Probability, Modality and Triviality

Antony Eagle

Exeter College & Oxford University
Oxford OX1 3DP
antony.eagle@philosophy.oxford.ac.uk

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Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Abstract

Abstract

Many philosophers accept the following three theses:

1. that probability is a modal concept;
2. that, if determinism is true, there would still be objective modal facts; and
3. that if determinism is true, there are no genuine objective probabilities (chances).

I argue that these 3 claims are inconsistent, and that their widespread acceptance is thus quite troubling. I suggest, as others have, that we should reject the last thesis: objective probability is perfectly compatible with determinism. Nevertheless we must still explain why this thesis seems attractive; I suggest that a subtle equivocation is to blame.

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Outline

Outline

Modality and Determinism

Triviality of Deterministic Probability

Probability and Modality

Contradiction Between Our Theses, and the Resolution

Further Issues: Equivocation and Objectivity

Thesis 1: The Compatibility of Objective Modality and Determinism

Thesis 1: Modality

If determinism were true, then there would still be objective non-trivial modal facts. The fact that there would be precisely one future evolution of the actual world would not render modal claims degenerately true or false.

Before the argument: it seems a Moorean fact that things might have been otherwise and objects might have behaved differently.

As a basic truth about possibility, it's not the kind of thing to be refuted by empirical considerations.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Determinism and Possibility

History, Laws, and the Definition of Determinism

For each possible world w, and any time t, there is a proposition $S_{t,w}$ which captures the complete state description of that world at that time. If we conjoin all these instantaneous state propositions, we get the complete history $\mathcal{H}_w$.

Let $\mathcal{L}_w$ stand for the dynamical laws that relate $S_{t,w}$ to $S_{t+\epsilon,w}$. (If we are Humeans, we might think that $\mathcal{L}_w$ supervenes on $\mathcal{H}_w$, but no-one thinks that $\mathcal{L}_w$ supervenes on the individual $S_{t,w}$.)

Definition (Determinism)

A world w, with laws of nature $\mathcal{L}_w$, is deterministic just in case for any time t, $S_{t,w}$ is compatible with exactly one total history $\mathcal{H}_w$.

In other words, if $S_{t,w} = S_{t,w'}$ then $w = w'$ [Earman (1986), 13].

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Determinism and Possibility

Possibilities in Deterministic Worlds

Since there are no worlds that share exactly the current state of a given world, there are no non-degenerate possibilities relative to holding fixed the precise current state and laws. The actual future is futuristically necessary.

But clearly, there are non-degenerate possibilities:

- ▶ The laws of nature are most often (though not invariably) taken to be contingent.
- ▶ Even if the current state is necessary relative to the current state, it is by no means necessary simpliciter: the current state might have been otherwise.

But these are quite distant possibilities, if determinism is true: for neither gives a sense in φ's being possible indicates that φ might come to pass—therefore neither connects with deliberation.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Determinism and Possibility

Interesting Relative Modalities in Deterministic Worlds

Is there an interesting and useful sense of possibility that is not trivialised in deterministic worlds—that plays the possibility role better than futuristic possibility?

> *To say that something can happen means that its happening is compossible with certain facts. Which facts? That is determined, but sometimes not determined well enough, by context. [Lewis (1976), 77]*

A proposal: consider not our judgements about global future states, but instead those concerning more local capacities of objects to bring future events about—to do things. Correspondingly, we should hold local facts fixed, rather than the global state that futuristic possibility demands.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Context and Capability

Capabilities of an Object to Do Something

> *If you make any counterfactual supposition and hold all else fixed you get a contradiction. The thing to do is rather to make the counterfactual supposition and hold all else as closed to fixed as you consistently can. [Lewis (1976), 79]*

So what do we hold fixed? Relevant facts about the abilities of the objects we are talking about, where relevance is contextually fixed:

- Intrinsic properties of the object;
- Salient (macroscopic) facts about the local and historical circumstances.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Context and Capability

Powers

Ordinary ascriptions of powers depend only on intrinsic properties (and not even all of those).

The ascription of powers is independent of whether or not the external situation allows for the expression of those powers.

Moreover, in ascribing powers we must rely on laws of nature, even deterministic onesotherwise how could we judge what capacities an object would display when trialled? (Hume)

In general, if O is able to (capable of) φ, then '$\Diamond\varphi$' is true: non-trivial capacities give rise to non-trivial possibilities.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Context and Capability

Circumstances

Is this this sense of possibility too weak (too subjective?), since it depends on ignoring certain relevant factors of circumstance?

This objection fails to respect the intuition that truths of mere possibility are in some way derivative from various actual truths. For anything to be possible, it must involve some variation from actuality.

What guides this variation—which factors must we hold fixed? We have no good reason to think that there must be a context-independent set of such facts which give the 'real' possibilities (inegalitarian modal realism?)

Rather, we hold fixed facts about the things we're talking about, pragmatically and contextually governed. (A necessary task elsewhere that I can appeal to here.)

Thesis 2: The Triviality of Deterministic Chance

Thesis 2: Triviality

If determinism were true, then there would be no genuine objective probabilities (chances). All chances would be degenerate, either zero or one.

> *To the question how chance can be reconciled with determinism... my answer is: it can't be done... If our world is deterministic, there are no chances in it, save chances of zero and one... If a determinist says a tossed coin is fair, and has an equal chance of falling heads or tails, he does not mean what I mean when he speaks of chance. [Lewis (1980), 118–20]*

Modality	**Triviality**	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Arguments in favour of Triviality

Arguing for the Triviality Thesis

Orthodoxy, yet infrequently argued for. Perhaps:

- ▶ If Determinism is true, then the state of the system at the current time fixes all other states;
- ▶ If all the states are fixed, every fact that will be true is fixed now;
- ▶ Therefore, there are no genuine chances, because any chancy fact is either necessitated or impossible given the current state.

That is, the conditional probability of any φ, given the laws and the current state, must be either 0 or 1, since either φ is entailed by the laws and the current state, or $\neg\varphi$ is.

Thesis 3: Probability is a Modality

Thesis 3: Probability

Objective probability (chance) is a modal concept—chances depend not just upon actual facts, but upon facts about other possibilities.

> *Chances measure a contingent and quantitative kind of possibility...No radium atom must decay, or must not decay, in any given time; it is merely possible for it to decay, a possibility that is both contingent and comes by degrees. The possibility is contingent on the structure and state of the atom's nucleus, and its degree is measured by the chance of decay. [Mellor (1995), 21]*

Modality	Triviality	**Probability**	Contradiction and Resolution	Further Issues	Conclusion
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Intuitive Grounds for the Thesis

The Classical Theory

Definition (Classical Theory of Probability)

The *probability* $\Pr(\varphi)$ of a proposition φ is the *proportion* of φ-favourable cases n_φ in a set of *n equipossible* outcomes $\left(\frac{n_\varphi}{n}\right)$.

Obviously, the probability of any event depends on which events happen to be (equi-)possible. This should at least suggest that facts about probability are modally involved, and not exhausted by non-modal facts.

It also suggests something more: that if a member of a (finite) partition is possible, there is some non-zero chance it might happen.

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Intuitive Grounds for the Thesis

Justifying Kolmogorov

One also often hears the axiom $Pr(\top) = 1$ justified by appeal to the
necessity of $\top$.

Implicitly, the appeal is to something like this [Mellor (1995), 31–2]:

(Necessity) $Pr(\varphi) = 1 \vDash \varphi$.

The thought is that (Necessity) shows that $Pr(\cdot) = 1$ functions like a
necessity operator; as such, since '$\Box\top$' is true, so also '$Pr(\top) = 1$' must
be true.

Modality	Triviality	**Probability**	Contradiction and Resolution	Further Issues	Conclusion
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Basic Chance Principles

The Basic Chance Principle

One might directly motivate the necessity axiom by the claim that $\Box\varphi \vDash \Pr(\varphi) = 1$, which given $\vDash \Box\top$ yields the axiom.

This claim is equivalent to the following:

The Basic Chance Principle (BCP)

$$\Pr(\varphi) > 0 \vDash \Diamond\varphi.$$

In general, if the chance of A is positive there must be a possible future in which A is true. [Bigelow et al. *(1993), 459*]

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Basic Chance Principles

Converse Basic Chance Principle

Intuitively, we may find the converse principle equally appealing:

Converse Basic Chance Principle

$$\Diamond\varphi \vDash \Pr(\varphi) > 0.$$

This cannot be right as it stands: a uniform distribution over infinite outcome spaces must give each outcome probability 0; if the Converse BCP were right, each outcome in such a space would be impossible, which is not the case.

Modality	Triviality	**Probability**	Contradiction and Resolution	Further Issues	Conclusion
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Basic Chance Principles

Infinitesimals

We could loosen our mathematics for probability, and introduce infinitesimal values for probabilities: probability values less than any standard real but still greater than zero [Bernstein and Wattenberg (1969)].

We could then keep extreme probability values for necessary and impossible propositions—regularity. Moreover, we could have well-defined conditional probabilities on these events, via the usual ratio rule.

But adopting infinitesimals doesn't solve our whole problem, since we still don't know which possibilities should count in assigning a non-zero probability, infinitesimal or not.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Basic Chance Principles

Which worlds?

We want to define a kind of 'probabilistic possibility', a relative modality that will secure the Converse BCP.

Not every metaphysical possibility should have non-zero chance of coming about—no chance of contravening the laws, for example.

Not every physical possibility should have non-zero chance. Different initial conditions are physically possible (compatible with the laws of nature), but have no chance of coming about.

This suggests that both history and laws are important.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Basic Chance Principles

Local chances and similar worlds

We shouldn't be too fixated on precise and exact match of laws and history, because chance is a local matter. The chance of a coin toss landing heads depends on the tossing device and the coin, and other local details, but not on distant matters.

Proposal: Local Similarity

'$\Diamond_{Pr}\varphi$' is true iff φ occurs in a world sufficiently similar to actuality in those respects of law and history relevant for bringing about the occurrence of φ. The Converse BCP holds of $\Diamond_{Pr}$.

Definition (Inevitability)

φ is inevitable iff $\neg\Diamond_{Pr}\neg\varphi$ (i.e. $\neg\Pr(\neg\varphi) > 0$, hence $\Pr(\varphi) = 1$).

Modality	Triviality	**Probability**	Contradiction and Resolution	Further Issues	Conclusion
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Basic Chance Principles

Consequences of the Proposal

Actual sensitivity Chance, though modal, is highly sensitive to actual details how φ is brought about—biased coins.

Counterfactuals Counterfactuals about stochastic situations come out right. Worlds 'sufficiently similar' are exactly the ones used in in assessing these counterfactuals ('had the experiment been run differently, it might have resulted in spin up': this cannot require perfect similarity). (Compare 'had this experiment been repeated, the outcome would have been the same'.)

Expectation Bishop Butler vindicated: chances will be important in expectations about what might happen, which depends on these counterfactuals.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Contradiction

Contradiction

The particular forms of our thesis that we end up defending look like this:

1. If w is deterministic, nevertheless there exist propositions such that, non-trivially, $w \vDash \Diamond\varphi$.
2. If w is deterministic, there do not exist propositions which have non-trivial chances.
3. Probability is a kind of graded possibility.

It's obvious already that, on the basis of determinism alone, we should not be able to rule out the possibility of non-trivial chances. Can we make a contradiction more explicit?

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Contradiction

Capacities and Probabilistic Possibilities

I think we can: recall that the constraints required to get non-trivial $\Diamond\Phi(O)$ were the intrinsic properties of O and the contextually salient circumstances.

But these are precisely the factors that delimit the relevant worlds in the converse BCP: sufficient similarity of local matters, relevant to the production of the outcome in question. In some sense, objective chances involve capacities of chance devices.

Let some chance device O possibly—but not actually—produce φ; then both $w \vDash \Diamond\varphi$ and $w \vDash \neg\varphi$. For some w', $w' \vDash \varphi$; w' is a relevant world for the production of φ, so $w \vdash \Diamond_{\Pr}(\varphi)$, and hence $w \vDash \Pr(\varphi) > 0$. By Triviality, $w \vDash \Pr(\varphi) = 1$; by (Necessity), $w \vDash \varphi$. Contradiction.

Modality	Trivality	Probability	**Contradiction and Resolution**	Further Issues	Conclusion
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Resolution of the Contradiction

Rejecting Triviality

I propose that we reject the Triviality thesis. Precedent: deterministic yet probability-involving special sciences (Bohm, meteorology, population genetics,. . .).

> *It seems to me that the issue of determinism versus indeterminism really ought to be (is) irrelevant to an interpretation of probability theory. [Clark (2001), 275]*

> *[We are] prepared to assert that there is a chance of* $1/2$ *that a particular coin tossed in a certain way will come up heads, a chance of* $1/2$ *that a certain molecule will be located in the left half of a container of gas at some particular time. . . To suppose that the truth of such statements depends on the truth of indeterminism flies in the face of virtually universal common and scientific usage. [Salmon (1979), 199]*

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Resolution of the Contradiction

Unsuccessful Denials

Frequentism If we are frequentists about chance, we never had any problem with determinism (indeed, easier to be frequentist!). Lots of problems [Hájek (1997)]. But, even apart from their other problems, they don't give us the right kinds of modal facts for our purposes, because they don't satisfy the Probability thesis.

Macroscopic 'Chance' Loewer defines [Loewer (2001), 618–9]:

$$\Pr_{\text{macro}}(\varphi) =_{\text{df}} \Pr_{\text{initial}}(\varphi|M_{w,t}).$$

But this surrogate doesn't give us non-trivial real chances (except for initial conditions); and, as the mention of macroscopic (observable) history emphasises, doesn't give us the objectivity we wish.

Modality	Triviality	Probability	**Contradiction and Resolution**	Further Issues	Conclusion
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Resolution of the Contradiction

Probability Ascriptions and Their Evaluation

Earlier, I claimed that when we ascribe probabilities to certain outcomes we are implicitly making some kind of claim about the system which generates that outcome:

- ▶ that the outcome is possible for that kind of system;
- ▶ that we should expect it to occur in such a system with a certain frequency;
- ▶ if we altered the system in certain ways that would alter the distribution of possibilities, etc.

When evaluating these specific claims, we do not always hold every actual fact about the system fixed: we don't hold the actual outcome fixed, nor do we hold external constraints fixed (Frankfurt-style examples), but we do hold fixed the particulars of the experimental apparatus.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Resolution of the Contradiction

Indeterminism and Particular Experiments

Determinism is a property of a world.

Admitting that surrounding facts aren't always fixed when evaluating probability claims, we already deny the necessary connection of indeterminism and chance.

Determinism doesn't entail local determinism (the instantaneous state of every part of the universe, plus the laws, fixes every state of that part)—but local determinism is required to trivialise most ordinary probability ascriptions to chance devices.

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Resolution of the Contradiction

Pragmatics, Hypotheticals, and Chance

How to explain the intuitive incoherence of 'this coin could land heads if tossed, but has no chance of landing heads if tossed'?

Consider the role of chance and possibility in hypothetical reasoning:

► The facts to be held fixed are those that make this trial of the same type as others that guide our hypothetical claims; and

► the salience of particular facts, and the classification, is contextually dependent.

Canonically, we fix on those facts that identify the system as the kind of setup it is, and dictate what outcomes can come about in a setup like that, on the basis of intrinsic properties that are robust across parametric variations in that kind of setup.

As such, we don't always hold fixed global complete particular state descriptions—and we don't trivialise chance.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Equivocation

Equivocation Over 'Has Some Chance of'

We seem committed to each of:

1. 'This coin has no chance of landing heads because given the history and the laws it is inevitable that it land tails';

2. 'This coin has some chance of landing heads because it, and the tossing system, are of the sort that can produce heads'.

These can both be true if 'having some chance' is equivocal.

Since it is possible for us to hold fixed different sets of facts in evaluating capabilities underlying probability ascriptions, it is possible for us to equivocate.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Equivocation

Capricious Probability

We can separate the concepts over which we equivocate:

Definition (Capricious Possibility)

$$\Pr_{\text{capricious}}(\varphi) =_{\text{df}} \Pr(\varphi | \mathcal{H}_{w,t} \wedge \mathcal{L}_w).$$

Thesis 4: Capricious Triviality

If determinism were true, then there would be no non-trivial objective capricious probabilities.

This thesis does seem true, but it does not undermine the existence of non-trivial chances in the more ordinary sense.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Equivocation

Explaining the Appeal of Triviality

I wish to explain the appeal of the Triviality thesis by suggesting that it has been confused with the Capricious Triviality thesis.

Consider when we might use the concept of capricious probability in discussions of chance. If what I've said is right, it is contexts where a very rich set of law and history facts is salient.

It is quite clear that this rich set of facts is salient when we discuss determinism! Hence it is natural to think that determinism rules out non-trivial chances.

But outside the philosophical context, when the salient facts are more limited, the normal chances we appeal to are perfectly compatible with determinism.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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Objectivity

Objectivity

Is this kind of context-dependent chance objective? In one sense, clearly: once the facts are fixed, the possibilities follow completely independently of us.

Yet some may worry that chance is not a fundamental feature of the world, but is relational and contextual, and hence is less than fully respectable.

But should we want any more than this? If actuality is primary, then modal claims are a little bit less respectable than actual truths— there must be an asymmetrical dependence of the former on the latter. Probabilities will be no exception.

Conclusion

- ▸ We examined three theses, commonly accepted, about the relationships between modality, objectivity, and chance:
  1. That determinism and non-trivial possibility are compatible;
  2. That determinism and non-trivial probability are incompatible; and
  3. That probability is a kind of possibility.
- ▸ We discerned a tension between them, that might be massaged into a contradiction.
- ▸ We resolved the contradiction by rejecting the Triviality thesis.
- ▸ We explained the appeal of that thesis by examining the plausibility of an equivocation over 'having some chance'.

Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	**Conclusion**
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Modality	Triviality	Probability	Contradiction and Resolution	Further Issues	Conclusion
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