Philosophy 57 — Day 8

- I will return quiz #2 on Tuesday
- There is no quiz on Tuesday (Quiz #3 is on the following Tuesday 3/04/03)
- Back to Chapter 4 — Categorical Statements
  - Brief Review of Terminology
  - Quality, Quantity, and Distribution of Categorical Statements
  - Venn Diagram Representations of Categorical Statements
  - Using Venn Diagram's to Study Simple Arguments
  - The Square of Opposition
  - Conversion, Obversion, and Contraposition
  - Later: Translating from English into Categorical Logic
  - **NOTE**: Sections 4.5–4.6 *skipped* (no Aristotelian stuff)

Chapter 4: Categorical Statements — Definition & Components

- A categorical statement (or proposition) relates two classes or categories, denoted by the subject term (S) and the predicate term (P). Categorical statements assert that either all or part of S is included in (excluded from) P.
- Categorical statements come in four standard forms:

(**A**) All S are P.	(**E**) No S are P.
(**I**) Some S are P.	(**O**) Some S are not P.

- The words "all", "no" and "some" are called quantifiers.
- The words "are" and "are not" are called the copula.

Example. All members of the American Medical Association are persons holding degrees from recognized academic institutions.

* quantifier = "all," S = "members of the AMA," P = "persons holding degrees from recognized academic institutions," copula = "are".

Chapter 4: Categorical Statements — Quality, Quantity & Distribution I

- The quality of a categorical claim is either affirmative or negative, depending on whether it *affirms* or *denies* class membership.
  - “All S are P” and “Some S are P” have *affirmative* quality.
  - “No S are P” and “Some S are not P” have *negative* quality.
- The quantity of a categorical claim is either universal or particular, depending on whether it makes a claim about *every* member or just *some* member of S.
  - “All S are P” and “No S are P” are *universal*.
  - “Some S are P” and “Some S are not P” are *particular*.
- A term X is distributed in a categorical statement if the statement asserts something about *every* member of the class X (otherwise, X is *un*distributed).
  - S is distributed in “All S are P” and “No S are P”.
  - P is distributed in “No S are P” and “Some S are not P”.
- Remember: **U**niversals distribute **S**ubjects. **N**egatives distribute **P**redicates.

Chapter 4: Categorical Statements — Quality, Quantity & Distribution II

Proposition	**Name**	**Quantity**	**Quality**	***S***	***P***
All *S* are *P*.	**A**	Universal	Affirmative	Distributed	Undistributed
No *S* are *P*.	**E**	Universal	Negative	Distributed	Distributed
Some *S* are *P*.	**I**	Particular	Affirmative	Undistributed	Undistributed
Some *S* are not *P*.	**O**	Particular	Negative	Undistributed	Distributed

- It may help to simply *memorize* the cases of distribution. The text offers two mnemonic devices for remembering the above facts about distribution.

  Mnemonic #1. **U**nprepared **S**tudents **N**ever **P**ass.

  Universals distribute **S**ubjects. **N**egatives distribute **P**redicates.

  Mnemonic #2. **A**ny **S**tudent **E**arning **B**'s **I**s **N**ot **O**n **P**robation.

  A distributes **S**ubject. **E** distributes **B**oth.

  I distributes **N**either. **O** distributes **P**redicate.

- I prefer to *deduce* these using Venn Diagrams and the *definition* of distribution. In Logic, answers can always be *deduced* from basic definitions.

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition I

- Ultimately, we will use Venn Diagrams to test categorical *arguments* (*syllogisms*) for validity and invalidity. First, we need to learn how to represent categorical *statements* using Venn Diagrams.
- We will always operate from the *modern*, *Boolean* standpoint. You can ignore the stuff in the book about the traditional, Aristotelain standpoint.
- The standard from categorical statements can be understood as follows:

  (A) All S are P. = No members of S are *outside* P.

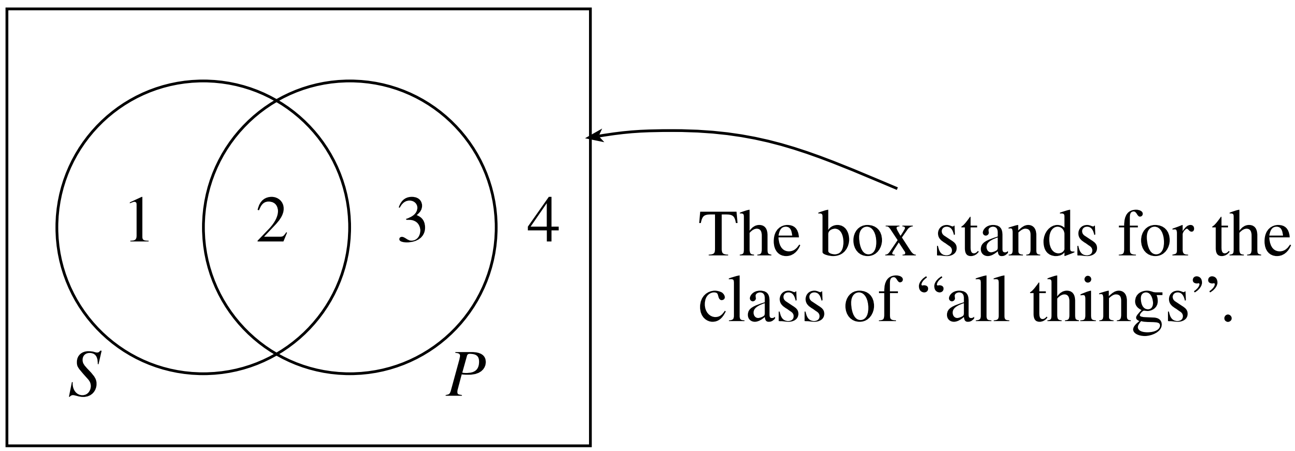
  (E) No S are P. = No members of S are *inside* P.

  (I) Some S are P. = At least one S exists, and that S is a P.

  (O) Some S are not P. = At least one S exists, and that S is not a P.
- **Note**: **A** and **E** do *not* imply that any S's *exist*! This is the modern, Boolean standpoint. On the Aristotelian view, **A** and **E** *do* imply that some S's exist.
- Consider "All unicorns are one-horned animals" (Boolean *vs* Aristotelian).

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition II

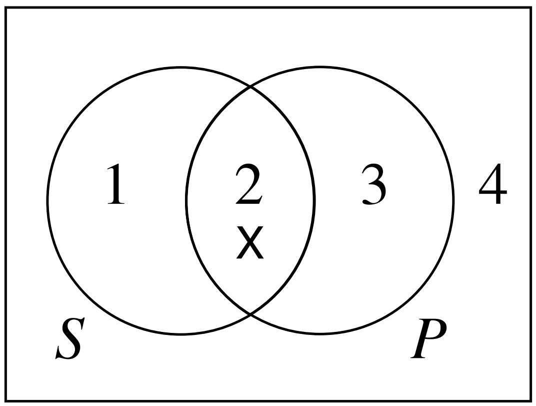
- To represent categorical statements using Venn Diagrams, we draw a box containing two overlapping circles. The box stands for "all things", and the two circles stand for the S and P classes in the claim being represented.



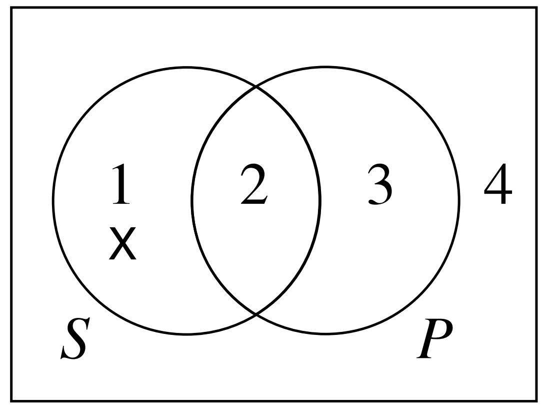
- It is helpful to think about which class of things are contained in each of 1–4.
- Region 1 = the class of things which are inside S but outside P.
  Region 2 = the class of things which are inside S and inside P.
  Region 3 = the class of things which are outside S and inside P.
  Region 4 = the class of things which are outside S and outside P.

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition III

- Next, we adopt the following two Venn Diagram conventions.
  1. If a region (*i.e.*, 1–4) is *empty*, we use *shading* (*hashing*) to indicate this.
  2. If a region contains *at least one thing*, we use an "X" to indicate this.
- Venn Diagrams for the *particular* claims **I** and **O** involve only "X"s:



(**I**) Some *S* are *P*.

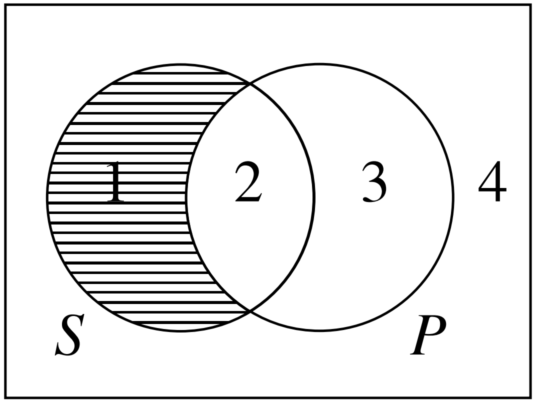


(**O**) Some *S* are not *P*.

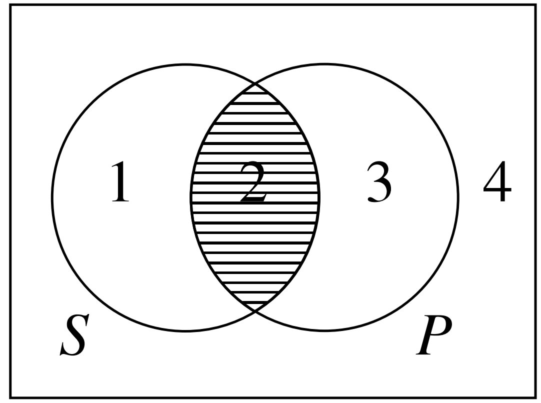
- It should be clear from these diagrams that the **I** and **O** claims *say different things*. We'll show below that *neither claim implies the other*.

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition IV

- The *universal* **A** and **E** claims require the *shading* (*hashing*) of regions.



(**A**) All *S* are *P*.



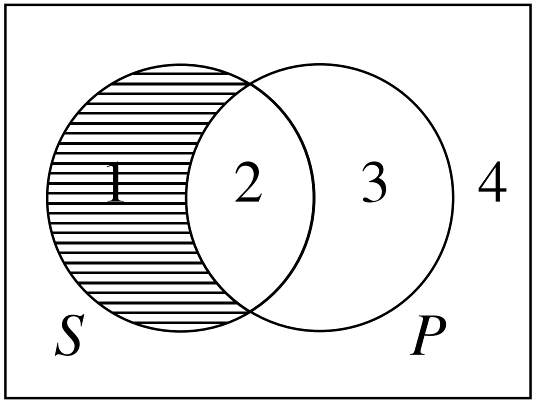
(**E**) No *S* are *P*.

- We can use these 2-circle Venn diagrams to investigate the *logical relationships between* the 4 standard-form categorical claims.
- For instance, we can already determine if the following four simple arguments are valid (Hurley calls these arguments "immediate inferences"):

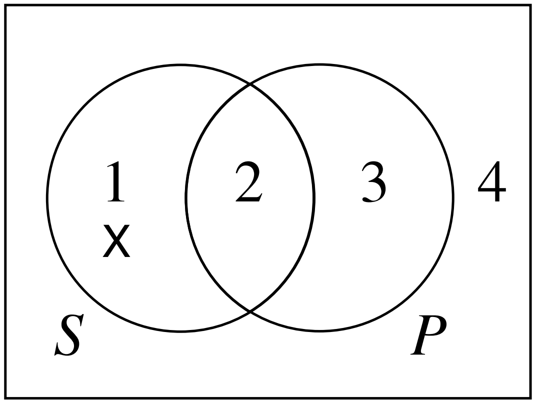
$$\frac{\textbf{A}}{\therefore\ \textbf{O}},\quad \frac{\textbf{A}}{\therefore\ \textit{not}\text{-}\textbf{O}},\quad \frac{\textbf{E}}{\therefore\ \textbf{I}},\quad \frac{\textbf{E}}{\therefore\ \textit{not}\text{-}\textbf{I}}$$

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition V

- Three steps: (1) Draw the Venn Diagram for the premise, (2) Draw the Venn Diagram for the conclusion, (3) Does the premise-diagram contain the information in conclusion-diagram? If so, then the inference is valid.
- Example: $\frac{\mathbf{A}}{\therefore \mathbf{O}}$. Putting the **A** and **O** diagrams side by side, we have:



(**A**) All *S* are *P*.

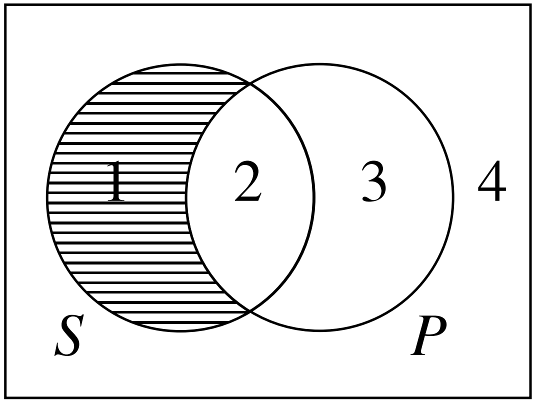


(**O**) Some *S* are not *P*.

- We can see that the premise-diagram does not contain the information of the conclusion diagram. So, the argument $\frac{\mathbf{A}}{\therefore \mathbf{O}}$ is *invalid* ($\mathbf{A} \not\Rightarrow \mathbf{O}$).
- What about the argument from **A** to the *denial* of **O**?

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition VI

- To draw the Venn diagram for the *denial* of a categorical claim, one marks the same regions as for the categorical claim itself — *but in the opposite ways*. Instead of putting an "X" in a region, one shades it (and *vice versa*).
- So, the *denial* of an **O** claim would look like this:

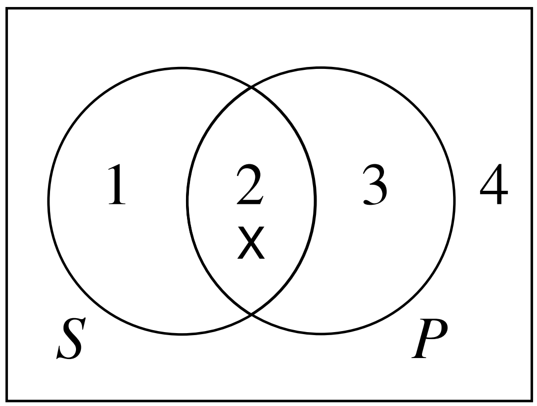


not–**O**: It is *not* the case that some *S* are not *P*.

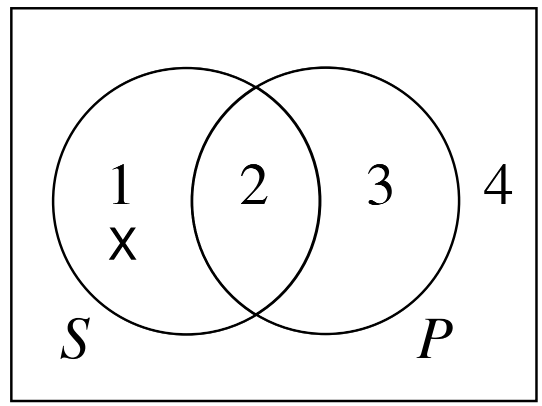
- But, this *is* just the **A**-diagram! That is, the **A**-diagram contains the information in the *not*-**O**-diagram. Hence, $\frac{\textbf{A}}{\therefore \textit{not-}\textbf{O}}$ *is valid* (**A** $\Rightarrow$ *not*-**O**).

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition VII

- We can use the same technique to analyze $\frac{\mathbf{E}}{\therefore \mathbf{I}}$ and $\frac{\mathbf{E}}{\therefore \textit{not}\text{-}\mathbf{I}}$. Blackboard exercise.
- Let's return to the inference from **I** to **O**. Recall, I said that "Some *S* are *P*" does *not* imply "Some *S* are not *P*". Look at the diagrams again:



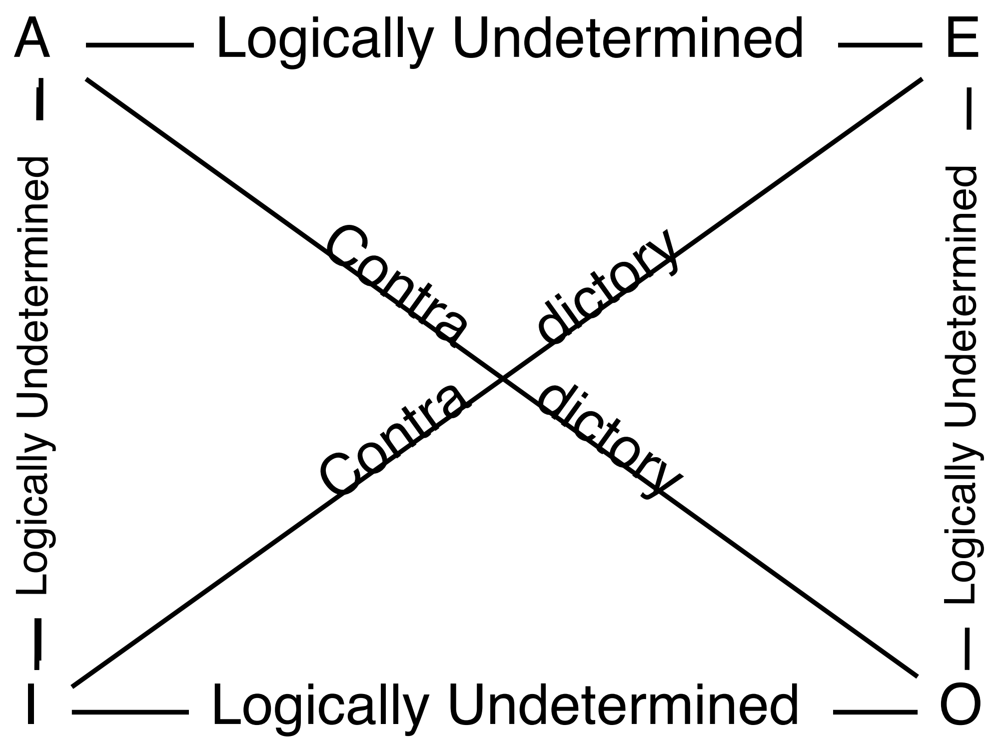
(**I**) Some *S* are *P*.



(**O**) Some *S* are not *P*.

- The **I**-diagram does *not* contain the information of the **O**-diagram. So, the argument $\frac{\mathbf{I}}{\therefore \mathbf{O}}$ is *invalid* ($\mathbf{I} \nRightarrow \mathbf{O}$). "Some *S* are *P*" $\nRightarrow$ "Some *S* are not *P*"
- Also: $\mathbf{I} \nRightarrow$ *not*-**O**, $\mathbf{A} \nRightarrow \mathbf{I}$, $\mathbf{A} \nRightarrow$ *not*-**I**, $\mathbf{E} \nRightarrow \mathbf{O}$, $\mathbf{E} \nRightarrow$ *not*-**O**. These logical relationships between **A**, **E**, **I**, **O** are summarized in the Square of Opposition.

Chapter 4: Categorical Statements — Venn Diagrams & The Square of Opposition VIII



- This Square is just a handy way of summarizing the following 12 logical relationships between the four standard form categorical claims:
  - **A** $\Rightarrow$ *not*-**O**, **O** $\Rightarrow$ *not*-**A**, **E** $\Rightarrow$ *not*-**I**, **I** $\Rightarrow$ *not*-**E**, **I** $\not\Rightarrow$ **O**, **I** $\not\Rightarrow$ *not*-**O**, **A** $\not\Rightarrow$ **I**, **A** $\not\Rightarrow$ *not*-**I**, **E** $\not\Rightarrow$ **O**, **E** $\not\Rightarrow$ *not*-**O**, **A** $\not\Rightarrow$ **E**, **A** $\not\Rightarrow$ *not*-**E**.

Chapter 4: Categorical Statements — Conversion, Obversion & Contraposition I

- Conversion, Obversion, and Contraposition are three important operations or transformations that can be performed on categorical statements.
- The Converse of a categorical statement is obtained by switching its subject and predicate terms. This switching process is called Conversion.

Proposition	Name	Converse
All *A* are *B*.	**A**	All *B* are *A*.
No *A* are *B*.	**E**	No *B* are *A*.
Some *A* are *B*.	**I**	Some *B* are *A*.
Some *A* are not *B*.	**O**	Some *B* are not *A*.

- Some statements are *equivalent to* (*i.e.*, *have the same Venn Diagram as*) their converses. Some statements are *not* equivalent to their converses.
- **E** and **I** claims are equivalent to their converses, whereas **A** and **O** claims are *not* equivalent to their converses. Let's *prove* this with Venn Diagrams.

Chapter 4: Categorical Statements — Conversion, Obversion & Contraposition II

- The complement of a term "X" is written "non-X", and it denotes the class of things *not* contained in the X-class. Do not confuse "not" and "non-". "not" is part of the *copula* "are not", but "non-" is part of a *term* "non-X" ("non-X" can be either the subject term or the predicate term of a categorical statement).
- The Obverse of a categorical statement is obtained by: (1) switching the quality (but *not* the quantity!) of the statement, and (2) replacing the predicate term with its complement. This 2-step process is called Obversion.

Proposition	**Name**	**Obverse**
All A are B.	**A**	No A are non-B.
No A are B.	**E**	All A are non-B.
Some A are B.	**I**	Some A are not non-B.
Some A are not B.	**O**	Some A are non-B.

- *All* categorical statements are logically equivalent to their obverses. Let's *prove* this for each of the four categorical claims, using Venn Diagrams.

Chapter 4: Categorical Statements — Conversion, Obversion & Contraposition III

- The Contrapositive of a categorical statement is obtained by: (1) *converting* the statement, and (2) replacing both the subject term and the predicate term with their complements. This 2-step process is called Contraposition.

Proposition	**Name**	**Contrapositive**
All *A* are *B*.	**A**	All non-*B* are non-*A*.
No *A* are *B*.	**E**	No non-*B* are non-*A*.
Some *A* are *B*.	**I**	Some non-*B* are non-*A*.
Some *A* are not *B*.	**O**	Some non-*B* are not non-*A*.

- Some statements are *equivalent to* (*i.e.*, *have the same Venn Diagram as*) their contrapositives. Some statements are *not* equivalent to their contrapositives.

- **A** and **O** claims are equivalent to their contrapositives, whereas **E** and **I** claims are *not* equivalent to their contrapositives. Let's *prove* this with Venn's.