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The Accuracy and Rationality of Imprecise Credences

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1. Precise and Imprecise Credences

An agent has precise credences if her belief state is representable by a credence function that assigns numbers to propositions. These numbers represent how confident the agent is in each of the propositions that the credence function is defined over. Representing the belief states of an ideally rational rational agent using a precise credence function has a variety of virtues. But some have thought that the precise model for (ideal) rationality should be abandoned.

One popular motivation for abandoning the precise credence model is the thought that, in response to certain kinds of evidence, *any* precise credence would be irrational.[1] Joyce (2005, 2010) and Sturgeon (2010) argue for this claim by pointing to cases like the following:

> FAIR COIN: The only evidence you have that is relevant to whether the coin in front you will land heads (we'll call this proposition "Heads") is that the coin is fair.

> MYSTERY COIN: The only evidence you have that is relevant to whether Heads is that the objective chance of Heads is between 0.05 and 0.95.

All parties in the debate agree that it's rational to have a 0.5 credence in Heads in FAIR COIN. But what about MYSTERY COIN? It may be tempting to think that you should assign a 0.5 credence to Heads in MYSTERY COIN as well. After all, you might think that in a case like MYSTERY COIN your credence in Heads should be the average of the credences you assign to the different chance hypotheses, weighted by your credences that these hypotheses obtain. Furthermore, one might argue, you should have an even probability distribution over the different chance hypotheses since you have no reason to privilege any over any of the others. If that's right, then the "weighted" average, and therefore, the rational credence in Heads, will be 0.5.

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[1] Advocates of this view include Levi (1974, 1985), Kaplan (1996), Joyce (2005, 2011) and Sturgeon (2010).

But the defenders of imprecise credences think that the above reasoning is mistaken. First, such reasoning faces well known difficulties (see, for example, van Frassen's (1989) cube factory example). Second, even if such problems could be solved, Joyce argues that a 0.5 credence in MYSTERY-COIN is simply not supported by the evidence. Joyce claims that an agent who assigns a 0.5 credence in such a case is "acting as if he has some reason to rule out those possibilities in which [the objective chance is not .5], even though none of his evidence speaks on the issue" (170).

If 0.5 is the *wrong* credence, what should the agent's credence be? Defenders of imprecise credences propose that sometimes our evidence warrants a response which is best represented as a *set* of credence functions, called "a representor" rather than a single one. Joyce says that an agent in the evidential situation described in Example #2 is being "epistemically irresponsible unless, for each *i* and for each *x* between [.05 and .95], his credal state $\boldsymbol{c_t}$ contains at least one credence function such that **c**[p] = x." (171).

The aim of this paper is to examine how considerations of accuracy bear on the question of whether, in cases like MYSTERY-COIN, the evidence really does support imprecise credences. I'm going to argue that if you accept a certain accuracy-based constraint on claims about rational requirements, the particular evidential motivation offered by Joyce and others for imprecise credences in cases like MYSTERY-COIN is unsuccessful. In what follows I'll call someone who thinks that one is sometimes rationally required to have imprecise credences, *and is motivated by cases like* MYSTERY-COIN, "the imprecise credence defender." (At the end of the paper I'll consider some alternative motivations).

I will assume that the imprecise credence defender endorses the following two claims:

A. For any probability distribution **p** over the set of propositions: {Heads, Tails} there is some evidential situation which makes it rational to adopt **p.**
B. For any set of probability distributions **b** over {Heads, Tails} there is some evidential situation which makes it rational to adopt **b.**

(A) follows from the a weak version of the Principal Principle: one's credences should match the objective chances when they are known, and the assumption that for any distribution of probabilities **p,** there is a body of evidence which contains only the information that the objective chances of the elements in the partition match the probabilities in **p**. The defenders of imprecise credences are clear that they don't mean to be saying anything incompatible with the Principal Principle. Indeed, they contrast the cases in which imprecise credences are appropriate with cases in which precise credences are appropriate by pointing out that our credences should match objective chances when they are known.

The reason I am interested in an imprecise credence defender who endorses (B) is as follows: I am interested in a defender of imprecise credences who is motivated by cases like MYSTERY-COIN. For set of probability functions, *S*, over {Heads, Tails} there is a possible body of evidence which includes only the information that the objective chances function for {Heads, Tails} is in *S*. The defender of imprecise credences motivated by MYSTERY-COIN will think that, in such cases, one's belief state should be represented by *S*.

2. Accuracy Based Arguments for Rational Requirements

It's common to think that there must be some connection between rationality and accuracy. Belief, in some sense, aims at the truth, and so it is natural to think that the norms governing belief also, in some sense, aim at the truth. Why is it rational to believe a proposition under consideration that one knows is entailed by one's evidence? One tempting thought is that (assuming evidence is factive), it is rational to believe the propositions one knows are entailed by one's evidence because one is guaranteed that the propositions entailed by one's evidence are *true*. An argument for the requirement to believe what one knows is entailed by one's evidence that appeals to the guaranteed truth of the resulting belief is what I'll call *an accuracy based argument*.

The argument that I will present against the claim that the evidence in MYSTERY-COIN requires imprecise credences will also be an accuracy-based argument, so it will be helpful to begin by getting clear on exactly what accuracy is and how we measure it. The setup I describe below is common in the literature on accuracy measures, but my version is inspired especially by Joyce (1998) and Gibbard (2007).[2]

Let's start by thinking about the accuracy of an agent's precise credence towards a proposition P. Intuitively, we can think of the accuracy of an agent's credence in P as measuring its "closeness to the truth."

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[2] I am also indebted to Seamus Bradley for some excellent suggestions concerning the best way to set up the argument that follows.

You're maximally accurate with respect to P if you have credence 1 in P and P is true, or if you have credence 0 in P and P is false. You're maximally *in*accurate with respect to P if you have credence 1 in P when P is false or credence 0 in P when P is true. The greater your confidence in P the *more* accurate you are when P is true, whereas the greater your confidence in P the *less* accurate you are when P is false. So we can define one increasing function, f_1, from [0,1] to [0,1] which assigns to each possible credence in [0,1] a number in [0,1] that represents the accuracy of that credence in a *true* proposition. We can also define a decreasing function f_0 from [0,1] to [0,1] which assigns to each possible credence in [0,1], a number in [0,1] that represents the accuracy of that credence in a *false* proposition. We'll let $f_1(1) = 1$ (the maximal accuracy score) and $f_1(0) = 0$ (the minimal accuracy score). This means that if one assigns credence 1 to a truth, one's accuracy score with respect to that proposition is 1, whereas if one assigns credence 0 to a truth, one's accuracy score with respect to that proposition is 0. Similarly we'll let $f_0(1) = 0$ and $f_0(0) = 1$. This means that if one assigns credence 1 to a falsehood one's accuracy score is 0 (the minimal accuracy score), whereas if one assigns credence 0 to a falsehood one's accuracy score is 1 (the maximal accuracy score).

$$f_1 : [0,1] \rightarrow [0,1]$$

f_1 is an increasing function representing the accuracy of a credence is in a true proposition

$$f_0 : [0,1] \rightarrow [0,1]$$

f_0 is a decreasing function representing the accuracy of a credence in a false proposition

Now, consider any partition of propositions X: $\{X_1 \ldots X_n\}$. C_x will be the set of all credence functions defined on X, P_x is the set of all probability functions defined on X, and V_x is the set of all consistent assignments of truth value to the elements of X. A consistent assignment of truth values to the elements of X is just a function which assigns to each member of X, a number, 0 or 1, which represents the truth value of that member. So each truth value assignment in V_x can be thought of as a possible world. A $v \in \mathit{V}_X$ which has $v(X_i) = 1$, represents a world in which X_i is true. A $v \in \mathit{V}_X$ which has $v(X_i) = 0$ represents a world in which X_i false.

We can now use f_1 and f_0 to define *a scoring rule* which takes a credence in a proposition, and the truth value of that proposition, and assigns the credence/truth value pair, a number between 0 and 1 that represents the accuracy of that credence given the truth value. For any proposition P we let:

$$\mathbf{S}(\mathbf{c}(P), v(P)) = v(P)f_1(\mathbf{c}(P)) + v^-(P)\, f_0(\mathbf{c}(P))$$

($v^-(P)$ is just the opposite truth value of $v(P)$. If $v(P)=1$, $v^-(P) = 0$, and if $v(P) = 0$, $v^-(P) = 1$).

Note that our definition of **S** has the consequence that the score of a credence in P, **c**(P), when P is true (that is, when $v(P)=1$) is $f_1(\mathbf{c}(P))$ – the accuracy of a credence in a true proposition. Similarly, the score of a credence in P, **c**(P), when P is false (that is when $v(P) = 0$) is $f_0(\mathbf{c}(P))$ – the accuracy of a credence in a false proposition.

$\mathbf{S}: [0,1] \times \{0,1\} \rightarrow [0,1]$
S tells us how accurate a credence in a proposition is given its truth value.

So far I've described a way of representing the accuracy of a particular *credence* in a proposition, given the proposition's truth value, using the scoring rule **S**. We now need a measure for the accuracy of a *credence function* over a partition, given a truth value assignment over that partition. We'll call such a measure, over a partition X, $\mathbf{G}_X$. .

I will assume that, at least sometimes, we can represent the accuracy of an agent's credence function as a whole as the average of the agent's accuracy scores with respect to each proposition in the partition. Although there might be some contexts in which one wants to assign accuracy to an agent's credence function in a more complex way, it will suffice for my purposes that, in at least some cases, perhaps very simple cases involving only a proposition and its negation, and in which neither of the two propositions is, in any way, more important than the other, the accuracy of an agent's credence function is the average of the accuracy of the agent's credences in each proposition. Since for the purposes of this paper, I am interested in the motivations for imprecise credences coming from cases like MYSTERY-COIN, I will be focusing specifically on these very simple cases (a partition with just two propositions, neither of which is more important than the other). So for the purposes of this paper I will assume that:

$$\mathbf{G}_x(\mathbf{c}, \nu) = \sum_{X_i \in X} (1/n)\, \mathbf{S}(\mathbf{c}(Xi), \nu(Xi))$$

So the accuracy of any sharp-credence function defined over the partition {Heads,Tails} (which I'll call H/T) will be:

$$\mathbf{G}_{H/T}(\mathbf{c}, \nu) = .5\ [\mathbf{S}(\mathbf{c}(\text{Heads}), \nu(\text{Heads})) + \mathbf{S}(\mathbf{c}(\text{Tails}), \nu(\text{Tails})]$$

$\mathbf{G}_{\mathbf{X}}: C_X \text{ X } V_X \rightarrow [0,1]$
$\mathbf{G}_{\mathbf{X}}$ tells us how accurate a credence function defined over X is given any consistent truth value assignment for the members of X.

2.1. Dominance and Non-Dominance Reasoning

Accuracy-based arguments for rational requirements frequently[3] rely on the following principle:

> *Dominance*: For any belief state **b,** defined over a partition X, if there exists a belief state **b'** defined over X that is more accurate than **b** for every $\nu \in V_X$, and no less accurate than **b** for any $\nu \in V_X$, **b** is rationally forbidden.

Is *Dominance* true? One might think that *Dominance* should be rejected because of the possibility of cases in which, although **b'** accuracy dominates **b**, **b** is required because **b** is *better supported by the evidence*. But those who favor *Dominance* will think that such cases are simply impossible. **b** simply *can't* be better supported by the evidence than **b'** and we can be certain of this *because* **b'** accuracy dominates **b.** On these views there is some constitutive connection between what the evidence supports and what the accuracy-based considerations favor which

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[3] For example, Joyce (1998) relies on such a principle in his argument for probabilistm.

doesn't allow the evidence to best support an accuracy-dominated belief state. I will not take a stand on *Dominance* here. My aim is to show that a similar line of reasoning, which I'll call *Non-Dominance*, can be used to argue against the claim that the one is rationally required to adopt imprecise credences in MYSTERY-COIN.

Here is the key principle:

> *Non-Dominance*: For any belief state **b** defined over a partition X, if there exists a belief state **b'** defined over X that is no less accurate than **b** in every world then **b** is not rationally required.

In other words: if one is rationally required to adopt **b** in some circumstance, and so forbidden to adopt an alternative belief state **b**', there must be at least one world which **b** is more accurate than **b'**

Since my argument against the rational requirement to have imprecise credences in MYSTERY-COIN will be a *non-dominance* argument (rather than a *dominance* argument) I will not be arguing that, for any imprecise credal state **i** defined over {Heads, Tails} there is a precise credal state **p** that accuracy dominates **i**, and that **i** is therefore *forbidden*. Rather, I will show that for any imprecise credal state **i** defined over {Heads, Tails} there is a precise credal state **p** that will *never* be less accurate than **i.** If *Non-Dominance* is true, this would show that **i** is not rationally *required*.

Is it a genuine constraint on a rational requirement to adopt **b** that there be at least *some world* in which **b** will be more accurate than a forbidden belief state **b'**? Here too, whether we think *Non-Dominance* is plausible depends on the degree to which we think rational requirements on belief are tied to considerations of accuracy. If we think that there are rational requirements on belief that will not, under *any* circumstances, lead to *any* advantage in the accuracy of our belief states, then the requirements of rationality go beyond what would be warranted by an interest in accuracy. An agent who was only interested in the truth of her beliefs or the accuracy of her credences would not be motivated to conform to this requirement. So why should such a requirement be conformed to? The idea would have to be that it is simply intrinsically better to have belief states that conform to this requirement than to have belief states that don't.

There are large issues here concerning the connection between rationality and accuracy – large issues that I will not tackle in this paper. Instead, I will simply show that, on any plausible way of measuring the accuracy of imprecise credences, there is always a precise credal state that will be no less accurate in every possible world than the imprecise state. So my conclusion will be that if imprecise credences are rationally required, a motivation to conform to the requirement will have to be based on something intrinsically valuable about these imprecise states. If all you care about is truth, or accuracy, imprecise credences won't look any better than precise ones. For ease of discussion, in what follows I'll simply assume that *Non-Dominance* is true, and so the argument will take the form of an argument for the claim that imprecise credences in cases like MYSTERY-COIN are not rationally required. But the reader should keep in mind that an alternative is to give up *Non-Dominance*, and to think that imprecise credences *are* required – just not for any reason that can be connected to our concern with accuracy or truth. Since anybody that accepts *Non-Dominance* will accept *Dominance*, I will assume *Dominance* in what follows as well.

3. The Accuracy of Imprecise Credences

In this section I present the non-dominance argument against the requirement to have imprecise credences in cases like MYSTERY-COIN.

3.1. Using Numbers

In this subsection, I focus on ways of representing the accuracy of imprecise credences using numbers. In the next, I will discuss the possibility of representing the accuracy of imprecise credences using intervals (or sets of numbers).

Recall that our measure for the accuracy of a precise credence *function* depended on the accuracy scores of the credences assigned by that function to each element of the partition. If we want to pursue a similar strategy in coming up with an accuracy measure for *sets of credence functions*, we'll need a way of talking about what an imprecise agent's attitude is towards a single proposition in the partition. At least in cases like MYSTERY-COIN this is easy to do. In MYSTERY-COIN, the imprecise credence defender thinks that one's belief state should consist of the set of functions that includes all and only functions that assign r to Heads, where $r \in [0.05, 0.95]$, and $1\text{-}r$ to Tails. Call this belief state **i**. We can represent **i**'s attitude towards Heads as the set of numbers which includes all and only the credences that members of **i** assign towards Heads: in this case, this will be the interval [0.05, 0.95]. **i**'s attitude towards Tails is also represented by the interval [0.05, 0.95], the set of numbers which includes all and only the credences that members of **i** assign to Tails. In what follows I will assume that an imprecise agent's attitude towards a proposition can be represented by an *interval* since an interval valued credence in each of {Heads, Tails} is what the imprecise credence defenders recommend in the case of interest: MYSTERY-COIN.

How do we score the accuracy of an interval-valued credence? Recall that our scoring rule for a precise credence in a particular proposition P was measured by **S** which took as its input *a number*: the agent's credence in P, and the truth value of P. In this subsection I explore the consequences of extending our scoring rule so that it can take as input interval-valued credences. For uniformity, let's now represent any precise credence *c* in P by the (trivial) interval [*c*, *c*].

To extend our scoring rule, let's let f_1* be the function that takes as input an interval, and outputs a number that represents how accurate that interval-valued credence is in a proposition when the proposition is true. f_0* will be the function that takes as input an interval, and outputs a number that represents how accurate that interval-valued credence is in a proposition when the proposition is false. Using f_1* and f_0* we can create an extended scoring rule, **S***, which will takes as input an *interval* and a truth value and outputs a number representing how accurate that interval-valued credence is in a proposition, given its truth value. Let **b** represent some imprecise belief state (a set of credence functions). And let **b**(P) be the interval [*a, b*] such that $r \in [a,b]$ if and only if, for some $\mathbf{p_i} \in \mathbf{b}$, $\mathbf{p_i}(P) = r$. Then:

$$\mathbf{S}^*(\mathbf{b}(P), \nu(P)) = \nu(P) f_1^*(\mathbf{b}(P)) + \nu^-(P)\, f_0^*(\mathbf{b}(P))$$

Let *I* be the set of intervals in [0,1].

$f_1^*: I \rightarrow [0,1]$
A function that represents the accuracy of an interval-valued credence in a proposition when that proposition is true.

$f_0^*: I \rightarrow [0,1]$
A function that represents the accuracy of an interval-valued credence in a proposition when that proposition is false.

$\mathbf{S}^*: I \times \{0,1\} \rightarrow [0,1]$
A function that represents the accuracy of an interval-valued credence in a proposition given its truth value.

Finally let **G*** be a generalization of **G** so that for an n-membered partition X,

$$\mathbf{G_X}^*(\mathbf{b}, v) = \sum_{X_i \in \mathbf{X}} (1/n)\, \mathbf{S}^*(\mathbf{b}(Xi), v\,(Xi))$$

Let B_X be the set of belief states in the imprecise model over the partition X. This will be the set of *sets of credence functions* defined over X. Then:

$\mathbf{G_X}^*: B_X \text{ X } V_X \rightarrow [0,1]$ $\mathbf{G}_X$ tells us how accurate a belief state defined over the partition X is, given a truth-value assignment for X.

I propose the following three constraints on **G*:**
First,

EXTENSION: **G*** should be an extension of some plausible accuracy measure, **G** for precise credences. **G** is bounded (let's say by 0 and 1) and continuous.[4]

Here's what I mean by being an "extension" of **G**: For all X, $X_i \in X$, $\mathbf{b} \in B_x$ and v in V_x if, $\mathbf{b}(X_i) = [r,r]$ for some r for each $X_i \in X$, then $\mathbf{G_X}^*(\mathbf{b}, v) = \mathbf{G_X}(\mathbf{b}', v)$, where **b**' is the single credence function which assigns to each X_i credence *r* if and only if $\mathbf{b}(X_i) = [r, r]$.

The motivation for EXTENSION is that we don't want our extension of the accuracy measure to deliver verdicts that we find unacceptable when comparing the accuracy of precise credal states with other precise credal states. The second constraint is:

BOUNDEDNESS: **G*** should be bounded (let's stipulate by 0 and 1).

The motivation for BOUNDEDNESS is that even in the imprecise model there is, intuitively, a maximally accurate belief state and a minimally accurate belief state. (The maximally accurate belief state has each function assign 1 to all truths and 0 to

[4] By "continuous" I mean that for any partition X, and any member of that partition X_i, small differences in the credence functions defined over X should result in small differences in the accuracy of those functions as evaluated at X_i.

all falsehoods. Vice versa for the minimally accurate belief state). And the final constraint is:

> **PROBABILISTIC ADMISSIBILITY**: A belief state is probabilistic if it a set that contains only probability functions. Take any partition X: $\{X_1 \ldots X_n\}$ and any probabilistic belief state **b** defined over X. There can be no belief state **b'** defined over X such that **b'** is more accurate than **b** in some X_i, and no less accurate than **b** in any X_i.

Why accept PROBABILISTIC ADMISSIBILITY? Joyce (2009) argues that this is a constraint on any plausible scoring rule. But it is especially plausible in the current dialectical context because I'm interested in an opponent who accepts claims (A) and (B) above. Recall that (A) and (B) say that for any probabilistic precise or imprecise belief state there is some body of evidence that rationalizes that belief state. But if **b** is more accurate than **p** in some worlds, and **b** is no less than **p** in *any* world, then, by *Dominance*, one can rule out **p** a priori.

With these constraints in mind, let's return to MYSTERY-COIN. I will now argue that on any scoring rule that satisfies these three constraints the following is true:

> COIN NON-DOMINANCE: For any imprecise belief state **i** defined over the partition H/T: {Heads, Tails} there will be a precise belief state **p,** that is at least as accurate as **i,** for any ν in $V_{H/T}$.

There may be elaborations of my argument that apply to more fine-grained partitions, but I am going to focus on a 2-cell partition because I'm specifically interested in whether the imprecise credence defenders can claim that the evidence supports imprecise credences in cases like MYSTERY-COIN. I will return to this point about the number of cells in the partition later.

Here's the argument: First, it follows from PROBABILISTIC ADMISSIBILITY that:

(1) For any imprecise belief state **i** and any precise belief state **p** in $B_{H/T}$:
If $\mathbf{G^*}_{H/T}$ (**i,** Heads) = $\mathbf{G^*}_{H/T}$(**p**, Heads), then $\mathbf{G^*}_{H/T}$ (**i,** Tails) = $\mathbf{G^*}_{H/T}$(**p**, Tails)

Here's why:
Suppose that $\mathbf{G^*}_{H/T}$ (**i,** Heads) = $\mathbf{G^*}_{H/T}$(**p**, Heads) but $\mathbf{G^*}_{H/T}$ (**i,** Tails) $\neq$ $\mathbf{G^*}_{H/T}$(**p**, Tails).

Then, either $\mathbf{G}^*_{\mathrm{H/T}}$ (**i,** Tails) > $\mathbf{G}^*_{\mathrm{H/T}}$(**p**, Tails) or $\mathbf{G}^*_{\mathrm{H/T}}$ (**i,** Tails) < $\mathbf{G}^*_{\mathrm{H/T}}$(**p**, Tails). If $\mathbf{G}^*_{\mathrm{H/T}}$ (**i,** Tails) > $\mathbf{G}^*_{\mathrm{H/T}}$(**p**, Tails) then **p** would be ruled out a priori, since **i** is more accurate than **p** in Tails worlds, and no less accurate than **p** in Heads worlds. A similar problem arises if we assume $\mathbf{G}^*_{\mathrm{H/T}}$ (**i,** Tails) < $\mathbf{G}^*_{\mathrm{H/T}}$(**p**, Tails). This time **i** is ruled out a priori. This is because **p** is more accurate than **i** in Tails worlds, and no less accurate than **i** in Heads worlds.

Now, take any imprecise belief state, **i**, defined over H/T. And let:

(2) $\mathbf{G}^*_{\mathrm{H/T}}$(**i,** Heads) = r.

By BOUNDEDNESS $r \in [0,1]$.

By EXTENSION, **G*** is an extension of **G** and **G** is bounded and continuous. Since **G** is bounded and continuous it follows from the intermediate value theorem that for any $r \in [0,1]$ there exists a precise belief state **p**, such that

(3) $\mathbf{G}^*_{\mathrm{H/T}}$(**p**, Heads) = r.

It follows from (2) and (3) that:

(4) $G^*_{\mathrm{H/T}}$(**i,** Heads) = $\mathbf{G}^*_{\mathrm{H/T}}$(**p**, Heads).

Finally, it follows from (1) and (4) that

(5). $\mathbf{G}^*_{\mathrm{H/T}}$ (**i,** Tails) = $\mathbf{G}^*_{\mathrm{H/T}}$(**p**, Tails).

Since **i** represents any imprecise belief state in $B_{\mathrm{H/T}}$, this proves COIN NON-DOMINANCE.

3.2. Using Intervals

Perhaps the defender of the requirement to have imprecise credences will think that it's more plausible to represent the accuracy of an imprecise credence with an *interval*.[5] After all, the whole point of imprecise credences, say their

[5] Or, more generally, a *set* of numbers corresponding to the accuracy scores of each member of the representor.

defenders, is that credences are too precise to represent the appropriate doxastic attitude in response to certain bodies of evidence. So once we allow for these "imprecise" attitudes, we shouldn't expect to be able to measure their accuracy with some precise number! The best thing to do would be to represent the accuracy of an imprecise credence [*a*,*b*] in P with an *interval*, spanning the range of accuracy scores for each *precise* credence in the interval [*a*,*b*]. Let's call a scoring rule that maps interval valued credences to interval valued accuracy scores **S****. We'll let f_1** describe the function that assigns interval valued scores to an interval valued credence in a true proposition, and f_0** will be a function that assigns interval valued scores to an interval valued credence in a false proposition.

I agree with the imprecise credence defender that **S**** is much more in the spirit of the view than **S***. But the move from **S*** to **S**** won't solve the non-dominance problem. To see this, let's return to MYSTERY-COIN. Let's compare two doxastic states. The first is represented by the set of probability functions which contains, for each *r* in [0.05, 0.95], a function which assigns *r* to Heads. Since each such function is a probability function, that function also assigns 1-*r* to Tails. The second state is represented by the single probability function which assigns 0.5 to Heads and 0.5 to Tails. I'll call the first the doxastic state **m** and the second doxastic state **s.**[6] We can now ask, in a world in which the coin lands Heads, which of **m** or **s** is more accurate?

You might think that it doesn't make sense to make such a comparison. Suppose, for example, that **S****(**m**(Heads), Heads) = [.01, .98] and **S****(**s**(Heads), Heads) = [.75, .75]. Which of [0.01, 0.98] or [.75, .75] is bigger? Frequently when we say that one interval [*a*,*b*] is larger than another interval [*c*,*d*], we mean that the *length* of the first interval (which is *b*-*a*) is larger than the length of the second (which is *d*-*c*). But this can't be what we mean here. For one thing, it would violate EXTENSION since it wouldn't be a plausible extension of a precise scoring rule. This is because all precise credences would get the same accuracy score (0), no matter what the world is like.

On the other hand, if one simply refuses to make any comparisons amongst the degrees of accuracy between doxastic states, when at least one of those states is imprecise, then its unclear what the purpose is of having an accuracy measure over

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[6] Since **i** and **p** are being used throughout to stand for generic imprecise and precise belief states, I am here using the first letter of the alternative terminology: *mushy* and *sharp* to describe the particular belief states in question.

imprecise credences to begin with. And certainly, it can't be claimed that imprecise credences ever have an advantage, as far as accuracy goes, over precise ones. In fact, if the imprecise credence defender wants to escape the non-dominance argument she must allow that for *any* imprecise state **i** and precise state **p** we can compare the accuracy of **i** and **p**. For if there was no way to compare the accuracy of **i** and **p** then, trivially, there would be no world in which **i** is more accurate than **p.** So **p** would remain non-dominated.

Thus, to avoid non-dominance we need our imprecise interval-valued accuracy scores rule to be comparable with our precise interval-valued scores. (which will be of the form [*r,r*]). This, in and of itself, isn't a problem. There are various orderings (complete and partial) that can be imposed on the set of intervals, *I*. If we want to measure the accuracy of representors (rather than just the accuracy of an interval-valued credence in a single proposition), we'll also need a way of summing intervals. This too is unproblematic. But non-dominance will arise even once these details are filled in.

Let's represent the accuracy measure over representors that assigns interval valued scores **G****. Let's now compare $\mathbf{G}_{\mathbf{H/T}}$**(**s**, Heads) with $\mathbf{G}_{\mathbf{H/T}}$**(**m**, Heads). Since we're assuming that these values are comparable they must either be equal to one another or one must be greater than the other.

If **s** and **m** are equally accurate when the coin lands Heads, they must also be equally accurate when the coin lands tails. For if $\mathbf{G}_{\mathbf{H/T}}$**(**s**, Tails) > $\mathbf{G}_{\mathbf{H/T}}$**(**m**, Tails) while $\mathbf{G}_{\mathbf{H/T}}$**(**s**, Heads) = $\mathbf{G}_{\mathbf{H/T}}$**(**m**, Heads) then **m** is ruled out a priori. In some worlds (Tails worlds) **s** is more accurate than **m** and in the other worlds (Heads worlds) **s** is no less accurate than **m**. Similarly, if $\mathbf{G}_{\mathbf{H/T}}$**(**s**, Tails) < $\mathbf{G}_{\mathbf{H/T}}$**(**m**, Tails) while $\mathbf{G}_{\mathbf{H/T}}$**(**s**, Heads) = $\mathbf{G}_{\mathbf{H/T}}$**(**m**, Heads) then **s** is ruled out a priori. For in some worlds (Tails worlds) it is less accurate than **m** and in the other worlds (Heads worlds), it's no worse. But if

$\mathbf{G}_{\mathbf{H/T}}$**(**s**, Heads) = $\mathbf{G}_{\mathbf{H/T}}$**(**m**, Heads)

And

$\mathbf{G}_{\mathbf{H/T}}$**(**s**, Tails) = $\mathbf{G}_{\mathbf{H/T}}$**(**m**, Tails)

it follows that **m** is non-dominated by **s.** So by *Non-Dominance* **m** can't be rationally required.

Since both incomparability and equality between the accuracy of precise and imprecise credences in Heads worlds, leads to non-dominance, it follows that if the defender of the requirement to have imprecise credences in MYSTERY-COIN is to avoid the non-dominance argument, she must think that either

$\mathbf{G}_{H/T}$**($\mathbf{s}$, Heads) < $\mathbf{G}_{H/T}$**($\mathbf{m}$, Heads)

Or

$\mathbf{G}_{H/T}$**($\mathbf{s}$, Heads) > $\mathbf{G}_{H/T}$**($\mathbf{m}$, Heads)

But going either of these routes will violate PROBABILISTIC ADMISSABILITY. Here's why: Suppose that:

(7) $\mathbf{G}_{H/T}$**($\mathbf{s}$, Heads) < $\mathbf{G}_{H/T}$**($\mathbf{m}$, Heads)

Since we're assuming that the accuracy of a belief state in $B_{\mathrm{H/T}}$ is the average of the scores the belief state gets for each proposition in the partition, we have that:

(8) **G****$_{H/T}$($\mathbf{m}$, Heads) = .5(**S****($\mathbf{m}$(Heads), Heads) + **S****($\mathbf{m}$(Tails), Heads)).

Now note that:

(9) **S****($\mathbf{m}$(Heads), Heads) = f_1**([0.05, 0.95])

(10) **S****($\mathbf{m}$(Tails), Heads) = f_0**([0.05, 0.95).

From (8), (9) and (10) it follows that:

(11) **G****$_{H/T}$($\mathbf{m}$, Heads) = .5(f_1**([0.05, 0.95]) + f_0**([0.05, 0.95])).

Similarly:

(12) **G****$_{H/T}$($\mathbf{s}$, Heads) = .5(f_1**([0.5, .5]) + f_0**([0.5, 0.5])).

Let's now consider the accuracy scores of **s** and **m** when the coin lands Tails. Once again, we have:

(13) **G****$_{H/T}$($\mathbf{m}$, Tails) = .5(**S****($\mathbf{m}$(Heads), Tails) + **S****($\mathbf{m}$(Tails), Tails)).

(14) **S****($\mathbf{m}$(Heads), Tails) = f_0**([0.05, 0.95])

(15) $\mathbf{S}^{**}(\mathbf{m}(\text{Tails}), \text{Tails}) = f_1^{**}([0.05, 0.95])$

It follows from (13), (14) and (15) that:

(16) $\mathbf{G}^{**}_{\mathbf{H/T}}(\mathbf{m}, \text{Tails}) = .5(f_0^{**}([0.05, 0.95]) + f_1^{**}([0.05, 0.95]))$

Switching the order of the terms being summed yields:

(17) $\mathbf{G}^{**}_{\mathbf{H/T}}(\mathbf{m}, \text{Tails}) = .5(f_1^{**}([0.05, 0.95]) + f_0^{**}([0.05, 0.95]))$

Similarly,

(18) $\mathbf{G}^{**}_{\mathbf{H/T}}(\mathbf{s}, \text{Tails}) = .5(f_1^{**}([0.5, .5]) + f_0^{**}([0.5, 0.5]))$.

Comparing (11) and (17) we can see that

(19) $\mathbf{G}^{**}_{\mathbf{H/T}}(\mathbf{m}, \text{Heads}) = \mathbf{G}^{**}_{\mathbf{H/T}}(\mathbf{m}, \text{Tails})$

Comparing (12) and (18) we can see that:

(20) $\mathbf{G}^{**}_{\mathbf{H/T}}(\mathbf{s}, \text{Heads}) = \mathbf{G}^{**}_{\mathbf{H/T}}(\mathbf{s}, \text{Tails})$.

It follows from (7), (19) and (20) that:

(21) $\mathbf{G}_{\mathbf{H/T}}^{**}(\mathbf{s}, \text{Tails}) < \mathbf{G}_{\mathbf{H/T}}^{**}(\mathbf{m}, \text{Tails})$

But now note that given (7) and (21), **s** is ruled out a priori. **m** is more accurate than **s** in every world. Parallel reasoning would show that if we assumed that **s** was more accurate than **m** in Heads worlds, **m** would be ruled out a priori.

It follows that the imprecise credence defender can't claim either that $\mathbf{G}_{\mathbf{H/T}}^{**}(\mathbf{s}, \text{Heads}) < \mathbf{G}_{\mathbf{H/T}}^{**}(\mathbf{m}, \text{Heads})$
Or that
$\mathbf{G}_{\mathbf{H/T}}^{**}(\mathbf{s}, \text{Heads}) > \mathbf{G}_{\mathbf{H/T}}^{**}(\mathbf{m}, \text{Heads})$
without violating PROBABILISTIC ADMISSIBILITY. This means that, assuming the two terms are comparable, $\mathbf{G}_{\mathbf{H/T}}^{**}(\mathbf{s}, \text{Heads}) = \mathbf{G}_{\mathbf{H/T}}^{**}(\mathbf{m}, \text{Heads})$. But I already argued that it follows from this equality that $\mathbf{G}_{\mathbf{H/T}}^{**}(\mathbf{s}, \text{Tails}) = \mathbf{G}_{\mathbf{H/T}}^{**}(\mathbf{m}, \text{Tails})$, and so **m** is

no more accurate than **s** in any world. *Non-Dominance* implies that **m** can't be rationally required.

In sum, whether we represent the accuracy scores of imprecise credences using numbers or intervals, there is a non-dominance argument against the claim that an agent is required to have imprecise credences in response to MYSTERY-COIN.

4. Conclusion: Why Have Imprecise Credences?

I showed in Section 3 that *Non-Dominance* implies that it is not rationally required to have imprecise credences in MYSTERY-COIN. Are there other cases in which imprecise credences might be required? Are there alternative ways of defending the requirement to have imprecise credences that don't fall prey to the *Non-Dominance* argument? In this section I explore some options.

First, as I mentioned at the outset, the defender of imprecise credences may choose to reject *Non-Dominance*. She may claim that imprecise credences are required given certain bodies of evidence even though one is guaranteed to never be more accurate as a result of adopting them.

The second thing to note is that the arguments given in section 3 relied on my usage of a two-cell partition, and the argument in section 3.2 in particular relied on the fact that the recommended imprecise doxastic state in MYSTERY-COIN contained interval valued credences centered at 0.5. I will not, in this paper, consider the prospects of generalizing the argument. So it may be worth thinking about whether there are reasons to adopt imprecise credences in cases in which there is a more fine grained partition, or when the imprecise credences in question aren't centered on 0.5. Nothing I have said shows that there is a non-dominance argument against a requirement to have imprecise credences in such cases. But it is worth noting that if there is such a requirement, it won't be for the reason that is typically given: that "incomplete" bodies of evidence warrant imprecise credences. MYSTERY-COIN is the poster child for the kind of "incomplete evidence" that the defenders of imprecise credences have in mind, and I've already shown that imprecise credences *can't* be required in MYSTERY-COIN if non-dominance reasoning is legitimate.

The defense of imprecise credences that I am most sympathetic with is one that appeals to our cognitive limitations. Traditional defenders of imprecise credences have been adamant that even ideally rational agents with no cognitive limitations will have imprecise credences.

Joyce (2005, 171) writes:

> …It is not just that sharp degrees of belief are psychologically unrealistic (though they are). Imprecise credences have a clear epistemological motivation: they are the proper response to unspecific evidence.

And here's Levi (1985, 396):

> …it should be emphasized that those who insist on the reasonableness of indeterminacy in probability judgment under the permissibility interpretation mean to claim that even superhumans ought not always to have credal states that are strictly Bayesian.

I find *Non-Dominance* plausible so I reject the claim that even superhumans are required to have imprecise credences, at least in cases like MYSTERY-COIN. But there may be good reason for agents like us, with limited cognitive capacities, to adopt imprecise credences.

How might such an argument be developed? Here is one thought: consider a case in which a detective receives a large and complex body of evidence concerning a murder that took place. It is reasonable to expect that no number will consciously "pop into the detective's mind" as her degree of confidence in the claim that Jones committed the murder. Nor will she end up with fixed behavioral dispositions that correspond to a single such number. So the first thing to note is that it may simply be *impossible* for agents like us to adopt precise credences in every proposition we consider. But even if it were possible – even if the detective insists on deliberately adopting a precise credence in such a case, and she succeeds – she will, essentially, be choosing a number arbitrarily. And there's no reason to think that this arbitrarily chosen number will be more accurate, or expectedly accurate, than the state of mind she'd be in if she didn't choose such a number. So it, at very least, seems that a requirement to adopt *precise* credences will not do much good for agents like us. In fact, there may be reason to think that we will do *worse* as a result of forcibly adopting precise credences in every proposition we consider. For even if we could succeed at going around arbitrarily picking numbers as our credences (which, again, is questionable) the prospects that we will end up in a coherent belief state as a result of this process are dim. Are the prospects better if "when no number pops to mind" we remain mushy? This is an interesting question which I will not explore here. For now, suffice it to say that, for agents like us, there seems to be no reason to expect that we will do *better* by always adopting precise credences than we would do otherwise. If what is rationally required of an agent can depend on her cognitive limitations, or on her evidence about her cognitive limitations, then imprecise credences might at least be *permitted*.[7] The "cognitive limitations strategy" has been traditionally frowned upon by defenders of the imprecise credence model. But if one thinks that there can't be a rational requirement to adopt a belief state that one knows will never be more accurate than an alternative, the cognitive-limitations strategy may be the best way forward for the imprecise credence advocate. At very least, we might hope that, for agents like us, imprecise credences will be *permitted*.

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[7] For further discussion of the "cognitive limitation" defense of imprecise credences see Schoenfield (2012)

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