Announcements & Overview

- Administrative Stuff
  - ☞ **Everything has been pushed back by a week** (including mid-term).
    - ∗ HW #1 grades now posted (with detailed solutions).
    - ∗ **HW #2 due *next* Friday (2/19).**
      - · Consult *Homework Guidelines & Tips* handout re HW #2
  - – HWs are now worth 25% of your final grade, and Participation is worth 8% (*via* ungraded TurningPoint Cloud quizzes).
  - – Consult the latest revision of the Syllabus & Website for details.
- Today: Introduction to Unit #2 (**L**anguage of **S**entential **L**ogic)
  - – Symbolizing English Sentences (four examples)
  - – Numerical expressions like "at least one of" or "exactly two of"
  - – Then: Symbolizing English *Arguments* (four examples)

English ↦ LSL X (&, →): Example #1

- 'John will study hard and also bribe the instructor, and if he does both then he'll get an "A", provided the instructor likes him.'
  - – Step 0: Decide on atomic sentences and letters.
    - S: John will study hard. A: John will get an "A".
    - B: John will bribe the instructor. L: The instructor likes John.
  - – Step 1: Substitute into English, yielding "Logish":
    - S and B, and if S and B then A, provided L.
  - – Step 2: Make the transition into LSL (in stages as well, perhaps):
    - S and B, and if L, then if S and B then A.
    - $(S \,\&\, B) \,\&\, (L \to$ (if S and B then A)).
    - Final Product: $(S \,\&\, B) \,\&\, (L \to ((S \,\&\, B) \to A))$

English ↦ LSL II (∼, &, ∨, →, ↔): Example #2

- 'Sara is going unless either Richard or Pam is going, and Sara is not going if, and only if, neither Pam nor Quincy are going.'
  - – Step 0: Decide on atomic sentences and letters.
    - P: Pam is going. Q: Quincy is going.
    - R: Richard is going. S: Sara is going.
  - – Step 1: Substitute into English, yielding "Logish":
    - S unless either R or P, and not S iff neither P nor Q.
  - – Step 2: Make the transition into LSL (in stages again):
    - S unless $(R \vee P)$, and $\sim S$ iff $(\sim P \,\&\, \sim Q)$
    - $(\sim(R \vee P) \to S) \,\&\, (\sim S \leftrightarrow (\sim P \,\&\, \sim Q))$
- It is also acceptable to replace the 'unless' with '∨', yielding:
  - $(S \vee (R \vee P)) \,\&\, (\sim S \leftrightarrow (\sim P \,\&\, \sim Q))$

English ↦ LSL II (∼, &, ∨, →, ↔): Example #3

- 'If you do not concentrate well unless you are alert, then provided that you are not a maniac, you will fly an airplane only if you are sober.'
  - – Step 0: Decide on atomic sentences and letters.
    - C: You concentrate well. M: You are a maniac.
    - A: You are alert. F: You will fly an airplane.
    - S: You are sober.
  - – Step 1: Substitute into English, yielding "Logish":
    - If not C unless A, then provided that not M, F only if S.
  - – Step 2: Make the transition into LSL (in stages again):
    - If $\sim C$ unless A, then if $\sim M$, then F only if S.
    - Final Product: $(\sim A \to \sim C) \to (\sim M \to (F \to S))$.
- It is also acceptable to replace the 'unless' with '∨', yielding:
  - Alternative Final Product: $(\sim C \vee A) \to (\sim M \to (F \to S))$

English ↦ LSL II (~, &, ↔): Example #4

- ‘If, but only if, they have made no commitment to the contrary, may reporters reveal their sources, but they always make such a commitment and they ought to respect it.’
  - Step 0: Decide on atomic sentences and letters.

  S: Reporters may reveal their sources.

  C: Reporters have made a commitment to protect their sources.

  R: Reporters ought to respect their commitment to protect sources.
  - Step 1: Substitute into English, yielding “Logish”:

    If, but only if, it is not the case that C, then S, but C and R.
  - Step 2: make the transition into LSL (in stages as well, perhaps):

    S iff not C, but C and R.

    Final Product: $(S \leftrightarrow \sim C) \,\&\, (C \,\&\, R)$

Symbolizing Numerical Expressions

- Let $A \stackrel{\text{def}}{=}$ Alice will attend the party, $B \stackrel{\text{def}}{=}$ Bill will attend the party, and $C \stackrel{\text{def}}{=}$ Carol will attend the party. And consider these

(1) *At least two of* Alice, Bill, and Carol will attend the party.

(2) *Exactly one of* Alice, Bill, and Carol will attend the party.

(3) *At most two of* Alice, Bill, and Carol will attend the party.

(4) *Exactly two of* Alice, Bill, and Carol will attend the party.

- Some hints:
  - “Exactly 2 of $\{A, B, C\}$ are true” is equivalent to a *disjunction of conjunctions*, in which each conjunction expresses *one specific way* in which exactly two of $\{A, B, C\}$ could be true.
  - “At least 2 of $\{A, B, C\}$ are true” is equivalent to: *either exactly* 2 of $\{A, B, C\}$ are true *or all three* of $\{A, B, C\}$ are true.

Symbolizing/*Reconstructing* Entire English Arguments

- Naïvely, an argument is “just a collection of sentences”. So, naïvely, one might think that symbolizing arguments should just boil down to symbolizing a bunch of individual sentences. It’s not so simple.
- An argumentative passage has more structure than an individual sentence. This makes argument *reconstruction* more subtle.
- We must now make sure we capture the inter-relations of content across the various sentences of the argument.
- To a large extent, these interrelations are captured by a judicious choice of atomic sentences for the reconstruction.
- It is also crucial to keep in mind the overall intent of the argumentative passage — the intended argumentative strategy.
- Forbes glosses over the art of (charitable!) argument reconstruction. I will be a bit more explicit about this today in some examples.

Symbolizing Entire Arguments: An Example

- ‘If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient. But, if God exists then He is none of these, and there is evil in the world. So, we must conclude that God does not exist.’
- Step 0: Decide on atomic sentences and letters.

  G: God exists. E: There is evil in the world.

  J: God is just. O: God is omnipotent.

  K: God is omniscient.
- Step 1: Identify (and symbolize) the *conclusion* of the argument:
  - ‘God does not exist.’ (which is just ‘$\sim G$’ in LSL)
- Step 2: Symbolize the premises (in this case, there are two):
  - Premise #1: ‘If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient.’

Symbolizing Arguments: Example #2

- Premise #1: 'If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient.'

  If G, then ($\sim E$ unless ($\sim J$ or ($\sim O$ or $\sim K$)))

  $G \to (\sim E \vee (\sim J \vee (\sim O \vee \sim K)))$
- Premise #2: 'If God exists then He is none of these (*i.e.*, He is *neither* unjust *nor*...), and there is evil in the world.'

  If G, then not not-J and not not-O and not not-K, and E.

  $[G \to (\sim\sim J \,\&\, (\sim\sim O \,\&\, \sim\sim K))] \,\&\, E$
- This yields the following (valid!) sentential form:

  $G \to (\sim E \vee (\sim J \vee (\sim O \vee \sim K)))$

  $[G \to (\sim\sim J \,\&\, (\sim\sim O \,\&\, \sim\sim K))] \,\&\, E$

  $\therefore \sim G$

Symbolizing Arguments: Example #2 Notes

- The sentential form:

  $G \to (\sim E \vee (\sim J \vee (\sim O \vee \sim K)))$

  $[G \to (\sim\sim J \,\&\, (\sim\sim O \,\&\, \sim\sim K))]$

  E

  $\therefore \sim G$

  with *three* premises is *equivalent* to the *two*-premise sentential form we wrote down originally (why?).
- Alternative for premise #1: '$G \to \{\sim[\sim J \vee (\sim O \vee \sim K)] \to \sim E\}$'.
- Moreover, if we had written '$(\sim\sim K \,\&\, (\sim\sim J \,\&\, \sim\sim O))$' rather than '$(\sim\sim J \,\&\, (\sim\sim O \,\&\, \sim\sim K))$' in premise #2, we would have ended-up with yet another *equivalent* sentential form (why?).
- All of these forms capture the meaning of the premises and conclusion, and all are close to the given form. So, all are OK.

Symbolizing Arguments: Example #2 More Notes

- Premise #1: If God exists, then there is no evil in the world unless God is unjust, or not omnipotent, or not omniscient.
- Two Questions: ① Why render this as (*i*) $\ulcorner p \to (q \text{ unless } r)\urcorner$, as opposed to (*ii*) $\ulcorner (p \to q) \text{ unless } r\urcorner$? ② *Does it matter* (*semantically*)?

① First, there's no comma after 'world'. Second, (*i*) is probably *intended*. The second answer assumes (*i*) and (*ii*) are *not* equivalent *in English*.

- That *may* be right, but it's not clear. It presupposes two things:
  (1) *In English*, $\ulcorner q \text{ unless } r\urcorner$ is equivalent to $\ulcorner$If not r, then $q\urcorner$.
  (2) *In English*, $\ulcorner$If p, then (if q then r)$\urcorner$ [*i.e.*, $\ulcorner p \to (q \to r)\urcorner$] is *not* equivalent to $\ulcorner$If (p and q), then $r\urcorner$ [*i.e.*, $\ulcorner (p \,\&\, q) \to r\urcorner$].
- We're *assuming* (1) in this class. (2) is controversial (but defensible).

② In LSL, (*i*) and (*ii*) *are equivalent*, *i.e.*, *in LSL* (2) is *false*. Thus, it seems to me that both readings are probably OK. This is a subtle case.

Symbolizing Arguments: Example #3

If Yossarian flies his missions then he is putting himself in danger, and it is irrational to put oneself in danger. If Yossarian is rational he will ask to be grounded, and he will be grounded only if he asks. But only irrational people are grounded, and a request to be grounded is proof of rationality. Consequently, Yossarian will fly his missions whether he is rational or irrational.

- Basic Sentences: Yossarian flies his missions (F), Yossarian puts himself in danger (D), Yossarian is rational (R), Yosarian asks to be grounded (A).
- Premise #1: If F then D, and if D then not R. $[(F \to D) \,\&\, (D \to \sim R)]$
- Premise #2: If R then A, and not F only if A. $[(R \to A) \,\&\, (\sim F \to A)]$
- Premise #3: But not F only if not R, and A implies R. $[(\sim F \to \sim R) \,\&\, (A \to R)]$
- Conclusion: Consequently, F whether R or not R. $[(R \to F) \,\&\, (\sim R \to F)]$. [Alternatively, the conclusion could be symbolized as: '$(R \vee \sim R) \to F$']
- Note: this is a valid form (we'll be able to prove this pretty soon).

Symbolizing Arguments: Example #4

Suppose no two contestants enter; then there will be no contest. No contest means no winner. Suppose all contestants perform equally well. Still no winner. There won't be a winner unless there's a loser. And conversely. Therefore, there will be a loser only if at least two contestants enter and not all contestants perform equally well.

- Step 0: Decide on atomic sentences and letters.

  T: At least two contestants enter. C: There is a contest.
  E: All contestants perform equally well.
  W: There is a winner. L: There is a loser.

- Step 1: Identify (and symbolize) the *conclusion* of the argument:
  - Conclusion: There will be a loser only if at least two contestants enter and not all contestants perform equally well.
    - ∗ "Logish": L only if T and not E.
    - ∗ LSL: $L \rightarrow (T \,\&\, {\sim}E)$.

- Step 2: Symbolize the premises (here, there are as many as five):

(1) Suppose no two contestants enter; then there will be no contest.
  - · "Logish": Suppose that not T; then it is not the case that C.
  - · LSL: '${\sim}T \rightarrow {\sim}C$'.

(2) No contest means no winner.
  - · "Logish": Not C means not W. [*i.e.*, not C *implies* not W.]
  - · LSL: '${\sim}C \rightarrow {\sim}W$'.

(3) Suppose all contestants perform equally well. Still no winner.
  - · "Logish": Suppose E. Still not W. [*i.e.*, E *also* implies not W.]
  - · LSL: '$E \rightarrow {\sim}W$'.

(4) There won't be a winner unless there's a loser. And conversely.
  - · "Logish": Not W unless L, *and conversely*.
  - · LSL: '$({\sim}L \rightarrow {\sim}W) \,\&\, ({\sim}W \rightarrow {\sim}L)$'. [*i.e.*, not W *iff* not L.]

∗ The final product is the following *valid* sentential form:
${\sim}T \rightarrow {\sim}C$. ${\sim}C \rightarrow {\sim}W$. $E \rightarrow {\sim}W$. ${\sim}L \leftrightarrow {\sim}W$. Therefore, $L \rightarrow (T \,\&\, {\sim}E)$.