The Reduction of Strategic Plasticity

Rory Smead

University of California, Irvine

Kevin J.S. Zollman

Carnegie Mellon University

Formal Epistemology Workshop
May 18, 2008

Overview

1. Background: Plasticity and Game Theory
2. The Modeling Framework
3. Reduction of Strategic Plasticity
4. The Simpson-Baldwin Effect

The Problem of Plasticity

Humans and other animals are behaviorally adaptive. But, plasticity is generally assumed to be somewhat *costly*...

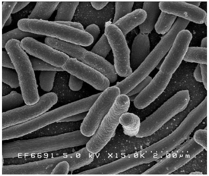
Maintenance cost

The Problem of Plasticity

Humans and other animals are behaviorally adaptive. But, plasticity is generally assumed to be somewhat *costly*...

Maintenance cost

Increased Replication Time

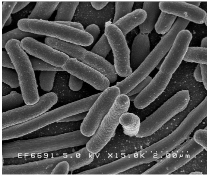


The Problem of Plasticity

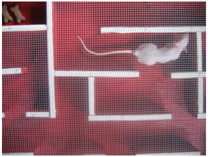
Humans and other animals are behaviorally adaptive. But, plasticity is generally assumed to be somewhat *costly*...

Maintenance cost

Increased Replication Time



Errors

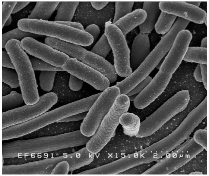


The Problem of Plasticity

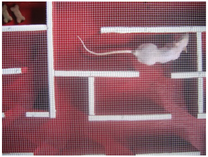
Humans and other animals are behaviorally adaptive. But, plasticity is generally assumed to be somewhat *costly*...

Maintenance cost

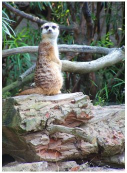
Increased Replication Time



Errors



Delay in action



The Problem of Plasticity

These considerations raise questions about the evolution of plasticity:

- Why hasn't selection eliminated plasticity?
- When will plasticity evolve and be maintained?

Similar questions also arise in the context of the evolution of cognition:

> *"Why has the expensive and delicate biological machinery underlying mental life evolved?" (Godfrey-Smith 2002)*

Evolution of Plasticity

Environmental complexity is one answer. (Godfrey-Smith 1996, 2002; Sterelny 2003; Ancel 1999).

But "environmental complexity" can come in different forms:

- Ecological Variation
- Population Interactions and Changes (Frequency Dependent Selection)

Can frequency dependent selection *alone* provide the selective pressure needed to evolve plasticity?

A Game consists of the following:

- A set of players $P = \{1, ..., n\}$
- A set of strategies for each player S_i
- A payoff function for each player π_i

We will restrict ourselves to finite, 2-player, symmetric games.

	c	d
c	$2, 2$	$0, 3$
d	$3, 0$	$1, 1$

We will also allow for mixed strategies and we can consider the expected utility with respect to such players: $u(x, y)$

Other important concepts:

- Best Response: $B(x) = \{y | u(y, x) \geq u(y', x) \forall y'\}$
- Nash Equilibrium: a set of strategies is a NE iff each strategy is a best response to the others.

Other important concepts:

- Best Response: $B(x) = \{y | u(y, x) \geq u(y', x) \forall y'\}$
- Nash Equilibrium: a set of strategies is a NE iff each strategy is a best response to the others.

Evolutionary Game Theory: Strategies in the game represent phenotypes in the population and we can think about the stability of populations.

Definition

A strategy set s^* is an evolutionarily stable strategy (ESS) if and only if:

- $u(s^*, s^*) \geq u(s, s^*)$ for all s *and*
- If $u(s^*, s^*) = u(s, s^*)$, then $u(s^*, s) > u(s, s)$ for $s \neq s^*$

The Modeling Framework

Individuals will be paired randomly from a population to play a game repeatedly with each other.

Each strategy of the game represents one phenotype.

The Modeling Framework

Individuals will be paired randomly from a population to play a game repeatedly with each other.

Each strategy of the game represents one phenotype.

We now introduce an additional phenotype: Learning ($\mathcal{L}$).

- $\mathcal{L}$ will learn to play a best response against a fixed strategy.
- $\mathcal{L}$ will learn to play a Nash equilibrium with other learners.

Payoffs for each type against the population will be the long-run payoffs of interactions.

The Modeling Framework

G

	c	d
c	$2, 2$	$0, 3$
d	$3, 0$	$1, 1$

The Modeling Framework

G

	c	d
c	$2, 2$	$0, 3$
d	$3, 0$	$1, 1$

G^L

	c	d	$\mathcal{L}$
c	2	0	0
d	3	1	1
$\mathcal{L}$	$3 - c$	$1 - c$	$1 - c$

The Modeling Framework

G

	c	d
c	$2, 2$	$0, 3$
d	$3, 0$	$1, 1$

G^L

	c	d	$\mathcal{L}$
c	2	0	0
d	3	1	1
$\mathcal{L}$	$3 - c$	$1 - c$	$1 - c$

Including Costs:

1. Costs are exogenous and only imposed on the learners.
2. Costs are exogenous and imposed on both learners and those interacting with learners.
3. Costs are endogenous, incurred by error.

Initial results show that the situation is not good for learners...

Theorem 1

For all games without a pareto dominant mixed strategy Nash equilibrium, learning is not an ESS of the extended game.

Exogenous costs on Learners

Initial results show that the situation is not good for learners...

Theorem 1

For all games without a pareto dominant mixed strategy Nash equilibrium, learning is not an ESS of the extended game.

Theorem 2

Suppose a symmetric two player game G, a strategy s of G such that s is a best response to itself, and mixed strategy Nash equilibrium s^* of G^L which includes s and $\mathcal{L}$. s^* is not an ESS.

Rock-Paper-Scissors

If we remove inflexible mixed strategies from the model, it is easy to generate a 3×3 game where $\mathcal{L}$ will be an ESS, consider Rock-Paper-Scissors:

	R	P	S
R	0	-1	1
P	1	0	-1
S	-1	1	0

Here, there is only one NE, so $\mathcal{L}$ will learn to randomize among the strategies when playing other learners and will best respond to any pure strategy (making $\mathcal{L}$ is an ESS):

$$u^L(\mathcal{L}, \mathcal{L}) = 1 > u^L(s, \mathcal{L}) = 0$$

Why the restiction?

Theorem 1

For all games without a pareto dominant mixed strategy Nash equilibrium, learning is not an ESS of the extended game.

- We have not specified what learning does against mixed strategies.
- How learning responds to mixed strategy Nash Equilibrium strategies is important.
- Learning can be made to be an ESS, but the only examples (we've found) are odd.

Generalized Exogenous Costs

By loosening the restriction on costs, learning can be an ESS. However, we need one of two things to occur:

1. There exists a “good” Nash equilibrium of G and the learners converge to that NE.

Generalized Exogenous Costs

By loosening the restriction on costs, learning can be an ESS. However, we need one of two things to occur:

1. There exists a “good” Nash equilibrium of G and the learners converge to that NE.

   or

2. It is more costly to be a non-learner in a world of learners than it is to be a learner.

Endogenous Costs

Here, we suppose costs are the result of $\mathcal{L}$ playing a non-best response.

For tractability we will make the following restrictions:

- 2×2 games with only pure strategies.
- The rate of errors for learners is constant.

Endogenous Costs

Here, we suppose costs are the result of $\mathcal{L}$ playing a non-best response.

For tractability we will make the following restrictions:

- 2×2 games with only pure strategies.
- The rate of errors for learners is constant.

Results: With one small exception, Learning cannot be an ESS.

	A	B
A	2	0
B	x	1

Where 2>x>1.

The Dynamical Picture

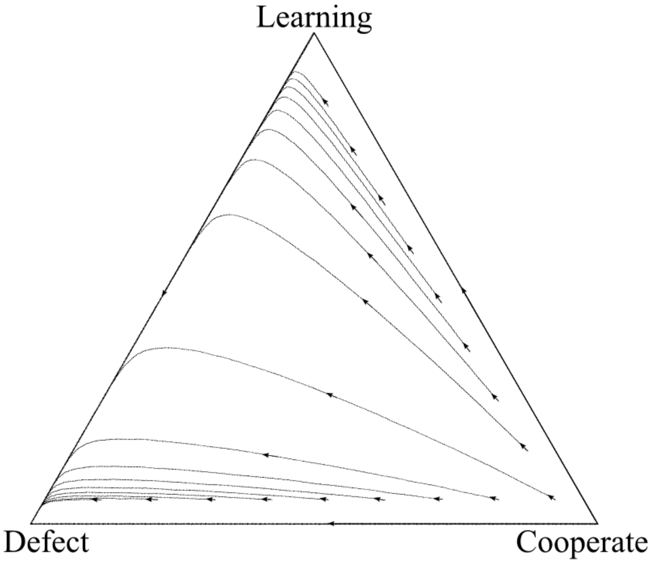
So far we have considered only equilibria, but evolution is really about change over time.

To study this we use computer simulations using the discrete-time *replicator dynamic*:

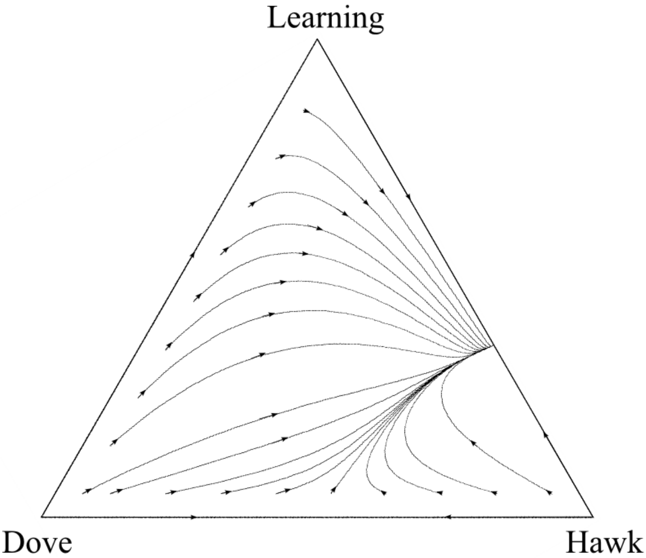
$$x^{t_{n+1}} = x^{t_n} \frac{f_x(X^t)}{\theta(X^t)}$$

We look at a variety of initial conditions and are able to map the evolutionary trajectories of populations...

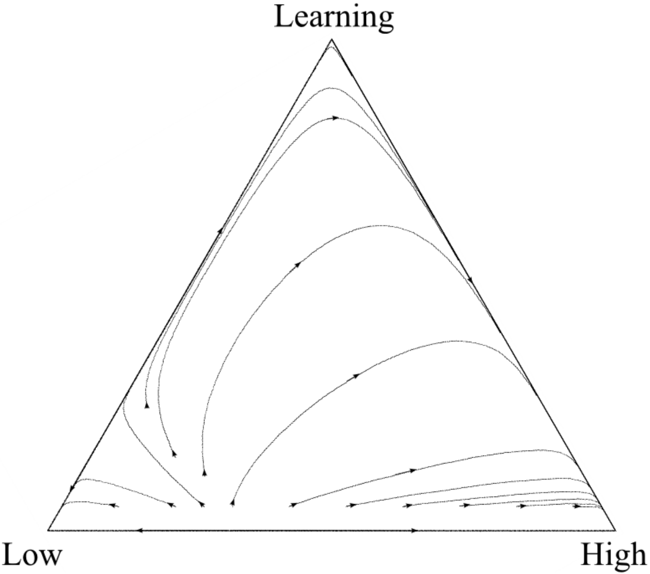
Prisoner's Dilemma



Hawk-Dove



Coordination Game



The Simpson-Baldwin Effect

"Characters individually acquired by members of a group of organisms may eventually, under the influence of selection, be reenforced or replaced by similar hereditary characters. That is the essence of the evolutionary phenomenon here called 'the Baldwin effect.'" (Simpson 1952, "The Baldwin Effect")

James Mark Baldwin (1861-1934)

George Gaylord Simpson (1902-1984)

Conditions for the Simpson-Baldwin Effect

For a Simpson-Baldwin effect to occur, several conditions need to be in place:

- Variation among the population with respect to the relevant plasticity.
- Enough environmental variation that plasticity (or behavioral flexibility) will be initially advantageous.
- Not so much environmental variation that plasticity remains advantageous.

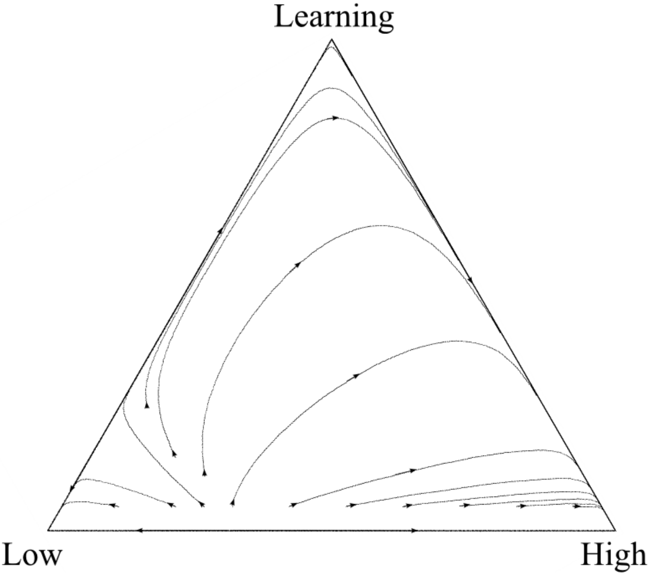
Conditions for the Simpson-Baldwin Effect

For a Simpson-Baldwin effect to occur, several conditions need to be in place:

- Variation among the population with respect to the relevant plasticity.
- Enough environmental variation that plasticity (or behavioral flexibility) will be initially advantageous.
- Not so much environmental variation that plasticity remains advantageous.

Peter Godfrey-Smith (2003): Frequency dependent selection may provide an ideal setting for the Baldwin effect.

Coordination Game



Conclusions

1. In general, strategic interaction alone will not be sufficient to provide the selective pressure necessary for the evolution of strategic plasticity.

Conclusions

1. In general, strategic interaction alone will not be sufficient to provide the selective pressure necessary for the evolution of strategic plasticity.
2. Dynamically, plasticity will often be initially selected for in mixed populations but will eventually be eliminated or reduced.

Conclusions

1. In general, strategic interaction alone will not be sufficient to provide the selective pressure necessary for the evolution of strategic plasticity.
2. Dynamically, plasticity will often be initially selected for in mixed populations but will eventually be eliminated or reduced.
3. For learning to persist and dominate the population, we will likely have to draw on environmental variation beyond what can be generated by frequency dependent selection alone.