Integrative Reduction, Confirmation, and the Syntax-Semantics Map

Kristina Liefke Sanders

Tilburg Center for Logic and Philosophy of Science

Formal Epistemology Workshop, USC, May 21, 2011

Outline

Why Care about Reduction?

Reductive relations are ubiquitous in science:

- Thermodynamics (TD) $\longrightarrow$ Statistical mechanics (SM)
- Chemistry $\longrightarrow$ Atomic physics
- Psychology $\longrightarrow$ Neuroscience

Advantages of reductive relations:

- Simplicity
- Explanation
- Consistency
- Confirmation (Dijzadji-Bahmani et al., 2010b)

The Nagelian Model of Reduction (Nagel, 1961), cf. (Dijzadji-Bahmani et al., 2010a)

Assume two theories:

$\mathbf{T}_{\mathrm{P}}$ the reduced, or phenomenological, theory (TD)

$\mathbf{T}_{\mathrm{F}}$ the reducing, or fundamental, theory (SM)

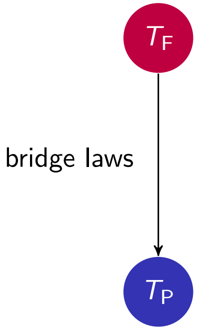
With each theory, we associate a set of propositions

$$\mathcal{T}_{\mathrm{P}} = \{T_{\mathrm{P}}^1, \dots, T_{\mathrm{P}}^n\} \text{ resp. } \mathcal{T}_{\mathrm{F}} = \{T_{\mathrm{F}}^1, \dots, T_{\mathrm{F}}^n\}$$

$\mathbf{T}_{\mathrm{P}}$ is reduced to $\mathbf{T}_{\mathrm{F}}$ in three steps:

1. Connect the vocabularies of $\mathbf{T}_{\mathrm{F}}$ and $\mathbf{T}_{\mathrm{P}}$ via bridge laws;
2. Substitute terms from $\mathbf{T}_{\mathrm{F}}$ by their correspondents from $\mathbf{T}_{\mathrm{P}}$;
3. Derive every proposition in $\mathbf{T}_{\mathrm{P}}$ from a proposition in $\mathbf{T}_{\mathrm{F}}$.

The 'Fundamental'-'Phenomenological' Map



(T_F) The kinetic theory of gases:
Newton's equations of motion, the def.s of pressure and kinetic energy

Bridge laws:
mean kinetic energy $\sim$ temperature

Auxiliary assumptions:
molecules interact only kinetically, the velocity distribution is isotropic

(T_P) The thermal theory of the ideal gas:
Boyle-Charles Law: $pV = kT$.

Assumption The connection bw $\mathbf{T}_F$- and $\mathbf{T}_P$-terms is bijective.

Montague's 'Two Theories' Theory (Montague, 1970; 1973)

Categorial Grammar (CG)

A triple $\langle \text{CAT}, \mathcal{E}, \mathbb{G}\rangle$, with

$$\begin{array}{lcll} \text{CAT} & = & \{\text{N(OUN)}, \text{V(ERB)}, \text{S(ENTENCE)}, \dots\} & \text{categories} \\ \mathcal{E} & = & \{\mathcal{E}_N, \mathcal{E}_V, \mathcal{E}_S, \dots\} & \text{expressions} \\ \mathbb{G} & = & \{\mathbb{G}_S, \dots\} & \text{syntactic rules} \end{array}$$

Syntax An algebra $\mathcal{A}_{CG} = \langle \mathcal{E}, \mathbb{G}_S, \dots\rangle$ over the set $\mathcal{E}$.

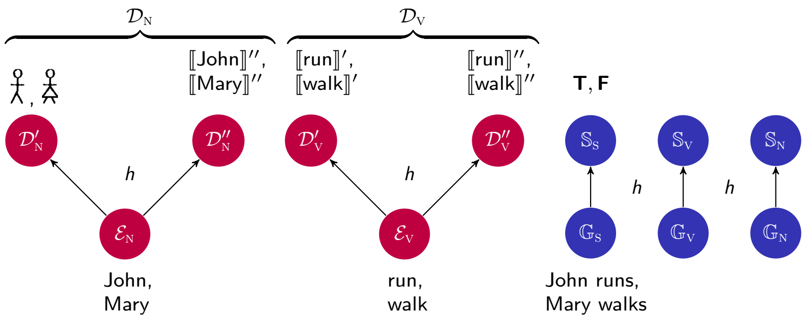
Model-Theoretic Semantics (MS)

A pair $\langle \mathcal{D}, \mathbb{S}\rangle$, with

$$\begin{array}{lcll} \mathcal{D} & = & \{\mathcal{D}_N, \mathcal{D}_V, \mathcal{D}_S, \dots\} & \text{objects} \\ \mathbb{S} & = & \{\mathbb{S}_N, \mathbb{S}_V, \mathbb{S}_S\} & \text{semantic rules} \end{array}$$

Semantics An algebra $\mathcal{A}_{MS} = \langle \mathcal{D}, \mathbb{S}_S, \dots\rangle$ over the set $\mathcal{D}$.

The Syntax-Semantics Map



$\mathbb{G}_S$. If $R \in \mathcal{E}_V$ and $j \in \mathcal{E}_N$, then $[jR'] \in \mathcal{E}_S$.

$\mathbb{S}_S$. If $[\![R]\!] \in \mathcal{D}_V$ and $[\![j]\!] \in \mathcal{D}_N$, then $[\![R]\!]([\![j]\!]) \in \mathcal{D}_S$.

A New Type of Intertheoretic Relation?

- The syntax-semantics pair instantiates a specific type of reduction relation.
- Commonalities with Nagelian reduction:
  - The map h connects objects of the two theories.
  - The map h enables us to derive propositions of one theory from propositions of the other theory.
- Differences from Nagelian reduction:
  - The relation between CG and MS is a directed relation.
- We assume Syntax the reduced theory ($\mathbf{T}_P$)
  Semantics the reducing theory ($\mathbf{T}_F$)

A New Type of Intertheoretic Reduction?

$$
\begin{tikzcd}
T_{\mathrm{F}} \arrow[d, "\text{bridge laws}"] & \mathbb{S}_{\mathrm{S}} = \{\mathbb{S}'_{\mathrm{S}}, \mathbb{S}''_{\mathrm{S}}\} \arrow[d, "\text{the map } h^{-1}"] \\
T_{\mathrm{P}} & \mathbb{G}_{\mathrm{S}}
\end{tikzcd}
$$

Caveats

- Our model of the syntax-semantics relation represents only one particular type of relation:

  Nagelian Reduction
  ↑
  Nagel-Schaffner Reduction
  ↑
  undirected dependency relations

- We distinguish 'Montague' Reduction (MR) from Integrative Reduction (IR).
- The syntax-semantics relation is a very weak relation:
  - Syntax is structurally richer than semantics.
  - Syntax and semantics have distinct domains of application.

Hypothesis Formation

Rules in $\mathbb{G}$, $\mathbb{S}$ are obtained by the scientific method:

1. Isolate syntactic structures in a given text sample;
2. Observe their common structural properties;
3. Propose a hypothesis about their formation;
4. Test the hypothesis by analyzing other samples.

An expression supports a hypothesis if

- The expression is intuitively well-formed.
- Its structure reflects the assumed formation process.

Montagovian Rules and Probabilities

- Rules in $\mathbb{G}$, $\mathbb{S}$ are subject to probability attributions.
- The probability of a rule is informed by frequentist data.
- The frequentist probability of a rule influences a linguist's psychological confidence in its descriptive adequacy.

Note
- Only syntactic rules are directly instantiated.
- Semantic rules derive their support from syntactic rules via the assumption of the map h.

Caveat We are NOT interested in a probabilistic extension of Montague Grammar.

Bayesian Networks

We analyze the relation between confirmation and reduction via Bayesian networks:

Bayesian network A directed acyclical graph, where

- **Nodes** propositional variables,
- **Arrows** probabilistic dependence relations between variables

Variables can take different values, assigned by **P**.

We use the following conventions:

- **Variables** H, E ;
- **Values** H, ¬H := ‘the proposition is true/false’;
  E, ¬E := ‘the evidence obtains/does not obtain’.

Bayesian Networks: Illustration

We frame the confirmatory relation between H and E:

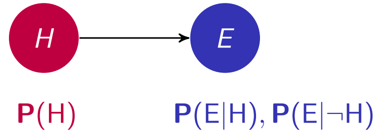
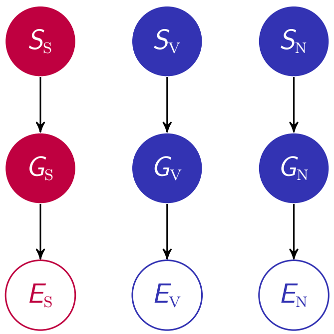


Figure: The dependence between E and H.

- The arrow denotes a direct influence of H on E.
- To turn the graph into a Bayesian network, we further need:
  - The marginal probability distribution for every ‘root’ variable;
  - The conditional prob’y distribution for every ‘child’ variable.

Confirmation and Reduction



The Single-Proposition Case

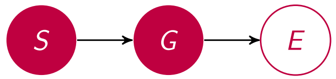


Figure: Montagovian dependencies of S, G, E.

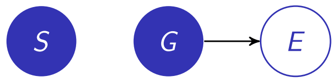
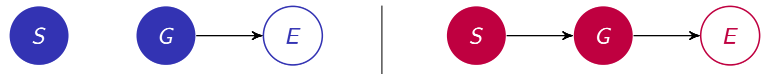


Figure: Pre-reductive dependencies of S, G, E.

The Pre- vs. Post-Reductive Situation (1)



$\mathbf{P}_1(\mathrm{S}) = \sigma$
$\mathbf{P}_1(\mathrm{G}) = \gamma$
$\mathbf{P}_1(\mathrm{E}|\mathrm{G}) = \pi\ ,\quad \mathbf{P}_1(\mathrm{E}|\neg\mathrm{G}) = \rho$

$\mathbf{P}_2(\mathrm{S}) = \sigma$
$\mathbf{P}_2(\mathrm{G}|\mathrm{S}) = 1\ ,\quad \mathbf{P}_2(\mathrm{G}|\neg\mathrm{S}) = 0$
$\mathbf{P}_2(\mathrm{E}|\mathrm{G}) = \pi\ ,\quad \mathbf{P}_2(\mathrm{E}|\neg\mathrm{G}) = \rho$

- We assume $\mathbf{P}_2(\mathrm{S}) = \mathbf{P}_1(\mathrm{S})$, $\mathbf{P}_2(\mathrm{G}) = \mathbf{P}_1(\mathrm{G})$, $\mathbf{P}_2(\mathrm{E}|\mathrm{G}) = \mathbf{P}_1(\mathrm{E}|\mathrm{G})$.
- Then, iff $\sigma \in (0,1)$ and $\pi > \rho$, the following holds:
  - $\mathbf{P}_2(\mathrm{S},\mathrm{G}) > \mathbf{P}_1(\mathrm{S},\mathrm{G})$
  - $\mathbf{P}_2(\mathrm{S},\mathrm{G}|\mathrm{E}) > \mathbf{P}_1(\mathrm{S},\mathrm{G}|\mathrm{E})$
  - $d_2 > d_1$

The Pre- vs. Post-Reductive Situation (1)

- We assume $\mathbf{P}_2(\mathrm{S}) = \mathbf{P}_1(\mathrm{S})$, $\mathbf{P}_2(\mathrm{G}) = \mathbf{P}_1(\mathrm{G})$, $\mathbf{P}_2(\mathrm{E}|\mathrm{G}) = \mathbf{P}_1(\mathrm{E}|\mathrm{G})$.
- Then, iff $\sigma \in (0,1)$ and $\pi > \rho$, the following holds:
  - $\mathbf{P}_2(\mathrm{S},\mathrm{G}) > \mathbf{P}_1(\mathrm{S},\mathrm{G})$
  - $\mathbf{P}_2(\mathrm{S},\mathrm{G}|\mathrm{E}) > \mathbf{P}_1(\mathrm{S},\mathrm{G}|\mathrm{E})$
  - $d_2 > d_1$

'Montague' Reduction increases the probabilities and effects a flow of confirmation between syntactic and semantic propositions.

The Problem

Triples $\langle \mathrm{S}_k, \mathrm{G}_k, \mathrm{E}_k \rangle$ (with $k \in \{\mathrm{S}, \mathrm{V}, \mathrm{N}\}$) remain probabilistically independent.

- The probability of the truth of propositions in $\mathbb{S}$, $\mathbb{G}$ corresponds to the product of their individual probabilities:

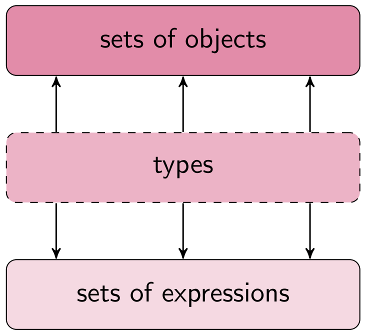
$$\begin{aligned} \mathbf{P}_2(\bigcap_k \langle \mathrm{S}_k, \mathrm{G}_k \rangle) &= \mathbf{P}_2(\mathrm{S}_\mathrm{S}, \mathrm{G}_\mathrm{S})\,\mathbf{P}_2(\mathrm{S}_\mathrm{V}, \mathrm{G}_\mathrm{V})\,\mathbf{P}_2(\mathrm{S}_\mathrm{N}, \mathrm{G}_\mathrm{N}) \\ &= \mathbf{P}_2(\mathrm{S}_\mathrm{S})\,\mathbf{P}_2(\mathrm{S}_\mathrm{V})\,\mathbf{P}_2(\mathrm{S}_\mathrm{N}) \end{aligned}$$

- Their joint probability converges to zero as their number increases.

The improvement of our model requires insight into the mutual dependencies between same-theory propositions.

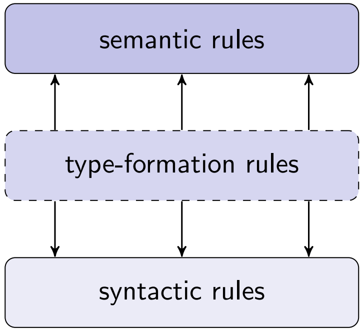
Montague's Solution (1)

Stipulate a level of types:



Montague's Solution (1)

Stipulate a level of types:



Types

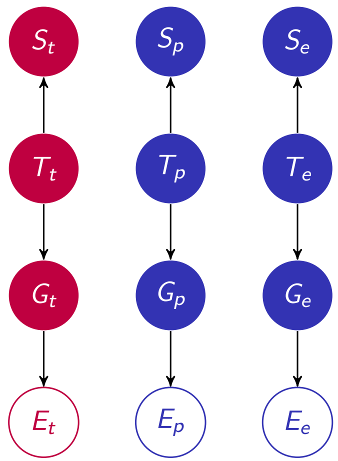
- Types are Montague's counterparts of bridge laws:
  - Types connect elements from the two theories.
- Types introduce a new (onto-)logical level.
- We call this new relation Integrative Reduction.
- Different ways of implementing types:
  1. Assume a separate type for each category, $|\mathrm{TY}| = |\mathrm{CAT}|$.
  2. Assume fewer types than categories, $|\mathrm{TY}| < |\mathrm{CAT}|$.
     - Assume $|\mathrm{TY}| > 1$.
     - Assume $|\mathrm{TY}| = 1$.

Case 1: Separate Types

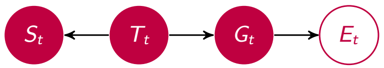
Assume types e, t, p, where

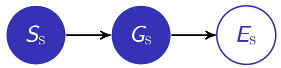
e := the type of objects in $\mathcal{D}_{\mathrm{N}}$, associated with nouns;
t := the type of objects in $\mathcal{D}_{\mathrm{S}}$, associated with sentences;
p := the type of objects in $\mathcal{D}_{\mathrm{V}}$, associated with verbs.

Case 1: Separate Types



The Three-Typed vs. Untyped Situation (1)





$\mathbf{P}_3(\mathrm{T}_t) = \tau$
$\mathbf{P}_3(\mathrm{S}_t|\mathrm{T}_t) = 1 \;,\; \mathbf{P}_3(\mathrm{S}_t|\neg\mathrm{T}_t) = 0$
$\mathbf{P}_3(\mathrm{G}_t|\mathrm{T}_t) = 1 \;,\; \mathbf{P}_3(\mathrm{G}_t|\neg\mathrm{T}_t) = 0$
$\mathbf{P}_3(\mathrm{E}_t|\mathrm{G}_t) = \pi \;,\; \mathbf{P}_3(\mathrm{E}_t|\neg\mathrm{G}_t) = \rho$

$\mathbf{P}_2(\mathrm{S}_S) = \sigma$
$\mathbf{P}_2(\mathrm{G}_S|\mathrm{S}_S) = 1, \mathbf{P}_2(\mathrm{G}_S|\neg\mathrm{S}_S) = 0$
$\mathbf{P}_2(\mathrm{E}_S|\mathrm{G}_S) = \pi, \mathbf{P}_2(\mathrm{E}_S|\neg\mathrm{G}_S) = \rho$

- If $\mathbf{P}_3(\mathrm{T}_t) = \mathbf{P}_2(\mathrm{S}_S)$, $\mathbf{P}_3(\mathrm{E}_t|\mathrm{G}_t) = \mathbf{P}_2(\mathrm{E}_S|\mathrm{G}_S)$, etc., then
  - $\mathbf{P}_3(\mathrm{T}_t, \mathrm{S}_t, \mathrm{G}_t) = \mathbf{P}_2(\mathrm{S}_S, \mathrm{G}_S)$
  - $\mathbf{P}_3(\mathrm{T}_t, \mathrm{S}_t, \mathrm{G}_t|\mathrm{E}_t) = \mathbf{P}_2(\mathrm{S}_S, \mathrm{G}_S|\mathrm{E}_S)$
  - $d_3 = d_2$

Montague's Solution (2)

Assume fewer types than categories, $|\mathrm{TY}| < |\mathrm{CAT}|$:

- Some expressions/objects are associated with basic types; others with constructions out of basic types (derived types).
- Derived types establish connections between same-theory propositions.
- Dependencies between same-theory propositions characterize Integrative Reduction.

Case 2: Two Types

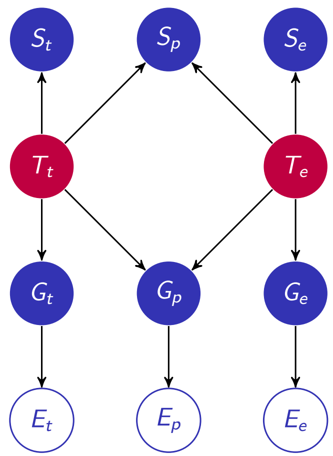
- Assume basic types e, t (not p), where

  e := the type of objects in $\mathcal{D}_N$, associated with nouns;
  t := the type of objects in $\mathcal{D}_S$, associated with sentences;

- Type-p objects are represented by functions $\mathcal{D}_N \to \{\mathbf{T}, \mathbf{F}\}$.
  - Let w be inhabited by , , and
  - $[\![\text{is a dog}]\!] := \{x \in \mathcal{D}_N \mid [\![\text{is a dog}]\!](x) = \mathbf{T}\} = \{\}$.

Case 2: Two Types

$\mathbf{P}_4(\mathrm{T}_t) = \tau \;,\; \mathbf{P}_4(\mathrm{T}_e) = \tau'$
$\mathbf{P}_4(\mathrm{S}_t|\mathrm{T}_t) = 1 \;,\; \mathbf{P}_4(\mathrm{S}_t|\neg\mathrm{T}_t) = 0$
$\mathbf{P}_4(\mathrm{S}_e|\mathrm{T}_e) = 1 \;,\; \mathbf{P}_4(\mathrm{S}_e|\neg\mathrm{T}_e) = 0$
$\mathbf{P}_4(\mathrm{S}_p|\mathrm{T}_e, \mathrm{T}_t) = 1 \;,\; \mathbf{P}_4(\mathrm{S}_p|\neg\mathrm{T}_e, \mathrm{T}_t) = 0$
$\mathbf{P}_4(\mathrm{S}_p|\mathrm{T}_e, \neg\mathrm{T}_t) = 0 \;,\; \mathbf{P}_4(\mathrm{S}_p|\neg\mathrm{T}_e, \neg\mathrm{T}_t) = 0$
$\vdots \qquad \vdots$
$\mathbf{P}_4(\mathrm{E}_t|\mathrm{G}_t) = \pi \;,\; \mathbf{P}_4(\mathrm{E}_t|\neg\mathrm{G}_t) = \rho$
$\mathbf{P}_4(\mathrm{E}_e|\mathrm{G}_e) = \pi' \;,\; \mathbf{P}_4(\mathrm{E}_e|\neg\mathrm{G}_e) = \rho'$
$\mathbf{P}_4(\mathrm{E}_p|\mathrm{G}_p) = \pi'' \;,\; \mathbf{P}_4(\mathrm{E}_p|\neg\mathrm{G}_p) = \rho''$



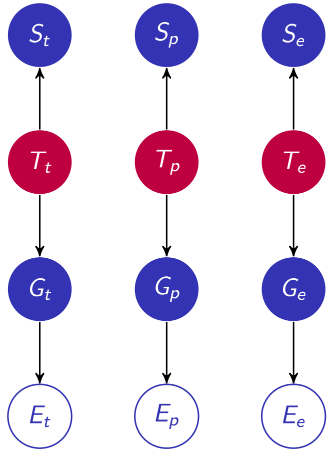
The Two- vs. Three-Typed Situation (1)

To compare both situations, we must first obtain

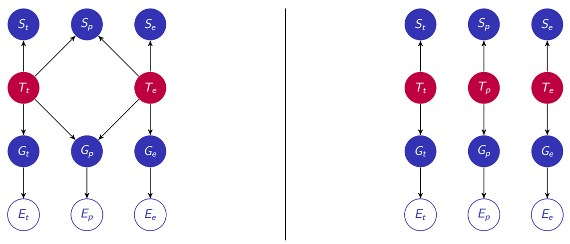
- the probabilities $\mathbf{P}_3(\bigcap_j \langle \mathrm{T_j}, \mathrm{S_j}, \mathrm{G_j}\rangle)$, $\mathbf{P}_3(\bigcap_j \langle \mathrm{T_j}, \mathrm{S_j}, \mathrm{G_j} | \mathrm{E_j}\rangle)$;
- the degree of confirmation
  $d_5 := \mathbf{P}_3(\bigcap_j \langle \mathrm{T_j}, \mathrm{S_j}, \mathrm{G_j} | \mathrm{E_j}\rangle) - \mathbf{P}_3(\bigcap_j \langle \mathrm{T_j}, \mathrm{S_j}, \mathrm{G_j}\rangle)$.

The Two- vs. Three-Typed Situation (1)

$$
\begin{array}{rcl}
\mathbf{P}_3(\mathrm{T}_t) = \tau & , & \mathbf{P}_3(\mathrm{T}_e) = \tau' \\
\mathbf{P}_3(\mathrm{T}_p) = \tau'' & , & \\
\mathbf{P}_3(\mathrm{S}_t|\mathrm{T}_t) = 1 & , & \mathbf{P}_3(\mathrm{S}_t|\neg\mathrm{T}_t) = 0 \\
\mathbf{P}_3(\mathrm{S}_e|\mathrm{T}_e) = 1 & , & \mathbf{P}_3(\mathrm{S}_e|\neg\mathrm{T}_e) = 0 \\
\mathbf{P}_3(\mathrm{S}_p|\mathrm{T}_p) = 1 & , & \mathbf{P}_3(\mathrm{S}_p|\neg\mathrm{T}_p) = 0 \\
\vdots & & \vdots \\
\mathbf{P}_3(\mathrm{E}_t|\mathrm{G}_t) = \pi & , & \mathbf{P}_3(\mathrm{E}_t|\neg\mathrm{G}_t) = \rho \\
\mathbf{P}_3(\mathrm{E}_e|\mathrm{G}_e) = \pi' & , & \mathbf{P}_3(\mathrm{E}_e|\neg\mathrm{G}_e) = \rho' \\
\mathbf{P}_3(\mathrm{E}_p|\mathrm{G}_p) = \pi'' & , & \mathbf{P}_3(\mathrm{E}_p|\neg\mathrm{G}_p) = \rho''
\end{array}
$$



The Two- vs. Three-Typed Situation (2)



- We assume $\mathbf{P}_4(\mathrm{T}_j) = \mathbf{P}_3(\mathrm{T}_j)$ and $\mathbf{P}_4(\mathrm{E}_j|\mathrm{G}_j) = \mathbf{P}_3(\mathrm{E}_j|\mathrm{G}_j)$.
- If $\mathbf{P}_i(\mathrm{T}_t) \cdot \mathbf{P}_i(\mathrm{T}_e) \cdot \mathbf{P}_i(\mathrm{T}_p) \in (0,1)$, $\mathbf{P}_i(\mathrm{E}_p|\mathrm{G}_p) > \mathbf{P}_i(\mathrm{E}_p|\neg\mathrm{G}_p)$, then
  - $\mathbf{P}_4(\mathrm{T}_t, \mathrm{T}_e, \mathrm{S}_t, \mathrm{S}_e, \mathrm{S}_p, \mathrm{G}_t, \mathrm{G}_e, \mathrm{G}_p) > \mathbf{P}_3(\bigcap_j \langle \mathrm{T}_j, \mathrm{S}_j, \mathrm{G}_j\rangle)$
  - $\mathbf{P}_4^* > \mathbf{P}_3^*$
  - $d_4 > d_3$ iff $(\mathbf{P}_4^* - \mathbf{P}_3^*) > \mathbf{P}_i(\mathrm{T}_t) \cdot \mathbf{P}_i(\mathrm{T}_e) \cdot \mathbf{P}_i(\mathrm{T}_p)$.

Integrative Reduction

Integrative Reduction (IR) A type of intertheoretic relation s.t.

1. There exist structural connections between same-theory objects and propositions.
2. The prob'y of the conjunction of propositions after the establishment of IR relations is higher than after the establishment of MR relations.

Conjecture

The propositions' probability and degree of confirmation is inversely proportional to their number of basic types.

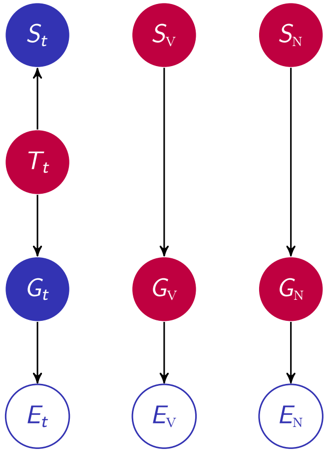
Case 3: One Type

- We cannot construct non-t rules.
- The assumption of a single type is probabilistically disadvantageous:

$$\mathbf{P}_1^{(*)} < \mathbf{P}_5^{(*)} < \mathbf{P}_3^{(*)}$$

- $\mathrm{S}_{\mathrm{N}}, \mathrm{S}_{\mathrm{V}}$ are not supported by evidence.
- The assumption is confirmationally disadvantageous: $d_1 < d_6 < d_5$
- The introduction of maps $\mathrm{S}_k \to \mathrm{G}_k$ raises the theories' probabilities and confirmation:

$$\mathbf{P}_1^{(*)} < \mathbf{P}_5^{(*)} = \mathbf{P}_3^{(*)}$$
$$d_1 < d_6 = d_5$$



Case 3: One Type

Let TY_m and TY_n be different basic-type sets such that $\mathrm{TY}_m \subseteq \mathrm{TY}_n$.

Theorem

If TY_m enables the construction of all linguistically relevant types, then

- $\bigcap_m \langle \mathrm{S}_m, \mathrm{G}_m \rangle$ *has a higher prior probability than* $\bigcap_n \langle \mathrm{S}_n, \mathrm{G}_n \rangle$.
- $\bigcap_m \langle \mathrm{S}_m, \mathrm{G}_m \rangle$ *has a higher posterior probability than* $\bigcap_n \langle \mathrm{S}_n, \mathrm{G}_n \rangle$.
- $\bigcap_m \langle \mathrm{S}_m, \mathrm{G}_m \rangle$ *may be better confirmed than* $\bigcap_n \langle \mathrm{S}_n, \mathrm{G}_n \rangle$ *under the difference measure.*

Note

1. The one-type case is optimal if the basic type is higher-order:

$$q := ((e \to t) \to t)$$

2. The success of Integrative Reduction is not conditional on the use of types.
3. The IR model can be adapted to accommodate 'corrected' versions of propositions (Schaffner, 1974), cf. (Nagel, 1977).

Wrap-Up

- We have identified a new type of intertheoretic relation, IR, inspired by (Montague, 1973).
- We have compared IR to Nagelian reduction.
- We have analyzed IR in the framework of confirmation theory.
- We have shown that IR is advantageous over NR:
  1. It raises the propositions' prior and posterior probability.
  2. It sometimes raises the propositions' degree of confirmation.
- This is due to constructivity relations bw same-theory objects, and dependency relations bw same-theory propositions.