Announcements and Such

- Administrative Stuff
  - **HW #5 will be graded soon (and I will post solutions soon)**
  - **HW #6 is due next Friday (April 22)**
    - Consists of two (sets of) probability problems: one involving general algebraic reasoning, one involving numerical calculation.
  - **I have posted a *Practice Final Exam*. We will go over this Practice Final in class on our last class day – next Tuesday (4/19).**
  - **I will also be doing *course evaluations* on Tuesday**
- Unit #4 — *Probability & Inductive Logic, Continued*
  - Measuring Factor #2 — measures of confirmation (relevance)
  - Prospects for measuring “Overall Argument Strength”?
  - Probabilism and the Accuracy of Credences
  - Time-Permitting: The Dutch Book Argument for Probabilism

Three Grades of Measurement

- Suppose we are measuring some numerical quantity. Two examples: *the temperature of an object o* [$\mathfrak{t}(o)$] and *the degree to which E confirms H* [$\mathfrak{c}(H, E)$]. Each of these cases involves *three grades* of measurement.

  ## Qualitative Measurement

  ∗ **Temperature.** This first grade of temperature measurement involves one object o being *warm* (or *cold*). This will correspond to the temperature of o being *above some threshold* t.

  · "Object o is *warm*" $\mapsto \mathfrak{t}(o) > t$.

  ∗ **Confirmation.** This first grade of confirmation measurement involves an argument being *strong* (or *weak*) — in the Factor #2 sense. This will correspond to the degree of confirmation that E provides for H being *above some threshold* t.

  · "$E \therefore H$ is *strong*" $\mapsto \mathfrak{c}(H, E) > t$.

Comparative Measurement

* **Temperature.** This first grade of temperature measurement involves one object o_1 being warm*er* (or cold*er*) than another object o_2.
  - "Object o_1 is warm*er* than object o_2" $\mapsto \mathfrak{t}(o_1) > \mathfrak{t}(o_2)$.
* **Confirmation.** This involves one argument $E_1 \therefore H_1$ being strong*er* (or weak*er*) than another argument $E_2 \therefore H_2$.
  - "$E_1 \therefore H_1$ is strong*er* than $E_2 \therefore H_2$" $\mapsto \mathfrak{c}(H_1, E_1) > \mathfrak{c}(H_2, E_2)$.

Numerical Measurement

* **Temperature.** This involves an object o having a precise numerical temperature.
  - "Object o is 32 degrees Fahrenheit" $\mapsto \mathfrak{t}(o) = 32^\circ$ Fahrenheit.
* **Confirmation.** This involves an argument $E \therefore H$ having a precise numerical degree of confirmation/strength (in the Factor #2 sense).
  - "The degree to which E confirms H is $1/2$.″ $\mapsto \mathfrak{c}(H, E) = 1/2$.

Measuring Factor 2: Degrees of Confirmation I

- *Dozens* of relevance/confirmation measures have been proposed in the literature. Here are the four most popular measures (each defined on a $[-1, +1]$ scale, for ease of comparison).
  - The *Difference*: $d(H, E) = \Pr(H \mid E) - \Pr(H)$
  - The *Ratio*: $r(H, E) = \dfrac{\Pr(H \mid E) - \Pr(H)}{\Pr(H \mid E) + \Pr(H)}$
  - The *Likelihood-Ratio*: $l(H, E) = \dfrac{\Pr(E \mid H) - \Pr(E \mid {\sim}H)}{\Pr(E \mid H) + \Pr(E \mid {\sim}H)}$
  - The *Normalized-Difference*:
  $$s(H, E) = \Pr(H \mid E) - \Pr(H \mid {\sim}E) = \frac{1}{\Pr({\sim}E)} \cdot d(H, E)$$
- *A fortiori*, *all* Bayesian confirmation measures agree on *qualitative* judgments like "E confirms/disconfirms/is irrelevant to H". But, these measures *disagree* with each other in various ways — *comparatively*.

Measuring Factor 2: Degrees of Confirmation III

- There is a relatively simple way of narrowing the field of competing measures of degree of confirmation, which is based on *thinking of inductive logic as a generalization of deductive logic*.
- The likelihood-ratio measure l stands out from the other relevance measures in the literature, since l is the only relevance measure that gets the (non-trivial) deductive cases right (as cases of *extreme relevance*).
- That is, l is the only measure (defined on the scale $[-1, +1]$) that satisfies:

$$\mathfrak{c}(H, E) \text{ should be } \begin{cases} +1 & \Leftarrow E \text{ entails } H \text{ (non-trivially).} \\ > 0 \text{ (confirmation)} & \Rightarrow \Pr(H \mid E) > \Pr(H). \\ = 0 \text{ (irrelevance)} & \Rightarrow \Pr(H \mid E) = \Pr(H). \\ < 0 \text{ (disconfirmation)} & \Rightarrow \Pr(H \mid E) < \Pr(H). \\ -1 & \Leftarrow E \text{ entails } {\sim}H \text{ (non-trivially).} \end{cases}$$

- Here, we assume that $\mathfrak{c}$ is *defined*, which constrains the unconditional Pr's.

Measuring Factor 2: Degrees of Confirmation V

- Here's how our 4 relevance measures handle non-trivial deductive cases.

- $$l(H,E) = \begin{cases} +1 & \text{if } E \vDash H,\ \Pr(E) > 0,\ \Pr(H) \in (0,1) \\ -1 & \text{if } E \vDash {\sim}H,\ \Pr(E) > 0,\ \Pr(H) \in (0,1) \end{cases}$$

- $$d(H,E) = \begin{cases} \Pr({\sim}H) & \text{if } E \vDash H,\ \Pr(E) > 0 \\ -\Pr(H) & \text{if } E \vDash {\sim}H,\ \Pr(E) > 0 \end{cases}$$

- $$r(H,E) = \begin{cases} \dfrac{1-\Pr(H)}{1+\Pr(H)} & \text{if } E \vDash H,\ \Pr(E) > 0,\ \Pr(H) > 0 \\ -1 & \text{if } E \vDash {\sim}H,\ \Pr(E) > 0,\ \Pr(H) > 0 \end{cases}$$

- $$s(H,E) = \begin{cases} \Pr({\sim}H \mid {\sim}E) & \text{if } E \vDash H,\ \Pr(E) \in (0,1) \\ -\Pr(H \mid {\sim}E) & \text{if } E \vDash {\sim}H,\ \Pr(E) \in (0,1) \end{cases}$$

- From an inductive-logical point of view, this favors l over the other measures. Other considerations can also be used to narrow the field.

Can We Measure *Argument Strength* (Numerically)? I

- We know how to measure Factor #1 — this is just the conditional probability of the conclusion, given the premise: $\Pr(C \mid P)$.
- We have some idea of how we might go about measuring Factor #2 — a measure like $l(C, P)$ seems a plausible candidate. Let's run with that.
- This allows us to give a *numerical* version of our "Two-Factor" Chart for graphing the two components of argument strength (next slide).
- Every argument will have associated with it an *ordered pair/vector*: $\langle \Pr(C \mid P), l(C, P) \rangle$, which records values for both Factors.
- However, it is not at all clear how we might *combine* these two measures to yield a *single measure* of *overall* argument strength.
- Presumably, such a measure would be *some function f of* $\Pr(C \mid P)$ and $l(C, P)$. The challenge is to say *which function f is*. Let's think about this, in terms of our *three grades of measurement*.

- We've already given a definition of *qualitative* argument strength. That's the *first grade of measurement* for "overall argument strength".

  Proposal #3. An argument $P \therefore C$ is *inductively strong* iff

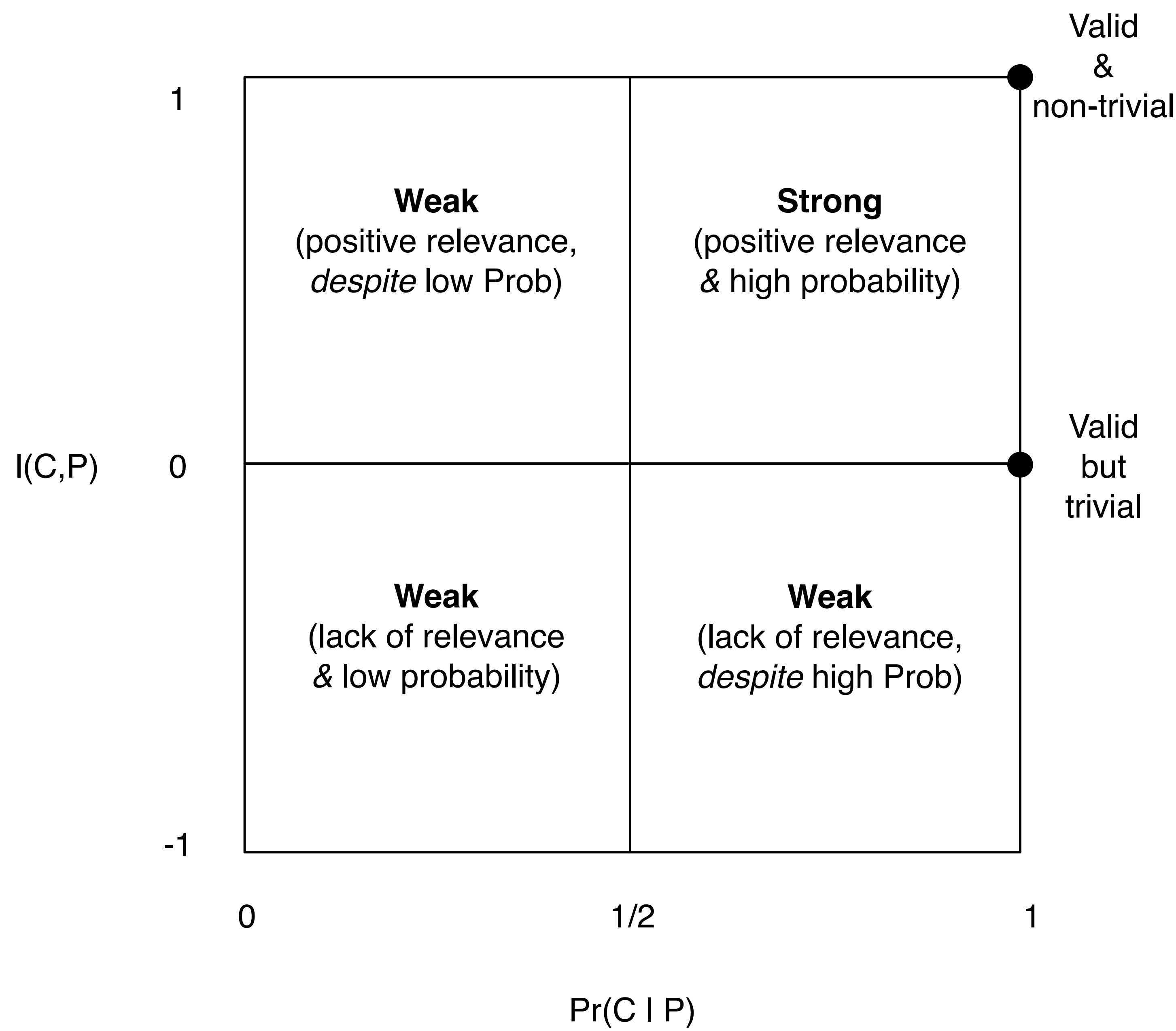
  (1) C is probable, *given* P, *i.e.*, $\Pr(C \mid P) > \frac{1}{2}$, *and*

  (2) P is *positively relevant* to C, *i.e.*, $\Pr(C \mid P) > \Pr(C)$.

- This places a strong constraint on the shape of f. Specifically, it requires that f be *above some threshold* t *in the upper-right quandrant of our 4-quadrant chart, and below* t *in the other three compartments*.

- We can visualize f as adding a *third dimension* to our 4-quadrant chart (imagine a z-axis, coming out of the chart). The height of each point in this third dimension will correspond to the value of $f(x, y)$.

- Of course, this qualitative constraint is not the end of the story. To get a better grip on f, we'd need to think about its *comparative* structure. This would involve thinking about various *pairs* $\langle x, y \rangle$ and their (intuitive) comparative relationship to each other…


Valid
&
non-trivial
1
Weak
(positive relevance,
despite low Prob)
Strong
(positive relevance
& high probability)
Valid
but
trivial
I(C,P)
0
Weak
(lack of relevance
& low probability)
Weak
(lack of relevance,
despite high Prob)
-1
0
1/2
1
Pr(C | P)

Probabilism and The Accuracy of Credences I

- Many philosophers have argued for **Probabilism**, which is the claim that one's degrees of confidence (*i.e.*, one's credences) *should obey the probability calculus*. I will discuss one argument for probabilism.
- In epistemology (the theory of knowledge and rational belief), it is typical to suppose that *accuracy* in one's judgments is a virtue.
- For instance, when it comes to (qualitative) *belief*, it is better to have true beliefs than false beliefs. If a belief is false, then it *misrepresents* the world, and this is generally agreed to be (epistemically) *bad*.
- Something similar can be said for credences. Here is a principle.

  The Principle of Gradational Accuracy (qualitative rendition). One ought to be more confident in truths than in falsehoods.
- Ideally, one would assign maximal confidence to all the truths and minimal confidence to all the falsehoods (think: omniscient agents).

Probabilism and The Accuracy of Credences II

- Of course, it would be far too strong to require all rational agents to live up to this ideal. But, we can use this ideal notion to generate an interesting argument for probabilism.
- Let's call the ideal credence function (in a possible world) *the vindicated credence function*. I will use $v_w(\cdot)$ to denote this ideal function.

$$v_w(p) = \begin{cases} 1 & \text{if } p \text{ is true in } w, \\ 0 & \text{if } p \text{ is false in } w. \end{cases}$$

- We can use $v_w(\cdot)$ to state a quantitative form of the PGA.

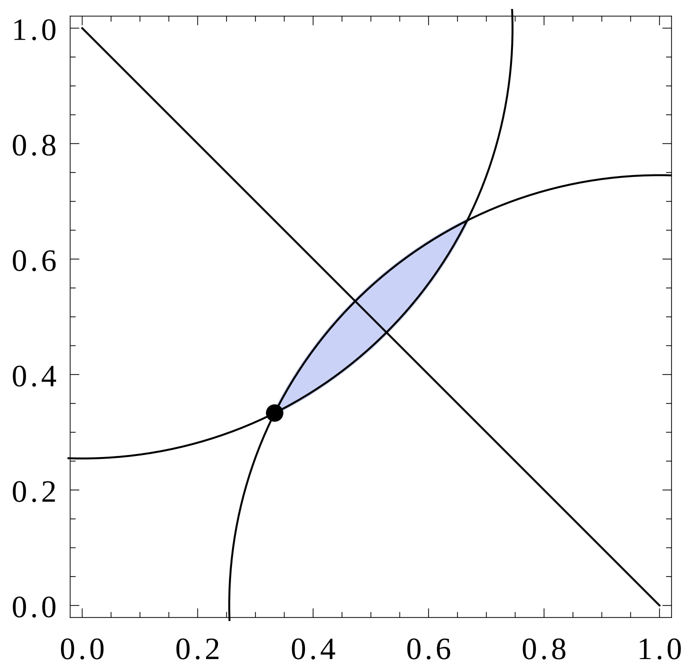
  The Principle of Gradational Accuracy (PGA, *quantitative* rendition). The closer a credence function $b(\cdot)$ is to $v_w(\cdot)$, the better.

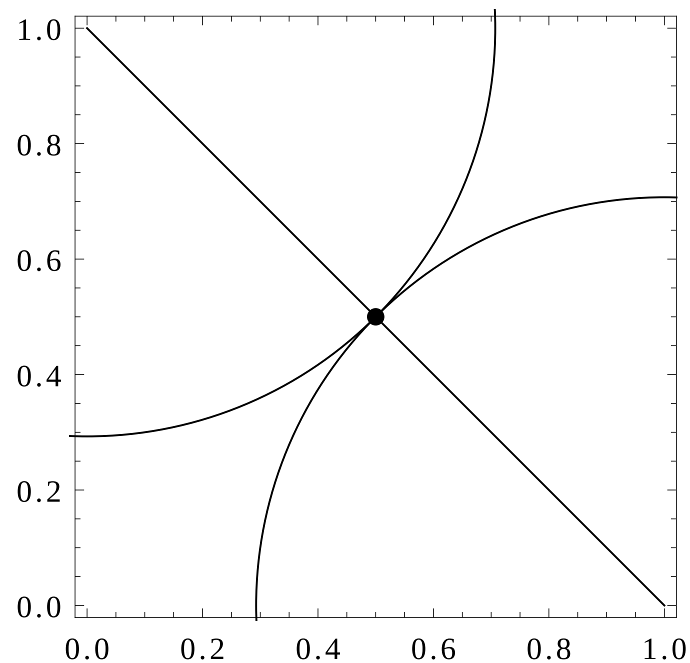
- To precisify PGA, we need a way to measure the *distance* between a credence function $b(\cdot)$ and the vindicated/ideal function $v_w(\cdot)$.

Probabilism and The Accuracy of Credences III

- Because we are only dealing with finite probability spaces, $b(\cdot)$ and $v_w(\cdot)$ will always be representable as *finite vectors of real numbers*.
- So, distance between $b(\cdot)$ and $v_w(\cdot)$ is just distance between finite vectors of real numbers. A very natural way to measure the distance between such vectors is *via* (squared) *Euclidean distance*.
- To make things easy, let's focus on the simplest possible example. Suppose we're assigning credences over a language with one atomic sentence: P. This means we'll have just *two states*: $\{P, \sim P\}$.
- So, any assignment of credence in this case will consist of vector containing two numbers: $\langle b(P), b(\sim P)\rangle$. This means we can visualize all such credences *via* a two-dimensional plot.
- On the next slide, I use such a plot to explain the simplest case of what I will call *the accuracy dominance argument for probabilism*.

Probabilism and The Accuracy of Credences IV





- The diagonal lines are the *probabilistic* b's (on $\langle P, \sim P\rangle$). The point $\langle 1, 0\rangle$ ($\langle 0, 1\rangle$) corresponds to the values assigned by $v_w(\cdot)$ in the P ($\sim P$) world.

  Theorem (de Finetti). b is *non*-probabilistic $\Leftrightarrow$ there exists a $b'(\cdot)$ which is (Euclidean) *closer to* $v_w(\cdot)$ *in every possible world*.

- The plot on the left (right) explains the $\Rightarrow$ ($\Leftarrow$) direction.

The Dutch Book Argument for Probabilism I

- The key assumptions/set-up of the Dutch Book argument are as follows:
  - For each proposition p that our agent (Mr. B) entertains at t, Mr. B must announce a number $q(p)$ — called his *betting quotient* on p, at t — and *then* Ms. A (the bookie) will choose the *stake* $\mathfrak{s}$ of the bet.
  - $|\mathfrak{s}|$ should be small in relation to Mr. B's total wealth (more on this later). But, it can be positive or negative (so, she can "switch sides").

$$\text{Mr. B's payoff (in \$) for a bet about } p = \begin{cases} \mathfrak{s} - q(p) \cdot \mathfrak{s} & \text{if } p \text{ is true.} \\ -q(p) \cdot \mathfrak{s} & \text{if } p \text{ is false.} \end{cases}$$

  - NOTE: If $\mathfrak{s} > 0$, then the bet is *on* p, if $\mathfrak{s} < 0$, then the bet is *against* p.
  - $q(p)$ is taken to be a measure of Mr. B's *degree of belief* in p (at t).
  - If there is a sequence of multiple bets on multiple propositions, then Mr. B's total payoff is the *sum* of the payoffs for each bet on each proposition. This is called "the package principle".

The Dutch Book Argument for Probabilism II

- The **Dutch Book Theorem** (DBT) has four parts [3 axioms for $\Pr(\cdot)$ plus 1 definition of $\Pr(\cdot \mid \cdot)$]. In each part, we prove that if $q(\cdot)$ [or $q(\cdot \mid \cdot)$] *violates* the axioms (or defn.), then $q(\cdot)$ is *in*coherent.

- **If Mr. B violates Axiom 1, then his q is incoherent**. Proof:
  - If $q(p) = a < 0$, then Ms. A sets $\mathfrak{s} < 0$, and Mr. B's payoff is $\mathfrak{s} - a\mathfrak{s} < 0$ if p, and $-a\mathfrak{s} < 0$ if $\sim p$. [If $q(p) \geq 0$, then Mr. B's payoff is $\mathfrak{s} - q\mathfrak{s} \geq 0$ if $\mathfrak{s} > 0$ and p is true, and $-q\mathfrak{s} \geq 0$ if $\mathfrak{s} < 0$ and $\sim p$, avoiding *this* Book.]

- **If Mr. B violates Axiom 2, then his q is incoherent**. Proof:
  - If Mr. B assigns $q(\top) = a < 1$, then Ms. A sets $\mathfrak{s} < 0$, and Mr. B's payoff is always $\mathfrak{s} - a\mathfrak{s} < 0$, since $\top$ cannot be false.
  - Similarly, if Mr. B assigns $q(\top) = a > 1$, then Ms. A sets $\mathfrak{s} > 0$, and Mr. B's payoff is always $\mathfrak{s} - a\mathfrak{s} < 0$, since $\top$ cannot be false.
    * NOTE: if $q(\top) = 1$, then Mr. B's payoff is always $\mathfrak{s} - \mathfrak{s} = 0$, which avoids *this particular* Dutch Book.

The Dutch Book Argument for Probabilism III

- Axiom 3 requires that

$$\Pr(p \vee r) = \Pr(p) + \Pr(r)$$

  if p and r are inconsistent (*i.e.*, if they can't both be true).

- The argument for this *additivity* axiom is more controversial. The main source of controversy is the "package principle".

- I will now go through the additivity case of the DBT.

- **Setup**: Let p and r be some pair of inconsistent propositions that the agent entertains at t. And, suppose Mr. B announces these q's:

$$q(p) = a,\ q(r) = b, \text{ and } q(p \vee r) = c, \text{ where } c \neq a + b.$$

- This leaves Mr. B susceptible to a *Dutch Book*. Next: the proof of this case of the DBT (note how this presupposes the "package principle").

The Dutch Book Argument for Probabilism IV

- **Case 1**: $c < a + b$. Ms. A asks Mr. B to make *all 3* of these bets ($\mathfrak{s} = +\$1$):
  1. Bet $\$a$ on p to win $\$(1 - a)$ if p, and to lose $\$a$ if $\sim p$.
  2. Bet $\$b$ on r to win $\$(1 - b)$ if r, and to lose $\$b$ if $\sim r$.
  3. Bet $\$(1 - c)$ *against* $p \vee r$ to win $\$c$ if $\sim(p \vee r)$, and lose $\$(1 - c)$ o.w.
- Since p and r are mutually exclusive (by assumption of the additivity axiom), the conjunction $p \,\&\, r$ cannot be true. $\therefore$ There are 3 cases:

Case	Payoff on (1)	Payoff on (2)	Payoff on (3)	Total Payoff
$p \,\&\, \sim r$	$1 - a$	$-b$	$-(1 - c)$	$c - (a + b)$
$\sim p \,\&\, r$	$-a$	$1 - b$	$-(1 - c)$	$c - (a + b)$
$\sim p \,\&\, \sim r$	$-a$	$-b$	c	$c - (a + b)$

- Since $c < a + b$, $c - (a + b)$ is negative. So, Mr. B loses $\$[c - (a + b)]$.
- **Case 2**: $c > a + b$. Ms. A simply reverses the bets ($\mathfrak{s} = -\$1$), and a parallel argument shows that the total payoff for Mr. B is $\$-[c - (a + b)] < 0$.
- Note: he can avoid *this* Book, by setting $c = a + b$.

The Dutch Book Argument for Probabilism V

- We also need to show that an agent's *conditional* betting quotients $q(\cdot \mid \cdot)$ are coherent only if they satisfy our ratio definition of $\Pr(\cdot \mid \cdot)$. There's a DBT for this too (note: this case *also* assumes the "package principle").
- Suppose Mr. B announces: $q(p \,\&\, r) = b$, $q(r) = c > 0$, and $q(p \mid r) = a$. Ms. A asks Mr. B to make *all 3* of these bets (stakes depend on quotients!):

1. Bet $\$(b \cdot c)$ on $p \,\&\, r$ to win $\$[(1 - b) \cdot c]$ if $p \,\&\, r$, and lose $\$(b \cdot c)$ o.w. [$\mathfrak{s} = c$]
2. Bet $\$[(1 - c) \cdot b]$ against r to win $\$(b \cdot c)$ if r, and lose $\$[(1 - b) \cdot c]$ o.w. [$\mathfrak{s} = b$]
3. Bet $\$[(1 - a) \cdot c]$ against p, conditional on r, to win $\$(a \cdot c)$ if $r \,\&\, p$, and lose $\$[(1 - a) \cdot c]$ if $r \,\&\, {\sim}p$. If ${\sim}r$, then the bet is *called off*, and payoff is \$0. [$\mathfrak{s} = c$]

Case	Payoff on (1)	Payoff on (2)	Payoff on (3)	Total Payoff
$p \,\&\, r$	$(1 - b) \cdot c$	$-[(1 - c) \cdot b]$	$-[(1 - a) \cdot c]$	$(a \cdot c) - b$
${\sim}p \,\&\, r$	$-(b \cdot c)$	$-[(1 - c) \cdot b]$	$a \cdot c$	$(a \cdot c) - b$
${\sim}r$	$-(b \cdot c)$	$b \cdot c$	0	0

- If $a < \frac{b}{c}$, then Mr. B loses *come what may*. If $a > \frac{b}{c}$, then Ms. A just asks Mr. B to take the other side on all three bets. So, coherence requires: $q(p \mid r) = \frac{q(p\&r)}{q(r)}$.