3 VALIDITY IN SENTENTIAL LOGIC

1. TAUTOLOGIES, CONTRADICTIONS, AND CONTINGENT FORMULAS

In Chapter 2 we saw how to construct the truth table for any formula in sentential logic. In doing the exercises, you may have noticed that in some cases the final (output) column has all T's, in other cases the final column has all F's, and in still other cases the final column has a mixture of T's and F's. There are special names for formulas with these particular sorts of truth tables, which are summarized in the following definitions.

> A formula $\mathcal{A}$ is a ***tautology***
> if and only if
> the truth table of $\mathcal{A}$ is such that
> every entry in the final column is T.

> A formula $\mathcal{A}$ is a ***contradiction***
> if and only if
> the truth table of $\mathcal{A}$ is such that
> every entry in the final column is F.

> A formula $\mathcal{A}$ is a ***contingent*** formula
> if and only if
> $\mathcal{A}$ is neither a tautology nor a contradiction.

The following are examples of each of these types of formulas.

A Tautology:

P	∨	~	P
T	T	F	T
F	T	T	F

A Contradiction:

P	&	~	P
T	F	F	T
F	F	T	F

A Contingent Formula:

P	→	~	P
T	F	F	T
F	T	T	F

In each example, the final column is shaded. In the first example, the final column consists entirely of T's, so the formula is a tautology; in the second example, the final column consists entirely of F's, so the formula is a contradiction; in the third example, the final column consists of a mixture of T's and F's, so the formula is contingent.

Given the above definitions, and given the truth table for negation, we have the following theorems.

> If a formula $\mathcal{A}$ is a tautology, then its negation $\sim\mathcal{A}$ is a contradiction.

> If a formula $\mathcal{A}$ is a contradiction, then its negation $\sim\mathcal{A}$ is a tautology.

> If a formula $\mathcal{A}$ is contingent, then its negation $\sim\mathcal{A}$ is also contingent.

By way of illustrating these theorems, we consider the three formulas cited earlier. In particular, we write down the truth tables for their negations.

$\sim$	(	P	$\vee$	$\sim$	P	)
F		T	T	F	T	
F		F	T	T	F	

$\sim$	(	P	&	$\sim$	P	)
T		T	F	F	T	
T		F	F	T	F	

$\sim$	(	P	$\rightarrow$	$\sim$	P	)
T		T	F	F	T	
F		F	T	T	F	

Once again, the final column of each formula is shaded; the first formula is a contradiction, the second is a tautology, the third is contingent.

2. IMPLICATION AND EQUIVALENCE

We can use the notion of tautology to define two very important notions in sentential logic, the notion of *implication*, and the notion of *equivalence*, which are defined as follows.

> Formula $\mathcal{A}$ ***logically implies*** formula $\mathcal{B}$
> if and only if
> the conditional formula $\mathcal{A}\rightarrow\mathcal{B}$ is a tautology.

> Formulas $\mathcal{A}$ and $\mathcal{B}$ are ***logically equivalent***
> if and only if
> the biconditional formula $\mathcal{A}\leftrightarrow\mathcal{B}$ is a tautology.

[Note: The above definitions apply specifically to sentential logic. A more general definition is required for other branches of logic. Once we have a more general definition, it is customary to refer to the special cases as *tautological implication* and *tautological equivalence*.]

Let us illustrate these concepts with a few examples. To begin with, we note that whereas the formula ~P logically implies the formula ~(P&Q), the converse is not true; i.e., ~(P&Q) does not logically imply ~P). This can be shown by constructing truth tables for the associated pair of conditionals. In particular, the question whether ~P implies ~(P&Q) reduces to the question whether the formula ~P→~(P&Q) is a tautology. The following is the truth table for this formula.

~	P	→	~	(	P	&	Q	)
F	T	T	F		T	T	T	
F	T	T	T		T	F	F	
T	F	T	T		F	F	T	
T	F	T	T		F	F	F	

Notice that the conditional ~P→~(P&Q) is a tautology, so we conclude that its antecedent logically implies its consequent; that is, ~P logically implies ~(P&Q).

Considering the converse implication, the question whether ~(P&Q) logically implies ~P reduces to the question whether the conditional formula ~(P&Q)→~P is a tautology. The truth table follows.

~	(	P	&	Q	)	→	~	P
F		T	T	T		T	F	T
T		T	F	F		F	F	T
T		F	F	T		T	T	F
T		F	F	F		T	T	F

The formula is false in the second case, so it is not a tautology. We conclude that its antecedent does not imply its consequent; that is, ~(P&Q) does not imply ~P.

Next, we turn to logical equivalence. As our first example, we ask whether ~(P&Q) and ~P&~Q are logically equivalent. According to the definition of logical equivalence, this reduces to the question whether the biconditional formula ~(P&Q)↔(~P&~Q) is a tautology. Its truth table is given as follows.

~	(	P	&	Q	)	↔	(	~	P	&	~	Q	)
F		T	T	T		T		F	T	F	F	T	
T		T	F	F		F		F	T	F	T	F	
T		F	F	T		F		T	F	F	F	T	
T		F	F	F		T		T	F	T	T	F	
*										*			

In this table, the truth value of the biconditional is shaded, whereas the constituents are marked by '*'. Notice that the biconditional is false in cases 2 and 3, so it is not a tautology. We conclude that the two constituents – ~(P&Q) and ~P&~Q – are *not* logically equivalent.

As our second example, we ask whether ~(P&Q) and ~P∨~Q are logically equivalent. As before, this reduces to the question whether the biconditional formula ~(P&Q)↔(~P∨~Q) is a tautology. Its truth table is given as follows.

~	(	P	&	Q	)	↔	(	~	P	∨	~	Q	)
F		T	T	T		T		F	T	F	F	T	
T		T	F	F		T		F	T	T	T	F	
T		F	F	T		T		T	F	T	F	T	
T		F	F	F		T		T	F	T	T	F	
*										*			

Once again, the biconditional is shaded, and the constituents are marked by '*'. Comparing the two *-columns, we see they are the same in every case; accordingly, the shaded column is true in every case, which is to say that the biconditional formula is a tautology. We conclude that the two constituents – ~(P&Q) and ~P∨~Q – are logically equivalent.

We conclude this section by citing a theorem about the relation between implication and equivalence.

> Formulas $\mathcal{A}$ and $\mathcal{B}$ are logically equivalent
> if and only if
> $\mathcal{A}$ logically implies $\mathcal{B}$
> and
> $\mathcal{B}$ logically implies $\mathcal{A}$.

This follows from the fact that $\mathcal{A}\leftrightarrow\mathcal{B}$ is logically equivalent to $(\mathcal{A}\rightarrow\mathcal{B})\&(\mathcal{B}\rightarrow\mathcal{A})$, and the fact that two formulas $\mathcal{A}$ and $\mathcal{B}$ are tautologies if and only if the conjunction $\mathcal{A}\&\mathcal{B}$ is a tautology.

3. VALIDITY IN SENTENTIAL LOGIC

Recall that an argument is valid if and only if it is impossible for the premises to be true while the conclusion is false; equivalently, it is impossible for the premises to be true without the conclusion also being true. Possibility and impossibility are difficult to judge in general. However, in case of sentential logic, we may judge them by reference to truth tables. This is based on the following definition of 'impossible', relative to logic.

> To say that it is ***impossible*** that S is to say that there is **no case** in which S.

Here, ø is any statement. the sort of statement we are interested in is the following.

S: the premises of argument $\mathcal{A}$ are all true, and the conclusion is false.

Substituting this statement for S in the above definition, we obtain the following.

> To say that it is impossible that {the premises of argument A are all true, and the conclusion is false} is to say that there is no case in which {the premises of argument A are all true, and the conclusion is false}.

This is slightly complicated, but it is the basis for defining validity in sentential logic. The following is the resulting definition.

> An argument $\mathcal{A}$ is ***valid***
> if and only if
> there is **no case** in which
> the premises are true
> and the conclusion is false.

This definition is acceptable provided that we know what "cases" are. This term has already arisen in the previous chapter. In the following, we provide the official definition.

> The ***cases*** relevant to an argument $\mathcal{A}$ are precisely all the possible combinations of truth values that can be assigned to the atomic formulas (P, Q, R, etc.), as a group, that constitute the argument.

By way of illustration, consider the following sentential argument form.

Example 1

(a1) $P \to Q$
$\sim Q$
$/ \sim P$

In this argument form, there are two atomic formulas – P, Q – so the possible cases relevant to (a1) consist of all the possible combinations of truth values that can be assigned to P and Q. These are enumerated as follows.

	P	Q
case1	T	T
case2	T	F
case3	F	T
case4	F	F

As a further illustration, consider the following sentential argument form, which involves three atomic formulas – P, Q, R.

Example 2

(a2) P → Q
Q → R
/ P → R

The possible combinations of truth values that can be assigned to P, Q, R are given as follows.

	P	Q	R
case1	T	T	T
case2	T	T	F
case3	T	F	T
case4	T	F	F
case5	F	T	T
case6	F	T	F
case7	F	F	T
case8	F	F	F

Notice that in constructing this table, the T's and F's are alternated in quadruples in the P column, in pairs in the Q column, and singly in the R column. Also notice that, in general, if there are n atomic formulas, then there are 2^n cases.

4. TESTING ARGUMENTS IN SENTENTIAL LOGIC

In the previous section, we noted that an argument is valid if and only if there is *no case* in which the premises are true and the conclusion is false. We also noted that the cases in sentential logic are the possible combinations of truth values that can be assigned to the atomic formulas (letters) in an argument.

In the present section, we use these ideas to test sentential argument forms for validity and invalidity.

The first thing we do is adopt a new method of displaying argument forms. Our present method is to display arguments in vertical lists, where the conclusion is at the bottom. In combination with truth tables, this is inconvenient, so we will henceforth write argument forms in horizontal lists. For example, the argument forms from earlier may be displayed as follows.

(a1) P → Q ; ~Q / ~P
(a2) P → Q ; Q → R / P → R

In (a1) and (a2), the premises are separated by a semi-colon (;), and the conclusion is marked of by a forward slash (/). If there are three premises, then they are separated by two semi-colons; if there are four premises, then they are separated by three semi-colons, etc.

Using our new method of displaying argument forms, we can form *multiple truth tables*. Basically, a multiple truth table is a collection of truth tables that all use the same guide table. This may be illustrated in reference to argument form (a1).

GuideTable: **Argument:**

	P	Q	P	→	Q	;	~	Q	/	~	P
case 1	T	T	T	T	T		F	T		F	T
case 2	T	F	T	F	F		T	F		F	T
case 3	F	T	F	T	T		F	T		T	F
case 4	F	F	F	T	F		T	F		T	F

In the above table, the three formulas of the argument are written side by side, and their truth tables are placed beneath them. In each case, the final (output) column is shaded. Notice the following. If we were going to construct the truth table for ~Q *by itself*, then there would only be two cases to consider. But in relation to the whole collection of formulas, in which there are two atomic formulas – P and Q – there are four cases to consider in all. This is a property of multiple truth tables that makes them different from individual truth tables. Nevertheless, we can look at a multiple truth table simply as a set of several truth tables all put together. So in the above case, there are three truth tables, one for each formula, which all use the same guide table.

The above collection of formulas is not merely a collection; it is also an argument (form). So we can ask whether it is valid or invalid. According to our definition an argument is valid if and only if there is no case in which the premises are all true but the conclusion is false.

Let's examine the above (multiple) truth table to see whether there are any cases in which the premises are both true and the conclusion is false. The starred columns are the only columns of interest at this point, so we simply extract them to form the following table.

	P	Q	P→Q	;	~Q	/	~P
case 1	T	T	T		F		F
case 2	T	F	F		T		F
case 3	F	T	T		F		T
case 4	F	F	T		T		T

In cases 1 through 3, one of the premises is false, so they won't do. In case 4, both the premises are true, but the conclusion is also true, so this case won't do either. Thus, there is no case in which the premises are all true and the conclusion is false. To state things equivalently, every case in which the premises are all true is also a case in which the conclusion is true. On the basis of this, we conclude that argument (a1) is *valid*.

Whereas argument (a1) is valid, the following similar looking argument (form) is *not* valid.

(a3) $P \rightarrow Q$
$\sim P$
$/ \sim Q$

The following is a concrete argument with this form.

(c3) if Bush is president, then the president is a U.S. citizen;
Bush is *not* president;
/ the president is *not* a U.S. citizen.

Observe that (c3) as the form (a3), that (c3) has all true premises, that (c3) has a false conclusion. In other words, (c3) is a counterexample to (a3); indeed, (c3) is a counterexample to any argument with the same form. It follows that (a3) is *not* valid; it is *invalid*.

This is one way to show that (a3) is invalid. We can also show that it is invalid using truth tables. To show that (a3) is invalid, we show that there is a case (line) in which the premises are both true but the conclusion is false. The following is the (multiple) truth table for argument (a3).

	P	Q	P	→	Q	;	~	P	/	~	Q
case 1	T	T	T	T	T		F	T		F	T
case 2	T	F	T	F	F		F	T		T	F
case 3	F	T	F	$\boxed{T}$	T		$\boxed{T}$	F		$\boxed{F}$	T
case 4	F	F	F	T	F		T	F		T	F

In deciding whether the argument form is valid or invalid, we look for a case in which the premises are all true and the conclusion is false. In the above truth table, cases 1 and 2 do not fill the bill, since the premises are not both true. In case 4, the premises are both true, but the conclusion is also true, so case 4 doesn't fill the bill either. On the other hand, in case 3 the premises are both true, and the conclusion is false. Thus, there is a case in which the premises are all true and the conclusion is false (namely, the 3rd case). On this basis, we conclude that argument (a3) is *invalid*.

Note carefully that case 3 in the above truth table demonstrates that argument (a3) is *invalid*; one case is all that is needed to show invalidity. But this is *not* to say that the argument is valid in the other three cases. This does not make any sense, for the notions of validity and invalidity do not apply to the individual cases, but to all the cases taken all together.

Having considered a couple of simple examples, let us now examine a couple of examples that are somewhat more complicated.

	P	Q		P	→	(	~	P	∨	Q	)	;	~	P	→	Q	;	Q	→	P	/	P	&	Q
1	T	T		T	T		F	T	T	T			F	T	T	T		T	T	T		T	T	T
2	T	F		T	F		F	T	F	F			F	T	T	F		F	T	T		T	F	F
3	F	T		F	T		T	F	T	T			T	F	T	T		T	F	F		F	F	T
4	F	F		F	T		T	F	T	F			T	F	F	F		F	T	F		F	F	F

In this example, the argument has three premises, but it only involves two atomic formulas (P, Q), so there are four cases to consider. What we are looking for is at least one case in which the premises are all true and the conclusion is false. As usual the final (output) columns are shaded, and these are the only columns that interest us. If we extract them from the above table, we obtain the following.

	P	Q	P→(~P∨Q)	;	~P→Q	;	Q→P	/	P&Q
1	T	T	T		T		T		T
2	T	F	F		T		T		F
3	F	T	T		T		F		F
4	F	F	T		F		T		F

In case 1, the premises are all true, but so is the conclusion. In each of the remaining cases (2-4), the conclusion is false, but in each of these cases, at least one premise is also false. Thus, there is no case in which the premises are all true and the conclusion is false. From this we conclude that the argument is *valid*.

The final example we consider is an argument that involves three atomic formulas (letters). There are accordingly 8 cases to consider, not just four as in previous examples.

	P	Q	R		P	∨	(	Q	→	R	)	;	P	→	~	R	/	~	(	Q	&	~	R	)
1	T	T	T		T	T		T	T	T			T	F	F	T		T		T	F	F	T	
2	T	T	F		T	T		T	F	F			T	T	T	F		F		T	T	T	F	
3	T	F	T		T	T		F	T	T			T	F	F	T		T		F	F	F	T	
4	T	F	F		T	T		F	T	F			T	T	T	F		T		F	F	T	F	
5	F	T	T		F	T		T	T	T			F	T	F	T		T		T	F	F	T	
6	F	T	F		F	F		T	F	F			F	T	T	F		F		T	T	T	F	
7	F	F	T		F	T		F	T	T			F	T	F	T		T		F	F	F	T	
8	F	F	F		F	T		F	T	F			F	T	T	F		T		F	F	T	F	

As usual, the shaded columns are the ones that we are interested in as far as deciding the validity or invalidity of this argument. We are looking for a case in which the premises are all true and the conclusion is false. So in particular, we are looking for a case in which the conclusion is false. There are only two such cases – case 2 and case 6; the remaining question is whether the premises both true in either of these cases. In case 6, the first premise is false, but in case 2, the premises are both true. This is exactly what we are looking for – a case with all true premises and a false conclusion. Since such a case exists, as shown by the above truth table, we conclude that the argument is *invalid*.

5. THE RELATION BETWEEN VALIDITY AND IMPLICATION

Let us begin this section by recalling some earlier definitions. In Section 1, we noted that a formula $\mathcal{A}$ is a tautology if and only if it is *true in every case*. We can describe this by saying that a tautology is a formula that is true no matter what. By contrast, a *contradiction* is a formula that is false in every case, or false no matter what. Between these two extremes contingent formulas, which are true under some circumstances but false under others.

Next, in Section 2, we noted that a formula $\mathcal{A}$ logically *implies* (or simply implies) a formula $\mathcal{B}$ if and only if the conditional formula $\mathcal{A}\rightarrow\mathcal{B}$ is a tautology.

The notion of implication is intimately associated with the notion of validity. This may be illustrated first using the simplest example – an argument with just one premise. Consider the following argument form.

(a1) ~P / ~(P&Q)

You might read this as saying that: it is not true that P; *so* it is not true that P&Q. On the other hand, consider the conditional formed by taking the premise as the antecedent, and the conclusion as the consequent.

(c1) ~P → ~(P&Q)

As far as the symbols are concerned, all we have done is to replace the '/' by '→'. The resulting conditional may be read as saying that: *if* it is not true that P, *then* it is not true that P&Q.

There seems to be a natural relation between (a1) and (c1), though it is clearly not the relation of identity. Whereas (a1) is a pair of formulas, (c1) is a single formula. Nevertheless they are intimately related, as can be seen by constructing the respective truth tables.

	P	Q	~	P	/	~	(P	&	Q)		~	P	→	~	(P	&	Q)
1	T	T	F	T		F	T	T	T		F	T	T	F	T	T	T
2	T	F	F	T		T	T	F	F		F	T	T	T	T	F	F
3	F	T	T	F		T	F	F	T		T	F	T	T	F	F	T
4	F	F	T	F		T	F	F	F		T	F	T	T	F	F	F

We now have two truth tables side by side, one for the argument ~P/~(P&Q), the other for the conditional ~P→~(P&Q).

Let's look at the conditional first. The third column is the final (output) column, and it has all T's, so we conclude that this formula is a tautology. In other words, no matter what, if it is not true that P, then it is not true that P&Q.

This is reflected in the corresponding argument to the left. In looking for a case that serves as a counterexample, we notice that every case in which the premise is true so is the conclusion. Thus, the argument is valid.

This can be stated as a general principle.

> Argument P/C is valid
> if and only if
> the conditional formula $P \rightarrow C$ is a tautology.

Since, by definition, a formula P implies a formula C if and only if the conditional $P \rightarrow C$ is a tautology, this principle can be restated as follows.

> Argument P/C is valid
> if and only if
> the premise P logically implies the conclusion C.

In order to demonstrate the truth of this principle, we can argue as follows. Suppose that the argument P/C is *not* valid. Then there is a case (call it case n) in which P is true but C is false. Consequently, in the corresponding truth table for the conditional $P \rightarrow C$, there is a case (namely, case n) in which P is true and C is false. Accordingly, in case n, the truth value of $P \rightarrow C$ is $T \rightarrow F$, i.e.,, F. It follows that $P \rightarrow C$ is *not* a tautology, so P does *not* imply C.

This demonstrates that if P/C is *not* valid, then $P \rightarrow C$ is *not* a tautology. We also have to show the converse conditional: if $P \rightarrow C$ is *not* a tautology, then P/C is *not* valid. Well, suppose that $P \rightarrow C$ isn't a tautology. Then there is a case in which $P \rightarrow C$ is false. But a conditional is false if and only if its antecedent is true and its consequent is false. So there is a case in which P is true but C is false. It immediately follows that P/C is not valid. This completes our argument.

[Note: What we have in fact demonstrated is this: the argument P/C is *not* valid *if and only if* the conditional $P \rightarrow C$ is *not* a tautology. This statement has the form: $\sim V \leftrightarrow \sim T$. The student should convince him(her)self that $\sim V \leftrightarrow \sim T$ is *equivalent* to $V \leftrightarrow T$, which is to say that $(\sim V \leftrightarrow \sim T) \leftrightarrow (V \leftrightarrow T)$ is a tautology.]

The above principle about validity and implication is not particularly useful because not many arguments have just one premise. It would be nice if there were a comparable principle that applied to arguments with two premises, arguments with three premises, in general to *all* arguments. There is such a principle.

What we have to do is to form a single formula out of an argument irrespective of how many premises it has. The particular formula we use begins with the premises, next forms a conjunction out of all these, next takes this conjunction and makes a conditional with it as the antecedent and the conclusion as the consequent. The following examples illustrate this technique.

	Argument	Associated conditional:
(1)	P_1; P_2 / C	$(P_1 \& P_2) \rightarrow C$
(2)	P_1; P_2; P_3 / C	$(P_1 \& P_2 \& P_3) \rightarrow C$
(3)	P_1; P_2; P_3; P_4 / C	$(P_1 \& P_2 \& P_3 \& P_4) \rightarrow C$

In each case, we take the argument, first conjoin the premises, and then form the conditional with this conjunction as its antecedent and with the conclusion as its consequent. Notice that the above formulas are not strictly speaking formulas, since the parentheses are missing in connection with the ampersands. The removal of the extraneous parentheses is comparable to writing ‘x+y+z+w’ in place of the strictly correct ‘((x+y)+z)+z’.

Having described how to construct a conditional formula on the basis of an argument, we can now state the principle that relates these two notions.

An argument $\mathcal{A}$ is **valid**
if and only if
the associated conditional is a **tautology**.

In virtue of the relation between implication and tautologies, this principle can be restated as follows.

Argument $P_1;P_2;...P_n/C$ is **valid**
if and only if
the conjunction $P_1\&P_2\&...\&P_n$
logically implies
the conclusion C.

The interested reader should try to convince him(her)self that this principle is true, at least in the case of two premises. The argument proceeds like the earlier one, except that one has to take into account the truth table for conjunction (in particular, P&Q can be true only if both P and Q are true).

6. EXERCISES FOR CHAPTER 3

EXERCISE SET A

Go back to Exercise Set 2C in Chapter 2. For each formula, say whether it is a **tautology**, a **contradiction**, or a **contingent** formula.

EXERCISE SET B

In each of the following, you are given a pair generically denoted $\mathcal{A}$, $\mathcal{B}$. In each case, answer the following questions:

(1) Does $\mathcal{A}$ **logically imply** $\mathcal{B}$?
(2) Does $\mathcal{B}$ **logically imply** $\mathcal{A}$?
(3) Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent?

1. $\mathcal{A}$: ~(P&Q)
   $\mathcal{B}$: ~P&~Q
2. $\mathcal{A}$: ~(P&Q)
   $\mathcal{B}$: ~P∨~Q
3. $\mathcal{A}$: ~(P∨Q)
   $\mathcal{B}$: ~P∨~Q
4. $\mathcal{A}$: ~(P∨Q)
   $\mathcal{B}$: ~P&~Q
5. $\mathcal{A}$: ~(P→Q)
   $\mathcal{B}$: ~P→~Q
6. $\mathcal{A}$: ~(P→Q)
   $\mathcal{B}$: P&~Q
7. $\mathcal{A}$: ~(P↔Q)
   $\mathcal{B}$: ~P↔~Q
8. $\mathcal{A}$: ~(P↔Q)
   $\mathcal{B}$: P↔~Q
9. $\mathcal{A}$: ~(P↔Q)
   $\mathcal{B}$: ~P↔Q
10. $\mathcal{A}$: P↔Q
    $\mathcal{B}$: (P&Q) & (Q→P)
11. $\mathcal{A}$: P↔Q
    $\mathcal{B}$: (P→Q) & (Q→P)
12. $\mathcal{A}$: P→Q
    $\mathcal{B}$: Q→P
13. $\mathcal{A}$: P→Q
    $\mathcal{B}$: ~P→~Q
14. $\mathcal{A}$: P→Q
    $\mathcal{B}$: ~Q→~P
15. $\mathcal{A}$: P→Q
    $\mathcal{B}$: ~P∨Q
16. $\mathcal{A}$: P→Q
    $\mathcal{B}$: ~(P&~Q)
17. $\mathcal{A}$: ~P
    $\mathcal{B}$: ~(P&Q)
18. $\mathcal{A}$: ~P
    $\mathcal{B}$: ~(P∨Q)
19. $\mathcal{A}$: ~(P↔Q)
    $\mathcal{B}$: (P&Q) → R
20. $\mathcal{A}$: (P&Q) → R
    $\mathcal{B}$: P→R
21. $\mathcal{A}$: (P∨Q) → R
    $\mathcal{B}$: P→R
22. $\mathcal{A}$: (P&Q)→R
    $\mathcal{B}$: P → (Q→R)
23. $\mathcal{A}$: P → (Q&R)
    $\mathcal{B}$: P→Q
24. $\mathcal{A}$: P → (Q∨R)
    $\mathcal{B}$: P→Q

EXERCISE SET C

In each of the following, you are given an argument form from sentential logic, splayed horizontally. In each case, use the method of truth tables to decide whether the argument form is **valid** or **invalid**. Explain your answer.

1. P→Q; P / Q
2. P→Q; Q / P
3. P→Q; ~Q / ~P
4. P→Q; ~P / ~Q
5. P∨Q; ~P / Q
6. P∨Q; P / ~Q
7. ~(P&Q); P / ~Q
8. ~(P&Q); ~P / Q
9. P↔Q; ~P / ~Q
10. P↔Q; Q / P
11. P∨Q; P→Q / Q
12. P∨Q; P→Q / P&Q
13. P→Q; P→~Q / ~P
14. P→Q; ~P→Q / Q
15. P∨Q; ~P→~Q / P&Q
16. P→Q; ~P→~Q / P↔Q
17. ~P→~Q; ~Q→~P / P↔Q
18. ~P→~Q; ~Q→~P / P&Q
19. P∨~Q; P∨Q / P
20. P→Q; P∨Q / P↔Q
21. ~(P→Q); P→~P / ~P&~Q
22. ~(P&Q); ~Q→P / P
23. P→Q; Q→R / P→R
24. P→Q; Q→R; ~P→R / R
25. P→Q; Q→R / P&R
26. P→Q; Q→R; R→P / P↔R
27. P→Q; Q→R / R
28. P→R; Q→R / (P∨Q)→R
29. P→Q; P→R / Q&R
30. P∨Q; P→R; Q→R / R

31. $P\rightarrow Q$; $Q\rightarrow R$; $R\rightarrow\sim P$ / $\sim P$

32. $P\rightarrow(Q\vee R)$; $Q\&R$ / $\sim P$

33. $P\rightarrow(Q\&R)$; $Q\rightarrow\sim R$ / $\sim P$

34. $P\&(Q\vee R)$; $P\rightarrow\sim Q$ / R

35. $P\rightarrow(Q\rightarrow R)$; $P\&\sim R$ / $\sim Q$

36. $\sim P\vee Q$; $R\rightarrow P$; $\sim(Q\&R)$ / $\sim R$

EXERCISE SET D

Go back to Exercise Set B. In each case, consider the argument $\mathcal{A}/\mathcal{B}$, as well as the converse argument $\mathcal{B}/\mathcal{A}$. Thus, there are a total of 48 arguments to consider. On the basis of your answers for Exercise Set B, decide which of these arguments are valid and which are invalid.

7. ANSWERS TO EXERCISES FOR CHAPTER 3

EXERCISE SET A

1. contingent
2. tautology
3. contradiction
4. contingent
5. contingent
6. contingent
7. contingent
8. tautology
9. contradiction
10. tautology
11. contradiction
12. contingent
13. tautology
14. tautology
15. contradiction
16. contingent
17. tautology
18. contingent
19. contingent
20. tautology
21. contingent
22. contingent
23. contingent
24. tautology
25. tautology

EXERCISE SET B

#1.

$\mathcal{A}$: | **$\mathcal{B}$:**

~	(P	&	Q)
F	T	T	T
T	T	F	F
T	F	F	T
T	F	F	F

~	P	&	~	Q
F	T	F	F	T
F	T	F	T	F
T	F	F	F	T
T	F	T	T	F

$\mathcal{A}$	→	$\mathcal{B}$
F	T	F
T	F	F
T	F	F
T	T	T

$\mathcal{B}$	→	$\mathcal{A}$
F	T	F
F	T	T
F	T	T
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? NO
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#2.

$\mathcal{A}$: | **$\mathcal{B}$:**

~	(P	&	Q)
F	T	T	T
T	T	F	F
T	F	F	T
T	F	F	F

~	P	∨	~	Q
F	T	F	F	T
F	T	T	T	F
T	F	T	F	T
T	F	T	T	F

$\mathcal{A}$	→	$\mathcal{B}$
F	T	F
T	T	T
T	T	T
T	T	T

$\mathcal{B}$	→	$\mathcal{A}$
F	T	F
T	T	T
T	T	T
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? YES

#3.

$\mathcal{A}$: | **$\mathcal{B}$:**

~	(P	∨	Q)
F	T	T	T
F	T	T	F
F	F	T	T
T	F	F	F

~	P	∨	~	Q
F	T	F	F	T
F	T	T	T	F
T	F	T	F	T
T	F	T	T	F

$\mathcal{A}$	→	$\mathcal{B}$
F	T	F
F	T	T
F	T	T
T	T	T

$\mathcal{B}$	→	$\mathcal{A}$
F	T	F
T	F	F
T	F	F
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? NO
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#4.

$\mathcal{A}$: | **$\mathcal{B}$:**

~	(P	∨	Q)
F	T	T	T
F	T	T	F
F	F	T	T
T	F	F	F

~	P	&	~	Q
F	T	F	F	T
F	T	F	T	F
T	F	F	F	T
T	F	T	T	F

$\mathcal{A}$	→	$\mathcal{B}$
F	T	F
F	T	F
F	T	F
T	T	T

$\mathcal{B}$	→	$\mathcal{A}$
F	T	F
F	T	F
F	T	F
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? YES

#5.

$\mathcal{A}$: | $\mathcal{B}$:

~	(	P	→	Q	)
F		T	T	T	
T		T	F	F	
F		F	T	T	
F		F	T	F	

~	P	→	~	Q
F	T	T	F	T
F	T	T	T	F
T	F	F	F	T
T	F	T	T	F

$\mathcal{A}$	→	$\mathcal{B}$
F	T	T
T	T	T
F	T	F
F	T	T

$\mathcal{B}$	→	$\mathcal{A}$
T	$\boxed{F}$	F
T	T	T
F	T	F
T	$\boxed{F}$	F

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? NO
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#6.

$\mathcal{A}$: | $\mathcal{B}$:

~	(	P	→	Q	)
F		T	T	T	
T		T	F	F	
F		F	T	T	
F		F	T	F	

P	&	~	Q
T	F	F	T
T	T	T	F
F	F	F	T
F	F	T	F

$\mathcal{A}$	→	$\mathcal{B}$
F	T	F
T	T	T
F	T	F
F	T	F

$\mathcal{B}$	→	$\mathcal{A}$
F	T	F
T	T	T
F	T	F
F	T	F

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? YES

#7.

$\mathcal{A}$: | $\mathcal{B}$:

~	(	P	↔	Q	)
F		T	T	T	
T		T	F	F	
T		F	F	T	
F		F	T	F	

~	P	↔	~	Q
F	T	T	F	T
F	T	F	T	F
T	F	F	F	T
T	F	T	T	F

$\mathcal{A}$	→	$\mathcal{B}$
F	T	T
T	$\boxed{F}$	F
T	$\boxed{F}$	F
F	T	T

$\mathcal{B}$	→	$\mathcal{A}$
T	$\boxed{F}$	F
F	T	T
F	T	F
T	$\boxed{F}$	F

Does $\mathcal{A}$ logically imply $\mathcal{B}$? NO
Does $\mathcal{B}$ logically imply $\mathcal{A}$? NO
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#8.

$\mathcal{A}$: | $\mathcal{B}$:

~	(	P	↔	Q	)
F		T	T	T	
T		T	F	F	
T		F	F	T	
F		F	T	F	

P	↔	~	Q
T	F	F	T
T	T	T	F
F	T	F	T
F	F	T	F

$\mathcal{A}$	→	$\mathcal{B}$
F	T	F
T	T	T
T	T	T
F	T	F

$\mathcal{B}$	→	$\mathcal{A}$
F	T	F
T	T	T
T	T	T
F	T	F

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? YES

#9.

$\mathcal{A}$:

~	(	P	↔	Q	)
F		T	T	T	
T		T	F	F	
T		F	F	T	
F		F	T	F	

$\mathcal{B}$:

~	P	↔	Q
F	T	F	T
F	T	T	F
T	F	T	T
T	F	F	F

$\mathcal{A}$	→	$\mathcal{B}$
F	T	F
T	T	T
T	T	T
F	T	F

$\mathcal{B}$	→	$\mathcal{A}$
F	T	F
T	T	T
T	T	T
F	T	F

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? YES

#10.

$\mathcal{A}$:

P	↔	Q
T	T	T
T	F	F
F	F	T
F	T	F

$\mathcal{B}$:

(	P	&	Q	)	&	(	Q	→	P	)
	T	T	T		T		T	T	T	
	T	F	F		F		F	T	T	
	F	F	T		F		T	F	F	
	F	F	F		F		F	T	F	

$\mathcal{A}$	→	$\mathcal{B}$
T	T	T
F	T	F
F	T	F
T	$\boxed{F}$	F

$\mathcal{B}$	→	$\mathcal{A}$
T	T	T
F	T	F
F	T	F
F	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? NO
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#11.

$\mathcal{A}$:

P	↔	Q
T	T	T
T	F	F
F	F	T
F	T	F

$\mathcal{B}$:

(	P	→	Q	)	&	(	Q	→	P	)
	T	T	T		T		T	T	T	
	T	F	F		F		F	T	T	
	F	T	T		F		T	F	F	
	F	T	F		T		F	T	F	

$\mathcal{A}$	→	$\mathcal{B}$
T	T	T
F	T	F
F	T	F
T	T	T

$\mathcal{B}$	→	$\mathcal{A}$
T	T	T
F	T	F
F	T	F
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? YES

#12.

$\mathcal{A}$:

P	→	Q
T	T	T
T	F	F
F	T	T
F	T	F

$\mathcal{B}$:

Q	→	P
T	T	T
F	T	T
T	F	F
F	T	F

$\mathcal{A}$	→	$\mathcal{B}$
T	T	T
F	T	T
T	$\boxed{F}$	F
T	T	T

$\mathcal{B}$	→	$\mathcal{A}$
T	T	T
T	$\boxed{F}$	F
F	T	F
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? NO
Does $\mathcal{B}$ logically imply $\mathcal{A}$? NO
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#13.

$\mathcal{A}$:

P	→	Q
T	T	T
T	F	F
F	T	T
F	T	F

$\mathcal{B}$:

~	P	→	~	Q
F	T	T	F	T
F	T	T	T	F
T	F	F	F	T
T	F	T	T	F

$\mathcal{A}$	→	$\mathcal{B}$
T	T	T
F	T	T
T	$\boxed{\text{F}}$	F
T	T	T

$\mathcal{B}$	→	$\mathcal{A}$
T	T	T
T	$\boxed{\text{F}}$	F
F	T	T
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? NO
Does $\mathcal{B}$ logically imply $\mathcal{A}$? NO
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#14.

$\mathcal{A}$:

P	→	Q
T	T	T
T	F	F
F	T	T
F	T	F

$\mathcal{B}$:

~	Q	→	~	P
F	T	T	F	T
T	F	F	F	T
F	T	T	T	F
T	F	T	T	F

$\mathcal{A}$	→	$\mathcal{B}$
T	T	T
F	T	F
T	T	T
T	T	T

$\mathcal{B}$	→	$\mathcal{A}$
T	T	T
F	T	F
T	T	T
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? YES

#15.

$\mathcal{A}$:

P	→	Q
T	T	T
T	F	F
F	T	T
F	T	F

$\mathcal{B}$:

~	P	∨	Q
F	T	T	T
F	T	F	F
T	F	T	T
T	F	T	F

$\mathcal{A}$	→	$\mathcal{B}$
T	T	T
F	T	F
T	T	T
T	T	T

$\mathcal{B}$	→	$\mathcal{A}$
T	T	T
F	T	F
T	T	T
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? YES

#16.

$\mathcal{A}$:

P	→	Q
T	T	T
T	F	F
F	T	T
F	T	F

$\mathcal{B}$:

~	(	P	&	~	Q	)
T		T	F	F	T	
F		T	T	T	F	
T		F	F	F	T	
T		F	F	T	F	

$\mathcal{A}$	→	$\mathcal{B}$
T	T	T
F	T	F
T	T	T
T	T	T

$\mathcal{B}$	→	$\mathcal{A}$
T	T	T
F	T	F
T	T	T
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? YES

#17.

$\mathcal{A}$: ~ P

~	P
F	T
F	T
T	F
T	F

$\mathcal{B}$: ~ (P & Q)

~	(	P	&	Q	)
F		T	T	T	
T		T	F	F	
T		F	F	T	
T		F	F	F	

$\mathcal{A} \rightarrow \mathcal{B}$

$\mathcal{A}$	$\rightarrow$	$\mathcal{B}$
F	T	F
F	T	T
T	T	T
T	T	T

$\mathcal{B} \rightarrow \mathcal{A}$

$\mathcal{B}$	$\rightarrow$	$\mathcal{A}$
F	T	F
T	$\boxed{F}$	F
T	T	T
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? NO
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#18.

$\mathcal{A}$: ~ P

~	P
F	T
F	T
T	F
T	F

$\mathcal{B}$: ~ (P ∨ Q)

~	(	P	∨	Q	)
F		T	T	T	
F		T	T	F	
F		F	T	T	
T		F	F	F	

$\mathcal{A} \rightarrow \mathcal{B}$

$\mathcal{A}$	$\rightarrow$	$\mathcal{B}$
F	T	F
F	T	F
T	$\boxed{F}$	F
T	T	T

$\mathcal{B} \rightarrow \mathcal{A}$

$\mathcal{B}$	$\rightarrow$	$\mathcal{A}$
F	T	F
F	T	F
F	T	T
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? NO
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#19.

$\mathcal{A}$: ~ (P ↔ Q)

~	(	P	↔	Q	)
F		T	T	T	
F		T	T	T	
T		T	F	F	
T		T	F	F	
T		F	F	T	
T		F	F	T	
F		F	T	F	
F		F	T	F	

$\mathcal{B}$: (P & Q) → R

(	P	&	Q	)	→	R
	T	T	T		T	T
	T	T	T		F	F
	T	F	F		T	T
	T	F	F		T	F
	F	F	T		T	T
	F	F	T		T	F
	F	F	F		T	T
	F	F	F		T	F

$\mathcal{A} \rightarrow \mathcal{B}$

$\mathcal{A}$	$\rightarrow$	$\mathcal{B}$
F	T	T
F	T	F
T	T	T
T	T	T
T	T	T
T	T	T
F	T	T
F	T	T

$\mathcal{B} \rightarrow \mathcal{A}$

$\mathcal{B}$	$\rightarrow$	$\mathcal{A}$
T	$\boxed{F}$	F
F	T	F
T	T	T
T	T	T
T	T	T
T	T	T
T	$\boxed{F}$	F
T	$\boxed{F}$	F

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? NO
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#20.

$\mathcal{A}$: $(P \,\&\, Q) \to R$

P	&	Q	→	R
T	T	T	T	T
T	T	T	F	F
T	F	F	T	T
T	F	F	T	F
F	F	T	T	T
F	F	T	T	F
F	F	F	T	T
F	F	F	T	F

$\mathcal{B}$: $P \to R$

P	→	R
T	T	T
T	F	F
T	T	T
T	F	F
F	T	T
F	T	F
F	T	T
F	T	F

$\mathcal{A} \to \mathcal{B}$

$\mathcal{A}$	→	$\mathcal{B}$
T	T	T
F	T	F
T	T	T
T	$\boxed{F}$	F
T	T	T
T	T	T
T	T	T
T	T	T

$\mathcal{B} \to \mathcal{A}$

$\mathcal{B}$	→	$\mathcal{A}$
T	F	T
F	T	F
T	T	T
F	T	T
T	T	T
T	T	T
T	T	T
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? NO
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#21.

$\mathcal{A}$: $(P \vee Q) \to R$

P	∨	Q	→	R
T	T	T	T	T
T	T	T	F	F
T	T	F	T	T
T	T	F	F	F
F	T	T	T	T
F	T	T	F	F
F	F	F	T	T
F	F	F	T	F

$\mathcal{B}$: $P \to R$

P	→	R
T	T	T
T	F	F
T	T	T
T	F	F
F	T	T
F	T	F
F	T	T
F	T	F

$\mathcal{A} \to \mathcal{B}$

$\mathcal{A}$	→	$\mathcal{B}$
T	T	T
F	T	F
T	T	T
F	T	F
T	T	T
F	T	T
T	T	T
T	T	T

$\mathcal{B} \to \mathcal{A}$

$\mathcal{B}$	→	$\mathcal{A}$
T	T	T
F	T	F
T	T	T
F	T	F
T	T	T
T	$\boxed{F}$	F
T	T	T
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? NO
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#22.

$\mathcal{A}$: $(P \,\&\, Q) \to R$

P	&	Q	→	R
T	T	T	T	T
T	T	T	F	F
T	F	F	T	T
T	F	F	T	F
F	F	T	T	T
F	F	T	T	F
F	F	F	T	T
F	F	F	T	F

$\mathcal{B}$: $P \to (Q \to R)$

P	→	Q	→	R
T	T	T	T	T
T	F	T	F	F
T	T	F	T	T
T	T	F	T	F
F	T	T	T	T
F	T	T	F	F
F	T	F	T	T
F	T	F	T	F

$\mathcal{A} \to \mathcal{B}$

$\mathcal{A}$	→	$\mathcal{B}$
T	T	T
F	T	F
T	T	T
T	T	T
T	T	T
T	T	T
T	T	T
T	T	T

$\mathcal{B} \to \mathcal{A}$

$\mathcal{B}$	→	$\mathcal{A}$
T	T	T
F	T	F
T	T	T
T	T	T
T	T	T
T	T	T
T	T	T
T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? YES

#23.

$\mathcal{A}$: **$\mathcal{B}$:**

P	→	(Q	&	R)		P	→	Q		$\mathcal{A}$	→	$\mathcal{B}$		$\mathcal{B}$	→	$\mathcal{A}$
T	T	T	T	T		T	T	T		T	T	T		T	T	T
T	F	T	F	F		T	T	T		F	T	F		T	F	F
T	F	F	F	T		T	F	F		F	T	T		F	$\boxed{T}$	F
T	F	F	F	F		T	F	F		F	T	F		F	T	F
F	T	T	T	T		F	T	T		T	T	T		T	T	T
F	T	T	F	F		F	T	T		T	T	T		T	T	T
F	T	F	F	T		F	T	F		T	T	T		T	T	T
F	T	F	F	F		F	T	F		T	T	T		T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? YES
Does $\mathcal{B}$ logically imply $\mathcal{A}$? NO
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

#24.

$\mathcal{A}$: **$\mathcal{B}$:**

P	→	(Q	∨	R)		P	→	Q		$\mathcal{A}$	→	$\mathcal{B}$		$\mathcal{B}$	→	$\mathcal{A}$
T	T	T	T	T		T	T	T		T	T	T		T	T	T
T	T	T	T	F		T	F	F		T	$\boxed{F}$	F		F	T	T
T	T	F	T	T		T	T	T		T	T	T		T	T	T
T	F	F	F	F		T	F	F		F	T	F		F	T	F
F	T	T	T	T		F	T	T		T	T	T		T	T	T
F	T	T	T	F		F	T	F		T	T	T		T	T	T
F	T	F	T	T		F	T	T		T	T	T		T	T	T
F	T	F	F	F		F	T	F		T	T	T		T	T	T

Does $\mathcal{A}$ logically imply $\mathcal{B}$? NO
Does $\mathcal{B}$ logically imply $\mathcal{A}$? YES
Are $\mathcal{A}$ and $\mathcal{B}$ logically equivalent? NO

EXERCISE SET C

1.

P	$\rightarrow$	Q	;	P	/	Q
T	T	T		T		T
T	F	F		T		F
F	T	T		F		T
F	T	F		F		F

VALID

2.

P	$\rightarrow$	Q	;	Q	/	P
T	T	T		T		T
T	F	F		F		T
F	$\boxed{T}$	T		$\boxed{T}$		$\boxed{F}$
F	T	F		F		F

INVALID

3.

P	$\rightarrow$	Q	;	~	Q	/	~	P
T	T	T		F	T		F	T
T	F	F		T	F		F	T
F	T	T		F	T		T	F
F	T	F		T	F		T	F

VALID

4.

P	$\rightarrow$	Q	;	~	P	/	~	Q
T	T	T		F	T		F	T
T	F	F		F	T		T	F
F	$\boxed{T}$	T		$\boxed{T}$	F		$\boxed{F}$	T
F	T	F		T	F		T	F

INVALID

5.

P	$\vee$	Q	;	~	P	/	Q
T	T	T		F	T		T
T	T	F		F	T		F
F	T	T		T	F		T
F	F	F		T	F		F

VALID

6.

P	$\vee$	Q	;	P	/	~	Q
T	$\boxed{T}$	T		$\boxed{T}$		$\boxed{F}$	T
T	T	F		T		T	F
F	T	T		F		F	T
F	F	F		F		T	F

INVALID

7.

~	(P	&	Q)	;	P	/	~	Q
F	T	T	T		T		F	T
T	T	F	F		T		T	F
T	F	F	T		F		F	T
T	F	F	F		F		T	F

VALID

8.

~	(P	&	Q)	;	~	P	/	Q
F	T	T	T		F	T		T
T	T	F	F		F	T		F
T	F	F	T		T	F		T
$\boxed{T}$	F	F	F		$\boxed{T}$	F		$\boxed{F}$

INVALID

9.

P	↔	Q	;	~	P	/	~	Q
T	T	T		F	T		F	T
T	F	F		F	T		T	F
F	F	T		T	F		F	T
F	T	F		T	F		T	F

VALID

10.

P	↔	Q	;	Q	/	P
T	T	T		T		T
T	F	F		F		T
F	F	T		T		F
F	T	F		F		F

VALID

11.

P	∨	Q	;	P	→	Q	/	Q
T	T	T		T	T	T		T
T	T	F		T	F	F		F
F	T	T		F	T	T		T
F	F	F		F	T	F		F

VALID

12.

P	∨	Q	;	P	→	Q	/	P	&	Q
T	T	T		T	T	T		T	T	T
T	T	F		T	F	F		T	F	F
F	$\boxed{T}$	T		F	$\boxed{T}$	T		F	$\boxed{F}$	T
F	F	F		F	T	F		F	F	F

INVALID

13.

P	→	Q	;	P	→	~	Q	/	~	P
T	T	T		T	F	F	T		F	T
T	F	F		T	T	T	F		F	T
F	T	T		F	T	F	T		T	F
F	T	F		F	T	T	F		T	F

VALID

14.

P	→	Q	;	~	P	→	Q	/	Q
T	T	T		F	T	T	T		T
T	F	F		F	T	T	F		F
F	T	T		T	F	T	T		T
F	T	F		T	F	F	F		F

VALID

15.

P	∨	Q	;	~	P	→	~	Q	/	P	&	Q
T	T	T		F	T	T	F	T		T	T	T
T	T	F		F	T	T	T	F		T	F	F
F	T	T		T	F	F	F	T		F	F	T
F	F	F		T	F	T	T	F		F	F	F

INVALID

16.

P	→	Q	;	~	P	→	~	Q	/	P	↔	Q
T	T	T		F	T	T	F	T		T	T	T
T	F	F		F	T	T	T	F		T	F	F
F	T	T		T	F	F	F	T		F	F	T
F	T	F		T	F	T	T	F		F	T	F

VALID

17.

~	P	→	~	Q	;	~	Q	→	~	P	/	P	↔	Q
F	T	T	F	T		F	T	T	F	T		T	T	T
F	T	T	T	F		T	F	F	F	T		T	F	F
T	F	F	F	T		F	T	T	T	F		F	F	T
T	F	T	T	F		T	F	T	T	F		F	T	F

VALID

18.

~	P	→	~	Q	;	~	Q	→	~	P	/	P	&	Q
F	T	T	F	T		F	T	T	F	T		T	T	T
F	T	T	T	F		T	F	F	F	T		T	F	F
T	F	F	F	T		F	T	T	T	F		F	F	T
T	F	T	T	F		T	F	T	T	F		F	F	F

INVALID

19.

P	∨	~	Q	;	P	∨	Q	/	P
T	T	F	T		T	T	T		T
T	T	T	F		T	T	F		T
F	F	F	T		F	T	T		F
F	T	T	F		F	F	F		F

VALID

20.

P	→	Q	;	P	∨	Q	/	P	↔	Q
T	T	T		T	T	T		T	T	T
T	F	F		T	T	F		T	F	F
F	T	T		F	T	T		F	F	T
F	T	F		F	F	F		F	T	F

INVALID

21.

~	(	P	→	Q	)	;	P	→	~	P	/	~	P	&	~	Q
F		T	T	T			T	F	F	T		F	T	F	F	T
T		T	F	F			T	F	F	T		F	T	F	T	F
F		F	T	T			F	T	T	F		T	F	F	F	T
F		F	T	F			F	T	T	F		T	F	T	T	F

VALID

22.

~	(	P	&	Q	)	;	~	Q	→	P	/	P
F		T	T	T			F	T	T	T		T
T		T	F	F			T	F	T	T		T
T		F	F	T			F	T	T	F		F
T		F	F	F			T	F	F	F		F

INVALID

23.

P	→	Q	;	Q	→	R	/	P	→	R
T	T	T		T	T	T		T	T	T
T	T	T		T	F	F		T	F	F
T	F	F		F	T	T		T	T	T
T	F	F		F	T	F		T	F	F
F	T	T		T	T	T		F	T	T
F	T	T		T	F	F		F	T	F
F	T	F		F	T	T		F	T	T
F	T	F		F	T	F		F	T	F

VALID

24.

P	→	Q	;	Q	→	R	;	~	P	→	R	/	R
T	T	T		T	T	T		F	T	T	T		T
T	T	T		T	F	F		F	T	T	F		F
T	F	F		F	T	T		F	T	T	T		T
T	F	F		F	T	F		F	T	T	F		F
F	T	T		T	T	T		T	F	T	T		T
F	T	T		T	F	F		T	F	F	F		F
F	T	F		F	T	T		T	F	T	T		T
F	T	F		F	T	F		T	F	F	F		F

VALID

25.

P	→	Q	;	Q	→	R	/	P	&	R
T	T	T		T	T	T		T	T	T
T	T	T		T	F	F		T	F	F
T	F	F		F	T	T		T	T	T
T	F	F		F	T	F		T	F	F
F	$\boxed{T}$	T		T	$\boxed{T}$	T		F	$\boxed{F}$	T
F	T	T		T	F	F		F	F	F
F	$\boxed{T}$	F		F	$\boxed{T}$	T		F	$\boxed{F}$	T
F	T	F		F	T	F		F	F	F

INVALID

26.

P	→	Q	;	Q	→	R	;	R	→	P	/	P	↔	R
T	T	T		T	T	T		T	T	T		T	T	T
T	T	T		T	F	F		F	T	T		T	F	F
T	F	F		F	T	T		T	T	T		T	T	T
T	F	F		F	T	F		F	T	T		T	F	F
F	T	T		T	T	T		T	F	F		F	F	T
F	T	T		T	F	F		F	T	F		F	T	F
F	T	F		F	T	T		T	F	F		F	F	T
F	T	F		F	T	F		F	T	F		F	T	F

VALID

27.

P	→	Q	;	Q	→	R	/	R
T	T	T		T	T	T		T
T	T	T		T	F	F		F
T	F	F		F	T	T		T
T	F	F		F	T	F		F
F	T	T		T	T	T		T
F	T	T		T	F	F		F
F	T	F		F	T	T		T
F	$\boxed{T}$	F		F	$\boxed{T}$	F		$\boxed{F}$

INVALID

28.

P	→	R
T	T	T
T	F	F
T	T	T
T	F	F
F	T	T
F	T	F
F	T	T
F	T	F

;

Q	→	R
T	T	T
T	F	F
F	T	T
F	T	F
T	T	T
T	F	F
F	T	T
F	T	F

/

(	P	∨	Q	)	→	R
	T	T	T		T	T
	T	T	T		F	F
	T	T	F		T	T
	T	T	F		F	F
	F	T	T		T	T
	F	T	T		F	F
	F	F	F		T	T
	F	F	F		T	F

VALID

29.

P	→	Q
T	T	T
T	T	T
T	F	F
T	F	F
F	T	T
F	T	T
F	T	F
F	T	F

;

P	→	R
T	T	T
T	F	F
T	T	T
T	F	F
F	T	T
F	T	F
F	T	T
F	T	F

/

Q	&	R
T	T	T
T	F	F
F	F	T
F	F	F
T	T	T
T	F	F
F	F	T
F	F	F

INVALID

30.

P	∨	Q
T	T	T
T	T	T
T	T	F
T	T	F
F	T	T
F	T	T
F	F	F
F	F	F

;

P	→	R
T	T	T
T	F	F
T	T	T
T	F	F
F	T	T
F	T	F
F	T	T
F	T	F

;

Q	→	R
T	T	T
T	F	F
F	T	T
F	T	F
T	T	T
T	F	F
F	T	T
F	T	F

/

R
T
F
T
F
T
F
T
F

VALID

31.

P	→	Q
T	T	T
T	T	T
T	F	F
T	F	F
F	T	T
F	T	T
F	T	F
F	T	F

;

Q	→	R
T	T	T
T	F	F
F	T	T
F	T	F
T	T	T
T	F	F
F	T	T
F	T	F

;

R	→	~	P
T	F	F	T
F	T	F	T
T	F	F	T
F	T	F	T
T	T	T	F
F	T	T	F
T	T	T	F
F	T	T	F

/

~	P
F	T
F	T
F	T
F	T
T	F
T	F
T	F
T	F

VALID

32.

P → (Q ∨ R) ; Q & R / ~ P

P	→	Q	∨	R	Q	&	R	~	P
T	T	T	T	T	T	T	T	F	T
T	T	T	T	F	T	F	F	F	T
T	T	F	T	T	F	F	T	F	T
T	F	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T	F
F	T	T	T	F	T	F	F	T	F
F	T	F	T	T	F	F	T	T	F
F	T	F	F	F	F	F	F	T	F

INVALID

33.

P → (Q & R) ; Q → ~ R / ~ P

P	→	Q	&	R	Q	→	~	R	~	P
T	T	T	T	T	T	F	F	T	F	T
T	F	T	F	F	T	T	T	F	F	T
T	F	F	F	T	F	T	F	T	F	T
T	F	F	F	F	F	T	T	F	F	T
F	T	T	T	T	T	F	F	T	T	F
F	T	T	F	F	T	T	T	F	T	F
F	T	F	F	T	F	T	F	T	T	F
F	T	F	F	F	F	T	T	F	T	F

VALID

34.

P & (Q ∨ R) ; P → ~ Q / R

P	&	Q	∨	R	P	→	~	Q	R
T	T	T	T	T	T	F	F	T	T
T	T	T	T	F	T	F	F	T	F
T	T	F	T	T	T	T	T	F	T
T	F	F	F	F	T	T	T	F	F
F	F	T	T	T	F	T	F	T	T
F	F	T	T	F	F	T	F	T	F
F	F	F	T	T	F	T	T	F	T
F	F	F	F	F	F	T	T	F	F

VALID

35.

P → (Q → R) ; P & ~ R / ~ Q

P	→	Q	→	R	P	&	~	R	~	Q
T	T	T	T	T	T	F	F	T	F	T
T	F	T	F	F	T	T	T	F	F	T
T	T	F	T	T	T	F	F	T	T	F
T	T	F	T	F	T	T	T	F	T	F
F	T	T	T	T	F	F	F	T	F	T
F	T	T	F	F	F	F	T	F	F	T
F	T	F	T	T	F	F	F	T	T	F
F	T	F	T	F	F	F	T	F	T	F

VALID

36.

~	P	∨	Q	;	R	→	P	;	~	(Q	&	R)	/	~	R
F	T	T	T		T	T	T		F	T	T	T		F	T
F	T	T	T		F	T	T		T	T	F	F		T	F
F	T	F	F		T	T	T		T	F	F	T		F	T
F	T	F	F		F	T	T		T	F	F	F		T	F
T	F	T	T		T	F	F		F	T	T	T		F	T
T	F	T	T		F	T	F		T	T	F	F		T	F
T	F	T	F		T	F	F		T	F	F	T		F	T
T	F	T	F		F	T	F		T	F	F	F		T	F

VALID

EXERCISE SET D

1. $\mathcal{A}$: ~(P&Q) $\mathcal{B}$: ~P&~Q
   (1)$\mathcal{A}$ / $\mathcal{B}$ INVALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

2. $\mathcal{A}$:~(P&Q) $\mathcal{B}$: ~P∨~Q
   (1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

3. $\mathcal{A}$: ~(P∨Q) $\mathcal{B}$: ~P∨~Q
   (1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ INVALID

4. $\mathcal{A}$: ~(P∨Q) $\mathcal{B}$: ~P&~Q
   (1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

5. $\mathcal{A}$: ~(P→Q) $\mathcal{B}$: ~P→~Q
   (1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ INVALID

6. $\mathcal{A}$: ~(P→Q) $\mathcal{B}$: P&~Q
   (1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

7. $\mathcal{A}$: ~(P↔Q) $\mathcal{B}$: ~P↔~Q
   (1) $\mathcal{A}$ / $\mathcal{B}$ INVALID (2) $\mathcal{B}$ / $\mathcal{A}$ INVALID

8. $\mathcal{A}$: ~(P↔Q) $\mathcal{B}$: P↔~Q
   (1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

9 $\mathcal{A}$: ~(P↔Q) $\mathcal{B}$: ~P↔Q
   (1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

10. $\mathcal{A}$: P↔Q $\mathcal{B}$: (P&Q) & (Q→P)
   (1) $\mathcal{A}$ / $\mathcal{B}$ INVALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

11. $\mathcal{A}$: P↔Q $\mathcal{B}$: (P→Q) & (Q→P)
   (1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

12. $\mathcal{A}$: P→Q $\mathcal{B}$: Q→P
   (1) $\mathcal{A}$ / $\mathcal{B}$ INVALID (2) $\mathcal{B}$ / $\mathcal{A}$ INVALID

13. $\mathcal{A}$: P→Q $\mathcal{B}$: ~P→~Q
   (1) $\mathcal{A}$ / $\mathcal{B}$ INVALID (2) $\mathcal{B}$ / $\mathcal{A}$ INVALID

14. $\mathcal{A}$: P→Q $\mathcal{B}$: ~Q→~P
(1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

15. $\mathcal{A}$: P→Q $\mathcal{B}$: ~P∨Q
(1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

16. $\mathcal{A}$: P→Q $\mathcal{B}$: ~(P&~Q)
(1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

17. $\mathcal{A}$: ~P $\mathcal{B}$ ~(P&Q)
(1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ INVALID

18. $\mathcal{A}$: ~P $\mathcal{B}$ ~(P∨Q)
(1) $\mathcal{A}$ / $\mathcal{B}$ INVALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

19. $\mathcal{A}$: ~(P↔Q) $\mathcal{B}$: (P&Q) → R
(1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ INVALID

20. $\mathcal{A}$: (P&Q) → R $\mathcal{B}$: P→R
(1) $\mathcal{A}$ / $\mathcal{B}$ INVALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

21. $\mathcal{A}$: (P∨Q) → R $\mathcal{B}$: P→R
(1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ INVALID

22. $\mathcal{A}$: (P&Q)→R $\mathcal{B}$: P → (Q→R)
(1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID

23. $\mathcal{A}$: P → (Q&R) $\mathcal{B}$: P→Q
(1) $\mathcal{A}$ / $\mathcal{B}$ VALID (2) $\mathcal{B}$ / $\mathcal{A}$ INVALID

24. $\mathcal{A}$: P → (Q∨R) $\mathcal{B}$: P→Q
(1) $\mathcal{A}$ / $\mathcal{B}$ INVALID (2) $\mathcal{B}$ / $\mathcal{A}$ VALID