Naturalized Epistemology:
A Sampler of Problems

0. Prolegomena: Price & Quine

I.
Is Visual Space Euclidean?

II.
Thinking, Automaticity and Computation

III.
Language Processing

IV.
Error, Ergodicity and Invariance

Nevertheless, despite the force of these criticisms, I hope that the book may still have some value as it stands. It may serve at least as a useful Aunt Sally, and beginners may exercise their minds by detecting its inadequacies. It may have a certain historical interest, as a typical product of a now remote philosophical epoch. Its whole approach to the problems of perception may be misguided, or over-narrow, or just mistaken. But the reader may still learn something by seeing what this approach leads to, when an attempt is made to work out its implications in detail.

H.H. Price, *Perception*, from
Preface to 1954 reprint

The idea of a self-sufficient sensory language as a foundation for science loses its lustre when we reflect that systematization of our sensory intake is the very business that science itself is engaged in. The memories that link our past experiences with present ones and induce our expectations are themselves mostly memories not of sensory intake but of essentially scientific posits, namely, things and events in the physical world. It was perhaps appreciation of this point that led Otto Neurath, Carnap's colleague in Vienna, to persuade Carnap to give up his methodological phenomenalism in favor of physicalism.

W.V. Quine, *From Stimulus to*

Science, 1995, p. 15.

The motivation is still philosophical, as motivation in natural science tends to be, and the inquiry proceeds in disregard of disciplinary boundaries but with respect for the disciplines themselves and appetite for their input. Unlike the old epistemologists, we seek no firmer basis for science than science itself; so we are free to use the very fruits of science in investigating its roots. It is a matter, as always in science, of tackling one problem with the help of our answers to others.

W.V. Quine, *From Stimulus*

to Science, 1995, p. 16.

II. Thinking, Automaticity and Computation

Thinking

Our mental concept of ourselves is above all that of self-aware thinking beings.

But contrary to much folklore psychology, we are almost entirely unaware or unconscious of our thinking processes.

What we have excellent knowledge of is the results of thinking.

Here is a brief survey of the many kinds of experimental studies supporting these conclusions.

THINKING

Richard E. Nisbett & Timothy D. Wilson, 1977, "Telling more than we can know: Verbal reports on mental processes", *Psychological Review,* **84**, 231-259.

Timothy Wilson, 1978, "Strangers to ourselves: The origins and accuracy of beliefs about one's own mental states." In J. H. Harvey and G. Weary (Eds.), *Attribution in contemporary psychology.* New York: Academic Press. pp. 9-36.

Inability to Answer Why Questions

Why they like particular political candidates (Gaudet, 1955)

Why they like certain detergents (Kornhauser & Lazarsfeld, 1935)

Why they choose a particular occupation (Lazarsfeld, 1931)

Why they go to graduate school (Davis, 1964)

Why they become a juvenile delinquent (Burt, 1925)

Why they got married or divorced (Goode, 1956)

Why they moved to a new home (Rossi, 1955)

Reports on Problem-Solving Processes

Ghilesin (1952) collecting data on creative problem solving from Picasso to Poincaré:

"Production by a process of purely conscious calculation seems never to occur."

Classic study of Maier (1931) on combining extension cords.

Widespread recognition that theorem-proving of any difficulty in mathematics depends on imaginative leaps very similar to memory retrieval, but clearly computational in character: the key idea comes into consciousness with no detailed trace at all of how it was arrived at.

Why Are We Unaware of Our Unawareness?

Confusion of Content and Process

Detailed private knowledge we have of ourselves:

- Private historical facts of thought and action
- Know focus of attention at any given moment
- Private intermittent awareness of various sensations

Capability to describe coarse intermediate steps in complex problem solving, but these steps are mainly intermediate results.

It is a profoundly erroneous truism, repeated by all copy-books and by eminent people making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of operations which we can perform without thinking about them. Operations of thought are like cavalry charges in a battle—they are strictly limited in number, they require fresh horses, and must only be made at decisive moments. A.N. Whitehead, 1911

General Theoretical Result on Self-Awareness

Under reasonable assumptions we can prove that any computational system that has complete awareness of its processes requires an infinite regress.

Under stronger assumptions, we can prove that real-time computational systems cannot be aware of a large part of their computations.

Awareness of Results Not Processes

Perceptual feature analyzers

Affective appraisals

Perceptual computations for motor control

Processes of problem solving

Memory retrieval

Language comprehension

Language production

AUTOMATICITY

Essential to complex problem solving, perception and judgment so routine steps require little focus.

Good recent survey, John Bargh and Tanya Chartrand, 1999, "The unbearable automaticity of being", *American Psychologist,* **54**, pp. 462-479.

Classic study, R.M. Schiffrin & W. Schneider, 1977, "Controlled and automatic human information processing: II. Perceptual learning, automatic attending, and a general theory". *Psychological Review,* **84**, pp. 127-190.

But William James (1890, II, pp. 496-497) stated the case very well long ago:

“It is a general principle in Psychology that consciousness deserts all processes where it can no longer be of use. The tendency of consciousness to a minimum of complication is in fact a dominating law. The law of parsimony in logic is only its best known case. We grow unconscious of every feeling which is useless as a sign to lead us to our ends, and where one sign will suffice others drop out, and that one remains, to work alone. We observe this in the whole history of sense-perception, and in the acquisition of every art. ...”

"... So in acquiring any art or voluntary function. The marksman ends by thinking only of the exact position of the goal, the singer only of the perfect sound, the balancer only of the point of the pole whose oscillations he must counteract. The associated mechanism has become so perfect in all these persons that each variation in the thought of the end is functionally correlated with the one movement fitted to bring the latter about. Whilst they were tyros, they thought of their means as well as their end: the marksman of the position of his gun or bow, or the weight of his stone; the pianist of the visible position of the note on the keyboard; the singer of his throat or breathing; the balancer of his feet on the rope, or his hand or chin under the pole. But little by little they succeeded in dropping all this supernumerary consciousness, and they became secure in their movements exactly in proportion as they did so."

SUMMARY

1. Many, if not most, brain computations need no conscious intervention.
2. Without automaticity, almost all high-level skills of thinking, perceiving and performing would not be possible.
3. Remove the automatic and unconscious part of most mental activities and the conscious mental part that is left can do nothing on its own.
4. In the usual process of thinking and doing, conscious mental control is fragmentary and fleeting.
5. Extended patches of automatic thinking and doing are not part of many traditional philosophical conceptions of the mind (e.g. Locke, Berkeley, Hume and Kant)

Major Area of Conflict: Ideas About Language

Computational processes of comprehension or production are almost entirely unconscious in nature.

Particular telling and important case: prosodic features of speaker's language in so far as they express anger, fear, contempt, … are more evident to listeners than the speaker even after they occur.

(For literal features of speech, we are aware of the results, i.e., speech after it has occurred.)

The result-thesis applies to implicit as well as actual speech.

Radical thesis: Detailed linguistic theories of parsing and generating grammars are mostly wrong as descriptions of brain processing of language.

Mental view: processing is sequential and highly intentional in character.

Brain computation view: processing is massively parallel and mainly unintentional, entirely so in the details.

Fact: in spite of massive efforts we have no adequate grammar of any natural language.

Likely fact: strong structural isomorphism between speech and corresponding brain wave.

Another likely fact: strong structural isomorphism between brain waves of different individuals for the same spoken speech.

More on Language

Folklore: We parse a sentence and then determine its meaning.

Brain computations: It all happens simultaneously and we have no awareness at all of how we compute the truth value.

Paris is not the capital of Poland.

Rome is north of London.

Time for the answer: About 100 ms after end of sentence.

II. Mental Representation of Language

1. Quine, Wittgenstein, Jonathan Lear
   "Quine, like Wittgenstein, categorically rejects the notion that meaning can essentially involve anything private to an individual, such as a hidden mental image. This is the myth of the museum—that words name specimen mental objects—which Quine rightly urges us to reject. If we are to explain language-mastery, and thus the meaning our words and sentences have, we must do it on the basis of our experience: the sensory evidence of all types to which we have over time been exposed and our sensitivity to it. Positing interior mental objects that are named by words only gets in the way of an explanation, for it merely papers over the gaps in our understanding of how language-mastery is acquired."

   Jonathan Lear, "Going Native", *Daedalus,* 1978, pp. 177—78.

2. John R. Anderson

Much on representation, but in two large works:

Language, Memory and Thought, 1976 (ACT Model)

and

The Atomic Components of Thought, (with Christian Labiere), 1998

No reference to “mental” or “conscious”.

3.

Chomsky again

“As I am using the term, knowledge may be unconscious and not accessible to conscious-ness. It may be “implicit” or “tacit.” No amount of intro-spection could tell us that we know, or cognize, or use certain rules or principles of grammar, or that use of language involves mental representations formed by these rules and principles. We have no privileged access to such rules and representations. This conclusion appears to be inconsistent with requirements that have been widely enun-ciated. Kant, for example, insisted that “All representations have a necessary relation to a *possible* empirical conscious-ness. For if they did not have this, and if it were altogether impossible to become conscious of them, this would practically amount to the admission of their non-existence." Similar doctrines are familiar in contemporary philosophy. John Searle writes that "It is in general character-istic of attributions of unconscious mental states that the attribution presupposes that the state can become conscious, and without this presupposition the attributions lose much of their explanatory power."

Chomsky, *Rules and Representations,* p. 128.

4. My Conclusion: Quine, Wittgenstein, Lear, Searle, Kant are Wrong.
   Chomsky is Right.

5. Our conscious mental representations of language poorly developed, especially almost all aspects of processing.

6. As in other areas: conscious of results not mental processes.

$$(\Omega, \Im, P) \text{ -- a probability space}$$

$T : \Omega \to \Omega$

T is measurable:

$$A \,\epsilon\, \Im \to T^{-1}A = \{\omega : T\omega \epsilon A\} \,\epsilon\, \Im$$

T is measure preserving:

$$P(T^{-1}A) = P(A)$$

T is invertible:

(i) T is $1 - 1$

(ii) $T\Omega = \Omega$

(iii) $A \,\epsilon\, \Im \to TA = \{T\omega : \omega \epsilon A\} \,\epsilon\, \Im$

Left Shift

If $\forall_n \qquad y_{n-1} = x_n$
then $\quad T(x) = y$

$$\begin{aligned} x &= \ldots x_0, x_1, x_2, x_3, x_4 \\ y &= \ldots y_0, y_1, y_2, y_3 \end{aligned}$$

A is *invariant* under T iff

$$TA = A$$

A is nontrivial iff $0 < P(A) < 1$.
T is ergodic iff there are no nontrivial invariant sets.

Stationarity

Let $\{\mathbf{X}_n : -\infty < n < \infty\}$ be a stochastic process. It is stationary iff the multivariate distribution of the random variables $\mathbf{X}_{t_1+h}, \ldots,$ $\mathbf{X}_{t_n+h}$ is independent of h for any $t_1, \ldots, t_n$.

ENTROPY

$$P(H) = p_1 = p,\ P(T) = (1-p) = p_2$$

$$\begin{aligned} \text{Entropy } H &= -\Sigma p_i \log p_i \\ &= -p \log p - (1-p)\log(1-p) \end{aligned}$$

For a process $\{\mathbf{X}_n : 0 < n < \infty\}$
entropy $\quad H = \lim \frac{1}{n} H(\mathbf{X}_1, \ldots, \mathbf{X}_n)$

For a Bernoulli Process χ

$$\begin{aligned} H(\chi) &= \lim nH(\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n) \\ &= \frac{nH(\mathbf{X}_1)}{n} = -\Sigma p_i \log p_i \end{aligned}$$

Markov Chain

$$\begin{aligned} H(\chi) &= \lim H(\mathbf{X}_n \mid \mathbf{X}_{n-1}, \ldots, \mathbf{X}_1) \\ &= H(\mathbf{X}_2 \mid \mathbf{X}_1) \\ &= -\Sigma_j p_{ij} \Sigma_i p_{ij} \log p_{ij} \end{aligned}$$

Given $(\Omega, \mathcal{F}, \mathcal{P}, \mathcal{T})$ and $\chi = \{\mathbf{X}_n : 0 < n < \infty\}$ a stochastic process on this space, χ is ergodic if T is.

Simplest example of a nonergodic process

$$\begin{array}{ccc} & H & T \\ H & 1 & 0 \\ T & 0 & 1 \end{array}$$

$$P_1(H) = P_1(T) = \frac{1}{2}$$

$$A = \{\omega_i = H,\ 1 \leq i < \infty\}$$

Then $\qquad TA = A$
but $P(A) = \frac{1}{2}$.

Types of Ergodic Processes

I. 0 Entropy

All measurements predictable

Example:

$$\begin{array}{ccc} & H & T \\ H & 0 & 1 \\ T & 1 & 0 \end{array} \qquad P(H_1) = 1$$

$HTHTHT\ldots$

II. Positive Entropy

Not all measurements predictable, but some may be.

Factor of a process χ – Restriction of the process to a sub-σ-algebra of events.

III. K Processes – no factor with 0 entropy.

Example: Sinai Billiards (Nothing predictable long-run)

IV. Bernoulli Processes

Extreme case of K Processes.

Sense in which an ergodic shift T replicates Ω. For almost any ω in Ω

$$\lim_{n \to \infty} \frac{1}{n} \Sigma_{k=0}^{n-1} I_A(T^k \omega) = P(A)$$

ISOMORPHISM

$$(\Omega, \Im, P), (\Omega', \Im', P') \text{ – probability spaces}$$

$$T \text{ and } T' \text{ – measure-preserving transformations}$$

Then

$$(\Omega, \Im, P, T) \textit{ isomorphic} \text{ to } (\Omega', \Im', P', T')$$

$$\text{iff } \ \exists \varphi : \Omega_0 \to \Omega_0' \ \text{ where}$$

$$\Omega_0 \,\epsilon\, \Im, \Omega_0' \,\epsilon\, \Im'$$

$$\text{and } P(\Omega_0) = P(\Omega_0') = 1$$

(i) φ is $1-1$,

(ii) If $A \subset \Omega_0 \quad \& \quad A' = \varphi A$ then

$$A \,\epsilon\, \Im \;\text{ iff }\; A' \,\epsilon\, \Im'$$

and if $A \,\epsilon\, \Im$

$$P(A) = P'(A')$$

(iii) $T\Omega_0 \subseteq \Omega_0 \quad \& \quad T'\Omega_0' \subseteq \Omega_0'$

(iv) For any ω in Ω_0

$$\varphi(T\omega) = T'\varphi(\omega).$$

Theorem 1. (Kolmogorov & sinai, 1958). If two Bernoulli or Markov processes are isomorphic then their entropies are the same.

Simple open problem until then:

Are B(1/2, 1/2) and B(1/3, 1/3, 1/3) isomorphic?

Theorem 2. (Ornstein, 1970). If two Bernoulli processes have the same entropy they are isomorphic.

Theorem 3. Any two irreducible, stationary, finite-state discrete Markov processes are isomorphic if and only if they have the same periodicity and the same entropy.

Corollary. An irreducible, stationary, finite-state discrete Markov process is isomorphic to a Bernoulli process of the same entropy if and only if it is aperiodic.

Surprising and Important Invariant Result

Entropy is a complete invariant for the measure-theoretic isomorphism of ergodic Markov processes.

In order to claim that intuitively the two kinds of analysis are indistinguishable from observation we need stricter concepts. To show this, we need not even consider something as complicated as the billiard example, but consider only a first-order Markov process and a Bernoulli process that have the same entropy rate and therefore are isomorphic in the measure-theoretic sense, but it is also easy to show by very direct statistical tests whether a given sample path of any length, which is meant to approximate an infinite sequence, comes from a Bernoulli process or a first-order Markov process. There is, for example, a simple chi-square test for distinguishing between the two. It is a test for first-order versus zero-order dependency in the process.

“Try to verify any law of nature, and you will find that the more precise your observations, the more certain they will be to show irregular departures from the law. We are accustomed to ascribe these, and I do not say wrongly, to errors of observation; yet we cannot usually account for such errors in any antecedently probable way. Trace their causes back far enough, and you will be forced to admit they are always due to arbitrary determination, or chance.”

Peirce, 1892, p. 329

Photons as Billiards

Rectangular box with reflecting sides

Classical law of reflection: angle of reflection equals angle of incidence.

Sinai billiards–add a convex obstacle, also reflecting.

Theorem. Motion of a photon as a Sinai billiard is ergodic.

Corollary. Motion of photon as a Sinai billiard is strongly chaotic.

α-Congruence

$$\begin{array}{ll} \textit{Let} & \mathbf{\Omega} = (\Omega, \mathcal{F}, \mathcal{P}, \mathcal{T}) \\ \textit{and} & \mathbf{\Omega}' = (\Omega', \mathcal{F}', P', T') \end{array}$$

be two discrete flows or processes, with ω_n, ω_n' in M, a compact metric space, such that Ω and Ω' are isomorphic under φ. Then Ω and Ω' are α-*congruent* iff for every ω in Ω and every n

$$d(\omega_n, \varphi(\omega_n)) < \alpha$$

except for a set of points in M of measure $< \alpha$.

Ω and Ω' look the same within experimental (or other) "error" when they are α-congruent for small α.

We also think of points in M with distance less than α as indistinguishable, due to inherent variability, experimental error, etc.

Theorem 4. (Ornstein) There are processes which can equally well be analyzed as deterministic systems of classical mechanics or as indeterministic Markov processes, no matter how many observations are made, if observations have only a bounded finite accuracy.

BILLIARD EXAMPLE WITH CONVEX OBSTACLE

1. Pick $\alpha > 0$.
2. Define a (biased) coin stochastic process $\mathcal{S}$.
3. Finitely partition the table.
4. Ball will always be in one element of the finite partition.
5. It stays in each element p of the partition for time $t(p)$.
6. The ball then jumps to one of a pair of points according to a flip of the coin.
7. The pair of points depend on p.

Theorem: Billiards, as we know it, & $\mathcal{S}$ are α-congruent.

INTERPRETATION OF ORNSTEIN THEOREM

1. Explicit theorems for concrete systems difficult, but seem likely to be true in a wide variety of cases.
2. Such results are important in making the case that any thesis of universal determinism, or, even much less than universal, is transcendental.
3. Transcendental not false.
4. Freedom, not determinism, is the empirical phenomenon.
5. Empirically, uncertainty is everywhere, especially in the arena of free human choices.

A new abstract view of mechanics and stochastic processes

	Continuous	Discrete
Determ.	M(C,D)	M(D,D)
Stochastic	M(C,S)	M(S,D)

$$\mathrm{M(C,D)} \simeq \mathrm{M(D,D)} \simeq \mathrm{M(C,S)} \simeq \mathrm{M(S,D)}$$

A new kind of invariance