On the Notion of Admissibility in Chance-Credence Principles:
A Comment on Vranas[1]

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I. Introduction

Lewis' Principal Principle (PP) aims at clarifying the connection between chance (i.e. objective probability) and credence (i.e. subjective probability). It is generally assumed that the chance that an event will occur does not depend on our credence in the occurrence of that event. Nevertheless, chances constrain our credence, and Lewis' PP is an attempt to capture this connection.

The precise formulation of PP requires some notation.[2] Let Ch be a function that assigns a number in the interval [0,1] to each proposition A. $<Ch(A) = x>$ is the proposition that Ch assigns the chance x to A. Let Cr be a function that assigns a number in the interval [0,1] to each proposition A, corresponding to a person's degree of belief (or credence) in A. Finally, let E be any proposition. With these definitions, we can now formulate PP:

Principal Principle (PP): If E is admissible with respect to $<Ch(A) = x>$,

then $Cr(A|E \wedge <Ch(A) = x>)$ should be x.

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[1] Comment on P. Vranas, "Have your Cake and Eat It too: The Old Principal Principle Reconciled with the New", delivered at the *First Annual Austin–Berkeley Formal Epistemology Workshop FEW 2004*, University of California at Berkeley, USA, May 21-23, 2004.

[2] Apart from a slight modification (I added the conjunction symbol "$\wedge$" between two propositions where necessary), I will follow Vranas' notation.

Obviously, the notion of admissibility is crucial in this definition,[3] as Lewis himself had already pointed out: "[T]he power of the Principal Principle depends entirely on how much is admissible" (Lewis 1980: 92). In his careful analysis, Lewis (1980) argued that two kinds of information are generally admissible. The first kind of admissible proposition is a proposition which states an event that occurred in the past: A proposition E is admissible at time t if E contains historical information until t. The second kind of admissible proposition is a proposition E that consists of hypothetical information about what the chance of another proposition A would be at time t', if the history before t' had been different.

Unfortunately, PP leads to a contradiction when dealing with (what is called in the literature) undermining futures.[4] Already in 1986 Lewis recognized the problem as so serious that he wrote:

> There is one big bad bug: chance. It is here, and here alone, that I fear defeat. (Lewis, 1986: xiv)

It took Lewis many years to propose a solution to this urging question. Finally, in 1994, he replaced PP by a new Principal Principle. This is the so-called New Principle (NP):

New Principle (NP): $Cr(A|H \wedge T_f)$ should be $f(A|\text{H} \wedge T_f)$,

where f is a probability measure, and H is a proposition that specifies the past and the present. T_f is, as Vranas (p. 3) explains, "the proposition that Ch is f, i.e. that (i) dom $Ch = \text{dom } f$ and (ii) for every A in dom f, $Ch(A) = f(A)$. T_f entails propositions (like $<Ch(A) = f(A)>$) that specify the chances of various propositions. For example: if $f(A)$ is 0.3, then T_f entails the proposition that $Ch(A)$ is 0.3 (so T_f is false if $Ch(A)$ is not 0.3)."

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[3] A chance-credence principle without any notion of admissibility can be found in Hall (1994). Moreover, Levi and Kyburg argue for different notions of admissibility (see Bogdan (1984).

[4] See Lewis (1994), Thau, (1994) and Hall (1994).

It is well known that Lewis felt uneasy when he suggested to replace PP by NP. After all, NP is less intuitive than PP, which Lewis (1994: 489) considers to be "the key to our concept of chance". Contrary to Lewis' intuition, Vranas attempts to vindicate the original PP by a careful investigation of the logical relations that hold between different chance-credence principles. More specifically, Vranas examines four additional principles that I shall introduce now. Let B be any proposition:

Minimal Principle (MP): $Cr(A|<Ch(A) = x>)$ should be x.

Conditional Principle (CP): $Cr(A|B \wedge <Ch(A \mid B) = x>)$ should be x.

General Principle (GP): If E is admissible with respect to $(B \wedge < Ch(A|B) = x>)$,

$\qquad$ then $Cr(A|E \wedge B \wedge <Ch(A|B) = x>)$ should be x.

Old Principle (OP): If $(H \wedge T_f)$ is admissible with respect to $<Ch(A) = f(A)>$,

$\qquad$ then $Cr(A|H \wedge T_f)$ should be $f(A)$.

With these definitions in hand, Vranas defends three main theses: The *first* thesis is that an inverse[5] form of NP is not "quite messy" (as Lewis thought (1994: 489)), but it is the simple CP (formulated by van Fraassen (1989)).[6] The *second* thesis is that PP and CP are special cases of GP. Since CP entails NP (first thesis), NP is a special case of GP as well. This allows Vranas to argue that PP and NP are compatible rather than rivals. The *third* thesis is that we should discard OP, while maintaining PP. Vranas thus claims that Lewis overreacted when stressing the problems of PP. Even if PP cannot be applied to cases of undermining, it can be applied to ordinary cases.

In this paper I will raise three objections: 1. According to Vranas, Lewis was wrong to believe that PP had to be discarded once NP was introduced. In fact, PP and NP are

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[5] "(A chance-credence principle) P_1 is an *inverse form* of P_2 exactly if P_1 entails P_2 and the standard formulation of P_2, unlike that of P_1, gives a credence conditional on the chances of all propositions." (p.1)

[6] This is a very nice result of Vranas. In fact, NP says that in order to assess the credence of a proposition A, one should know the chance of *all* propositions. Therefore NP is not only far less intuitive than PP, but it is also impossible to apply. Instead CP, the inverse form of NP, gives a credence conditional on the chance of the proposition $(B \wedge <Ch(A|B) = x>)$.

compatible. Yet, in the paper no effort is made to define the notion of compatibility, which renders Vranas' whole argument questionable. 2. The definition of admissibility offered in the paper, and not the principle GP, is the real "general unifier" from which all chance-credence principles can be derived. 3. In the proof that GP entails both CP and PP, Vranas uses, without argument, a tautology as an admissible proposition E. This is problematic as a tautology reduces the whole idea of admissibility of E, with respect to A, to the irrelevance of E on the chance of A. The fact that the time (and the relation between propositions) determines which propositions are admissible and which are not is not taken into account. I will address these objections in the following three sections in turn.

II. What is meant by compatibility?

Vranas proves that GP entails both PP and CP, and that CP entails NP. From this result, he concludes that (contrary to what Lewis thought) NP and PP are compatible. The notion of compatibility, though, is not defined. This is a serious problem, since appealing to an intuitive notion of compatibility is not sufficient to support the claim that Lewis was mistaken about PP.

Vranas claims that NP and PP are compatible *because* they can be derived from the same principle (GP):

> My second main thesis is that both PP and CP, and thus both PP and NP, are special cases of GP. (…) PP and NP are thus compatible rather than competing. (p.2)

If one wants a definition of compatibility in terms of derivation from a more general principle, as Vranas understands it, one needs to come up with a logical interpretation of that notion. An obvious proposal is to say that PP and NP are compatible, if they are *logically consistent*, provided that GP itself is not self-contradictory. Following Vranas, I assume that GP is logically consistent. To show that PP and NP are logically consistent provides a first hint that PP and NP are compatible. Furthermore, it has to be shown that the assumptions made in the proof that PP and CP follow from GP and

that NP follows from CP are also logically consistent with each other.[7] It seems that these two conditions are indeed satisfied, but it is important to note that CP entails NP under a very strong assumption (acknowledged by Vranas himself), which is that ($H \land T_f$) is in dom f. So it is necessary to assume that ($H \land T_f$) is a proposition that has a chance!

To summarize, the lack of a formal definition of compatibility weakens Vranas' argument. It is certainly sensible to call two principles compatible because they are special cases of a common principle. Nevertheless I believe that, to make his claim, Vranas needs to undertake two further steps: the first is to provide us with a formal definition of compatibility, and the second is to show that the conditions I have spotted above are satisfied.

III. Is admissibility the real unifier?

Vranas proves two of his main claims (viz. the compatibility – whatever this means – of PP and NP, and that CP is the inverse form of NP) by showing how the respective principles are interconnected. The relations between chance-credence principles are explored in terms of logical derivations of one principle from another principle. A nice result is that a general principle (GP), which entails all the other principles, can be defined. As we have seen in the previous section, the fact that GP entails both PP and NP is the reason for PP and NP to be declared compatible.

Contrary to Vranas I argue that it is not GP, but the definition of admissibility offered in the paper that is the real unifier of all principles. To show that the definition of admissibility is more general than GP, let us recall that Vranas (footnote 5, p.3) gives the following definition of admissibility: "*E* is admissible with respect to $<Ch(A) = x>$ exactly if $Cr(A|E \land <Ch(A) = x>)$ should be x". Vranas acknowledges that such a definition "does not provide a substantive account because it does not say for *which* proposition $Cr(A|E \land <Ch(A) = x>)$ should be x" (*ibid.*). He also recognizes that "PP is just a consequence of" this definition of admissibility (*ibid.*). Now, if we replace the

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[7] Thanks to Dov Gabbay here.

proposition $<Ch(A) = x>$ by ($B \wedge <Ch(A|B) = x>$) in the definition of admissibility, we obtain that E is admissible with respect to ($B \wedge <Ch(A|B) = x>$) if and only if $Cr(A|B \wedge E \wedge <Ch(A|B) = x>)$ should be x. It thus follows that, not only PP, but GP *as well* is just a consequence of the definition of admissibility provided in the paper. Hence, we can modify the diagram drawn in the paper such that the definition of admissibility is the real unifier of all principles:
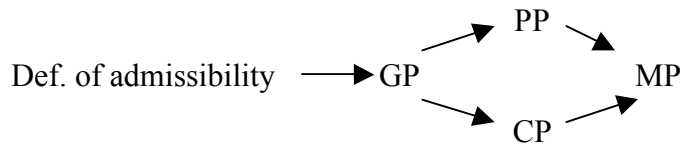
If this is correct, the logical relations between the chance-credence principles hold only by virtue of Vranas' definition of admissibility. The same applies to the supposed compatibility of PP and NP, which turns out to depend entirely on the notion of admissibility we use.

IV. Is a tautology admissible?

Let us now turn to my third and last objection to Vranas, namely the use of a tautology to prove that GP entails both CP and PP. In the proof, Vranas substitutes the same proposition I in GP twice: First, for the admissible proposition E to obtain CP, and then for B to obtain PP. Note that the proposition I is of a particular kind: It is a true proposition and it has chance one", i.e., I is a tautology, as Vranas' example on p.3 shows: "let I be the proposition that either 971 is a prime number or it is not". Let us now examine this issue more carefully.

To begin with, a tautology is *always* admissible. Vranas does not argue why he takes such a special case of an admissible proposition. Why doesn't he rather substitute a proposition that we know to be true (such as a proposition that states a fact about the past)? Note that both the credence and the chance of propositions of this kind are

one.[8] Vranas affirms that we need to know that the proposition I is true and our credence in it should be one. There are plenty of such (non-tautological) propositions, such as "yesterday I ate a tasty omelette". Why a tautology? To see that this is not a pedantic question, note that the chance and credence of a proposition, as well as whether or not a proposition is admissible, depend on time. As a tautology does not depend on time, it is always admissible. Therefore it is also always admissible with respect to any other proposition such as $<Ch(A) = x>$, and this observation renders the expression "with respect to" in the definition of admissibility meaningless.

The formal proof of Vranas' results is not disputed here. My concern is rather that substituting a tautology I for an admissible proposition E flattens the whole idea of an admissible proposition. The concept of admissibility is thus reduced to the lack of influence of E on the assessment of the chance of a proposition A. However, a proposition is admissible (or inadmissible) at a certain time, and it is admissible (or inadmissible) with respect to other propositions. All this is lost when we appeal to a tautology.

Moreover, I believe that a notion of admissibility makes sense when it can distinguish informative but irrelevant propositions from uninformative and irrelevant ones. To know that the last flip of a fair coin was heads is informative but irrelevant with respect to the next coin flip. The proposition that "29 is either a prime number or not" is neither informative nor relevant. Hence I suggest that a proposition is admissible if it says something related to A, but which turns out to have no effect on the assessment of the chance of A, and therefore (through some chance-credence principle) on the credence of A. Yet, I see that such an understanding of admissibility can be hardly captured by a formal definition like the one required in Vranas' paper.

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[8] To avoid Ned Hall's (1994) objections, we can also assume that no crystal ball exists. In fact, Hall noticed that the past can also carry information about the future, provided that well working crystal balls belong to the furniture of the world.

V. Conclusion

Lewis has long stressed that admissibility is the central notion in chance-credence principles:

> The power of the PP depends entirely on how much is admissible. If nothing is admissible it is vacuous. If everything is admissible it is inconsistent. (Lewis 1986: 92)

The logical relations between the chance-credence principles proven by Vranas are appealing, but they are based on a formal definition of admissibility that is too weak and too strong at the same time. It is too weak for it does not provide a substantive account of admissibility, i.e. it does not say which propositions do indeed satisfy the condition of admissibility. It is too strong for it appears to be the driving force behind the logical relations between chance-credence principles. Not only is PP just a consequence of the definition of admissibility but also, and more crucially, GP (from which all four other principles – viz. PP, CP, NP, and MP – can be derived) is a consequence of Vranas' definition of admissibility, as I have shown above (in sec. 3). Vranas' main result that NP and PP are compatible rather than competing is therefore doubtful. On the one hand, the notion of compatibility is left undefined (as I have pointed out in sec. 2). On the other hand, it still needs to be shown that the compatibility of the two principles does not depend on the specific definition of admissibility used by Vranas. Finally, in the last section, I have raised an objection against the use of a very special type of proposition (a tautology) in one of Vranas' proofs. I have shown that Vranas does not give any reason for this, and I have argued that this is (again) a consequence of the unrefined definition of admissibility he provides.

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