Practice Final Examination (With Solutions)

Philosophy 1115
April 15, 2016

1 Ten Cumulative True/False Questions

T $\boxed{\text{F}}$ 1. All unsound arguments are invalid.

$\boxed{\text{T}}$ F 2. Every sound argument has a true conclusion.

$\boxed{\text{T}}$ F 3. Some invalid arguments have a true conclusion.

T $\boxed{\text{F}}$ 4. If the conclusion of a valid argument is false, then all of its premises are false as well.

T $\boxed{\text{F}}$ 5. The following argument is sound: "Justin Bieber is a professional football player. If Justin Bieber is a professional football player, then Justin Bieber is bald. Therefore, Justin Bieber is bald."

$\boxed{\text{T}}$ F 6. The following LSL argument is valid: $\boxed{\begin{array}{l} A \,\&\, {\sim}A \\ \therefore\ B \end{array}}$

T $\boxed{\text{F}}$ 7. All inductively strong arguments are valid arguments.

T $\boxed{\text{F}}$ 8. All valid arguments are inductively strong arguments (in *my* "third proposal" sense).

T $\boxed{\text{F}}$ 9. All valid arguments are inductively strong arguments (in *Skyrms's* "second proposal" sense).

T $\boxed{\text{F}}$ 10. The following LSL sentence is *contingent*: '$[(A \rightarrow B) \rightarrow A] \rightarrow A$'.

2 Testing Two LSL Sentences for Equivalence, Contradictoriness, and Consistency

Consider the following pair of LSL sentences:

- '$(P \rightarrow Q) \vee (Q \rightarrow P)$'
- '$[(P \rightarrow Q) \rightarrow P] \rightarrow P$'

This question has three parts. In order to answer these questions, you should do the full truth-table for the pair of sentences, and compare them.

1. Are these two LSL sentences *logically equivalent*? **Yes, they are both tautologies.**
2. Are these two two LSL sentences *contradictory*? **No, they always have *the same* truth-value.**
3. Are these two two LSL sentences *consistent*? **Yes, in fact, they are *always* both true.**

3 Symbolizing an LSL Argument

Symbolize the following English argument in LSL (hint: use the 5 letters suggested at the end of the passage — but be explicit about your atomic sentence lexicon).

> John will not get a seat at the front of the class unless he arrives on time. The instructor will call roll right at the beginning of class provided she herself is not late. If she's not on time and John does get a seat at the front, he'll be able to finish his homework. So, if John isn't late in getting to class, then if the instructor does not call role right at the beginning of class, then he will get his homework done. (S, O, R, L, F)

Here is one correct LSL symbolization of this argument. First, the lexicon of atomic sentences.

- S = **John gets a seat at the front of the class.**
- O = **John arrives to class on time.**
- R = **The instructor calls roll right at the beginning of class.**
- L = **The instructor is late to class.**
- F = **John finishes his homework.**

And, here is the LSL symbolization of the argument.

$$\begin{array}{l} {\sim}S \vee O. \\ {\sim}L \rightarrow R. \\ (L \,\&\, S) \rightarrow F. \\ \therefore\ O \rightarrow ({\sim}R \rightarrow F). \end{array}$$

4 Testing Symbolized Argument for Validity

Is the LSL argument you arrived at above (in question 8) valid? Use any legitimate truth-table technique to answer this question (if it's valid, you need to give an exhaustive account; if it's invalid, then a single row of the truth-table will suffice).

The above argument is *in*valid. Here is a counterexample row from the truth-table of the argument.

F	L	O	R	S	${\sim}S \vee O$	${\sim}L \rightarrow R$	$(L \,\&\, S) \rightarrow F$	$O \rightarrow ({\sim}R \rightarrow F)$
$\bot$	$\top$	$\top$	$\bot$	$\bot$	$\top$	$\top$	$\top$	$\bot$

5 Proving a General Probabilistic Claim (Theorem) Algebraically

The following equation is true for all probability functions (*i.e.*, it is a theorem of probability calculus).

$$\Pr(H \mid E) = \frac{\Pr(E \mid H) \cdot \Pr(H)}{\Pr(E \mid H) \cdot \Pr(H) + \Pr(E \mid {\sim}H) \cdot \Pr({\sim}H)}$$

Use the following (generic) probabilistic truth-table (and our definitions of/rules for conditional and unconditional probability) to (a) translate both sides of this equation into their algebraic counterparts (in terms of the variables $a_1, \ldots, a_4$), and then (b) use algebraic simplification, *etc.* to show that the resulting algebraic equation reduces to an algebraic identity (assuming only that the a_i's are on $[0, 1]$ and that they sum to 1).

E	H	$\Pr(s_i)$
$\top$	$\top$	a_1
$\top$	$\bot$	a_2
$\bot$	$\top$	a_3
$\bot$	$\bot$	a_4

If we apply our definitions of conditional and unconditional probability to all of the constituents of the left and right hand sides of this equation, then we get the following algebraic equation:

$$\begin{aligned} \frac{a_1}{a_1 + a_2} &= \frac{\left(\frac{a_1}{a_1+a_3}\right) \cdot (a_1 + a_3)}{\left(\frac{a_1}{a_1+a_3}\right) \cdot (a_1 + a_3) + \left(\frac{a_2}{a_2+a_4}\right) \cdot (a_2 + a_4)} \\ &= \frac{\left(\frac{a_1}{\cancel{a_1+a_3}}\right) \cdot \cancel{(a_1 + a_3)}}{\left(\frac{a_1}{\cancel{a_1+a_3}}\right) \cdot \cancel{(a_1 + a_3)} + \left(\frac{a_2}{\cancel{a_2+a_4}}\right) \cdot \cancel{(a_2 + a_4)}} \\ &= \frac{a_1}{a_1 + a_2} \end{aligned}$$

6 Calculating Inductive Probabilities from a Probabilistic Truth-Table

I have an urn containing 30 objects. I'm going to sample an object o from the urn at random. Let:

- L = o is large ($\sim L$ = o is small).
- W = o is white ($\sim W$ = o is black).

Assume the following probabilistic truth-table (*i.e.*, inductive probability assignment):

L	W	world	$\Pr(s_i)$
$\top$	$\top$	s_1	$\frac{3}{30}$
$\top$	$\bot$	s_2	$\frac{12}{30}$
$\bot$	$\top$	s_3	$\frac{2}{30}$
$\bot$	$\bot$	s_4	$\frac{13}{30}$

Calculate the following inductive probabilities, using this probabilistic truth-table.

1. $\Pr(L \mid W)$.
   - **Answer**: $\Pr(L \mid W) = \dfrac{\Pr(L \,\&\, W)}{\Pr(W)} = \dfrac{\Pr(s_1)}{\Pr(s_1) + \Pr(s_3)} = \dfrac{\frac{3}{30}}{\frac{3}{30} + \frac{2}{30}} = \dfrac{3/30}{5/30} = \mathbf{\dfrac{3}{5}}$.
2. $\Pr(W \to L)$.
   - **Answer**: $\Pr(W \to L) = \Pr(s_1) + \Pr(s_2) + \Pr(s_4) = \dfrac{3}{30} + \dfrac{12}{30} + \dfrac{13}{30} = \dfrac{28}{30} = \mathbf{\dfrac{14}{15}}$.[1]
3. $\Pr(L)$.
   - **Answer**: $\Pr(L) = \Pr(s_1) + \Pr(s_2) = \dfrac{3}{30} + \dfrac{12}{30} = \dfrac{15}{30} = \mathbf{\dfrac{1}{2}}$.

7 Using Calculated Inductive Probabilities to Assess Inductive Strength

Using the above inductive probabilities (from question 9), answer the following questions about the inductive strength of the argument "o is white. Therefore, o is large" (*i.e.*, $\ulcorner W \therefore L \urcorner$).

1. Is the argument $\ulcorner W \therefore L \urcorner$ strong, according to Proposal #1?
   - **According to Proposal #1, $\ulcorner W \therefore L \urcorner$ is strong iff $\Pr(W \to L) > \frac{1}{2}$. From (2) above, we know that $\Pr(W \to L) = \frac{14}{15} > \frac{1}{2}$. Therefore, $\ulcorner W \therefore L \urcorner$ *is* strong, according to proposal #1.**
2. Is the argument $\ulcorner W \therefore L \urcorner$ strong, according to Proposal #2?
   - **According to Proposal #2, $\ulcorner W \therefore L \urcorner$ is strong iff $\Pr(L \mid W) > \frac{1}{2}$. From (1) above, we know that $\Pr(L \mid W) = \frac{3}{5} > \frac{1}{2}$. Therefore, $\ulcorner W \therefore L \urcorner$ *is* strong, according to proposal #2.**
3. Is the argument $\ulcorner W \therefore L \urcorner$ strong, according to Proposal #3?
   - **According to Proposal #3, $\ulcorner W \therefore L \urcorner$ is strong iff *both* $\Pr(L \mid W) > \frac{1}{2}$, *and* $\Pr(L \mid W) > \Pr(L)$. From (1) and (3) above, we know that $\Pr(L \mid W) = \frac{3}{5} > \frac{1}{2} = \Pr(L)$. Therefore, $\ulcorner W \therefore L \urcorner$ *is* strong, according to proposal #3.**

[1] Alternatively, you could calculate this as $\Pr(W \to L) = 1 - \Pr(\sim(W \to L)) = 1 - \Pr(s_3) = 1 - \frac{2}{30} = \frac{28}{30}$.