PHIL 424: HW #1 Solutions

09/25/14

1 Comparative Entailment and Contradictions

Explain why (given a classical logical entailment relation) Comparative Entailment requires a rational agent to be equally confident of every contradiction

Suppose p, q are any two contradictions. Let the degree of confidence that the agent has in p be symbolized by $\mathrm{cr}(p)$ and, similarly, let the degree of confidence that the agent has in q be symbolized by $\mathrm{cr}(q)$.

For classical entailment relations, we have p entails q. So by Comparative Entailment $\mathrm{cr}(q) \geq \mathrm{cr}(p)$. By the same reasoning, q entails p so (again, by Comparative Entailment) $\mathrm{cr}(p) \geq \mathrm{cr}(q)$. Conjoining these claims, we have $\mathrm{cr}(q) \geq \mathrm{cr}(p)$ and $\mathrm{cr}(p) \geq \mathrm{cr}(q)$. But that implies that $\mathrm{cr}(p) = \mathrm{cr}(q)$. Thus, the agent must be exactly as confident in p as in q.

2 Lottery Paradox

What do you *think the agent in the Lottery Paradox should believe? In particular, should she believe of each ticket in the lottery that that ticket will lose? Does it make a difference how many tickets there are in the lottery? Explain and defend your answers.*

A wide range of answers are acceptable.

3 Figure 1.1 and Comparative Entailment

Suppose we have a confidence ordering consisting of only the relations depicted by arrows in Figure 1.1. So, for example, the agent in question is more confident in "W" than "not-W"? (because there?s an arrow from the latter to the former), but is not more confident in "W" than "W and not-W"? (because there is no arrow connecting the two).

3.1 Why doesn't Figure 1.1 Satisfy Comparative Entailment?

Explain why this ordering does not satisfy Comparative Entailment.

In order to satisfy Comparative Entailment, an agent must assign at least as high a degree of confidence to p as q for any p that is entailed by q.

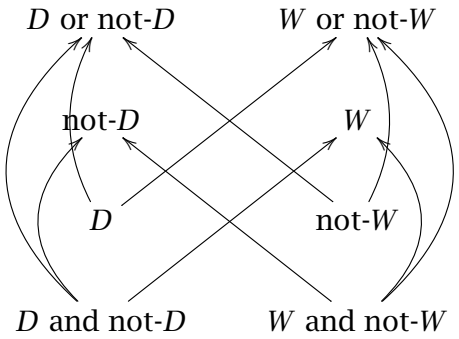
An arrow from q to p ($q \longrightarrow p$) represents being *strictly more* confident in p as in q. A successful answer to this question identifies any pair of propositions q, p so that (i) q entails p and (ii) the confidence in q is not the same as the confidence in p but (iii) there is no arrow from p to q. Here is a comprehensive list of examples:

1. (i) W and not-W (ii) W
2. (i) W and not-W (ii) W or not-W
3. (i) W and not-W (ii) not-D
4. (i) W and not-W (ii) D or not-D
5. (i) D and not-D (ii) not-D
6. (i) D and not-D (ii) D or not-D
7. (i) D and not-D (ii) W
8. (i) D and not-D (ii) W or not-W
9. (i) D (ii) D or not-D
10. (i) D (ii) W or not-W
11. (i) not-W (ii) D or not-D
12. (i) not-W (ii) W or not-W

3.2 How to Fix 1.1?

Describe all the arrows that would have to be added to the diagram to make the ordering satisfy Comparative Entailment.

The following diagram shows **only** which arrows to add to make Figure 1.1 satisfy Comparative Entailment.



Arrows connect contradictions to every other proposition. Arrows connect any proposition to tautologies. Since arrows represent "more confident than", there should not be any arrow between the contradictions or between the tautologies: by Comparative Entailment the agent is equally confident of each contradiction and of each tautology.

4 Assigning Confidence Values to Figure 1.1

Assign numerical confidence values (between 0% and 100%, inclusive) to each of the propositions mentioned in Figure 1.1. These confidence values should be arranged so that if there?s an arrow in Figure 1.1 from one proposition to another, then the first proposition has a lower confidence value than the second

A wide range of answers are acceptable. Here are the bare minimum constraints:

- The degree of D and not-D should be lower than the degree of any other proposition except W and not-W
- The degree of W and not-W should be lower than the degree of any other proposition except D and not-D
- The degree of D and not-D can be equal to the degree of W and not-W, but they don't have to be.
- The degree of D or not-D should be higher than the degree of any other proposition except W or not-W
- The degree of W or not-W should be higher than the degree of any other proposition except D or not-D
- The degree of W or not-W can be equal to the degree of D or not-D, but they don't have to be.
- D should have a lower degree than not-D
- not-W should have a lower degree than W