Comments on Lara Buchak, "Risks and Tradeoffs"

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Introduction: an ancestral FEW session, 2005

Buchak's paper on treating risk sensitivity in rational decision theory; Armendt's comments included:

- What are the *prob*, *des* functions, given that *EU* theory is being revised? Can we be sure that the account is consistent when it is fully applied? In short, is there an axiomatic foundation for the account?
- Shouldn't the approach capture risk-seeking as well as risk-aversion?
- Why do the *r* functions have exactly the features and influence Buchak describes? Why are they purely functions of probability/belief, fixed over all choices with the same probability profiles?
- Buchak finds no effective arguments that risk-sensitivity is irrational. Is there an effective positive argument that it is consistent with rationality? Is there a good motivation for including this much risk-sensitivity in rational decision theory, but no more?

Buchak has met the first and second challenges exceptionally well. She has provided a preference-theoretic foundation for a general account of risk-sensitivity, one that encompasses a wide range of possible risk-sensitive attitudes to be captured by *r* functions of many shapes.

This is a remarkable achievement, and the book displays a sophisticated theory, together with a rich discussion of related literature and relevant philosophical issues.

The third and fourth challenges have certainly been addressed, and Buchak has gone a long way toward meeting them, too. My comments today mainly seek to hear a little more about them.

Motivation for REU

Knowing how much he values the various ends involved and how likely each act is to lead to each end is not enough to determine what the agent should do in these cases: the agent must make a judgment not only about how much he cares about *particular* ends, and how effective his actions will be in realizing *each* of these ends, but about which sort of *strategy* to take towards realizing his ends *as a whole*: how to structure the realization of his aims. This involves deciding whether to prioritize definitely ending up with something of some value or instead to prioritize possibly ending up with something of extraordinarily high value, and by how much: specifically, he must decide the extent to which he is generally willing to accept the risk of something worse in exchange for the possibility of something better. This judgment corresponds to considering global or structural properties of gambles. (Buchak, p.5)

REU is the normative theory of how to do this. The function *r* , together with the combination rule *REU*, describes how the influences of beliefs (probabilities) are modified/filtered in light of their relevance to the set of strategies the decision-maker considers, and among which he chooses.

Together with *p* and *U*, *r* provides a fuller characterization of an agent's rational efforts to make the best choice.

Comment on the quoted passage:

> '…he must decide the extent to which he is generally willing to accept the risk of something worse in exchange for the possibility of something better. This judgment corresponds to considering global or structural properties of gambles.'

Notice the use here (and elsewhere) of 'global.' It contrasts nicely with 'particular', but so might an adjective like 'regional.'

REU shares features of other formal theories, and in a way synthesizes two other formal results.

It is innovative in providing that synthesis, and in providing the formal account of risk-sensitivity for a subjective *EU* theory, where *p* is to be consistently interpreted as subjective degree of belief.

Weights given to outcomes in deliberation are subjective beliefs, filtered through the weighting function *r*.

The combination rule that yields *REU* is a cumulative utility theory, which provides a bottom-up recipe for calculating the effects of *r:*

> *REU*(act) is the utility of its worst outcome plus the sum of the risk-weighted increments of utility attributable to the possibility of getting the next-worse outcome, and the next-worst after that, and so on.

Buchak's theory establishes the existence of a unique *r* that, in the context of the combination rule, does this.

REU is a natural way to capture how an agent who focuses on the minimum guarantee might think: Start with the guaranteed minimum outcome, and upgrade the act's utility by the other possible outcomes it may yield; each increment is the extra utility that outcome would provide over the next-worst one, weighted by the risk-colored probability of getting that outcome.

An alternative top-down construction, could be given (and has been, elsewhere), and could be seen as a natural way to capture the thinking of agents who focus first on the best possible outcome, and decrement from there.

Given that *r* can be concave or convex, either way can capture risk aversion or risk seeking. And one can be transformed into the other by modifying the risk function.

Note that after the theory both chooses a combination rule, and provides a unique global *r* for a given rational agent, it is committed to being able to use one of these ways, not both, to informally capture the agent's thinking throughout the preference system.

Aside on Rabin's result

When risk aversion is represented by concave U, mild local concavity everywhere yields extreme large-scale concavity:

If mild risk aversion over moderate stakes, e.g.

reject \$110 on heads, -\$100 on tails

is present regardless of initial wealth level,

it forces extreme risk aversion at higher stakes

reject \$$x$ on heads, -\$1000 on tails *no matter how large x.*

- Significant for economic models—population of agents, single representative curve.
- But for individual decision-maker, whose preferences may evolve over time: when a U curve is his at t, the equivalent condition would be mild local concavity at all points representing counterfactual wealth levels.
- That strong condition is doubtful.

Suppose I would reject \$110 on heads, -\$100 on tails.

Would I now also say that if I had \$100,000 more than I do, I would reject \$110 on heads, -\$100 on tails?

> Not at all obvious, in fact I doubt it. That much local concavity everywhere is implausible.

Note that I can now judge both that,

- conditional on my being richer, I should take the gamble, and
- were I lot richer, my preferences could shift in a way that would lead me to then reject the gamble.

My point is not to claim, *contra* Rabin, Buchak, that concave/convex *U* accounts for all risk-sensitivity.

But Rabin's result is not as dire as it may seem for subjective *EU* in rational choice theory for individuals.

On the global character of risk-sensitivity

REU establishes, for an agent satisfying its axioms, a unique, global risk function *r*. Are there good normative reasons for expecting a unique global *r*?

A flimsy analogy to norms of motoring:

- (Ohio): obey the speed limit
- (California): lots of ways of deviating from the speed limit are OK, but always do it in the same way no matter what the driving occasion
  same way = same function of the speed limit, many candidates

(California) is clearly more lenient. But it surely elicits the question: why the same way every time? What's the normative justification of that?

- (Nevada): Drive in a way that's inspired by the speed limit, as fits the occasion…

REU is a generalization of *EU*, *REU* is *EU* when *r* is the identity function.

So *REU* allows a wider range of rational preference systems than does *EU*.

So it may seen inappropriate to ask why *REU* imposes a norm of global risk-sensitivity, when *EU* imposes the same norm, only more stringently, requiring global risk-neutrality. (Allowing no risk-sensitivity of the sort *REU* captures.)

Still, when we attend to the requirement, it is natural to ask,

> "Why is it a norm of rational preference that an agent's risk-sensitivity applies globally in this way?"

That's a question about a *result* that the formal theory provides. A great virtue of having a formal theory is that we can look for an answer in the *axioms*.

So, a corresponding question, intended not as a challenge, but a request for help:

What is the axiomatic source of the unique, global *r* function?

Buchak's paper has a good discussion of what *makes room* for the *r* function in the theory (p. 20-23): the restriction of the tradeoff consistency requirement to sets of pairwise comonotonic acts (comoncones), rather than requiring tradeoff consistency over all acts.

But (speaking autobiographically) the axiomatic source of the uniqueness of the global *r* is more obscure, and more guidance would help (me) judge the normative plausibility of that axiomatic source, as opposed to possible alternatives.

Is risk sensitivity uniform over risky choices? Should it be?

An illustration from our previous session that may still serve here:

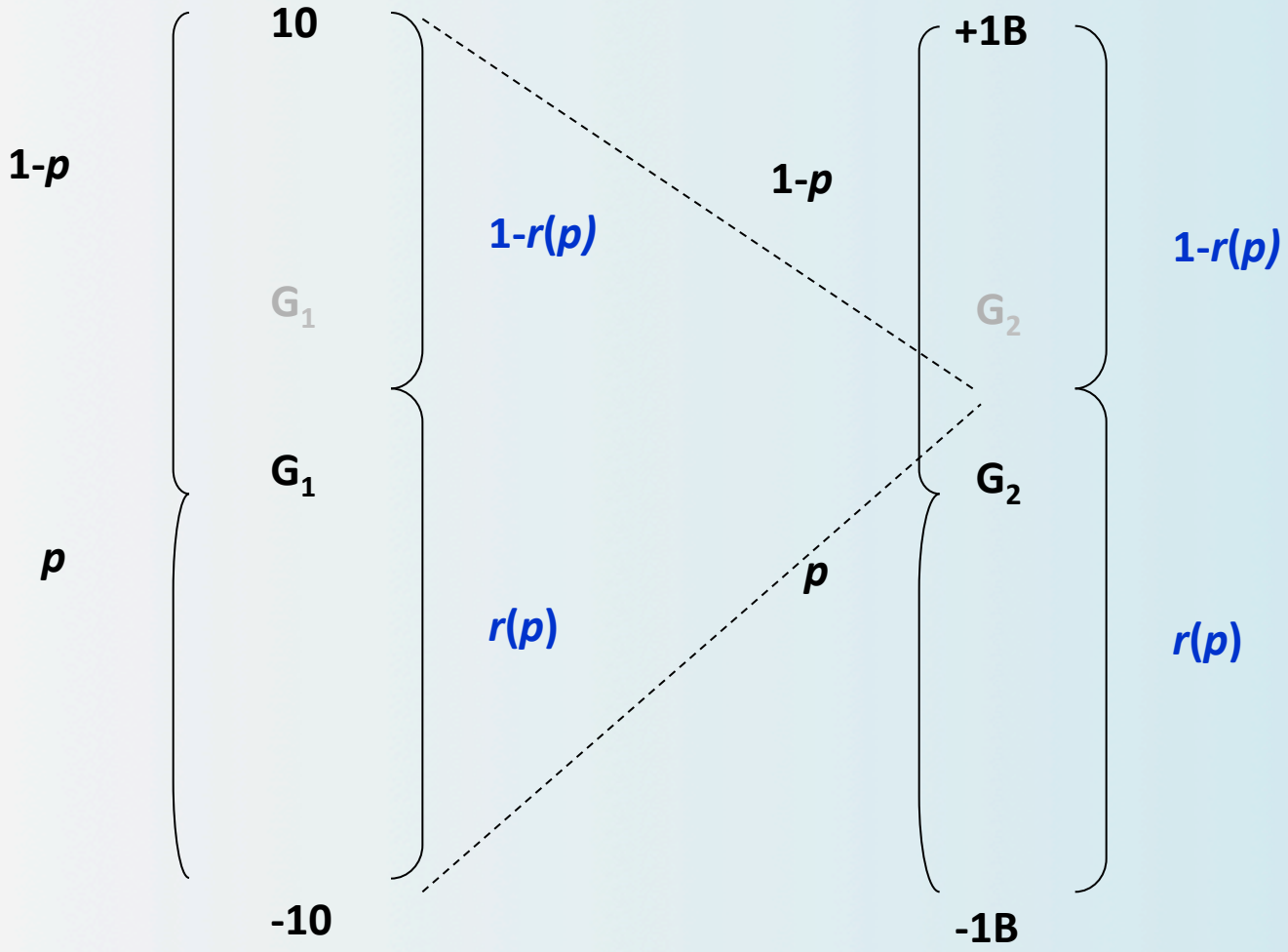
Consider gambles, G_1, G_2, each has two possible outcomes. I describe their payoffs in utility, with cues about what those utilities might measure.

G_1: p(10 utiles) ⊻ (1-p)(-10 utiles)

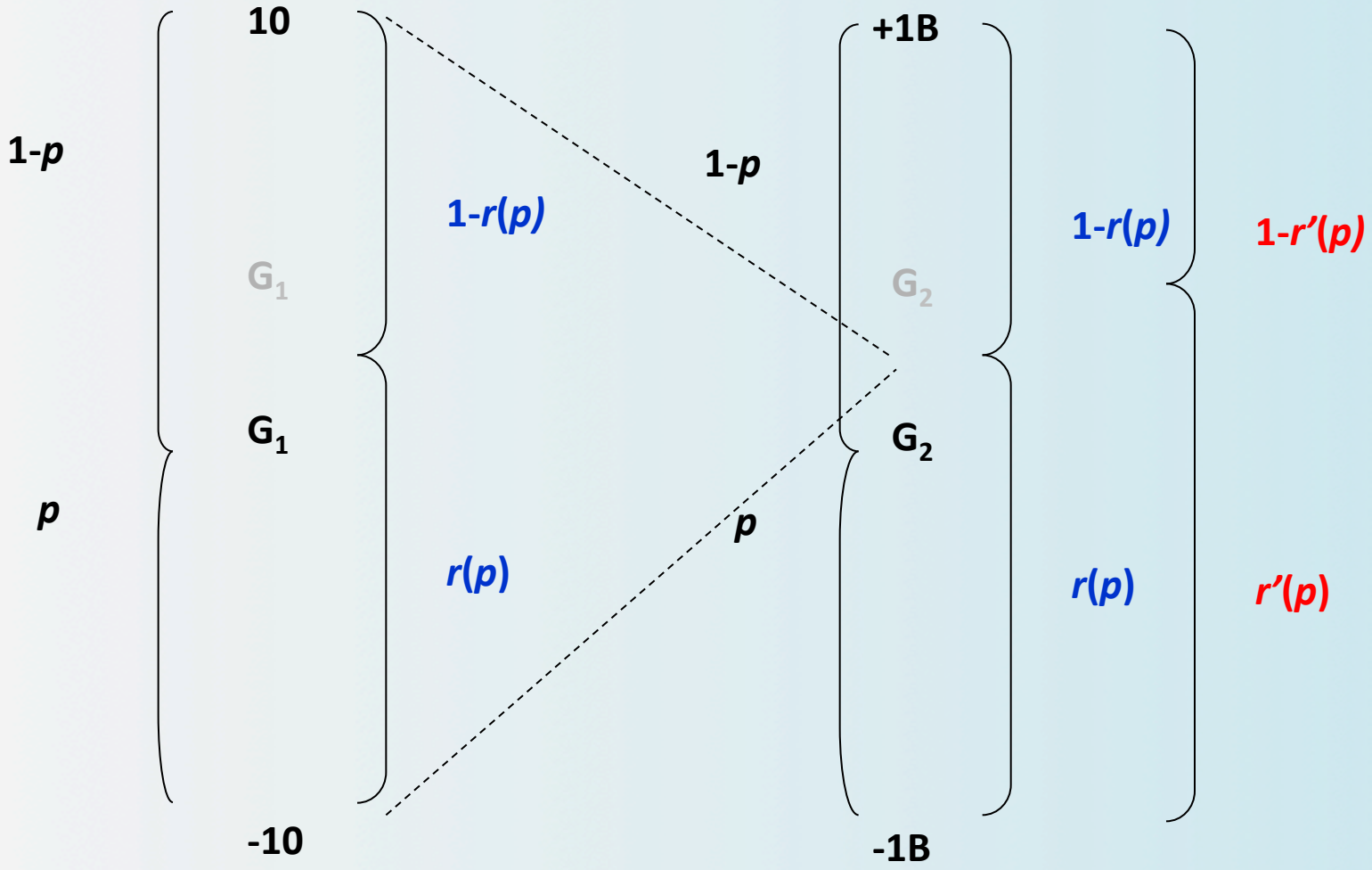
G_2: p(1 billion utiles) ⊻ (1-p)(-1billion utiles)

Cues: 10 utiles is the value of receiving an unexpected $100
- 1 billion utiles is something like slow death by torture, brutal incarceration for life

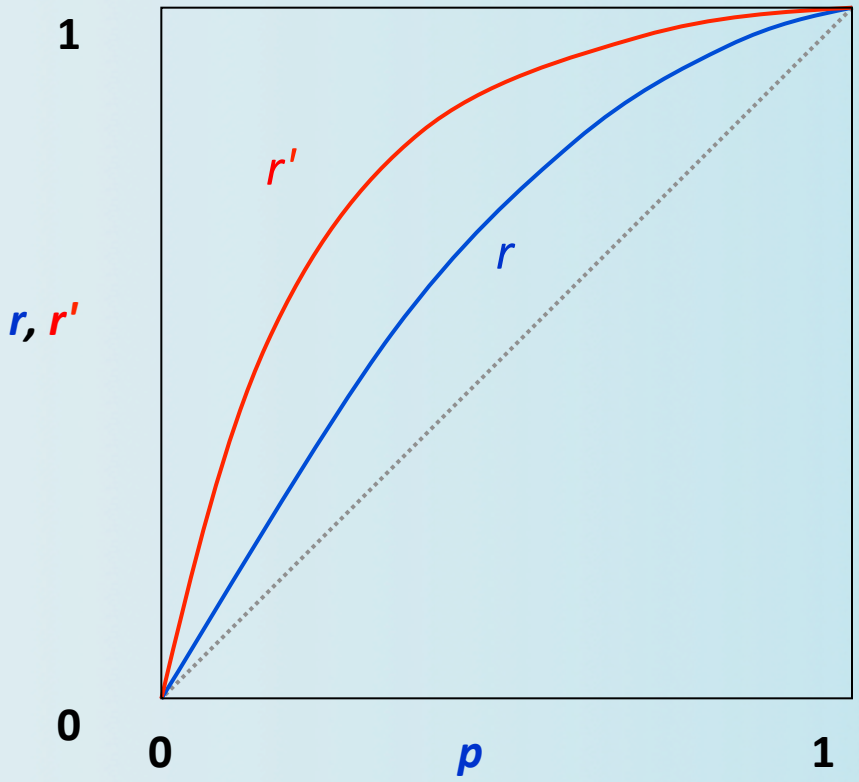
Will G_1 be treated the same way as G_2?



Or might G_2 be discounted more, by a different *r'*?



- Is this a plausible suggestion?
- If there is a difference in risk-sensitivity, the shape of the utility curve over goods/bads wouldn't explain it.
- Is risk-sensitivity a more regional matter than *REU* allows?



Gains and losses

There is strong empirical evidence that gains and losses do not weigh equally in our deliberations. That is a descriptive point.

Decision theories that treat outcomes as prospects, or deviations from the status quo, are an alternative to theories that treat outcomes as total wealth levels, or complete worlds.

Best known: Tversky & Kahneman's prospect theory, and its successor, cumulative prospect theory (designed for many-outcome choices).

Prospect theory is, first of all, intended to be a good descriptive theory, and it is designed to represent a variety of features of our decision-making, including some that would be very hard to accept as rational.

A key feature: the asymmetry between prospect of gain and prospect of loss.

Gains and losses matter to risk-sensitivity; we are

- risk-seeking with regard to small probabilities of gain (people buy lottery tickets from profit-seeking governments),
- risk-averse with regard to small probabilities of loss (people buy insurance from profitable companies),
- risk-averse with regard to larger probabilities of gain (we take the sure $490 over a 50:50 chance of $1000), and
- risk-seeking with regard to larger probabilities of loss (we take the 50:50 chance of losing $1000 over the sure loss of $490).

These are not effects that can all be captured by a given risk-function *r*, used with a consistent build-from-the-bottom-up combination rule. Buchak does not seek to subsume them into *REU*.

What counts as a gain and what as a loss depends on a *point of reference*.

Another feature of deliberation that TK famously explored are the effects of *framing*. Framing of outcomes compares them to a point of reference.

It is well established that descriptively, we are very flexible, and influenced by many things, in framing. Different descriptions of problems can lead us to choose different points of reference, to treat hypothetical points of reference as the status quo, and to reverse our choices when arguably everything about the problem is the same except for the way in which it is described.

I do not suggest that it is rational to do that.

But the idea that a reference point matters to us, and to our sensitivity to risk, need not commit us to the view that any way we choose it is rational, and need not imply the irrationality of treating risks of gains and losses differently.

The influences of a point of reference on risk-sensitivity are pervasive.

They also seem relevant to the recommendation Buchak made in the passage quoted earlier, that a rational agent can/should legitimately care about the structure of his attainable outcomes, about the strategy he chooses, and not just about the values of the outcomes considered individually.

After all, a strategy has a starting point; it is far from obvious that the point from which a strategy is executed is no significant part of it, and not obvious that it makes no contribution to the assessment of the strategy's value within the decision structure at hand.

So we might think that the general motivation Buchak offers for the rationality of risk-sensitivity arising from probability-weights applies equally to risk-sensitivity tied to a point of reference.

Conclusion

REU and its theorem that provide a demonstration of the tenability of treating an identifiable part of the complex influences of risk on deliberation, the nonlinear weighting of probabilities/degrees of belief, within rational subjective *EU* theory.

REU is an important contribution to the pursuit of a normative subjective theory for more-realistically-modeled agents.

Suppose we suspect that there may be further sources of risk-sensitivity, even rationally acceptable risk-sensitivity (not sure that Buchak entirely rules that out).

My point is **NOT**, "Well this theory is cool, but why isn't there even more?"

Instead, it is that Buchak's *REU* moves the boundaries of what is counted as normatively acceptable when it comes to preferring and deciding. Is there good *normative* motivation for setting the boundaries exactly there?

I know that Buchak has more to say about this than is in the paper.

I raised questions about Buchak's paper and theory, as a commenter should.

But in closing: this is exceptionally good work.

If you're interested in rational choice theory, you should pay attention to it.

See the book!