Accuracy and Evidence

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ABSTRACT

In "A Nonpragmatic Vindication of Probabilism", Jim Joyce argues that our credences should obey the axioms of the probability calculus by showing that, if they don't, there will be alternative credences that are guaranteed to be more accurate than ours. But it seems that accuracy is not the only goal of credences: there is also the goal of matching one's credences to one's evidence. I will consider four ways in which we might make this latter goal precise: on the first, the norms to which this goal gives rise act as 'side constraints' on our choice of credences; on the second, matching credences to evidence is a goal that is weighed against accuracy to give the overall cognitive value of credences; on the third, as on the second, proximity to the evidential goal and proximity to the goal of accuracy are both sources of value, but this time they are incomparable; on the fourth, the evidential goal is not an independent goal at all, but rather a byproduct of the goal of accuracy. All but the fourth way of making the evidential goal precise are pluralist about credal virtue: there is the virtue of being accurate and there is the virtue of matching the evidence and neither reduces to the other. The fourth way is monist about credal virtue: there is just the virtue of being accurate. The pluralist positions lead to problems for Joyce's argument; the monist position avoids them. I endorse the latter.

Jim Joyce has argued that our credences should obey the axioms of the probability calculus by showing that, if they don't, there will be alternative credences that are guaranteed to be more accurate than ours (Joyce 1998). But it seems that accuracy is not the only goal of credences: there is also the goal of matching one's credences to one's evidence. I will consider four ways in which we might make this latter goal precise: on the first, the norms to which this goal gives rise act as 'side constraints' on our choice of credences; on the second, matching credences to evidence is a goal that is weighed against accuracy to give the overall cognitive value of credences, so that one might trade off the evidential goal against the goal of accuracy in the service of obtaining greater overall cognitive value; on the third, as on the second, proximity to the evidential goal and proximity to the goal of accuracy are both sources of value, but this time they are incomparable, so they cannot be weighed against one another; on the fourth, the evidential goal is not an independent goal at all, but rather a byproduct of the goal of accuracy. All but the fourth way of making the evidential goal precise are pluralist about credal virtue: there is the virtue of being accurate and there is the virtue of matching the evidence and neither reduces to the other. The fourth way is monist about credal virtue:

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there is just the virtue of being accurate; any norms that seem to follow from the evidential goal in fact follow from the goal of accuracy. The pluralist positions lead to problems for Joyce's argument; the monist position avoids them. I endorse the latter.

1. Joyce's argument for probabilism

We represent an agent's cognitive state at a given time by her *credence function* at that time: this is the function c that takes each proposition about which the agent has an opinion and returns the real number that measures her *degree of belief* or *credence* in that proposition. By convention, we represent minimal credence by 0 and maximal credence by 1. Thus, c is defined on the set $\mathcal{F}$ of propositions about which the agent has an opinion; and it takes values in [0, 1]. If X is in $\mathcal{F}$, then $c(X)$ is our agent's degree of belief or credence in X. Throughout, we assume that $\mathcal{F}$ is finite. With this framework in hand, we can state the norm of Probabilism:

> **Probabilism** At any time in an agent's credal life, it ought to be the case that her credence function c at that time is a probability function.[1]

How do we establish this norm? Jim Joyce offers the following argument: It is often said that the aim of full belief is truth. One way to make this precise is to say that the ideal doxastic state is that in which one believes every true proposition about which one has an opinion, and one disbelieves every false proposition about which one has an opinion. That is, the ideal doxastic state is the *omniscient doxastic state* (relative to the set of propositions about which one has an opinion). We might then measure how good an agent's doxastic state is by its proximity to this omniscient state.[2]

Joyce's argument – as I will present it – is based on an analogous claim about credences. We say that the ideal credal state is that in which our agent assigns credence 1 to each true proposition in $\mathcal{F}$ and credence 0 to each false proposition in $\mathcal{F}$. By analogy with the doxastic case, we might call this the *omniscient credal state* (relative to the set of propositions about which she has an opinion). Let $\mathcal{W}_{\mathcal{F}}$ be the set of possible worlds relative to $\mathcal{F}$: that is, the set of consistent assignments of truth values to the propositions in $\mathcal{F}$. Now, let w be a world in $\mathcal{W}_{\mathcal{F}}$. Then let v_w be the omniscient credal state at w: that is,

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[1] If $\mathcal{F}$ is an algebra and $c : \mathcal{F} \to [0, 1]$, then c is a probability function if

(i) $c(\bot) = 0$ and $c(\mathrm{T}) = 1$.

(ii) $c(A \lor B) = c(A) + c(B)$ if A and B are mutually exclusive.

If $\mathcal{F}$ is not an algebra and $c : \mathcal{F} \to [0, 1]$, then c is a probability function if it can be extended to a probability function over an algebra that contains $\mathcal{F}$.

[2] See Easwaran (ms) for a fascinating description of the consequences of such an account of full beliefs.

$$v_w(X) := \begin{cases} 0 & \text{if } X \text{ is false at } w \\ 1 & \text{if } X \text{ is true at } w \end{cases}$$

We then measure how good an agent's credal state is by its proximity to the omniscient state. Following Joyce, we call this the *gradational accuracy* of the credal state (or, to abbreviate, its *accuracy*). To do this, we need a measure of distance between credence functions. Many different measures will do the job, but here I will focus on the most popular, namely, Squared Euclidean Distance. Suppose c and c' are two credence functions. Then define the Squared Euclidean Distance between them as follows ('Q' stands for 'quadratic'):

$$Q(c, c') := \sum_{X \in \mathcal{F}} (c(X) - c'(X))^2$$

That is, to obtain the distance between c and c', we consider each proposition X in $\mathcal{F}$ on which they are defined; we take the difference between the credences they assign to X; we square that difference; and we sum the results. Thus, given a possible world w in $\mathcal{W}_\mathcal{F}$, the cognitive badness or disvalue of the credence function c at w is given by its inaccuracy; that is, the distance between c and v_w, namely, $Q(c, v_w)$. We call this the *Brier score* of c at w, and we write it $B(c, w)$. So the cognitive value of c at w is the negative of the Brier score of c at w; that is, it is $-B(c, w)$. Thus, B is a measure of inaccuracy; $-B$ is a measure of accuracy.

  With this measure of cognitive value in hand, Joyce argues for Probabilism by appealing to a standard norm of traditional decision theory:

Dominance Suppose $\mathcal{O}$ is a set of options, $\mathcal{W}$ is a set of possible worlds, and U is a measure of the value of the options in $\mathcal{O}$ at the worlds in $\mathcal{W}$. Suppose o, o' in $\mathcal{O}$. Then we say that

(a) *o strongly U-dominates o'* if $U(o', w) < U(o, w)$ for all worlds w in $\mathcal{W}$
(b) *o weakly U-dominates o'* if $U(o', w) \leq U(o, w)$ for all worlds w in $\mathcal{W}$ and $U(o', w) < U(o, w)$ for at least one world w in $\mathcal{W}$.

Now suppose o, o' in $\mathcal{O}$ and

 (i) *o strongly U-dominates o'*;
(ii) There is no o'' in $\mathcal{O}$ that weakly U-dominates o.

Then o' is irrational.

Of course, in standard decision theory, the options are practical actions between which we wish to choose. For instance, they might be the various environmental policies that a government could pursue; or they might be the medical treatments that a doctor may recommend. But there is no reason why **Dominance** or any other decision-theoretic norm can only determine the irrationality of such options. They can equally be used to establish the irrationality of accepting a particular scientific theory or, as we will see, the irrationality of particular credal states. When they are

put to use in the latter way, the options are the possible credal states an agent might adopt; the worlds are, as above, the consistent assignments of truth values to the propositions in $\mathcal{F}$; and the measure of value is $-B$, the negative of the Brier score. Granted that, which credal states does **Dominance** rule out? As the following theorem shows, it is precisely those that violate **Probabilism**.

THEOREM 1. For all credence functions c:

(I)   *If c is not a probability function, then there is a credence function c* that is a probability function such that c* strongly Brier-dominates c.*

(II)   *If c is a probability function, then there is no credence function c* such c* weakly Brier-dominates c.*

This, then, is Joyce's argument for **Probabilism**:

(1)   The cognitive value of a credence function is given by its proximity to the ideal credence function:
         (i) The ideal credence function at world w is v_w.
         (ii) Distance is measured by the Squared Euclidean Distance.
         Thus, the cognitive value of a credence function at a world is given by the negative of its Brier score at that world.

(2)   **Dominance**

(3)   Theorem 1

Therefore,

(4)   **Probabilism**

In fact, Joyce weakens (1)(ii) and thus strengthens the argument (Joyce, 1998, section 4). But that is not relevant for our purposes, so we leave it as it is.

2. Alethic goals and evidential goals

In the remainder of the paper, I wish to consider the following sort of objection to Joyce's argument: Joyce's argument seems to rely on the assumption that the only epistemic virtue is accuracy, or proximity to the omniscient credences; but, one might think, there is at least one other epistemic virtue, namely, the virtue of matching one's credences to one's evidence. That is, Joyce's argument appeals only to the *alethic goal* of credences, namely, the goal of getting closer to the omniscient credences; but there is also an *evidential goal*, namely, the goal of proportioning belief to evidence. Thus, this objection runs, Joyce's argument for **Probabilism** is flawed in the way that a scientist's reasoning is flawed if she argues for her favoured hypothesis by pointing out only that it is simpler than all existing rivals. Since there are scientific virtues other than simplicity, her argument holds little weight. Simi- larly, the objection goes, Joyce's argument holds little weight, since it appeals only

to the virtue of accuracy when there are other relevant virtues. This objection was considered and quickly dismissed in Leitgeb and Pettigrew (2010, 244–245): it was stated with much greater force and precision in Easwaran and Fitelson (2012). I follow Easwaran and Fitelson in calling it the *evidentialist objection*.

In what follows, I will consider four views of the evidential goal of credences: the *Side Constraints View*; the *Competing Virtues View*; the *Incomparable Virtues View*; and the *Reductive View*. On the Side Constraints View, the goal of matching the evidence trumps all other goals. That is, there is no trade-off between the extent to which one achieves that goal and the extent to which one achieves another goal, such as the alethic goal: no proximity to the alethic goal can ever justify deviation from the evidential goal. In this respect, it is quite unlike the way in which scientific virtues are treated: in that situation, even empirical adequacy is not treated as a side constraint. Thus, as its name suggests, the Side Constraints View is analogous to certain Kantian or rights-based views in ethics in which the categorical imperative or certain basic rights behave as side constraints in any moral decision-making; they cannot be trumped by consequentialist considerations. The Competing Virtues View, on the other hand, is closer to (certain sorts of) consequentialism. On this view, the degree to which the evidential goal is achieved is weighed against the degree to which the alethic goal is achieved to give the overall cognitive value of the credences in question. The Incomparable Virtues View is akin to a view in ethics on which the utilities of individual agents are incomparable. On this view, it is valuable to be accurate and it is valuable to respect one's evidence. But these values are incomparable; they cannot be combined to give an overall cognitive value, and they cannot be weighed against one another. Finally, the Reductive View resembles hedonistic versions of utilitarianism in ethics. On this view, the evidential goal is eliminated, just as the talk of rights and the categorical imperative is eliminated in hedonistic utilitarianism. The only virtue is accuracy, just as for the utilitarians, the only virtue is high hedonistic utility. Any norm that might initially seem to follow from the virtue of matching credences to the evidence is instead derived from the virtue of accuracy, just as the utilitarian might attempt to recover certain norms that seem to follow from rights or from the categorical imperative.

I will argue that the Side Constraints View, the Competing Virtues View, and the Incomparable Virtues View create insurmountable problems for Joyce's argument for Probabilism. But the Reductive View saves the argument. I endorse the Reductive View.

3. The side constraints view

Easwaran and Fitelson (2012) raise a powerful objection to Joyce's argument based on the Side Constraints View. In this section, I spell out their objection more fully; I consider an objection and rebut it.

Probabilism is a very general norm: it applies to any agent at any time in her credal life. Thus, if it is to work, Joyce's argument for that norm must apply to any agent at any time. Easwaran and Fitelson object that it doesn't. Consider the following agent:

> **Taj** Taj has opinions about only two propositions, X and $\neg X$. Her credence function is $c(X) = 0.6$, $c(\neg X) = 0.6$.

According to Joyce's argument, Taj is irrational because there are credal states that strongly accuracy dominate hers and that are not themselves weakly accuracy dominated: for instance, $c^*(X) = 0.5$, $c^*(\neg X) = 0.5$, as Figure 1 illustrates.

However, suppose that Taj knows that the chance of X is 0.6 – and suppose that she does not thereby know that the chance of $\neg X$ is 0.4. Suppose, moreover, that
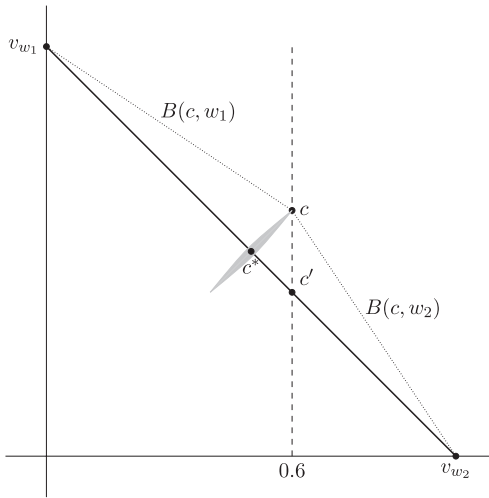

Figure 1. Since Taj has credences in only two propositions, we can represent her credence function c – as well as the two omniscient credence functions v_{w_1} and v_{w_2} – by points on the Euclidean plane. On this representation, the Brier score of c at v_{w_1}, for instance, is the square of the Euclidean distance between the points that represent them. The thick diagonal line represents the credence functions that satisfy **Probabilism**; the dashed vertical line represents the credences that satisfy the **Principal Principle**; and the grey shaded area represents the credence functions that accuracy dominate c. Joyce's argument turns on the fact that the grey area and the thick line intersect. Thus, there are probabilistic credence functions that accuracy dominate c. Indeed, c^* is such a credence function. Easwaran and Fitelson's objection turns on the fact that the shaded grey area and the dashed line do not intersect. Thus, none of the credence functions that accuracy dominate c satisfy the **Principal Principle**. Indeed, the only credence function that satisfies Probabilism and the Principal Principle is $c'(X) = 0.6$, $c'(X) = 0.4$.

she knows nothing more. Then, according to another norm that governs credences, David Lewis' Principal Principle, our agent ought to have credence 0.6 in *X* (Lewis 1980):[3]

> **Principal Principle** If an agent knows that the chance of *X* is *r*, and nothing more, then she ought to have $c(X) = r$.

Now, as Figure 1 makes clear, while Taj's current credence function satisfies **Principal Principle**, none of the credences that dominate it do. Thus, while *c* is strongly dominated by c^*, and c^* is not weakly dominated by anything, c^* is ruled out because it violates **Principal Principle**. This, Easwaran and Fitelson contend, blocks the application of dominance reasoning, and thus the argument to **Probabilism**.

> Joyce's argument tacitly presupposes that – for any incoherent agent *S* with credence function *b* – some (coherent) functions *b′* that [accuracy] dominate *b* are always 'available' as 'permissible alternative credences' for *S*. (Easwaran and Fitelson 2012, 430)

Thus, Easwaran and Fitelson seem to be proposing three things: first, we need to weaken **Dominance** to ensure that a dominated option is only ruled out as irrational if it is dominated by another option that is *both* not itself dominated *and* 'available' or a 'permissible alternative'; second, they claim that an evidential norm renders any credence function that violates it 'unavailable' or 'impermissible' in the sense relevant to the dominance norm; and third, they claim that there are evidential norms – such as **Principal Principle** – that permit incoherent credence functions that are dominated only by credence functions that violate the norm.[4] Here is the reformulation of **Dominance**:

> **Dominance⁻** Suppose $\mathcal{O}$ is a set of options, $\mathcal{W}$ is a set of possible worlds, and *U* is a measure of the value of the options in $\mathcal{O}$ at the worlds in $\mathcal{W}$. Suppose *o*, *o′* in $\mathcal{O}$ and
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>   (i) *o* strongly dominates *o′*
> (ii) There is no *o″* in $\mathcal{O}$ that weakly dominates *o*.
> (iii) *o* is 'available' or 'permissible'.
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> Then *o′* is irrational.

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[3] Note that this is the formulation of the Principal Principle to which Easwaran and Fitelson appeal. Lewis' formulation is in terms of conditional probabilities and it applies only to an agent's initial credence function. Together with Conditionalization, it gives the norm discussed here.

[4] It is this final claim with which Joyce takes issue (Joyce ms). He denies that there can be such an evidential norm. If he is right, the evidentialist objection to the accuracy argument for Probabilism is blocked. In this paper, I explore what can be said if, instead, Easwaran and Fitelson are correct to say that **Principal Principle** is an evidential norm.

If **Dominance⁻** is the strongest norm in the vicinity, and if evidential norms can indeed rule out credence functions as impermissible in the sense relevant to **Dominance⁻**, then Joyce's argument is blocked.

Easwaran and Fitelson's argument is based on what I call the Side Constraints View of evidential norms. There is a clear analogy between this view and Nozick's appeal to side constraints in ethics (Nozick 1974, chapter 3). On Nozick's view, we temper consequentialism by appealing to the rights of the agents affected by our actions: these rights impose side constraints on action in much the same way that Easwaran and Fitelson claim evidential norms impose side constraints on credence functions. Thus, in the trolley problem, we might save five track workers by turning the trolley and killing one of their co-workers, thereby causing a better (if nonetheless still tragic) consequence. But, on Nozick's view, we must not do that. The dominating action in this case is ruled out because it violates the side constraints imposed by the rights of the co-worker.

Before moving on to the Competing Virtues View of evidential norms, we stop to consider an objection to Easwaran and Fitelson's argument. The objection agrees with Easwaran and Fitelson that evidential norms impose side constraints on our credences. And it agrees that any credence function that violates these side constraints is irrational. But it denies that they are thereby impermissible *in the sense relevant to* **Dominance⁻**. Thus, on this view, **Dominance⁻** and the evidential norms are distinct norms that function entirely independently of one another. **Dominance⁻** rules out as impermissible any option that is dominated by one that isn't itself dominated; and the evidential norms rule out anything that violates them. An option is then irrational if it is ruled out by one or by the other or by both. Thus, in the case of Taj, the only credence function that isn't ruled irrational is $c'(X) = 0.6$, $c'(\neg X) = 0.4$ (see Figure 1).

Admittedly, this account gives what we take to be the correct result in the case of Taj: we agree that only c' is rational. But it does not do so in general. For instance, when this account of side constraints and dominance norms is applied to the trolley problem, it rules out both actions – killing one or letting five die – as impermissible: letting five die is dominated by killing one; but killing one violates the side constraints. Thus, applied in general, this account violates the meta-normative principle of ought-can: no norm can rule out every option as irrational.

However, a deeper problem emerges when we see the effects of this account in cases such as Taj's, where it does not violate ought-can. In her situation, this account of how side constraints contribute to rational choice renders c irrational, but it doesn't render c' irrational. However, if we compare the features of c and c', it isn't clear what makes the latter better than the former.

- c and c' both satisfy the evidential side constraints.
- c is dominated, but only by options that violate the side constraints; c' is not dominated by any options.
- c' is more inaccurate than c at w_1 but less inaccurate than it at w_2 – i.e., $B(c', w_1) > B(c, w_1)$ but $B(c, w_2) > B(c', w_2)$. Thus, c' exposes the agent to a risk of higher inaccuracy than c does; though it also presents the possibility of lower inaccuracy.

So, to say that c' is better than c in some sense strong enough to render the latter irrational and not the former, the proponent of this account of side constraints must say that being dominated is bad in itself and indeed bad enough to render the risk of greater inaccuracy mandatory. And here I think we come to the crux of the matter. Being dominated is *not* bad in itself. The badness of being dominated depends on the status of the options that do the dominating. To see this, consider our initial formulation of the dominance norm as **Dominance**:

Dominance Suppose o, o' in $\mathcal{O}$ and

 (i) o strongly dominates o'
 (ii) There is no o'' in $\mathcal{O}$ that weakly dominates o.

Then o' is irrational.

If being dominated were bad in itself, we needn't have included condition (ii). We might have strengthened **Dominance** by weakening (ii) to give:

(ii′) There is o'' in $\mathcal{O}$ that is not weakly dominated by any option in $\mathcal{O}$.

Call the resulting norm **Dominance⁺**. It obeys the ought-can principle; and it treats being dominated as bad in itself. However, here is a counterexample to **Dominance⁺**:

- $\mathcal{O} = \{o, o_1, o_2, o_3, o_4, \ldots\}$
- $\mathcal{W} = \{w_1, w_2\}$

U	o	o_1	o_2	o_3	o_4	$\ldots$	o_n	$\ldots$
w_1	2	1	2	3	4	$\ldots$	n	$\ldots$
w_2	2	1	$2 - \dfrac{1}{2}$	$2 - \dfrac{1}{4}$	$2 - \dfrac{1}{8}$	$\ldots$	$2 - \dfrac{1}{2^{n-1}}$	$\ldots$

In this example, o is the only option that is not strongly dominated. But this shouldn't rule out the others as irrational. One way to see this is to observe that, for each o_n ($n \geq 3$), there is a non-extremal probability function that assigns a greater expected utility to o_n than to o.

The lesson of this example is that what renders an option irrational is being dominated by an option *that isn't itself then ruled out because it is also dominated.*

o_3 is dominated by each o_n ($n > 3$); but, since each o_n ($n > 3$) is dominated by each o_m ($m > n$), this does not rule out o_3 as irrational. Analogously, c shouldn't be ruled out as irrational just because it is dominated; after all, the options that dominate c are all themselves ruled out because they violate the side constraints. This supports the move to **Dominance⁻**; and it supports Easwaran and Fitelson's contention that side constraints rule out options as impermissible *in the sense required by Dominance⁻*. Thus, the objection is defeated and, on the Side Constraints View, Joyce's argument is blocked.

4. The competing virtues view

In ethics and in epistemology, a side constraint is a constraint that must not be violated no matter how much else of value could be acquired by doing so. Thus, on the Side Constraints View of evidential norms, no gain in accuracy however large could justify violating the Principal Principle. And in ethics, Nozick must say that it is wrong to kill one to save any number of others. In ethics, this is characteristic of Kantian views. The Competing Virtues View of evidential norms, which we consider in this section, is, by contrast, analogous to a traditional sort of consequentialist view. On such a view, while a person's rights are important and it is valuable to respect them, they are not inviolable: there are other sources of value – such as pleasure, beauty, equality – and we might rationally choose to violate a person's rights in order to acquire more of the value those sources can provide.[5] Analogously, we might think that while it is valuable to satisfy the evidential norms, one can rationally opt to violate them in order to obtain more accuracy. Let's see how we might make this precise.

Evidence constrains our credences. For instance, according to **Principal Principle**, knowing that the chance of X is r (and nothing more) constrains an agent to have $c(X) = r$. Thus, let us represent a body of evidence by the set of credence functions that count as responding properly to that evidence. So we represent the evidence provided by knowing that the chance of X is r (and nothing more) by the following set:

$$\mathcal{E} = \{c : c(X) = r\}$$

With this representation in hand, we might make the evidential goal of credences precise by saying that is it a goal of a credal state to lie in the set $\mathcal{E}$ of credence functions that respond properly to the available evidence. And we might say that the goodness of a credence function is determined at least in part by its proximity to that goal. How might we measure that proximity? Using the standard

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[5] Not all consequentialist views have this feature. Indeed, many can replicate the deontic consequences of a 'side constraints' view such as Nozick's. See Brown (2011) for the strengths and limitations of this 'consequentializing' strategy.

mathematical definition of the distance between a point and a set of points, we might naturally define:[6]

$$E(c, \mathcal{E}) := \inf\{Q(c, c') : c' \in \mathcal{E}\}$$

where Q is Squared Euclidean Distance, as defined earlier. And we might say that $E(c, \mathcal{E})$ is the distance from c to the evidential goal and $-E(c, \mathcal{E})$ is its proximity to that goal.

We might then say that the overall cognitive value of a credence function c at a world w in $\mathcal{W}_{\mathcal{F}}$ and in the presence of evidence $\mathcal{E}$ is:

$$C(c, \mathcal{E}, w) := -B(c, w) - E(c, \mathcal{E})$$

That is, it is the sum of its proximity to the *alethic goal of credences* – that is, matching the omniscient credences – and its proximity to *the evidential goal of credences* – that is, responding properly to the evidence.

Consider Taj again. This is the case that poses problems for Joyce's argument. Taj's credence function is $c(X) = 0.6$, $c(\neg X) = 0.6$; and she knows that the chance of X is 0.6. So

$$\mathcal{E} = \{c : c(X) = 0.6\}$$

Now, let $c^*(X) = 0.5$, $c^*(\neg X) = 0.5$. Then it is straightforward to calculate

C	w_1	w_2
c	−0.52	−0.52
c^*	−0.51	−0.51

Thus, c^* dominates c relative to C. Moreover, c^* is not itself even weakly dominated by any other credence function. Thus, on this account, c is indeed rendered irrational by **Dominance**, as Joyce requires.

However, this victory is short-lived. Consider the credence function $c_1(X) = 0.8$, $c_1(\neg X) = 0$. It clearly violates **Probabilism**, yet it is not dominated by any other credence function: it is, in fact, the credence function for which $C(\cdot, \mathcal{E}, w_1)$ achieves its maximum. Similarly, $c_2(X) = 0.6$, $c_2(\neg X) = 1$ is the credence function for which $C(\cdot, \mathcal{E}, w_2)$ achieves its maximum. Thus, c_1 and c_2 are not dominated. So, while we might appeal to C to provide a response to Easwaran and Fitelson's chosen example, it will not take us any further. Thus, on this version of the Competing Virtues View, Joyce's argument fails.

Of course, this is not the only way to make the Competing Virtues View precise. We could combine the evidential value $-E(c, \mathcal{E})$ of a credence function

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[6] The *infimum* of a set of real numbers S (written inf S) is the greatest lower bound on S.

c with its accuracy $-B(c, w)$ in some way other than by adding them together. We could take a weighted sum or some other binary function; or, we could alter our measure of proximity to the evidential goal. It turns out that opting for a weighted sum will not help (providing one gives at least some weight to both $-B$ and $-E$): there will always be incoherent credence functions that are not dominated relative to such an overall cognitive value function. Perhaps another alternative will help. But she who defends Joyce's argument in this way will have to motivate that alternative on independent grounds; moreover, she must argue that the way of combining $-B$ and $-E$ considered in this section – that is, taking their straight sum or taking a weighted sum – is *impermissible* as an overall cognitive value function; it is not sufficient to show merely that some other way of combining those two functions is permissible.

5. The incomparable virtues view

The Incomparable Virtues View shares much in common with the Competing Virtues View.[7] On both, we have (i) a measure B of the distance of a credence function from the goal of accuracy, and thus $-B$ as a measure of one sort of value that a credence function can have; and (ii) we have a measure E of the distance of that credence function from the evidential goal, and thus $-E$ as a measure of a different sort of value that a credence function can have. On the Competing Virtues View, we can combine $-B$ and $-E$ to give an overall cognitive value function because, while they are measures of different sorts of values, they are nonetheless comparable. On the Incomparable Virtues View, on the other hand, there is no way of combining $-B$ and $-E$ to give an overall value function: on this view, they measure different and, moreover, incomparable values. This is analogous to a position in ethics that measures the goodness of an outcome for each individual in exactly the way that utilitarianism does (that is, by using that agent's own hedonistic utility function); but which denies that the outcome can be assigned an overall value by summing (or in some other way combining) the values it has for each individual.

The problem with such a view in ethics or in epistemology is to say how it constrains what we ought to do. If there is no way of weighing one good against another, we can't always compare all options with respect to overall goodness. Of course, that's not to say that we can't sometimes compare some options. For instance, if $c*$ is closer than c to both the evidential goal and the goal of accuracy at all worlds, then $c*$ is to be preferred to c. But the ordering of credence functions that this gives is typically very far from being complete. How are we to adjudicate between two credence functions that are not ordered by this condition?

The orthodox answer – which dates at least to Good (1952), and which is endorsed by Weirich (2004, section 4.4) – is this:

> **Caprice** An option is irrational if it is ruled out as irrational relative to *every* way of combining the two incomparable utility functions to give an overall value function.[8]

The idea behind Caprice is this: What is irrational is that which one has positive reason not to do.

This is a very permissive norm. Indeed, as those who endorse it are aware, it must be made stricter if it is to be used at different times (Weirich 2004; Weatherson ms). For our purposes, however, we can use the simpler norm as it is stated above. So stated, Caprice is so permissive that it blocks Joyce's argument. After all, as we saw in section 4, there is a way of combining $-B$ and $-E$ to give an overall cognitive value function – e.g., by summing – such that there are incoherent credence functions that are not ruled out as irrational relative to this value function. Thus, on the Incomparable Virtues View, Joyce's argument for Probabilism is blocked.

6. The reductive view

We have been considering the following objection to Joyce's argument for **Probabilism**: Joyce's argument assumes that the only goal of a credal state is accuracy; but there is at least one other goal, namely, matching the evidence. That is, Joyce's argument assumes a version of *credal virtue monism*. It is the version that Goldman (2001) calls "veritism". In that paper, Goldman defends the position by objecting to rival accounts of the relationship between alethic and evidential virtues. In this section, I wish to defend the monism required for Joyce's argument rather differently.

The major stumbling block for this version of credal value monism is that there *seem* to be evidential norms; and their force *seems* to stem from the evidential goals at which they demand that we aim. There are two ways to overcome this obstacle: on the first, we give an error theory of evidential norms and explain why we mistakenly think they hold; on the second, we agree that the evidential norms hold, but we show that they follow from the goal of accuracy alone, not from a distinct evidential goal. I will take the latter route. I call this the Reductive View. It is analogous to those versions of utilitarianism that insist that only utility is valuable, but try to recover some of the Kantian or rights-based norms by showing that, in the long run, actions that violate them have lower total utility.

My strategy is this: Recall the form of Joyce's argument for **Probabilism** as I presented it above. It has two components: the first is the claim that the measure of

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[8] Weirich calls this "Quantization". "Caprice" is the name given by Weatherson (ms) to the analogous rule for decision makers with indeterminate (or imprecise) probabilities.

accuracy – namely, the negative Brier score – measures cognitive value; the second is a decision-theoretic norm, namely, **Dominance**. From these, via a mathematical theorem, Joyce derives **Probabilism**. I claim that any evidential norm can be derived in the same manner. We retain the first component, namely, the identification of cognitive value with accuracy – this is the core of veritism. But we change the decision-theoretic norm. Different decision-theoretic norms give rise to different credal norms, some of which are the evidential norms we wish to recover.

Consider, for instance, the decision-theoretic norm of **Maximin**:

Maximin We say

(c) *o worst-case dominates o′ with respect to U* if the minimum utility of *o* is greater than the minimum utility of *o′*. That is,

$$\min\{U(o', w) : w \in \mathcal{W}\} < \min\{U(o, w) : w \in \mathcal{W}\}$$

Now suppose *o*, *o′* in $\mathcal{O}$ and

(i) *o* worst-case dominates *o′*;
(ii) There is no *o″* in $\mathcal{O}$ such that *o″* worst-case dominates *o*.

Then *o′* is irrational.

Thus, roughly, an option *o* is irrational if there is another option *o′* whose minimum value is greater than the minimum value of *o*. That is, **Maximin** asks us to look at the worst-case scenario for each option and pick one of those whose worst-case scenario is best. Then, as shown in Pettigrew (ms), when this norm is applied to the negative Brier score measure of cognitive value, we can derive the **Principle of Indifference**:

Principle of Indifference Suppose $\mathcal{F}$ is finite. Then, in the absence of evidence, the agent ought to have the uniform distribution over $\mathcal{F}$ as her credence function.

That is, in the absence of evidence, each possible world ought to be assigned the same credence.

Next, consider the following norm:

Chance Dominance Suppose $\mathcal{C} = \{ch_1, \dots, ch_n\}$ is the set of possible chance functions. We say

(d) *o chance dominates o′ with respect to U* if the objective expected utility of *o* is greater than the objective expected utility of *o′* relative to any chance function in $\mathcal{C}$. That is,

$$\sum_{w \in \mathcal{W}} ch_i(w)U(o, w) > \sum_{w \in \mathcal{W}} ch_i(w)U(o', w)$$

for all ch_i in $\mathcal{C}$.

Now suppose *o*, *o′* in $\mathcal{O}$ and

(i) *o* chance dominates *o'* with respect to *U*;

(ii) There is no *o''* in $\mathcal{O}$ such that *o''* chance dominates *o*.

Then *o'* is irrational.

Thus, roughly, if every possible chance function expects option *o* to be better than it expects option *o'* to be, then *o'* is irrational. Then, as shown in Pettigrew (2013), when this norm is applied to the negative Brier score, we derive something close to Lewis' version of the **Principal Principle**:

Principal Principle In the absence of evidence, an agent ought to have a credence function *c* such that

$$c(X \mid E \ \& \ \text{the objective chances are given by } ch_i) = ch_i(X|E)$$

This allows us to identify what is really wrong with Taj. It is not that her credence function *c* is dominated by credence functions such as *c** that are not themselves dominated. It is rather that there is another credence function – in fact, it is *c'* – such that the only possible chance function expects *c* to be more inaccurate than it expects *c* to be.[9]

Finally, consider the most familiar decision-theoretic norm:

Maximize Subjective Expected Utility Suppose our agent has probabilistic credence function *c*. We say

(e) *c expects o to have higher utility than it expects o' to have with respect to U* if

$$\sum_{w \in \mathcal{W}} c(w)U(o, w) > \sum_{w \in \mathcal{W}} c(w)U(o', w)$$

Now suppose *o*, *o'* in $\mathcal{O}$ and

(i) *c* expects *o* to have higher utility than it expects *o'* to have with respect to *U*;

(ii) There is no *o''* in $\mathcal{O}$ such that *c* expects *o''* to have higher utility than it expects *o* to have with respect to *U*.

Then *o'* is irrational.

Thus, roughly, an option *o'* is irrational if there is another option *o* that one's own probabilistic credence function expects to have higher utility than it expects *o'* to have. We will use this to establish a norm that governs not credences, but updating rules for credences – that is, rules that take an agent's current credence function and a new piece of evidence and return the posterior credence function the agent ought to have if she learns only that evidence. Thus, we need to adapt our measure of cognitive value for credence functions to give a measure of cognitive value for updating rules. But that is easily done. Suppose our agent knows that she will

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[9] In unpublished work, Joyce also endorses the diagnosis of Taj's cognitive failing given in Pettigrew (2013) – cf. Joyce (ms).

receive evidence from the partition $\mathcal{E} = \{E_1, \ldots, E_n\}$. Then, for each world, there is a unique piece of evidence she might obtain. Then the cognitive value of an updating rule at a given world is the cognitive value, at that world, of the credence function to which the updating rule would give rise if the agent were to learn the unique piece of evidence that she will obtain at that world. With this in hand, it is shown in Oddie (1997) – and, more fully, in Greaves and Wallace (2006) – that this decision-theoretic norm together with this measure of cognitive value for updating rules entails **Conditionalization**:

> **Conditionalization** If the agent has credence function c and knows she is about to receive evidence from the partition $\mathcal{E} = \{E_1, \ldots, E_n\}$, then she ought to plan to have credence function $c(\cdot|E_i)$ upon receipt of evidence E_i.

The **Principle of Indifference**, the **Principal Principle**, and **Conditionalization** are three of the most important and widely used evidential norms for credences. The results just cited show that they follow from the goal of accuracy. There is no need to appeal to a distinct goal of matching the evidence. As we might hope, by pursuing the goal of accuracy properly and in line with the decision-theoretic norms just stated, we thereby satisfy the evidential norms that are often cited in favour of a pluralistic view of epistemic virtue.[10]

Thus, on the Reductive View, there are evidential norms. But this doesn't show that credences aim to match the evidence. Rather, they aim to match the omniscient credal state. But, that very goal, together with standard norms that govern decisions between options, entails the evidential norms.

On the picture I have been sketching, Joyce's argument for **Probabilism** is fundamental. After all, **Dominance** is the weakest decision-theoretic norm of all: **Maximin**, **Chance Dominance**, and **Maximize Subjective Expected Utility** all entail **Dominance**. Thus, any other norm we apply will entail a norm that is itself stronger than **Probabilism**. **Probabilism** is the norm that can be deduced from any application of decision theory in the theory of credences. Thus, on the Reductive View, Joyce's argument for that norm is safe.

7. Conclusion

In sum: on the Side Constraints View and on the Competing Virtues View of evidential norms, Joyce's argument for Probabilism is blocked. However, on the Reductive View it succeeds. All that remains is to say which view is correct.

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[10] It might be objected that the decision-theoretic norms Maximin and Maximize Subjective Expected Utility are in tension: for certain choices, they will jointly rule out all options as irrational. But note that they are deployed here in different contexts: Maximin is used at the beginning of an agent's epistemic life, before she has set her credences – indeed, Maximin is what she uses to set those initial credences; Maximize Subjective Expected Utility is used after that – once the agent has a credence function in place – in order to fix her updating policy. For a discussion of the possible tension between Maximin and Chance Dominance, see Pettigrew (ms, section 6.3).

   I endorse the Reductive View. Of course, what I presented above is not itself an argument for that view. I have shown that three major evidential norms follow from the goal of accuracy. But that does not rule out the possibility that there is also an evidential goal that gives rise to the same norms. That is, it might be that the evidential norms are overdetermined: they follow from the evidential goal *and* from the alethic goal. Yet, in the light of the considerations above, parsimony tells in favour of veritism. The above arguments remove the motivation for positing an evidential goal as well as an alethic goal; and parsimony demands that we not multiply credal virtues unnecessarily.

   As well as parsimony, these arguments endow veritism with explanatory power. To explain evidential norms by positing an evidential goal for credences is akin to explaining the soporific effect of opium by positing a 'dormitive virtue', whereas there is considerable explanatory power in a theory that derives evidential norms directly from alethic goals. If we adapt Joyce's style of argument by replacing **Dominance** by other decision-theoretic norms, such as **Maximin**, **Chance Dominance**, and **Maximize Subjective Expected Utility**, we have such a theory.*

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