Announcements & Overview

- Administrative Stuff
  - **HW #3 grades & solutions have been posted**
    - People generally did very well on this one. (See Histogram.)
  - **The mid-term grades have also been posted**
    - Again, the class did very well, generally. (See Histogram.)
  - **HW #4 has been posted — due on Friday, March 25**
    - This one consists of six (6) validity testing problems. The last 3 *require* the "short method" (either method is OK for the first 3).
  - **Consult my "Short Method" handout for detailed examples of the "short method" and its presentation (to be discussed today).**
  - **My Friday office hours this week will be 3:30–4:45 (not 12–1:15).**
- Today: Truth-tables and their applications (continued)

The "Short" Method for Constructing Interpretations: Handout Problem #1

- Question: $A \to (C \vee E), B \to D \overset{?}{\vDash} (A \vee B) \to (C \to (D \vee E))$.
- Answer: $A \to (C \vee E), B \to D \nvDash (A \vee B) \to (C \to (D \vee E))$.
- Step 1: Assume there is an interpretation on which the premises is $\top$ but the conclusion is $\bot$. This leads to the following partial row:

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
					$\top$	$\top$	$\bot$

- Step 2: There's only one way the conclusion can be $\bot$, which leads to:

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
					$\top$	$\top$	$\top$ $\bot$ $\bot$

- Step 3: There's only one way $C \to (D \vee E)$ can be $\bot$, which leads to:

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
		$\top$			$\top$	$\top$	$\top$ $\bot$ $\top$ $\bot$ $\bot$

- Step 4: There's only one way $D \vee E$ can be $\bot$, which leads to:

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
		$\top$	$\bot$	$\bot$	$\top$	$\top\ \bot$	$\top\ \ \bot\ \top\ \bot\ \ \bot$

- Step 5: Since D is $\bot$, the only way $B \to D$ can be $\top$ is if B is $\bot$:

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
	$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\bot\ \top\ \bot$	$\top\ \ \bot\ \top\ \bot\ \ \bot$

- Step 6: Now, the only way to make the conclusion $\bot$ is to make 'A' $\top$, which yields the following counterexample to validity (check this!):

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top\ \top\ \ \top$	$\bot\ \top\ \bot$	$\top\ \ \bot\ \top\ \bot\ \ \bot$

- When reporting your answer, all you need to do is give the single row that serves as a counterexample. Here, I recommend you include the quasi-columns that you used to calculate the truth-values in the row.

- Verbal explanations are optional. Here's the detailed handout solution.

Answer. $A \to (C \vee E), B \to D \nvDash (A \vee B) \to (C \to (D \vee E))$

Explanation.[a] Assume that '$A \to (C \vee E)$' is $\top$, '$B \to D$' is $\top$, and '$(A \vee B) \to (C \to (D \vee E))$' is $\bot$. In order for '$(A \vee B) \to (C \to (D \vee E))$' to be $\bot$, both '$A \vee B$' and 'C' must be $\top$, and both 'D' and 'E' must be $\bot$. This *guarantees* that the first premise is $\top$ (since '$A \to (C \vee E)$' *must*, at this point, have a $\top$ consequent). We can also make the second premise $\top$, simply by making 'B' $\bot$. Finally, by making 'A' $\top$, we can ensure that the conclusion is $\bot$, which yields the following interpretation on which '$A \to (C \vee E)$' and '$B \to D$' are $\top$, but '$(A \vee B) \to (C \to (D \vee E))$' is $\bot$. *QED*.

A	B	C	D	E	$A \to (C \vee E)$	$B \to D$	$(A \vee B) \to (C \to (D \vee E))$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\top\ \boldsymbol{\top}\ \top\ \top\ \bot$	$\bot\ \boldsymbol{\top}\ \bot$	$\top\ \top\ \bot\ \boldsymbol{\bot}\ \top\ \bot\ \bot\ \bot\ \bot$

[a] You do *not* have to show *all* of your reasoning in cases like this one, where the argument is *in*valid (*i.e.*, where $\nvDash$). I am just showing you *all* of *my* reasoning to give you more information about how these kinds of problems are solved. All you *need* to do here is report an interpretation (*i.e.*, a single-row) which invalidates the inference. But, when you do so, I recommend filling-in all of the quasi-columns to make explicit all of the calculations required.

The “Short” Method for Constructing Interpretations: Handout Problem #2

- Question: $A \leftrightarrow (B \vee C), B \rightarrow D, D \leftrightarrow C \overset{?}{\vDash} A \leftrightarrow D$.
- Answer: $A \leftrightarrow (B \vee C), B \rightarrow D, D \leftrightarrow C \vDash A \leftrightarrow D$.
- Step 1: Assume there is an interpretation on which the premises is $\top$ but the conclusion is $\bot$. This leads to the following partial row:

A	B	C	D	$A \leftrightarrow (B \vee C)$	$B \rightarrow D$	$D \leftrightarrow C$	$A \leftrightarrow D$
				$\top$	$\top$	$\top$	$\bot$

- Already, we have to break this down into cases, since there are ($\geq$) two ways each premise can be $\top$ and also two ways the conclusion can be $\bot$.
  - Case 1: A is $\top$ and D is $\bot$.
  - Case 2: A is $\bot$ and D is $\top$.

- Step 2 (Case 1): If A is $\top$ and D is $\bot$, then we have the following:

A	B	C	D	$A \leftrightarrow (B \vee C)$	$B \to D$	$D \leftrightarrow C$	$A \leftrightarrow D$
$\top$			$\bot$	$\top\ \top$	$\top\ \bot$	$\bot\ \top$	$\top\ \bot\ \bot$

- Step 3 (Case 1): Now, the only way for $B \to D$ to be $\top$ is for B to be $\bot$. And, the only way for $D \leftrightarrow C$ to be $\top$ is for C to be $\bot$, which yields:

A	B	C	D	$A \leftrightarrow (B \vee C)$	$B \to D$	$D \leftrightarrow C$	$A \leftrightarrow D$
$\top$	$\bot$	$\bot$	$\bot$	$\top\ \top\ \bot\ \ \bot$	$\bot\ \top\ \bot$	$\bot\ \top\ \bot$	$\top\ \bot\ \bot$

- Step 4 (Case 1): But, we need $A \leftrightarrow (B \vee C)$ to be $\top$, which means we need $B \vee C$ to be $\top$. However, this contradicts our assumptions — dead end!

A	B	C	D	$A \leftrightarrow (B \vee C)$	$B \to D$	$D \leftrightarrow C$	$A \leftrightarrow D$
$\top$	$\bot$	$\bot$	$\bot$	$\top\ \top\ \bot$ $\top/\bot$!! $\bot$	$\bot\ \top\ \bot$	$\bot\ \top\ \bot$	$\top\ \bot\ \bot$

- As usual, we cannot infer — yet — that this argument is valid.
- We must continue on with an examination of Case 2 …

- Step 2 (Case 2): If A is $\bot$ and D is $\top$, then we have the following:

A	B	C	D	$A \leftrightarrow (B \vee C)$	$B \to D$	$D \leftrightarrow C$	$A \leftrightarrow D$
$\bot$			$\top$	$\bot\ \top$	$\top\ \top$	$\top\ \top$	$\bot\ \bot\ \top$

- Step 3 (Case 2): Now, the only way for $D \leftrightarrow C$ to be $\top$ is for C to be $\top$, which forces $B \vee C$ to be $\top$, contradicting our assumptions — dead end!

A	B	C	D	$A \leftrightarrow (B \vee C)$	$B \to D$	$D \leftrightarrow C$	$A \leftrightarrow D$
$\bot$		$\top$	$\top$	$\bot\ \top/\bot!!\ \ \top\ \top$	$\top\ \top$	$\top\ \top\ \top$	$\bot\ \bot\ \top$

- Since *both* of the two possible cases lead to a dead-end (*i.e.*, a contradiction), we may (finally) infer that this argument is *valid*.

- For valid arguments, you must give a verbal explanation of your "short" method answers. The handout contains two model solutions.

- Here's what the model solution on the handout looks like for this problem. Note: there are no "partial rows" included in the solution. You *may* include these (as in the lecture notes above), but you *need not*.

Answer. $A \leftrightarrow (B \vee C), B \to D, D \leftrightarrow C \vDash A \leftrightarrow D$.

Explanation. Assume '$A \leftrightarrow (B \vee C)$' is $\top$, '$B \to D$' is $\top$, '$D \leftrightarrow C$' is $\top$, and '$A \leftrightarrow D$' is $\bot$. There are *exactly two* ways in which '$A \leftrightarrow D$' can be $\bot$:

1. 'A' is $\top$, and 'D' is $\bot$. In this case, in order for '$D \leftrightarrow C$' to be $\top$, 'C' must be $\bot$. And, in order for '$B \to D$' to be $\top$, 'B' must be $\bot$. This means that the *disjunction* '$B \vee C$' must be $\bot$. So, in order for '$A \leftrightarrow (B \vee C)$' to be $\top$, we must have 'A' $\bot$ as well, which contradicts our assumption. So, in this first case, we have been forced into a *contradiction*.

2. 'A' is $\bot$, and 'D' is $\top$. In this case, in order for '$D \leftrightarrow C$' to be $\top$, 'C' must be $\top$. But, if 'C' is $\top$, then so is '$B \vee C$'. Hence, if '$A \leftrightarrow (B \vee C)$' is going to be $\top$, then 'A' must be $\top$, which contradicts our assumption. So, in this second (and *last*) case, we have been forced into a *contradiction*.

$\therefore$ There are no interpretations on which '$A \leftrightarrow (B \vee C)$', '$B \to D$', and '$D \leftrightarrow C$' are all $\top$ and '$A \leftrightarrow D$' is $\bot$. So, $A \leftrightarrow (B \vee C), B \to D, D \leftrightarrow C \vDash A \leftrightarrow D$. $\square$

Presenting Your "Short-Method" Truth-Table Tests

- In any application of the "short" method, there are two possibilities:

1. You find an interpretation (*i.e.*, a row of the truth-table) on which all the premises $p_1, \ldots, p_n$ of an argument are true and the conclusion q is false. *All you need to do here* is (*i*) write down the relevant row of the truth-table, and (*ii*) say "Here is an interpretation on which $p_1, \ldots, p_n$ are all true and q is false. So, $p_1, \ldots, p_n \therefore q$ is *in*valid."
2. You discover that there is *no possible way* of making $p_1, \ldots, p_n$ true and q false. Here, you need to *explain all of your reasoning* (as I do in lecture, or as Forbes does, or as I do in my handout). It must be clear that you have *exhausted all possible cases*, before concluding that $p_1, \ldots, p_n \therefore q$ is *valid*. This can be rather involved, and should be spelled out in a step-by-step fashion. Each salient case has to be examined.

- Consult my handout and lecture notes for model answers of both kinds.

Two Chapter 2 Examples — In Light of Chapter 3

If Yossarian flies his missions then he is putting himself in danger, and it is irrational to put oneself in danger. If Yossarian is rational he will ask to be grounded, and he will be grounded only if he asks. But only irrational people are grounded, and a request to be grounded is proof of rationality. So, Yossarian will fly his missions whether he is rational or irrational.

- Basic Sentences: Yossarian flies his missions (F), Yossarian puts himself in danger (D), Yossarian is rational (R), Yosarian asks to be grounded (A).
- We reconstructed this argument as having the following form:

$$
(1)\quad
\begin{array}{l}
(F \to D) \,\&\, (D \to {\sim}R) \\
(R \to A) \,\&\, ({\sim}F \to A) \\
({\sim}F \to {\sim}R) \,\&\, (A \to R) \\
\therefore\ (R \to F) \,\&\, ({\sim}R \to F)
\end{array}
$$

- (1) is valid. This can be verified using various truth-table techniques.

(1) $\begin{array}{l} (F \to D) \,\&\, (D \to {\sim}R) \\ (R \to A) \,\&\, ({\sim}F \to A) \\ ({\sim}F \to {\sim}R) \,\&\, (A \to R) \\ \therefore\ (R \to F) \,\&\, ({\sim}R \to F) \end{array}$ is valid, since its corresponding conditional is a tautology.

A D F R | (((F→D) & (D→ ~R)) & (((R→A) & (~F→A)) & ((~F→ ~R) & (A→R)))) → ((R→F) & (~R→F))

A	D	F	R	→	&	→	~	&	→	&	~	→	&	~	→	~	&	→	→	→	&	~	→
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- If we replace '$\sim F$' with 'G' throughout the argument, and then add the additional premise '$G \to \sim F$', then the resulting argument is *not* valid.

$(F \to D) \,\&\, (D \to \sim R)$

$(R \to A) \,\&\, (G \to A)$

(2) $(G \to \sim R) \,\&\, (A \to R)$ is *not* valid — see the truth-table on the following slide.

$G \to \sim F$ [implicit]

$\therefore (R \to F) \,\&\, (\sim R \to F)$

- What is needed is the other direction of '$G \to \sim F$', as in the following:

$(F \to D) \,\&\, (D \to \sim R)$

$(R \to A) \,\&\, (G \to A)$

(3) $(G \to \sim R) \,\&\, (A \to R)$

$\sim F \to G$ [implicit]

$\therefore (R \to F) \,\&\, (\sim R \to F)$

- As an exercise, use truth-table methods to show that (3) is valid. [Of course, the argument is also valid if we use the biconditional '$G \leftrightarrow \sim F$'.]

A	D	F	G	R	((((F→D)	&	(D→	~R))	& (((	R→A)	&	(G→A))	& ((	G→	~R)	&	(A→R))))	&	(G→	~F))	→	((R→F)	&	(~	R→F))
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⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊤	⊥
⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊤
⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊤
⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤
⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥
⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊤	⊥

Chapter 2 in Light of Chapter 3: Example #2

Suppose no two contestants enter; then there will be no contest. No contest means no winner. Suppose all contestants perform equally well. Still no winner. There won't be a winner unless there's a loser. And conversely. Therefore, there will be a loser only if at least two contestants enter and not all contestants perform equally well.

- Here are the atomic sentences:

  T: At least two contestants enter. C: There is a contest.

  P: All contestants perform equally well.

  W: There is a winner. L: There is a loser.

- The resulting sentential form of the argument is as follows:

  $\sim T \to \sim C$. $\sim C \to \sim W$. $P \to \sim W$. $\sim L \leftrightarrow \sim W$. Therefore, $L \to (T \,\&\, \sim P)$.

- This is a valid form, as can be seen via the following truth-table, which shows that its corresponding conditional is tautologous:

C	L	P	T	W	((~T	→	~C)	&	(~C	→	~W)	&	(P→	~W)	&	(~L	↔	~W))	→	(L→	(T&	~P))
⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	**⊤**	⊥	⊥	⊥
⊤	⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	**⊤**	⊥	⊥	⊥
⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	**⊤**	⊥	⊥	⊥
⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	**⊤**	⊥	⊥	⊥
⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊥	**⊤**	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	**⊤**	⊤	⊤	⊤
⊤	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	**⊤**	⊥	⊥	⊤
⊤	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	**⊤**	⊥	⊥	⊤
⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	**⊤**	⊤	⊥	⊥
⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	**⊤**	⊤	⊥	⊥
⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	**⊤**	⊤	⊥	⊥
⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	**⊤**	⊤	⊥	⊥
⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	**⊤**	⊤	⊤	⊤
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⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	**⊤**	⊥	⊥	⊥
⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	**⊤**	⊥	⊥	⊥
⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	**⊤**	⊥	⊥	⊥
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⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	**⊤**	⊥	⊥	⊤
⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤	**⊤**	⊥	⊥	⊤
⊥	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	**⊤**	⊤	⊥	⊥
⊥	⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	**⊤**	⊤	⊥	⊥
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⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	**⊤**	⊤	⊥	⊥
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- We saw the following premise our last argument: 'There won't be a winner unless there's a loser. And conversely.' I symbolized it as:
  - "Logish": If not L, then not W, *and conversely*. [*i.e.*, not L *iff* not W.]
  - LSL: '$\sim L \leftrightarrow \sim W$', *equivalently*: '$(\sim L \to \sim W)\ \&\ (\sim W \to \sim L)$'.
- Why not interpret the "and conversely" to be operating on the *unless* operator itself? This yields the following *different* symbolization:
  - "Logish": not W unless L, and L unless not W.
  - LSL: '$(\sim L \to \sim W)\ \&\ (\sim\sim W \to L)$', *equivalently*: '$(\sim L \to \sim W)\ \&\ (W \to L)$'.
- Answer: This is a *redundant* symbolization in LSL, since '$\sim L \to \sim W$' *is equivalent to* '$W \to L$'. Moreover, the resulting argument *isn't* valid.
- If we replace '$\sim L \leftrightarrow \sim W$' with '$\sim L \to \sim W$', then the resulting sentential form is not valid — see the truth-table on the following slide.
- **Principle of Charity**. If an argument $\mathscr{A}$ has ($\geq$) two *plausible but semantically distinct* LSL symbolizations (where neither is *obviously* preferable) — and *only one of them is valid — choose the valid one*.

C	L	P	T	W	((~T	→	~C)	&	(~C	→	~W)	&	(P →	~W)	&	(~L	→	~W))	→	(L →	(T &	~P))
⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊤	⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥
⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊤	⊤	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊥
⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤
⊤	⊤	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊤
⊤	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊥	⊥	⊤
⊤	⊥	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊤	⊥	⊥	⊤	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊤	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊤	⊥	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤
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⊤	⊥	⊥	⊥	⊤	⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤
⊤	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊥	⊤
⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊥	⊤	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥
⊥	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊥
⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊥
⊥	⊤	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊤	⊤	⊤	⊤
⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊤	⊥	⊥	⊤
⊥	⊤	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊥	⊥	⊤
⊥	⊥	⊤	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊥
⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊥
⊥	⊥	⊥	⊤	⊤	⊥	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤
⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤
⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊥	⊤	⊥	⊥	⊥	⊤	⊥	⊥	⊤	⊥	⊥	⊤	⊤	⊥	⊤
⊥	⊥	⊥	⊥	⊥	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊤	⊥	⊤

PHIL 201 & The LSAT: A Sample Question

A university library budget committee must reduce exactly five of eight areas of expenditure--G, L, M, N, P, R, S, and W--in accordance with the following conditions:

If both G and S are reduced, W is also reduced.
If N is reduced, neither R nor S is reduced.
If P is reduced, L is not reduced.
Of the three areas L, M, and R, exactly two are reduced.

Which one of the following could be a complete and accurate list of the areas of expenditure reduced by the committee?

(A) G, L, M, N, W
(B) G, L, M, P, W
(C) G, M, N, R, W
(D) G, M, P, R, S
(E) L, M, R, S, W

- Formalization of given information in LSL:
  - $(G \& S) \to W$
  - $N \to (\sim R \& \sim S)$
  - $P \to \sim L$
  - $((((L \& M) \vee (L \& R)) \vee (M \& R)) \& \sim(L \& (M \& R)))$
- Ruling-out answers:

(A) G, L, M, N, W

(B) G, L, M, P, W [impossible, since $P \to \sim L$]

(C) G, M, N, R, W [impossible, since $N \to (\sim R \& \sim S)$]

(D) G, M, P, R, S [impossible, since $(G \& S) \to W$]

(E) L, M, R, S, W [impossible, since $\sim(L \& (M \& R))$]

- The question is asking: which of (A)–(E) is *consistent* (in the LSL sense!) with the given information. Hint: (B)–(E) can be *ruled-out* quickly (shortcuts!).
- So, there is no need to *prove* (A) is consistent with the given information. To do that, one would produce a truth-table *row* in which G, L, M, N, W all come out $\top$, and such that all four given sentences also come out $\top$.

Final Topic From Chapter 3: Expressive Completeness

- In LSL, we have five connectives: $\langle\sim, \&, \vee, \rightarrow, \leftrightarrow\rangle$. But, we don't "need" all five. We can express all the same propositions with fewer connectives.
- If a set of connectives is sufficient to express all the propositions expressible in LSL, then we say that set is *expressively complete*.
- To show that a set is expressively complete, all we need to do is show that we can emulate all five LSL connectives using just that set.
- **Fact.** The set of 4 connectives $\langle\sim, \&, \vee, \rightarrow\rangle$ is expressively complete.
  - All we need to do is explain how $\langle\sim, \&, \vee, \rightarrow\rangle$ allows us to express all statements that involve '$\leftrightarrow$' — *i.e.* — to *define* '$\leftrightarrow$' using $\langle\sim, \&, \vee, \rightarrow\rangle$.
  - There are many ways we could do this. Here's one:

$$\ulcorner p \leftrightarrow q \urcorner \mapsto \ulcorner (p \rightarrow q) \,\&\, (q \rightarrow p) \urcorner$$

  - This works because: $\ulcorner p \leftrightarrow q \urcorner \mathrel{=\!\!|\!\!\models} \ulcorner (p \rightarrow q) \,\&\, (q \rightarrow p) \urcorner$.

- **Fact.** The set of 3 connectives $\langle \sim, \&, \vee \rangle$ is expressively complete.
  - Since we already know that $\langle \sim, \&, \vee, \rightarrow \rangle$ is expressively complete, all we need to do is explain how $\langle \sim, \&, \vee \rangle$ allows us to emulate '$\rightarrow$'.
  - Again, there are many ways to do this. The most obvious is:

$$\ulcorner p \rightarrow q \urcorner \mapsto \ulcorner \sim p \vee q \urcorner$$

- **Fact.** The pairs $\langle \sim, \& \rangle$ and $\langle \sim, \vee \rangle$ are both expressively complete.
  - For $\langle \sim, \& \rangle$, we just need to show how to express '$\vee$':

$$\ulcorner p \vee q \urcorner \mapsto \ulcorner \sim(\sim p \;\&\; \sim q) \urcorner$$

  - The $\langle \sim, \vee \rangle$ strategy is similar [$\ulcorner p \;\&\; q \urcorner \mapsto \ulcorner \sim(\sim p \vee \sim q) \urcorner$].

- Consider the binary connective '$|$' such that $\ulcorner p | q \urcorner \mathrel{=\!\!|\!\models} \ulcorner \sim(p \;\&\; q) \urcorner$.

- **Fact.** '$|$' *alone* is expressively complete! How to express $\langle \sim, \& \rangle$ using '$|$':

$$\ulcorner \sim p \urcorner \mapsto \ulcorner p | p \urcorner, \text{ and } \ulcorner p \;\&\; q \urcorner \mapsto \ulcorner (p|q)|(p|q) \urcorner$$

  - I called '$|$' 'NAND' in a previous lecture. NOR is also expressively complete.

Expressive Completeness: Additional Remarks and Questions

- **Q.** How can we define $\leftrightarrow$ in terms of $|$? **A.** If you naïvely apply the schemes I described last time, then you get a *187 symbol monster*:

  $\ulcorner p \leftrightarrow q \urcorner \mapsto A|A$, where A is given by the following *93 symbol* expression:

  $$(((p|(q|q))|(p|(q|q)))|((p|(q|q))|(p|(q|q))))|(((q|(p|p))|(q|(p|p)))|((q|(p|p))|(q|(p|p))))$$

- There are *simpler* definitions of $\leftrightarrow$ using $|$. *E.g.*, this *43 symbol* answer:

  $$\ulcorner p \leftrightarrow q \urcorner \mapsto ((p|(q|q))|(q|(p|p)))|((p|(q|q))|(q|(p|p)))$$

- Can anyone give an *even simpler* definition of $\leftrightarrow$ using $|$? Extra-Credit!
- How could you show that the pair $\langle \rightarrow, \sim \rangle$ is expressively complete?
- **Fact.** No subset of $\langle \sim, \&, \vee, \rightarrow, \leftrightarrow \rangle$ that does *not* contain negation $\sim$ is expressively complete. [This is proved in our advanced logic class.]
- Let $\curlywedge$ denote the $\perp$ truth-function (*i.e.*, the trivial function that *always* returns $\perp$). How could you show that $\langle \rightarrow, \curlywedge \rangle$ is expressively complete?

Epilogue: A Famous Set of Logic Puzzles

- The island of Knights and Knaves has two types of inhabitants, Knights who always tell the truth, and Knaves who always lie (no knaves are knights).
- Suppose A is the proposition person a is a knight and suppose a makes a statement S. Then, A is true if and only if S is true, since A is equivalent to S.
- That is, $A \approx S$. So, whenever an inhabitant x makes a claim S, we can infer that $X \leftrightarrow S$. That is, we can infer that x is a knight if and only if S is true.
- If a says "I am a Knight" then we can infer from this statement that $A \leftrightarrow A$. But, since this is *logically true*, we get no information from such a statement.
- A native *cannot* say "I am a Knave", since if this were true, then it would be false and if it were false, then it would be true (and, no Knights are Knaves).
- If a says "I am the same type as b," then we can infer $A \leftrightarrow (A \leftrightarrow B)$ which is equivalent to B (that is, $B \mathrel{=\!|\!\models} A \leftrightarrow (A \leftrightarrow B)$, which you can prove using truth-tables). So, this statement allows us to infer that person b is a Knight!