

Homework #4 Solutions

Philosophy 1115

April 5, 2016

Part One

1. $A \rightarrow C$
 $B \rightarrow C$
 $A \vee B$ is a *valid* argument. There are no rows in which all of its premises are true and its conclusion is false:
 $\therefore C$

A	B	C	$A \rightarrow C$	$B \rightarrow C$	$A \vee B$	C
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	T	F	T	F	T	F
F	F	T	T	T	F	T
F	F	F	T	T	F	F

2. $I \rightarrow N$
 $(\sim K \vee D) \leftrightarrow N$
 $D \rightarrow \sim I$ is an *invalid* argument. Here's a row that makes all its premises true and its conclusion false:
 $\therefore \sim I \rightarrow (N \rightarrow K)$

D	I	K	N	$I \rightarrow N$	$(\sim K \vee D) \leftrightarrow N$	$D \rightarrow \sim I$	$\sim I \rightarrow (N \rightarrow K)$
T	F	F	T	T	T	T	F

3. $(\sim O \rightarrow \sim S) \& (O \rightarrow (M \& \sim I))$
 $\sim I \rightarrow \sim M$ is a *valid* argument. For the conclusion ' $\sim S$ ' to be false, we require ' S ' to be true. Now,
 $\therefore \sim S$

for the first premise of this argument to be true, its first conjunct ' $\sim O \rightarrow \sim S$ ' must be true. Hence, the antecedent ' $\sim O$ ' of this first conjunct must be false (since its consequent ' $\sim S$ ' has already been assumed to be false). So ' O ' must be true. But, if ' O ' is true, then in order for the second conjunct of the first premise to be true, its consequent ' $M \& \sim I$ ' must be true. This forces ' M ' to be true and ' I ' to be false. But, that forces the second premise ' $\sim I \rightarrow \sim M$ ' to be false. Therefore, there is no way to make both premises true while the conclusion is false, and the argument is valid.

Part Two: Page 66 I

1. **Answer.** $A \rightarrow B, B \rightarrow (C \vee D), \sim D \models A \rightarrow C$.

"Short Method" Explanation. For the conclusion ' $A \rightarrow C$ ' to be false, we require ' A ' to be true, and ' C ' to be false. For premise ' $\sim D$ ' to be true, we require ' D ' to be false. Therefore, for premise ' $B \rightarrow (C \vee D)$ ' to be true we require its antecedent ' B ' to be false (since our assumptions so far already force its consequent ' $C \vee D$ ' to be false). But, all of this forces premise ' $A \rightarrow B$ ' to be false, which means it's impossible for all the premises of this sequent to be true while its conclusion is false. Therefore, the conclusion is a semantic consequence of the premises.

5. **Answer.** $A \vee (B \& C), C \vee (D \& E), (A \vee C) \rightarrow (\sim B \vee \sim D) \models B \& D$.

"Short Method" Explanation. For the conclusion to be false at least one of ' B ' and ' D ' must be false, so at least one of ' A ' and ' C ' must be true if the first two premises are to be true. This means premise three ' $(A \vee C) \rightarrow (\sim B \vee \sim D)$ ' has a true antecedent, but its consequent is true anyway, so the conclusion is *not* a semantic consequence of the premises. Specifically, here is an interpretation that makes all the premises true and the conclusion false.

A	B	C	D	E	$A \vee (B \& C)$	$C \vee (D \& E)$	$(A \vee C) \rightarrow (\sim B \vee \sim D)$	$B \& D$
T	F	T	F	T	T	T	T	F

8. **Answer.** $(A \leftrightarrow B) \vee (B \leftrightarrow C) \models A \leftrightarrow (B \vee C)$.

"Short Method" Explanation. For the conclusion to be false, either (a) ' A ' is true and ' $B \vee C$ ' is false, or (b) ' A ' is false and ' $B \vee C$ ' is true. In case (a) the premise is true since its second disjunct ' $B \leftrightarrow C$ ' is true (because ' B ' and ' C ' are both false), so the conclusion is *not* a semantic consequence of the premises. Specifically, here is an interpretation that makes all the premises true and the conclusion false.

A	B	C	$(A \leftrightarrow B) \vee (B \leftrightarrow C)$	$A \leftrightarrow (B \vee C)$
T	F	F	T	F