Philosophy 57 — Day 24

- Quiz #6 to be Returned Next Week
- Next Week, I will also post the "Curve for the course to this point"
  - This will tell you where you need to be on the final, etc.
- Extra-Credit Problems Posted on Website (5 problems, each worth 1 point!)
  - Extra-Credit Problems are due by Tuesday 05/20/03
  - No partial credit within problems (but you can do fewer than 5 problems)
  - You may use any tools/references you like to do these (but individually!)
  - Stay Tuned for Hints, etc. [these are all "chapter 6" problems]
- Back to Chapter 6
  - Definitions of Truth-Functional Connectives
  - Truth-Tables for Claims
  - Truth-Tables for Arguments

Chapter 6 — Propositional Logic: Truth Functions – Review

- Negation (just like English "not"), and Conjunction (just like English "and"):

p	$\sim p$
T	F
F	T

p	q	$p \bullet q$
T	T	T
T	F	F
F	T	F
F	F	F

- Disjunction is *similar* to English "or", but *not* in the "exclusive" sense:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- But, we can express the English exclusive "A or B, but not both", as:

$$(A \vee B) \bullet \sim(A \bullet B)$$

- So, "$\sim$", "$\bullet$", and "$\vee$" do seem to match English usage for "not", "and", "or".

Chapter 6 — Propositional Logic: Truth Functions – $\supset$

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

- The truth-functional definition of $\supset$ is farther from the English "only if". A PL conditional is false iff its antecedent is true and its consequent is false.
- In English, conditionals can be false, even if their antecedents are false. Moreover, English conditionals can be false even if their consequents are true.
  - If New York is in New Zealand, then $2 + 2 = 4$.
  - If New York is in the U.S.A., then WWII ended in 1945.
  - If WWII ended in 1941, then gold is an acid.
- So, $\supset$ does *not* capture the English "if". We'll see later that $p \supset q \approx \sim p \vee q$.
- But, I will explain later why this is the *only* acceptable *truth-functional* choice.

Chapter 6 — Propositional Logic: Truth Functions – $\equiv$

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

- The truth-functional definition of $\equiv$ is far from the English "if and only if". A PL biconditional is true iff both of its components have the same truth value.
- Consider these two biconditionals. [M = the moon is made of green cheese, U = there are unicorns, E = life exists on Earth, and S = the sky is blue]
  - The moon is made of green cheese if and only if there are unicorns.
  - Life exists on earth if and only if the sky is blue.
- The PL translations of these sentences are both true. $M \equiv U$ is true because M and U are false. $E \equiv S$ is true because E and S are true. This does *not* capture the English "if and only if". We'll see that $p \equiv q \approx (p \bullet q) \vee (\sim p \bullet \sim q)$.

Chapter 6 — Propositional Logic: Truth Tables I

- With the truth-table definitions of the five connectives in hand, we can now construct truth tables for arbitrary compound PL statements.
- The procedure for constructing the truth-table of p is as follows:
  1. Determine the number of rows in the truth-table. This is 2^n, where n is the number of atomic sentences in the compound statement p.
  2. The table will have $n + 1$ main columns: n columns for the atomic sentences in p, and one for the truth-values of p itself.
  3. The table will also have some "quasi-columns" — one for each PL statement occurring in the compound p — which needn't be drawn explicitly, but which will go into the determination of the truth values of p.
  4. Place the atomic symbols in the left most columns, going in alphabetical order from left to right. And place p in the right most column.
  5. Write in all possible combinations of truth-values for the atomic statements. There will be 2^n of these — one for each row of the table.

6. The convention here is to start on the nth column (farthest down the alphabet) with the pattern TFTF . . . repeated until the column is filled. Then, go TTFF . . . in the $n - 1$st column. And, TTTTFFFF . . . in the $n - 2$nd column, etc. . . , until the very first column has been completed.
7. Next, we need to compute the truth-values of p in each row of the table. Here, we start from the inside-out. We first copy the truth-values of the atoms, then we compute the negations, conjunctions, etc. which compose p. Finally, we will be in a position to compute the value of the main connective of p, at which point we will be done with p's truth table.

- Example: Step-By-Step Truth-Table Construction of "$A \equiv (B \bullet A)$."

A	B	A	$\equiv$	$(B$	$\bullet$	$A)$
T	T	T	T	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	F	F
F	F	F	T	F	F	F

Chapter 6 — Propositional Logic: Truth Tables II

- A statement is said to be logically true (or tautologous) if it is true regardless of the truth-values of its components. Example: $p \equiv p$ is logically true.

p	p	$\equiv$	p
T	T	T	T
F	F	T	F

- A statement is logically false (or self-contradictory) if it is false regardless of the truth-values of its components. Example: $p \bullet \sim p$ is logically false.

p	p	$\bullet$	$\sim$	p
T	T	F	F	T
F	F	F	T	F

- A statement is contingent if its truth-value varies depending on the truth-values of its components. Example: A (or *any* atom) is contingent.

A	A
T	T
F	F

Chapter 6 — Propositional Logic: Truth Tables III

- Classify the following statements as logically true (tautologous), logically false (self-contradictory), or contingent (exercise 6.3.I):
  1. $N \supset (N \supset N)$
  2. $(G \supset G) \supset G$
  3. $(S \supset R) \bullet (S \bullet \sim R)$
  4. $((E \supset F) \supset F) \supset E$
  6. $(M \supset P) \vee (P \supset M)$
  11. $[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim(P \vee R)$
  12. $[(H \supset N) \bullet (T \supset N)] \supset [(H \vee T) \supset N]$
  15. $[(F \vee E) \bullet (G \vee H)] \equiv [(G \bullet E) \vee (F \bullet H)]$

Chapter 6 — Propositional Logic: Truth Tables IV

- Here is a completed truth-table for #11, $[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim(P \vee R)$:

P	Q	R	$[(Q$	$\supset$	$P)$	$\bullet$	$(\sim$	Q	$\supset$	$R)]$	$\bullet$	$\sim$	$(P$	$\vee$	$R)$
T	T	T	T	T	T	T	F	T	T	T	F	F	T	T	T
T	T	F	T	T	T	T	F	T	T	F	F	F	T	T	F
T	F	T	F	T	T	T	T	F	T	T	F	F	T	T	T
T	F	F	F	T	T	F	T	F	F	F	F	F	T	T	F
F	T	T	T	F	F	F	F	T	T	T	F	F	F	T	T
F	T	F	T	F	F	F	F	T	T	F	F	T	F	F	F
F	F	T	F	T	F	T	T	F	T	T	F	F	F	T	T
F	F	F	F	T	F	F	T	F	F	F	F	T	F	F	F

- Therefore, the statement "$[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim(P \vee R)$" is *logically false*.

Chapter 6 — Propositional Logic: Truth Tables V

- Two statements are said to be equivalent (written $p \approx q$) if they have the same truth-value in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, $A \supset B \approx \sim A \vee B$:

A	B	A	$\supset$	B	$\sim$	A	$\vee$	B
T	T	T	T	T	F	T	T	T
T	F	T	F	F	F	T	F	F
F	T	F	T	T	T	F	T	T
F	F	F	T	F	T	F	T	F

- Two statements are said to be contradictory if they have opposite truth-values in all possible worlds (*i.e.*, in all rows of a simultaneous truth-table of both statements). For instance, A and $\sim A$ are contradictory:

A	A	$\sim$	A
T	T	F	T
F	F	T	F

- Two statements are inconsistent if they are never both true in any possible world (*i.e.*, in any row of a simultaneous truth-table of both statements). For instance, $A \equiv B$ and $A \bullet \sim B$ are inconsistent (but *not* contradictory!):

A	B	A	$\equiv$	B	A	$\bullet$	$\sim$	B
T	T	T	T	T	T	F	F	T
T	F	T	F	F	T	T	T	F
F	T	F	F	T	F	F	F	T
F	F	F	T	F	F	F	T	F

- Two statements are consistent if they are both true in at least one possible world (*i.e.*, in at least one row of a simultaneous truth-table of both statements). For instance, $A \bullet B$ and $A \vee B$ are consistent:

A	B	A	$\bullet$	B	A	$\vee$	B
T	T	T	T	T	T	T	T
T	F	T	F	F	T	T	F
F	T	F	F	T	F	T	T
F	F	F	F	F	F	F	F

Chapter 6 — Propositional Logic: Truth Tables VI

- Use truth-tables to determine whether the following pairs of statements are logically equivalent, contradictory, consistent, or inconsistent (exercise 6.3.II).

2. $F \bullet M$ and $\sim(F \vee M)$
4. $R \vee \sim S$ and $S \bullet \sim R$
6. $H \equiv \sim G$ and $(G \bullet H) \vee (\sim G \bullet \sim H)$
8. $N \bullet (A \vee \sim E)$ and $\sim A \bullet (E \vee \sim N)$
10. $W \equiv (B \bullet T)$ and $W \bullet (T \supset \sim B)$
12. $R \bullet (Q \vee S)$ and $(S \vee R) \bullet (Q \vee R)$
14. $Z \bullet (C \equiv P)$ and $C \equiv (Z \bullet \sim P)$
15. $Q \supset \sim(K \vee F)$ and $(K \bullet Q) \vee (F \bullet Q)$

Chapter 6 — Propositional Logic: Truth Tables VII

- Here is a simultaneous truth-table which establishes that

$$A \equiv B \approx (A \bullet B) \vee (\sim A \bullet \sim B)$$

A	B	A	$\equiv$	B	$(A$	$\bullet$	$B)$	$\vee$	$(\sim$	A	$\bullet$	$\sim$	$B)$
T	T	T	T	T	T	T	T	T	F	T	F	F	T
T	F	T	F	F	T	F	F	F	F	T	F	T	F
F	T	F	F	T	F	F	T	F	T	F	F	F	T
F	F	F	T	F	F	F	F	T	T	F	T	T	F

- Can you prove the following equivalences with simultaneous truth-tables?
  - $\sim(A \bullet B) \approx \sim A \vee \sim B$
  - $\sim(A \vee B) \approx \sim A \bullet \sim B$
  - $A \approx (A \bullet B) \vee (A \bullet \sim B)$
  - $A \approx A \bullet (B \supset B)$
  - $A \approx A \vee (B \bullet \sim B)$