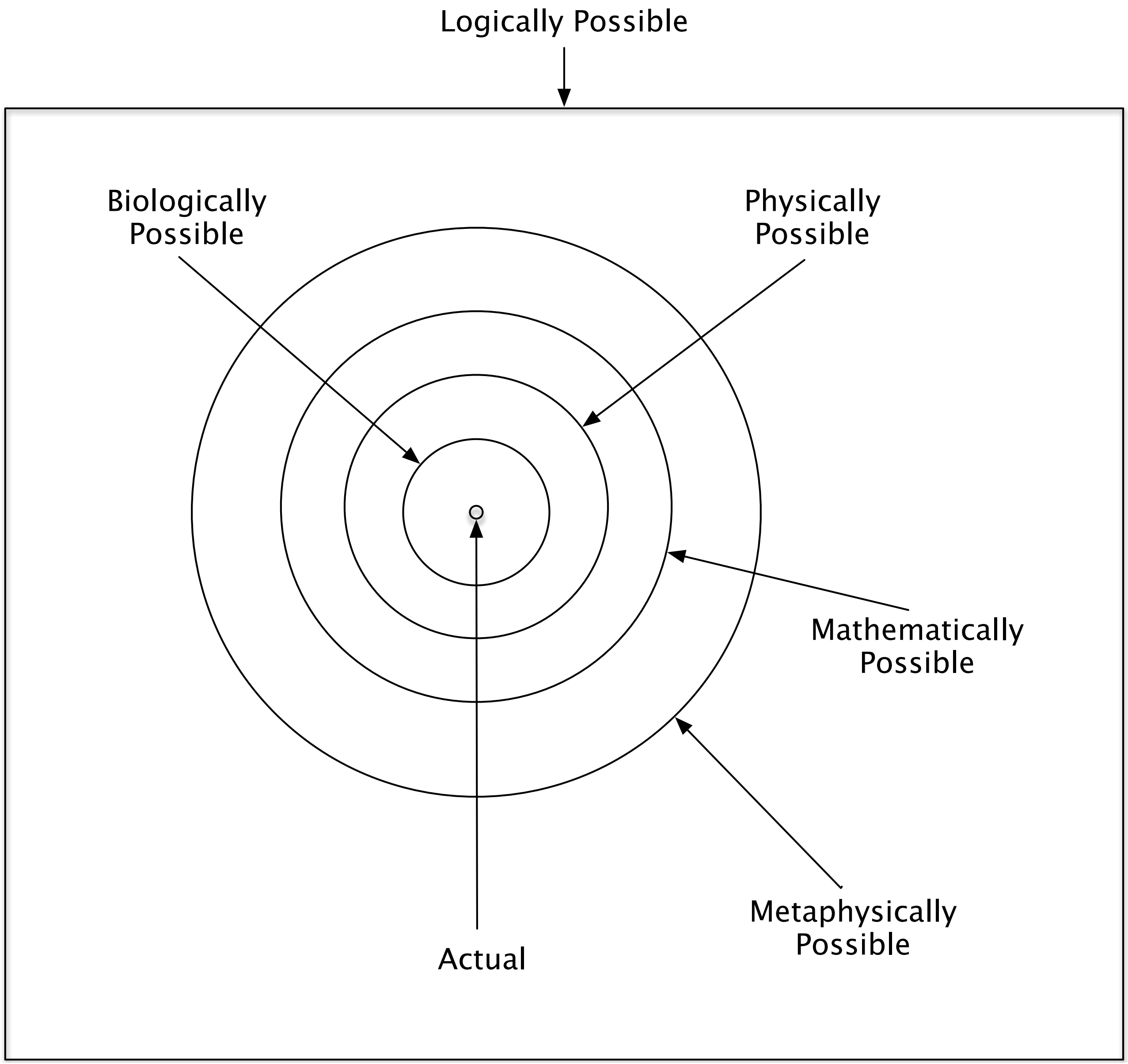
Overview of Today's Lecture

- Administrative Stuff
  - Last Time: Course Website/Syllabus
    - Please get a copy of the syllabus if you weren't here last time
    - Note: my office hours are 3:30–4:15 Tuesdays & 12–1:15 Fridays.
  - ☞ **HW #1 Assigned (see website). Due in 2 weeks (*via* `Blackboard`).**
- Unit #1: Basic Underlying Concepts of Logic (Chapter 1 of Forbes)
  - Sentences, Propositions, and Arguments (the building blocks)
  - Actual, Possible, and Necessary Truth (key basic concepts)
  - Deductive Validity of Arguments (*the* central concept of Part I)
  - Validity, Soundness, and "Goodness" of Arguments
  - Absolute *vs* Sentential Validity and the notion of *logical form*
  - Glimpses beyond sentential validity

Background 2: Actual, Possible, and Necessary Truth

- Some propositions are actually true (Snow is white), and some are not (Al Gore is President of the United States in 2007).
- Other propositions are not *actually* true, but still *possibly* true. Al Gore is not *actually* our President in 2007, but he *might have been*. As such, it is *possibly* true that Al Gore is President in 2007.
- Some propositions are not even *possibly* true. For instance:
  1. My car has traveled faster than the speed of light.
  2. $2 + 2 = 5$.
  3. Branden weighs 200 lbs and Branden does not weigh 200 lbs.
- (1) violates the laws of physics: it is *physically impossible*. (2) violates the laws of arithmetic: it is *arithmetically* impossible.
- (3) violates the laws of *logic*: it is *logically* impossible.

- This is the kind of impossibility that interests the logician. In slogan form, we might call this "the strongest possible kind of impossibility."
- Some propositions are not only *actually* true, but (logically) *necessarily* true. These *must* be true, on pain of *self-contradiction*:
  - Either Branden weighs 200lbs or he does not weigh 200lbs.
  - If Branden is a good man, then Branden is a man.
- Logical possibility and logical necessity are central concepts in this course. We will make extensive use of them.
- We will look at two precise, formal logical theories in which the notion of logical necessity will have a more precise meaning.
- But, before we get into our formal theorizing, we will look informally at the *following-from* relation between propositions.
- As we will see, understanding the following-from relation will require a grasp of the notions of logical necessity (and logical truth).


Logically Possible
Biologically Possible
Physically Possible
Mathematically Possible
Metaphysically Possible
Actual

Bakckground 3: The "Logical Constants"

- If logical possibility does not depend on content (*i.e.*, on which objects are being talked about, or which properties are involved), then what does it depend on? The answer will be "logical form."
- We'll talk a lot about logical form in Part I of the course (in a way, that's *the* central concept of Part I). But, first, it's helpful to identify some "logical constants" in our language(s). [These determine logical forms.]
- The logical constants (see "Logical Constants", which is now linked from our course materials page) are *terms with meanings that do not depend on which objects the sentences in which they occur are about*.
- Prime examples: the *truth-functional* (*a.k.a.*, *Boolean*) *connectives*, which are expressed in English using, *e.g.*, "and", "or", "not", "if… then… "
- Their meanings/referents do not vary across sentences that are about different objects. The meanings of these connectives are *functions of the truth-values of the statements to which they are applied*.

- *E.g.*, *any* claim of the *form* "P and not P" is *impossible*. It doesn't matter which statement P we're talking about (or which objects P is about). If P is true, then "not P" is false and if "not P" is true, then P is false.
- Not all connectives are logical constants. Indeed, not all connectives are even *truth-functional*. For instance, consider the connective "because".
- "P because Q" is true *only* if both P and Q are true. But, some instances of "P because Q" are *false*, *even though* both P and Q *are* true.
- *E.g.*: let P be the (true) claim that George Bush was president in 2001, and let Q be the (true) claim that it snowed in Boston in February 2015.
- In this case, "P because Q" is *false*, even though both P and Q are true. Therefore, "because" is *not* truth-functional. "Because" depends on which objects (and on which times) the claims P and Q are about.
- [The formal part of] Part I of the course will be all about the truth-functional connectives, and truth-functional logical forms (*a.k.a.*, the *sentential* logical forms). But, let's not get ahead of ourselves…

Bakckground 4: Arguments, Following-From, and Validity

- An *argument* is a collection of propositions, one of which (the *conclusion*) is supposed to *follow from* the rest (the *premises*).

  If John is a bachelor, then John is unmarried.

  John is a bachelor.

  ∴ John is unmarried.

- If the conclusion of an argument *follows from* its premises, then the argument is said to be *valid* (otherwise, it's *in*valid).

☞ **Definition**. An argument $\mathscr{A}$ is *valid* if and only if:

Rendition #1. It is (logically!) *necessary* that *if* all of the premises of $\mathscr{A}$ are true, *then* the conclusion of $\mathscr{A}$ is also true.

Rendition #2. It is (logically!) *impossible* for both of the following to be true simultaneously: (1) all of the premises of $\mathscr{A}$ are true, *and* (2) the conclusion of $\mathscr{A}$ is false. [For us, this will be *equivalent* to #1.]

Background 5: Validity, Soundness, and "Good" Arguments

- A "good" argument is one in which the conclusion follows from the premises. But, intuitively, there is more to a "good" argument (all things considered) than mere validity.
- Ideally, arguments should also have (actually) *true premises*. If the premises of an argument are (actually) false, then (intuitively) the argument isn't very "good" — even if it is valid. *Why not*?

☞ **Definition**. An argument $\mathscr{A}$ is *sound* if and only if *both*: (*i*) $\mathscr{A}$ is valid, *and* (*ii*) all of $\mathscr{A}$'s premises are (actually) true.

- So, there are two components or aspects of "good" arguments:
  - Logical Component: Is the argument valid?
  - Non-Logical Component: Are the premises (actually) true?
- This course is only concerned with the *logical* component.

Is it *possible* that all of $\mathscr{A}$'s premises are true, *but* $\mathscr{A}$'s conclusion is false?

- YES
  - $\mathscr{A}$ is *in*valid.
  - $\mathscr{A}$ is *un*sound.
- NO
  - $\mathscr{A}$ is valid.
  - Are all of $\mathscr{A}$'s premises *actually true*?
    - YES
      - $\mathscr{A}$ is *sound*.
    - NO
      - $\mathscr{A}$ is *un*sound.

Why study logic *formally* or *symbolically*?

- Ultimately, we want to decide whether arguments expressible in *natural* languages are valid. But, in this course, we will only study arguments expressible in *formal* languages. And, we will use *formal* tools. *Why?*
- Analogous question: What we want from natural science is explanations and predictions about *natural* systems. But, our theories (strictly) apply only to systems faithfully describable in *formal*, *mathematical* terms.
- Although formal models are *idealizations* which abstract away some aspects of natural systems, they are *useful idealizations* that help us understand *many* natural relationships and regularities.
- Similarly, studying arguments expressible in formal languages allows us to develop powerful tools for testing validity. We won't be able to capture *all* valid arguments this way. But, we can grasp many.

A Subtle Argument, and the Notion of Logical Form

(i)
John is a bachelor.
$\therefore$ John is unmarried.

- Is (i) valid? Well, this is tricky. Intuitively, being unmarried is part of the *meaning* of "bachelor". So, it *seems* like it is (intuitively) logically impossible for the premise of (i) to be true while its conclusion is false
- This suggests that (i) is (intuitively/absolutely) valid.
- On the other hand, consider the following argument:

(ii)
If John is a bachelor, then John is unmarried.
John is a bachelor.
$\therefore$ John is unmarried.

- The correct judgment about (ii) seems *clearly* to be that it is valid – *even if we don't know the meaning of "bachelor" (or "unmarried")*.
- This is clear because the logical form of (ii) is *obvious* [(i)'s form is not].

Logical Form II

- This suggests the following additional “conservative” heuristic:
  - ☞ We should conclude that an argument $\mathscr{A}$ is valid only if we can see that $\mathscr{A}$'s conclusion follows from $\mathscr{A}$'s premises *without appealing to the meanings of the predicates involved in* $\mathscr{A}$.
- But, if validity does not depend on the meanings of predicates, then what *does* it depend on? This is a deep question about logic. We will not answer it here. That's for more advanced philosophical logic courses.
- What we will do instead is adopt a conservative methodology that only classifies *some* “intuitively/absolutely valid” arguments as valid.
- The strategy will be to develop some *formal* methods for *modeling* intuitive/abolsute validity of arguments expressed in English.
- We won't be able to capture *all* intuitively/absolutely valid arguments with our methods, but this is OK. [Analogy: mathematical physics.]

Logical Form III

- We will begin with *sentential logic*. This will involve providing a characterization of valid *sentential forms*. Here's a paradigm example:

Dr. Ruth is a man.
(1) If Dr. Ruth is a man, then Dr. Ruth is 10 feet tall.
∴ Dr. Ruth is 10 feet tall.

- (1) is a set of sentences with a valid sentential form. So, whatever argument it expresses is a valid argument. What's its *form*?

p.
(1_f) If p, then q.
$\therefore q$.

- (1)'s valid *sentential form* (1_f) is so famous it has a name: *Modus Ponens*. [Usually, latin names are used for the *valid* forms.]

☞ **Definition.** The *sentential form* of an argument (or, the sentences faithfully expressing an argument) is obtained by replacing each basic (or, atomic) sentence in the argument with a single (lower-case) letter.

- What's a "basic" sentence? A basic sentence is a sentence that doesn't contain any sentence as a proper part. How about these?

  (a) Branden is a philosopher and Branden is a man.

  (b) It is not the case that Branden is 6 feet tall.

  (c) Snow is white.

  (d) Either it will rain today or it will be sunny today.

- Sentences (a), (b), and (d) are *not* basic (we'll call them "complex" or "compound"). Only (c) is basic. We'll also use "atomic" for basic.
- What's the sentential form of the following argument (is it valid?):

  If Tom is at his Fremont home, then he's in California.

  Tom is in California.

  ∴ Tom is at his Fremont home.

Two "Strange" Valid Sentential Forms

(†) p. Therefore, either q or not q.

- (†) is valid because it is (logically) *impossible* that *both*:

  (i) p is true, *and*
  (ii) "either q or not q" is false.

  This is impossible because *(ii) alone* is impossible.

(‡) p and not p. Therefore, q.

- (‡) is valid because it is (logically) *impossible* that *both*:

  (iii) "p and not p" is true, *and*
  (iv) q is false.

  This is impossible because *(iii) alone* is impossible.

- We'll soon see why we have these "oddities". They stem from our semantics for "If … then" statements (and our first def. of validity).

Some Valid and Invalid Sentential Forms

Sentential Argument Form	Name	Valid/Invalid
p If p, then q $\therefore q$	*Modus Ponens*	Valid
q If p, then q $\therefore p$	Affirming the Consequent	Invalid
It is not the case that q If p, then q $\therefore$ It is not the case that p	*Modus Tollens*	Valid
It is not the case that p If p, then q $\therefore$ It is not the case that q	Denying the Antecedent	Invalid
If p, then q If q, then r $\therefore$ If p, then r	Hypothetical Syllogism	Valid
It is not the case that p Either p or q $\therefore q$	Disjunctive Syllogism	Valid

Logical Form IV — Beyond Sentential Form

- The first half of the course involves developing a precise *theory* of *sentential* validity, and several rigorous techniques for *deciding* whether a sentential form is (or is not) valid. This only takes us so far.
- Not all (absolutely) valid arguments have valid *sentential* forms, *e.g.*:

All men are mortal.
(2) Socrates is a man.
∴ Socrates is mortal.

- The argument expressed by (2) seems clearly valid. But, the sentential form of (2) is not a valid form. Its sentential form is:

p.
(2_f) q.
$\therefore r$.

- In this first course, we will not be studying predicate/quantifier logic, which gives a formal theory of validity that covers such forms.
- In that more general theory, one can recognize that (2) has something like the following (non-sentential!) logical form:

All Xs are Ys.

(2_f*) a is an X.

$\therefore$ a is a Y.

- We will leave such arguments (called *syllogisms*) for a future, more sophisticated theory of logical validity.
- In Part I of the course, we'll learn a (simple) purely formal language for talking about *sentential* forms, and then we'll develop some rigorous methods for determining whether sentential forms are valid.
- As we will see, the fit between our simple formal sentential language and English (or other natural languages) will not be perfect.

Validity and Soundness of Arguments — Some Non-Sentential Examples

- Can we classify the following according to validity/soundness?

1) All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage.	**5)** All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine.
2) All wines are whiskeys. Chardonnay is a wine. Therefore, chardonnay is a whiskey.	**6)** All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine.
3) All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink.	**7)** All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine.
4) All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey.	**8)** All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine.

	Valid	**Invalid**
True premises True conclusion	All wines are beverages. Chardonnay is a wine. Therefore, chardonnay is a beverage. [sound]	All wines are beverages. Chardonnay is a beverage. Therefore, chardonnay is a wine. [unsound]
True premises False conclusion	**Impossible** None exist	All wines are beverages. Ginger ale is a beverage. Therefore, ginger ale is a wine. [unsound]
False premises True conclusion	All wines are soft drinks. Ginger ale is a wine. Therefore, ginger ale is a soft drink. [unsound]	All wines are whiskeys. Chardonnay is a whiskey. Therefore, chardonnay is a wine. [unsound]
False premises False conclusion	All wines are whiskeys. Ginger ale is a wine. Therefore, ginger ale is a whiskey. [unsound]	All wines are whiskeys. Ginger ale is a whiskey. Therefore, ginger ale is a wine. [unsound]

- See, also, our validity and soundness handout …

Some Brain Teasers Involving Validity and Soundness

- Here are two very puzzling arguments:

$(\mathscr{A}_1)$ Either $\mathscr{A}_1$ is valid or $\mathscr{A}_1$ is invalid.

$\therefore$ $\mathscr{A}_1$ is invalid.

$(\mathscr{A}_2)$ $\mathscr{A}_2$ is valid.

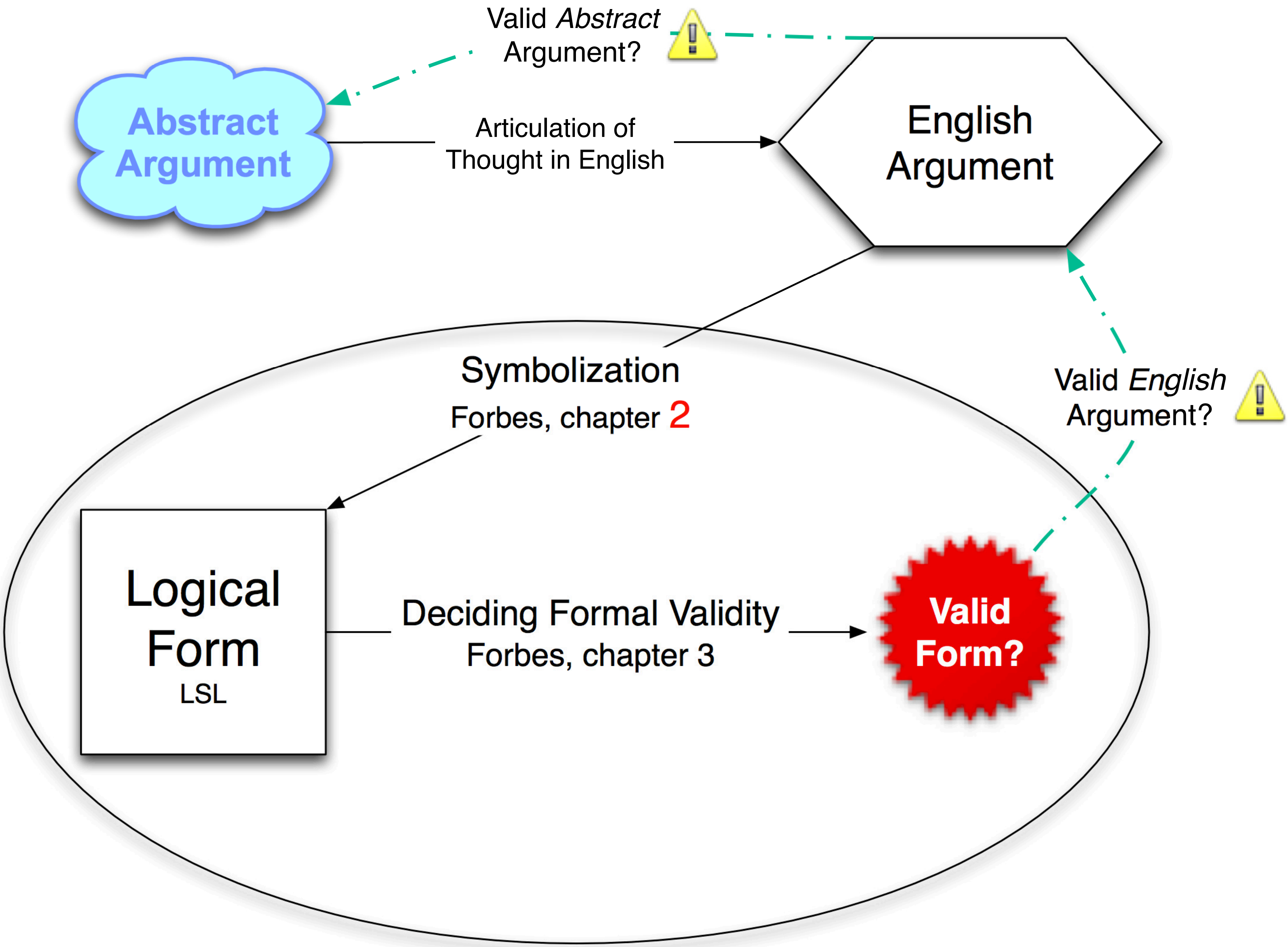
$\therefore$ $\mathscr{A}_2$ is invalid.

- I'll discuss $\mathscr{A}_2$ ($\mathscr{A}_1$ is left as an exercise).
  - If $\mathscr{A}_2$ is valid, then it has a true premise and a false conclusion. But, this means that if $\mathscr{A}_2$ is valid, then $\mathscr{A}_2$ invalid!
  - If $\mathscr{A}_2$ is invalid, then its conclusion must be true (as a matter of logic). But, this means that if $\mathscr{A}_2$ is invalid then $\mathscr{A}_2$ is valid!
  - This *seems* to imply that $\mathscr{A}_2$ is *both valid and invalid*. But, remember our conservative validity-principle. What is the *logical form* of $\mathscr{A}_2$?

Absolute Validity *vs* Formal Validity

- Forbes calls the general, informal notion of validity “absolute validity”.
- Our notion is a bit more conservative than his, since we’ll only call an argument valid if one of our *formal theories* captures it as falling under a valid *form*. Our first formal theory (LSL) is about *sentential* validity.
- An argument is *sententially* valid if it has a valid *sentential form*.
- Sentential form is obtained by replacing each basic or atomic sentence in an argument with a corresponding lower-case letter.
- Once we know the sentential form of an argument (chapter 2), we will be able to apply purely formal, mechanical methods (chapters 3 and 4) for determining whether that sentential form is valid.

☞ Even if an argument fails to be *sententially* valid, it could still be valid according to a richer logical theory than LSL. I’ll mention some other, more sophisticated theories of logical form later in the course.


Valid *Abstract*
Argument?
Abstract
Argument
Articulation of
Thought in English
English
Argument
Symbolization
Forbes, chapter 2
Valid *English*
Argument?
Logical
Form
LSL
Deciding Formal Validity
Forbes, chapter 3
Valid
Form?