

Philosophy 101

(2/22/11)

- **HW #2 to be returned today** (end of class)
- **I will be grading on a “curve” after all.** [more soon]
- **Solutions to HW #2 posted (later today)**
- **HW #3 assigned last week** (due next Thursday)
- **Quiz #2 on this Thursday** (on *rational belief*)
- **Today: Chapter 3, Continued**
 - **Two subtle aspects of formal validity**
 - **Cogency (of invalid arguments)**
 - **Next: Chapter 4 — Strong Arguments**

Chapter 3: Well-Formed Arguments 32

• **Validity — Predicate Logic (some *invalid* forms)**

- | | |
|--|------------------------------|
| 1. All <i>As</i> are <i>Bs</i> . | 1. All men are mortal. |
| 2. <u><i>x</i> is not an <i>A</i>.</u> | 2. <u>Fido is not a man.</u> |
| 3. <i>x</i> is not a <i>B</i> . | 3. Fido is not mortal. |
| 1. All <i>As</i> are <i>Bs</i> . | 1. All men are mortal. |
| 2. <u><i>x</i> is a <i>B</i>.</u> | 2. <u>Fido is mortal.</u> |
| 3. <i>x</i> is an <i>A</i> . | 3. Fido is a man. |

Another important Example:

| |
|--------------------------------|
| Most <i>As</i> are <i>Bs</i> . |
| <i>x</i> is an <i>A</i> . |
| ----- |
| <i>x</i> is a <i>B</i> . |

Chapter 3: Well-Formed Arguments 33

- **Validity — A Clarification of the Definition**
- Our initial definition of validity was a bit unclear. You can see this unclarity in certain more subtle examples, such as:

Argument 3.12

1. Jones is a mother.
2. Jones is female.

- Is this argument valid? One might *think* it is, because it might seem that it would be a *logical* contradiction for the premise of this argument to be true *while* its conclusion is false.
- But, *strictly speaking*, we will *not* classify this argument as *valid*.
 - ➡ This is because we have *no logical theory* (sentential or predicate) according to which this argument has a *valid form*.
- This leads to an important *clarification* of our definition.

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- **Validity — A Clarification of the Definition**
- Here is a clarified definition of validity:
 - **D3.1b:** An argument is valid iff the argument has *some logical form* such that it is impossible for any argument with that form to have true premises and a false conclusion.
- That is, *all valid arguments must have **valid logical forms***.
- The following two (“equivalent”) arguments **are** valid:
 1. Jones is a female and Jones is a parent.

 2. Jones is a parent.

1. Jones is a mother.
2. All mothers are females.

 3. Jones is a female.

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• Validity — A Further Clarification

- In our revised definition of validity, we only require that the argument instantiate *some* valid logical form.

- Because we have two different notions/theories of logical form (*sentential*-logical form and *predicate*-logical form), we must be careful about certain cases where they *come apart*.

- Consider the following argument:

1. All men are mortal.
2. Socrates is a man.
3. Therefore, Socrates is mortal.

- As we have seen, this argument has a *valid predicate-logical form*, and so it is (according to our *refined* definition) *valid*.

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• Validity — A Further Clarification

1. All Xs are Ys. [X = men, Y = mortals]
2. s is an X. [s = Socrates]
3. Therefore, s is a Y.

- But, ask yourself: what is its **sentential**-logical form?

- Remember, from the point of view of sentence logic, this argument contains *three distinct “atomic” sentences*!

- This is because *none of the sentences in the argument contains any of the 5 sentential connectives* (and, or, ~, if, iff). So, we have:
 1. P.
 2. Q.
 3. Therefore, R.

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• Validity — A Further Clarification

- The moral of this story is that some arguments do *not* have a valid *sentential* form, but they *do* have a valid *predicate* form.

- Such arguments are (still) valid, but in order to see that they are valid, one needs to look at predicate-logical form.

- Sentence-logical form is “coarse grained” or “zoomed out”. It is not capable of “seeing” *subject-predicate structure*.

- Predicate-logical form is “finer-grained” or “zoomed in”. It is capable of “seeing” *subject-predicate structure*.

- On the other hand, if an argument *does* have a valid *sentence*-logical form, it *must* have a valid *predicate*-logical form as well.

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• Cogency

- Some invalid arguments are better than others (from a logical point of view). Some are *cogent*, while others are not.

- In fact, cogency comes in *degrees*. Here is an example:

1. Boris is a student at State U.
2. Almost all students at State U. voted.
-
3. Boris voted.

- This argument is *invalid*, because it is *possible* for its conclusion to be *false*, even *given the truth of its premises*.

- But, this argument is **cogent**, since its conclusion is *probable*, *given the truth of all of its premises*. That leads to our definition.

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• Cogency

- **D3.2a:** An argument is **cogent** iff it is not valid, but its conclusion is *probable, given* the truth of its premises.
- As a result of this definition, *cogency comes in degrees*. The more probable the conclusion is (*given the truth of all the premises!*), the *stronger* the cogency of the argument is. *Form:*
 1. $n\%$ of As are Bs.
 2. x is an A.
 -
 3. x is a B.
- If $n > 50$, then the argument is *cogent* (if $n = 100$, the argument is *valid*). If $n \leq 50$, the argument is *not* (even) cogent.

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• Well-formed vs Ill-Formed Arguments

- Like validity, cogency is a matter of *form*. If an argument is cogent, then it must instantiate a *cogent form*. *Examples:*

| Pattern | Example |
|---|---|
| A. 1. Most As are Bs. 2. x is an A. 3. x is a B. | 1. Most Americans watch the Super Bowl. 2. Ann Landers is an American. 3. Ann Landers watches the Super Bowl. |
| B. 1. x is an A. 2. x is a B. 3. Most ABs are Cs. 4. x is a C. | 1. Tiger is healthy. 2. Tiger is a cat. 3. Most healthy cats like to chase mice. 4. Tiger likes to chase mice. |

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• Well-formed vs Ill-Formed Arguments

- If an argument has either a valid or a cogent form, then the argument is said to be **well-formed**, otherwise, the argument is said to be **ill-formed**. Here is a summarizing table:

| Well-Formed Arguments | | Ill-Formed Arguments |
|---|---|--|
| Valid arguments | Cogent arguments | |
| Truth of premises guarantees truth of conclusion. | Truth of premises does not guarantee truth of conclusion, but the conclusion is probably true if the premises are true. | Premises do not even make conclusion probable. |

- We can use a scale of from 0 to 1. If the conclusion *must* be true (*given the truth of its premises!*), then it is *valid* (1 on our scale), if its probability is $> 1/2$, then it's *cogent*, o.w., it's *ill-formed*.
 - [Analogous to the "scale" we used for degrees of belief.]

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• Well-formed vs Ill-Formed Arguments

- There are also (many) **non-cogent** forms. *Examples:*

| Pattern | Example |
|--|--|
| A. 1. Most As are Bs. 2. x is not an A. 3. x is not a B. | 1. Most Americans have never gone to the moon. 2. <u>The Queen of England is not an American.</u> 3. The Queen of England has (=not never) gone to the moon. |
| B. 1. Most As are Bs. 2. x is a B. 3. x is an A. | 1. Most robins can fly. 2. <u>Tweety can fly.</u> 3. Tweety is a robin. |

- Such arguments are *ill-formed*, because they do not instantiate a valid form, and they do not instantiate a cogent form.
- We can use "diagrams" to think about cogency as well...

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• Well-formed vs Ill-Formed Arguments

- Here is a summary of the key definition of this section:

- D3.1b:** An argument is *valid* if and only if the argument follows a pattern such that it is impossible for any argument following that pattern to have true premises and a false conclusion.
- D3.2b:** An argument is *cogent* if and only if it is not valid but it follows a pattern such that all arguments following that pattern have a conclusion that is probably true if the premises are true.
- D3.3:** An argument is *ill-formed* if and only if it is neither valid nor cogent.
- D3.4:** An argument is *well-formed* if and only if it is either valid or cogent.

Chapter 4: Strong Arguments 1

• Deductive Strength

- If an argument is valid, it is good — *from a logical perspective*. But, validity is only *part* of the story.
- Ideally, an argument would be valid *and* it would also have premises that are (known to be) true.
- More generally, we will speak of the *strength* of arguments.
- The basic idea is that an argument will be *strong for a person S* just in case the argument is *both* (a) *well-formed*, and (b) *it is rational for S to believe all of the arguments premises*.
- We will have one definition of strength for valid arguments, and a different definition of strength for cogent arguments.
 - We'll discuss the deductive/valid case first.

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• Well-formed vs Ill-Formed Arguments (Q's)

- Suppose that an argument is ill-formed. What can you conclude about the truth value of its conclusion?
- Suppose that an argument is valid. What can you conclude about the truth value of its conclusion?
- Suppose that an argument is valid and that its conclusion is false and that one of its two premises is true. What can you conclude about the truth value of its other premise?
- Suppose an argument is cogent and its conclusion is true. What can you conclude about the truth value of its premises?
- Suppose an argument is valid and has true premises. What can you conclude about the truth value of its conclusion?

Chapter 4: Strong Arguments 2

• Deductive Strength

- We define *deductive strength* as follows:

- D4.1:** An argument is *deductively strong* for a person if and only if
1. it is deductively valid; and
 2. it is reasonable for the person to believe all the argument's premises.
- If an argument is not deductively strong, then we say it is *deductively weak*. There are two main ways in which an argument may be deductively weak for a person *S*.
 - The argument may be *invalid*.
 - It may not be rational for *S* to believe *all* of its premises.
 - This includes cases in which it is reasonable to believe *each* premise *individually*, but *not* when taken *altogether*.

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- **Deductive Strength**

- It is important that we require that it is rational for *S* to believe *the conjunction of all of the premises*, and not merely rational for *S* to believe *each* premise, *taken individually*.
- Here is an example that illustrates the importance of this:
 - Suppose you enter a lottery, which has exactly one winner and 1 million tickets. For *each* ticket # *i*, it would be rational for you to believe that ticket #*i* will lose.
 - But, it is *not* rational for you to believe that *all* the tickets will lose, since this *contradicts* the setup of the case, in which it is assumed that there is *exactly one winning ticket*.
 - This kind of case is sometimes called the *lottery paradox*.

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- **Deductive Strength**

- What should we say about *circular* arguments, such as this?
 1. The earth is round.

 2. Therefore, the earth is round.
- First, we must ask whether the argument is *valid*.
 - Yes, it is clearly valid (*why* is “*p*, therefore *p*” *valid*).
- Then, we must ask whether *it would be rational for us to believe the premise*.
 - In this case, it would indeed be rational for us to believe the premise (assuming we have the usual evidence).
- So, this argument is *deductively strong*. But, is it *useful*?