Comments on Antony Eagle, "Probability, Modality and Triviality"

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Antony's first thesis holds that "objective probability is a modal concept". I'll begin with some formal questions about what it would mean for this to be the case. The most precise implementation of the thesis in Antony's paper is the following. We define a modal operator $\Box_{Pr}$ which is a box-like operator over a limited class of metaphysically possible worlds, with the goal of achieving the biconditionals:

- $\Box_{Pr}\phi$ iff $Pr(\phi) = 1$

- $\Diamond_{Pr}\phi$ iff $Pr(\phi) > 0$

To do this, $\Box_{Pr}$ will have to be associated with a set of worlds W_{Pr} such that each w in W_{Pr} verifies every claim of probability 1 in the actual worlds. However, under plausible assumptions, there will be no non-empty such set of worlds. Suppose, for example, that a certain event in the actual world creates an object of a mass falling within some continuous range, with the probability distribution even across that continuum. Then claims of the form "The object does not have mass m", for specific m within the continuum, will have probability 1, but no possible world will verify all of those claims. More generally, if there's any way of partitioning the probability space into an exhaustive collection of measure 0 sets, we can associate propositions with the complements of each of those sets. The resulting propositions will each be of probability 1, but cannot all be made true together.

Antony, though, gives an independent characterization of the class of worlds W_{Pr}, which would make it non-empty. He suggests that W_{Pr} contain worlds in which we "[hold] fixed facts about the properties of a particular chance setup that make it possible to produce a phi outcome, plus the laws which dictate how that setup produces its outcomes". Now, there's an initial difficulty here, since the resulting set W_{Pr} would appear to be relative to a particular choice of ϕ, which would mean that no genuine operator $\Box_{Pr}$ is defined. Setting this point aside, the more significant difficulty is that, given the earlier observation, this set won't generate a modality verifying the biconditionals:

- $\Box_{Pr}\phi$ iff $Pr(\phi) = 1$

- $\Diamond_{Pr}\phi$ iff $Pr(\phi) > 0$

(That is, W_{Pr} won't give us the BCP and the Converse BCP). We could, of course, drop these biconditionals as desiderata for our modality, but then (a) it's not clear in what sense we are showing that probability is a modal, and (b) Antony's argument for the incompatibility of the three theses, which uses these biconditionals, will fail.

Another option here is to take W_{Pr} as Antony defines it, and then make $\Box_{Pr}$ into something short of a universal quantifier over worlds. W_{Pr} will, given the first observation, include worlds at which $Pr(\phi) < 1$. So we want our necessity operator not to require the truth of ϕ at those worlds. But taking this route carries a cost. The resulting "modal operators" will fail to verify the **K** axiom, making it dubious whether they are genuine modal operators at all.

A final option is to directly define a modality out of the probability structure, by simply defining $\Box_{Pr}$ to mean $Pr(\phi) = 1$, without introducing a corresponding collection of worlds. The resulting structure would appear to produce the modal logic **D**. However, it doesn't give any clear interpretation to iterated modalities, since such modalities require assessing the probabilities of probability claims. It's thus again not clear whether a genuine modal operator has been defined. More immediately problematic, there's no clear reason why this modality will interact with the modality of metaphysical possibility, as appealed to in the second thesis, to create a problem of the sort Antony wants.

I don't think anything decisively problematic for Antony's argument comes out of this line of considerations, but it does show that we don't have a fully precise picture of what it means for probability to be a modal concept. The more general idea that Thesis 1 simply tells us that probability facts depend not just on the status of the actual world, but on that of various possible worlds as well, I find less helpful, since it relies on a problematic separation of modal features from the actual world. Since I don't think there are any possible worlds, I don't think even metaphysical modality facts depend on anything other than the actual world, so I don't see how probability facts are to be differentiated from run-of-the-mill facts in that way.

I now want to turn to the question of the collective status of Antony's three theses. Antony argues that we should respond to the tension among them by rejecting the third. I want to suggest a different approach. It seems to me that if we endorse the third thesis and the world is in fact deterministic (say then that things are *strongly deterministic*), then the arguments for the first thesis become quite unpersuasive. Consider the arguments:

- Justifications of the Kolmogorov axioms in terms of principles such as the Necessity principle ($Pr(\phi) = 1 \Rightarrow \phi$) show us that "we should regard

propositions of intermediate probability as somehow also falling between necessity and impossibility". But if things are strongly deterministic, there are no propositions of intermediate probability, so this consideration loses its bite.

- Surprisingness, defined probabilistically, does not supervene on actual truth value. But when things are strongly deterministic, both probabilities and surprisingness do supervene on actual truth value.

- The BCP and converse BCP are explained by the modality of probability. But when things are strongly deterministic, we have:

  – $Pr(\phi) > 0 \Rightarrow Pr(\phi) = 1 \Rightarrow \phi \Rightarrow \Diamond\phi$
  – $\Diamond\phi \Rightarrow Pr(\phi) > 0 \Rightarrow Pr(\phi) = 1 \Rightarrow \phi$

  And neither of these lines of reasoning lends support to treatment of the probabilities as modals.

- Probabilities guide expectation and rational decision making, and do so in a way that is continuous with the role of possible outcomes in expectation and rational decision making. But, of course, the idea of rational decision making is, first of all, notoriously difficult to make full sense of in the light of determinism, and second, surely not deeply integrated with probabilistic considerations when things are strongly deterministic.

- We should expect events to occur with a frequency matching that of their probability, so when the actual frequency diverges from the probability, we must then appeal to the distribution of possible events to regain the link between probability and frequency. But when things are strongly deterministic, the actual frequency cannot diverge from the probability.

So it looks plausible to me for someone who believes that things are strongly deterministic then to reject Antony's Thesis 1. That claim is then contingent, and fails in those situations needed for the construction of the contradiction. But the point can be put somewhat differently, in a way that allows us to hold on to all three theses. Instead, when things are strongly deterministic, of rejecting the claim that probability is a modality, we can hold that it is a *trivial* modality. Following Antony's suggestion, we can treat probability as a restricted metaphysical modal, where there is (for each world) a set of probabilistically relevant worlds W_{Pr}. (I here prescind from the difficulties with this analysis of the modal that I raised earlier.) One could then hold that, in a deterministic world w, the class W_{Pr} reduces to $\{w\}$ alone. Then the probabilistic modal would continue to track the deterministic probabilities (restricted, according to Thesis 3, to 0 and 1), via BCP and the Converse BCP. But there would be no contradiction, because the modality spoken of in Thesis 2 would be full-fledged metaphysical possibility, and not the same as the probabilistic modality.

Hence, in Antony's derivation of the contradiction, the move from $\Diamond\phi$ to $Pr(\phi) > 0$ would commit a fallacy of equivocation. The Converse BCP will take us from the probabilistic possibility $\Diamond_{Pr}$ to the non-zero probability, but Thesis 2 will give us only the metaphysical possibility $\Diamond$.

This style of analysis gives us the possibility of holding on to all three theses, and thus removes the pressure to reject one. But it also - being essentially the analysis that Antony proposes - gives us a framework that lets us see how Thesis 3 might be rejected. Once we think of probability as a restricted modal, we can ask - as Antony does - why we ought, in a deterministic world, to restrict that modality to a trivial set of worlds. We can, as Antony suggests, let the probabilistic modal applied to the claim "The coin comes up heads", range over all worlds which agree about certain overt features of the coin flipping scenario.

The last point I want to make, though, is that while this picture is certainly compatible with the rejection of Thesis 3, I don't see that it helps us make sense of that rejection. To be given a space of worlds that hold constant facts relevant to the probabilistic situation is not so far to be given tools for assigning non-trivial propositions. We can't make the transition from the distribution of truth values over possible worlds to the value of probabilities without some sort of measure on the space of worlds. And I take it that the theoretical pressures which incline people toward trivial probability distributions under determinism will also incline them toward a trivializing possible worlds measure under determinism. W_{Pr} may, in the deterministic world, contain worlds in which the microphysical conditions underlying the coin flip are realized in many different ways. But, given the determinism, we may be inclined to regard all but one of these ways as having probabilistic measure 0. We take these worlds to be possible, and perhaps even relevant to the determination of probabilities, but also to be so unlikely to occur that they need be given no non-trivial weight in that determination.

I don't mean to say that we must set up the measure in this way - I only want to observe that there's still a problem here. Here's one way of having non-trivial probabilities in a deterministic world - we can simply insist that the chance of the coin coming up heads is .5, despite the determinism. That's fine, and consistent, and it gives us non-trivial probabilities, but it doesn't respond to philosophical pressures against that assignment. In the end, Antony's relative modality approach seems to me to amount to the same move.