

## Philosophy 57 — Day 20

- Quiz Today (3-Circle Venn Diagram Technique)
- New Versions of LogicCoach Available Online. See:  
<http://philosophy.wisc.edu/fitelson/57/links.htm>
  - LogicCoach is very useful for chapter 6 problems
- Extra-Credit Problems to be Posted Within 10 Days
  - Will Be Problems from Chapter 6
  - May Use Computer Programs (or any other tools!)
- Back to Chapter 6
  - Review of Basic Terminology and Set-up
  - Translation from English into Propositional Logic



## Chapter 6 — Propositional Logic Review I

- Capital letters denote **simple** or **atomic** statements. These are combined with **connectives** (and, or, if, not, iff) to form **compound** or **molecular** statements.

### Simple Statements (symbolic):

- Today is Halloween.  
 $\mapsto H$ .
- The monk seal is endangered.  
 $\mapsto M$ .
- James Joyce wrote *Ulysses*.  
 $\mapsto J$ .
- Parakeets are colorful birds.  
 $\mapsto P$ .
- Butane is a hydrogen compound.  
 $\mapsto B$ .

### Compound Statements (pseudo-symbolic):

- Either it's raining or it's snowing.  
 $\mapsto$  Either  $R$  or  $S$ .
- If Dell introduces a new line, then Apple will also.  
 $\mapsto$  If  $D$ , then  $A$ .
- Snow is white and the sky is blue.  
 $\mapsto W$  and  $L$ .
- is not the case that Emily Bronte wrote *Jane Eyre*.  
 $\mapsto$  It is not the case that  $E$ .
- John is a bachelor if and only if he is unmarried.  
 $\mapsto I$  if and only if  $U$ .

- Symbolic Connectives: “ $\sim$ ” for “not”, “ $\vee$ ” for “or”, “ $\bullet$ ” for “and”, “ $\supset$ ” for “if”, and “ $\equiv$ ” for “if and only if”. Here's a table of the five PL connectives:



Operator	Name	Logical Function	Used to translate
$\sim$	tilde	negation	not, it is not the case that
$\bullet$	dot	conjunction	and, also, moreover, but
$\vee$	wedge	disjunction	or, unless, either ... or ...
$\supset$	horseshoe	implication	if ... then ..., only if
$\equiv$	triple bar	equivalence	if and only if

- Using these symbols, we can put our compound examples in symbolic form:

#### Pseudo-Symbolic:

- Either  $R$  or  $S$ .
- If  $D$ , then  $A$ .
- $W$  and  $L$ .
- It is not the case that  $E$ .
- $I$  if and only if  $U$ .

#### Full Symbolic:

- $R \vee S$ .
- $D \supset A$ .
- $W \bullet L$ .
- $\sim E$ .
- $I \equiv U$ .

- Our first topic is *translation* from English into PL.



## Chapter 6 — Propositional Logic Review II

Statement Form:	Statement Name:	Statement Components:
$\sim p$	negation	—
$p \bullet q$	conjunction	$p, q$ called <b>conjuncts</b>
$p \vee q$	disjunction	$p, q$ called <b>disjuncts</b>
$p \supset q$	conditional	$p$ called <b>antecedent</b> , $q$ called <b>consequent</b>
$p \equiv q$	biconditional	—

- NOTE: I use lower-case letters  $p, q$ , etc. as *variables*, ranging over sentences in PL. Lower-case letters are *not* part of the PL language!
- PL sentences can be as complex as you like. But, they are always constructed out of upper-case letters, and connectives (or operators).
- The **main operator** in a (compound) PL sentence is the operator that governs the largest component(s) in the statement. What are the following?

“ $\sim(G \supset H) \supset (I \vee J)$ ”    “ $(A \equiv F) \bullet (C \equiv G)$ ”    “ $\sim A \supset$ ”    “ $A \supset B \vee C \bullet D$ ”



## Chapter 6 — Propositional Logic Translations I

- The following are all examples of **negations**:
  - Rolex does not make computers.  $\mapsto \sim R$
  - It is not the case that Rolex makes computers.  $\mapsto \sim R$
  - It is false that Rolex makes computers.  $\mapsto \sim R$
- We always place the tilde *in front* of the proposition it negates. The following are also **negations**, since their main connectives are **tildes**:

$\sim B$

$\sim(G \supset H)$

$\sim[(A \equiv F) \bullet (C \equiv G)]$

- The following are *not* negations. Watch the parentheses! What *are* these?

$\sim B \bullet C$

$\sim(G \supset H) \vee J$

$\sim P \supset Q$



## Chapter 6 — Propositional Logic Translations II

- The following are all examples of **conjunctions**:
  - Tiffany sells jewelry and Gucci sells cologne.  $\mapsto T \bullet G$
  - Tiffany sells jewelry, but Gucci sells cologne.  $\mapsto T \bullet G$
  - Tiffany sells jewelry, however, Gucci sells cologne.  $\mapsto T \bullet G$
  - Tiffany and Ben Bridge sell jewelry.  $\mapsto T \bullet B$
- The following are all **conjunctions**, since their main connectives are **dots**:
 
$$K \bullet \sim L$$

$$(E \vee F) \bullet (G \vee H)$$

$$[(R \supset T) \vee (S \supset U)] \bullet [(W \equiv X) \vee (Y \equiv Z)]$$
- The following are *not* conjunctions. Watch the parentheses! What *are* these?
 
$$(A \bullet B) \supset C$$

$$(A \bullet (B \equiv C)) \vee (A \bullet D)$$

$$\sim(A \bullet B)$$



## Chapter 6 — Propositional Logic Translations III

- The following are all examples of **disjunctions**:
  - Cigna expands operations or Aetna does.  $\mapsto C \vee A$
  - Either Cigna or Aetna expands operations.  $\mapsto C \vee A$
  - Cigna expands operations **unless** Aetna does.  $\mapsto C \vee A$
  - **Unless** Cigna expands operations, Aetna does.  $\mapsto C \vee A$
- The following are all **disjunctions**, since their main connectives are **wedges**:
  - $\sim C \vee \sim D$
  - $(F \bullet H) \vee (\sim K \bullet \sim L)$
  - $[S \bullet (T \supset U)] \vee [X \bullet (Y \equiv Z)]$
- The following are *not* disjunctions. Watch the parentheses! What *are* these?
  - $(A \vee B) \supset C$
  - $(A \vee (B \equiv C)) \bullet (A \vee D)$
  - $\sim(A \vee B)$



## Chapter 6 — Propositional Logic Translations IV

- The following are all examples of **conditionals**:
  - If Intel raises prices, then so does Compaq.  $\mapsto I \supset C$
  - Compaq raises prices if Intel does.  $\mapsto I \supset C$
  - Intel raises prices **only if** Compaq does.  $\mapsto I \supset C$
  - Delta lowers fares **provided that** United does.  $\mapsto U \supset D$
  - Delta lowers fares **on condition that** United does.  $\mapsto U \supset D$
  - United's lowering fares **implies that** Delta does.  $\mapsto U \supset D$
  - Hilton's opening a new hotel is a **sufficient condition** for Marriott's doing so.  $\mapsto H \supset M$
  - Hilton's opening a new hotel is a **necessary condition** for Marriott's doing so.  $\mapsto M \supset H$
- The following are all **conditionals** — their main connectives are **horseshoes**:  

$$H \supset \sim J \qquad (A \vee C) \supset (D \bullet E) \qquad [K \vee (S \bullet \sim T)] \supset [\sim F \vee (M \bullet O)]$$





## Chapter 6 — Propositional Logic Translations V

- The following are examples of **biconditionals**:
  - Kodak introduces a new film **if and only if** Fuji does.  $\mapsto K \equiv F$
  - Kodak's introducing a new film is a **necessary and sufficient condition** for Fuji's doing so.  $\mapsto K \equiv F$
- Sidebar: Use the mnemonic device "SUN" to remember the important distinction between **sufficient** *versus* **necessary** conditions.
  - If you rotate the "U" in "SUN" to the left, then you get " $S \supset N$ ," which means that **sufficient conditions always imply necessary conditions**.
  - **Sufficient conditions** are antecedents. **Necessary conditions** are consequents.
- The following are **biconditionals** — their main connectives are **triple bars**:
$$M \equiv \sim T$$
$$(B \vee D) \equiv (A \bullet C)$$
$$[K \vee (F \supset I)] \equiv [\sim L \bullet (G \vee H)]$$



## Chapter 6 — Propositional Logic Translations VI

- Whenever 3 or more letters are appear, parentheses (or brackets or braces) must be used carefully to indicate the proper range of the connectives.
- For instance, the string of symbols “ $A \bullet B \vee C$ ” is *ambiguous*. It could represent either “ $(A \bullet B) \vee C$ ” or “ $A \bullet (B \vee C)$ ”. These have *different meanings*!
- A **well-formed formula** (WFF, for short) is a grammatical PL sentence. In English, “Porch on the is cat a there” is ungrammatical. And, in PL, the following strings of symbols are not WFFs, because they are ungrammatical:  

$$“A \supset \vee B” \qquad “A \bullet B \vee C” \qquad “A \supset B \supset C” \qquad “\sim \vee B(\vee C)”$$
- Here are some examples, to illustrate the importance of proper grouping.
  1. Prozac relieves depression and Allegra combats allergies, or Zocor lowers cholesterol.  $\mapsto (P \bullet A) \vee Z$
  2. Prozac relieves depression, and Allegra combats allergies or Zocor lowers cholesterol.  $\mapsto ??$



3. Either Prozac relieves depression and Allegra combats allergies or Zocor lowers cholesterol.  $\mapsto ??$
  4. Prozac relieves depression and either Allegra combats allergies or Zocor lowers cholesterol.  $\mapsto ??$
  5. Prozac relieves depression or both Allegra combats allergies and Zocor lowers cholesterol.  $\mapsto ??$
  6. If Merck changes its logo, then if Pfizer increases sales, then Lilly will reorganize.  $\mapsto ??$
  7. If Merck's changing its logo implies that Pfizer increases sales, then Lilly will reorganize.  $\mapsto ??$
  8. If Schering and Pfizer lower prices or Novartis downsizes, then Warner will expand production.  $\mapsto ??$
- Do not confuse the following three statement forms:  
“A if B”  $\mapsto$  “ $B \supset A$ ”      “A only if B”  $\mapsto$  “ $A \supset B$ ”      “A if and only if B”  $\mapsto$  “ $A \equiv B$ ”



## Chapter 6 — Propositional Logic Translations VII

- The tilde “ $\sim$ ” operates *only* on the unit that immediately follows it. In “ $\sim K \vee M$ ,”  $\sim$  affects only “ $K$ ”; in “ $\sim(K \vee M)$ ,”  $\sim$  affects the entire “ $K \vee M$ ”.
- “It is not the case that  $K$  or  $M$ ” is *ambiguous* between “ $\sim K \vee M$ ,” and “ $\sim(K \vee M)$ .” **Convention:** “It is not the case that  $K$  or  $M$ ”  $\mapsto$  “ $\sim K \vee M$ ”.
- “Not both  $S$  and  $T$ ”  $\mapsto$  “ $\sim(S \bullet T)$ ”. As we will see later (**DeMorgan rule**), “ $\sim(S \bullet T)$ ”  $\approx$  “ $\sim S \vee \sim T$ ”. But, “ $\sim(S \bullet T)$ ”  $\not\approx$  “ $\sim S \bullet \sim T$ ”.
- Similarly, “Not either  $S$  or  $T$ ”  $\mapsto$  “ $\sim(S \vee T)$ ”. And, (**DeMorgan rule** again) “ $\sim(S \vee T)$ ”  $\approx$  “ $\sim S \bullet \sim T$ ”, but “ $\sim(S \vee T)$ ”  $\not\approx$  “ $\sim S \vee \sim T$ ”.
- Here are some examples involving  $\sim$ ,  $\bullet$ , and  $\vee$  (not, and, or):
  1. Shell is not a polluter, but Exxon is.  $\mapsto$  ??
  2. Not both Shell and Exxon are polluters.  $\mapsto$  ??
  3. Both Shell and Exxon are not polluters.  $\mapsto$  ??



4. Not either Shell or Exxon is a polluter.  $\mapsto ??$

5. Neither Shell nor Exxon is a polluter.  $\mapsto ??$

6. Either Shell or Exxon is not a polluter.  $\mapsto ??$

- Summary of translations involving  $\sim$ ,  $\bullet$ , and  $\vee$  (not, and, or):

### Pseudo-Symbolic

### Propositional Logic (PL)

Not either  $A$  or  $B$ .

$\sim(A \vee B)$

Either not  $A$  or not  $B$

$\sim A \vee \sim B$

Not both  $A$  and  $B$ .

$\sim(A \bullet B)$

Both not  $A$  and not  $B$ .

$\sim A \bullet \sim B$

- **DeMorgan rules** (we will *prove* these rules later in the chapter):

$$\sim(p \vee q) \approx \sim p \bullet \sim q$$

$$\sim(p \bullet q) \approx \sim p \vee \sim q$$

- But,  $\sim(p \vee q) \not\approx \sim p \vee \sim q$  and  $\sim(p \bullet q) \not\approx \sim p \bullet \sim q$ .



## Chapter 6 — Propositional Logic Translations VIII

English Expression	PL Operator
not, it is not the case that, it is false that	$\sim$
and, yet, but, however, moreover, nevertheless, still, also, although, both, additionally, furthermore	$\bullet$
or, unless, either ... or ...	$\vee$
if ... then ..., only if, given that, in case, provided that, on condition that, sufficient condition for, necessary condition for (Note: do not confuse antecedents and consequents!)	$\supset$
if and only if (iff), is equivalent to, sufficient and necessary condition for, necessary and sufficient condition for	$\equiv$



## Chapter 6 — Propositional Logic Translations IX

- A Bunch of Translation Problems:
  1. California does not allow smoking in restaurants.
  2. Jennifer Lopez becomes a superstar given that *I'm Real* goes platinum.
  3. Mary-Kate Olsen does not appear in a movie unless Ashley does.
  4. Either the President supports campaign reform and the House adopts universal healthcare or the Senate approves missile defense.
  5. Neither Mylanta nor Pepcid cures headaches.
  6. If Canada subsidizes exports, then if Mexico opens new factories, then the United States raises tariffs.
  7. If Iraq launches terrorist attacks, then either Peter Jennings or Tom Brokaw will report them.
  8. Tom Cruise goes to the premiere provided that Penelope Cruz does, but Nicole Kidman does not.



9. It is not the case that either Bart and Lisa do their chores or Lenny and Karl blow up the power plant.
10. N'sync winning a grammy is a sufficient condition for the Backstreet Boys to be jealous, only if Destiny's Child getting booed is a necessary condition for TLC's being asked to sing the anthem.
11. Dominos' delivers for free if Pizza Hut adds new toppings, provided that Round Table airs more commercials.
12. If evolutionary biology is correct, then higher life forms arose by chance, and if that is so, then it is not the case that there is any design in nature and divine providence is a myth.
13. Kathie Lee's retiring is a necessary condition for Regis's getting a new co-host; moreover, Jay Leno's buying a motorcycle and David Letterman's telling more jokes imply that NBC's airing more talk shows is a sufficient condition for CBS's changing its image.

