Epistemology formalized

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As we often tell our undergraduates, epistemology is the study of *knowledge*. Given just this simple definition, 'formal epistemology' seems like a misnomer for the philosophical program inspired by Thomas Bayes and developed in the twentieth century by Ramsey 1926, de Finetti 1937, Jeffrey 1983 and others. Bayesians articulate constraints on rational credences: synchronic constraints on what credences you may have, and diachronic constraints on how your credences must evolve. Like traditional epistemologists, Bayesians are concerned with norms governing your doxastic state. But in modeling your doxastic state, Bayesians do not represent what full beliefs you have.[2] And so they do not have the resources to talk about which of those beliefs constitute knowledge.

This paper develops a formal extension of traditional epistemology for which 'epistemology' is not a misnomer. I accept the traditional claim that beliefs can constitute knowledge. But I argue for an apparently radical thesis about the doxastic states that Bayesians care about: some of these states can also constitute knowledge. For example, suppose you are playing an ordinary poker game, and you have just been dealt some middling cards face down. Your justifiedly low credence that you have been dealt four aces may constitute knowledge. The thesis appears radical because knowledge is ordinarily assumed to be a factive attitude, where an attitude is *factive* if and only if necessarily, one bears it only to truths. Since the proposition that you have been dealt four aces is not a truth, this assumption entails that your low credence in that proposition cannot constitute knowledge. The aim of this paper is

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1. Thanks to Allan Gibbard, Jim Joyce, Jason Stanley, Seth Yalcin, and the Michigan Crop Circle for helpful comments, as well as audiences at Pittsburgh and the Michigan Formal Epistemology Working Group.

2. At most, one might be able to derive facts about what full beliefs you have from facts that Bayesians do represent. But even this "Lockean Thesis" is contentious; see Foley 1993 and Huber & Schmidt-Petri 2009 for further discussion.

to set out the strongest possible case for the apparently radical thesis in light of this simple argument against it. I argue that the thesis ultimately does not challenge our core assumptions about the nature of knowledge. And I argue that the thesis yields simple solutions to some difficult problems.

In §1, I present the first of these problems: giving a semantics for factive attitude reports embedding language of subjective uncertainty. In §2, I present a second problem: identifying what is wrong with the credences of agents in probabilistic analogs of Gettier cases. In §3, I state my thesis and demonstrate that it can solve these problems. In §4, I argue that the thesis is not as radical as it appears, since several core epistemological notions naturally extend to states other than full beliefs. Even if knowledge is not always a relation to propositions, there is nevertheless a sense in which it may be factive, safe, and sensitive. In §5, I flag several decision points in the development and application of the notion of non-propositional knowledge. I conclude in §6 by outlining a number of further problems that may be solved by accepting non-propositional knowledge, including problems that have been recently raised by opponents of pragmatic encroachment and knowledge-based norms for action.

1 The first problem: ascriptions embedding language of subjective uncertainty

The language of subjective uncertainty is pervasive in ordinary conversation:

(1) John might be in his office.
(2) Paris is probably the largest city in France.
(3) Marseille is probably not the largest city in France.
(4) If the second largest British city is not Leeds, then it is probably Birmingham.
(5) John must have gone home early today.
(6) If John is still in the building, he is in his office.
(7) It is more likely that John is in London than that he is in Paris.

And yet it is notoriously difficult for traditional semantic theories to model such language. For instance, KRATZER 1977 gives a powerful truth-conditional semantics for necessity and possibility modals. KRATZER 1981a extends the account to some "graded" modal vocabulary, and KRATZER 1978, 1981b discusses conditionals. On the theory that Kratzer develops in these papers, sentences containing language of subjective uncertainty are context-sensitive. In particular, modals quantify over a contextually supplied domain of possibilities. This approach provides an elegantly uniform treatment of different modalities: *epistemic* modals are just those that quantify over *epistemic* possibilities.

However, many have raised serious objections to this prevailing truth conditional theory of expressions of subjective uncertainty. For instance, Yalcin 2007 argues that standard truth conditional theories of epistemic modals fail to predict the behavior of embedded modals. For example:

(8)   #Suppose that it is not raining and it might be raining.

(9)   #Suppose that it is raining and it is probably not raining.

(10)   #If it is not raining and it is probably raining, then. . .

According to standard truth conditional theories, Yalcin argues, 'it might be raining' is true just in case certain contextually determined evidence does not rule out that it is raining. It is perfectly acceptable to suppose both that it is not raining and that certain evidence does not rule out that it is raining. So standard truth conditional theories incorrectly predict that it is acceptable to suppose as in (8).

A second objection is that truth conditional theories flout our intuitions about the subject matter of sentences containing language of subjective uncertainty. For example, consider the famous Sly Pete example from Gibbard 1981. Suppose you observe Sly Pete advance to the last round of a poker game. Just then you leave the room, but only after seeing that the unscrupulous Pete has looked at his opponent's hand. On this basis, you may utter the following:

(11)   If Pete called, he won.

Kratzer 1986 says that in order to account for Gibbardian stand-offs, truth conditional theories must claim that you use (11) to report facts about your own evidential state. But many have rejected this result as intuitively unacceptable. For instance, Bennett 2003 provides the following argument about a close variant of the Sly Pete case: "Common sense and the Ramsey test both clamour that [the speaker of (11)] is not assuring me that her value for a certain conditional probability is high, but is assuring me of that high value. . . She aims to convince me of that probability, not the proposition *that* it is her probability" (90). Yalcin 2009 adds that the *reasons* that I give in support of my utterance 'it is probably raining' concern only the proposition that it is raining, rather than any contextually determined body of evidence.

A third objection follows from an observation in Adams 1965 and Stalnaker 1970, namely that your degree of belief in an indicative conditional should match your degree of conditional belief in the consequent given the antecedent.[3] Unfortunately for the truth-conditional semanticist, the triviality results of Lewis 1976 demon-

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3. An alternative interpretation understands Adams 1965 as defending a claim about appropriateness of utterances rather than degrees of belief. Hall & Hájek 1994 provide a helpful assessment of interpretations of Adams' Thesis, as well as a catalog of further triviality results.

strate that indicative conditionals cannot express propositions such that your credence in the conditional proposition matches your conditional credence in the consequent given the antecedent. Many have responded by rejecting the claim that indicative conditionals express propositions.[4]

These objections merely scratch the surface.[5] In response to scores of compelling arguments, many have developed alternative semantic theories of the language of subjective uncertainty. Broadly speaking, some advocate a dynamic semantics. Others advocate a relativist semantics. And others advocate an expressivist semantics.[6] On each of these accounts, (1)–(7) do not have straightforward truth conditions.

For ease of exposition, I shall assume a very simple expressivist semantics in my discussion of language of subjective uncertainty, a semantics along the lines of that defended in SWANSON 2006 and YALCIN 2007.[7] On this simple semantics, assertions express advice concerning credal states. In particular, the semantic value of a sentence is a constraint on your credences, and my assertion of the sentence expresses the advice that your credal state conform to that constraint. The resulting semantics fits neatly with Bayesian doctrine: the semantic value of a sentence is a set of probability measures, and my assertion expresses the advice that your credence distribution be among the members of that set. For example:

(3)   Marseille is probably not the largest city in France.

(4)   If the second largest British city is not Leeds, then it is probably Birmingham.

(7)   It is more likely that John is in London than that he is in Paris.

(3) advises you to give low credence to the proposition that Marseille is the largest city in France. (4) advises you to give high conditional credence to the proposition that the second largest British city is Birmingham, conditional on the proposition that it is not Leeds. (7) advises you to give more credence to the proposition that John is in London than to the proposition that he is in Paris.

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4. For sympathetic discussion and a catalog of relevant literature, see EDGINGTON 1995.

5. For additional arguments against truth conditional theories, see the case of the missing car keys in SWANSON 2006 and VON FINTEL & GILLIES 2010, the "speaker inclusion constraint" in EGAN et al. 2005 and WEATHERSON 2008, the eavesdropping cases in EGAN 2007, the discussion of assertability and disagreement in YALCIN 2010, and the discussion of retraction and disputes in MACFARLANE 2010.

6. For relativist theories, see EGAN et al. 2005, EGAN 2007, STEPHENSON 2007, WEATHERSON 2008, and MAC-FARLANE 2010. For dynamic theories, see VELTMAN 1985, 1996 and GILLIES 2004, 2009. For expressivist theories, see ADAMS 1975, EDGINGTON 1995, BENNETT 2003, SWANSON 2006, YALCIN 2007, and SCHNIEDER 2010. For decisive arguments against force modifier theories, see SWANSON 2010.

7. It is necessary to make simplifying assumptions; developing a more nuanced expressivist theory, or arguing for an expressivist semantics, would take us too far afield of the present project. However, my discussion is ultimately neutral between various semantics for language of subjective uncertainty. In §5, I briefly discuss the relevance of my arguments for advocates of dynamic and relativist theories.

Expressivist theories provide a very natural account of attitude ascriptions embedding language of subjective uncertainty. A belief ascription simply says that the credences of the subject fit the constraint that is the semantic value of the prejacent. For example, (7) says that Bob gives more credence to the proposition that John is in London than to the proposition that he is in Paris:

(7) Bob believes that it is more likely that John is in London than that he is in Paris.

On this analysis, attitude ascriptions ascribe relations not just to propositions, but to constraints. And that is just as the Bayesian would have it, since she endorses the claim that propositions do not suffice to characterize the contents of our attitudes.

This natural account of attitude ascriptions constitutes yet another argument against the semantics of epistemic modals advocated in KRATZER 1977, 1981a. As YALCIN 2010 points out, the way that Bayesians informally describe credal states suggests that Bayesians already tacitly accept this sort of account of attitude ascriptions (§9). For instance, Bayesians use (7) in order to describe credences in first-order propositions about John, not in order to describe beliefs in propositions about the likelihood of first-order propositions given a certain body of evidence. In fact, the expressivist account of ascriptions is so intuitive that advocates of truth conditional theories have aimed to replicate its verdicts. For instance, KRATZER 2010 develops a semantics according to which (12) and (13) do not straightforwardly ascribe relations to propositions:

(12) Bob believes that John might be in his office.
(13) Bob believes that John must have gone home early today.

On the traditional truth conditional semantics, (12) says that in every world compatible with what Bob believes, a certain body of evidence does not rule out that John in his office. On the revised semantics in KRATZER 2010, (12) simply says that in some worlds compatible with what Bob believes, John is in his office. (13) says that in all worlds compatible with what Bob believes, John went home early.

The non-truth-conditional innovations I have described constitute major progress in the semantics of the language of subjective uncertainty. But they also give rise to a serious problem. The problem arises because language of subjective uncertainty is not only commonly embedded in belief reports, but in reports of knowledge and other factive attitudes. For example:

(14) If you give a clear, understandable direction... and the child does not comply, then you know that it is more likely due to compliance issues than

lack of understanding.[8]

(15)   The Fellahs advanced till they saw that it was probably a large tomb.[9]

(16)   I couldn't figure out how they had the entire day free to go tracking down Seth, until I remembered it was most likely a Sunday.[10]

(17)   By monitoring your home a couple of times a year, you can observe fading and cracking issues, which lets you know that it is probably getting close to that time again.[11]

(18)   I hereby let you know that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi*.[12]

(19)   I realized that I probably liked that guy as more than a friend.[13]

For the expressivist, the semantic value of the prejacent of the attitude report in (18) is the set of all probability measures that give higher credence to the proposition that the specimens under discussion are *G. hackmani* than to the proposition that they are *G. balachowskyi*. The attitude report ascribes a relation to that semantic value. But that spells trouble, for it is traditionally taken as a platitude that knowledge is an attitude that one can bear only to true propositions.[14] Hence our first problem: knowledge is an attitude towards truths; (18) ascribes knowledge; yet (18) does not ascribe an attitude towards truths.[15]

The same problem arises for any theory that says that attitude reports ascribe attitudes to the contents of their embedded clauses, and that sentences containing expressions of subjective uncertainty do not have truth conditional contents. In short, nearly everyone faces some form of the problem, including advocates of dynamic, relativist, and expressivist semantic theories, as well as KRATZER 2010. Our traditional understanding of knowledge does not fit our contemporary understanding of assertion and the ascription of attitudes.

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8. WEBB et al. 2007, p.113

9. http://ascendingpassage.com/tomb-of-Seti-I-Belzoni.htm

10. http://forums.televisionwithoutpity.com/index.php?showtopic=3180931&st=885

11. http://www.jdhostetter.com/news/detail.asp?news_id=3

12. http://www.diptera.info/print.php?type=F&thread=782

13. http://www.dearcupid.org/question/i-know-i-cant-have-him-so-what.html

14. I focus on knowledge-that rather than objectual knowledge, knowledge how, or knowledge-wh. Ascriptions of the latter attitudes raise interesting issues that are largely orthogonal to the present discussion.

15. Here I use '(18)' to refer to the token utterance cited above. It is important to appreciate that a sentence containing language of subjective uncertainty may have multiple readings, including readings for which a contextualist semantics is appropriate. Anti-contextualist arguments do not establish that no utterance containing language of subjective uncertainty expresses a proposition about a contextually determined body of evidence, but merely that not all utterances fit this mold. I discuss particular sentences for ease of exposition, but my arguments ultimately concern any embedded language of subjective uncertainty not accommodated by contextualist accounts.

2 The second problem: probabilistic analogs of Gettier cases

A second problem accompanies a contemporary formal understanding of learning. The following story is a familiar one: according to tradition, you should update your credence distribution by conditionalizing it on the propositions you learn. But some instances of learning intuitively call for other updating procedures. JEFFREY 1965 argues that seeing a cloth in dim light may lead you to rationally assign .7 credence to the proposition that the cloth is green, and VAN FRAASSEN 1980 suggests that the soldier Judy Benj amin may directly update her conditional credences upon hearing sentences such as:

(20)   If you are in Red Army territory, you are .75 likely in their Headquarters Company Area.

On the resulting picture, you should update your credence distribution by adopting credences that conform to constraints delivered by your experiences. JOYCE 2005 sums up the picture as follows: "at any time there should be some set of constraints that specify those invariant features of a person's credal state that are directly imposed by her evidence" (158). For VAN FRAASSEN 1981, these features include "deliverances of experience" that constrain the credences that you can rationally have (375). Indeed, it is intuitively evident that experience can impose constraints on the credences you have in all the same ways that it can impose constraints on the propositions that you believe. The examples (15)–(19) given in the previous section are naturally understood as statements about constraints on credences provided by perception, memory, inference, testimony, and introspection, respectively.

It is notoriously difficult to defend general procedures for directly updating credences on constraints.[16] But it is also necessary to appreciate the powerful reasons for endorsing alternatives to strict conditionalization. It is not just that our intuitions about particular cases call for alternative procedures. Strict conditionalization results in certainties, and certainties make agents prefer bets at arbitrarily risky odds, and maintain opinions that cannot be altered by further learning. It may simply seem unreasonable for ordinary agents to have this degree of "blind faith" in propositions. This point is recognized as early as QUINE 1951, where Quine argues that even observational propositions can be undermined by theoretical considerations. Even without endorsing any Quinean confirmational holism, many contemporary theorists accept that ordinary agents should not be perfectly certain of non-trivial propositions. Learning should not by its very nature require an agent to become dogmatically inflexible

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16. For further discussion, see DIACONIS & ZABELL 1982, JAYNES 1978, SKYRMS 1987, JOYCE 1999, and GRÜNWALD & HALPERN 2003.

with respect to some opinion.

Just as beliefs that result from experience can be justified or unjustified, so can properties of your credence distribution. This claim is not meant to be surprising; others have defended the similar claim that your having a particular degree of belief in a proposition may or may not be justified.[17] The present claim is simply that even if learning proceeds by directly updating on constraints—rather than by acquiring a special epistemic relation to individual propositions—resulting properties of your credence distribution may be justified. In other words, you may be justified in having credences with the property of satisfying the constraint on which you updated. As with full belief, this justification comes in degrees. Even if your experience justifies your giving .3 credence to a given proposition, you may gain further justification for that same credence through further experience. Judy Benjamin may call helicopter pilots to confirm the testimony of her first informant, for instance. Or she may spot flags on nearby trees, and recall that such markers are exactly three times as prevalent in the Red Army Headquarters Area as in the rest of the Red Army territory.

There are many perspicuous parallels between justified beliefs and justified properties of credence distributions. But not every parallel between beliefs and properties of credence distributions is so easily understood. In particular, there are cases in which justified properties of credence distributions fail to be epistemically good, and they fail in just the same way that justified beliefs fail to constitute knowledge. Recall the following sample ascriptions involving constraints on credences provided by testimony and introspection:

(18) I hereby let you know that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi*.

(19) I realized that I probably liked that guy as more than a friend.

Here are some examples where similar testimony and introspection intuitively fail to yield knowledge:

> Several assistants at a reputable entomology supply company select specimens from cultures in response to mail order requests for cereal flies. There are two cultures, one consisting mainly of *G. hackmani* specimens and one consisting mainly of *G. balachowskyi* specimens. A diligent lab assistant sends out specimens from the former culture along with a letter to the recipient saying that their specimens are more likely *G. hackmani* than *G. balachowskyi*. Meanwhile, several disgruntled lab assistants have started sending out the very same sort of letter along with specimens from the latter culture. On receiving your letter, which happens to be from the diligent lab assistant, you assign more credence to the proposition that your

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17. For some representative examples, see p.200 of WILLIAMSON 2000, §3.2 of MAHER 2004, p.129 of CONEE & FELDMAN 2010, and §2 of KVANVIG 2010.

specimens are *G. hackmani* than to the proposition that they are *G. balachowskyi*.

> Sue and her friend Bob enter a psychology study. The study proceeds as follows: each woman is given questionnaires that indicate whether she finds her friend attractive. If she does, she is injected with an anxiety-producing drug before meeting her friend. If she does not, she is injected with a saline solution. Sue is not told about the nature of the experiment. She does find Bob attractive. On receiving the anxiety-producing drug and meeting Bob, Sue reflects on her fluttering nerves and raises her credence that she finds Bob attractive.[18]

In both cases, agents rationally update their credences on a constraint delivered by their experience. In the entomology case, your relative confidence that your specimens are *G. hackmani* is justified on the basis of the letter you receive. In the psychology case, Sue's confidence that she finds Bob attractive is justified on the basis of her fluttering nerves. And one may suppose that in both cases, agents arrive at just the credences that they should have, objectively speaking. In other words, they arrive at the credences that they would have if they were entirely enlightened about their situation.

And yet: in both cases, there is something epistemically incorrect about these credences. As a symptom of this incorrectness, notice that the following variants of (18) and (19) are intuitively false:

(21) On receiving your flies from the entomology supply company, you know that it is more likely that they belong to *G. hackmani* than to *G. balachowskyi*.

(22) Sue knows that she probably likes Bob as more than a friend.

The entomology case resembles the fake barn case by Carl Ginet, cited by GOLDMAN 1976. The psychology case resembles the traditional Gettier cases in GETTIER 1963. Both instances of resemblance raise our second problem, namely how to give a uniform account of the epistemic incorrectness of Ginet beliefs and the credences in (21), and Gettier beliefs and the credences in (22). As PRITCHARD 2005 would put it, the former result from "environmental luck" and the latter result from "intervening luck." A less-than-satisfying response would be to say that the absence of luckiness is simply a primitive epistemic virtue that properties of credence distributions may exhibit. A more satisfying theory would identify positive epistemic properties that properties of credence distributions may share with beliefs, and explain why these properties are not instantiated in the above examples.

A natural conservative response to the examples is to try to explain the epistemic incorrectness of the relevant credences in terms of the epistemic incorrectness

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18. See DUTTON & ARON 1974 for a similar experiment demonstrating the misattribution of arousal by study participants.

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of full beliefs. For instance, one might say that the incorrect credences in each case are grounded in full beliefs that fail to constitute knowledge. I discuss similar strategies in detail in §5.2, but it may be useful to note in advance that the conservative response faces several challenges. For instance, the most natural way of spelling out the claim that your credences are grounded in certain propositional beliefs is to say that your credences are the result of your conditionalizing on certain propositions. But this claim conflicts with our initial rejection of strict conditionalization as an acceptable updating procedure. Furthermore, the conservative response forecloses on the possibility of a certain sort of "Probability First" epistemology, according to which all epistemic facts about agents can ultimately be understood in terms of facts about their credence distributions and value functions.[19] And even for opponents of Probability First theories, it is difficult to point out the full beliefs that allegedly ground the incorrect credences in the above examples. The fact that Sue gives a certain amount of credence to the proposition that she likes Bob may not even supervene on facts about her full beliefs.

3 A solution: probabilistic knowledge

The problems raised in §§1–2 are obviously intertwined. For instance: in the §2 cases, one ascribes epistemically incorrect credences using belief reports embedding language of subjective uncertainty, and corresponding knowledge reports are infelicitous. Also: if we adopt an expressivist semantics for assertions in order to address the problems raised in §1, then assertions such as (20) constitute evidence that directly constrains our credences:

(20)   If you are in Red Army territory, you are .75 likely in their Headquarters Company Area.

And that means that one can easily manufacture more cases like those in §2, since it is easy to manufacture deviant circumstances under which testimony imparts justification but not knowledge.

At this point, the most straightforward solution to both problems is relatively conspicuous: *properties of your credence distribution can constitute knowledge.* In other words: it is commonly said that some beliefs amount to knowledge, or count as knowledge, or constitute knowledge. In just this same sense, partial beliefs—i.e. credences—can constitute knowledge.[20] The same goes for conditional credences, and more generally

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19. The 'Probability First' handle is due to Weatherson 2005.

20. I follow many authors in using 'constitutes' for the relevant relationship between your doxastic and epistemic states. To cite a few examples: Alston 1988 mentions the requirement that a "grounding

for arbitrary properties of your credence distribution.

Given this simple thesis, we can accept the self-evident claim that ascriptions such as (21) ascribe knowledge, while also accepting the compelling evidence that (21) does not ascribe an attitude towards truths.

> (21)   You know that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi*.

We can also give an attractively simple theory of the cases discussed in §2. The relevant properties of credence distributions are epistemically incorrect because they fail to constitute knowledge. They are exactly like traditional Gettier beliefs in this respect, and that is what unifies the §2 cases with more standard cases where an agent has a justified true belief without having knowledge.

One could argue for the necessity of probabilistic knowledge by canvassing alternative answers to the §§1–2 problems and arguing that none of them is viable. I briefly discuss some alternative answers in §5.2. But my main aim in this paper is more modest: I build a positive case for a theory of probabilistic knowledge. A positive case for a theory has two parts: expounding the virtues of the theory and tempering its flaws. A virtue of probabilistic knowledge is that it yields simple solutions to problems that I discuss in §§1–2 and §6. An apparently serious flaw of probabilistic knowledge is that it seems to fly in the face of our intuition that knowledge is a factive attitude, as well as traditional epistemological claims that take that intuition for granted. In the next section, I argue that this flaw is not as serious as it first appears.

4 Factivity, safety, and sensitivity

Let us examine the argument against probabilistic knowledge in more detail. It is widely agreed that knowledge is a factive mental state and that 'knows' is a factive attitude verb.[21] In fact, a natural thought is that knowledge ascriptions are valuable in part because knowledge is factive, and ascriptions of knowledge can therefore communicate information about subjects other than their subject. By learning that John knows that it is raining, you learn that it is raining. By saying that John knows that it is raining, I can tell you that it is raining.

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belief *constitute* knowledge" (270); PLANTINGA 1996 says that in a Gettier case, "your belief is justified and true, but doesn't *constitute* knowledge" (309); WILLIAMSON 2004 claims that "a flat-out belief is fully justified if and only if it *constitutes* knowledge" (284); and FUMERTON 2006 discusses whether "an apparently justified true belief *constitutes* knowledge" (26). Like these authors, I use 'constitutes' for a metaphysically innocuous relation.

21. See STJERNBERG 2009, HAZLETT 2010 for some expressions of dissent. Their objections are orthogonal to the present project.

Some have used the following schema in order to say what it is for 'knows' to be factive (see for instance WILLIAMSON 2000, 34), where 'S' is to be replaced by the name of an agent and 'p' by a declarative sentence:

(FACTIVE₁)   The inference from 'S knows that p' to 'p' is deductively valid, i.e. for any world w, if 'S knows that p' is true as evaluated at w, then 'p' is true as evaluated at w.

An inference will generally fail to be deductively valid if its conclusion is not truth-apt, and so instances of (FACTIVE₁) are generally false where the semantic value of 'p' is a constraint on credence distributions.[22] The opponent of probabilistic knowledge concludes: agents cannot bear the knowledge relation to properties of credence distributions.

The advocate of probabilistic knowledge has a ready response. Our ordinary use of knowledge ascriptions does not constitute evidence for (FACTIVE₁) as opposed to the following underspecified claim:

(FACTIVE₂)   The inference from 'S knows that p' to 'p' is valid.

If we restrict our attention to instances of (FACTIVE₂) where the semantic value of 'p' is a proposition, then (FACTIVE₂) should be equivalent to (FACTIVE₁). But a more general notion of validity is required in order to capture our intuitions about instances of (FACTIVE₂) where the semantic value of 'p' is a constraint on credences. For example, suppose I tell you the following:

(23)   John knows that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi*.

Just as in the propositional case, this ascription communicates information about subjects other than John. By learning that John knows that it is more likely that your specimens are *G. hackmani* than that they are *G. balachowskyi*, you learn that it is more likely that your specimens are *G. hackmani*. By uttering (23), I communicate that it is more likely that your specimens are *G. hackmani*. In other words, according to the expressivist, I advise you to give more credence to the proposition that your specimens are *G. hackmani* than to the proposition that they are *G. balachowskyi*. If I do not endorse this advice, it is inappropriate for me to utter (23).

This example points to an appropriate interpretation of (FACTIVE₂) for the expres-

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22. More precisely, such instances of (FACTIVE₁) are true only when 'S knows that p' is false as evaluated at every world.

sivist. The expressivist need not say that a factive ascription is true only if its prejacent is true. She may say that 'S knows that p' entails 'p' in the following sense: if you follow the advice 'S knows that p' expresses, you thereby follow the advice that 'p' expresses. In other words, every credal state that satisfies the constraint expressed by a factive ascription also satisfies the constraint expressed by its prejacent. For example: if the semantic value of a sentence is a set of credence distributions, as on the naïve expressivist theory outlined in §1, then an ascription entails its prejacent because the semantic value of the former is a subset of the semantic value of the latter.

This liberal understanding of factivity is not necessarily at odds with (FACTIVE$_1$). Given a carefully chosen theory of truth, an expressivist may ultimately accept both characterizations of factivity stated above. For instance, the expressivist may say that uttering a factive construction commits you to the *truth* of its complement, where this commitment merely involves a commitment to the *complement itself*. In particular, an expressivist about epistemic modals may endorse an expressivist theory of truth, according to which "to believe that S is true is just to agree with S, and to believe that S is false is just to disagree with S" (SCHROEDER 2010, 10). She could then characterize factivity by saying that you should utter a factive construction only if you agree with its complement. GIBBARD 2003 and SCHROEDER 2008 develop accounts of truth on behalf of expressivists about ethical discourse, and their accounts are similarly hospitable to (FACTIVE$_1$).[23] The present point is merely that (FACTIVE$_1$) is not essential to our notion of factivity. The main requirement on a probabilistic definition of factivity is simply that it should yield the standard definition of factivity as a special case, so that the probabilistic definition still applies when restricted to factive operators not embedding language of subjective uncertainty.

Using a suitably general interpretation of (FACTIVE$_2$) to define factivity lets us predict judgments about when we can utter probabilistic knowledge ascriptions. For instance: the expressivist should accept some norms of assertion, such as the norm that you should not utter a sentence unless you endorse the advice that it expresses. On the most straightforward expressivist account of endorsing advice, that just means that your credal state must itself be a member of the semantic value of the ascription. From (FACTIVE$_2$), it follows that your credal state is a member of the semantic value of the prejacent. Hence: you should not utter a factive ascription unless you endorse the advice expressed by its prejacent. And this conclusion is borne out by our judgments about factive ascriptions, as noted with (23) above.

Using (FACTIVE$_2$) to define factivity also lets us predict judgments about the sorts

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23. For further discussion, see §§11.4–11.6 of SCHROEDER 2008. I shall continue presupposing a naïve expressivist semantics for ease of exposition, though the following discussion could be tailored to accommodate a variety of more nuanced expressivist theories.

of constraints that can be embedded in factive ascriptions. A speaker should not utter a knowledge ascription unless she thinks that its prejacent expresses good advice. In some cases, we have direct intuitions about when a sentence expresses good advice. For instance, an ordinary sentence such as 'it is raining' expresses good advice just in case it is raining. It is natural to think that in order for a constraint to constitute good advice, it must contain at least some probability measure that gives at least some credence to the actual world. In other words, good advice cannot demand that you become certain of some false proposition. This simple condition already allows us to predict a number of natural judgments. For example: since 'it is probably raining' and 'it is probably not raining' have incompatible constraints as their semantic values, no probability measure can satisfy both. This reasoning accounts for why (24) is infelicitous, while ascriptions such as (25) sound fine:

(24) #John knows that it is probably raining, and Bob knows that it probably isn't.

(25) John thinks that it is probably raining, and Bob thinks that it probably isn't.

Given expressivist theories of particular expressions, we may predict further judgments. For example, the following utterance is also infelicitous:

(26) #John knows that it is very probably raining, and Bob knows that if it is more likely than not to be raining, then it is Sunday. But it is not Sunday.

In order to derive this result, it is necessary to have some semantics in place for indicatives with constraints as antecedents. In the spirit of YALCIN 2007, let us suppose that the semantic value of an indicative contains a probability measure just in case the result of updating that measure on the antecedent constraint satisfies the consequent constraint. First, note that any measure that satisfies the semantic value of 'it is very probably raining' satisfies the semantic value of 'it is more likely than not to be raining'. Second, note that updating a measure on a constraint that the measure already satisfies should yield the measure itself as a result.[24] It follows that any measure that satisfies the semantic value of 'it is very probably raining' and 'if it is more likely than not to be raining, then it is Sunday' must satisfy the semantic value of 'it is Sunday'. But if it is not Sunday, then 'it is Sunday' expresses bad advice. Hence: if it is not Sunday, one should not ascribe knowledge that it is very probably raining and knowledge that if it is more likely than not to be raining, then it is Sunday. To sum up: any expressivist must develop notions of consequence and validity suitable

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24. It is difficult to give a procedure for updating an arbitrary probability measure on an arbitrary constraint, for reasons mentioned in §3. But it should be relatively uncontroversial that the stated result will be a feature of any reasonable updating procedure.

for her semantic theory. And using these notions, the expressivist can account for ordinary language judgments about knowledge ascriptions by endorsing the claim that one can validly infer from a knowledge ascription to its complement.

One might object that factivity should be defined using (FACTIVE$_1$) rather than (FACTIVE$_2$), since *factive* verbs should relate their subjects to *facts*. Ultimately, I am not interested in settling a terminological dispute over the most apt definition of 'factive'. (FACTIVE$_1$) is already an idiosyncratic definition compared with the standard use of 'factive' in the linguistics literature. Following an early discussion in KIPARSKY & KIPARSKY 1970, semanticists distinguish *factive* from *entailing* verbs. A verb is factive just in case its content clause complement is normally presupposed. A verb is entailing just in case its complement is entailed by positive declarative sentences containing the verb.[25] In this terminology, the epistemologically important feature of knowledge characterized by (FACTIVE$_1$) is that it is an *entailing* mental state. Whatever this epistemologically important feature is called, my claim is that (FACTIVE$_2$) effectively captures our intuitions about the feature, and thereby lets us predict many ordinary language judgments about probabilistic knowledge.

In light of the cases given in §2, some theorists may be pessimistic about analyses of propositional knowledge. But even without attempting an analysis of probabilistic knowledge, one can investigate the qualities that distinguish it from other mental states. Factivity is one example. Safety is another. Factive attitudes are valuable in part because learning that an agent is in a certain factive mental state constrains our opinions about subjects other than that agent. Safe attitudes are valuable because they preclude a certain sort of epistemic fragility exhibited by the justified credences in the §2 cases. And just as with factivity, our traditional notion of safety naturally extends to probabilistic knowledge. Consider the following simple statement of the safety condition from WILLIAMSON 2000:

(SAFE$_1$) For all cases α and β, if β is close to α and in α one knows that C obtains, then in β one does not falsely believe that C obtains. (128)

It is not difficult to reformulate (SAFE$_1$) so that it applies to probabilistic knowledge:

(SAFE$_2$) For all cases α and β, if β is close to α and in α one knows that C, then the following is not the case in β: that one believes that C and it is not the case that C.

True instances of (SAFE$_1$) result from replacing 'C' by an expression that refers to a

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25. For further discussion of factive and entailing verbs, see HUDDLESTON & PULLUM 2002, §7.4.

proposition. True instances of (Safe₂) result from replacing 'C' by a sentence. That sentence may contain language of subjective uncertainty. If it does, the resulting instance of (Safe₂) will contain expressions of subjective uncertainty embedded under four operators: the universal quantifier, conditional, and intensional operator 'in β' as well as 'believes' or negation. Any complete semantics for expressions of subjective uncertainty should settle how they interact with these operators. But even without using a semantic theory to generate truth conditions for instances of (Safe₂), we can rely on ordinary language intuitions in assessing its instances in particular cases. And just as with (Factive₂), instances of (Safe₂) let us predict judgments about probabilistic knowledge ascriptions. For instance, recall that the following ascriptions are felicitous in some cases:

(18)   I hereby let you know that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi*.

(19)   I realized that I probably liked that guy as more than a friend.

But in the deviant entomology and psychology cases in §2, the following variants of (18) and (19) are intuitively false:

(21)   You know that it is more likely that your specimens belong to *G. hackmani* than to *G. balachowskyi*.

(22)   Sue knows that she probably likes Bob as more than a friend.

The safety condition (Safe₂) accounts for our judgments that (21) and (22) are false. In the entomology case, you might easily have received your specimens from a disgruntled lab assistant instead of the diligent assistant. And so it might easily have been that your specimens were not more likely *G. hackmani* than *G. balachowskyi*, but you still believed that they were more likely *G. hackmani* on the basis of testimony from the entomology supply company. In the psychology case, it might easily have been that the psychology researchers injected subjects with anxiety-producing drugs just in case they were not attracted to their friends. And so it might easily have been that Sue in fact probably didn't like Bob, but still believed that she probably liked him on the basis of her fluttering nerves.

Finally, notions of sensitivity also extend to probabilistic knowledge. For instance, Nozick 1981 states the following sensitivity condition on knowledge:

(Sensitive₁)   *S* knows, via method (or way of believing) *M*, that *p* only if: if *p* weren't true and *S* were to use *M* to arrive at a belief whether (or not) *p*, then *S* wouldn't believe, via *M*, that *p*.

The advocate of probabilistic knowledge may endorse the following deflationist variant of (Sensitive₁):

> (Sensitive₂)   *S* knows, via method (or way of believing) *M*, that *p* only if: if it were not the case that *p* and *S* were to use *M* to arrive at a belief whether (or not) *p*, then *S* wouldn't believe, via *M*, that *p*.

Here again, the expressivist may capture the spirit of a statement concerning the truth conditions of ordinary propositions by endorsing a principle in which schematic letters are to be replaced by sentences containing expressions of subjective uncertainty.[26]

The deflationist strategy applied repeatedly in this section is not open just to the expressivist. Advocates of dynamic and relativist semantic theories may endorse similar deflationist theories of factivity, safety, and sensitivity. In just this sense, my defense of probabilistic knowledge has been neutral between various semantic theories of language of subjective uncertainty.[27] However, the deflationist strategy does serve to remind us of the fact that it is no small feat to give a complete semantics for expressions of subjective uncertainty. In particular, such expressions commonly occur not only in belief reports, but under overt intensional operators (as in (Safe₂)), and in the antecedents of subjunctives (as in (Sensitive₂)). Furthermore, I have only considered a small handful of definitions of factivity, safety, and sensitivity; other definitions may contain other intensional operators. It is not at all obvious what an intensional semantics for non-truth-conditional expressions should look like; on a traditional semantics, intensional operators just are those operators that shift the world at which their argument is evaluated for truth.

At this juncture, some might take intensional contexts as an insuperable difficulty for non-truth-conditional analyses of the language of subjective uncertainty, preferring instead to wrestle with the host of arguments against truth-conditional analyses canvassed in §1. I do not aim to refute that position here, but simply to set out a strong positive case for probabilistic knowledge. For the many theorists that endorse non-truth-conditional theories of the language of subjective uncertainty, it is good news that the core features of factivity, safety, and sensitivity can be applied to probabilistic mental states.

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26. As noted above, some expressivists may endorse a deflationist notion of truth and thereby automatically reinterpret (Safe₁) as (Safe₂), (Sensitive₁) as (Sensitive₂), and so on for similar pairs of principles.

27. It is beyond the scope of this paper to review the many nuanced theories of truth developed by advocates of dynamic and relativist theories. For further discussion of factive constructions in dynamic frameworks, see Karttunen 1974 and Chapter 6 of Beaver 2001. For the development of a disquotational notion of truth in a relativist framework, see Egan et al. 2005.

5 Several decision points for advocates of probabilistic knowledge

The foregoing discussion raises several questions for advocates of probabilistic knowledge. Can other properties of mental states constitute knowledge, aside from properties of your credence distribution? Can probabilistic knowledge be analyzed in terms of propositional knowledge or other familiar epistemic notions? And finally, what less radical morals can be taken from this discussion by those who resolutely endorse the claim that the objects of knowledge must have truth conditions? In this section, I discuss each of these questions in turn.

5.1 Non-probabilistic non-propositional knowledge

Some expressions of subjective uncertainty are not easily modeled by the naïve expressivist theory outlined in §1. The less tractable expressions include three of our original examples:

(1)   John might be in his office.
(5)   John must have gone home early today.
(6)   If John is still in the building, he is in his office.

For the naïve expressivist, the most natural account of these expressions appeals to threshholds. (1) advises you to give at least a certain small amount of credence to the proposition that John is in his office. (5) advises you to give at least a certain large amount of credence to the proposition that John went home early. (6) advises you to give at least a certain large amount of credence to the proposition that John is in his office, conditional on his still being in the building. But unfortunately, unless the threshhold credence in question is 0 for 'might' and 1 for 'must' and conditionals, these proposals fail to validate intuitive inference rules, such as:

(27)   From 'might (p or q)', infer: 'might p or might q'.
(28)   From 'must p' and 'must q', infer: 'must (p and q)'.
(29)   From 'if p, q' and 'if p, r', infer: 'if p, (q and r)'.[28]

And it would be a mistake for the naïve expressivist to accept extreme threshholds for these expressions. To do so would conflate epistemic possibility with nonzero probability.[29] This problem for the naïve expressivist has a familiar shape, of course.

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28. It is generally assumed that (27) is likewise valid; for further discussion, see KAMP 1974, ZIMMERMANN 2000, SIMONS 2005, and GEURTS 2005. (28) is a theorem of **K** (cf. HUGHES & CRESSWELL 1996, p.27), and (29) is a similarly basic rule.

29. See p.1016 of YALCIN 2007 for a related discussion, and McGEE 1994 and HÁJEK 2003 for corroborating arguments.

In asking what minimal conditional credence one ought to have in order to count as having updated on an indicative conditional, one wants to say: credence sufficient for conditional belief. But identifying such a threshhold runs afoul of familiar problems for the Lockean project of analyzing belief in terms of sufficient credence: belief does not entail certainty, and so the Lockean must deny closure for belief.

A better expressivist account of (1), (5), and (6) requires semantic resources other than sets of probability measures. For instance, the expressivist might use sets of possibilities as part of her representation of agents' doxastic states, à la Hintikka 1962. The expressivist could then say that modals and conditionals are used to express advice about that set of doxastic possibilities: (1) ensures that it contains some worlds where John is in his office, (5) ensures that it contains only worlds where John went home early, and (6) ensures that if it contains a world where John is still in the building, that world is one where John is still in the building. Another expressivist strategy for indicatives simply accepts conditional belief as a primitive mental state and says that (6) expresses advice regarding instances of that state. Either of these strategies will prompt a non-probabilistic account of knowledge ascribed by sentences such as:

(30)   Bob knows that John might be in his office.
(31)   Bob knows that John must have gone home early today.
(32)   Bob knows that if John is still in the building, he is in his office.

In other words, there may be *non-propositional* knowledge, other than *probabilistic* knowledge. Fortunately, some arguments in §§2–4 extend easily to non-probabilistic non-propositional knowledge. For instance: an expressivist may say that the semantic value of a sentence like (1) is a set of sets of possible worlds, and that in order for such a constraint to constitute good advice, its semantic value must contain at least some set of worlds that contains the actual world. In other words, good advice cannot demand that you rule out the actual world as a doxastic possibility. This simple condition lets us predict infelicity judgments such as:

(33)   #John knows that it can't be raining, and Bob knows that it might be.
(34)   #John knows that it might be raining, and Bob knows that if it is raining, then it is Sunday. But it can't be Sunday.

Judging the bounds of non-probabilistic non-propositional knowledge is a vast project. Expressivists about ethical discourse have developed quasi-realist accounts of moral vocabulary in knowledge ascriptions and other factive environments.[30] It is just as important to develop accounts of factive environments if one rejects truth-conditional

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30. For discussion, see Blackburn 1996 and 1998, Gibbard 2003, Ridge 2007, Schroeder 2008, and Chrisman 2010.

semantics for predicates of personal taste, future contingents, or aesthetic discourse. The scope of this paper is limited to the case for probabilistic knowledge, which is especially well-supported by contemporary theories of the semantics of subjective uncertainty and the epistemology of updating. In §6, I give arguments that more specifically target probabilistic knowledge, presenting problems that are uniquely addressed by the claim that properties of credence distributions can constitute knowledge.

5.2 Analyzing probabilistic knowledge

It is worth investigating whether probabilistic knowledge can be analyzed in terms of more familiar epistemic notions. For instance, traditional epistemologists frequently discuss the *epistemic probability* of propositions. And for some epistemologists, epistemic probability is by definition closely tied with propositional knowledge. Such theorists might propose that your credence in a proposition constitutes knowledge just in case it equals your epistemic probability for that proposition.

In order to evaluate this proposal, it is necessary to have a clear definition of 'epistemic probability' at hand. There is a family resemblance among the numerous definitions of 'epistemic probability' in the literature. A proposition having high epistemic probability for you is generally associated with your having justification or evidence for believing that proposition:

> It is a truism that a belief is justified if and only if its epistemic probability is sufficiently high. (Pollock & Cruz 1999, 110)

> This is what we shall mean by 'evidential probability': the evidential probability of a statement S, relative to a body of knowledge Γ, is the interval $[p,q]$ determined by the unsharpenable *evidence* Δ contained in Γ bearing on S. (Kyburg & Teng 2001, 219)

Another common thread is that your epistemic probability for a proposition is the degree of belief in that proposition that it is rational for you to have:

> ['Degree of rational belief'] is ambiguous between 'degree of rationality of belief', i.e., epistemic probability, and 'degree to which one ought to believe', which may or may not be the same thing. (Pollock 1983, 245)

> Relative to K, p is epistemically more probable than q, where K is an epistemic situation and p and q are propositions, just in case any fully rational person in K would have a higher degree of belief in p than in q. (Draper 1989, 349)

In the same vein, van Inwagen 1996 says that the epistemic probability of proposition relative to an epistemic situation equals the odds that a "fully rational ideal bookmaker" would give to the proposition if he were in that epistemic situation (221).

These definitions point to a central difficulty for saying that properties of your credences are knowledge just in case they are properties of your epistemic probabilities. For many accounts of justification, evidence, or rational belief, the identification returns the wrong verdict in probabilistic analogs of Gettier cases. Subjects in Gettier cases have justification and evidence for their beliefs, and their beliefs constitute rational responses to that evidence. This claim is not only entailed by a number of theories of justification but also demanded by our intuitions about the cases. For example: in the psychology case, Sue intuitively has evidence for the proposition that she likes Bob as more than a friend, namely that her nerves start fluttering as soon as she sees him, and so Sue has a high epistemic probability for the proposition that she likes Bob. But her high credence in the proposition that she likes Bob does not constitute knowledge. Hence: it is not the case that properties of your credence distribution constitute knowledge just in case they are properties of your epistemic probability distribution. This problem for the proposed analysis of probabilistic knowledge does not depend on an internalist conception of evidence. For instance: suppose (à la WILLIAMSON 2000) that your evidential probability function is the result of conditionalizing a distinguished initial credence distribution on all and only the propositions that you know. In the psychology case, Sue knows that her nerves start fluttering as soon as she sees Bob. So even on this account, her evidential probability that she likes Bob may match her high credence, without that high credence constituting knowledge.

There are several further problems for an objective Bayesian definition of probabilistic knowledge in terms of evidential probability, according to which you know exactly those properties of a distinguished initial credence distribution conditionalized on your evidence. Problems arise even if we adopt an externalist theory of evidence according to which you do not have evidence for your true beliefs in Gettier cases. First, it is not clear that any facts exist that could entirely determine the identity of the distinguished initial distribution. Second, the proposal does not generalize easily to properties of other mental states that could constitute knowledge, such as conditional beliefs (cf. §5.1). Third, the proposal contradicts the intuitive claim that your high credence in some evidence proposition may constitute probabilistic knowledge. For example: suppose that Sue knows that it is .99 likely that her nerves started fluttering. The proposal entails that her epistemic probability for the proposition that her nerves started fluttering is merely .99. Hence the proposition cannot be evidence for Sue. But intuitively, such propositions can indeed constitute evidence for Sue. Fourth, it is not obvious that your actual credences ever exactly match the result of conditionalizing a distinguished credence distribution on your evidence, especially given that the initial distribution is notoriously difficult to define. But on the present proposal, that means

that it is not obvious that your actual credences ever constitute knowledge.

The objective Bayesian could address the last two concerns by introducing another proposal, namely that a property of your credence distribution constitutes knowledge just in case it closely enough resembles some property of your epistemic probability function. But this second proposal precludes plausible closure principles for probabilistic knowledge. For example, the proposal suggests that one may know that it is probably snowing, and that it is not July if it is probably snowing, while failing to know that it is not July. Closure may be recovered by yet another proposal, namely that a property of your credence distribution constitutes knowledge just in case it is a property of the distribution that best estimates your epistemic probability function, given your credences about your epistemic probabilities. But this third proposal yields counterintuitive verdicts. For instance, the simple knowledge that it is probably raining outside is intuitively available even to subjects that are incapable of estimating epistemic probabilities.

Finally, a general problem arises for nearly any analysis of probabilistic knowledge in terms of epistemic probability. On many accounts of epistemic probability, your credence in a proposition might equal your epistemic probability for that proposition simply as a matter of coincidence. And in such cases, it is not clear that any property of your credence distribution should thereby constitute knowledge.

5.3 Probabilistic quasi-knowledge

Analyses of probabilistic knowledge in terms of more familiar epistemic notions do not seem forthcoming. But even if they were, one might still endorse the central claims of this paper: that probabilistic knowledge can help us solve several problems, and that it can do so without overturning our core intuitions about the nature of knowledge. Some may resist the latter claim on conceptual grounds, however. They may claim that it is an analytic truth that properties of credence distributions simply cannot constitute knowledge. Their resistance raises important metasemantic questions, such as whether it is analytic that knowledge is a relation to propositions, and how we should settle disputes about such analyticity claims. These questions are comparable to questions about plan-laden judgments raised in Gibbard 2003:

> Can we, then, sometimes know what to do? When we do, is this real knowledge; is it knowledge in the same sense as with natural features of our surroundings? Knowledge or quasi-knowledge—which it is I won't try saying. In crucial respects, though, plan-laden judgments can at least parallel the clearest and most literal cases of knowledge. (235)

Gibbard adopts a reasonably modest stance about knowledge ascriptions embedding ethical vocabulary. Similarly, it is beyond the scope of this paper to settle metasemantic questions about knowledge ascriptions embedding language of subjective uncertainty. In order to remain as neutral as possible, I shall instead offer a few more modest alternative theses about properties of credence distributions, claims that should be more palatable to conceptually cautious audiences. Alternative thesis one: knowledge is a member of a natural kind of epistemically good mental states, and that natural kind also includes mental states that are relations to properties of credence distributions. Alternative thesis two: beliefs that constitute knowledge have a certain primitive epistemic virtue, and properties of credence distributions can have the very same virtue.

Advocates of these alternative theses may respond to the problem in §1 by saying that we use 'knows' to relate subjects to epistemically good properties of credence distributions. They may agree that 'knows' ascriptions are factive in the sense of (Factive$_2$), and this may help them explain why we use the term 'knows' in such ascriptions, despite the fact that the ascriptions do not concern knowledge but rather some other related mental state. Advocates of the alternative theses may also endorse the claim that the quasi-knowledge relation is safe in the sense of (Safe$_2$), and the fact that quasi-knowledge is factive and safe may help them capture our intuition that there are strong and important similarities between propositional and probabilistic Gettier cases. To sum up: accepting probabilistic knowledge is the simplest and most natural way to resolve the problems in §§1–2. And the discussion in §4 demonstrates that one can accept probabilistic knowledge at little cost. This constitutes a positive case for probabilistic knowledge. It is compatible with my having made this case that for some theorists, the cost of accepting probabilistic knowledge may still be high enough to outweigh concerns of simplicity and naturalness. For such theorists, the moral of the present paper is that some properties of credence distributions may be epistemically good in the same way that beliefs are good when they constitute knowledge.

6 Further applications for probabilistic knowledge

In assessing theoretical reasoning, it is natural to talk about whether an agent knows a proposition, e.g. whether an agent knows the premises of her argument, or knows that the rules of inference she uses are valid. A number of recent papers have argued that whether an agent knows a proposition also affects our assessment of her *practical* reasoning. In particular, many have argued that there is an intimate connection

between what you know and what you may treat as your reasons for doing something.[31] In this section, I briefly discuss principles connecting knowledge and action defended in Hawthorne & Stanley 2008 and Weatherson 2010b. Both principles face significant problems. Accepting probabilistic knowledge provides tidy solutions to both problems.

Suppose that Allan has a hunch that he will not get sick this year. He declines reasonably priced health insurance, saying:

(35)   I should decline the health insurance, since I am not going to get sick this year.

Intuitively, there is something wrong with Allan declining insurance for the reason he gives in (35), i.e. that he is not going to get sick this year. Stanley and Hawthorne argue that since Allan should not decline insurance for that reason because he does not know that he is not going to get sick. Abstracting from particular cases, they argue for a general principle connecting knowledge and rational action: that an agent should act only on the basis of reasons that he knows.

In response to Stanley and Hawthorne, some have objected that when an action is informed by rational credences, an agent can act for reasons that do not constitute knowledge. For instance, Schiffer 2007 objects that "the following sort of example is very common: you are completely justified in carrying an umbrella even though you don't know that it will rain but merely believe to degree .4 that it will rain" (189). To take another example, suppose that Alice has one of 10,000 tickets in a lottery with a $5 first prize. Alice decides to sell her ticket for a penny, saying:

(36)   I should sell my ticket, since it is merely slightly likely that my ticket is the winner.

The objection to Stanley and Hawthorne is that Alice sells her ticket for a perfectly good reason in this case, though the credence that informs her action does not constitute knowledge. Stanley and Hawthorne respond to this sort of objection by arguing that Alice uses (36) to say that it is merely slightly likely on her evidence that her ticket is the winner, and that this known fact about her evidence constitutes her reason for selling her ticket. But the anti-contextualist arguments in §1 should caution the reader against hastily adopting this response. For instance, saying that Alice sells her ticket on the basis of a fact about her own epistemic state seems to misidentify the subject matter of her reason for selling her ticket.[32]

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31. For starters, see Unger 1974, Hyman 1999, Fantl & McGrath 2002, Hawthorne 2004, Stanley 2005, and Fantl & McGrath 2007.

32. For instances of the contextualist response to the Schiffer objection, see Hawthorne 2004, p.135; Stan-

WEATHERSON 2010b advocates a second principle connecting knowledge and rational action. The central claim is that one may accurately represent a decision problem without representing states of the world to which the agent gives some credence, if the agent knows that those states of the world do not obtain. Hence an agent may rationally apply the Sure-Thing Principle to choose an action that dominates alternatives in every possibility consistent with her knowledge. Conversely, if some state of the world is consistent with her knowledge, that state must be included in any appropriate representation of her decisions. According to Weatherson, that is why the Sure-Thing Principle cannot tell Allan to decline insurance: it is consistent with his knowledge that he is going to get sick, and so declining insurance does not dominate accepting it. In slogan form, the general principle is that "knowledge structures decision problems."

Trouble is just around the corner for this knowledge-first approach to decision theory. Suppose that Barry is sitting in his apartment when he hears a familiar sound outside. It is his favorite street musician, Beth. He is hurrying down to meet her when a genie appears and offers him a free bet. If he takes the bet and the musician is indeed Beth, he will get $10. But if it isn't Beth, he will be tortured for several hours. A dilemma ensues. It seems that Barry should decline the bet. But according to the claim that knowledge structures decision problems, if Barry knows that the street musician is Beth, his decision is appropriately represented by a table according to which taking the bet is the dominant option. Weatherson summarizes the problem:

> If you accept that the bet should be declined, then there are three options available it seems to me.
> 1. Barry never knew that the musician was Beth.
> 2. Barry did know that the musician was Beth, but this knowledge was destroyed by the genie's offer of the bet.
> 3. States of the world that are known not to obtain should still be represented in decision problems, so taking the bet is not a dominating option. (7)

Weatherson concludes that in order to avoid skepticism, the knowledge-first decision theorist must embrace option (2): Barry's knowledge is *interest-relative*. Weatherson argues elsewhere that epistemic interest-relativity is not as bad as it first seems (cf. WEATHERSON 2010a), so that this is not a costly outcome for the knowledge-first decision theorist. But many have argued that interest-relativity is extremely counterintuitive, and they may count the above argument as a strong reason to reject a knowledge-first decision theory.[33]

LEY 2005, p.10; and HAWTHORNE & STANLEY 2008, p.13.

33. Even those who accept interest-relativity often admit that it is an extremely counterintuitive claim. For further discussion, see STANLEY 2007, p.2; FANTL & McGRATH 2009, p.15; and DEROSE 2009, p.189.

Both Schiffer and Weatherson raise significant problems for knowledge-based norms of action and decision-making. But in both cases, the advocate of probabilistic knowledge has an easy way out. Suppose that Stanley and Hawthorne accept that properties of credence distributions may constitute knowledge. Then they may endorse a probabilistic-knowledge-based norm of action: an agent must act on the basis of properties of her credence distribution that constitute knowledge. The case of Alice is consistent with this norm: Alice may sell her ticket, as long as she *knows* that it is merely slightly likely that it is the winner. That is, Alice may sell her ticket if her very low credence that it is the winner constitutes knowledge.

One may similarly replace the knowledge-based norms in Weatherson 2010b with less problematic norms concerning probabilistic knowledge. For instance, the knowledge-first decision theorist may claim that if an action has maximal expected value according to properties of your credence distribution that constitute knowledge, you should perform that action. For example: in representing Barry's initial decision to go downstairs, it is permissible to appeal to the premise that it is at least .8 likely that Beth is outside, precisely because this premise is known by Barry. Since this premise entails that going downstairs has maximal expected value, Barry should go downstairs. On the other hand, in representing the decision to take the genie's bet, it is not permissible to appeal to the premise that it is at least .9999 likely that Beth is outside, precisely because this premise is not known by Barry. In both cases, what Barry should do depends on what probabilistic knowledge he has. But that does not mean that his probabilistic knowledge is interest-relative. Barry does not lose his knowledge that it is .8 likely that Beth is outside when the genie offers him a bet. His declining the bet is rational because he lacks much stronger probabilistic knowledge, knowledge that he never had to begin with.

The aim of the present discussion of knowledge-based norms is modest. I have not argued for knowledge-based norms of action and decision-making. I have simply outlined ways in which the advocate of probabilistic knowledge can defend such norms against the challenges articulated above. Insofar as knowledge-based norms of action and decision-making are compelling, such arguments constitute reasons to accept probabilistic knowledge. Summing up: the claim that properties of credence distributions can constitute knowledge allows us to solve significant problems. I argued in §§1–2 that the claim allows us to reconcile work in traditional epistemology with necessarily sophisticated models of assertion and updating. And here I have sketched how the claim allows us to develop knowledge-based norms of action and decision-making without accepting contextualist accounts of reasons statements, or the counterintuitive claim that knowledge is interest-relative.

This is just the beginning. Once we accept probabilistic knowledge, we can construct probabilistic analogs of a number of traditional claims about the role of knowledge, e.g. that knowledge is the aim of belief, that it is the norm of assertion, and that it constitutes your evidence. Advocates of probabilistic knowledge may also construct theories about epistemically virtuous credences that are informed by traditional theories of propositional knowledge. For instance: foundationalists may say that properties of your credence distribution constitute knowledge just in case they are justified by certain constraints directly imposed on your credences; reliabilists may say that properties constitute knowledge just in case they result from cognitive processes that generally produce accurate credences; and so on. In short, accepting probabilistic knowledge allows us to fully explore nascent attempts to merge traditional epistemological theories and formal representations of doxastic states.[34] And these benefits are achieved at little cost to our intuitions about core features of knowledge. The Bayesian can accommodate both the assumptions and the insights of the traditional study of epistemology. Knowledge may be first, but that does not mean that credences must be second.

34. This conclusion resembles the brief, optimistic suggestion in Joyce 2005 that "almost everything epistemologists have had to say about the nature of evidence can be imported into the Bayesian framework" (157). A helpful case study is the externalist account of justification for credences found in Goldman 1986, who praises a credence-producing system as "well calibrated if and only if, for each degree D, the truth ratio of propositions believed to degree D is approximately D" (114).

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