CONDITIONALIZATION AND KNOWLEDGE *DE PRAESENTI*[*]

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Abstract

This paper argues that the principle of conditionalization is not, as some have recently claimed, falsified by possible rational changes in our temporally indexical opinions. Instead, temporally indexical credence is shown to fall entirely outside the scope of the principle of conditionalization because it is impossible for the relevant object of new temporally indexical certainty to be in the domain of an agent's credence function at any prior time. In addition to saving the principle of conditionalization from absurd implications, this result pays substantial dividends in connection with the Sleeping Beauty problem. Many philosophers who have addressed this problem have claimed that one can conditionalize on temporally indexical knowledge when one is, like Beauty upon her experimental awakenings, uncertain of the time. The falsehood of this claim provides a clear diagnosis of the error in the arguments for two standard positions on Sleeping Beauty.

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1. *Introduction*

Those with a broadly Bayesian perspective have often claimed that an ideally rational agent ought, upon becoming newly certain of the truth of a proposition *e* (and nothing stronger), to alter her credence in each other proposition *h* by conditionalization—adopting, as her new credence for *h*, her prior credence for *h* conditional on *e*. While this "principle of conditionalization" is a normative principle regarding how our opinions ought to be revised upon the receipt of new knowledge, it is closely linked with a natural account of when such knowledge counts as evidence for another proposition. For according to a standard account of incremental confirmation, *e* confirms *h* when $P(h/e) > P(h)$, and one way of demonstrating that this inequality obtains is to demonstrate that a rational agent would have a higher degree of belief in *h* after conditionalizing on *e*.[1]

Frameworks for probabilistic epistemology have also traditionally attributed to agents credence functions defined over propositions. More specifically, they have appealed to credence functions defined over propositions as traditionally conceived—as objects of belief or partial belief which do not change their truth value from time to time or person to person and which are, at least in principle, cognitively accessible to all rational agents at all times. However, it has become increasingly clear that an adequate account of our opinions must allow that we have essentially indexical opinions which cannot be captured in the traditional framework of propositions. Moreover, essentially temporally indexical credence appears to

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[1]   I suppress reference to background knowledge.

raise significant new questions about the principle of conditionalization and the related account of evidence.

Consider a clock-watcher who sits merely watching the time pass on a clock which she is certain is perfectly accurate. She believes first that it is 1:00 PM, and then that it is 1:01 PM, and so on. She is, it appears, certain of one proposition and then certain of an incompatible proposition. As some have noted, such changes of credence cannot be brought about by conditionalization (Arntzenius 2003, 367; Hitchcock 2004, 416). However, the existence of a change of confidence not produced by conditionalization is not itself problematic, as Bayesians have always held that one's confidence can change other than by conditionalization—this is exactly what occurs when one makes an observation. The difficulty, rather, is the apparent replacement of one certainty with an incompatible certainty because if one allows that the clock-watcher has adopted full credence in a proposition in which she previously had a zero credence, a straightforward application of the principle of conditionalization would yield the absurd result that all her posterior credences should be undefined (Titelbaum 2008, 566).

My goal in this paper is to move some distance toward an adequate Bayesian treatment of such essentially indexical credence. Because they have so frequently been addressed together, I shall discuss both essentially temporally indexical credence (credence *de praesenti* or *de nunc*) and essentially personally indexical credence (credence *de se*). My focus, however, will be primarily on credence *de praesenti*. In particular, I will argue that the principle of conditionalization is not, as has been claimed, falsified by cases like that of the clock-watcher. Rather, I shall argue that essentially temporally indexical credence falls entirely outside the scope of the principle of conditionalization because it is impossible for the relevant object of

new temporally indexical certainty to be in the domain of an agent's credence function at any

prior time.[2] In addition to saving the principle of conditionalization from counterexample, this

result pays substantial dividends in connection with the Sleeping Beauty problem, for many

philosophers who have addressed this problem have claimed that one can conditionalize on

temporally indexical knowledge when one is, unlike the clock-watcher, initially uncertain of the

time. Given the connection between conditionalization and evidential relevance just noted, this

possibility would appear to justify the conclusion that essentially temporally indexical

knowledge can be evidentially relevant. However, if I am right about the impossibility of

conditionalizing on temporally indexical credence, not only do we have a clear diagnosis of the

error in the arguments for two standard positions on Sleeping Beauty, but such a route to

demonstrating evidential relevance is foreclosed.

2. *Sleeping Beauty and Essentially Temporally Indexical Knowledge*

Sleeping Beauty is informed on Sunday that she will be put into a dreamless sleep for

the next two days. She will be awakened on Monday morning, shortly thereafter told that it is

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[2]   I use "object of credence" and similar phrases to refer to an item in the domain of a credence

function. I recognize that the notion of an "object" of credence is not ideally suited to the

present discussion as it applies most clearly to the second relatum in a dyadic cognitive

relation between a person and a proposition while (a) a state of credence is not clearly a

dyadic relation between a person and a proposition, and (b) some of the views of indexical

binary belief discussed in this paper explicitly reject the dyadic propositionalist picture.

Monday and then, if the toss of a fair coin lands heads, returned to sleep until Wednesday. If

the toss of the coin lands tails, she will have her memory of Monday morning erased prior to

being awakened again on Tuesday morning in a state subjectively indiscernible from her

Monday awakening.[3] When she awakens on Monday, what should her credence be that the

coin lands heads?

'Thirders" hold that Beauty's credence that the coin lands heads should be 1/3.[4]

"Halfers" maintain that it should remain 1/2.[5] Let P be Beauty's credence function immediately

after awakening on Monday and let P^+ be her credence function immediately after she is told by

the experimenters that it is Monday. Let HEADS be the proposition that the coin lands heads,

MONDAY the proposition that it is now Monday, and TUESDAY the proposition that it is now

Tuesday. Then the following initially plausible claims are inconsistent, given the probability

calculus:

(1) P(HEADS) = 1/2

(2) P^+(HEADS) = 1/2

(3) P^+(HEADS) = P(HEADS/MONDAY)

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[3]   We must also suppose that Beauty knows on Sunday exactly what every experimental

       awakening will be like.

[4]   Thirders include Elga (2000), Dorr (2002), Monton (2002), Arntzenius (2003), Horgan (2004),

       Weintraub (2004), Hitchcock (2004), Dieks (2007), Stalnaker (2008), and Titelbaum (2008).

[5]   Halfers include Lewis (2001), Bostrom (2007), Hapern (2007), and Meacham (2008).

(4) P(HEADS/not-MONDAY) = 0

(5) $0 < P(MONDAY) < 1$[6]

In light of the indiscriminability of her Monday and (possible) Tuesday awakenings and the

stipulation that Beauty knows with certainty the experimental set-up, (4) and (5) are agreed by

all to be unassailable. Elga (2000) and Lewis (2001) both accept (3), endorsing the view that

Beauty ought to change her degree of belief in HEADS (a non-indexical proposition) upon

learning MONDAY (a temporally indexical proposition). Elga accepts (2) and so must deny (1).

However, Beauty was certain in advance that she would be awakened during the experiment

with no recollection of a previous experimental awakening and hence intuitively seems to gain

no new evidence upon awakening on Monday morning. [7] Lewis accepts (1) and so must deny

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[6]   Dorr (2002) provides the following proof:

P(HEADS) = P(HEADS/MONDAY) * P(MONDAY) + P(HEADS/not-MONDAY) * P(not-

MONDAY), by (5).

P(HEADS) = P(HEADS/MONDAY) * P(MONDAY), by (4).

P(HEADS) = P+(HEADS) * P(MONDAY), by (3).

1/2 = 1/2 * P(MONDAY), by (1) and (2).

P(MONDAY) = 1, contradicting (5).

[7]   White (2006) argues that both Elga's argument for 1/3 and the Dorr-Arntzenius argument for

1/3 (discussed later in this paper) generalize in an unacceptable way. However, White

(2). However, Beauty, having been told that it is Monday, is then certain that the fair coin toss

lies in the future and hence intuitively seems to have strong reason to regard heads and tails as

equiprobable.[8]

Both of these counterintuitive results could be avoided if (3) is rejected. If (3) is false,

then Elga's and Lewis' arguments give us no reason to reject the view, that of the "double-

halfer," that Beauty's credence in HEADS ought to remain 1/2 for the duration of the

experiment.[9] However, it may seem that (3) is supported by the principle of conditionalization.

Indeed, Elga and Lewis claim that (3) is true because Beauty should, upon learning from the

experimenters that it is Monday, conditionalize on MONDAY in order to arrive at P+. So,

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explicitly admits that he has not diagnosed "the exact error" in either argument and

provides no suggestion as to how the halfer could avoid following Lewis in claiming that (2)

is false. I hope to have done both in this paper while also supporting the plausible verdict

on White's generalization of the problem.

[8]   Lewis and Elga also accept, on the basis of a "restricted principle of indifference,"

(6) P(MONDAY/TAILS) = P(TUESDAY/TAILS).

Hence, Lewis must claim that P+(HEADS) = 2/3 and Elga must claim that P(HEADS) = 1/3.

[9]   Bostrom (2007), Halpern (2007), and Meacham (2008) defend the double-halfer position on

grounds distinct from those in the present paper.

among others, do Dorr (2002), Arntzenius (2003), Dieks (2007) and Titelbaum (2008).[10] I'll argue

that (3) cannot be so supported because it is entirely impossible to conditionalize on a

temporally indexical certainty. Hence, we can accept the universal applicability of the principle

of conditionalization while rejecting (3).

Even though they endorse Elga's original argument, Dorr (2002) and Arntzenius (2003)

each provide an additional argument for 1/3. Their arguments, however, also require

conditionalization on temporally indexical knowledge. In the original case, when Beauty

awakens on Monday, she is certain that she is in one of the following three situations:

HEADS & MONDAY, TAILS & MONDAY, and TAILS & TUESDAY. The Dorr and Arntzenius

arguments each involve appeal to a variant case in which HEADS & TUESDAY is a possibility

with some positive epistemic probability upon Beauty's Monday awakening. Dorr's variant

case is one in which Beauty is certain that she will be awakened on both Monday and Tuesday

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[10] Titelbaum (2008) provides an admirably clear and detailed modeling framework for *de se*

and *de praesenti* credence and utilizes it to defend 1/3. However, he explicitly avoids

thorough examination of the nature of indexical credence, couching his framework in terms

of sentences (558). Titelbaum claims that, in circumstances in which the temporally

indexical *sentence* representing an agent's *de praesenti* credence has (and is known to have)

the same truth value at t_1 and t_2 (though it is not known what the truth value is at t_1), one can

conditionalize on the new *de praesenti* certainty at t_2 (583). If my arguments in the remainder

of this paper are correct, Titelbaum errs in neglecting the fact that the sentence in question

necessarily represents, at each distinct time, a different item in which credence is invested.

and the coin flip determines whether she will be administered a drug that entirely erases her memories of Monday (tails) or one that merely delays their onset for one minute upon her Tuesday awakening (heads). Arntzenius' case is exactly like the original case except that Beauty is such that if she is not awakened on a given morning, she dreams that she is awakened. This dream is indistinguishable from genuine awakening except that if she "pinches herself" while dreaming, she feels nothing and does not wake up and if she pinches herself while awake, it hurts.

Arntzenius and Dorr make the following claims about their respective variant cases: (I) Upon awakening on Monday, Beauty ought to have equal credence of 1/4 in HEADS & MONDAY, HEADS & TUESDAY, TAILS & MONDAY, and TAILS & TUESDAY. (II) After failing to have the experience which she would certainly have if HEADS & TUESDAY were true (the flood of memories or the painless "pinch"), she ought to have 0 credence in that claim and 1/3 credence in the remaining three possibilities. (III) As Beauty has the same total (relevant) evidence at some suitable point in the original and variant cases, P(HEADS) = 1/3 in the original case [i.e. ~(1)]. Dorr and Arntzenius defend (II) with the claim that Beauty's failure to have the relevant experience *now* (or ~(HEADS & TUESDAY) which that failure entails) should be conditionalized upon (Arntzenius 2003, 364; Dorr 2002, 294). If, as I shall argue, it is impossible to conditionalize upon such temporally indexical certainty, then Dorr's and Arntzenius' arguments for 1/3 also fail.[11]

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[11]   As will become clearer below, I do not deny that Beauty has, in the variant cases, learned

        something that is inconsistent with one of the four claims about her (then) current situation.

3. *Conditionalization and Credence De Praesenti*

Thus far, I have referred to the objects over which Beauty's credence function is defined as "propositions." However, important features of Beauty's epistemic predicament during the course of the experiment can only be characterized indexically and, on some views of propositions, essentially indexical belief cannot be thought of as a dyadic relation relating a person to a proposition (and, presumably, essentially indexical credence cannot be thought of as a triadic relation relating a person to a proposition and a number from zero to one). Beauty, by construction, knows in advance all of the non-indexical truths (and all of the merely *de se* indexical truths) about her predicament from Sunday through Monday evening. She knows, for example, that she will sleep Sunday night, will awaken on Monday morning without any recollection of a previous awakening, will be told that it is Monday, will be returned to sleep, etc. Her ignorance regarding her situation when she awakens on Monday can *only* be properly characterized in temporally indexical terms, as ignorance of whether it is *now* Monday or *now* Tuesday.

A great deal of ink has been spilt on the question of precisely how to understand indexical thought. According to the extremely influential *argument from cognitive significance* (Perry 1979; Castañeda 1968), indexical statements cannot be translated into standard *de dicto*

I do deny that she has ruled out an antecedently grasped possibility. Notice that if my claims about conditionalization and temporally indexical credence are correct, they also undermine Arntzenius' treatment of the case of The Prisoner (2003, 357-363).

statements free of indexicals. Two famous illustrations of this form of argument are provided

by John Perry:

THE SHOPPER

I once followed a trail of sugar on a supermarket floor, pushing my cart down

the aisle on one side of a tall counter and back the aisle on the other, seeking the

shopper with the torn sack to tell him he was making a mess. With each trip

around the counter, the trail became thicker. But I seemed unable to catch up.

Finally it dawned on me. I was the shopper I was trying to catch. I believed at

the outset that the shopper with a torn sack was making a mess. And I was right.

But I didn't believe that I was making a mess. That seems to be something I came

to believe. (Perry 1979, 3)

THE MEETING

[A] professor, who desires to attend the department meeting at noon, sits

motionless in his office at that time. Suddenly, he begins to move. What

explains his action? A change in belief. He believed all along that the

department meeting starts at noon; he came to believe . . . that the meeting starts

now. (Perry 1979, 4).

Perry argues that there is no acceptable *de dicto* translation of the shopper's new belief free of the

personal indexical "I" and no acceptable *de dicto* translation of the professor's new belief free of

the temporal indexical "now." In particular, Perry claims that no account free of the relevant indexicals has the appropriate rationalizing explanatory force. These sorts of cases are the standard justification of the claim (which I assume in what follows) that at least some knowledge is "essentially indexical."

According to a standard formulation of the principle of conditionalization, when one learns e (and nothing stronger) at t_2, $P_{t2}(h)$ should be equal to one's previous degree of belief in h conditional on e, i.e. $P_{t1}(h/e)$. The principle of conditionalization, then, requires that the proposition newly certain at t_2 *had some value in one's prior credence distribution* at t_1. Hence, if there is some reason why even an ideally rational agent must lack *any* prior credence (even zero) for some proposition of which she becomes certain, then it will be impossible to conditionalize on that proposition. In the next section, I shall show that on the most straightforward account of temporally indexical knowledge, it cannot be conditionalized on for precisely this reason. I'll then turn, in subsequent sections, to the two main alternative accounts of indexical belief and, by extension, degrees of belief. It will turn out that each of these alternative accounts of indexical credence requires compensatory alterations to the synchronic and diachronic constraints on rational credence and that each suitably modified principle of conditionalization also precludes conditionalizing on temporally indexical credence. Given that the major frameworks for understanding indexical opinions preclude such conditionalization, we have strong reasons for thinking it impossible.

4. Propositions of Limited Temporal Accessibility

The most straightforward way of dealing with the phenomena of essential personal and temporal indexicality, and the only one that clearly preserves the traditional view that belief is a dyadic relation between a person at a time and a proposition,[12] is simply to accept that there are *propositions of limited accessibility*—propositions which only a particular person can grasp or which can only be grasped at particular times.[13] Though Perry rejects the view, he provides a nice summary of its central tenets,

> One may take all that has been said so far as an argument for the existence of a special class of propositions, propositions of limited accessibility. . . . [A]t noon on the day of the meeting, we could all express the proposition the tardy professor expressed by those words at that time. But once that time has past, the proposition becomes inaccessible. We can still identify it, as the proposition which was expressed by those words at that time. But we cannot express it with

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[12] Some who reject propositions of limited accessibility hold that belief is a dyadic relation between a person and a proposition but claim that it holds only when a different triadic relation with an additional relatum holds. Rational belief doesn't, on this picture, behave in traditional ways as it behaves in ways governed by the underlying triadic relation. See Section 5 of the present paper for further discussion.

[13] For acceptance of propositions of limited accessibility see Chisholm (1976), Castañeda (1980), Peacocke (1981), Pollock (1982), Forbes (1987), and Markie (1988).

those words any longer, for with each passing moment they express a different

proposition. And we can find no other words to express it. (Perry 1979, 15-16)

There are a variety of accounts of such propositions of limited accessibility. One account

takes them to involve *haecceities*—individual non-qualitative or non-repeatable properties of

particulars (Chisholm 1976). In the personal case, it is claimed that each person has the

property of being herself, *S**, such that nothing else has or could have had that property. To

believe of oneself as oneself that one is wise is to believe *S** and wisdom to be co-exemplified or

to believe a singular proposition predicating wisdom of *S** (Sosa 1983, 132). The temporal case

would be treated analogously in postulating an individual property, *T**, had by each time such

that no other time could have had that property. To believe of a time as the then present time

that it is the time when the meeting starts is to believe the property of being a time when the

meeting starts to be co-exemplified with *T** or a singular proposition predicating that property

with respect to that temporal *haecceity*.

This way with essentially indexical belief preserves the elements of the traditional

theory of propositions as abstract necessary objects which are true or false objectively or

absolutely, and it preserves the traditional dyadic account of the psychological attitudes as

relating a person and a proposition. However, in order to account for cases like THE MEETING

and THE SHOPPER it must hold that such propositions cannot be grasped by other persons or

at other times, presumably because each person can grasp only their own essence and only the

essence of the present moment may be grasped.

An alternative account is provided by neo-Fregeans such as Peacocke (1981) and Forbes
(1987), who take Fregean thoughts to be the content of propositional attitudes and the bearers of
truth-values.[14] They postulate, in light of the phenomena of essential indexicality, token modes
of presentation obtained by indexing with a person or time a certain type of way of thinking of
a person or a time. Peacocke, for example, holds that there is a particular temporal type of
mode of presentation "under tokens of which *at any given time one can think only of that time and
no other*" (1981, 190, emphasis added), though it is one which may form thoughts "to which
many people can have attitudes." Similarly, the Peacocke-Forbes account appeals to modes of
presentation of the self which only that person can take direct attitudes toward, token modes of
presentation "under which any person can think only of himself, and under which no one else
can think of him" (Peacock 1981, 190).[15]

The various proponents of propositions of limited accessibility do not hold that no other
person can think about a person's personally indexical thoughts or that persons at times cannot
think about temporally indexical thoughts entertained at other times. The Peacocke-Forbes

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[14] Kripke (2008) offers another neo-Fregean account clearly committed to propositions of
limited temporal accessibility.

[15] Perhaps this neo-Fregean story makes the propositions or thoughts employing indexical
modes of presentation contingent. However, as Peacocke notes (1981, 197), Frege's general
opposition to such a view was derived from his requirement that the individuation of senses
be answerable to facts about cognitive significance, which facts Peacocke's account
endeavors to respect.

version distinguishes between *employing* a mode of presentation and *referring to* a mode of presentation. As Peacocke notes, "From the fact that only John can think thoughts containing [his token mode of presentation] it does not follow that we cannot know which thoughts he thinks, or that we cannot think about the constituent modes of presentation of his thoughts" (191). Pollock, another proponent of propositions of limited accessibility, draws a similar distinction between *entertaining* a proposition and *thinking about* the proposition. Even Perry, who rejects propositions of limited accessibility, allows in the passage quoted above that our inability to *express* such a proposition does not imply an inability to *refer to* the proposition.

Any version of the theory of propositions of limited accessibility clearly implies that it is impossible to conditionalize on a new certainty regarding what is now true. The theory holds that the temporally indexical proposition grasped at *t* when one considers 'It is *now* Monday' cannot be grasped at a distinct moment in time, *t'*. Recall that (3) is justified by the *diachronic* claim that Beauty's credence in HEADS on Monday afternoon ought to be equal to her earlier credence in HEADS conditional on the proposition newly certain for her—that it is *now* Monday. However, on the doctrine of propositions of limited accessibility, the proposition newly certain at *t* could not even have been grasped at an earlier time and so Beauty could have no credence invested in it at the earlier time. On this view, while one can learn that a temporally indexical proposition is true, one cannot have had previous uncertainty regarding

that very proposition.[16] So, while Beauty learns something when she is told that it is Monday, she cannot update by conditionalization on this new knowledge.[17]

5. *Essentially Synchronic Proposition-Guise Pairs*

Although those with a generally Fregean outlook appear forced by the phenomena of essential indexicality to endorse propositions of limited accessibility, many philosophers now endorse the view that all propositions (or at least the propositions relevant to the present discussion) are purely "Russellian" in that the constituents of a proposition are the very objects (individuals, properties and relations) the proposition is about.[18] Such views generally hold, for

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[16] Notice that a precisely parallel point will hold for a temporally indexical proposition previously grasped as one cannot now grasp the proposition of which one was previously uncertain.

[17] There is a tendency in the literature to formulate the relevant hypotheses regarding Beauty's temporal location in terms of days (e.g. "It is Monday") rather than moments. This obscures the fact that when Beauty considers whether it is Monday, she is really considering whether it is *now* Monday, i.e. whether the present moment occurs on Monday. When she is informed that it is Monday, she is actually learning that the (then) present moment occurs on Monday, a proposition which was, on the present view, previously inaccessible to her.

[18] Despite their differences, most direct reference and hidden-indexical theorists would endorse the views discussed in this section. Some prominent examples are Perry (1979), Salmon (1986; 1989), Braun (1998), and Richard (1983).

example, that the proposition that 'Hesperus = Hesperus' and the proposition that 'Hesperus = Phosphorus' are in fact the same proposition. They also typically entail that Beauty's Monday morning utterance of "I'm awake now" and her Sunday utterance of "I am awake on Monday" or "I'll be awake on Monday" express the same proposition.

As this account of propositions and their identity conditions is, if belief is fundamentally a dyadic relation to propositions, *prima facie* inadequate as a response to the intuitions about cognitive significance which motivated Frege's theory—that, for example, a fully rational thinker need not take the same attitude toward 'Hesperus = Hesperus' and 'Hesperus = Phosphorus'—many theorists hold that belief is either (a) a triadic relation between a person, a Russellian proposition, and a *way of believing* the proposition or a *propositional guise*, or (b) a dyadic relation which is analyzed in terms of a triadic relation (belief*) with the aforementioned three relata and which behaves in ways governed by the third relatum.[19] In what follows, I'll assume the former view simply to avoid needless qualification and repeated reference to the underlying belief* relation posited by the latter view. Different "triadic propositionalists" provide different accounts of ways of believing or guises, some incorporating something very much like Fregean modes of presentation, others sentences in the language of thought, and still

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[19] Salmon (1989) and Braun (1998) hold that it is the existential generalization of such a triadic relation. So, for S to believe p is for there to be some guise under which S believes p. A particular credence or degree of belief would have to be treated as a quadratic relation or a suitable existential generalization of such a relation.

others modes of access to a proposition or kinds of acquaintance with the constituents of the proposition. The differences between these accounts do not matter for present purposes.

On triadic propositionalism, one believes a proposition only when one believes the proposition in some particular way. Therefore, while a rational agent can have contradictory attitudes toward the same proposition at the same time, Fregean concerns about cognitive significance are accommodated by requiring that the proposition be believed and denied in different ways or under different guises. Furthermore, cases like THE MEETING and THE SHOPPER are accommodated by maintaining that only a given person can believe propositions about herself in the personally indexical way and only at a particular time can one believe propositions about that time in the temporally indexical way (Perry 1979; Richard 1983). Someone other than John might believe that John is making a mess and John might realize he himself is making a mess. Even if the same proposition is believed, say the Russellian proposition <John, making a mess>, it is believed, as Perry says, "in different ways." Similarly, even if the professor believes all along the Russellian proposition <Noon, meeting begins>, he believes it in different ways at different times.

Whatever else one thinks of this apparatus, its adoption obviously requires a reformulation of standard synchronic constraints on rational credence. For example, while the triadic theorist will deny that a rational agent who has degree of belief of 3/4 in a proposition p must have degree of belief of 1/4 in ~p, she will hold that something like such a principle is true, when guises are appropriately incorporated. In this case, the amended principle would be that any rational agent who has a degree of belief of 3/4 in p under guise w must have degree of belief of 1/4 in ~p under the negation of that guise. More generally, any of the constraints on

rational belief or rational credence invoked by the dyadic propositionalist theorist of belief will be rejected by such triadic propositionalists about belief in favor of an analogous constraint governing belief or degrees belief under guises.

By now, however, it will be clear that *diachronic* principles of rational credence must also incorporate appeal to guises or ways of believing. In particular, any acceptable principle of conditionalization must hold the guises of propositions diachronically constant. That is, it must hold that when one becomes certain of e under guise g_e, one ought to adopt, as one's new credence for h under guise g_h, one's previous degree of belief in h under guise g_h conditional on e under guise g_e. For non-temporally-indexical belief no new problem is produced by introducing diachronic constraints on guises. However, if temporally indexical guise-proposition pairs are *limited in their temporal availability*, our earlier difficulty simply reappears in this more complex framework. Indeed, as noted above, the doctrine at hand accounts for the phenomenon of essentially temporally indexical credence by holding that a temporally indexical "*now*-guise" necessarily relates one to a proposition about the current time. This is how it accounts for the professor's psychological change when he comes to believe that the meeting starts now. So, when it comes to knowledge *de praesenti*, the temporally indexical guise under which one believes a proposition at a given time must be distinct from the guise under which one believes that very proposition at any other time.

Beauty, on this account, believes the same propositions from Sunday through Monday evening but believes those propositions under different guises. Hence, as one must update relative to a guise, updating on the current temporally indexical guise of a given proposition will be impossible because the current temporally indexical guise cannot have been the

previous guise of that proposition. While this account avoids positing propositions of limited

temporal accessibility, it trades them for proposition-guise pairs of limited temporal

accessibility and, given a suitable corresponding revision of the principles of diachronic and

synchronic credence, yields again the conclusion that one cannot conditionalize on temporally

indexical knowledge.

6. Essentially Synchronic Objects of Attribution and the Inadequacy of Centered Worlds

Roderick Chisholm and David Lewis famously took cases like THE MEETING and THE

SHOPPER to support the radical thesis that belief and the other propositional attitudes are not

really propositional at all. They are not, that is, relations between a person and a proposition,

whether or not such a relation has (or is analyzed in terms of a relation having) an additional

relatum. Rather, Chisholm and Lewis claim, belief in general is a relation between a person and

a property or, in cases of belief *de praesenti*, either a person, a property and a time, or a temporal

part of a person and a property.

Chisholm's (1981) account of *de se* belief holds that to believe of oneself as oneself that

one is wise is to *self-attribute* wisdom. Ernest Sosa suggests that "a parallel solution for our

consciousness of the present would replace present-tense propositions with *present-attribution* of

corresponding properties of times" (1983, 134).[20] On this view, when *S* believes of the present

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[20]   Note that "self-attribution" (by a person or a temporal part) and "present-attribution" are

taken as theoretical primitives by these theorists. See Boër & Lycan (1980, 445) for an

expression of skepticism regarding the coherence of such primitives.

time as the present time that it is on Monday, *S* is said to engage in *present-attribution* of the

property of being on Monday. As Sosa rightly notes, one would have to allow combined joint

"self-cum-present-attribution" of properties in order to capture present-tense first-person belief.

Lewis' view of *de se* belief is much like Chisholm's in that he treats essentially indexical

de se belief as the self-attribution of properties. However, Lewis claims that temporally

indexical belief should be treated as a special case of belief *de se*, belief in which the believer is

not a normal continuant but "a more-or-less momentary time-slice thereof" (Lewis 1979, 527).

Imagining an insomniac who wonders all through a given night what time it is, Lewis suggests:

> To understand how he wonders, we must recognize that it is time-slices of him
>
> that do the wondering. A slice of the insomniac may locate the whole of the
>
> insomniac well enough in logical space and space and time. Yet that slice may
>
> fail to locate itself in space, in time, and in the population of slices of the well-
>
> located continuant insomniac. The slice at 3:49 A.M. may self-ascribe the
>
> property of being one slice of an insomniac who lies awake all night on such-
>
> and-such date at such-and-such place at such-and-such a kind of world, and yet
>
> may fail to self-ascribe the property of being at 3:49. That is how this slice may
>
> be ignorant, and wonder what time it is, without failing in any relevant way to
>
> locate the continuant to which it belongs. It is the slice, not the continuant, that
>
> fails to self-ascribe a property. (Lewis 1979, 527)

And, elsewhere he writes:

By treating the subjects of belief as momentary, we can subsume belief about

what time it is as a special case of egocentric belief. (Lewis 1986, 29)

Lewis' account requires endorsing the doctrine of temporal parts with respect to

persons. However, one could endorse the doctrine of temporal parts while also appealing to

propositions of limited temporal accessibility or proposition-guise pairs of limited temporal

accessibility. When it comes to conditionalizing on temporally indexical knowledge, those

proposals would have the difficulties I've already outlined. So, what is essential to treating

Lewis' proposal as a genuinely distinct one is careful attention to the combination of the

doctrine temporal parts with the view that belief *de praesenti* is self-attribution of properties *by a*

temporal part.

Of course, given these alternative accounts of belief and credence as non-propositional,

we require a different set of constraints on rational belief and credence. Whereas many

standard synchronic constraints on rational belief and degrees of belief are derived from the

logical relations between the propositional content of such opinions (Christensen 2004, 12-18),

the required alternative set of constraints must be based on the necessary relations between self-

attributed or present-attributed properties. However, while propositions are true or false,

properties are not. Hence, such an account must, in order to mimic the truth-functional logical

relations determined by propositions, appeal to the necessary relations between properties

taken as jointly possessed by or co-instantiated in a fixed object. For example, rather than

holding that a rational agent must not believe both p and $\sim p$ at t, it must require that a rational

agent not attribute both a property and its negation to the same object at t. Rather than holding

that a rational agent's credence in p at t and ~p at t must sum to 1, it must hold that a rational

agent's partial attributions of F to o at t and of ~F to o at t must (suitably normalized) sum to 1.[21]

Furthermore, just as any plausible version of the standard diachronic principle of

conditionalization requires grasp of the same proposition by the same person at two points in

time, any similarly plausible modified diachronic principle of conditionalization suitable for use

by the property theorist would require both a constant attributed property and *a constant object*

of attribution. It is easy to overlook this last requirement as identity of the object of attribution is

automatically guaranteed for all *de se* beliefs had by a given continuant person at any time and

for all *de praesenti* beliefs at a given time. Hence, specification of the object of attribution can

almost always be neglected without loss. However, the required identity of the object of

attribution cannot obtain in the special case of *de praesenti* beliefs at different times.

Given the Chisholm-inspired account of temporally indexical belief outlined above, it is

impossible to conditionalize on a new temporally indexical certainty as the moment to which

one fully present-attributes a given property is, of necessity, a different moment from that to

which one previously less-than-fully present-attributed the property. Given Lewis' account of

temporally indexical belief, conditionalization will be impossible because such credence is self-

attribution by a temporal part—a part which cannot exist at multiple times. To hold the object

---

[21]   I argue in [material omitted for blind review] that degrees of belief or confidence cannot be

captured properly by the intuitive notion of attribution or ascription because that notion

seems not to allow for degrees. I set that problem aside here.

of attribution constant, Lewis must require that if a temporal part S_2 fully self-attributes E at t_2,

then S_2 ought to self-attribute H to the degree S_2 self-attributed H conditional on E at t_1.

However, S_2, being a mere temporal part, didn't exist at t_1 and so the principle would have the

difficulties already noted. Chisholm and Lewis, then, avoid propositions of limited temporal

accessibility and essentially synchronic proposition-guise pairs, but trade them for *objects of*

attribution (times or believers) of momentary duration.[22]

It is well worth pausing at this juncture to dwell briefly upon the fact that much of the

contemporary debate about credence *de se* and *de praesenti* attributes to agents credence

functions defined over Lewis' "centered worlds." Such worlds are treated formally as ordered

triples of possible worlds, persons and times, or as ordered pairs of worlds and temporal parts

of persons. However, we can now see that to explicate an agent's credences merely by appeal to

---

[22]   Space considerations preclude a detailed examination of the thesis that credence *de se* and *de*

praesenti are just a special sort of credence *de re* of oneself and of the present time (Boër &

Lycan 1980). Notice, however, that austere accounts of *de re* belief (Schiffer 1978) hold that

only beliefs of oneself and of the present moment are genuinely *de re* and so will still yield

my conclusions. Furthermore, a typical feature of more latitudinarian accounts of *de re*

belief is a failure to support standard deductive and probabilistic constraints, rendering

them inappropriate for Bayesian treatment. Latitudinarian accounts of *de re* belief which are

so constrained by the addition of something like modes of presentation (Davidson 1985) also

clearly encounter the problems with conditionalizing on temporal indexicals that I've been

discussing.

such possible worlds with "designated centers" is to represent only the properties attributed and to neglect the issues of *who* is doing the attributing of properties and *to what or to whom* they are attributing the properties. As just noted, this neglect is entirely inconsequential for modeling all the opinions of an agent at a time and every non-temporally-indexical opinion of an agent at multiple times. Perhaps this is why it is easy to make the common mistake of supposing that a continuant person's credence function could be fully captured by a probability distribution over the same set of centered worlds at two distinct points in time. While the set of centered worlds is not limited in its temporal accessibility, to take a person's temporally indexical opinions to be adequately captured by a probability distribution over such a set at two distinct times is to ignore the crucial fact that *temporally indexical credence requires either present-attribution to the present time or self-attribution by a temporal part*. Hence, an adequate representation of an agent's credence function (or at least that part featuring *de praesenti* opinion) at a time requires either that the time at which the agent has the credence or the temporal part of the agent with the credence be an additional element in each *n*-tuple over which credence is distributed. Once this is done, however, it is again clear that the function cannot have a constant domain over time.

Here is Lewis apparently making the mistake against which I have just warned:

> [I]t is interesting to ask what happens to decision theory if we take all attitudes
>
> as *de se*. Answer: very little. We replace the space of worlds by the space of
>
> centered worlds, or by the space of inhabitants of worlds. All else is just as
>
> before. Whatever the points of the space of probabilities may be, we have
>
> probability distributions over the space and assignments of utility values to the

points. For any rational agent at any time there is a pair of a probability

distribution and a utility assignment. The probabilities change under the impact

of his perception. (1979, 534).

If we restrict ourselves to attitudes *de se* of continuants, Lewis is correct. When it comes

to credence *de praesenti*, however, all else is not as before. This is not because, as some have

suggested in connection with cases like the clock-watcher, conditionalizing on newly acquired

de praesenti certainties can yield unpalatable results. Rather, it is because the space of centered

worlds is simply inadequate as a representation of a continuant's *de praesenti* opinions.

7. Credence De Se and the Doppelganger Case

The literature on Sleeping Beauty contains other cases, structurally similar in various

respects to the original case, but in which the uncertainty at issue is (continuant) *de se*

uncertainty rather than *de praesenti* uncertainty (Elga 2004; Arntzenius 2003; Kierland & Monton

2005; Titelbaum 2008). In one such variant, "the Doppelganger Case," the experimenters

awaken Beauty on Monday morning if a fair coin falls heads and, if the coin falls tails, rather

than awakening her on an additional occasion, they duplicate her and awaken both Beauty and

her doppelganger (separately) on Monday morning. Such cases are often treated as mere

variations on the original case and thought to require the same arguments and verdicts. We are

now in a position to see that this is a mistake.

Consider how such a case would have to be structured for an argument analogous to

Elga's to apply. The case would have to be one in which the experimenters tell Beauty, soon

after awakening, that she is Beauty. Let BEAUTY be the proposition that one is oneself Sleeping Beauty. Let P and P+ be Beauty's credence function upon awakening and upon being informed that she is Beauty. The set of claims analogous to the original inconsistent set would then be:

(1D) $P(HEADS) = 1/2$

(2D) $P^+(HEADS) = 1/2$

(3D) $P^+(HEADS) = P(HEADS/BEAUTY)$

(4D) $P(HEADS/not\text{-}BEAUTY) = 0$

(5D) $0 < P(BEAUTY) < 1$

Taken as epistemic probability claims, it seems to me that (4D) and (5D) are as overwhelmingly plausible as their analogs in the original Sleeping Beauty case. However, my argument against the possibility of conditionalizing on MONDAY rests on the fact that MONDAY is a temporally indexical claim whereas BEAUTY is not a temporally indexical claim, but a personally indexical one. Nothing in my argument implies that there is any difficulty in conditionalizing on (mere) knowledge *de se*. The essential difficulty with conditionalizing on credence *de praesenti*—the radically synchronic nature of such credence—simply does not afflict credence *de se*.[23] Hence, there appears no barrier to conditionalizing on BEAUTY and my

---

[23]   Though some *de se* knowledge cannot be conditionalized on for a quite different reason, as is

    shown in Pust (2007).

argument against (3) does not apply to (3D). Friends of conditionalization must, it seems,

choose between (1D) and (2D).

While this is sufficient to demonstrate that the original case and the Doppelganger case

should be treated differently, notice also that (2D) lacks the intuitive grounds adduced in favor

of (2). Beauty is, in the Doppelganger case, certain on Monday that the coin toss lies in the past

rather than, as in the original case, the future. Furthermore, if one thinks (2D) plausible because

in the Doppelganger case Beauty has the same total (relevant) evidence on Sunday and on

Monday afternoon (having been told she is Beauty), this supports the denial of (1D) only by

appeal to a fact which does not obtain in the original case as it amounts to treating the

Doppelganger case as a novel case of forgetting. While I would resist the claim that Beauty has

forgotten when she awakens on Monday morning in original case, on the grounds that

forgetting requires going from full (or high) credence in a proposition to some lower credence

in the very same proposition, these grounds do not apply to the Doppelganger case.[24] So long

as we are considering *merely de se* and not *de se cum de praesenti* knowledge, it is natural to

suppose that Beauty has, upon awakening in the Doppelganger case, forgotten who she is.

After all, she knows who she is, and then she does not. Hence, there is nothing surprising about

the view that her credence upon awakening ought to be different from her Sunday credence

and this supports the denial of (1D).

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[24]   See Monton (2002) for an attempt to defend the thirder on the grounds that Beauty forgets

her temporal location from Sunday to Monday morning. See Draper (2007) for a version of

the objection I offer above.

So, upon examination, the Doppelganger case is, while in some sense "structurally identical" to the original Sleeping Beauty case, different in significant ways. I have provided an independent argument, based on the nature of credence *de praesenti*, against the claim that (3) follows from the principle of conditionalization, but (mere) credence *de se* is not essentially synchronic and so no similar argument can be produced against (3D). Furthermore, (2D) is only plausible, if at all, on grounds quite different from those that support (2) and such grounds also support the denial of (1D). Hence, the differences are significant enough that we ought not be tempted to endorse the same verdicts about the original Sleeping Beauty case and the Doppelganger case.

8. *The Irrelevance of Temporally Indexical Knowledge*

I have made the case that temporally indexical knowledge simply cannot be conditionalized upon. Any plausible normative constraint on the temporal evolution of a person's credence function must hold that the credence function is that of the same person at two distinct times. On the doctrine of propositions of limited temporal accessibility, we may suppose that we have the same person at two distinct points in time, but conditionalization on such propositions is precluded by the fact that we cannot take the domain of the function to be constant. On the triadic propositionalist model invoking temporally indexical guises, we may suppose we have the same person and the same propositions at two distinct points in time, but the domain of the (more complex) function involves a distinct element (temporally indexical guises) which necessarily relate a person to different propositions at different times and so we again lack a constant domain. The same is true on the present-attribution and self-attribution

by temporal parts models. On the present-attribution model, we have constant properties and

constant persons, but lack constant objects of attribution (i.e. times) over time. On the model

featuring self-attribution by temporal parts of persons, we have, waiving objections to a

temporal parts metaphysic of continuant persons, constant persons and properties, but lack

constant objects of attribution (i.e. temporal parts) over time.

   As noted at the beginning of this paper, many have thought that temporally indexical

credence falsifies the principle of conditionalization because they have assumed that rational

changes in one's temporally indexical views can produce, as Arntzenius claims, a "shifting" of

one's credence function over a constant space of possibilities. That is, they have assumed that in

cases like the clock-watcher, an agent's new credence function assigns 1 to an item the agent's

previous credence function assigned 0 and assigns 0 to an item the agent's previous credence

function assigned 1.[25] It should now be clear that there simply is no such thing as shifting in

this sense, as the set of possibilities over which temporally indexical credence is distributed

cannot remain constant over time. Hence, temporally indexical credence doesn't falsify the

principle of conditionalization by implying the absurd result that all agents ought to have

entirely undefined credences. That absurdity follows only if we assume that all objects of

credence are equally accessible to all rational agents at all times. Instead, in cases of newly

acquired temporally indexical knowledge, the new item in which credence is invested was *not*

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[25]   I conjecture that many philosophers have been misled by adopting the centered worlds

   framework without attending to its property-attributionist interpretation and motivation.

   See, for example, Arntzenius (2003, 367), Hitchcock (2004, 411), and Meacham (2008, 246).

even in the domain of one's previous credence function.[26] So, the principle faces no threat from temporally indexical knowledge as one can have no prior probability whatsoever (not even zero) for that item with a present probability of one.[27] Furthermore, insofar as we link evidential relevance to the possibility of conditionalizing, it also appears clear that temporally indexical knowledge is, in itself, evidentially irrelevant. Whether such knowledge can ever be evidentially relevant seems to me to depend on the possibility of direct inference based upon it.

In closing, let us return briefly to Sleeping Beauty. Thirders claim, plausibly, that P^+(HEADS) = 1/2. However, P^+ cannot be the product of conditionalization and so the Elga and Arntzenius-Dorr arguments provide no reason to hold, contrary to Lewis, that P(HEADS) ≠ 1/2. Lewis claims, plausibly, that P(HEADS) = 1/2. Again, however, without the claim that Beauty should conditionalize to arrive at P^+, there is no reason to hold that, contrary to the thirders, P^+(HEADS) ≠ 1/2. Without the link provided by (3) we are free to hold that P(HEADS) =

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[26] This account is further supported by the fact that standard diachronic Dutch Book arguments for conditionalization fail when it comes to *de praesenti* opinion as it is impossible gain the diachronic access to the same proposition, proposition-guise pair or object of property-attribution required for betting at two distinct times.

[27] It also follows that van Fraassen's Reflection principle (1995) is, contrary to Arntzenius (2003, 370) and Elga (2000, 146), unthreatened by temporally indexical credence.

P^+(HEADS) = 1/2. We are, in other words, free to endorse the intuitively plausible (1) *and* the intuitively plausible (2) and thereby to accept 1/2 without embarrassment.[28]

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[28] A complete defense of the double-halfer would require discussion of other arguments for the thirder position such as Horgan's (2004) "Synchronic Updating" argument, Hitchcock's (2004) diachronic Dutch book argument, and Pollock and colleagues' (Arico *et. al.* 2008) direct inference argument. Horgan's account avoids appeal to a diachronic conditionalization principle but must appeal to something like logical probabilities defined over indexical possibilities (Horgan 2008). I argue in [reference omitted for blind review] that the accounts of indexical credence surveyed in the present paper yield logical probability values different from those which Horgan's proposal requires. Furthermore, given the arguments of the present paper, a diachronic Dutch book argument against the halfer is successful only if causal decision theory is correct. (See Draper & Pust (2008). They are mistaken, however, in claiming (286, fn. 5) that there is a diachronic Dutch book argument against the position of the double-halfer which even an evidential decision theorist must accept). A response to Pollock's argument must await another occasion as it would require a detailed discussion of difficult issues surrounding direct inference and indexical reference.

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