#### Philosophy 57 — Day 24

- Quiz #6 to be Returned Next Week
- Next Week, I will also post the "Curve for the course to this point"
  - This will tell you where you need to be on the final, etc.
- Extra-Credit Problems Posted on Website (5 problems, each worth 1 point!)
  - Extra-Credit Problems are due by Tuesday 05/20/03
  - No partial credit within problems (but you can do fewer than 5 problems)
  - You may use any tools/references you like to do these (but individually!)
  - Stay Tuned for Hints, etc. [these are all "chapter 6" problems]
- Back to Chapter 6
  - Definitions of Truth-Functional Connectives
  - Truth-Tables for Claims
  - Truth-Tables for Arguments



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#### Chapter 6 — Propositional Logic: Truth Functions – Review

• Negation (just like English "not"), and Conjunction (just like English "and"):

• Disjunction is *similar* to English "or", but *not* in the "exclusive" sense:

• But, we can express the English exclusive "A or B, but not both", as:

$$(A \lor B) \bullet \sim (A \bullet B)$$

• So, "~", "•", and "V" do seem to match English usage for "not", "and", "or".



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#### **Chapter 6** — **Propositional Logic: Truth Functions** – ⊃

p	q	$p \supset q$
Т	Т	Т
Τ	F	F
F	Т	Т
F	F	Т

- The truth-functional definition of ⊃ is farther from the English "only if". A PL conditional is false iff its antecedent is true and its consequent is false.
- In English, conditionals can be false, even if their antecdents are false.

  Moreover, English conditionals can be false even if their consequents are true.
  - If New York is in New Zealand, then 2 + 2 = 4.
  - If New York is in the U.S.A., then WWII ended in 1945.
  - If WWII ended in 1941, then gold is an acid.
- So,  $\supset$  does *not* capture the English "if". We'll see later that  $p \supset q \approx \neg p \lor q$ .
- But, I will explain later why this is the only acceptable truth-functional choice.

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Chapter 6 — Propositional Logic: Truth Functions –  $\equiv$ 

p	q	$p \equiv q$
Т	Т	Т
Т	F	F
F	Т	F
F	F	Т

- The truth-functional definition of ≡ is far from the English "if and only if". A PL biconditional is true iff both of its components have the same truth value.
- Consider these two biconditionals. [M = the moon is made of green cheese, U = there are unicorns, E = life exists on Earth, and S = the sky is blue]
  - The moon is made of green cheese if and only if there are unicorns.
  - Life exists on earth if and only if the sky is blue.
- The PL translations of these sentences are both true.  $M \equiv U$  is true because M and U are false.  $E \equiv S$  is true because E and E are true. This does *not* capture the English "if and only if". We'll see that  $p \equiv q \approx (p \bullet q) \lor (\sim p \bullet \sim q)$ .

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#### Chapter 6 — Propositional Logic: Truth Tables I

- With the truth-table definitions of the five connectives in hand, we can now construct truth tables for arbitrary compound PL statements.
- The procedure for constructing the truth-table of p is as follows:
  - 1. Determine the number of rows in the truth-table. This is  $2^n$ , where n is the number of atomic sentences in the compound statement p.
  - 2. The table will have n + 1 main columns: n columns for the atomic sentences in p, and one for the truth-values of p itself.
  - 3. The table will also have some "quasi-columns" one for each PL statement occurring in the compound p — which needn't be drawn explicitly, but which will go into the determination of the truth values of p.
  - 4. Place the atomic symbols in the left most columns, going in alphabetical order from left to right. And place p in the right most column.
  - 5. Write in all possible combinations of truth-values for the atomic statements. There will be  $2^n$  of these — one for each row of the table.



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- 6. The convention here is to start on the *n*th column (farthest down the alphabet) with the pattern TFTF ... repeated until the column is filled. Then, go TTFF ... in the n-1st column. And, TTTTFFFF ... in the n-2nd column, etc..., until the very first column has been completed.
- 7. Next, we need to compute the truth-values of p in each row of the table. Here, we start from the inside-out. We first copy the truth-values of the atoms, then we compute the negations, conjunctions, etc. which compose p. Finally, we will be in a position to compute the value of the main connective of p, at which point we will be done with p's truth table.
- Example: Step-By-Step Truth-Table Construction of " $A \equiv (B \bullet A)$ ."

$\boldsymbol{A}$	B	A	=	(B	•	A)
Т	Т	Т	Т	•	Т	Т
Т	F			F	F	T
F	Т		Т		F	F
F	F	F	Т	F	F	F

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Chapter 6 — Propositional Logic: Truth Tables II

# • A statement is said to be logically true (or tautologous) if it is true regardless

of the truth-values of its components. Example:  $p \equiv p$  is logically true.

$$\begin{array}{c|cccc} p & p & \equiv & p \\ \hline T & T & T & T \\ \hline F & F & T & F \end{array}$$

• A statement is logically false (or self-contradictory) if it is false regardless of the truth-values of its components. Example:  $p \bullet \sim p$  is logically false.

• A statement is contingent if its truth-value varies depending on the truth-values of its components. Example: A (or any atom) is contingent.

$$\begin{array}{c|c|c}
A & A \\
\hline
T & T \\
\hline
F & F
\end{array}$$

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#### Chapter 6 — Propositional Logic: Truth Tables III

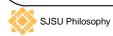
- Classify the following statements as logically true (tautologous), logically false (self-contradictory), or contingent (exercise 6.3.I):
  - 1.  $N \supset (N \supset N)$
  - 2.  $(G \supset G) \supset G$
  - 3.  $(S \supset R) \bullet (S \bullet \sim R)$
  - 4.  $((E \supset F) \supset F) \supset E$
  - 6.  $(M \supset P) \lor (P \supset M)$
- 11.  $[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim (P \lor R)$
- 12.  $[(H \supset N) \bullet (T \supset N)] \supset [(H \lor T) \supset N]$
- 15.  $[(F \lor E) \bullet (G \lor H)] \equiv [(G \bullet E) \lor (F \bullet H)]$

#### Chapter 6 — Propositional Logic: Truth Tables IV

• Here is a completed truth-table for #11,  $[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim (P \lor R)$ :

P	Q	R	[(Q	$\supset$	<i>P</i> )	•	(~	Q	$\supset$	<i>R</i> )]	•	~	( <i>P</i>	٧	R)
Т	Т	Т	Т	Т	Т	Т	F	Т	Т	Т	F	F	Т	Т	Т
Т	Т	F	Т	Т	Т	Т	F	Т	Т	F	F	F	Т	Т	F
Т	F	Т	F	Т	Т	Т	Т	F	Т	Т	F	F	Т	Т	Т
Т	F	F	F	Т	Т	F	Т	F	F	F	F	F	Т	Т	F
F	Т	Т	Т	F	F	F	F	Т	Т	Т	F	F	F	Т	Т
F	Т	F	Т	F	F	F	F	Т	Т	F	F	Т	F	F	F
F	F	Т	F	Т	F	Т	Т	F	Т	Т	F	F	F	Т	Т
F	F	F	F	Т	F	F	Т	F	F	F	F	Т	F	F	F

• Therefore, the statement " $[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim (P \lor R)$ " is *logically false*.



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## Chapter 6 — Propositional Logic: Truth Tables V

• Two statements are said to be equivalent (written  $p \approx q$ ) if they have the same truth-value in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance,  $A \supset B \approx \sim A \vee B$ :

<u>A</u>	$\boldsymbol{B}$	A	$\supset$	В	~	A	V	<u>B</u>
Т	Т	Т	Т	Т	F	Т	Т	Т
	F							
F	Т							
F	F	F	Т	F	Т	F	Т	F

• Two statements are said to be contradictory if they have opposite truth-values in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, *A* and ~*A* are contradictory:



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• Two statements are inconsistent if they are never both true in any possible world (*i.e.*, in any row of a simultaneous truth-table of both statements). For instance,  $A \equiv B$  and  $A \bullet \sim B$  are inconsistent (but *not* contradictory!):

A	В	A	≡	В	A	•	~	В
Т	Т	Т	Т	Т	Т	F	F	Т
Т	F			F				
F	Т	F						
F	F	F	Т	F	F	F	Т	F

• Two statements are consistent if they are both true in at least one possible world (*i.e.*, in at least one row of a simultaneous truth-table of both statements). For instance, *A* • *B* and *A* ∨ *B* are consistent:

A	B	A	•	В	A	٧	В
Т	Т	Т	Т	Т	Т	Т	Т
Т	F			F	1		
F	Т	F	F	Т	F	Т	Т
F	F	F	F	F	F	F	F

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#### Chapter 6 — Propositional Logic: Truth Tables VI

• Use truth-tables to determine whether the following pairs of statements are logically equivalent, contradictory, consistent, or inconsistent (exercise 6.3.II).

2. 
$$F \bullet M$$
 and  $\sim (F \lor M)$ 

4. 
$$R \lor \sim S$$
 and  $S \bullet \sim R$ 

6. 
$$H \equiv \sim G$$
 and  $(G \bullet H) \lor (\sim G \bullet \sim H)$ 

8. 
$$N \bullet (A \lor \sim E)$$
 and  $\sim A \bullet (E \lor \sim N)$ 

10. 
$$W \equiv (B \bullet T)$$
 and  $W \bullet (T \supset \sim B)$ 

12. 
$$R \bullet (Q \lor S)$$
 and  $(S \lor R) \bullet (Q \lor R)$ 

14. 
$$Z \bullet (C \equiv P)$$
 and  $C \equiv (Z \bullet \sim P)$ 

15. 
$$Q \supset \sim (K \lor F)$$
 and  $(K \bullet Q) \lor (F \bullet Q)$ 

### Chapter 6 — Propositional Logic: Truth Tables VII

• Here is a simultaneous truth-table which establishes that

$$A \equiv B \approx (A \bullet B) \lor (\sim A \bullet \sim B)$$

A	В	A	≡	B	(A	•	B)	V	(~	$\boldsymbol{A}$	•	~	<i>B</i> )
					Т								
					Т								
					F								
F	F	F	Т	F	F	F	F	Т	Т	F	Т	Т	F

• Can you prove the following equivalences with simultaneous truth-tables?

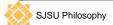
$$-\sim (A\bullet B)\approx \sim A\vee \sim B$$

$$-\sim (A\vee B)\approx \sim A\bullet \sim B$$

$$-A \approx (A \bullet B) \vee (A \bullet \sim B)$$

$$-A \approx A \bullet (B \supset B)$$

$$-A \approx A \vee (B \bullet \sim B)$$



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