Announcements & Overview

- Administrative Stuff
  - Last Week: Course Website/Syllabus (see me if you have questions)
  - ☞ **HW #1 Assigned (see website) – due next Friday (*via* `Blackboard`).**
- Today: Basic Underlying Concepts of Logic (Chapter 1, Cont'd)
  - Why model logical concepts mathematically/formally?
  - A Subtle Argument, and the Notion of Logical Form
  - A Conservative Principle About Attributions of Validity
  - Sentential Logical Form
  - Beyond Sentential Form
  - Two "Strange" Valid Sentential Forms
  - Validity and Soundness of Arguments – Some Examples
  - A "Big Picture" View of Part I of the Course
  - Time Permitting: Preamble for Chapter 2 (language of sentential logic)

Why model logic concepts *formally* or *symbolically*?

- Ultimately, we want to decide whether arguments expressible in *natural* languages are valid. But, in this course, we will only study arguments expressible in *formal* languages. And, we will use *formal* tools. *Why?*
- Analogous question: What we want from natural science is explanations and predictions about *natural* systems. But, our theories (strictly) apply only to systems faithfully describable in *formal*, *mathematical* terms.
- Although formal models are *idealizations* which abstract away some aspects of natural systems, they are *useful idealizations* that help us understand *many* natural relationships and regularities.
- Similarly, studying arguments expressible in formal languages allows us to develop powerful tools for testing validity. We won't be able to capture *all* valid arguments this way. But, we can grasp many.
- *Why* or *how* mathematical/formal methods *are* helpful for such understanding is a deep question in the philosophy of science.

A Subtle Argument, and the Notion of Logical Form

(i) John is a bachelor.
$\therefore$ John is unmarried.

- Is (i) valid? Well, this is tricky. Intuitively, being unmarried is part of the *meaning* of "bachelor". So, it *seems* like it is (intuitively) logically impossible for the premise of (i) to be true while its conclusion is false
- This suggests that (i) is (intuitively/absolutely) valid.
- On the other hand, consider the following argument:

(ii) If John is a bachelor, then John is unmarried.
John is a bachelor.
$\therefore$ John is unmarried.

- The correct judgment about (ii) seems *clearly* to be that it is valid – *even if we don't know the meaning of "bachelor" (or "unmarried").*
- This is clear because the logical form of (ii) is *obvious* [(i)'s form is not].

A Conservative Principle About Attributions of Validity

- This suggests the following additional "conservative" heuristic:
  - ☞ We should conclude that an argument $\mathscr{A}$ is valid only if we can see that $\mathscr{A}$'s conclusion follows from $\mathscr{A}$'s premises *without appealing to the meanings of the predicates involved in* $\mathscr{A}$.
- But, if validity does not depend on the meanings of predicates, then what *does* it depend on? This is a deep question about logic. We will not answer it here. That's for more advanced philosophical logic courses.
- What we will do instead is adopt a conservative methodology that only classifies *some* "intuitively/absolutely valid" arguments as valid.
- The strategy will be to develop some *formal* methods for *modeling* intuitive/abolsute validity of arguments expressed in English.
- We won't be able to capture *all* intuitively/absolutely valid arguments with our methods, but this is OK. [Analogy: mathematical physics.]

Sentential Logical Form

- We will begin with *sentential logic*. This will involve providing a characterization of valid *sentential forms*. Here's a paradigm example:

   Dr. Ruth is a man.
(1) If Dr. Ruth is a man, then Dr. Ruth is 10 feet tall.
   ∴ Dr. Ruth is 10 feet tall.

- (1) is a set of sentences with a valid sentential form. So, whatever argument it expresses is a valid argument. What's its *form*?

   p.
(1_f) If p, then q.
   ∴ q.

- (1)'s valid *sentential form* (1_f) is so famous it has a name: *Modus Ponens*. [Usually, latin names are used for the *valid* forms.]

☞ **Definition**. The *sentential form* of an argument (or, the sentences faithfully expressing an argument) is obtained by replacing each basic (or, atomic) sentence in the argument with a single (lower-case) letter.

- What's a "basic" sentence? A basic sentence is a sentence that doesn't contain any sentence as a proper part. How about these?

(a) Branden is a philosopher and Branden is a man.
(b) It is not the case that Branden is 6 feet tall.
(c) Snow is white.
(d) Either it will rain today or it will be sunny today.

- Sentences (a), (b), and (d) are *not* basic (we'll call them "complex" or "compound"). Only (c) is basic. We'll also use "atomic" for basic.
- What's the sentential form of the following argument (is it valid?):

   If Tom is at his Fremont home, then he's in California.
   Tom is in California.
   ∴ Tom is at his Fremont home.

Two "Strange" Valid Sentential Forms

(†) p. Therefore, either q or not q.

- (†) is valid because it is (logically) *impossible* that *both*:
   (i) p is true, *and*
   (ii) "either q or not q" is false.

   This is impossible because *(ii) alone* is impossible.

(‡) p and not p. Therefore, q.

- (‡) is valid because it is (logically) *impossible* that *both*:
   (iii) "p and not p" is true, *and*
   (iv) q is false.

   This is impossible because *(iii) alone* is impossible.

- We'll soon see why we have these "oddities". They stem from our semantics for "If ... then" statements (and our first def. of validity).

Some Valid and Invalid Sentential Forms

Sentential Argument Form	Name	Valid/Invalid
p If p, then q ∴ q	*Modus Ponens*	Valid
q If p, then q ∴ p	Affirming the Consequent	Invalid
It is not the case that q If p, then q ∴ It is not the case that p	*Modus Tollens*	Valid
It is not the case that p If p, then q ∴ It is not the case that q	Denying the Antecedent	Invalid
If p, then q If q, then r ∴ If p, then r	Hypothetical Syllogism	Valid
It is not the case that p Either p or q ∴ q	Disjunctive Syllogism	Valid

Beyond Sentential Form

- The first half of the course involves developing a precise *theory* of *sentential* validity, and several rigorous techniques for *deciding* whether a sentential form is (or is not) valid. This only takes us so far.
- Not all (absolutely) valid arguments have valid *sentential* forms, *e.g.*:

  All men are mortal.
  (2) Socrates is a man.
  ∴ Socrates is mortal.

- The argument expressed by (2) seems clearly valid. But, the sentential form of (2) is not a valid form. Its sentential form is:

  p.
  (2_f) q.
  $\therefore r$.

- In this first course, we will not be studying predicate/quantifier logic, which gives a formal theory of validity that covers such forms.
- In that more general theory, one can recognize that (2) has something like the following (non-sentential!) logical form:

  All Xs are Ys.
  (2_f*) a is an X.
  $\therefore a$ is a Y.

- We will leave such arguments (called *syllogisms*) for a future, more sophisticated theory of logical validity (*viz.*, *predicate logic*).
- In Part I of the course, we'll learn a (simple) purely formal language for talking about *sentential* forms, and then we'll develop some rigorous methods for determining whether/which sentential forms are valid.
- As we will see, the fit between our simple formal sentential language and English (or other natural languages) will not be perfect. First, let's think a bit harder about the above "Barbara" Aristotelian form.

- As an illustration of the subtlety of determining what "the" sentential form of an argument is, let's reconsider our Socrates syllogism.

  All men are mortal.
  Socrates is a man.
  ∴ Socrates is mortal.

- Notice that (intuitively) the first premise of this syllogism entails a conditional claim about each individual object. Specifically, for each object o, the first premise entails the following conditional proposition.

  If o is a man, *then* o is mortal.

- So, to be more specific, the first premise entails the following conditional claim about the object named "Socrates."

  If Socrates is a man, *then* Socrates is mortal.

- In other words, the first premise entails a conditional claim about Socrates that — together with the second premise — yields a *modus ponens* argument about Scorates, which *is sententially valid*!

Validity and Soundness of Arguments — Some Non-Sentential Examples

- Can we classify the following according to validity/soundness?

1)	All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage.	**5)**	All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine.
2)	All wines are whiskeys. Chardonnay is a wine. Therefore, chardonnay is a whiskey.	**6)**	All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine.
3)	All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink.	**7)**	All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine.
4)	All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey.	**8)**	All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine.

	Valid	**Invalid**
True premises True conclusion	All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage. [sound]	All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine. [unsound]
True premises False conclusion	**Impossible** None exist	All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine. [unsound]
False premises True conclusion	All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink. [unsound]	All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine. [unsound]
False premises False conclusion	All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey. [unsound]	All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine. [unsound]

- See, also, our validity and soundness handout ...

Some Brain Teasers Involving Validity and Soundness

- Here are two very puzzling arguments:

($\mathscr{A}_1$) Either $\mathscr{A}_1$ is valid or $\mathscr{A}_1$ is invalid.
$\therefore$ $\mathscr{A}_1$ is invalid.

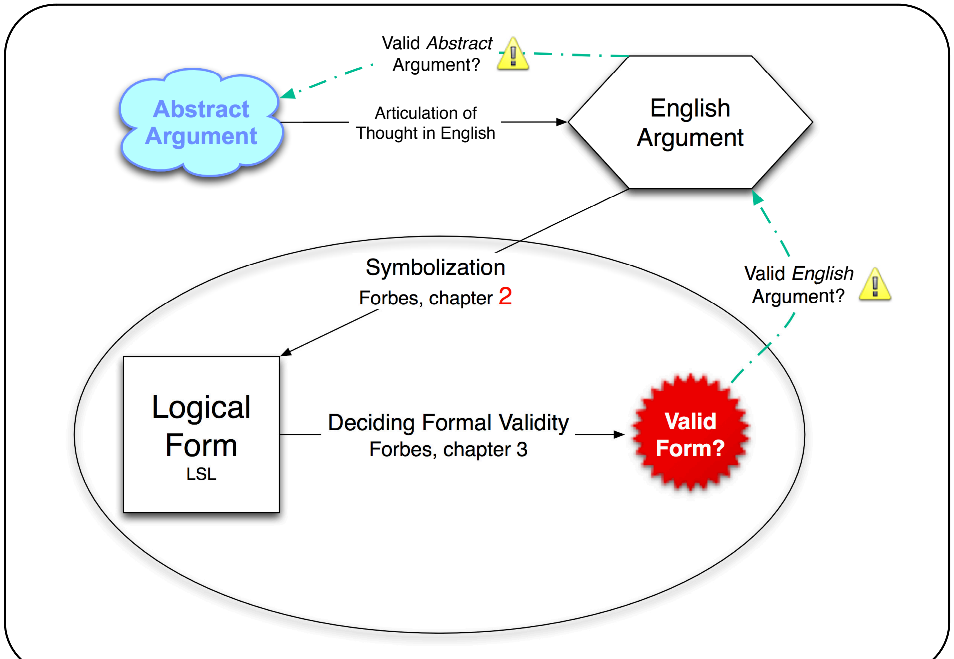
($\mathscr{A}_2$) $\mathscr{A}_2$ is valid.
$\therefore$ $\mathscr{A}_2$ is invalid.

- I'll discuss $\mathscr{A}_2$ ($\mathscr{A}_1$ is left as an exercise).
  - If $\mathscr{A}_2$ is valid, then it has a true premise and a false conclusion. But, this means that if $\mathscr{A}_2$ is valid, then $\mathscr{A}_2$ invalid!
  - If $\mathscr{A}_2$ is invalid, then its conclusion must be true (as a matter of logic). But, this means that if $\mathscr{A}_2$ is invalid then $\mathscr{A}_2$ is valid!
  - This *seems* to imply that $\mathscr{A}_2$ is *both valid and invalid*. But, remember our conservative validity-principle. What is the *logical form* of $\mathscr{A}_2$?

Absolute Validity *vs* Formal Validity

- Forbes calls the general, informal notion of validity "absolute validity".
- Our notion is a bit more conservative than his, since we'll only call an argument valid if one of our *formal theories* captures it as falling under a valid *form*. Our first formal theory (LSL) is about *sentential* validity.
- An argument is *sententially* valid if it has a valid *sentential form*.
- Sentential form is obtained by replacing each basic or atomic sentence in an argument with a corresponding lower-case letter.
- Once we know the sentential form of an argument (chapter 2), we will be able to apply purely formal, mechanical methods (chapters 3 and 4) for determining whether that sentential form is valid.

☞ Even if an argument fails to be *sententially* valid, it could still be valid according to a richer logical theory than LSL. I'll mention some other, more sophisticated theories of logical form later in the course.



Preamble for Chapter 2: The Use/Mention Distinction

- Consider the following two sentences:
  (1) California has more than nine residents.
  (2) 'California' has more than nine letters.
- In (1), we are *using* the word 'California' to talk about the State of California. But, in (2), we are merely *mentioning* the word 'California' (*i.e.*, we're talking about *the word itself*).
- If Jeremiah = 'California', which of these sentences are true?
  (3) Jeremiah has (exactly) eight letters [false].
  (4) Jeremiah has (exactly) ten letters [true].
  (5) 'Jeremiah' has eight letters [true].
  (6) 'Jeremiah' is the name of a state [false].

Preamble for Chapter 2: More on Use/Mention and ' ' *versus* ⌜ ⌝

- Consider the following two statements about LSL sentences
  (*i*) If p and q are both sentences of LSL, then so is $\ulcorner(p \,\&\, q)\urcorner$.
  (*ii*) If p and q are both sentences of LSL, then so is '$(p \,\&\, q)$'.
- As it turns out, (*i*) is true, but (*ii*) is *false*. The string of symbols '$(p \,\&\, q)$' *cannot* be a sentence of LSL, since 'p' and 'q' are *not* part of the lexicon of LSL. They allow us to talk about LSL *forms*.
- The trick is that $\ulcorner(p \,\&\, q)\urcorner$ abbreviates the long-winded phrase:
  - The symbol-string which results from writing '(' followed by p followed by '&' followed by q followed by ')'.
- In (*ii*), we are merely *mentioning* 'p' and 'q' (in '$(p \,\&\, q)$'). But, in (*i*), we are *using* 'p' and 'q' (in $\ulcorner(p \,\&\, q)\urcorner$) to talk about (forms of) sentences in LSL. In (*i*), 'p' and 'q' are *used* as *metavariables*.

Preamble for Chapter 2: Object language, Metalanguage, *etc.* ...

- LSL is the *object language* of our current studies. The symbol string '$(A \,\&\, B) \vee C$' is a sentence of LSL. But, the symbol string '$(p \,\&\, q) \vee r$' is *not* a sentence of LSL. Why?
- We use a *metalanguage* to talk about the object language LSL. This metalanguage is not formalized. It's mainly English, plus *metavariables* like 'p', 'q', 'r', and *selective quotes* '⌜' and '⌝'.
- If p = '$(A \vee B)$', and q = '$(C \to D)$', then what are the following?
  - $\ulcorner p \,\&\, q\urcorner$ [$(A \vee B) \,\&\, (C \to D)$], '$p \,\&\, q$' [$p \,\&\, q$], '$p$' [$p$], '$q$' [$q$]
- And, which of the following are true?
  - p has five symbols [true]. 'p' has five symbols [false].
  - $\ulcorner p \,\&\, q\urcorner$ is a sentence of LSL [true]. So is '$\ulcorner p \,\&\, q\urcorner$' [false].

Introduction to the Syntax of the LSL: The Lexicon

- The syntax of LSL is quite simple. Its lexicon has the following symbols:
  - Upper-case letters 'A', 'B', ... which stand for *basic sentences*.
  - Five *sentential connectives/operators* (one *unary*, four *binary*):

Operator	Name	Logical Function	Used to translate
'$\sim$'	tilde	negation	not, it is not the case that
'&'	ampersand	conjunction	and, also, moreover, but
'$\vee$'	vee	disjunction	or, either ... or ...
'$\to$'	arrow	conditional	if ... then ..., only if
'$\leftrightarrow$'	double arrow	biconditional	if and only if

  - Parentheses '(', ')', brackets '['. ']', and braces '{', '}' for grouping.
- If a string of symbols contains anything else, then it's not a sentence of LSL. And, only *certain* strings of these symbols are LSL sentences.
- Some LSL symbol strings aren't *well-formed*: '$(A \,\&\, B$', '$A \,\&\, B \vee C$', *etc.*