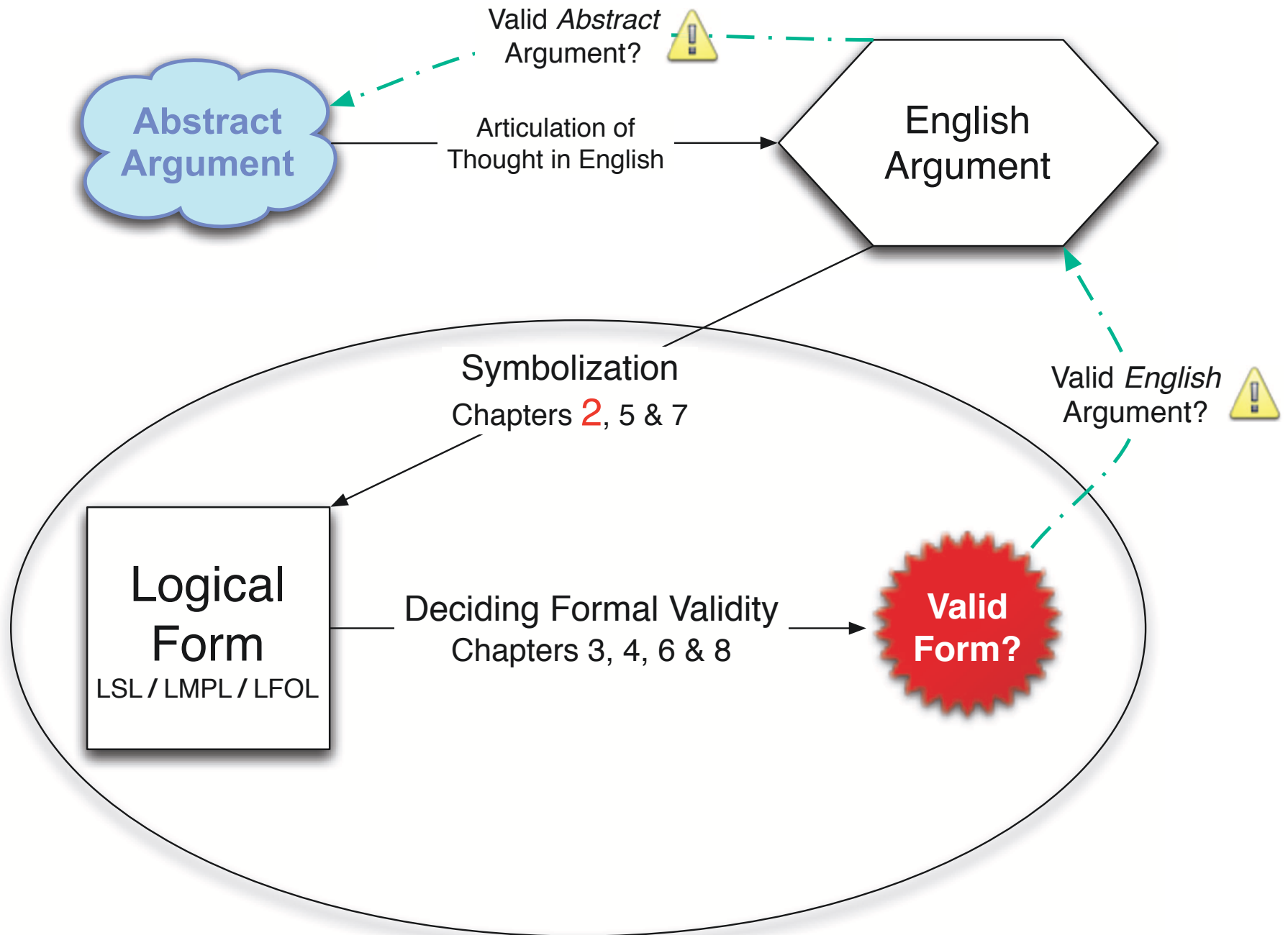


Announcements & Such

- *Miles Davis & John Coltrane: So What*
- Administrative Stuff
 - Permanent locations for all sections are now known (see website).
 - HW #1 is due today (by 4pm in the 12A drop box outside 301 Moses).
 - HW #1 will be returned Tues. Re-sub due Thursday (4pm, drop box).
- ☞ **Homework formatting. Please put the following information:**
 - * **Name, GSI, section time, and date.**
 - on all assignments and exams (upper-right corner of first page).**
 - Branden will not have office hours today.
- Introduction to the Course & Chapter 1 of Forbes
 - Chapter 2 — The Language of Sentential Logic (LSL)
 - Today: syntax, use/mention, and intro. to symbolization



Introduction to the Syntax of the LSL: The Lexicon

- The syntax of LSL is quite simple. Its lexicon has the following symbols:
 - Upper-case letters ‘A’, ‘B’, ... which stand for *basic sentences*.
 - Five *sentential connectives/operators* (one *unary*, four *binary*):

Operator	Name	Logical Function	Used to translate
‘~’	tilde	negation	not, it is not the case that
‘&’	ampersand	conjunction	and, also, moreover, but
‘∨’	vee	disjunction	or, either ... or ...
‘→’	arrow	conditional	if ... then ..., only if
‘↔’	double arrow	biconditional	if and only if

- Parentheses ‘(’, ‘)’, brackets ‘[’, ‘]’, and braces ‘{’, ‘}’ for grouping.
- If a string of symbols contains anything else, then it’s not a sentence of LSL. And, only *certain* strings of these symbols are LSL sentences.
- Some LSL symbol strings aren’t *well-formed*: ‘(A & B’, ‘A & B ∨ C’, etc.

Digression: The Use/Mention Distinction

- Consider the following two sentences:
 - (1) California has more than nine residents.
 - (2) 'California' has more than nine letters.
- In (1), we are *using* the word 'California' to talk about the State of California. But, in (2), we are merely *mentioning* the word 'California' (*i.e.*, we're talking about *the word itself*).
- If Jeremiah = 'California', which of these sentences are true?
 - (3) Jeremiah has (exactly) eight letters [false].
 - (4) Jeremiah has (exactly) ten letters [true].
 - (5) 'Jeremiah' has eight letters [true].
 - (6) 'Jeremiah' is the name of a state [false].

Digression: More on Use/Mention and ‘ ’ versus ‘ ’

- Consider the following two statements about LSL sentences
 - (i) If p and q are both sentences of LSL, then so is $\ulcorner (p \ \& \ q) \urcorner$.
 - (ii) If p and q are both sentences of LSL, then so is $\langle (p \ \& \ q) \rangle$.
- As it turns out, (i) is true, but (ii) is *false*. The string of symbols $\langle (p \ \& \ q) \rangle$ *cannot* be a sentence of LSL, since ‘ p ’ and ‘ q ’ are *not* part of the lexicon of LSL. They allow us to talk about LSL *forms*.
- The trick is that $\ulcorner (p \ \& \ q) \urcorner$ abbreviates the long-winded phrase:
 - The symbol-string which results from writing ‘(’ followed by p followed by ‘&’ followed by q followed by ‘)’.
- In (ii), we are merely *mentioning* ‘ p ’ and ‘ q ’ (in $\langle (p \ \& \ q) \rangle$). But, in (i), we are *using* ‘ p ’ and ‘ q ’ (in $\ulcorner (p \ \& \ q) \urcorner$) to talk about (forms of) sentences in LSL. In (i), ‘ p ’ and ‘ q ’ are *used* as *metavariables*.

Digression: Object language, Metalanguage, *etc.* ...

- LSL is the *object language* of our current studies. The symbol string ' $(A \& B) \vee C$ ' is a sentence of LSL. But, the symbol string ' $(p \& q) \vee r$ ' is *not* a sentence of LSL. Why?
- We use a *metalanguage* to talk about the object language LSL. This metalanguage is not formalized. It's mainly English, plus *metavariables* like ' p ', ' q ', ' r ', and *selective quotes* ' \ulcorner ' and ' \urcorner '.
- If $p = '(A \vee B)'$, and $q = '(C \rightarrow D)'$, then what are the following?
 - ' $p \& q$ ' [$(A \vee B) \& (C \rightarrow D)$], ' $p \& q$ ' [$p \& q$], ' p ' [p], ' q ' [q]
- And, which of the following are true?
 - p has five symbols [true]. ' p ' has five symbols [false].
 - ' $p \& q$ ' is a sentence of LSL [true]. So is ' $\ulcorner p \& q \urcorner$ ' [false].

The Five Kinds (Forms) of *Non-Basic* LSL Sentences

- Sentences of the form ' $p \ \& \ q$ ' are called *conjunctions*, and their constituents (p , q) are called *conjuncts*.
- Sentences of the form ' $p \ \vee \ q$ ' are called *disjunctions*, and their constituents (p , q) are called *disjuncts*.
- Sentences of the form ' $p \rightarrow q$ ' are called *conditionals*. p is called the *antecedent* of ' $p \rightarrow q$ ', and q is called its *consequent*.
- Sentences of the form ' $p \leftrightarrow q$ ' are called *biconditionals*. p is called the *left-hand side* of ' $p \leftrightarrow q$ ', and q is its *right-hand side*.
- Sentences of the form ' $\sim p$ ' are called *negations*. The sentence p is called the *negated sentence*.
- These 5 kinds of sentences (+ *atoms*) are the *only* kinds in LSL.
- Next, we begin to think about “translation” from English into LSL.

English \mapsto LSL I: Basic Steps Toward Symbolization

- Sentences with *no* connectives are *trivial* to “translate” or symbolize:
 - ‘It is cold.’ \mapsto ‘*C*’.
 - ‘It is rainy.’ \mapsto ‘*R*’.
 - ‘It is sunny.’ \mapsto ‘*S*’.
- Sentences with just one sentential connective are also pretty easy:
 - ‘It is cold and rainy.’ \mapsto ‘*C & R*’. [why two atomic letters?]
- ☞ Try to give the most *precise* (fine-grained) LSL rendition you can, and try to come as close as possible to capturing the meaning of the original.
- Sentences with two connectives can be trickier:
 - ‘Either it is sunny or it is cold and rainy.’ \mapsto ‘*S \vee (C & R)*’.
- Q: Why is ‘(*S \vee C*) & *R*’ incorrect? A: The English is *not* a conjunction.

English \mapsto LSL II: Symbolizing in Two Stages

☞ When symbolizing English sentences in LSL (especially complex ones), it is useful to perform the symbolization in (at least) *two stages*.

Stage 1: Replace all basic sentences (explicit or implicit) with atomic letters. This yields a sentence in “Logish” (neither English nor LSL).

Stage 2: Eliminate remaining English by replacing English connectives with LSL connectives, and properly grouping the resulting symbolic expression (w/parens, *etc.*) to yield pure LSL.

- Here are some simple examples involving only single connectives:

English:

Either it's raining or it's snowing.

If Dell introduces a new line, then Apple will also.

Snow is white and the sky is blue.

It is not the case that Emily Bronte wrote *Jane Eyre*.

John is a bachelor if and only if he is unmarried.

“Logish”:

Either R or S .

If D , then A .

W and B .

It is not the case that E .

J if and only if not M .

LSL:

$R \vee S$

$D \rightarrow A$

$W \& B$

$\sim E$

$J \leftrightarrow \sim M$

English \mapsto LSL III: Symbolizations involving ‘&’ and ‘ \vee ’

- We use ‘&’ to symbolize a variety of English connectives, including:
 - ‘and’, ‘yet’, ‘but’, ‘however’, ‘moreover’, ‘nevertheless’, ‘still’, ‘also’, ‘although’, ‘both’, ‘additionally’, ‘furthermore’ (and others)
- There is often more to the meaning of ‘but’, ‘nevertheless’, ‘still’, ‘although’, ‘however’ (and other such English connectives) than merely ‘and’. But, in LSL, the closest we can get to these connectives is ‘&’.
- On the other hand, there are fewer English expressions that we will symbolize using ‘ \vee ’. Typically, these involve either ‘or’ or ‘either ... or’.
- But, less typically and more controversially, there is one other English connective we will symbolize as ‘ \vee ’, and that is ‘unless’. Seem strange?
- Intuitively, ‘ p unless q ’ means something like ‘if not q , then p ’. But, in LSL, ‘ $\sim q \rightarrow p$ ’ is *equivalent* to (means the same as) ‘ $p \vee q$ ’. [Ch. 3.]

English \mapsto LSL IV: Symbolizations involving ' \rightarrow ' (and ' \leftrightarrow ')

☞ We will use ' \rightarrow ' to symbolize *many* different English expressions. These will be among the most tricky of our LSL symbolizations. It is very important that you remember these various expressions involving ' \rightarrow '!

- 'if p then q ' \mapsto ' $p \rightarrow q$ '
- ' p implies q ' \mapsto ' $p \rightarrow q$ '
- ' p only if q ' \mapsto ' $p \rightarrow q$ '
- ' q if p ' \mapsto ' $p \rightarrow q$ '
- ' p is a sufficient condition for q ' \mapsto ' $p \rightarrow q$ '
- ' q is a necessary condition for p ' \mapsto ' $p \rightarrow q$ '
- ' q provided p ' \mapsto ' $p \rightarrow q$ '
- ' q whenever p ' \mapsto ' $p \rightarrow q$ '
- ' p is contingent upon q ' \mapsto ' $p \rightarrow q$ '
- ' $p \leftrightarrow q$ ' is equivalent to ' $(p \rightarrow q) \& (q \rightarrow p)$ ' (so mastering ' \rightarrow ' is key)

English \rightarrow LSL V: More on Conditionals & Biconditionals

- 'if p then q ' and ' q if p ' both get translated as ' $p \rightarrow q$ '.
- 'if p then q ', ' q if p ' and ' $p \rightarrow q$ ' are all ways of saying p is a *sufficient condition* for q (or q is a *necessary condition* for p).
- ' q only if p ', however, is symbolized ' $q \rightarrow p$ ', and says that p is a *necessary condition* for q (or q is a *sufficient condition* for p).
- It is important not to confuse necessary conditions with sufficient conditions (or, 'if' with 'only if'). Helpful examples:
 'Your computer will work *only if* it is plugged in.' (true)
 versus
 'Your computer will work *if* it is plugged in.' (false!)
- Prerequisites are *necessary* but *not* sufficient for getting into a course. *If* you get in, *then* you've satisfied the prerequisites (\leftarrow).

English \rightarrow LSL VI: More on \rightarrow and \leftrightarrow , Continued

- In English, there are many ways to say 'if p then q ', e.g., ' q , provided p ' and ' q , whenever p '. These all become ' $p \rightarrow q$ '.
- ' p unless q ' and 'unless q , p ' both get translated as ' $\sim q \rightarrow p$ ' (or as ' $q \vee p$ '). In chapter 3, we'll see why these are *equivalent*.
- 'Your computer will *not* work *unless* it is plugged in' says your computer being plugged in is a *necessary condition* for your computer to work (' $\sim W$ unless P ' \mapsto ' $\sim P \rightarrow \sim W$ ' \approx ' $W \rightarrow P$ ').
- Necessary conditions N are consequents, and sufficient conditions S are antecedents: ' $S \rightarrow N$ ' (a useful mnemonic).
- 'if p then q and if q then p ' (i.e., ' p if and only if q ', or, for short, ' p iff q ') gets translated into the *biconditional* ' $p \leftrightarrow q$ '.
- ' $p \leftrightarrow q$ ' says that p is *both* necessary *and* sufficient for q .
- ' $p \leftrightarrow q$ ' is basically an *abbreviation* for ' $(p \rightarrow q) \& (q \rightarrow p)$ '.

English \rightarrow LSL VII: Grouping Two or More Binary Connectives

- Whenever three or more LSL sentence letters appear in an LSL sentence, parentheses (or brackets or braces) must be used (carefully!) to indicate the intended *scope* of the connectives. Otherwise, problems ensue ...
- *E.g.*, ' $A \& B \vee C$ ' is *not* an LSL sentence. It is *ambiguous* between ' $(A \& B) \vee C$ ' and ' $A \& (B \vee C)$ ', which are *distinct* LSL sentences.
- In this case, ' $(A \& B) \vee C$ ' and ' $A \& (B \vee C)$ ' have *different meanings*. We'll see precisely why they have different meanings in chapter 3.
- **NOTE:** We **must** group expressions when we have two or more connectives — *even if the alternative groupings have the same meaning*.
 - ' $A \vee (B \vee C)$ ' and ' $(A \vee B) \vee C$ ' have the same meaning, and
 - ' $A \& (B \& C)$ ' and ' $(A \& B) \& C$ ' have the same meaning.
- But, we must choose one of these groupings when symbolizing. It doesn't matter *which* one we choose, but we must choose one.

English \mapsto LSL VIII: Negation, Conjunction, and Disjunction

- The tilde ' \sim ' operates *only* on the unit that *immediately* follows it. In ' $\sim K \vee M$,' \sim affects only ' K '; in ' $\sim(K \vee M)$,' \sim affects the entire ' $K \vee M$ '.
- 'It is not the case that K or M ' is *ambiguous* between ' $\sim K \vee M$,' and ' $\sim(K \vee M)$.' **Convention:** 'It is not the case that K or M ' \mapsto ' $\sim K \vee M$ '.
- 'Not both S and T ' \mapsto ' $\sim(S \& T)$ '. [Chapter 3: ' $\sim(S \& T)$ ' *means the same as* ' $\sim S \vee \sim T$ '. But, ' $\sim(S \& T)$ ' does *not* mean the same as ' $\sim S \& \sim T$ '.]
- 'Not either S or T ' \mapsto ' $\sim(S \vee T)$ '. [Chapter 3: ' $\sim(S \vee T)$ ' *means the same as* ' $\sim S \& \sim T$ ', but ' $\sim(S \vee T)$ ' does *not* mean the same as ' $\sim S \vee \sim T$ '.]
- Here are some examples involving \sim , $\&$, and \vee (not, and, or):
 1. Shell is not a polluter, but Exxon is. \mapsto ??
 2. Not both Shell and Exxon are polluters. \mapsto ??
 3. Both Shell and Exxon are not polluters. \mapsto ??

4. Not either Shell or Exxon is a polluter. $\mapsto ??$

5. Neither Shell nor Exxon is a polluter. $\mapsto ??$

6. Either Shell or Exxon is not a polluter. $\mapsto ??$

- Summary of translations involving \sim , $\&$, and \vee (not, and, or):

“Logish”

LSL

Not either A or B .

$\sim(A \vee B)$

Either not A or not B

$\sim A \vee \sim B$

Not both A and B .

$\sim(A \& B)$

Both not A and not B . (Neither A nor B .)

$\sim A \& \sim B$

- DeMorgan Laws (we will *prove* these laws in Chapter 3):

$\lceil \sim(p \vee q) \rceil$ is equivalent to (means the same as) $\lceil \sim p \& \sim q \rceil$

$\lceil \sim(p \& q) \rceil$ is equivalent to (means the same as) $\lceil \sim p \vee \sim q \rceil$

- But, $\lceil \sim(p \vee q) \rceil$ is *not* equivalent to $\lceil \sim p \vee \sim q \rceil$.
- And, $\lceil \sim(p \& q) \rceil$ is *not* equivalent to $\lceil \sim p \& \sim q \rceil$.

English \rightarrow LSL IX: Summary of the LSL Connectives

English Expression

LSL Connective

not, it is not the case that, it is false that

\sim

and, yet, but, however, moreover, nevertheless, still, also,
although, both, additionally, furthermore

$\&$

or, unless, either ... or ...

\vee

if ... then ..., only if, given that, in case, provided that, on
condition that, sufficient condition, necessary condition,
unless (**Note:** don't confuse antecedents/consequents!)

\rightarrow

if and only if (iff), is equivalent to, sufficient and necessary
condition for, necessary and sufficient condition for

\leftrightarrow

English \mapsto LSL X (&, \rightarrow): Example #1

- ‘John will study hard and also bribe the instructor, and if he does both then he’ll get an “A”, provided the instructor likes him.’
 - Step 0: Decide on atomic sentences and letters.
 S : John will study hard. A : John will get an “A”.
 B : John will bribe the instructor. L : The instructor likes John.
 - Step 1: Substitute into English, yielding “Logish”:
 S and B , and if S and B then A , provided L .
 - Step 2: Make the transition into LSL (in stages as well, perhaps):
 S and B , and if L , then if S and B then A .
 $(S \ \& \ B) \ \& \ (L \rightarrow (\text{if } S \text{ and } B \text{ then } A))$.

Final Product: $(S \ \& \ B) \ \& \ (L \rightarrow ((S \ \& \ B) \rightarrow A))$

English \mapsto LSL II (\sim , $\&$, \leftrightarrow): Example #2

- ‘If, but only if, they have made no commitment to the contrary, may reporters reveal their sources, but they always make such a commitment and they ought to respect it.’

- Step 0: Decide on atomic sentences and letters.

S: Reporters may reveal their sources.

C: Reporters have made a commitment to protect their sources.

R: Reporters ought to respect their commitment to protect sources.

- Step 1: Substitute into English, yielding “Logish”:

If, but only if, it is not the case that *C*, then *S*, but *C* and *R*.

- Step 2: make the transition into LSL (in stages as well, perhaps):

S iff not *C*, but *C* and *R*.

Final Product: $(S \leftrightarrow \sim C) \& (C \& R)$

English \mapsto LSL II (\sim , $\&$, \vee , \rightarrow , \leftrightarrow): Example #3

- ‘Sara is going unless either Richard or Pam is going, and Sara is not going if, and only if, neither Pam nor Quincy are going.’
 - Step 0: Decide on atomic sentences and letters.

P : Pam is going. Q : Quincy is going.
 R : Richard is going. S : Sam is going.
 - Step 1: Substitute into English, yielding “Logish”:

S unless either R or P , and not S iff neither P nor Q .
 - Step 2: Make the transition into LSL (in stages again):

S unless $(R \vee P)$, and $\sim S$ iff $(\sim P \& \sim Q)$
 $(\sim(R \vee P) \rightarrow S) \& (\sim S \leftrightarrow (\sim P \& \sim Q))$
- It is also acceptable to replace the ‘unless’ with ‘ \vee ’, yielding:

$(S \vee (R \vee P)) \& (\sim S \leftrightarrow (\sim P \& \sim Q))$

English \mapsto LSL VIII: Some More Problems to Try

- A Bunch of LSL Symbolization Problems:
 1. California does not allow smoking in restaurants.
 2. Jennifer Lopez becomes a superstar given that *I'm Real* goes platinum.
 3. Mary-Kate Olsen does not appear in a movie unless Ashley does.
 4. Either the President supports campaign reform and the House adopts universal healthcare or the Senate approves missile defense.
 5. Neither Mylanta nor Pepcid cures headaches.
 6. If Canada subsidizes exports, then if Mexico opens new factories, then the United States raises tariffs.
 7. If Iraq launches terrorist attacks, then either Peter Jennings or Tom Brokaw will report them.
 8. Tom Cruise goes to the premiere provided that Penelope Cruz does,

but Nicole Kidman does not.

9. It is not the case that either Bart and Lisa do their chores or Lenny and Karl blow up the power plant.
10. N'sync winning a grammy is a sufficient condition for the Backstreet Boys to be jealous, only if Destiny's Child getting booed is a necessary condition for TLC's being asked to sing the anthem.
11. Dominos' delivers for free if Pizza Hut adds new toppings, provided that Round Table airs more commercials.
12. If evolutionary biology is correct, then higher life forms arose by chance, and if that is so, then it is not the case that there is any design in nature and divine providence is a myth.
13. Kathie Lee's retiring is a necessary condition for Regis's getting a new co-host; moreover, Jay Leno's buying a motorcycle and David Letterman's telling more jokes imply that NBC's airing more talk shows is a sufficient condition for CBS's changing its image.

Symbolizing/*Reconstructing* Entire English Arguments

- Naïvely, an argument is “just a collection of sentences”. So, naïvely, one might think that symbolizing arguments should just boil down to symbolizing a bunch of individual sentences. It’s not so simple.
- An argumentative passage has more structure than an individual sentence. This makes argument *reconstruction* more subtle.
- We must now make sure we capture the inter-relations of content across the various sentences of the argument.
- To a large extent, these interrelations are captured by a judicious choice of atomic sentences for the reconstruction.
- It is also crucial to keep in mind the overall intent of the argumentative passage — the intended argumentative strategy.
- Forbes glosses over the art of (charitable!) argument reconstruction. I will be a bit more explicit about this today in some examples.

Symbolizing Entire Arguments: Example #1

- ‘If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient. But, if God exists then He is none of these, and there is evil in the world. So, we must conclude that God does not exist.’
- Step 0: Decide on atomic sentences and letters.
 - G : God exists. E : There is evil in the world.
 - J : God is just. O : God is omnipotent.
 - K : God is omniscient.
- Step 1: Identify (and symbolize) the *conclusion* of the argument:
 - ‘God does not exist.’ (which is just ‘ $\sim G$ ’ in LSL)
- Step 2: Symbolize the premises (in this case, there are two):
 - Premise #1: ‘If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient.’

Symbolizing Arguments: Example #1 (Cont'd)

- Premise #1: 'If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient.'

If G , then $(\sim E \text{ unless } (\sim J \text{ or } (\sim O \text{ or } \sim K)))$

$$G \rightarrow (\sim E \vee (\sim J \vee (\sim O \vee \sim K)))$$

- Premise #2: 'If God exists then He is none of these (*i.e.*, He is *neither* unjust *nor*...), and there is evil in the world.'

If G , then not not- J and not not- O and not not- K , and E .

$$[G \rightarrow (\sim\sim J \& (\sim\sim O \& \sim\sim K))] \& E$$

- This yields the following (valid!) sentential form:

$$G \rightarrow (\sim E \vee (\sim J \vee (\sim O \vee \sim K)))$$

$$[G \rightarrow (\sim\sim J \& (\sim\sim O \& \sim\sim K))] \& E$$

$$\therefore \sim G$$

Symbolizing Arguments: Example #1 Notes

- The sentential form:

$$G \rightarrow (\sim E \vee (\sim J \vee (\sim O \vee \sim K)))$$

$$[G \rightarrow (\sim\sim J \& (\sim\sim O \& \sim\sim K))]$$

$$E$$

$$\therefore \sim G$$

with *three* premises is *equivalent* to the *two*-premise sentential form we wrote down originally (why?).

- Alternative for premise #1: ' $G \rightarrow \{\sim[\sim J \vee (\sim O \vee \sim K)] \rightarrow \sim E\}$ '.
- Moreover, if we had written ' $(\sim\sim K \& (\sim\sim J \& \sim\sim O))$ ' rather than ' $(\sim\sim J \& (\sim\sim O \& \sim\sim K))$ ' in premise #2, we would have ended-up with yet another *equivalent* sentential form (why?).
- All of these forms capture the meaning of the premises and conclusion, and all are close to the given form. So, all are OK.

Symbolizing Arguments: Example #1 More Notes

- Premise #1: If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient.
- Two Questions: ① Why render this as (i) ' $p \rightarrow (q \text{ unless } r)$ ', as opposed to (ii) ' $(p \rightarrow q) \text{ unless } r$ '? ② *Does it matter (semantically)?*
- ① First, there's no comma after 'world'. Second, (i) is probably *intended*. The second answer assumes (i) and (ii) are *not* equivalent *in English*.
- That *may* be right, but it's not clear. It presupposes two things:
 - (1) *In English*, ' $q \text{ unless } r$ ' is equivalent to 'If not r , then q '.
 - (2) *In English*, 'If p , then (if q then r)' [*i.e.*, ' $p \rightarrow (q \rightarrow r)$ '] is *not* equivalent to 'If (p and q), then r ' [*i.e.*, ' $(p \& q) \rightarrow r$ '].
- We're *assuming* (1) in this class. (2) is controversial (but defensible).
- ② In LSL, (i) and (ii) *are* equivalent, *i.e.*, in LSL (2) is *false*. Thus, it seems to me that both readings are probably OK. This is a subtle case.