Two Approaches to Belief Revision

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- Today, we'll sketch a new approach to (qualitative) belief revision based on *epistemic utility theory* (EUT) and contrast it with the traditional AGM theory of belief revision.
- The EUT approach involves a (normative) *Lockean thesis*. It's well known (lottery and preface paradoxes, *etc.* [9, 2]) that Lockean approaches to full belief fail to satisfy **Cogency**.

  Cogency. An agent's belief set **B** should (at any given time) be deductively consistent and closed under logic.

- Our main focus will be on divergences between EUT & AGM that are *orthogonal* to the classic debates about **Cogency**.
- That is, we will investigate the ways in which EUT and AGM diverge regarding diachronic constraints on *cogent* agents.
- The upshot will be that — as a constraint on cogent agents, and from an EUT perspective — AGM is *epistemically risk-seeking* (at least, in one sense). First, some setup.

- Our agents possess *both* numerical credence functions, $b(\cdot)$, *and* qualitative belief sets, **B**. When $p \in \mathbf{B}$, we write $\mathsf{B}(p)$. We're interested in (*non-reductive!*) *joint constraints* on $b/\mathbf{B}$
- Our agents revise *both* their b's *and* their **B**'s, upon learning (exactly) some proposition E. On the credence side:
  (1) $b(\cdot)$ is a classical (Kolmogorov) probability function.
  (2) given a *prior* $b(\cdot)$, the *posterior* $b'(\cdot)$ is generated *via conditionalizing* $b(\cdot)$ on E — *i.e.*, $b'(\cdot) = b(\cdot \mid E)$.[1]
- On the belief side, our agents entertain (classical, possible worlds) propositions on some finite *agenda* $\mathcal{A}$.
  (3) **B** is the set of members of $\mathcal{A}$ that our agent believes; and
  (4) given a *prior* belief set **B**, the *posterior* belief set **B**′ is generated by revising the prior by E — *i.e.*, $\mathbf{B}' = \mathbf{B} \star E$.

[1]Our results generalize beyond $b'(p) = b(p \mid E)$. Any "minimum distance" [4] Bayesian update (on E) satisfying (i) $b'(E) > b(E)$, (ii) $b'(E) > t$ (where t is the agent's EUT Lockean threshold), and (iii) $b(E \supset X) \geq b'(X)$ will suffice.

- The fundamental EUT principle [12, 15]: **B** should *maximize expected epistemic utility* — as calculated using $b(\cdot)$.
- We take a *veritistic* (*i.e.*, *accuracy-centered*) approach to epistemic utility according to which *the only feature of epistemic attitudes that matters is their accuracy*.
- More precisely, we will adopt the following naïve, accuracy-centered *epistemic utility function* for belief.

$$u(\mathsf{B}(p), w) \stackrel{\text{def}}{=} \begin{cases} \mathfrak{r} & \text{if } p \text{ is true at } w \\ -\mathfrak{w} & \text{if } p \text{ is false at } w \end{cases}$$

- The only constraint we will impose on $\mathfrak{r}$ and $\mathfrak{w}$ is

$$(\dagger) \qquad 1 \geq \mathfrak{w} > \left(\frac{1+\sqrt{5}}{2}\right) \cdot \mathfrak{r} > 0.^2$$

[2] We assume $\mathfrak{r} > 0$, and $\mathfrak{w} > \phi \cdot \mathfrak{r}$ (where ϕ is the *Golden Ratio*) since these assumptions imply threshold ranges for (*cogent*) EUT agents, which allow them (in some cases) to *violate* (some) AGM postulates (as we explain below).

- The *expected epistemic utility* (*EEU*) of a *belief* $\mathsf{B}(p)$, from the point of view of a credence function $b(\cdot)$, is given by

$$EEU(\mathsf{B}(p), b) \stackrel{\text{def}}{=} \sum_{w \in W} b(w) \cdot u(\mathsf{B}(p), w)$$

- The *overall EEU* of an agent's belief *set* $\mathbf{B}$, from the point of view of her credence function $b(\cdot)$ is defined as

$$EEU(\mathbf{B}, b) \stackrel{\text{def}}{=} \sum_{p \in \mathbf{B}} EEU(\mathsf{B}(p), b)$$

Theorem (Dorst [5], Easwaran [7]) A belief set $\mathbf{B}$ (on $\mathcal{A}$) *maximizes EEU relative to b* if and only if, for every $p \in \mathbf{B}$

$$b(p) > \frac{\mathfrak{w}}{\mathfrak{r} + \mathfrak{w}}.$$

☞ *MEEU* entails (normative) Lockeanism, with threshold $\frac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}$.[3]

[3] This explains (†), since (a) $\mathfrak{w} \leq \phi \cdot \mathfrak{r}$ permits $\mathsf{B}(p)$ when $b(p) \leq \phi - 1$, and (b) allowing $\mathfrak{r} = 0$ implies an EUT threshold of 1. See, also, *fn.* 2 and slide 14.

- We just explained how EUT implies a *synchronic* coherence constraint — specifically, a normative Lockean thesis.
- Since our agents are (Bayesian) *conditionalizers*, this immediately suggests a natural *diachronic* requirement.
- Our diachronic requirement will be that — upon learning E *via* conditionaliation — our agent beleives exactly those propositions that are sufficiently probable, *a posteriori*.

EUT Revision. If an agent with a prior belief set $\mathbf{B}$ learns (exactly) E, then her posterior $\mathbf{B}'$ should maximize *EEU* relative to her *conditional* credence function $b(\cdot \mid E)$.

- Formally, $\mathbf{B}' = \mathbf{B} \circledast E$, where

$$\mathbf{B} \circledast E \stackrel{\text{def}}{=} \left\{ p \;\middle|\; b'(p) = b(p \mid E) > \tfrac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}} \right\}.^{4}$$

[4] As we mentioned above (in *fn.* 1), our main results generalize to any "minimum distance" [4] Bayesian update (on E), subject to the following three constraints: (i) $b'(E) > b(E)$, (ii) $b'(E) > \frac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}$, and (iii) $b(E \supset X) \geq b'(X)$.

- To get a feel for how EUT Revision works, it is instructive to note that $\circledast$ does *not* generally satisfy the following (AGM) principle, which has recently been employed by Leitgeb [14].

(P2) If an agent learns something that she *already* believes, then her belief set should *remain unchanged*.

[More formally, $X \in \mathbf{B} \Rightarrow \mathbf{B}' = \mathbf{B} \star X = \mathbf{B}$.]

- Informally, the reason $\circledast$ violates (P2) is that — even if an agent already believes q — learning q can lower her credence in some other p proposition she also believes.
- Indeed, learning something one already believes (q) can drop one's credence in another proposition one also believes (p) *below one's EUT Lockean threshold.*
- However, there is a precise upper-bound on the "degree" to which (P2) can fail from the perspective of EUT.

- The following proposition provides a bound on *how much* an EUT agent's credence in one of her beliefs can be lowered by learning something else that she already believes.

Proposition. Suppose $b(p) > \frac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}$ and $b(q) > \frac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}$ (*i.e.*, that our EUT agent believes both p and q) and $1/2 < \frac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}} \leq 1$. Then, $b(p \mid q) > \frac{\mathfrak{w}-\mathfrak{r}}{\mathfrak{w}}$, and $b(p \mid q) - b(p) < \frac{\mathfrak{r}^2}{\mathfrak{r}\mathfrak{w}+\mathfrak{w}^2}$.

- And, in the limit as an agent's credences in p and q approach 1, (P2) *will* be satisfied by EUT revision.
- This goes some way toward explaining why (P2) may *seem* like a plausible diachronic constraint on full belief, since it is "approximately" true if full beliefs have sufficiently high credence (and it is *exactly* true in the extremal case).
- More generally, *extremal* EUT agents (*i.e.*, agents such that $\mathfrak{w} = 1$ and $\mathfrak{r} = 0$, who would have Lockean thresholds of 1) will *always* satisfy *all* of the AGM constraints [10, 11].

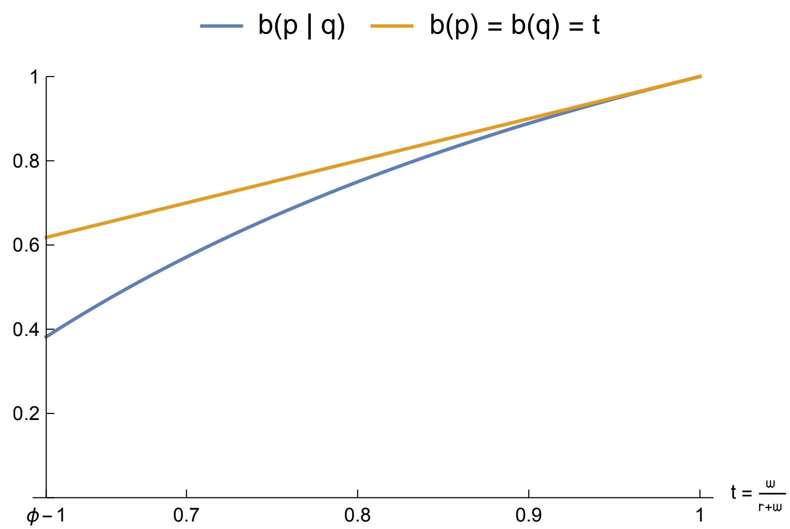


Figure: The “degree” to which EUT belief updating can violate Leitgeb’s (P2), as a function of the EUT Lockean threshold $t = \frac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}$.

- The AGM theory of belief revision (**A**lchourron, **G**ärdenfors & **M**akinson [1]) is the most widely investigated and influential account of qualitative belief revision.
- AGM’s underlying principle is the principle of *Conservativity* (also sometimes called the principle of *informational economy*, or *minimal mutilation*).

  > **Conservativity**. When an agent learns E, she should revise to a posterior belief set $\mathbf{B}'$ such that (a) $\mathbf{B}'$ *accommodates* E, (b) $\mathbf{B}'$ is *deductively cogent*, and (c) $\mathbf{B}'$ constitutes the *minimal change* to $\mathbf{B}$ which satisfies (a) and (b).

- A precise way to understand **Conservativity** is: $\mathbf{B}'$ should be such that (a) $E \in \mathbf{B}'$, (b) $\mathbf{B}'$ is deductively cogent, and (c) among those sets satisfying (a) and (b), $\mathbf{B}'$ is *closest to* $\mathbf{B}$.[5]
- AGM revision can be *axiomatized*. . .

[5] Here, distance between belief sets may be measured using Hamming distance, or any of a wide variety of other distance measures [13, 3, 6].

Basic AGM postulates:

(*1)	$\mathbf{B} * E = \mathsf{Cn}(\mathbf{B} * E)$	**Closure**
(*2)	$E \in \mathbf{B} * E$	**Success**
(*3)	$\mathbf{B} * E \subseteq \mathsf{Cn}(\mathbf{B} \cup \{E\})$	**Inclusion**
(*4)	If E is consistent with $\mathbf{B}$, then $\mathbf{B} * E \supseteq \mathsf{Cn}(\mathbf{B} \cup \{E\})$	**Vacuity**
(*5)	If E is not a contradiction, then $\mathbf{B} * E$ is consistent	**Consistency**
(*6)	If $X \Leftrightarrow Y$, then $\mathbf{B} * X = \mathbf{B} * Y$	**Extensionality**

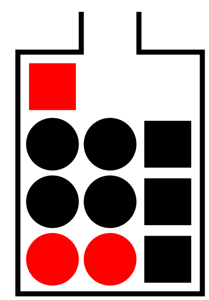
Supplementary AGM postulates:

(*7)	$\mathbf{B} * (X \wedge Y) \subseteq \mathsf{Cn}((\mathbf{B} * X) \cup \{Y\})$	**Superexpansion**
(*8)	If Y is consistent with $\mathsf{Cn}(\mathbf{B} * X)$, then $\mathbf{B} * (X \wedge Y) \supseteq \mathsf{Cn}((\mathbf{B} * X) \cup \{Y\})$	**Subexpansion**

(*4) If E is consistent with $\mathbf{B}$, then $\mathbf{B} \star E \supseteq \mathsf{Cn}(\mathbf{B} \cup \{E\})$ **Vacuity**

Proposition. $\divideontimes$ does *not* satisfy **Vacuity**.

Proof: Let $\mathfrak{w} = 0.17$ & $\mathfrak{r} = 0.03$ (*i.e.*, $t = 0.85$). Consider a simple *urn model*, where we will be sampling an object at random from the urn depicted on the right. Then let E and X be interpreted as follows:



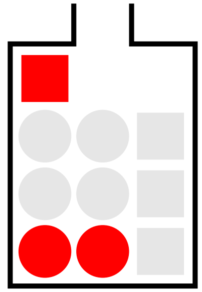
- $E \overset{\text{def}}{=}$ ‘The object sampled will be red’
- $X \overset{\text{def}}{=}$ ‘The object sampled will be a circle’

Note: $E \supset X$ is *the only* proposition with probability above 0.85. So, the rational Bayesian prior belief set is the *singleton*:

$$\mathbf{B} = \{E \supset X\}.$$

[*See Extras Slide 23 for the full probability distribution*]

Conditionalizing on E yields the "posterior urn" depicted on the right. Note: the proposition $E \supset X$ *drops below threshold*.



$$b(E \supset X \mid E) = {}^2/_3$$

Thus, when our Bayesian revises by E, she ends up with the following posterior:

$$\mathbf{B}' = \mathbf{B} \divideontimes E = \{E, E \vee X, E \vee \neg X\}.$$

So, we have the following facts in this case:

- Both $\mathbf{B}$ and $\mathbf{B}'$ are deductively cogent.
- E is consistent with $\mathbf{B} = \{E \supset X\}$.
- $X \in \mathsf{Cn}(\mathbf{B} \cup \{E\})$, since $\mathbf{B} = \{E \supset X\}$.
- But, $X \notin \mathbf{B} \divideontimes E$. □

- Counterexamples to **Vacuity** for (*cogent*) EUT agents are only possible for certain Lockean threshold (t) ranges.
- Examples of the kind we reported above (with 4 worlds) must have a Lockean threshold of *at least* $\frac{1}{\sqrt{2}} \approx 0.707$.
- Thus, 4-world EUT counterexamples to **Vacuity** can only exist for EUT agents who are such that: $\mathfrak{w} > (1 + \sqrt{2}) \cdot \mathfrak{r}$ (*i.e.*, $\mathfrak{w}$ must be greater than approximately 2.414 times $\mathfrak{r}$).
- There are also 3-world EUT counterexamples to **Vacuity**, and some of these have lower EUT thresholds than $\frac{1}{\sqrt{2}}$.
- But, we have established the following *lower bound*:
- **Theorem**. All (*cogent*) EUT agents with Lockean thresholds such that $t < \phi - 1 \approx 0.618$ *must satisfy* **Vacuity**.
- An immediate corollary of this theorem (and well-known results regarding *extremal* $t = 1$ agents [10, 11]) is:

☞ Cogent EUT agents *are* AGM agents — *unless* $t \in [\phi - 1, 1)$.

(*3) $\mathbf{B} \ast E \subseteq \mathsf{Cn}(\mathbf{B} \cup \{E\})$ **Inclusion**

- Intuitively, **Inclusion** requires that a revision does not include any *more* than the logical closure of the union of the original beliefs with the learned proposition.

  Proposition. $\mathbf{B} \divideontimes E \subseteq \mathsf{Cn}(\mathbf{B} \cup \{E\})$

  Proof: Suppose $X \in \mathbf{B} \divideontimes E$. Then, $b(X \mid E) > t$. And, it is a theorem of probability calculus that $\Pr(E \supset X) \geq \Pr(X \mid E)$. Therefore, $b(E \supset X) > t$. So, $E \supset X \in \mathbf{B}$. Hence, by *modus ponens* (for material implication), $X \in \mathsf{Cn}(\mathbf{B} \cup \{E\})$. □

- A similar argument shows that EUT revision satisfies the more general principle **Superexpansion**. Suppose $P \in \mathbf{B} \divideontimes (X \wedge Y)$. Then, $b(P \mid X \wedge Y) > t$. It is a theorem of probability calculus that $\Pr((X \wedge Y) \supset P) \geq \Pr(P \mid X \wedge Y)$. Therefore, $b((X \wedge Y) \supset P) > t$. So, $(X \wedge Y) \supset P \in \mathbf{B}$. So, by **Success** and *modus ponens*, $P \in \mathsf{Cn}((\mathbf{B} \divideontimes X) \cup \{Y\})$. □

- The crucial difference between AGM and EUT (aside, of course, from **Cogency**) is **Vacuity**/**Subexpansion**.
- Because EUT satisfies **Inclusion**, it will never require (cogent) agents to *gain more* new beliefs than AGM. But, EUT may require (cogent) agents to *lose* more beliefs than AGM.
- This feature (in conjunction with the fact that AGM *may* require an agent to gain more new beliefs than EUT) shows that EUT is (in a sense) *less demanding* of (cogent) agents.
- In this sense, AGM's diachronic requirements are *more epistemically risk-seeking* than EUT's are.[6]
- We close with a final theorem, which illuminates the tight connection between EUT's violations of **Vacuity** and its "risk aversion" (*vs* AGM, and as a constraint on cogent agents).

[6] Pettigrew [16] has independently argued (*via* the use of an epistemic *Hurwicz Criterion*) that **Cogency** implies its own variety of *risk-seeking*.

Theorem

EUT violates Vacuity (wrt $\mathbf{B}, E$) $\Leftrightarrow$ E is consistent with $\mathbf{B}$ and

$$\mathbf{B} \circledast E \subset \mathbf{B} * E.$$

Proof.

($\Rightarrow$) Suppose EUT violates Vacuity (wrt $\mathbf{B}$ and E). Then, (a) E is consistent with $\mathbf{B}$; and, (b) $\mathbf{B} \circledast E \not\supseteq \mathsf{Cn}(\mathbf{B} \cup \{E\})$. By (b), there exists an X such that $X \in \mathsf{Cn}(\mathbf{B} \cup \{E\})$ but $X \notin \mathbf{B} \circledast E$. It follows from (a), Vacuity and Inclusion that $\mathsf{Cn}(\mathbf{B} \cup \{E\}) = \mathbf{B} * E$. Therefore, $X \in \mathbf{B} * E$ and $X \notin \mathbf{B} \circledast E$. And, by Inclusion, $\mathbf{B} \circledast E \subseteq \mathsf{Cn}(\mathbf{B} \cup \{E\}) = \mathbf{B} * E$. □

($\Leftarrow$) Suppose E is consistent with $\mathbf{B}$ and $\mathbf{B} \circledast E \subset \mathbf{B} * E$. Then, there exists an X such that $X \in \mathbf{B} * E$ but $X \notin \mathbf{B} \circledast E$. Because E is consistent with $\mathbf{B}$, Vacuity and Inclusion imply that $\mathbf{B} * E = \mathsf{Cn}(\mathbf{B} \cup \{E\})$. Therefore, $X \in \mathsf{Cn}(\mathbf{B} \cup \{E\})$; but, $X \notin \mathbf{B} \circledast E$. □

- It would be useful to investigate (general) EUT Revision from a "non-classical probability" perspective (*e.g.*, Popper functions [10, 11], imprecise probability functions, *etc.*).
- It would be nice to have a purely qualitative characterization/axiomatization of EUT Revision. Ideally, we'd like to have one for arbitrary Lockean thresholds.
  - Jan van Eijck & Bryan Renne [8] recently provided a modal logic for belief given a Lockean threshold of 1/2. A near-term task is to investigate how their modal logic may be used to define a system of belief revision for a threshold of 1/2.
- Because we can state both EUT [4] and AGM [13] in terms of "minimal distance" revision, this yields a general "geodesic update" framework in which we can also define *contraction*.
  1. Let $b^\star$ be **the closest probability function** to b s.t. $b^\star(p) \leq \frac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}$
  2. $\mathbf{B} \div p \overset{\text{def}}{=} \left\{p \;\middle|\; b^\star(p) > \frac{\mathfrak{w}}{\mathfrak{r}+\mathfrak{w}}\right\}$
- We are working out the consequences of this definition...

[1] C. Alchourron, P. Gärdenfors, and D. Makinson. *On the logic of theory change: Partial meet contraction and revision functions*, 1985.

[2] D. Christensen, *Putting Logic in its Place*, 2007.

[3] M. Deza and E. Deza, *Encyclopedia of Distances*, 2009.

[4] P. Diaconis and S. Zabell, *Updating Subjective Probability*, 1982.

[5] K. Dorst, *Lockeans Maximize Expected Accuracy*, 2015.

[6] C. Duddy and A. Piggins, *A measure of distance between judgment sets*, 2012.

[7] K. Easwaran, *Dr. Truthlove or: How I Learned to Stop Worrying and Love Bayesian Probability*, 2015.

[8] J. van Eijck and B. Renne, *Belief as Willingness to Bet*, 2014.

[9] R. Foley, *Working Without a Net*, 1992.

[10] W. Harper, *Rational belief change, popper functions and counterfactuals*, 1975.

[11] J. Hawthorne, *A primer on rational consequence relations, popper functions, and their ranked structure*, 2013.

[12] C. Hempel, *Deductive-nomological vs. statistical explanation*, 1962.

[13] K. Georgatos, *Geodesic Revision*, 2008.

[14] H. Leitgeb, *The review paradox: On the diachronic costs of not closing rational belief under conjunction*, 2013.

[15] I. Levi, *Gambling with Truth*, 1967.

[16] R. Pettigrew, *Accuracy and Risk: a Jamesian investigation in formal epistemology*, 2015.

- Given the other AGM axioms, **Superexpansion** and **Subexpansion** imply **Inclusion** and **Vacuity**, respectively, assuming only the following weak additional postulate.
  - **Idempotence.** $\mathbf{B} * \top = \mathbf{B}$.

1. $Y \in \mathbf{B} * X$	Assumption
2. $\mathbf{B} * X = \mathbf{B} * (\top \wedge X)$	(1), **Extensionality**
3. $Y \in \mathbf{B} * (\top \wedge X)$	(1), (2), Logic
4. $Y \in \mathsf{Cn}((\mathbf{B} * \top) \cup \{X\})$	(3), Logic, **Superexpansion**
5. $\mathsf{Cn}((\mathbf{B} * \top) \cup \{X\}) = \mathsf{Cn}(\mathbf{B} \cup \{X\})$	**Idempotence**, Logic
6. $Y \in \mathsf{Cn}(\mathbf{B} \cup \{X\})$	(4), (5), Logic □

1. X is consistent with $\mathbf{B}$	Assumption
2. $Y \in \mathsf{Cn}(\mathbf{B} \cup \{X\})$	Assumption
3. $Y \in \mathsf{Cn}((\mathbf{B} * \top) \cup \{X\})$	(2), **Idempotence**, Logic
4. $Y \in \mathbf{B} * (\top \wedge X)$	(3), **Subexpansion**
5. $Y \in \mathbf{B} * X$	(4), **Extensionality** □

Alternative Axiomatization of AGM, using **Idempotence**.

(*1)	$\mathbf{B} * E = \mathsf{Cn}(\mathbf{B} * E)$	**Closure**
(*2)	$E \in \mathbf{B} * E$	**Success**
(*5)	If E is not a contradiction, then $\mathbf{B} * E$ is consistent	**Consistency**
(*6)	If $X \Leftrightarrow Y$, then $\mathbf{B} * X = \mathbf{B} * Y$	**Extensionality**
(*7)	$\mathbf{B} * (X \wedge Y) \subseteq \mathsf{Cn}((\mathbf{B} * X) \cup \{Y\})$	**Superexpansion**
(*8)	If Y is consistent with $\mathsf{Cn}(\mathbf{B} * X)$, then $\mathbf{B} * (X \wedge Y) \supseteq \mathsf{Cn}((\mathbf{B} * X) \cup \{Y\})$	**Subexpansion**
(*9)	$\mathbf{B} * \top = \mathbf{B}$	**Idempotence**

1. $\mathbf{B}$ is consistent.	Assumption
2. $\mathbf{B}$ is closed, *i.e.*, $\mathbf{B} = \mathsf{Cn}(\mathbf{B})$.	Assumption
3. $X \in \mathbf{B}$.	Assumption
4. X is consistent with $\mathbf{B}$.	(1), (3), Logic
5. $\mathbf{B} * X = \mathsf{Cn}(\mathbf{B} \cup \{X\})$.	(4), **Vacuity**, **Inclusion**
6. $\mathbf{B} * X = \mathsf{Cn}(\mathbf{B})$.	(5), (3), Logic
7. $\mathbf{B} * X = \mathsf{Cn}(\mathbf{B} * X)$	**Closure**
8. $\mathsf{Cn}(\mathbf{B} * X) = \mathsf{Cn}(\mathbf{B})$	(6), (7), Logic
9. $\mathbf{B} * X = \mathbf{B}$	(7), (8), (2), Logic $\square$

Figure: Derivation of (P2) from **Closure**, **Inclusion**, and **Vacuity**

p	$b(p)$	$b(p \mid E)$	$p \in \mathbf{B}$?	$p \in \mathbf{B} \circledast E$?	$p \in \mathbf{B} * E$?	$p \in \mathsf{Cn}(\mathbf{B} \cup \{E\})$?
$E \wedge X$	2/10	2/3	No	**No**	**Yes**	**Yes**
$E \wedge \neg X$	1/10	1/3	No	No	No	No
$\neg E \wedge X$	4/10	0	No	No	No	No
$\neg E \wedge \neg X$	3/10	0	No	No	No	No
E	3/10	1	No	Yes	Yes	Yes
X	6/10	2/3	No	**No**	**Yes**	**Yes**
$E \equiv X$	5/10	2/3	No	**No**	**Yes**	**Yes**
$E \equiv \neg X$	5/10	1/3	No	No	No	No
$\neg E$	7/10	0	No	No	No	No
$\neg X$	4/10	1/3	No	No	No	No
$E \vee X$	7/10	1	No	Yes	Yes	Yes
$E \vee \neg X$	6/10	1	No	Yes	Yes	Yes
$\neg E \vee X$	9/10	2/3	Yes	No	No	**Yes**
$\neg E \vee \neg X$	8/10	1/3	No	No	No	No

Table: Full counterexample to **Vacuity** for EUT Revision

- It is sometimes claimed by AGM-ers that they *only* need to assume deductive consistency as a *diachronic* requirement (as encoded in the **Consistency** axiom of AGM).
- This is incorrect — AGM-ers must (on pain of absurdity) assume consistency as a *standing, synchronic* requirement.
- To see why, consider the closed, but *in*consistent, belief set $\mathbf{B} = \{P, \neg P, \top, \bot\}$, where P is a contingent (atomic) claim.
- **Consistency** implies that $\mathbf{B} * \top$ must be consistent. Thus, according to AGM, if an agent starts out with the prior belief set $\mathbf{B}$ and then "revises by a tautology $\top$," they must abandon either their belief in P or their belief in $\neg P$ (since, otherwise, $\mathbf{B} * \top$ will *violate* **Consistency**).
- This means the AGM-er is faced with a dilemma: *either reject* **Idempotence** *or* assume consistency as a *universal* requirement. But, such a rejection of **Idempotence** would be absurd. $\therefore$ AGM-ers need *all* belief sets to be consistent.