Comments on Carl Wagner's *Jeffrey Conditioning and External Bayesianity*

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Outline

1. Mathematical considerations

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2 Philosophical hesitations

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- A weighted average, essentially, over classically updating on the alternatives.
- Unlike classical Bayesian conditioning, this allows learning to be unlearned.

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- He shows that this paramaterization preserves commutativity.
- (unlike measuring learning by the posterior evidential probabilities)

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- Wagner argues for this rule with a pile of mathematical elegance.
- Today he showed how it can capture commutativity of pooling operators.
- Elsewhere he extends Field's result to infinite sample spaces with countable partitions.

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- Finally, it has a nice tie with a recent plausible metric from Chan & Darwiche for probability measures over a finite sample space.

Chan-Darwiche distance and the uniformity rule

$$
\begin{aligned}
\mathrm{CD}(p, q) &= \log \max_{\omega \in \Omega} \frac{q(\omega)}{p(\omega)} - \log \min_{\omega \in \Omega} \frac{q(\omega)}{p(\omega)} \\
&= \log \frac{\max_{\omega \in \Omega} q(\omega)/p(\omega)}{\min_{\omega' \in \Omega} q(\omega')/p(\omega')} \\
&= \max_{\omega, \omega' \in \Omega} \log \frac{q(\omega)/p(\omega)}{q(\omega')/p(\omega')} \\
&= \max_{\omega, \omega' \in \Omega} \log \frac{q(\omega)/q(\omega')}{p(\omega)/p(\omega')} \\
&= \max_{\omega, \omega' \in \Omega} \log \beta_{q,p}(\{\omega\} : \{\omega'\}) \\
&= \max_{A, B \in \mathscr{P}(\Omega) - \emptyset} \log \beta_{q,p}(A : B)
\end{aligned}
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- There are a number of cases that seem to show this is still a messy notion.

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  repeated applications of Bayes factors, drive you toward
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- Wagner: we should therefore divorce identical learning from
  sense experiences; "we learn nothing new from repeated
  glances and so all Bayes factors beyond the first are equal to
  one."

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    - Therefore our Bayes factors differ.
    - Therefore we didn't learn the same thing.
- In some sense maybe this is right—but in some important sense we surely *did* learn the same thing.

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- "I could disconfirm lots of theories just by running around at night."

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- Thus the Wagner-Field uniformity rule looks appropriate.

- Lange's suggestion: "...two agents are undergoing the same sensory experience exactly when it is the case that *had* the two agents begun with the *same* prior probability distribution, then they *would* as a result of their actual sensory experiences have imposed exactly the *same* constraints on that distribution, ... no matter what the two agents' common prior probability distribution had been."

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- (So the Garber case actually involves different sensory experiences?!)

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  - One is a scowl-into-laugh, another a laugh-into-scowl.

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- We could admit them as different elements in the sample space, and do classical conditioning—but how plausible is that?

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- In both these cases, sure looks like cheating to say that it's a different piece of evidence when it happens in a different order.

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  - Such a notion cannot be revived.