Announcements & Overview

- Administrative Stuff
  - Last Week: Course Website/Syllabus (see me if you have questions)
  - ☞ **HW #1 Assigned (see website) – due next Friday (*via* `Blackboard`).**
- Today: Basic Underlying Concepts of Logic (Chapter 1, Cont'd)
  - Why model logical concepts mathematically/formally?
  - A Subtle Argument, and the Notion of Logical Form
  - A Conservative Principle About Attributions of Validity
  - Sentential Logical Form
  - Beyond Sentential Form
  - Two "Strange" Valid Sentential Forms
  - Validity and Soundness of Arguments – Some Examples
  - A "Big Picture" View of Part I of the Course
  - Time Permitting: Preamble for Chapter 2 (language of sentential logic)

Why model logic concepts *formally* or *symbolically*?

- Ultimately, we want to decide whether arguments expressible in *natural* languages are valid. But, in this course, we will only study arguments expressible in *formal* languages. And, we will use *formal* tools. *Why?*
- Analogous question: What we want from natural science is explanations and predictions about *natural* systems. But, our theories (strictly) apply only to systems faithfully describable in *formal*, *mathematical* terms.
- Although formal models are *idealizations* which abstract away some aspects of natural systems, they are *useful idealizations* that help us understand *many* natural relationships and regularities.
- Similarly, studying arguments expressible in formal languages allows us to develop powerful tools for testing validity. We won't be able to capture *all* valid arguments this way. But, we can grasp many.
- *Why* or *how* mathematical/formal methods *are* helpful for such understanding is a deep question in the philosophy of science.

A Subtle Argument, and the Notion of Logical Form

(i) John is a bachelor.
∴ John is unmarried.

- Is (i) valid? Well, this is tricky. Intuitively, being unmarried is part of the *meaning* of “bachelor”. So, it *seems* like it is (intuitively) logically impossible for the premise of (i) to be true while its conclusion is false
- This suggests that (i) is (intuitively/absolutely) valid.
- On the other hand, consider the following argument:

(ii) If John is a bachelor, then John is unmarried.
John is a bachelor.
∴ John is unmarried.

- The correct judgment about (ii) seems *clearly* to be that it is valid – *even if we don’t know the meaning of “bachelor” (or “unmarried”)*.
- This is clear because the logical form of (ii) is *obvious* [(i)’s form is not].

A Conservative Principle About Attributions of Validity

- This suggests the following additional "conservative" heuristic:
  - ☞ We should conclude that an argument $\mathscr{A}$ is valid only if we can see that $\mathscr{A}$'s conclusion follows from $\mathscr{A}$'s premises *without appealing to the meanings of the predicates involved in* $\mathscr{A}$.
- But, if validity does not depend on the meanings of predicates, then what *does* it depend on? This is a deep question about logic. We will not answer it here. That's for more advanced philosophical logic courses.
- What we will do instead is adopt a conservative methodology that only classifies *some* "intuitively/absolutely valid" arguments as valid.
- The strategy will be to develop some *formal* methods for *modeling* intuitive/abolsute validity of arguments expressed in English.
- We won't be able to capture *all* intuitively/absolutely valid arguments with our methods, but this is OK. [Analogy: mathematical physics.]

Sentential Logical Form

- We will begin with *sentential logic*. This will involve providing a characterization of valid *sentential forms*. Here's a paragidm example:

   Dr. Ruth is a man.

   (1) If Dr. Ruth is a man, then Dr. Ruth is 10 feet tall.

   ∴ Dr. Ruth is 10 feet tall.

- (1) is a set of sentences with a valid sentential form. So, whatever argument it expresses is a valid argument. What's its *form*?

   p.

   (1_f) If p, then q.

   ∴ q.

- (1)'s valid *sentential form* (1_f) is so famous it has a name: *Modus Ponens*. [Usually, latin names are used for the *valid* forms.]

☞ **Definition.** The *sentential form* of an argument (or, the sentences faithfully expressing an argument) is obtained by replacing each basic (or, atomic) sentence in the argument with a single (lower-case) letter.

- What's a "basic" sentence? A basic sentence is a sentence that doesn't contain any sentence as a proper part. How about these?

  (a) Branden is a philosopher and Branden is a man.

  (b) It is not the case that Branden is 6 feet tall.

  (c) Snow is white.

  (d) Either it will rain today or it will be sunny today.

- Sentences (a), (b), and (d) are *not* basic (we'll call them "complex" or "compound"). Only (c) is basic. We'll also use "atomic" for basic.

- What's the sentential form of the following argument (is it valid?):

  If Tom is at his Fremont home, then he's in California.

  Tom is in California.

  ∴ Tom is at his Fremont home.

Two "Strange" Valid Sentential Forms

(†) p. Therefore, either q or not q.

- (†) is valid because it is (logically) *impossible* that *both*:

  (i) p is true, *and*

  (ii) "either q or not q" is false.

  This is impossible because *(ii) alone* is impossible.

(‡) p and not p. Therefore, q.

- (‡) is valid because it is (logically) *impossible* that *both*:

  (iii) "p and not p" is true, *and*

  (iv) q is false.

  This is impossible because *(iii) alone* is impossible.

- We'll soon see why we have these "oddities". They stem from our semantics for "If … then" statements (and our first def. of validity).

Some Valid and Invalid Sentential Forms

Sentential Argument Form	Name	Valid/Invalid
p If p, then q —————— $\therefore q$	*Modus Ponens*	Valid
q If p, then q —————— $\therefore p$	Affirming the Consequent	Invalid
It is not the case that q If p, then q —————— $\therefore$ It is not the case that p	*Modus Tollens*	Valid
It is not the case that p If p, then q —————— $\therefore$ It is not the case that q	Denying the Antecedent	Invalid
If p, then q If q, then r —————— $\therefore$ If p, then r	Hypothetical Syllogism	Valid
It is not the case that p Either p or q —————— $\therefore q$	Disjunctive Syllogism	Valid

Beyond Sentential Form

- The first half of the course involves developing a precise *theory* of *sentential* validity, and several rigorous techniques for *deciding* whether a sentential form is (or is not) valid. This only takes us so far.
- Not all (absolutely) valid arguments have valid *sentential* forms, *e.g.*:

All men are mortal.
(2) Socrates is a man.
∴ Socrates is mortal.

- The argument expressed by (2) seems clearly valid. But, the sentential form of (2) is not a valid form. Its sentential form is:

p.
(2_f) q.
∴ r.

- In this first course, we will not be studying predicate/quantifier logic, which gives a formal theory of validity that covers such forms.
- In that more general theory, one can recognize that (2) has something like the following (non-sentential!) logical form:

All Xs are Ys.

(2_f*) a is an X.

$\therefore$ a is a Y.

- We will leave such arguments (called *syllogisms*) for a future, more sophisticated theory of logical validity (*viz.*, *predicate logic*).
- In Part I of the course, we'll learn a (simple) purely formal language for talking about *sentential* forms, and then we'll develop some rigorous methods for determining whether/which sentential forms are valid.
- As we will see, the fit between our simple formal sentential language and English (or other natural languages) will not be perfect. First, let's think a bit harder about the above "Barbara" Aristotelian form.

- As an illustration of the subtlety of determining what "the" sentential form of an argument is, let's reconsider our Socrates syllogism.

All men are mortal.

Socrates is a man.

∴ Socrates is mortal.

- Notice that (intuitively) the first premise of this syllogism entails a conditional claim about each individual object. Specifically, for each object *o*, the first premise entails the following conditional proposition.

If *o* is a man, *then* *o* is mortal.

- So, to be more specific, the first premise entails the following conditional claim about the object named "Socrates."

If Socrates is a man, *then* Socrates is mortal.

- In other words, the first premise entails a conditional claim about Socrates that — together with the second premise — yields a *modus ponens* argument about Scorates, which *is sententially valid*!

Validity and Soundness of Arguments — Some Non-Sentential Examples

- Can we classify the following according to validity/soundness?

1) All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage.	**5)** All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine.
2) All wines are whiskeys. Chardonnay is a wine. Therefore, chardonnay is a whiskey.	**6)** All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine.
3) All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink.	**7)** All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine.
4) All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey.	**8)** All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine.

	Valid	Invalid
True premises True conclusion	All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage. [sound]	All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine. [unsound]
True premises False conclusion	**Impossible** None exist	All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine. [unsound]
False premises True conclusion	All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink. [unsound]	All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine. [unsound]
False premises False conclusion	All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey. [unsound]	All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine. [unsound]

- See, also, our validity and soundness handout . . .

Some Brain Teasers Involving Validity and Soundness

- Here are two very puzzling arguments:

$(\mathscr{A}_1)$ Either $\mathscr{A}_1$ is valid or $\mathscr{A}_1$ is invalid.
$\therefore$ $\mathscr{A}_1$ is invalid.

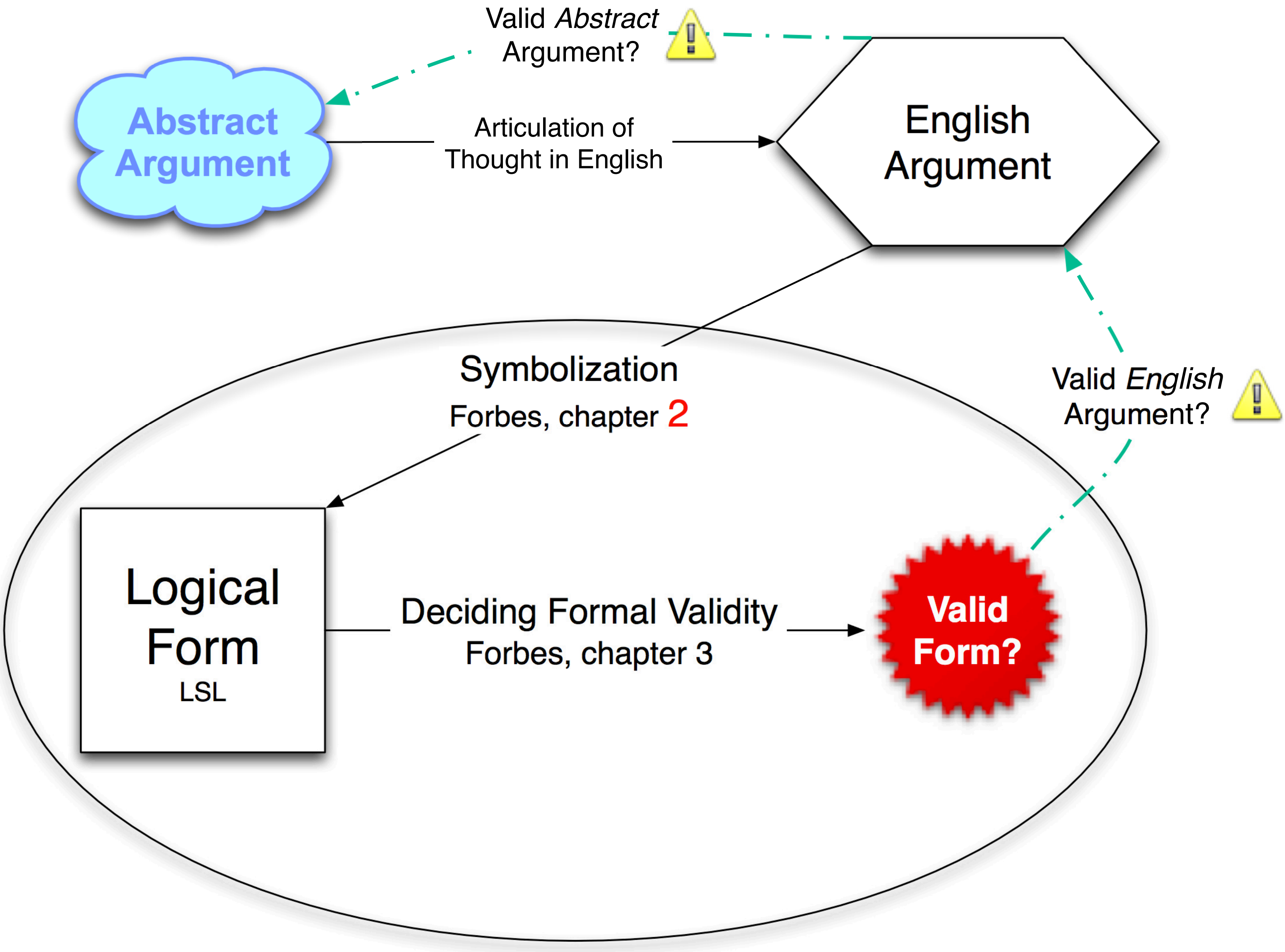
$(\mathscr{A}_2)$ $\mathscr{A}_2$ is valid.
$\therefore$ $\mathscr{A}_2$ is invalid.

- I'll discuss $\mathscr{A}_2$ ($\mathscr{A}_1$ is left as an exercise).
  - If $\mathscr{A}_2$ is valid, then it has a true premise and a false conclusion. But, this means that if $\mathscr{A}_2$ is valid, then $\mathscr{A}_2$ invalid!
  - If $\mathscr{A}_2$ is invalid, then its conclusion must be true (as a matter of logic). But, this means that if $\mathscr{A}_2$ is invalid then $\mathscr{A}_2$ is valid!
  - This *seems* to imply that $\mathscr{A}_2$ is *both valid and invalid*. But, remember our conservative validity-principle. What is the *logical form* of $\mathscr{A}_2$?

Absolute Validity *vs* Formal Validity

- Forbes calls the general, informal notion of validity "absolute validity".
- Our notion is a bit more conservative than his, since we'll only call an argument valid if one of our *formal theories* captures it as falling under a valid *form*. Our first formal theory (LSL) is about *sentential* validity.
- An argument is *sententially* valid if it has a valid *sentential form*.
- Sentential form is obtained by replacing each basic or atomic sentence in an argument with a corresponding lower-case letter.
- Once we know the sentential form of an argument (chapter 2), we will be able to apply purely formal, mechanical methods (chapters 3 and 4) for determining whether that sentential form is valid.

☞ Even if an argument fails to be *sententially* valid, it could still be valid according to a richer logical theory than LSL. I'll mention some other, more sophisticated theories of logical form later in the course.


Valid *Abstract* Argument?
Abstract Argument
Articulation of Thought in English
English Argument
Symbolization
Forbes, chapter 2
Valid *English* Argument?
Logical Form
LSL
Deciding Formal Validity
Forbes, chapter 3
Valid Form?

Preamble for Chapter 2: The Use/Mention Distinction

- Consider the following two sentences:

(1) California has more than nine residents.

(2) ‘California’ has more than nine letters.

- In (1), we are *using* the word ‘California’ to talk about the State of California. But, in (2), we are merely *mentioning* the word ‘California’ (*i.e.*, we’re talking about *the word itself*).

- If Jeremiah = ‘California’, which of these sentences are true?

(3) Jeremiah has (exactly) eight letters [false].

(4) Jeremiah has (exactly) ten letters [true].

(5) ‘Jeremiah’ has eight letters [true].

(6) ‘Jeremiah’ is the name of a state [false].

Preamble for Chapter 2: More on Use/Mention and ‘ ’ *versus* ⌜ ⌝

- Consider the following two statements about LSL sentences

  (*i*) If p and q are both sentences of LSL, then so is $\ulcorner(p \,\&\, q)\urcorner$.

  (*ii*) If p and q are both sentences of LSL, then so is ‘$(p \,\&\, q)$’.

- As it turns out, (*i*) is true, but (*ii*) is *false*. The string of symbols ‘$(p \,\&\, q)$’ *cannot* be a sentence of LSL, since ‘p’ and ‘q’ are *not* part of the lexicon of LSL. They allow us to talk about LSL *forms*.

- The trick is that $\ulcorner(p \,\&\, q)\urcorner$ abbreviates the long-winded phrase:

  – The symbol-string which results from writing ‘(’ followed by p followed by ‘&’ followed by q followed by ‘)’.

- In (*ii*), we are merely *mentioning* ‘p’ and ‘q’ (in ‘$(p \,\&\, q)$’). But, in (*i*), we are *using* ‘p’ and ‘q’ (in $\ulcorner(p \,\&\, q)\urcorner$) to talk about (forms of) sentences in LSL. In (*i*), ‘p’ and ‘q’ are *used* as *metavariables*.

Preamble for Chapter 2: Object language, Metalanguage, *etc.* . . .

- LSL is the *object language* of our current studies. The symbol string '$(A \,\&\, B) \vee C$' is a sentence of LSL. But, the symbol string '$(p \,\&\, q) \vee r$' is *not* a sentence of LSL. Why?
- We use a *metalanguage* to talk about the object language LSL. This metalanguage is not formalized. It's mainly English, plus *metavariables* like 'p', 'q', 'r', and *selective quotes* '$\ulcorner$' and '$\urcorner$'.
- If $p =$ '$(A \vee B)$', and $q =$ '$(C \to D)$', then what are the following?
  - $\ulcorner p \,\&\, q \urcorner$ [$(A \vee B) \,\&\, (C \to D)$], '$p \,\&\, q$' [$p \,\&\, q$], '$p$' [$p$], '$q$' [$q$]
- And, which of the following are true?
  - p has five symbols [true]. 'p' has five symbols [false].
  - $\ulcorner p \,\&\, q \urcorner$ is a sentence of LSL [true]. So is '$\ulcorner p \,\&\, q \urcorner$' [false].

Introduction to the Syntax of the LSL: The Lexicon

- The syntax of LSL is quite simple. Its lexicon has the following symbols:
  - Upper-case letters 'A', 'B', ... which stand for *basic sentences*.
  - Five *sentential connectives/operators* (one *unary*, four *binary*):

Operator	**Name**	**Logical Function**	**Used to translate**
'$\sim$'	tilde	negation	not, it is not the case that
'&'	ampersand	conjunction	and, also, moreover, but
'$\vee$'	vee	disjunction	or, either ... or ...
'$\rightarrow$'	arrow	conditional	if ... then ..., only if
'$\leftrightarrow$'	double arrow	biconditional	if and only if

  - Parentheses '(', ')', brackets '['. ']', and braces '{', '}' for grouping.
- If a string of symbols contains anything else, then it's not a sentence of LSL. And, only *certain* strings of these symbols are LSL sentences.
- Some LSL symbol strings aren't *well-formed*: '$(A \& B$', '$A \& B \vee C$', *etc.*