Phil 424: HW #6 Solutions

Point Values

There were 5 questions. Each was worth 20 points. Partial credit was awarded.

1 Craps

There are 36 possible outcomes of dice rolls. Let Sn be the proposition that the sum of the dice is exactly n.

There are 6 equiprobable ways for the dice to add up to 7. So $\mathrm{cr}(S7) = 6/36$. There is only one way for the dice to add up to 2, so $\mathrm{cr}(S2) = 1/36$. There are 2 ways for the dice to add up to 3, so $\mathrm{cr}(S3) = 2/36 = 1/18$. There is only way for the dice to add up to 12, so $\mathrm{cr}(S12) = 1/36$

$E\$(BR) = \$4 \cdot \mathrm{cr}(S7) + (-\$1) \cdot \mathrm{cr}(\sim S7) = \$4 \cdot \frac{1}{6} + (-\$1) \cdot \frac{5}{6} = -\$\frac{6}{36} = -\$\frac{1}{6}$

$E\$(C) = \$7 \cdot \mathrm{cr}(S2) + \$7 \cdot \mathrm{cr}(S3) + \$7 \cdot \mathrm{cr}(S12) + (-\$1) \cdot \mathrm{cr}(\sim(S2 \vee S3 \vee S12)) = \$7 \cdot \frac{1}{36} + \$7\frac{2}{36} + \$7 \cdot \frac{1}{36} + (-\$1) \cdot \frac{-32}{36} = -\$\frac{4}{36} = -\$\frac{1}{9}$

$E\$(SE) = \$30 \cdot \mathrm{cr}(S2) + (-\$1) \cdot \mathrm{cr}(\sim S2) = \$30 \cdot \frac{1}{36} + (-\$1) \cdot \frac{35}{36} = -\$\frac{5}{36}$

So the ranking of the bets by their expected dollar values is C, SE, BR

2 4 Gambles

2.a x and y

$\text{EU}_{\text{SAV}}(1) = \text{u}(1 \,\&\, P) \cdot \text{cr}(P) + \text{u}(1 \,\&\, {\sim}P) \cdot \text{cr}({\sim}P) = x \cdot \text{cr}(P) + y \cdot \text{cr}({\sim}P)$

$\text{EU}_{\text{SAV}}(2) = \text{u}(2 \,\&\, P) \cdot \text{cr}(P) + \text{u}(2 \,\&\, {\sim}P) \cdot \text{cr}({\sim}P) = y \cdot \text{cr}(P) + x \cdot \text{cr}({\sim}P)$

$$
\begin{aligned}
x \cdot \text{cr}(P) + y \cdot \text{cr}({\sim}P) &= y \cdot \text{cr}(P) + x \cdot \text{cr}({\sim}P) && \text{Assumption} \\
\text{cr}(P) \cdot (x - y) &= \text{cr}({\sim}P) \cdot (x - y) && \text{Algebra} \\
\text{cr}(P) &= \text{cr}({\sim}P) && \text{Algebra, assumption that } x \neq y \\
\text{cr}(P) &= 1/2 && \text{Probability axioms}
\end{aligned}
$$

2.b $a, z,$ and m

$\text{EU}_{\text{SAV}}(3) = \text{u}(3 \,\&\, P) \cdot \text{cr}(P) + \text{u}(3 \,\&\, {\sim}P) \cdot \text{cr}({\sim}P) = 100 \cdot 1/2 + (-100) \cdot 1/2 = 50 + (-50) = 0$

$\text{EU}_{\text{SAV}}(4) = \text{u}(4 \,\&\, P) \cdot \text{cr}(P) + \text{u}(4 \,\&\, {\sim}P) \cdot \text{cr}({\sim}P) = m \cdot 1/2 + m1/2 = m$

This immediately entails $m = 0$

3 Jeffrey

3.a B is Not Highest

No. This is because EDT doesn't respect the Dominance Principle. Consider the following utility table:

	S	$\sim S$
A	10	0
B	1	-5
C	8	0

With the credence table:

	S	$\sim S$
A	0.01	0.99
B	0.99	0.01
C	0.01	0.99

Then $EU_{EDT}(A) = u(A \,\&\, S) \cdot cr(S|A) + u(A \,\&\, {\sim}S) \cdot cr({\sim}S|A) = 10 \cdot 0.01 + 0 \cdot 0.99 = 0.1$

$EU_{EDT}(B) = u(B \,\&\, S) \cdot cr(S|B) + u(B \,\&\, {\sim}S) \cdot cr({\sim}S|B) = 1 \cdot 0.99 + (-5) \cdot 0.01 = 0.94$

$EU_{EDT}(C) = u(C \,\&\, S) \cdot cr(S|C) + u(C \,\&\, {\sim}S) \cdot cr({\sim}S|C) = 8 \cdot 0.01 + 0 \cdot 0.99 = 0.08$

Since $EU_{EDT}(B) > EU_{EDT}(A) > EU_{EDT}(C)$, the agent will choose action B. But $B \,\&\, S$ does not have the highest utility of the S outcomes and $B \,\&\, {\sim}S$ does not have the highest utility of the ${\sim}S$ outcomes.

3.b B is Lowest

No, the decision problem above also shows this.