Rules Handout (Final Edition)

Symbolization Rules for LSL

English Expression	LSL Connective
not, it is not the case that, it is false that	$\sim$
and, yet, but, however, moreover, nevertheless, still, also, although, both, additionally, furthermore	&
or, unless, either ... or ...	$\vee$
if ... then ..., only if, given that, in case, provided that, on condition that, sufficient condition, necessary condition, unless (**Note**: don't confuse antecedents/consequents!)	$\rightarrow$
if and only if (iff), is equivalent to, sufficient and necessary condition for, necessary and sufficient condition for	$\leftrightarrow$

Detailed Symbolization Rules for $\rightarrow$

- $\ulcorner$if p then $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ implies $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ only if $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner q$ if $p\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ is a sufficient condition for $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner q$ is a necessary condition for $p\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner q$ provided $p\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner q$ whenever $p\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ is contingent upon $q\urcorner \mapsto \ulcorner p \rightarrow q\urcorner$
- $\ulcorner p$ unless $q\urcorner \mapsto \ulcorner \sim q \rightarrow p\urcorner$

Rules for Calculating Probabilities

Unconditional Probability Rule. The unconditional probability of a claim p is the sum of the probabilities of the states in which p is true.

$$\Pr(p) \stackrel{\text{def}}{=} \sum_{s_i \vDash p} \Pr(s_i)$$

Conditional Probability Rule. The conditional probability p, *given* q is the *ratio* $\frac{\Pr(p\&q)}{\Pr(q)}$, ***if*** $\Pr(q) > 0$.

$$\Pr(p \mid q) \stackrel{\text{def}}{=} \frac{\Pr(p \,\&\, q)}{\Pr(q)}, \textbf{\textit{ if}}\ \Pr(q) > 0.$$

2 Constraints on the state probabilities $\Pr(s_i) = a_i$.

(i) Each of the a_i is on the unit interval $[0, 1]$.

(ii) The $\{a_i\}$ sum to one. [*i.e.*, $\sum_i a_i = 1$]

Truth-Table Definitions of LSL Connectives

p	$\sim p$
$\top$	$\bot$
$\bot$	$\top$

p	q	$p \,\&\, q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$

p	q	$p \vee q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$

p	q	$p \rightarrow q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\top$

p	q	$p \leftrightarrow q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\top$