSL is the *object language* of our current studies. The symbol string '$(A \,\&\, B) \vee C$' is a sentence of LSL. But, the symbol string '$(p \,\&\, q) \vee r$' is *not* a sentence of LSL. Why?
- We use a *metalanguage* to talk about the object language LSL. This metalanguage is not formalized. It's mainly English, plus *metavariables* like 'p', 'q', 'r', and *selective quotes* '⌜' and '⌝'.
- If p = '$(A \vee B)$', and q = '$(C \to D)$', then what are the following?
  - ⌜$p \,\&\, q$⌝ [$(A \vee B) \,\&\, (C \to D)$], '$p \,\&\, q$' [$p \,\&\, q$], '$p$' [$p$], '$q$' [$q$]
- And, which of the following are true?
  - p has five symbols [true]. 'p' has five symbols [false].
  - ⌜$p \,\&\, q$⌝ is a sentence of LSL [true]. So is '⌜$p \,\&\, q$⌝' [false].

Introduction to the Syntax of the LSL: The Lexicon

- The syntax of LSL is quite simple. Its lexicon has the following symbols:
  - Upper-case letters 'A', 'B', ... which stand for *basic sentences*.
  - Five *sentential connectives/operators* (one *unary*, four *binary*):

Operator	Name	Logical Function	Used to translate
'$\sim$'	tilde	negation	not, it is not the case that
'&'	ampersand	conjunction	and, also, moreover, but
'$\vee$'	vee	disjunction	or, either ... or ...
'$\to$'	arrow	conditional	if ... then ..., only if
'$\leftrightarrow$'	double arrow	biconditional	if and only if

  - Parentheses '(', ')', brackets '['. ']', and braces '{', '}' for grouping.
- If a string of symbols contains anything else, then it's not a sentence of LSL. And, only *certain* strings of these symbols are LSL sentences.
- Some LSL symbol strings aren't *well-formed*: '$(A \,\&\, B$', '$A \,\&\, B \vee C$', *etc.*

The Five Kinds (Forms) of *Non-Basic* LSL Sentences

- Sentences of the form ⌜$p \,\&\, q$⌝ are called *conjunctions*, and their constituents (p, q) are called *conjuncts*.
- Sentences of the form ⌜$p \vee q$⌝ are called *disjunctions*, and their constituents (p, q) are called *disjuncts*.
- Sentences of the form ⌜$p \to q$⌝ are called *conditionals*. p is called the *antecedent* of ⌜$p \to q$⌝, and q is called its *consequent*.
- Sentences of the form ⌜$p \leftrightarrow q$⌝ are called *biconditionals*. p is called the *left-hand side* of ⌜$p \leftrightarrow q$⌝, and q is its *right-hand side*.
- Sentences of the form ⌜$\sim p$⌝ are called *negations*. The sentence p is called the *negated sentence*.
- These 5 kinds of sentences (+ *atoms*) are the *only* kinds in LSL.
- Next, we begin to think about "translation" from English into LSL.

English $\mapsto$ LSL I: Basic Steps Toward Symbolization

- Sentences with *no* connectives are *trivial* to "translate" or symbolize:
  - 'It is cold.' $\mapsto$ 'C'.
  - 'It is rainy.' $\mapsto$ 'R'.
  - 'It is sunny.' $\mapsto$ 'S'.
- Sentences with just one sentential connective are also pretty easy:
  - 'It is cold and rainy.' $\mapsto$ '$C \,\&\, R$'. [why two atomic letters?]

☞ Try to give the most *precise* (fine-grained) LSL rendition you can, and try to come as close as possible to capturing the meaning of the original.

- Sentences with two connectives can be trickier:
  - 'Either it is sunny or it is cold and rainy.' $\mapsto$ '$S \vee (C \,\&\, R)$'.
- Q: Why is '$(S \vee C) \,\&\, R$' incorrect? A: The English is *not* a conjunction.

English ↦ LSL II: Symbolizing in Two Stages

☞ When symbolizing English sentences in LSL (especially complex ones), it is useful to perform the symbolization in (at least) *two stages*.

Stage 1: Replace all basic sentences (explicit or implicit) with atomic letters. This yields a sentence in "Logish" (neither English nor LSL).

Stage 2: Eliminate remaining English by replacing English connectives with LSL connectives, and properly grouping the resulting symbolic expression (w/parens, *etc.*) to yield pure LSL.

- Here are some simple examples involving only single connectives:

English:	**"Logish":**	**LSL:**
Either it's raining or it's snowing.	Either R or S.	$R \vee S$
If Dell introduces a new line, then Apple will also.	If D, then A.	$D \rightarrow A$
Snow is white and the sky is blue.	W and B.	$W \,\&\, B$
It is not the case that Emily Bronte wrote *Jane Eyre*.	It is not the case that E.	$\sim E$
John is a bachelor if and only if he is unmarried.	J if and only if not M.	$J \leftrightarrow \sim M$

English ↦ LSL III: Symbolizations involving '&' and '∨'

- We use '&' to symbolize a variety of English connectives, including:
  - 'and', 'yet', 'but', 'however', 'moreover', 'nevertheless', 'still', 'also', 'although', 'both', 'additionally', 'furthermore' (and others)
- There is often more to the meaning of 'but', 'nevertheless', 'still', 'although', 'however' (and other such English connectives) than merely 'and'. But, in LSL, the closest we can get to these connectives is '&'.
- On the other hand, there are fewer English expressions that we will symbolize using '∨'. Typically, these involve either 'or' or 'either ... or'.
- But, less typically and more controversially, there is one other English connective we will symbolize as '∨', and that is 'unless'. Seem strange?
- Intuitively, $\ulcorner p$ unless $q\urcorner$ means something like $\ulcorner$if not q, then $p\urcorner$. But, in LSL, $\ulcorner \sim q \rightarrow p\urcorner$ is *equivalent* to (*means the same as*) $\ulcorner p \vee q\urcorner$. [Ch. 3.]

English ↦ LSL IV: Symbolizations involving '→' (and '↔')

☞ We will use '→' to symbolize *many* different English expressions. These will be the most controversial and tricky of our LSL symbolizations. *E.g.*:

- $\ulcorner$if p then $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ implies $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ only if $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner q$ if $p\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ is a sufficient condition for $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner q$ is a necessary condition for $p\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner q$ provided $p\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner q$ whenever $p\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ is contingent upon $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$

- $\ulcorner p \leftrightarrow q\urcorner$ is equivalent to $\ulcorner (p \rightarrow q) \,\&\, (q \rightarrow p)\urcorner$ (so mastering '→' is key)

English ↦ LSL V: Grouping Two or More Binary Connectives

- Whenever three or more LSL sentence letters appear in an LSL sentence, parentheses (or brackets or braces) must be used (carefully!) to indicate the intended *scope* of the connectives. Otherwise, problems ensue ...
- *E.g.*, '$A \,\&\, B \vee C$' is *not* an LSL sentence. It is *ambiguous* between '$(A \,\&\, B) \vee C$' and '$A \,\&\, (B \vee C)$', which are *distinct* LSL sentences.
- The term "*well-formed formula of LSL*" ("LSL WFF") is synonymous with "*LSL sentence*." Non-well-formed strings of symbols aren't sentences.
- In English, the string of English words 'Porch on the is cat a there' is ungrammatical — it is *not well-formed.* All of its *constituent parts* are English words/letters, but (*as a whole*) it's not an English sentence.
- Similarly, in LSL, the following strings of symbols are not WFFs:

'$A \rightarrow \vee B$' '$A \,\&\, B \vee C$' '$A \rightarrow B \rightarrow C$' '$\sim \vee B(\vee C)$' '$A \,\&\, B \,\&\, C$'

English ↦ LSL VI: Negation, Conjunction, and Disjunction

- The tilde '$\sim$' operates *only* on the unit that *immediately* follows it. In '$\sim K \vee M$,' $\sim$ affects only 'K'; in '$\sim(K \vee M)$,' $\sim$ affects the entire '$K \vee M$'.
- 'It is not the case that K or M' is *ambiguous* between '$\sim K \vee M$,' and '$\sim(K \vee M)$.' **Convention**: 'It is not the case that K or M' ↦ '$\sim K \vee M$'.
- 'Not both S and T' ↦ '$\sim(S \,\&\, T)$'. [Chapter 3: '$\sim(S \,\&\, T)$' *means the same as* '$\sim S \vee \sim T$'. But, '$\sim(S \,\&\, T)$' does *not* mean the same as '$\sim S \,\&\, \sim T$'.]
- 'Not either S or T' ↦ '$\sim(S \vee T)$'. [Chapter 3: '$\sim(S \vee T)$' *means the same as* '$\sim S \,\&\, \sim T$', but '$\sim(S \vee T)$' does *not* mean the same as '$\sim S \vee \sim T$'.]
- Here are some examples involving $\sim$, &, and $\vee$ (not, and, or):
  1. Shell is not a polluter, but Exxon is. ↦ ??
  2. Not both Shell and Exxon are polluters. ↦ ??
  3. Both Shell and Exxon are not polluters. ↦ ??

4. Not either Shell or Exxon is a polluter. ↦ ??
5. Neither Shell nor Exxon is a polluter. ↦ ??
6. Either Shell or Exxon is not a polluter. ↦ ??

- Summary of translations involving $\sim$, &, and $\vee$ (not, and, or):

"Logish"	**LSL**
Not either A or B.	$\sim(A \vee B)$
Either not A or not B	$\sim A \vee \sim B$
Not both A and B.	$\sim(A \,\&\, B)$
Both not A and not B. (Neither A nor B.)	$\sim A \,\&\, \sim B$

- DeMorgan Laws (we will *prove* these laws is Chapters 3 & 4):

  $\ulcorner\sim(p \vee q)\urcorner$ is equivalent to (means the same as) $\ulcorner\sim p \,\&\, \sim q\urcorner$

  $\ulcorner\sim(p \,\&\, q)\urcorner$ is equivalent to (means the same as) $\ulcorner\sim p \vee \sim q\urcorner$

- But, $\ulcorner\sim(p \vee q)\urcorner$ is *not* equivalent to $\ulcorner\sim p \vee \sim q\urcorner$.
- And, $\ulcorner\sim(p \,\&\, q)\urcorner$ is *not* equivalent to $\ulcorner\sim p \,\&\, \sim q\urcorner$.

English ↦ LSL VII: Summary of the LSL Connectives

English Expression	**LSL Connective**
not, it is not the case that, it is false that	$\sim$
and, yet, but, however, moreover, nevertheless, still, also, although, both, additionally, furthermore	&
or, unless, either … or …	$\vee$
if … then …, only if, given that, in case, provided that, on condition that, sufficient condition, necessary condition, unless (**Note**: don't confuse antecedents/consequents!)	$\rightarrow$
if and only if (iff), is equivalent to, sufficient and necessary condition for, necessary and sufficient condition for	$\leftrightarrow$