Lecture 2

Applications in Epistemology

Motivation

- Lecture 1 introduced two paradigmatic models of partially reliable information sources, now it is time to apply them to philosophical questions.
- *Epistemology* is concerned with the question of how our beliefs can be justified. An answer to this question comes from the Cartesian Skeptic: There is no justification!
- In response, the *coherentist* points out that our system of beliefs about the world hangs together well. Different beliefs support each other nicely, and this is what justifies our whole system of beliefs.
- But: This kind of talk is rather vague! How can it be more precise? Probabilistic modeling.

Overview

I. Lecture 1: Bayesian Networks

1. Probability Theory
2. Bayesian Networks
3. Modeling Partially Reliable Information Sources

II. Lecture 2: Applications in Epistemology

1. Is Coherence Truth-Conducive?
2. How Can One Measure the Coherence of an Information Set?
3. Open Problems

III. Lecture 3: Applications in Philosophy of Science

1. Does the Variety-of-Evidence Thesis Hold?
2. What Is a Scientific Theory?
3. Open Problems

1. Is Coherence Truth-Conducive?

- According to the Coherence Theory of Justification, coherence is an indicator of truth.
- Or: The more coherent a set of information is, the higher its degree of confidence.
- My goals:

  - Make this claim more precise

  - Challenge it!

The Problem

When we receive information from independent and partially reliable sources, what is our degree of confidence that this information is true?

- Independence?
- Partial reliability?

Independence

REP_i

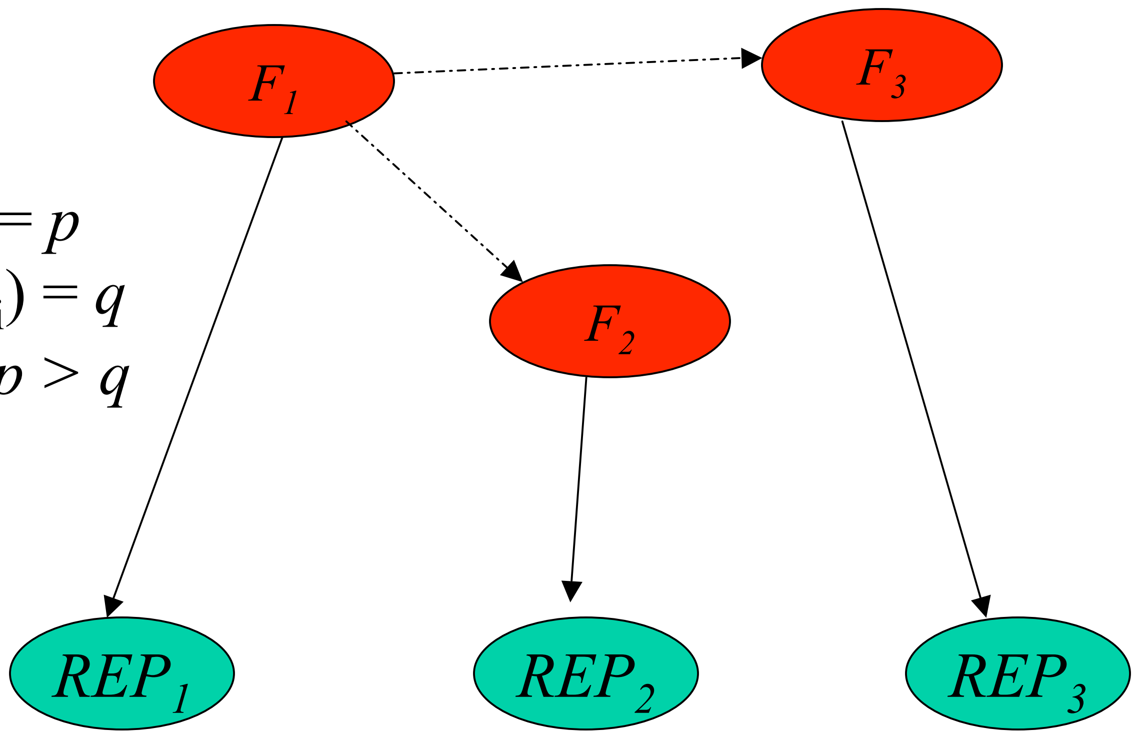
is independent of

$F_1, REP_1, F_{i-1}, REP_{i-1}, F_{i+1}, REP_{i+1}, F_n, REP_n$

given F_i

A Bayesian Network Model

$P(\mathrm{REP}_i|\mathrm{F}_i) = p$
$P(\mathrm{REP}_i|\neg \mathrm{F}_i) = q$
for $p > q$



Determinants of the Degree of Confidence

1. How *expected* is the information?

2. How *reliable* are the sources?

3. How *coherent* is the information?

Expectancy

- Example: Medical tests: Locus of gene
- There is some *prior knowledge* about the locus of the gene
- Compare two cases with two tests each
- Case 1: Overlapping area is expected
- Case 2: Overlapping area is not expected

- Case 1 has a higher degree of confidence.

Reliability

- Same setup
- Only difference:

  The tests in case 1 are more reliable than in case 2.

- Case 1 has a higher degree of confidence.

Coherence

- Same setup
- Only difference:

  Let the overlapping area be the same in cases 1 and 2, yet the non-overlapping area in case 2 is larger than the non-overlapping area in case 1.

- Case 1 has a higher degree of confidence.

A Conjecture

The three determinants are *separable*, i.e.

1. The more reliable the information sources are, the greater our degree of confidence, *ceteris paribus*.
2. The more plausible the information is, the greater our degree of confidence, *ceteris paribus*.
3. The more coherent the information is, the greater our degree of confidence, *ceteris paribus*.

My Goal

Challenge conjecture 3

(i.e. a central claim of the Coherence Theory of Justification)

Reliability

$P(\mathrm{REP}_i|\mathrm{F}_i) = p$ (for all $i = 1,\ldots, n$)

$P(\mathrm{REP}_i|\neg\mathrm{F}_i) = q$ with $p > q$

$r := 1 - q/p$

Randomization Full Reliability

0---1

Expectancy and Coherence

a_i with i counting the # of negative values

$a_0 = .05$

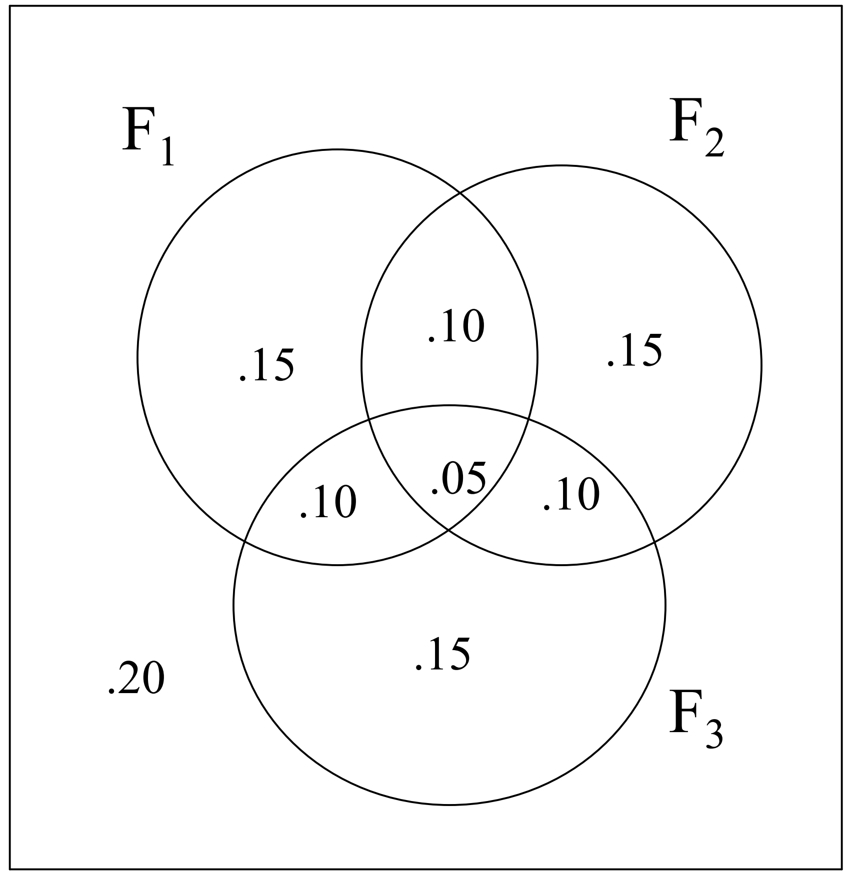
$a_1 = 3 \times .10 = .30$

$a_2 = 3 \times .15 = .45$

$a_3 = .20$

Expectancy □ a_0

Coherence □ $f(<a_1, a_2, a_3>)$



Degree of Confidence

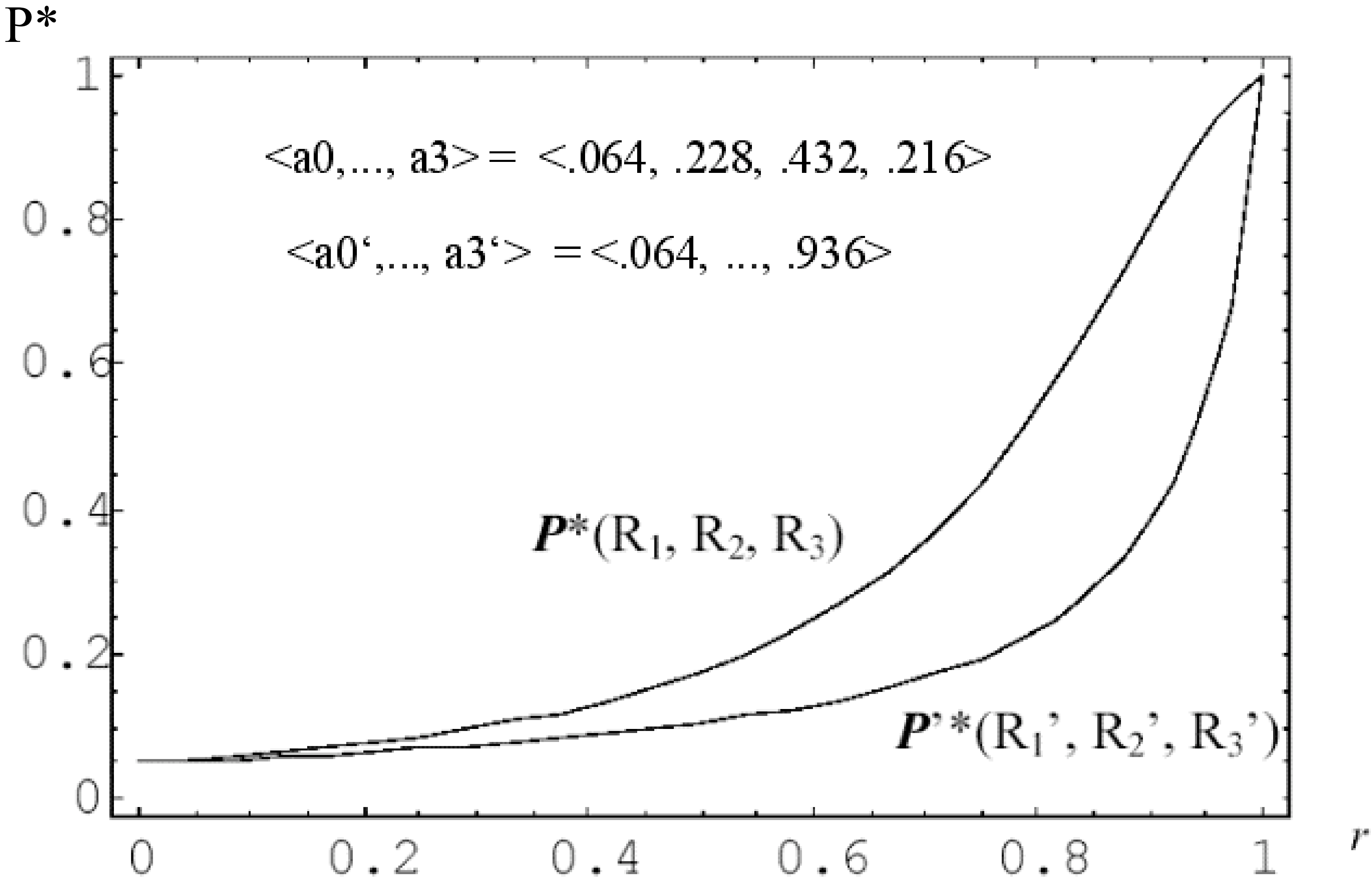
$$P^* = P^*(\mathrm{F}_1,\ldots,\mathrm{F}_n) = P(\mathrm{F}_1,\ldots,\mathrm{F}_n|\mathrm{REP}_1,\ldots,\mathrm{REP}_n) =$$

$$\frac{a_0}{\sum_{i=0}^{n} a_i (1 - r)^i}$$

An Example for $n = 3$

F_1 = The culprit was wearing Coco Chanel shoes.
F_2 = The culprit had a French accent.
F_3 = The culprit drove a Renault.

F_1' = The culprit was a woman.
F_2' = The culprit had a Flemish accent.
F_3' = The culprit drove a Ford.

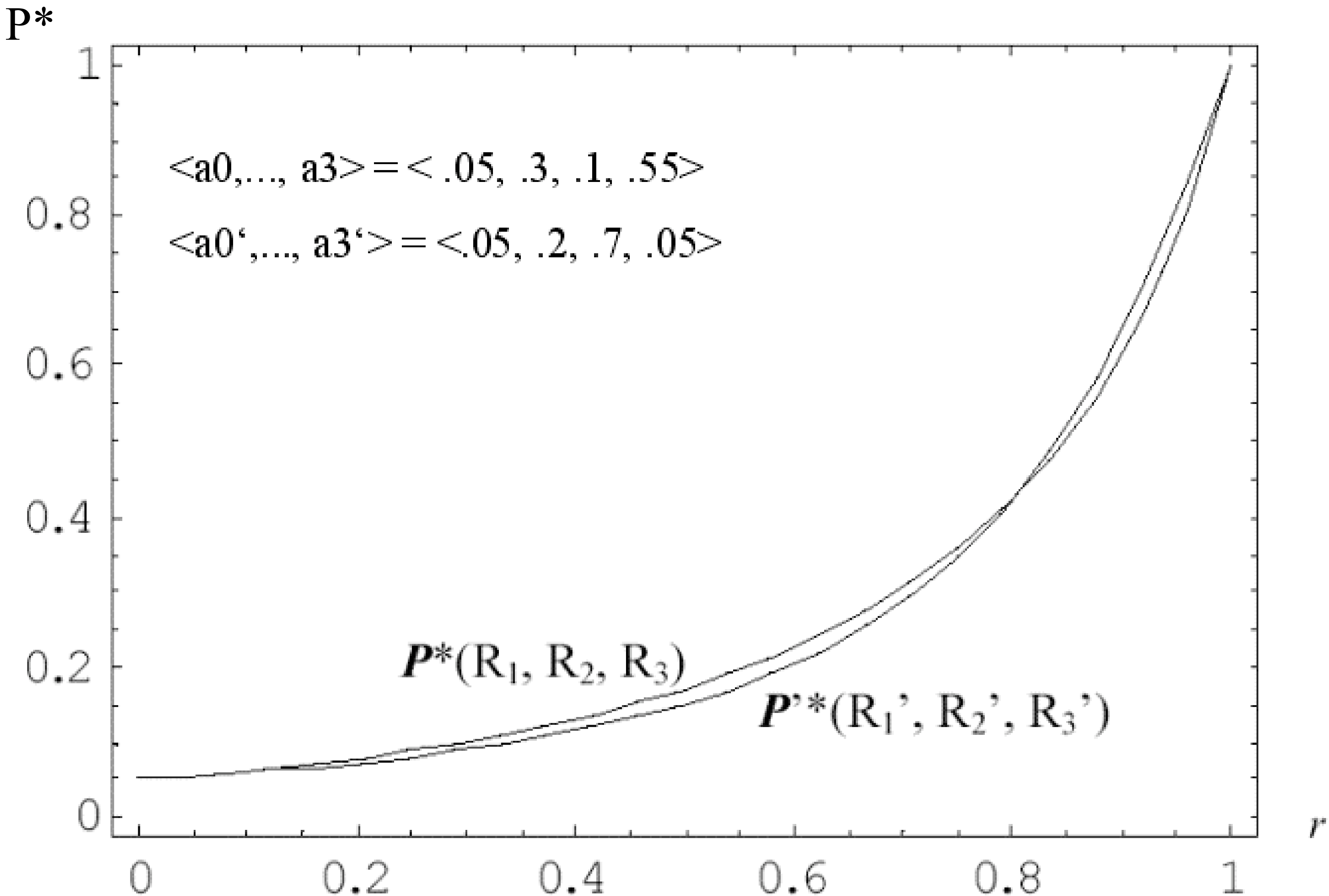

P*
<a0,..., a3> = <.064, .228, .432, .216>
<a0‘,..., a3‘> = <.064, ..., .936>
P*(R1, R2, R3)
P’*(R1’, R2’, R3’)
0
0.2
0.4
0.6
0.8
1
r

Note: One counter example is enough to show that conjecture 3 is false.

Assumptions:

(1) There are no further determinants.

(2) The coherence only depends on $<a_0,\ldots, a_n>$.


P*
<a0,…, a3> = < .05, .3, .1, .55>
<a0‘,..., a3‘> = <.05, .2, .7, .05>
$\boldsymbol{P}^*(R_1, R_2, R_3)$
$\boldsymbol{P}'^*(R_1', R_2', R_3')$
r
0
0.2
0.4
0.6
0.8
1

Separability Challenged

Separability: The more coherent the information is, the greater our degree of confidence, *ceteris paribus*.

(1) Suppose that S is more coherent than S':
Separability fails when we set a_0 = .05 and r = .90.

(2) Suppose that S' is more coherent than S:
Separability fails when we set a_0 = .05 and r = .10.
 Separability fails

An Example

- F_1 = The culprit is a woman.
- F_2 = The culprit has a Flemish accent.
- F_3 = The culprit drove a Ford.

- F_1‘ = The culprit wore Coco Chanel shows.
- F_2‘ = The culprit has a French accent.
- F_3‘ = The culprit drove a **Ford**.

What to Do?

Do we have to give up separability? This would have severe consequences for the Coherence Theory of Justification.

Check all assumptions we made!

Four *prima facie* plausible assumptions characterize *Bayesian Coherentism*...

(i) Separability

The more coherent the information is, the greater our degree of confidence, *ceteris paribus*.

(ii) Probabilism

The coherence of new information items is a function of probabilistic features of the information items.

(iii) Holism

The relation ‘is more coherent than’ is defined over information sets of size $n \geq 2$.

(iv) Ordering

The relation ‘is more coherent than’ is an ordering, i.e. it is reflexive, transitive and complete.

Which Assumption Should be Given Up?

- *Separability*: Drop the Coherence Theory of Justification.
- *Probabilism*: Drop the Bayesian Coherence Theory of Justification.
- *Holism*: Lehrer > Bonjour
- *Ordering*: □ Quasi-Ordering

3. How Can one Measure the Coherence of an Information Set?

Goal of this section: Construct a measure that induces a quasi-ordering of information sets.

Case I

- S = {[the culprit was a woman], [the culprit had a Flemish accent], [the culprit drove a Ford]}

- S’ = {[the culprit was wearing Coco Chanel shoes], [the culprit had a French accent], [the culprit drove a Renault]}.

- S” = {[the culprit was wearing Coco Chanel shoes], [the culprit had a French accent], [the culprit drove a Ford]}.

□ S’ > S □ S” > S and S > S”

Case II

- B’ = {[all ravens are black], [this bird is a raven], [this bird is black]}
- B = {[this chair is brown],[electrons are negatively charged], [today is Thursday]}

 B’ > B

Case III

- T' = {[Tweety is a bird], [Tweety cannot fly], [Tweety is a penguin]}
- T = {[Tweety is a bird], [Tweety cannot fly]}

□ T' > T

Lewis's Proposal

$\{R_1,\ldots, R_{i-1}, R_{i+1},\ldots, R_n\}$ is coherent (or congruent)

iff

$\boldsymbol{P}(R_i|R_1,\ldots, R_{i-1}, R_{i+1},\ldots, R_n) > \boldsymbol{P}(R_i)$ for all $i = 1,\ldots, n$.

Olsson's Proposal

$\{R_1, \ldots, R_m\} > \{R_1', \ldots, R_n'\}$

iff

$$\frac{\boldsymbol{P}(R_1, \ldots, R_m)}{\boldsymbol{P}(R_1 \quad \ldots \quad R_m)} \geq \frac{\boldsymbol{P}(R_1', \ldots, R_n')}{\boldsymbol{P}(R_1' \quad \ldots \quad R_n')}$$

Shogenji's Proposal

$\{R_1, \ldots, R_m\} > \{R_1', \ldots, R_n'\}$

iff

$$\frac{\boldsymbol{P}(R_1, \ldots, R_m)}{\prod_{i=1}^{n} \boldsymbol{P}(R_i)} \geq \frac{\boldsymbol{P}(R_1', \ldots, R_n')}{\prod_{i=1}^{n} \boldsymbol{P}(R_i')}$$

Fitelson's Proposal

K-O Measure of Confirmation:

$$F(R_1,R_2) = \frac{P(R_1 \mid R_2) - P(R_1 \mid \neg R_2)}{P(R_1 \mid R_2) + P(R_1 \mid \neg R_2)}$$

$\{R_1, R_2\} > \{R_1, R_2'\}$

iff

$$\frac{F(R_1,R_2) + F(R_2,R_1)}{2} \geq \frac{F(R_1',R_2') + F(R_2',R_1')}{2}$$

Moral

Intuitive proposals give us no more than a partial elucidation of certain aspects of coherence (positive relevance, overlap,…) but do not lead to a unitary account.

We need a measure that that weights these factors in an appropriate way, and there is no principled way to do this.

Way out: Think about the *function* of coherence.

Coherence of What?

Ant hills ~ fitness

Law firms ~ productivity

Families ~ happiness

Coherence of … is the property of … which increases ___ and is the neighborhood of our pre-theoretical notion of coherence.

Coherence of Information Sets

The coherence of an information set $\{R_1,\ldots, R_n\}$ is the property of this information set which increases the confidence boost that results from being informed by independent of partially reliable sources that respectively $R_1, \ldots, R_n$ are the case.

Tentative Proposal

We measure the coherence by the degree-of-confidence boost, i.e. the ratio of the prior over the posterior probability

$$c_r^{tent}(\mathrm{S}) = \frac{\boldsymbol{P}^*(\mathrm{F}_1,\ldots,\mathrm{F}_\mathrm{n})}{\boldsymbol{P}(\mathrm{F}_1,\ldots,\mathrm{F}_\mathrm{n})}$$

Problem #1

- Let S contain highly coherent information with $\boldsymbol{P}(F_1, F_2) \square 1$ and let S' contain highly incoherent information with $\boldsymbol{P'}(F_1', F_2') \square 0$

- Then for any value of r, $c_r^{tent}(S') > c_r^{tent}(S)$

Solution: Normalization

- We measure the coherence by the degree of confidence boost that actually obtained over the degree of confidence boost that would have obtained, had the same information been presented as maximally coherent information.

The Measure

$$c_r(\Sigma) = \frac{\frac{P^*(\mathrm{F}_1,\ldots,\mathrm{F}_n)}{P(\mathrm{F}_1,\ldots,\mathrm{F}_n)}}{\frac{P^{max*}(\mathrm{F}_1,\ldots,\mathrm{F}_n)}{P^{max}(\mathrm{F}_1,\ldots,\mathrm{F}_n)}}$$

$$= \frac{a_0 + (1-a_0)(1-r)^n}{\sum_{i=1}^{n} a_i (1-r)^i}$$

Problem #2

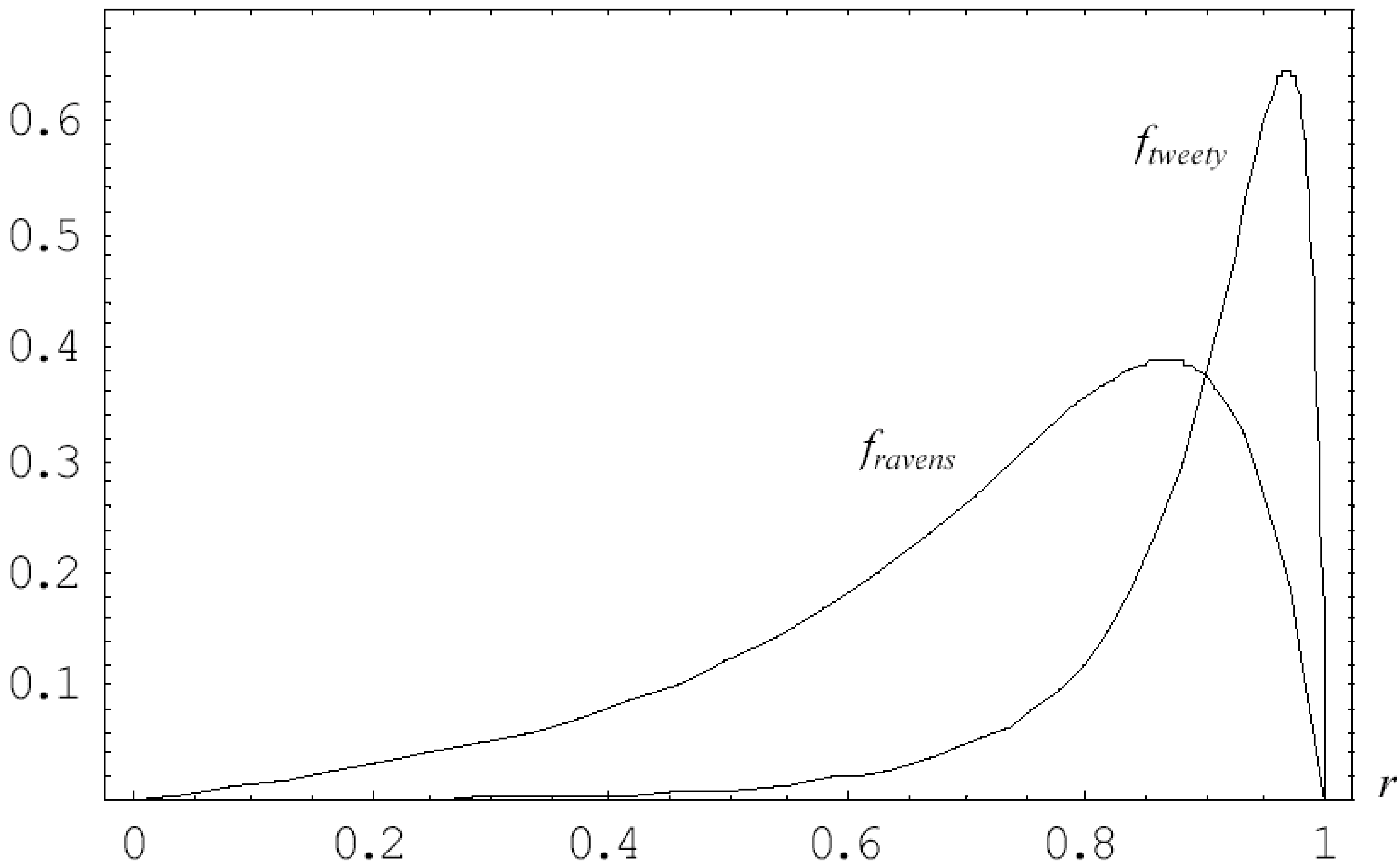
A coherence measure should not be dependent on the reliability of the informants r.

Solution

S > S' iff

$c_r(\mathrm{S}) > c_r(\mathrm{S'})$ for all $r \in (0, 1)$

Construct $f_r(\mathrm{S}, \mathrm{S'}) := c_r(\mathrm{S}) - c_r(\mathrm{S'})$


0.6
0.5
0.4
0.3
0.2
0.1
0
0.2
0.4
0.6
0.8
1
r
f tweety
f ravens

Taking Stock

- We proved an impossibility theorem and discussed various ways out.
- We suggest to give up that there is a coherence ordering over information sets.
- We proposed a measure for the coherence of an information set that induces a partial ordering.
- The measure does not obtain from intuitive ideas about the nature of coherence, but about its function.

3. Open Problems

1. Come up with alternative coherence measures.
2. Testimony: Various independent witnesses report the same unlikely event.
3. Systematically explore dependencies between reports: how do they affect the degree of confidence?
4. Belief revision: Represent an information set by a Bayesian Network; new and possibly conflicting information comes in. What shall one do?

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