

## Notes on Field's "What is the Normative Role of Logic?"

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I suspect that this paper is written in a way that doesn't reflect how Field *actually came to* the views he has about the normativity of logic. It seems likely to me that *first* (i) Field convinced himself that the relation of logical consequence could not be (even co-extensional with) the relation of necessary truth preservation (NTP), for reasons he lays out in PART TWO of the paper, *then* (ii) the only viable alternative Field saw to (NTP) was the view that the relation of logical consequence is a primitive epistemic relation (PER), and *finally* (iii) Field then proceeded to work out what he thought was the best (PER) story, which is described in PART ONE of the paper. For this reason, I will actually discuss PART TWO first, and PART ONE second.

## 1 PART TWO: Logical Consequence $\neq$ Necessary Truth Preservation

### 1.1 Meta-Theoretic Argument #1: The Gödel/Tarski Argument

In PART TWO, Field gives various arguments for the following claim:

$\sim$ (NTP) The relation of logical consequence is not (indeed, is not even co-extensional with) the relation of necessary truth preservation.

Before getting into the details of these arguments in PART TWO, I want to make an over-arching observation about them. They are all based on various limitative meta-theorems from classical logic. By my count, Field appeals to a total of three (3) classical meta-theoretic results in PART TWO: (1) Gödel's Second Incompleteness Theorem, (2) Tarski's "Undefinability of Truth" Theorem, and (3) Curry's Paradox (truth-theoretic version). Specifically, each of his arguments in this section makes use of at least one of these classical meta-theorems as a premise. Presumably, then, Field presupposes that we should all believe that these meta-theorems are *true*. I won't dwell on this too much, but one might ask at this point: *Why* should we believe that these meta-theorems are true? Is it because they are *logical consequences* of meta-*axioms* that we should believe are true? But, doesn't that presuppose that "logical consequence" — at least, in *these* cases — is a relation that *preserves truth*? That is to say, when we look at the "proofs" of these meta-theoretic results, they will involve some formal, meta-theoretic (or meta-meta-theoretic) "logical consequence relation" ( $\vdash$ ). Is Field supposing that this *theoretical*  $\vdash$ -relation is co-extensional (in its domain of application) with the *informal* relation of "logical consequence" in which he is interested? If not, then properties of  $\vdash$  seem *irrelevant*. If so, then I wonder how (dialectically) he gets to just *assume* this, without *argument*. I'll return to this question, below. Moreover, once we accept the truth of these meta-theoretic results, they get used as premises in arguments for  $\sim$ (NTP). Again, I presume that Field wants us to believe that  $\sim$ (NTP) is true, on the basis of these arguments, which include these meta-theoretic results as premises. But, why should we believe that  $\sim$ (NTP) is true, unless (a) we believe that the premises of Field's arguments for  $\sim$ (NTP) are true, *and* (b) we believe that  $\sim$ (NTP) follows-from them *in a way that preserves truth*? To be fair to Field, all he needs for (b) is that these relations of logical consequence he is using and discussing in PART TWO preserve truth *in the context of their use in his PART TWO arguments for  $\sim$ (NTP)*. OK, I won't dwell on (b) too much.

What I would like to dwell on a bit more is (a). Specifically, I am rather worried about the relation between the theoretical (or *formal*) concepts which appear in these classical meta-theoretic results and the informal concept "logical consequence", which Field thinks is not co-extensional with necessary truth preservation. Let's take Gödel's and Tarski's theorems first. What Gödel shows is that no (sufficiently interesting) mathematical theory can prove its own consistency. Specifically, no (interesting) *formal* theory  $T$  will be such that both of the following are provable in  $T$  ( $T$  has its own *formal* "entailment" relation  $\vdash_T$ ):

(Ai) that all the axioms of  $T$  are true.

(Aii) that all the rules of inference of  $T$  preserve truth.

Tarski provides the diagnosis. He shows that no (non-trivial) formal theory  $T$  can even *have* a truth predicate which satisfies the “natural” properties of “is true”. Thus, (Ai) and (Aii) can not even be *formulated* (much less *proved*!) in such *formal* theories  $T$ . I could, at this point, get picky about Tarski’s “proof”, and whether the “rules” he assumes in the meta-meta-theory (in order to “prove” his meta-theorem) are *themselves* truth-preserving. But, I won’t go there. Rather, let’s continue on with Field’s discussion of these classical limitative meta-theoretic results. Field explains that we can go ahead and add truth-predicates to theories  $T$ , but we’ll be forced to choose between rejecting (Ai) and (Aii). Field opts for rejecting (Aii) and maintaining (Ai). He is willing to live with (formal) theories  $T$  which have rules that do not always preserve truth, but he doesn’t like the idea of having axioms of  $T$  that are “not true by  $T$ ’s own lights”. Here’s the difference (for him):

While this [rejecting (Aii)] might at first seem just as counterintuitive as rejecting the truth of some of one’s axioms [*viz.*, rejecting (Ai)], I don’t think this is so. The reason is that with most such theories, there is no reason to doubt that all the rules preserve truth *when it matters*. That is, the rejection of the claim that a rule like modus ponens (or like the inference from  $\text{True}(\langle A \rangle)$  to  $A$ ) preserves truth generally arises because of a rejection of the claim that it preserves truth in certain instances (that is, with certain choices of premises for the rule) instances involving “ungrounded” occurrences of predicates like ‘True’. But these are all instances in which, if the theory is consistent, the premises of the rule could never be established or be rationally believable. In that case, the rejection of the claim that the rule preserves truth generally doesn’t seem to me to undermine the use of the rule.

I find this rather puzzling. Here, Field seems sensitive to the fact that there is something *good* and *important* about a deductive inference rule *preserving truth* (more on that later). Indeed, Field is explicitly trying to *downplay* the significance of any counterexamples to truth preservation. This suggests a *revision* of (NTP):

(NTP’) The logical consequence relation ( $q$  is a logical consequence of  $\Gamma$ ) is co-extensional with the relation of truth-preservation — in those cases where each member of  $\Gamma$  (and perhaps the conjunction of  $\Gamma$ ’s members?) “can be established or is rationally believable”.

Field only briefly discusses (NTP’). First, he complains that (NTP’) is “too vague for a definition of validity”. Then, he points out that the Gödel/Tarski worries would still apply to (NTP’), since a formal theory  $T$  could never be in a position to *settle* (in general) questions about whether  $p$  “can be established or is rationally believable” — on pain of  $T$  being able to settle its own consistency, which would violate Gödel’s theorem. There is another problem with (NTP’). It is *explicitly epistemic*, and hence not really a clean (“non-normative”) alternative to a “normative conception” of logical consequence. Before moving on to PART ONE, I want to digress briefly to discuss formal vs informal concepts, and the nature of *explication* in semantics/logic.

## 1.2 Digression: Informal, Formal, *Explicandum*, *Explicatum*, and all that ...

I prefer Carnap’s way of approaching topics like these. His idea was that we are engaged in *formal*, philosophical *explication* of some *informal* concept(s). The basic concept that is underlying Field’s discussion is the concept of *truth*. The informal truth concept (*explicandum*) is, I would say, a property of propositions. Informally, then, some propositions are true, others are not. Now, we might seek an *explication* of our informal truth concept. This involves developing a formal *explicatum*. For Field, this explicatum will be a *truth predicate* of some *formal, logical theory*  $T$ , which consists of the usual apparatus of any logical theory, plus a truth predicate ‘True’, which applies to *sentences* of  $T$ . So, in “explicandum-talk”, we speak of propositions and truth (in the *informal* sense, and in some *natural* language such as English), but in “explicatum-talk” we work with *sentences* and a *truth predicate* (in the *formal* language of a logical theory  $T$ ). When we are developing an *explicatum* ‘True’, we will add certain formal principles (*desiderata*) to our theory  $T$ , with an eye toward ensuring our *explicatum* is “similar” to our *explicandum*, and that it has other virtues such as theoretical fruitfulness, simplicity, *etc.* We shouldn’t expect our explicatum to be *perfectly* similar to our explicandum. But, we will want certain *desiderata* to be (demonstrably) satisfied by the explicatum. Once we arrive at our *best explicatum* for “truth”, we can then use it for various purposes. Let’s just suppose that we’re taking Field’s favorite explicatum of “truth” as our best explicatum. In Field’s framework  $T$ , some *rules of inference* of  $T$  will not “preserve truth” — by its own lights (note: this sort of thing happens because of the *liar paradox*, but I won’t get into *that*). For Field, this *means*  $\exists$  sentence schemas  $p, q, r \in T$ , s.t.:

- $p, q \vdash_T r$
- $\not\vdash_T [\text{True}(\langle p \rangle) \ \& \ \text{True}(\langle q \rangle)] \rightarrow \text{True}(\langle r \rangle)$

Field invites us to conclude (from this *formal fact about T*) that (NTP) is false — that *logical consequence is not co-extensional with necessary truth preservation*. There are several ways in which this inference may be a bit hasty. First, is it always appropriate to conclude that an *explicandum* has a certain feature from the fact that its *best explicatum* has the (analogous) feature? Perhaps not. Perhaps this (inevitable) feature of *T* (merely) reflects a *limitation of the formal framework* we are adopting for constructing our *explicata* for “truth” (in light of the liar paradox). Putting this general question about limitations of *explicata vs explicanda* to one side, there is another, more pressing worry lurking here. Notice how Field seems to just *assume* here that  $\vdash_T$  is the best *explicatum* for *logical consequence*. Why think *that*? Remember, our aim was to give our best *explicatum* of “truth” (in light of the liar paradox). Why should it be the case that the relation  $\vdash_T$  of our formal theory *T* (in which our best explicatum of “truth” happens to be formulated) turns out to be the best explicatum for *logical consequence*, or, even the best explicatum for logical consequence *that is formulable in T*? Why couldn’t it be that the best explicatum for “logical consequence” does *not* coincide with  $\vdash_T$  in this sense? After all, if “logical consequence” is *inherently normative* (as Field himself proposes!), then why expect its best explication to emerge from our best explication of “truth” (perhaps “truth” — in light of the liar paradox — is also inherently normative, or perhaps this is just a coincidence)? Moreover, even if the best explicatum for “logical consequence” *does* turn out to be formulable in *T*, why must it be the relation  $\vdash_T$  of *T*? Why not some *other* relation formulable in *T*? Here is another relation formulable in *T*, for instance:

$$\bullet \quad p, q \models_T r \stackrel{\text{def}}{=} \vdash_T [\text{True}(\langle p \rangle) \& \text{True}(\langle q \rangle)] \rightarrow \text{True}(\langle r \rangle)$$

By *definition* (given what Field *means* by “a truth-preserving relation of *T*”), the relation  $\models_T$  will be a truth-preserving relation of *T*. Moreover, we can define still other “entailment” relations in *T*, which will behave differently than either  $\vdash_T$  or  $\models_T$ . And, we might also want to explore other possible meanings of “a truth-preserving relation of *T*”, *etc.* The question here is: why suppose that any particular one of these relations will be the *best explicatum* for “logical consequence” (in the sense Field has in mind in his own essay), or even the best one *formulable in T*? Indeed, why even suppose that the best explicatum for “logical consequence” will be formulable in *any* formal theory of this sort (such as Field’s *T*) that *has* a truth predicate? Maybe the best explicatum for “logical consequence” is the the good-old  $\models$ -relation from classical, first-order logic (*without* a truth-predicate). And, in any case, perhaps we should think of *that* explication project as being *distinct* from the project of explicating “truth” (in light of the liar paradox). OK, that ends my little digression on *explication*. Next, one more topic from Field’s discussion in PART TWO, and then it’s on to PART ONE.

### 1.3 Meta-Theoretic Argument #2: The Curry Argument

Field gives another Meta-Theoretic Argument regarding  $\sim(\text{NTP})$ , based on a truth-predicate version of the Curry Paradox. He points out that the following argument might *seem* like a good argument *for* (NTP):

The validity of the inference from  $A_1, \dots, A_n$  to  $B$ .

is equivalent to

the validity of the inference from  $\text{True}(\langle A_1 \rangle), \dots, \text{True}(\langle A_n \rangle)$  to  $\text{True}(\langle B \rangle)$ ,

by the usual truth rules. That in turn is equivalent to

the validity of the inference from  $\text{True}(\langle A_1 \rangle)$  and ... and  $\text{True}(\langle A_n \rangle)$  to  $\text{True}(\langle B \rangle)$ ,

by the usual rules for conjunction. And that in turn is equivalent to

the validity of the sentence If  $\text{True}(\langle A_1 \rangle)$  and ... and  $\text{True}(\langle A_n \rangle)$ , then  $\text{True}(\langle B \rangle)$ ,

by the usual rules for the conditional. But validity of a sentence is necessary truth (by virtue of form), so this last is just the claim that the inference necessarily preserves truth by virtue of form.

As Field explains, however, Curry’s Paradox arises here, since this argument is based in *inconsistent* principles. Specifically, the (“*T*-schema”) principles governing the truth-predicate, which underlie the first step are inconsistent with the (classical) principles governing the conditional, which underlie the last step. At best, however, these Curry Paradox considerations *undermine* one argument *for* (NTP). That’s a far cry from

providing an argument *against* (NTP), or *for*  $\sim$ (NTP).<sup>1</sup> I suppose the Gödel/Tarski considerations are meant to be more directly probative in this sense. In any event, we're now ready to move on to PART ONE.

## 2 PART ONE: Logical Consequence as a “Primitive Epistemic Relation”

In PART ONE, Field tackles several worries raised by Harman, concerning the thesis that deductive logic (the relation of logical consequence) is normative for thought. Before getting into (parts of) that dialectic, I should first point out that Field seems to think that there are only two viable options concerning logical consequence: (a) it is co-extensional with necessary truth-preservation, or (b) it is a primitive epistemic relation (*i.e.*, it is normative for thought). Having taken himself to have ruled-out (a), *via* meta-theoretic considerations (discussed above), Field seems to think (b) is the only viable alternative. This is what inspires him to try to give *some* accounting of *how* logical consequence is normative for thought, despite Harmanian worries to the contrary. Harman's four main worries about the normativity of deductive logic are (briefly):

1. Reasoning doesn't follow the pattern of logical consequence. When one has beliefs  $A_1, \dots, A_n$ , and realizes that they together entail  $B$ , sometimes the best thing to do isn't to believe  $B$  but to drop one of the beliefs  $A_1, \dots, A_n$ .
2. We shouldn't clutter up our minds with irrelevancies, but we'd have to if whenever we believed  $A$  and recognized that  $B$  was a consequence of it we believed  $B$ .
3. It is sometimes rational to have beliefs even while knowing they are jointly inconsistent, if one doesn't know how the inconsistency should be avoided.
4. No one can recognize all the consequences of his or her beliefs. Because of this, it is absurd to demand that one's beliefs be closed under consequence. For similar reasons, one can't always recognize inconsistencies in one's beliefs, so even putting aside point (3) it is absurd to demand that one's beliefs be consistent.

Field tackles (1) first. He concedes that (1) is a good reason to reject:

- (I) If one realizes that  $A_1, \dots, A_n$  together entail  $B$ , then if one believes  $A_1, \dots, A_n$ , one ought to believe  $B$ .

He says that the problem with (I) is that its “ought” takes *narrow-scope* — but, the *wide-scope* version is OK:

- (II) If one realizes that  $A_1, \dots, A_n$  entail  $B$ , then one ought not [believe  $A_1, \dots, A_n$ , and not believe  $B$ ].

It is important to contrast (II) with a different wide-scope norm:

- (III) If one realizes that  $A_1, \dots, A_n$  entail  $B$ , then one ought not [believe  $A_1, \dots, A_n$ , and believe  $\sim B$ ].

I presume that Field would also endorse (III), *modulo* preface-y considerations (to which we will return when we discuss Harman's worry (3), below). OK, but *why* — according to Field — are these wide-scope norms correct (*qua* norms)? Here's one possible explanation of *why* such wide-scope norms are correct.

We realize that believing that  $A$  is equivalent to believing that  $A$  is true. If one realizes that  $A$  entails  $B$ , then one realizes that the inference from  $A$  to  $B$  preserves truth. So, suppose you realize that  $A$  entails  $B$ , and you believe that  $A$ . Now, in such a situation, if you were to believe that  $\sim B$ , then your beliefs would be inconsistent, which is bad because inconsistency entails that some of your beliefs are false; and, *qua* believers we ought (*ceteris paribus*) to avoid being in a situation where we know *a priori* some of our beliefs are false. Why? Because truth and the avoidance of falsehood are *fundamental epistemic aims of belief*. Moreover, if you were to *withhold judgment* on  $B$  (in this situation), this would also be rather odd, since you would be aware that the truth of  $B$  must obtain if the truth of  $A$  (which you now believe) obtains. So, from the point of view of epistemic rationality, what grounds could you have for withholding belief?

I don't mean to defend this “explanation”. But, it doesn't sound crazy to me (it sounds no less crazy to me than the norms themselves). The important point here is that this explanation is *not open to Field*. He *rejects* the claim that realizing that  $A$  entails  $B$  is tantamount to realizing that the inference from  $A$  to  $B$  *preserves truth*. More generally, any explanation that is tied essentially to whether the inference from  $A$  to  $B$  *preserves truth* is not open to Field. This means that the standard *truth-connection* stories are not available to him. I think that's *not a good thing*. To my mind, the truth-connection is quite important in epistemology, and it's difficult to see how we're going to explain the correctness or goodness of these sorts of norms, without appealing to it. As I'll explain, this is a persistent, general worry I have about *all* the norms Field discusses.

Next: Harman's worry (3). Harman points out that it (sometimes) seems reasonable to have beliefs that one knows are inconsistent (*e.g.*, the global preface paradox case). Field suggests alternatives to (II) and (III):

<sup>1</sup>Moreover, it's not clear to me that a defender of (NTP) would need to endorse inconsistent principles in order to endorse this argument. The principles that are inconsistent have to do with certain assumptions of logical equivalence of *formulas*, not logical equivalence of *claims involving the validity of inferences*. So, strictly speaking, it's not true that the above argument is sound only if incompatible principles about equivalence of formulas are presupposed. But, that's a rather nit-picky point, which I'll let slide.

(II\*) If one realizes that  $A_1, \dots, A_n$  entail  $B$ , then one ought not [believe  $A_1 \& \dots \& A_n$ , and not believe  $B$ ].

(III\*) If one realizes that  $A_1, \dots, A_n$  entail  $B$ , then one ought not [believe  $A_1 \& \dots \& A_n$ , and believe  $\sim B$ ].

Even the preface paradox cases are (intuitively) *not* counterexamples to (II\*) or (III\*). *Why not?* First, let's think a bit about the preface cases. *Why* does it seem (intuitively) OK in some preface cases to believe  $A_1, \dots, A_n$  and *also* to believe  $\sim B$ , where you know that  $B$  follows from  $A_1, \dots, A_n$ ? In a global preface case, the  $A_1, \dots, A_n$  are *all* of your beliefs, which you (because of your knowledge of human nature and the complexity of your beliefs  $A_1, \dots, A_n$ , etc.) believe contain *some* errors (i.e., you must have *some* false beliefs). Because *that* belief is *itself* one of the  $A_i$ 's, you thus realize that your beliefs  $A_1, \dots, A_n$  are *inconsistent* — they can't all be true. Now, Harman thinks this is a counterexample to (III), because *he* thinks that entailment is necessary truth-preservation. And, on *that* conception of entailment, *everything* (including  $B$ ) follows from  $A_1, \dots, A_n$ . Now, let us suppose further that you know  $B$  is false (here, Harman presupposes that having inconsistent beliefs doesn't preclude you from having *some* knowledge, and that seems right). Then, you *should* believe  $\sim B$ , since you should believe things that you know (*everyone* accepts *that*, even Williamson<sup>2</sup>). Hence, this is why Harman thinks there are counterexamples to (III).<sup>3</sup> Now, how does (III\*) avoid such preface counterexamples? Nobody (Harman included) thinks it's OK for you to have *particular* beliefs which you *know* to be (necessarily) false. And, you know that in a preface case, believing  $A_1 \& \dots \& A_n$  is believing something that is (necessarily) false. The key to the preface case is that you have no idea *which* of your beliefs  $A_1, \dots, A_n$  is false, and this is why you are under no rational pressure to revise any of your *particular* beliefs. But, if you believed  $A_1 \& \dots \& A_n$ , then you *would* have rational pressure to revise *that very belief*, since you *know it (a priori) to be (necessarily) false*. This is why (III\*) is, from Harman's perspective, able to avoid preface-type counterexamples. Note, again, the appeals to *believing truths* and *avoiding believing falsehoods*, which seem (to me) to be *crucial* parts of these “explanations” of what's going on in preface cases. Harman is *all about the truth-connection*, because *he* thinks entailment is *necessary truth-preservation*. Field, on the other hand, rejects this account of entailment. And, so, to my ear, Field's epistemic stories and explanations aren't as compelling as Harman's in this sense. Moreover, I find it difficult to even *construct* a *clear* preface-type case — *from Field's perspective* — since I don't know precisely what the extension of his “entailment” relation is (see footnote 3). In fact, this is a general problem I faced throughout my readings of the paper.

Moving on from Harman's worries (1) and (3), I will discuss (some of) Field's responses to worry (4) next. Field breaks (4) down into two component worries. Worry (4a) is what he calls the “computational aspect” of problem (4). There is a *prima facie* compelling reason to *strengthen* (II\*) and (III\*), as follows:

(II\*<sub>s</sub>) If  $A_1, \dots, A_n$  entail  $B$ , then one ought not [believe  $A_1 \& \dots \& A_n$ , and not believe  $B$ ].

(III\*<sub>s</sub>) If  $A_1, \dots, A_n$  entail  $B$ , then one ought not [believe  $A_1 \& \dots \& A_n$ , and believe  $\sim B$ ].

The reason for wanting to remove the “one realizes that” caveat from (II\*)/(III\*) is given by MacFarlane:

[one natural reading of (II\*)/(III\*) suggests that] the more ignorant we are of what follows logically from what, the freer we are to believe whatever we please however logically incoherent it is. But this looks backward. We seek logical knowledge so that we know how we ought to revise our beliefs: not just how we *will be* obligated to revise them when we acquire this logical knowledge, but how we *are* obligated to revise them even now, in our state of ignorance.

But, if we move to (II\*<sub>s</sub>)/(III\*<sub>s</sub>), this seems to be an *over*-reaction to MacFarlane's worry about (II\*)/(III\*), since it is *not humanly possible* to believe all the logical consequences of one's beliefs (even in the *wide-scope* sense implied by II\*<sub>s</sub>/III\*<sub>s</sub>). And, this seems to drain the resulting “norms” of their *normative force*. Or, to put it more positively, it seems quite clear (but not for Williamson?) that it can be reasonable to believe things which *turn out* to be inconsistent, for some very subtle reason that has eluded all investigators for many years.<sup>4</sup> Field proposes the following “obviousness cheat” as a “middle-way” of amending (II\*) and (III\*):

(II\*<sub>o</sub>) If  $A_1, \dots, A_n$  *obviously* entail  $B$ , then one ought not [believe  $A_1 \& \dots \& A_n$ , and not believe  $B$ ].

(III\*<sub>o</sub>) If  $A_1, \dots, A_n$  *obviously* entail  $B$ , then one ought not [believe  $A_1 \& \dots \& A_n$ , and believe  $\sim B$ ].

<sup>2</sup>Note: Williamson will *reject* the claim that it's OK for you to believe each of the  $A_1, \dots, A_n$ , since you *cannot know* each of them!

<sup>3</sup>Why does Field think so? Well, this is not as clear. Field *rejects* the (NTP) account of entailment. So, you might wonder how *he* gets from the inconsistency of  $A_1, \dots, A_n$  to the claim that  $A_1, \dots, A_n$  jointly entail  $B$  (which you know to be false). This is a lacuna in Field's discussion, which he does not seem to notice. I suppose all he needs is *one case* of this kind, which I trust he can give. But, without a clear sense of what the *extension* of Field's “entailment” relation is, it's difficult (for me) to clearly *demarcate* cases.

<sup>4</sup>Field gives the example of the discovery of continuous functions mapping the unit interval onto the unit square, which everyone assumed were impossible, until Peano *demonstrated* such functions. This also happens in *logic* (think Russell, Gödel, etc.).

At some stage of the discussion here, Field switches from talking about norms involving *full* belief to norms involving *degrees* of belief. He proposes the following norm relating entailment and degree of belief:

(D) If  $A_1, \dots, A_n$  together obviously entail  $B$ , then one's degrees of belief in  $A_1, \dots, A_n$  and  $B$  [which he denotes  $P(A_1), \dots, P(A_n), P(B)$ ] should be related as follows:  $P(B) \geq P(A_1) + \dots + P(A_n) - (n - 1)$ .

In the special case where  $n = 1$ , this reduces to the following norm:

(D<sub>1</sub>) If  $A$  obviously entails  $B$ , then one's degrees of belief should be such that:  $P(A) \leq P(B)$ .

Before moving on to some other issues addressed by Field, I want to voice my "truth connection" worry one last time, now with respect to (D<sub>1</sub>). Ask yourself *why* (D<sub>1</sub>) seems to be a correct norm for *degree* of belief. Here's *one* possible "explanation" of the correctness of (D<sub>1</sub>) that is (once again) *not open to Field*:

If  $A$  obviously entails  $B$ , then it is obviously the case that all situations/possible worlds in which  $A$  is true are also situations/possible worlds in which  $B$  is true. And, if it's obvious to you that whenever  $A$  is true,  $B$  must also be true, then you should not assign a degree of belief to  $A$  that is greater than the degree of belief you assign to  $B$ . Why? Because, in such a situation, it is obvious to you that there are *strictly more possibilities* in which  $B$  is true than there are possibilities in which  $A$  is true. And, this implies that the likelihood of  $B$ 's truth *cannot be less than* the likelihood of  $A$ 's truth. Finally, degree of belief just *is* a measure of (one's epistemically rational assessment of) *likelihood of truth*.

Once again, I'm not defending this "explanation", but (a) it seems no less crazy than the norm (D<sub>1</sub>) itself, (b) it is not open to Field, and (c) I can't think of any *alternative* explanation of the correctness of (D<sub>1</sub>), which would be open to Field, and which would make more sense. That's not an argument, it's a challenge. Having said that, two interesting things Field points out at this stage are (i) (D) has the virtue of being "neutral to the underlying logic", and (ii) if we assume *classical* logic, then (D) is a consequence of *Bayesianism* (which requires that rational degrees of belief obey *classical* probability calculus). That's right, and this is one reason why my "classical explanation" of (D<sub>1</sub>) sounds as plausible as it does. Field has a discussion of "the right logic" vs "the logic one accepts" that I don't really understand (partly because he's an *expressivist*). I'll say a bit more about that, at the end of these notes. Meanwhile, let's look at what Field says about worry (2).

The way Field handles the "clutter avoidance" problem is by making an *implicit/explicit* belief distinction. And, in the degree of belief case, he talks about *implicit constraints* on degrees of belief. He proposes:

(\*\*) If  $A_1, \dots, A_n$  together obviously entail  $B$ , then one shouldn't *explicitly* believe  $A_1, \dots, A_n$  without (at least) *implicitly* believing  $B$ .

(D\*) If  $A_1, \dots, A_n$  together obviously entail  $B$ , then *one ought to impose the constraint that*  $P(B) \geq P(A_1) + \dots + P(A_n) - (n - 1)$ , in any circumstance where  $A_1, \dots, A_n$  and  $B$  are in question.

To handle the "obviousness cheat", Field proposes laying down specific rules, single applications of which are to count as "the obvious entailments", and restricting the domain of application of (D\*) to those rules:

(D\*<sub>alt</sub>) If  $B$  follows from  $A_1, \dots, A_n$  *by a single application of a rule on the list*, then one ought to impose the constraint that  $P(B)$  is to be at least  $P(A_1) + \dots + P(A_n) - (n - 1)$ , in any circumstance where  $A_1, \dots, A_n$  and  $B$  are in question.

(D\*<sub>alt</sub>) is the final norm he proposes. Then, he discusses the (subtle and confusing) "correct logic" issue (4b). Field asks: "should the facts of logical implication impose an obligation on those who don't accept the logic, especially those who have serious (even though not ultimately correct) reasons for not accepting it?"

On what is probably the most natural interpretation of (D\*<sub>alt</sub>), the "simple rules" it talks about are simple rules of *the correct logic*. In that case, (D\*<sub>alt</sub>) answers 'yes'. But there is a case to be made that this consequence of (D\*<sub>alt</sub>) is incorrect. Suppose that classical logic is in fact correct, but that Bob has made a very substantial case for weakening it (we may even suppose that no advocate of classical logic has yet to give an adequate answer to his case). Suppose that usually Bob reasons in accordance with the non-classical logic he advocates, but that occasionally he slips into classical reasoning that is not licensed by his own theory. Isn't it when he *slips and reasons classically* that he is violating rational norms? But (D\*<sub>alt</sub>) (on the natural interpretation) says that it is the other occasions, when he follows the logic he believes in, that he is violating norms. That's problem 4b.

[I'm not sure what to say about this part of the paper. Here are some inchoate thoughts. I find this last bit confusing (and it is followed by an expressivist proposal that I do not understand). How can we suppose that classical logic is "the correct logic"? Isn't Field arguing that it is *not*? And, then, wouldn't this require supposing something logically impossible? Moreover, what does "the correct logic" *mean*? I suppose, for Field, it means something *normative*. As far as I can tell, though, none of the norms Field proposes are *incompatible* with the view that *logic is necessary truth-preservation*. So, how are we to get traction on the "correct logic" question, *even if we grant* that logic is normative for thought? Alas, I wish I had a better grasp on (even just the *extension* of) *what Field means by* "logical consequence".]