Philosophy 57 — Extra-Credit Problems

May 1, 2003 (due Tuesday, May 20, 2003)

1 Truth-Tables & PL Arguments

Use either full or "indirect" truth-tables (as described in §6.5) to determine whether the following PL-arguments are valid. Your answers should include *both* your truth-table reasoning *and* your verdict. If the argument is *in*valid, then all you need to report is a single row of the full truth-table (the one in which the premises are true and the conclusion is false). But, if the argument is *valid*, then you need to report either a *complete* truth-table, or an explanation of validity using the "indirect" truth-table method described in §6.5 of the text.

1. $$\begin{array}{l} I \supset N \\ (\sim K \vee D) \equiv N \\ D \supset \sim I \\ \hline \therefore \sim I \supset (N \supset K) \end{array}$$

2. $$\begin{array}{l} (\sim O \supset \sim S) \bullet (O \supset (M \bullet \sim I)) \\ \sim I \supset \sim M \\ \hline \therefore \sim S \end{array}$$

2 Knights, Knaves & Truth-Tables

The island of Knights and Knaves has two types of inhabitants, Knights who always tell the truth, and Knaves who always lie. Suppose A is the proposition person a is a knight and suppose a makes a statement S. Then, A is true if and only if S is true, since A is equivalent to S. That is, $A \approx S$. So, whenever an inhabitant x makes a claim S, we can infer that $X \equiv S$. That is, we can infer that x is a knight if and only if S is true. Here are some examples putting this to use:

If a says "I am a Knight" then we can infer from the statement that $A \equiv A$. But, since this is always true (it's a *tautology*), we get no information from such a statement. Similarly, it cannot be the case that a native says "I am a Knave" because we could then conclude $A \equiv \sim A$, which is always false (it's self-contradictory). If a says "I am the same type as b," then we can infer $A \equiv (A \equiv B)$ which is equivalent to B (that is, $B \approx A \equiv (A \equiv B)$, which you can prove using truth-tables). So, this statement allows us to infer that person b is a Knight!

Given this set-up, use truth-tables (either complete or "indirect") to justify your answers to the following three questions about Knights and Knaves. Explain your answers as fully as possible, and use truth-tables to establish the logical relationships (*e.g.*, equivalences) you use.

3. It is rumored that there is gold buried on the island (G). You ask one of the natives, a, whether there is gold on the island. He makes the following response: "There is gold on this island if and only if I am a Knight." Is there gold buried on the island?

4. Inhabitant a says "Either I am a Knave or b is a Knight." What can we infer about a and b?

5. Three of the inhabitants — a, b and c — were standing together in the garden. A stranger passed by and asked a, "Are you a Knight or a Knave?". a answered, but rather indistinctly, so the stranger could not make out what he said. The stranger then asked b, "What did a say?". b replied, "a said that he is a Knave." At this point the third man, c, said "Don't believe b; he's lying!". The question is, what are b and c?