Part (a) of Hunter's Proof of Henkin's Completeness Theorem for *PS*

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The first part of Henkin's completeness proof involves proving the following seven *theorem schemas*.

1. $\vdash_{PS} A \supset A$
2. $\vdash_{PS} A \supset (B \supset A)$
3. $\vdash_{PS} (A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$
4. $\vdash_{PS} {\sim}A \supset (A \supset B)$
5. $\vdash_{PS} A \supset ({\sim}B \supset {\sim}(A \supset B))$
6. $\vdash_{PS} (A \supset B) \supset ((A \supset {\sim}B) \supset {\sim}A)$
7. $\vdash_{PS} ({\sim}A \supset B) \supset (({\sim}A \supset {\sim}B) \supset A)$

We have already seen a proof of (1). (2) and (3) are (PS1) and (PS2). So, we just need proofs of the four theorem schemas (4)–(7). Here is a *sketch* (!) of one such proof (4 goals in **boldface**). Exercise: figure out the substitution instances of the formulas (listed on the right) required to generate each MP step.

PS1	$A \supset (B \supset A)$	
PS2	$A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$	
PS3	$({\sim}A \supset {\sim}B) \supset (B \supset A)$	
1.	$A \supset (B \supset (C \supset B))$	[MP, PS1, PS1]
2.	$((A \supset (B \supset C)) \supset (A \supset B)) \supset ((A \supset (B \supset C)) \supset (A \supset C))$	[MP, PS2, PS2]
3.	$A \supset (({\sim}B \supset {\sim}C) \supset (C \supset B))$	[MP, PS3, PS1]
4.	$(A \supset ((B \supset A) \supset C)) \supset (A \supset C)$	[MP, 2, 1]
5.	$(A \supset ({\sim}B \supset {\sim}C)) \supset (A \supset (C \supset B))$	[MP, 3, PS2]
6.	$\boldsymbol{{\sim}A \supset (A \supset B)}$	**[MP, 5, PS1]**
7.	${\sim}{\sim}A \supset (B \supset A)$	[MP, 6, 5]
8.	${\sim}{\sim}A \supset A$	[MP, 7, 4]
9.	$A \supset {\sim}{\sim}A$	[MP, 8, PS3]
10.	$(({\sim}{\sim}A \supset B) \supset A) \supset (({\sim}{\sim}A \supset B) \supset B)$	[MP, 8, PS2]
11.	$A \supset ({\sim}{\sim}B \supset B)$	[MP, 8, PS1]
12.	$A \supset (B \supset {\sim}{\sim}B)$	[MP, 9, PS1]
13.	$A \supset ((({\sim}{\sim}B \supset C) \supset B) \supset (({\sim}{\sim}B \supset C) \supset C))$	[MP, 10, PS1]
14.	$({\sim}{\sim}A \supset (A \supset B)) \supset ({\sim}{\sim}A \supset B)$	[MP, 11, 2]
15.	$A \supset (({\sim}{\sim}B \supset (B \supset C)) \supset ({\sim}{\sim}B \supset C))$	[MP, 12, PS2]
16.	$(A \supset B) \supset (A \supset {\sim}{\sim}B)$	[MP, 13, 4]
17.	$({\sim}A \supset B) \supset ({\sim}B \supset A)$	[MP, 14, PS1]
18.	$A \supset (({\sim}B \supset C) \supset ({\sim}C \supset B))$	[MP, 15, 5]
19.	$A \supset (({\sim}{\sim}A \supset B) \supset B)$	[MP, 16, 5]
20.	$A \supset (B \supset {\sim}(A \supset {\sim}B))$	[MP, 17, 4]
21.	$(A \supset B) \supset (A \supset {\sim}(A \supset {\sim}B))$	[MP, 18, PS1]
22.	$\boldsymbol{({\sim}A \supset B) \supset (({\sim}A \supset {\sim}B) \supset A)}$	**[MP, 19, PS2]**
23.	$(A \supset B) \supset ({\sim}{\sim}A \supset B)$	[MP, 20, 5]
24.	$(A \supset {\sim}B) \supset (B \supset {\sim}A)$	[MP, 21, PS2]
25.	$A \supset ((B \supset {\sim}C) \supset (C \supset {\sim}B))$	[MP, 22, 5]
26.	$(A \supset ({\sim}B \supset C)) \supset (A \supset ({\sim}C \supset B))$	[MP, 23, PS1]
27.	$\boldsymbol{A \supset ({\sim}B \supset {\sim}(A \supset B))}$	**[MP, 24, 16]**
28.	$(A \supset (B \supset {\sim}C)) \supset (A \supset (C \supset {\sim}B))$	[MP, 26, PS2]
29.	$\boldsymbol{(A \supset B) \supset ((A \supset {\sim}B) \supset {\sim}A)}$	**[MP, 28, 22]**