PHIL 424: HW #2 Solutions

October 6, 2014

1 Propositons

State-Descriptions

List all eight state-descriptions available in a language with the three atomic sentences P, Q, and R.

1. $P \,\&\, Q \,\&\, R$
2. $P \,\&\, Q \,\&\, {\sim}R$
3. $P \,\&\, {\sim}Q \,\&\, R$
4. $P \,\&\, {\sim}Q \,\&\, {\sim}R$
5. ${\sim}P \,\&\, Q \,\&\, R$
6. ${\sim}P \,\&\, Q \,\&\, {\sim}R$
7. ${\sim}P \,\&\, {\sim}Q \,\&\, R$
8. ${\sim}P \,\&\, {\sim}Q \,\&\, {\sim}R$

Point Values

This question was worth 12.5 points. Equal weight was given to each of the eight state-descriptions.

Disjunctive Normal Form

Give the disjunctive normal form of $(P \vee Q) \supset R$

to express the sentence as a disjunction of conjunctions. First, let's draw the truth table for this sentence.

P	Q	R	state-description	$P \vee Q$	$(P \vee Q) \supset R$
T	**T**	**T**	$P \& Q \& R$	**T**	**T**
T	**T**	**F**	$P \& Q \& {\sim}R$	**T**	**F**
T	**F**	**T**	$P \& {\sim}Q \& R$	**T**	**T**
T	**F**	**F**	$P \& {\sim}Q \& {\sim}R$	**T**	**F**
F	**T**	**T**	${\sim}P \& Q \& R$	**T**	**T**
F	**T**	**F**	${\sim}P \& Q \& {\sim}R$	**T**	**F**
F	**F**	**T**	${\sim}P \& {\sim}Q \& R$	**F**	**T**
F	**F**	**F**	${\sim}P \& {\sim}Q \& {\sim}R$	**F**	**T**

We see that $(P \vee Q) \supset R$ is true in states 1, 3, 5, 7, and 8. So the disjunctive normal form equivalent is

$$(P \& Q \& R) \vee (P \& {\sim}Q \& R) \vee ({\sim}P \& Q \& R) \vee ({\sim}P \& {\sim}Q \& R) \vee ({\sim}P \& {\sim}Q \& {\sim}R).$$

Point Values

This question was worth 12.5 points. Using a "good" method earned half credit, properly applying the method and getting the right answer earned the other half. Equal weight (1.25) was given to each of the five state-descriptions, with that many points deducted for each wrong state-description included.

5 Stochastic Truth-Tables

Consider the probabilistic credence distribution specified by this stochastic truth-table:

P	Q	R	$\mathrm{cr}(\cdot)$
T	**T**	**T**	0.1
T	**T**	**F**	0.2
T	**F**	**T**	0
T	**F**	**F**	0.3
F	**T**	**T**	0.1
F	**T**	**F**	0.2
F	**F**	**T**	0
F	**F**	**F**	0.1

Calculate each of the following values

cr$(P \equiv Q)$:

$P \equiv Q$ is true just in case P has the same truth value as Q. This occurs on rows 1, 2, 7, and 8. So we have $\mathrm{cr}(P \equiv Q) = 0.1 + 0.2 + 0 + 0.1 = 0.4$

cr$(R \supset Q)$:

$R \supset Q$ is false just in case R is true and Q is false. This occurs on rows 3 and 7. So we have $\mathrm{cr}({\sim}[R \supset Q]) = 0 + 0 = 0$. So by Negation, $\mathrm{cr}(R \supset Q) = 1 - \mathrm{cr}({\sim}[R \supset Q]) = 1 - 0 = 1$.

$\mathbf{cr}(P \,\&\, R) - \mathbf{cr}({\sim}P \,\&\, R)$:

$P \,\&\, R$ is true on rows 1 and 3. So $\mathrm{cr}(P \,\&\, R) = 0.1 + 0 = 0.1$. $P \,\&\, {\sim}R$ is true on rows 4 and 7. So $\mathrm{cr}(P \,\&\, {\sim}R) = 0.1 + 0 = 0.1$. And their difference is 0.

$\mathbf{cr}(P \,\&\, Q \,\&\, R)/\mathbf{cr}(R)$:

$P \,\&\, Q \,\&\, R$ is true on row 1. So $\mathrm{cr}(P \,\&\, Q \,\&\, R) = 0.1$. R is true on rows 1, 3, 5, and 7. So $\mathrm{cr}(R) = 0.1 + 0 + 0.1 + 0 = 0.2$. And $0.1/0.2 = 1/2$.

Point Values

This question was worth 25 points. Each part of the question was given equal weight.

6 Checking a Credence Distribution

Can a probabilistic credence distribution assign cr$(P) = 0.5$*, cr*$(Q) = 0.5$*, and cr*$({\sim}P \,\&\, {\sim}Q) = 0.8$*? Explain why or why not.*

No.

Suppose there were a credence distribution with those values. P and ${\sim}P \,\&\, {\sim}Q$ are mutually exclusive, so by Finite Additivity $\mathrm{cr}(P \vee [{\sim}P \,\&\, {\sim}Q]) = \mathrm{cr}(P) + \mathrm{cr}({\sim}P \,\&\, {\sim}Q) = 0.5 + 0.8 = 1.3$. However, by Maximality $\mathrm{cr}(P \vee [{\sim}P \,\&\, {\sim}Q]) \leq 1$. Contradiction.

Point Values

This question was worth 25 points. Partial credit was given based on “goodness” of method used. Partial credit was deducted based on “badness” of method used.

7 Checking Another Credence Distribution

Can an agent have a probabilistic cr-distribution meeting all of the following constraints?

- The agent is certain of $A \supset (B \equiv Q)$.
- The agent is equally confident of B and ${\sim}B$.
- The agent is twice as confident of C as $C \,\&\, A$.
- $\mathrm{cr}(B \,\&\, C \,\&\, {\sim}A) = 1/5$.

If not, prove that it's impossible. If so, provide a stochastic truth-table and demonstrate that the resulting distribution satisfies each of the four constraints.

The easiest way to answer this problem is to try to construct a stochastic truth-table subject to the constraints. If you can do it, then you have your answer. If not, then you just have to explain what went wrong. First, by Non-Negativity, Negation, and Entailment, we know that anything inconsistent with $A \supset (B \equiv Q)$ gets credence 0. By propositional logic, there are two such state-descriptions: $A \,\&\, B \,\&\, {\sim}C$ and $A \,\&\, {\sim}B \,\&\, C$.

A	B	C	$\mathrm{cr}(\cdot)$
T	**T**	**T**	a
T	**T**	**F**	0
T	**F**	**T**	0
T	**F**	**F**	b
F	**T**	**T**	$1/5$
F	**T**	**F**	c
F	**F**	**T**	d
F	**F**	**F**	e

If this is probabilistic, then $a+b+1/5+c+d+e = 1$. By the second constraint, $a+1/5+c = b+d+e$. $\mathrm{cr}(C \,\&\, A) = a$ and $\mathrm{cr}(C) = a + 1/5 + d$. So by the third constraint, $a = 1/5 + d$. The first and second constraints together imply that $a + 1/5 + c = 1/2 = b + d + e$. Doing some algebra reveals that this has several solutions. Here is one:

A	B	C	$\mathrm{cr}(\cdot)$
T	**T**	**T**	$1/4$
T	**T**	**F**	0
T	**F**	**T**	0
T	**F**	**F**	$1/5$
F	**T**	**T**	$1/5$
F	**T**	**F**	$1/20$
F	**F**	**T**	$1/20$
F	**F**	**F**	$1/4$

This is a probability distribution: the credences add up to 1 and they are all non-negative. Verifying that it has the appropriate properties: All non-zero rows are state-descriptions where $A \supset (B \equiv Q)$ is true, and they add up to 1. So the agent is certain of $A \supset (B \equiv Q)$. $\mathrm{cr}(B) = 1/4 + 1/5 + 1/20 = 1/2 = 1/5 + 1/20 + 1/4 = \mathrm{cr}({\sim}B)$. So the second condition holds. $\mathrm{cr}(C) = 1/4 + 1/5 + 1/20 = 1/2 = 2 \times 1/4 = \mathrm{cr}(C \,\&\, A)$. The last condition is obvious.

Point Values

This question was worth 25 points. A correct answer (with work) was worth 20 points. Verifying that your answer has the required properties is worth 5 points.