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Long-run patterns in the discovery of the adjacent possible

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The notion of the "adjacent possible" has been advanced to theorize the generation of novelty across many different research domains. This study is an attempt to examine in what way the notion can be made empirically useful for innovation studies. A new theoretical framework is construed based on the notion of innovation as a search process where knowledge is recombined to discover the adjacent possible. The framework makes testable predictions about the rate of innovation, the distribution of innovations across organizations, and the rate of diversification of product portfolios. The empirical section examines how well this framework predicts long-run patterns of new product introductions in Sweden, 1908–2016, and explores the long-run evolution of the product space of Swedish organizations. The results suggest that, remarkably, the rate of innovation depends linearly on cumulative innovations, which explains advantages of incumbent firms but excludes the emergence of winner-take-all distributions. The results also suggest that the rate of development of new types of products follows Heaps' law, where the share of new product types within organizations declines over time. Finally, the study also demonstrates that the structure of the Swedish product space carries important information about adjacent possible innovations.

JEL Classification: O31, N84, C02, D83, L25

1. Introduction

It has long been recognized that the emergence of novelty is a core aspect of evolutionary processes, propelling language, ecological, social, technological, and economic systems toward new configurations. A branch of studies has attempted to theorize the emergence of novelty across these different fields and, with some success, investigate the presence of common general patterns and mechanisms (e.g. Tria *et al.*, 2014; Loreto *et al.*, 2016; Marengo and Zeppini, 2016; Iacopini *et al.*, 2018; Tacchella *et al.*, 2020; Ubaldi *et al.*, 2021).

Up until recently, most cross-field exchange between evolutionary biology and innovation studies or evolutionary economics (Nelson and Winter, 1982; Klepper, 1997) has been centered on variety and selection, evolution as adaptive hill climbing on fitness landscapes (Kauffman, 1993; Levinthal, 1997), or evolution as a form of "tinkering" or trial and error process (Jacob, 1977; Wagner and Rosen, 2012). While broadly useful, the focus on adaptive search is restrictive if the goal is to explain the emergence of novelty (Felin *et al.*, 2014; Felin and Kauffman, 2023). In the economics of innovation, it has long been recognized that innovations are new (re-)combinations of previous innovations, that come into economic use (Schumpeter, 1911; compare OECD, 2005). Hence, the innovation process is usually conceptualized as a search process to find better combinations (Weitzman, 1998; Fleming and Sorenson, 2001; Fleming and Sorenson, 2004; Arthur, 2007; Bergh, 2008; Arthur, 2009; Zeppini and Bergh, 2011). Furthering such intuitions,

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the concept of the "adjacent possible" was introduced by Stuart Kauffman (2000a) to explain the emergence of novelty in complex adaptive systems such as the biosphere, where life creates new niches and opportunities into which it expands. The notion of an adjacent possible divides the space of innovations into three conceptual categories:

1. those that have been discovered,
2. those that can currently be discovered from (recombining) those that have already been discovered, and
3. innovations that are, as it were, "out of reach," but may become possible to discover in the future.

This framework is simple but powerful. Recently, several studies have drawn on this notion to examine how novelties give rise to other novelties, suggesting the existence of statistical laws for the rate of novelty generation in a broad set of phenomena (Tria *et al.*, 2014; Loreto *et al.*, 2016; Marengo and Zeppini, 2016; Monechi *et al.*, 2017; Iacopini *et al.*, 2018; Ubaldi *et al.*, 2021). Specifically, the so-called urn models of Tria *et al.* (2014) and Loreto *et al.* (2016) model the generation of novelty by assuming that novelty can be represented by balls with different colors in an urn, showing how novelties (new colors) can trigger other novelties through a simple reinforcement mechanism, i.e., that a drawn ball with a certain color will increase the probability that it is drawn again. These models predict statistical laws for the rate at that novelties happen, known as Heaps' law (Heaps, 1978), and the frequency distribution of different types of novelties (colors drawn), known as Zipf's law. Other models have explored novelty generation on social and innovation networks (Iacopini *et al.*, 2018; Ubaldi *et al.*, 2021). These studies showcase a pathway to better understand patterns in the emergence of novelties across a broad set of systems.

Recent studies have argued that one may model the adjacent possible on the basis of the number of recombinations that are possible to make from available knowledge (Steel *et al.*, 2010; Cortês *et al.*, 2022; Koppl *et al.*, 2023). This model, called the theory of the adjacent possible (TAP), strongly suggests that innovation is "super-exponential" in the long run, that is, grows faster than exponential curves over time. This suggestion aligns with the "hockey-stick" shape of long-run economic growth (Steel *et al.*, 2010; Koppl *et al.*, 2023).

There are, however, important challenges in operationalizing the concept of the adjacent possible.

First, as noted recently, the urn model (Tria *et al.*, 2014; Loreto *et al.*, 2016) is restricted as there is no "cross-talk" between product types, or "elements" (Kauffman, 2019, p. 139).[1] There is, in other words, a challenge in reuniting the predictions of Tria *et al.* (2014), with several other observations from theory, namely that innovation is the result of recombinant search. While the TAP framework is based on recombinant growth, it has not been shown to reproduce Heaps' law, or other statistical regularities as regards the emergence of novelty.

Second, in the context of long-run innovation dynamics, the recombinant perspective leads to a puzzle. If the adjacent possible of organizations grows faster than exponentially (Steel *et al.*, 2010; Solè *et al.*, 2016; Cortês *et al.*, 2022) and organizations innovate at a rate governed by the adjacent possible, it is easy to see that the long-run industrial dynamics should give way to a winner-take-all phenomenon (for a formal proof of the link between super-linear attachment kernels and winner-take-all distributions, see Krapivsky *et al.*, 2001). This does not align with various studies that have argued that long-run innovation is subject to natural or physical constraints (Ayres, 1994), resource constraints (Weitzman, 1998), or towering complexity (Strumsky *et al.*, 2010; Arnold *et al.*, 2019; Bloom *et al.*, 2020). This suggests that the TAP framework makes overly strong predictions for certain applications and that the probing of the adjacent possible has considerable constraints, as suggested by Weitzman (1998), for example, in terms of limited capability and resources available for search.

[1] The full quote: "The model is lovely, but does not yet answer our needs, for it is one of a branching set of independent lineages of descendant colored balls. A red ball gives rise to an orange ball, which gives rise to a blue ball. There is no cross-talk between lineages augmenting the combinatorial formation of new colors as there is in the economic web's evolution with new complements and substitutes arising from old ones by new jury-rigged combinations of one or several prior goods. I hope that a good model or set of models can be constructed" (Kauffman, 2019, p. 139).

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Third, a fundamental challenge is how to operationalize the adjacent possible, seeing as it may be fundamentally "unprestatable" (Kauffman, 2019), or, in other words, it may be impossible to predict in any detail what novelties will crop up. Relatedly, there is no way of directly estimating the size of the adjacent possible at a point in time, unless all of the possibilities in the adjacent possible were in fact discovered.

This study takes up the challenge of how to theorize and operationalize the adjacent possible by addressing these crucial challenges. The aim of this study is threefold. Most importantly, this study aims to accomplish a unification of the concepts of recombinant search, the limitations introduced by Weitzman (1998), and the notion of the adjacent possible, to make testable predictions about the long-run rate of innovation, the distribution across firms, and the rate of diversification of firms.

Second, I examine whether this framework is useful in explaining empirical patterns of innovation, leveraging historical data on new product introductions in the Swedish engineering industry during the period of 1908–2016 (Kander *et al.*, 2019; Taalbi, 2021). This data is used to test predictions on long-run innovation, including the rate of innovation, distribution across organizations, and patterns of diversification.

As discussed above, a key question for the TAP is also whether novelty generation can be predicted or is fundamentally "unprestatable," that is impossible to predict in detail. For this reason, this study also analyzes the product space of co-occurrence of products within Swedish organizations in order to discuss whether the structure of product space has information about the firms' adjacent possible innovations.

The rest of the study is organized as follows. In Section 2, a theoretical framework is proposed that unites the notion of the adjacent possible with innovation as recombination (Weitzman, 1998; Kauffman, 2000a; Arthur, 2009). The model produces testable predictions as regards the rate of innovation over time and across organizations and the diversification of product portfolios, as also suggested by (Tria *et al.*, 2014). Section 3 introduces the data on Swedish innovation output and Section 4 analyzes the rate of innovation, product diversification, and product space for Swedish organizations for the period 1908–2016. Section 5 concludes.

2. A theoretical framework

2.1 Main framework

To build a framework, I depart from the following basic considerations. Previous work has modeled the generation of novelty by representing novel entities as balls with different colors in an urn. Innovators discover novelties by drawing balls from the urn, with essentially two types of outcomes of a draw. Either a new color is discovered, or the draw results in a copy of a previously known color (Tria *et al.*, 2014; Loreto *et al.*, 2016).

The basic intuition of this study is very similar. The core assumption of this work is that new products developed by organizations embody different types of knowledge in a process of "learning by innovation" (Geroski and Mazzucato, 2002). Organizations have a varying number of *product types* in their repertoire, that is what is usually understood in terms of product classes, e.g. machinery equipment, software, computers, or plastics. From the organization's point of view, making new product types expands the organization's knowledge base to new fields (March, 1991; Geroski and Mazzucato, 2002; Katila and Ahuja, 2002). Hence, new product types are thought of as "new colors." Each organization also has a history of products made in the respective fields, representing improvements in knowledge *within* those fields, henceforth referred to as *product improvements*. This category can be thought of as "copies" of an already drawn color.

The number of product types in an organization's portfolio can be referred to as its *product diversity* (see e.g. Gort, 1962; Varadarajan, 1986), here denoted as D. To this comes the number of product improvements. The total cumulative number of innovations, i.e. new product types, and improvements, is denoted as k, embodying the cumulated knowledge base of the organization at some point in time t. Together the set of "product types" (colors) and improvements (copies) in the firm's portfolio defines the set of drawn elements $\mathcal{S}$.

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Now, with these basic definitions, the framework should be able to deliver predictions about three aspects of long-run innovation dynamics:

- The contribution of cumulative innovations k to the organization's rate of innovation dk/dt
- The rate of diversification of the product portfolio dD/dk, defined as the rate of discovery of new elements (dD/dt) as compared to the total rate of innovation (dk/dt) (cf. Tria *et al.*, 2014)
- The relative frequency of innovations across organizations, $P(k)$, i.e., the fraction of organizations that have made k innovations at some point in time

To make these predictions, this study makes use of the notion of the adjacent possible as discussed previously. For each organization, there is a set of product types, or "new colors," $\mathcal{U}_{NC}$, not yet produced but possible to produce given the current knowledge base. There is also a set of product improvements, or "copies," $\mathcal{U}_C$, not yet discovered, that an organization can develop given its current knowledge base. $\mathcal{U} = \mathcal{U}_{NC} \cup \mathcal{U}_C$ is the set of adjacent possible innovations.

The question is now only how, more precisely, the set of adjacent possible innovations $\mathcal{U}$ depends on the set of available knowledge of the organization, embodied in earlier innovations, the set of drawn elements $\mathcal{S}$. The key is *recombination*. To fix ideas, Figure 1 illustrates the interaction of the set of drawn elements $\mathcal{S}$ and the adjacent possible innovations $\mathcal{U}$ in an urn model where agents search by recombining from a subspace of drawn elements. The main sequence is (i) *Recombination:* recombination of already drawn elements $\mathcal{S}$ to discover products in the adjacent possible $\mathcal{U}$, (ii) *Expansion of search space:* in time $t+1$ the discovered element is added to the set of drawn elements that can be recombined, (iii) *Expansion of the adjacent possible:* in time $t+1$ the adjacent possible $\mathcal{U}$ expands with copies and new colors made possible by the addition of the newly discovered element. The details of this model are motivated in the rest of this section.

The view of innovation as a process of recombination naturally suggests that one should look at the set of all possible combinations, whose size is $\sum_k \alpha_L \frac{D!}{L!(D-L)!}$, where α_L is the relative frequency of an innovation among combinations of length L. This type of combinatorial model forms the basis of studies such as Weitzman (1998) and Koppl *et al.* (2023). However, organizations face well-known constraints to search and innovation that motivate a distinction between the universe of all adjacent possible innovations, and an *effective* adjacent possible that results from various types of constraints.

First, facing costs of integration, search complexity is limited to some maximal product length λ. If λ is small relative to D the number of possible combinations is well approximated by the number of combinations of the largest product length $\frac{D!}{\lambda!(D-\lambda)!}$.[2] It is also useful to note the well-known approximation of a combination D choose λ as $\frac{1}{\lambda!}D^{\lambda}$ for large D and small λ.

Second, the organizations' ability to recognize, assimilate, and apply new knowledge inputs, their "absorptive capacity" (Cohen and Levinthal, 1990), will determine the scope of search activities (cf. Katila and Ahuja, 2002). Hence, there is a more or less narrow 'window' of product types that are produced within an organization (for a similar argument, compare Dam and Frenken, 2020). Moreover, resource constraints or the "ability of the research facility to test or to process the materials" (Weitzman, 1998, p. 353) may imply that search takes place in a subset of possible knowledge recombinations. This set of recombinations is denoted $\mathcal{R}$. In other words, due to resource constraints, organizations do not necessarily search for combinations among *all* D knowledge types in their portfolio but rather search a smaller number of knowledge types $D^* \leq D$.

With these modifications, the relevant space of adjacent possible product types can be written as

$$|\mathcal{U}_{NC}| = \nu \frac{D^*!}{\lambda!\,(D^* - \lambda)!\,)} = \nu|\mathcal{R}| \tag{1}$$

[2] Consider the ratio of the number of combinations for the largest product length λ and the second largest $\lambda - 1$. It is straightforward to show that this ratio is equal to $\frac{D-\lambda+1}{\lambda}$. This means that the number of combinations for the leading product length is bigger than the second largest by a factor of approximately D/λ. Hence, for large D and small λ the total number of combinations will be dominated by the largest combination length λ.

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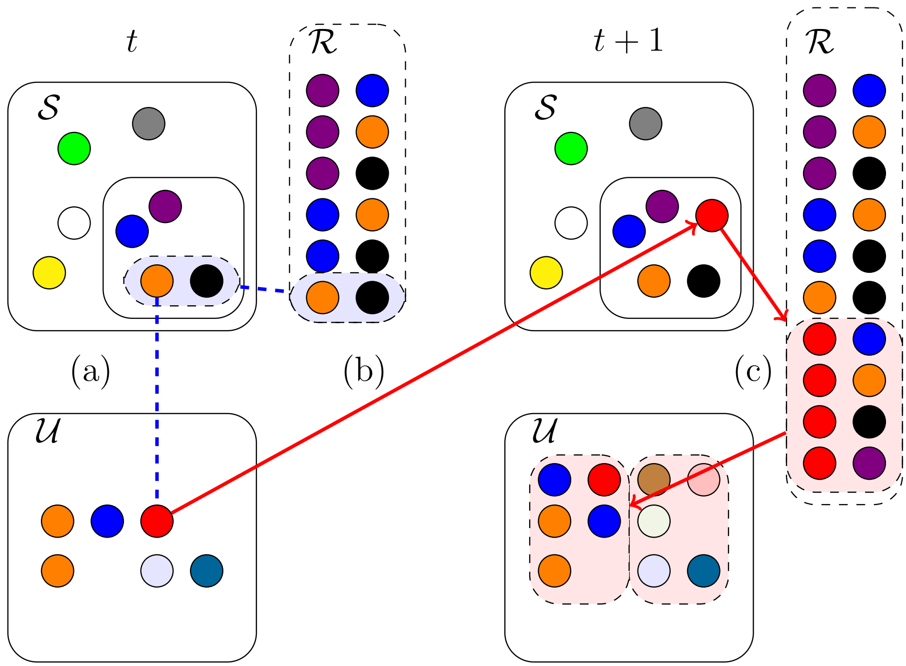

Figure 1. Urn model with recombination. An agent recombines elements to discover new products in the adjacent possible $\mathcal{U}$. (a) *Recombination:* In time t, an element (the red ball) is discovered by an agent by recombination of two of the available elements $\mathcal{S}$ (the black and orange balls) in a subspace that the agent focuses on. The possible recombinations of length two constitute the set $\mathcal{R}$. (b) *Expansion of search space:* At time $t+1$ the new element (the red ball) is added to the set of drawn elements $\mathcal{S}$. (c) *Expansion of the adjacent possible:* If the new element (the red ball) is included in the subspace that the agent searches, this expands the set of possible recombinations $\mathcal{R}$. A number of copies of previously drawn colors and previously unavailable colors are added to the adjacent possible $\mathcal{U}$. *Note:* The exact development of the system is governed by recombination length, here $\lambda = 2$, and the parameters ν and ρ. Here, $\nu = \rho = \frac{1}{2}$. The number of adjacent new colors is $\nu = \frac{1}{2}$ times the number of possible recombinations $|\mathcal{R}|$. The number of copies in the adjacent possible depends on $\rho = \frac{1}{2}$, the number of copies k drawn per element D and the number of possible recombinations $|\mathcal{R}|$ according to $\rho\frac{k}{D}|\mathcal{R}|$. In the above illustration k and D are both 8 at the outset. See the main text for further motivation.

where λ is the number of products that are recombined, and $|\mathcal{R}|$ is the number of recombinations of different knowledge types searched by an organization. $0 \leq \nu \leq 1$ is a parameter representing the fraction of recombinations that lead to the discovery of new elements.

To complete the framework, it is necessary to consider the fact that organizations also rely on their experience in given fields to produce *product improvements*, copies of a color. Both entirely new product types and product improvements may lead to product improvements in other fields. Every innovation made can be recombined with $D^* - 1$ elements within the search space and has a probability of D^*/D being among the product types that are actively searched. Every innovation made then contributes with $\frac{D^*}{D}\frac{(D^*-1)!}{(\lambda-1)!(D^*-1-(\lambda-1))!}$ new possibilities.[3] With k cumulative innovations the total number of adjacent possible improvements can be re-expressed as

$$|\mathcal{U}_C| = \mu\lambda\frac{k}{D}\frac{D^*!}{\lambda!\,(D^*-\lambda)!} = \mu\lambda\frac{k}{D}|\mathcal{R}| \tag{2}$$

[3] Say that a product improvement is made in product type A, and that the search space consists of types A, B, C, D, and E with search length 3. If A is in the search space it adds possible innovations to ABC, ABD, ABE, ACD, ACE, and ADE. The number of combinations added is equal to the number of combinations of length 2 ($\lambda - 1$) of the 4 other product types (B,C,D, and E) that A can combine with ($D^* - 1$): $4!/2!/2! = 6$.

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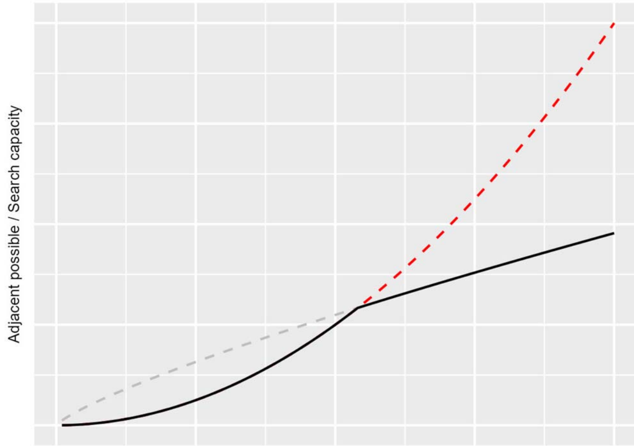

Figure 2. Stylized illustration of how the rate of innovation (solid line) is determined by the constraint of either the size of the adjacent possible (upper dashed line) that grows in a super-linear fashion, or resource constraints (the lower dashed line) that is sublinear or linear in the cumulative number of innovations (Weitzman, 1998).

where μ is a parameter expressing the probability of finding a new product improvement. For notational simplicity, the rest of this work replaces $\mu\lambda$ with the parameter $\rho \geq 0$, following similar notation in other works (see e.g. Tria *et al.*, 2014).

Combining equation (1) and equation (2), the total number of effectively adjacent possible innovations, that is improvements and new product types, is given by

$$|\mathcal{U}| = |\mathcal{U}_{\mathrm{NC}}| + |\mathcal{U}_{\mathrm{C}}| = \left(\nu + \rho\frac{k}{D}\right)|\mathcal{R}| \tag{3}$$

2.2 Long-run rate of innovation

A key assumption of the TAP is that the rate of introduction of novelties, here both new product types and product improvements, should be related to the size of the (effective) adjacent possible $\frac{dk}{dt} = |\mathcal{U}|$ (Cortês *et al.*, 2022; Koppl *et al.*, 2023).

Following Weitzman (1998) there are, however, two extreme cases depending on resources available, illustrated in Figure 2. The rate of innovation is initially limited by the size of the adjacent possible, but eventually becomes constrained by resources available for search. Under complete lack of constraints, organizations have enough absorptive and search capacity to explore all of the possible recombinations among all elements, that is $D^* = D$. In general, the rate of innovation can be re-expressed as a superlinear function of cumulative innovations by exploiting that D must grow linearly or sub-linearly with k and that D^* must grow linearly or sub-linearly with D. To arrive at a testable equation, one may stipulate that since $0 \leq D^* \leq D$, D^* grows as D^β, and similarly that D, in turn, grows as k^γ, with $0 < \beta \leq 1$ and $0 < \gamma \leq 1$. Recalling that the number of recombinations $|\mathcal{R}|$ can be approximated as $D^{*\lambda}/\lambda!$ and evaluating equation (3) for large k one can write[4]

$$\frac{dk}{dt} \propto k^{1-\gamma} D^{*\lambda} \tag{4}$$

[4] Note that if k grows faster than D in equation (3) one can ignore the ν term. Using $D \sim k^\gamma$ one can write $\frac{k}{D}$ as $k^{1-\gamma}$. Approximating $|\mathcal{R}|$ with $D^{*\lambda}/\lambda!$ one obtains $|\mathcal{U}| \sim \frac{\rho}{\lambda!} k^{1-\gamma} D^{*\lambda}$. The equations are written as proportions without the parameters ρ and $\lambda!$ for simplicity.

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This equation provides a way to estimate the recombination length λ, noting that if the average firm is recombining knowledge elements in the way theorized here, the estimate λ should, *ceteris paribus* be around 2 or higher.

It is also possible to use the assumptions about $D^* \sim D^\beta$ to simplify equation (4) further to

$$\frac{dk}{dt} \propto k^\sigma \tag{5}$$

with $\sigma = 1 - \gamma + \gamma\beta\lambda$. In this generic formulation, recombination length λ determines whether innovation rates follow exponential or super-exponential curves in time.[5] In the most extreme case of recombinant growth, recombination length λ is completely unconstrained, following the number of available elements D, quickly resulting in faster than exponential rates of innovation (Cortês *et al*., 2022).

One could be satisfied here, but it is possible to make more precise predictions for the case of recombination under resource constraints. In this case, organizations can only make innovations as fast as they can secure a living from doing so (Kauffman, 2019, p. 156), and there are important resource constraints that make it difficult to search all possible combinations and costly to explore and integrate new types of knowledge. To make predictions, the most straightforward approach is to consider resources to constrain the search space D^* to a limiting rate of growth η, i.e., $D^*(t) \propto \eta^t$. This also includes the case where $\eta = 1$ and search space has a maximum. In this case, we can consider equation (4) and formulate

$$\frac{dk}{dt} \sim k^{1-\gamma}\eta^{\lambda t} \tag{6}$$

This differential equation can be solved and rearranged to express the rate of innovation both as a function of time t and as a function of the previous innovations of a firm k. In Appendix B it is shown that this differential equation can be rearranged to

$$\frac{dk}{dt} \propto k \ln \eta \tag{7}$$

In other words, the rate of innovation for an organization is *linearly* dependent on cumulative innovations and the limiting rate of growth η, which varies across organizations. This result has direct parallels to the Bianconi-Barabási model (Bianconi and Barabási, 2001), used to model network phenomena. In our case, k represents a "rich-get-richer" mechanism and $\ln \eta$ represents the "fitness" of an organization.

Solving equation (7) (see Appendix B, equation (B7)) suggests that, on average, innovation within organizations follows exponential curves in time, rather than super-exponential growth.

This discussion can be summarized in the following hypotheses:

H1a: The rate of innovation is initially super-linear, but, due to resource constraints, eventually linear in the cumulative number of innovations.

H1b: The rate of innovation is positively dependent on the size of the search space, with a coefficient higher than 2

2.3 Distribution of innovation across organizations

If innovation rates scale with cumulative innovations k as in the general equations (4–5), the distribution of organizations that have made k innovations $P(k)$ depends on the exponent of k. If the rate of innovation has sub-linear attachment kernels this yields a stretched exponential distribution. Linear attachment kernels, as in equation (7), yield a power-law distribution with

[5] With an unconstrained search space and $\beta = 1$ and the number of elements growing on par with k, $\gamma = 1$, the recombination length governs the dynamics. With $\lambda = 1$ the rate of innovation has exponential solutions in time, but for any $\lambda > 1$ the rate of innovation is super-exponential in time.

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exponent ≈ -2 (as derived in Appendix C), whereas a super-linear attachment kernel leads to winner-take-all distributions (see Krapivsky *et al*., 2001). Since equation (7) and hypothesis H1a posit a linear attachment kernel, it follows that the overall distribution of innovations across organizations also follows a power law with exponent ≈ -2 according to

$$P_k \sim k^{-2} \tag{8}$$

However, this prediction is based on assuming that random elements across firms are negligible. As discussed in Appendix C, non-negligible variations in the growth rates of the search space (η) across firms in equation (7) produce a log-normal distribution. This discussion leads to the following main hypothesis:

H1c: The distribution of innovations across organizations follows a power law (with an exponent of approximately −2).

2.4 Product diversification

Following earlier research, it is natural to assume that the number of unique products (colors) that an organization finds through search depends on their relative frequency in the adjacent possible (Tria *et al*., 2014; Loreto *et al*., 2016). Hence, the rate of product diversification should on average be given by the fraction of new product types (unique colors) in the adjacent possible:

$$\frac{dD}{dk} = \frac{|\mathcal{U}_{NC}|}{|\mathcal{U}|} = \frac{\nu|\mathcal{R}|}{\nu|\mathcal{R}| + \rho\frac{k}{D}|\mathcal{R}|} \tag{9}$$

Multiplying numerator and denominator by $D/|\mathcal{R}|$, one can rewrite in a simpler form and recover the dynamic equation from Tria *et al.* (2014):

$$\frac{dD}{dk} = \frac{\nu D}{\nu D + \rho k} \tag{10}$$

This equation gives different solutions depending on parameter s ν and ρ (see Appendix A for full derivation of expected dynamics).

The cases of interest are when $\nu < \rho$, what may be called a deepening regime, and when $\nu > \rho$, what may be called a widening regime (see Breschi *et al*., 2000, on technological regimes). In the former case, distant or explorative search is more difficult or costly, such that firms are more unlikely to discover entirely new types of products, and search activity is dominated by local search, more associated with product improvements rather than radically new innovations (March, 1991; Katila and Ahuja, 2002). In the latter case, it is more likely (or less costly) to discover new types of products, due to the emergence of new technological opportunities, or exhausted opportunities within a current technological trajectory.

The received literature unanimously suggests that the former situation is the norm (Dosi, 1988; March, 1991), but widening search patterns among organizations may emerge under episodes of strong external pressure and during paradigm shifts. Since the interest of this study lies in the long-run dynamics, one may expect a deepening search regime to dominate the empirical picture. In a deepening search regime, the differential equation (10) produces Heaps' law (Heaps, 1978) with

$$D \sim (\rho - \nu)^{\nu/\rho} k^{\nu/\rho} \tag{11}$$

For the widening search regime, product diversity approaches a fixed share of the cumulative number of innovations (see Appendix A).

H2: In the long run, search is more likely to lead to product improvements than new product types, and the rate of product diversification follows Heaps' law.

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2.5 Predicting the adjacent possible

The above framework is deliberately general in the sense that it makes no special assumptions or restrictions on precisely what elements ("colors") can be recombined, nor does it posit specific correlations between the elements (colors) combined and the product types (colors) that emerge in the adjacent possible.

There are reasons to be cautious in this regard. Kauffman cautioned that the adjacent possible is "unprestatable," and, e.g. as regards the biosphere, "we cannot mathematize [its] detailed becoming" (Kauffman, 2000b, p. 3). A similar point of view is given by Fleming (2001) who argued that "inventors can recombine any components within their purview" and "[c]learly, no technology evolves independently of the entire world of made things. At any point in technological evolution, any component is at risk of being recombined with any other component. The made world evolves as a holistic, continuous, and interdependent web, and not as a disjoint assortment of separate trajectories or product life cycles" (Fleming, 2001, p. 118–119). In other words, there are no constraints on what can be combined and no strong correlations between the elements combined and the corresponding elements in the adjacent possible.

However, these views appear to be at odds with empirical studies that propose that the space of diversification of products or skills is strongly characterized by constraints. This type of dynamics has been explored, for example through constructing a "product space" based on skill-relatedness among firms (Neffke and Henning, 2013) or a global product space based on trade networks and the co-production of goods (Hidalgo *et al.*, 2007; Mealy and Teytelboym, 2022). These studies have shown that depending on where a firm or country is situated in the product space, i.e. what products are currently produced, it is more or less easy to reach other parts of the network, which e.g. explains the difficulties of developing countries to produce more sophisticated products (Hidalgo *et al.*, 2007).

Both of these perspectives are consistent with the general model presented in Sections 2.1–2.4, but it is also possible to argue that these perspectives operate at different time scales and *can both be correct*. If recombination is constrained by technological trajectories, the subset searched by firms in a particular industry will tend to be similar and there will be a high correlation between a firm's set of capabilities and the probability that it diversifies to certain products in a given time period. This will create subgroups in the product space, where some "colors" are more likely to recombine with, or produce some other "colors." This leads to the following main hypothesis:

H3a: The product space is constrained and characterized by community structure at shorter time spans

If Fleming (2001) is correct, however, community structures are likely to shift and ultimately disappear in the long run.

Similarly, one may expect that the firms' product portfolio and their positions in product space let us predict the future products of firms. However, in a longer time perspective, if Fleming (2001) is correct and the product space is dense, firms can potentially reach any product in only a few steps. Hence, one may conjecture:

H3b: At a given point in time, the firms' positions in the product space helps predict the adjacent possible, in terms of the firms' future product groups.

3. Methods and data

3.1 LBIO methodology

To analyze long-run innovation dynamics across organizations, this study uses longitudinal data on new products and commercialized processes for the Swedish engineering industry for the period 1908–2016. The data after 1970 was introduced in Sjöö *et al.*, 2014 and Kander *et al.*, 2019, and data before 1970 was introduced in Taalbi, 2021. In recent years, historians and economists have made strides in producing long-run historical data, providing insights about innovation patterns and industrial dynamics, based on trade-journal literature, prize, and awards data, machine learning methods applied to patent data or combinations thereof (Klepper, 2002;

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Taalbi, 2017; Ortiz-Villajos and Sotoca, 2018; Taalbi, 2019; Kelly *et al.*, 2021; Taalbi, 2021; Capponi *et al.*, 2022).

The data on innovation output used in the current paper is based on the screening of trade journals, i.e. magazines targeting industry members and specialists, according to the so-called Literature-Based Innovation Output (LBIO) method (Kleinknecht and Reijnen, 1993). There are different views on what articles and sections to include from the trade journals, but the currently employed innovation database rests firmly on the principle that all articles must be independent and edited. Consequently, the data does not include product announcements or advertisements, but edited articles typically written by journalists with some expertise in the field. In other words, advertisements and notification lists, based on press releases of firms, are not included.

The database, originally constructed for the period 1970–2007, is based on 15 trade journals covering the Swedish manufacturing industry and ICT services (Sjöö *et al.*, 2014; Kander *et al.*, 2019). The contemporary database includes almost 5000 innovations, commercialized by Swedish firms, whose characteristics and innovation biographies are described in detail in the trade journal articles.

Assembling this innovation data has advantages over patent and R and D statistics in that it captures actual innovations, rather than inventions, some of which are strategic or have little or no economic value. A drawback of the LBIO methodology is that manual collection of innovations is time and resource intensive. Another drawback is that in-house process innovations that do not enter the market are not generally reported (Van Der Panne, 2007).

To establish a long-run analysis of organizations' innovation activity, the present paper focuses on the two most important trade journals for the period studied. *Teknisk tidskrift* started in 1871, was published by the Swedish Association of Technologists, and was Sweden's foremost publisher of findings in engineering. In 1967, its weekly edition was continued under the name *Ny Teknik*, published by the Swedish Association of Graduate Engineers. The second journal, *Tidningen Verkstäderna* was founded in 1905 as the journal of the engineering industry's employer's association (Sveriges Verkstadsförening). Together these two journals reported 53% of the total number of innovations during the period 1970–2016.

An understanding of the coverage of these journals may be obtained by comparing them with other studies or lists of significant innovations. 54 out of 71 (76%) major innovations in engineering listed by Wallmark and McQueen (1991), are included in the current dataset. Similarly, 40 out of 49 (81%) engineering innovations listed in another list of major innovations (Sedig and Olson, 2002) are included in the current dataset. Those that were not covered in the current dataset are mostly innovations marketed by foreign companies and specialized machinery (e.g. for the paper and pulp, publishing and printing or chemical industries).

In other words, it is reasonable to assume that the current dataset captures innovations by organizations active in the engineering industry and ICT services, except for some types of specialized suppliers of machinery. However, the product types that are covered in the two magazines are not limited to engineering products but also include products across the board, including ceramics, wood and paper, chemicals and plastics, and software.

Nevertheless, diversification by engineering firms into these fields is underrepresented in the current data. Meanwhile, full data for the manufacturing and ICT services are available for the period 1970–2016. For this reason, as robustness checks, Appendix D also presents the main results of this study using the full data for all 15 trade journals for the sub-period 1970–2016.

3.2 Organizational boundaries and continuity

In order to analyze the impact of the history of innovation on future innovation, the data used in this paper includes both information on firms and innovations. Long-run series of firm-level innovation activity invariably encounters the problem of organizational change—i.e. mergers, splits, and acquisitions—making it necessary to devise a definition of organizations and organizational continuity. Previous studies on Swedish firms have used flows of employees to trace mergers, acquisitions, and firm survival (Eriksson and Kuhn, 2006; Andersson and Klepper, 2013). Such data is, however, not available for the long time period studied here. Therefore, this study employs a more heuristic approach based on company histories. Essentially, there are two pathways. One possibility is to define a firm as a single organizational unit that is

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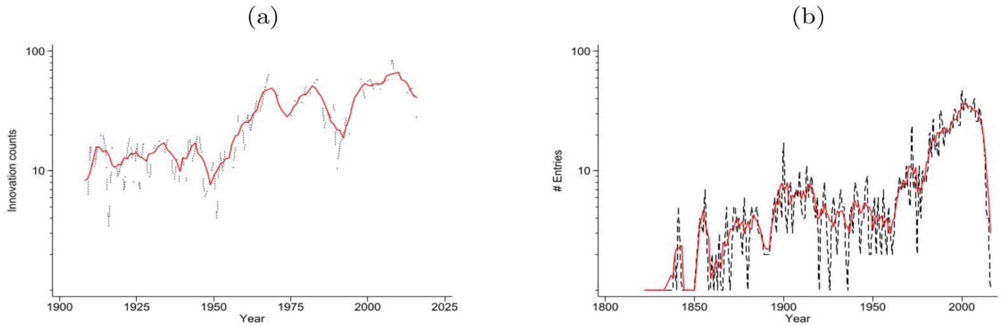

Figure 3. (a) Innovation counts per year of commercialization, (b) Number of entries of innovative firms, by entry date. Five year centered moving averages. Vertical axis in logarithmic scale.

discontinued under any merger, split, or acquisition. This definition ensures that any organization refers to a coherent unit with coherent competencies, but has the downside of leading to biases and inconsistencies after the event. For example, a new merger combines competencies and capabilities from two pre-existing firms. Likewise, a split does not render the new firms memoryless. Hence, this way of defining organizations would underestimate the historical experience of firms.

The other option is to collapse organizational units to a higher level when motivated by company histories. For example, Volvo is a corporate group consisting of several divisions with origins in diverse, originally independent, firms: the marine propulsion systems originate in AB Pentaverken, and the production of tractors originates in AB Bolinder-Munktell, in turn a merger of two previous firms. The main problem involved is that this creates a bias before merger events. This may, to some extent, be forgivable since firms rely on collaborations with similar firms or firms downstream in the supply chain, and such collaborations are frequently a predictor of later mergers and acquisitions. In the absence of more refined aggregation methods, this paper uses the latter strategy.

3.3 Variables used for regressions

The data used to test hypotheses H1a–H2 is based on four variables: (i) the product categories, (ii) the commercialization year of innovations, (iii) the starting years, and (iv) the known exit dates of firms. Based on the trade journal articles, each innovation is given a commercialization year. In the vast majority of cases, the trade journal article explicitly mentions a commercialization year. In the small minority of cases where the commercialization year was not mentioned, the year of the journal article was used as a proxy.

The journals used in this work (*Verkstäderna* and *Teknisk tidskrift* and its successor *Ny Teknik*) together collect 3086 innovations launched by 1493 distinct organizations in the period 1908–2016. Most of these innovations were developed since the 1970s (Figure 3a, see also Taalbi, 2021). Similarly, most of these organizations started after the 1970s (Figure 3). For each of these innovations, a product code (ISIC Rev. 3) has been assigned. The current work uses the 3-digit level codes to distinguish between product types.

Data on firms' entry and exit dates were collected from Statistics Sweden's company registers for the period 1970–2016. All earlier data was collected from company histories (annual reports, firm biographies, and Svensk Industrikalender). Since the data from Statistics Sweden does not capture splits and acquisitions, the data was cross-checked with these sources.

Finally, additional data has been collected for patented innovations (Taalbi, 2022) in order to estimate the role of the size of the search space (D^*). This data covers slightly less than 50% of the innovations for the period 1970–2016 (Taalbi, 2022). The measure of search size is defined on the basis of the number of unique CPC classes cited by a firm. The model introduced in section 2 assumes for simplicity that the search space is strictly cumulative. Accordingly, it would be possible to estimate D^* as the cumulative number of unique CPC classes cited by the patents of a firm. However, while some elements may be added to the search space, others may drop out as

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Table 1. Summary statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
New inno	0.06	0.322	0	7	25843
Cumulative innovations (log)	0.46	0.871	0	5.075	25843
Av. growth rate η (log)	0.053	0.074	0	0.805	9506
Δ Search space	0.062	0.227	0	2.485	7687
Search space $(t-5)$	1.043	0.712	0	3.135	7687
Age	47.144	56.201	1	436	25843

Table 2. Correlation table

Variables	New inno	Cum. inno. (log)	η (log)	Δ Search space	Search space $(t-5)$
New inno	1.000				
Cumulative inno. (log)	0.360	1.000			
Av. growth rate η (log)	−0.064	−0.255	1.000		
Δ Search space	0.139	0.149	0.143	1.000	
Search space $(t-5)$	0.150	0.358	0.300	−0.073	1.000
Age	0.096	0.362	−0.341	0.074	0.108

knowledge may expire rather quickly (Katila and Ahuja, 2002). To test the presence of knowledge depreciation, the econometric specification uses a simple decomposition of search space $\ln D^*(t)$ into (i) the increase of the search space over the past 5 years and (ii) the size of the (logarithm of) search space 5 years ago:

$$\ln D^*(t) \equiv \ln \frac{D^*(t)}{D^*(t-5)} + \ln D^*(t-5) = \Delta\text{Search space} + \text{Search space}\ (t-5) \qquad (12)$$

To test equation (7), the average growth rate of the search space is also included as an alternative specification, defined as the average geometric growth rate of the cumulative number of unique CPC classes cited by a firm from the first observation date: $\eta = D^{*1/t}$.

Table 1 gives summary statistics for the main regression variables, and Table 2 summarizes the Pearson correlation coefficients. Correlations do not indicate multicollinearity (Table 2).

3.4 Constructing a product space

The structure of "cross-product" relationships in the discovery of the adjacent possible, is here examined by constructing a "product space" (Hidalgo *et al.*, 2007; Pugliese *et al.*, 2019; Tacchella *et al.*, 2023), here understood as the relatedness between products produced by firms, that may help predict how firms diversify their product portfolios through innovation. The product network is crucially based on a proximity measure ϕ between products. A natural approach is to follow Hidalgo *et al.* (2007) in measuring proximity ϕ_{ij} as the probability that a firm produces a good i if it produces another j. However, this approach is limited in the sense that it is based on the (binary) presence or absence of a product in the firm's product portfolio. Moreover, in the present data, some products are far more common than others, and binary co-occurrences of products tend to be dominated by pairs of products that are common.

To avoid these issues, proximity is defined on the basis of the co-occurrence of two classes i and j, specifically as the share of product j in firm l's history, weighted by the firm's share of innovations of type i. Hence, if the firm has made only a minor contribution to a product class i the weight is small. Similarly, proximity is also lower if product j is only a small part of the firm's product portfolio. Formally, let k_{jl} be the cumulative number of times an organization l has developed an innovation in product group j, and k_l the cumulative number of innovations of the organization. The fraction k_{jl}/k_l then represents the historical importance of product group j for the organization in a period of time. The proximity of product j to another product i is

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obtained by taking a weighted sum over all firms producing product i, using the share of the firm's innovations in product group i (k_{il}/k_i) as weights:

$$\phi_{ij} = \sum_{l} \frac{k_{il}}{k_i} \frac{k_{jl}}{k_l} \tag{13}$$

For example, say firm A has developed 80% of all automotive innovations in a period and 50% of its innovation portfolio consists of electric motor innovations. Firm B has developed 20% of all automotive innovations in the period but has no prior electric motor innovations. The proximity to automotive and electric motors is then $0.8 \times 0.5 + 0.2 \times 0$, that is 0.4.

3.5 Density and evaluation metrics

The product space so defined is used to analyze the structure of the product space and test hypothesis H3a. To test whether the product space predicts innovation activity, hypothesis H3b, four different measures are used to calibrate the findings, following Tacchella *et al.* (2023) in spirit. For three of the measures, I use Hidalgo *et al.*'s (2007) "density" ω_{jl} that describes how close a firm l and a product j are, given the firm's product portfolio. Formally,

$$\omega_{jl} = \frac{\sum_i x_{il}\phi_{ij}}{\sum_i \phi_{ij}} \tag{14}$$

where ϕ_{ij} is a proximity measure and x_{il} is 1 if firm l produces product i, otherwise 0. The proximity measures used here are (i) Hidalgo *et al.* (2007)'s binary co-occurrence, (ii) the weighted co-occurrence, and (iii) a random benchmark, following Tacchella *et al.* (2023). The latter is based on a random graph calculated from a homogeneous distribution with all self-loops equal to 1.

To further calibrate the quality of the predictions from the product space, as a fourth measure, I also use XGBoost in R (Chen *et al.*, 2015; compare Tacchella *et al.*, 2023), a tree-based machine learning algorithm, trained to perform a binary classification task with the firms' product portfolios (k_{jl}) in period $t-1$ as inputs and firm-years with innovations in product j as the "labels." In other words, the model learns to associate a given product portfolio with the presence of innovations in the subsequent period. The model is trained separately for each product group and hence is likely to have significant advantages over a global index such as the density indices discussed above.

All prediction metrics in period $t-1$ are evaluated against the presence of innovations in product j in the subsequent period t. The dataset consists of 54,670 observations of firm-year products, with positive outcomes in a small minority of the cases (1.7% corresponding to 933 innovations). Firm-years with no previous innovations are excluded from the analysis.

The performance of these indicators in predicting whether firms have an innovation in a product group in time t is assessed through evaluation metrics that are standard for imbalanced datasets. Sensitivity is the fraction of true positives out of all positives, that is all firm-year products with at least one innovation: Sensitivity $= \frac{\text{true positives}}{\text{true positives+false negatives}}$. Specificity is the fraction of true negatives out of all negatives, that is all firm-year products with no innovations: Specificity $= \frac{\text{true negatives}}{\text{true negatives+false positives}}$. Since the dataset is strongly imbalanced, the model performance is assessed through the so-called G-mean and "balanced accuracy," defined respectively as the geometric and arithmetic means of sensitivity and specificity.

4. Results

4.1 Main results

This section provides tests of hypotheses H1a–H2. The main results are presented in Figure 4 and Tables 3 and 4. Figure 4 first of all shows how new innovations scale with cumulative

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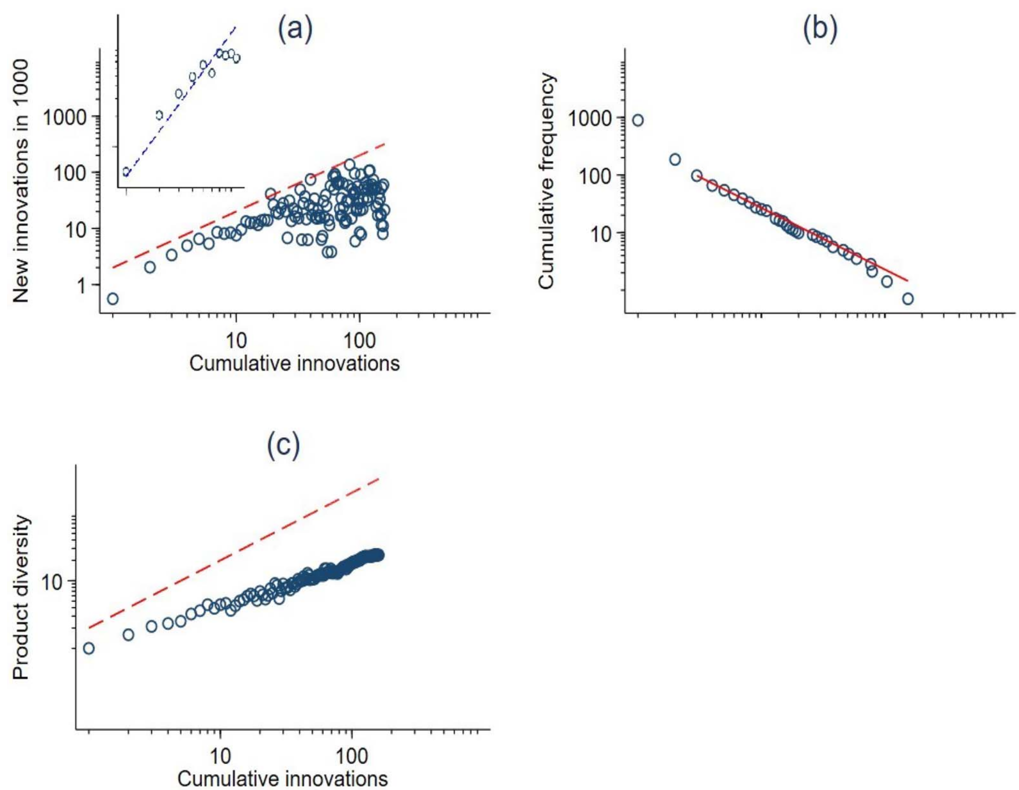

Figure 4. (a) Relative rate of new innovations by total cumulative innovations (k). Dashed lines demarcate linearity. The inset graph shows an early superlinear exponent of 1.5, (b) Distribution of the cumulative number of innovations $P(\kappa \geq k)$. The straight line shows the fitted power-law based on minimization of Kolmogorov-Statistics, (c) Product diversity (D) vs total cumulative innovation (k). The dashed line demarcates linearity.

innovations. The rate of innovation is overall linear in cumulative innovations, but initially superlinear as the inset graph shows in Figure 4a. Figure 4 also shows the distribution of innovations across firms, giving the impression of a power law, in line with hypothesis H1c, and shows a log-log relationship between product types and cumulative innovations in line with Heaps' law (hypothesis H2).

These first results naturally need further analysis. Table 3 conducts formal tests of hypothesis H1a that innovations are linear in cumulative innovations k, and dependent on the size of search space D^*. The regression models explicitly follow equation 4 and equation (5) and equation (7). Since innovations are overdispersed count data, I use a standard negative binomial regression model, which models an independent variable y as a function of a dependent variable x as $Pr(Y = y|x) = \frac{\mu^y e^{-\mu}}{y!}$ where $\mu = exp(\beta x + \epsilon)$ and ϵ being a Gamma-distributed random variable. If the independent variables are log-transformed, β describes log-log elasticities. Using equation (4), I simply regress the number of innovations in (log) cumulative innovations (k), including controls for the size of the search space (D^*).

The results, reported in Table 3. strongly suggest that innovation rates have a *linear* dependence on cumulative innovations as suggested by the theoretical framework and hypothesis H1a. This applies in pooled negative binomial regressions (models 1–5) regardless of control variables. Models 6–8 collapse the data by the number of past cumulative innovations, focusing on the average patterns (compare Newman, 2001 and Figure 4a). Model 6 suggests a baseline sublinear rate of innovation, but linearity is within the 95% confidence interval when search space is included in Models 7–8. Taken together with Figure 4a the results strongly suggest that innovations are linear in cumulative innovations, in line with hypothesis H1a.

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Table 3. Negative binomial regressions

VARIABLES	(1) New inno	(2) New inno	(3) New inno	(4) New inno	(5) New inno	(6) New inno	(7) New inno	(8) New inno
Cumulative innovations (log)	0.997	0.950	0.915	0.942	0.993	0.645	0.679	0.704
	(0.0216)	(0.0299)	(0.0395)	(0.0474)	(0.0221)	(0.123)	(0.153)	(0.153)
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Av. growth rate η (log)		1.934						
		(0.783)						
		[0.014]						
Δ Search space (log)			1.142	1.803			0.0682	0.0627
			(0.152)	(0.212)			(0.310)	(0.311)
			[0.000]	[0.000]			[0.826]	[0.840]
Search space $(t-5)$ (log)			0.125	0.252			−0.0608	−0.0193
			(0.0829)	(0.125)			(0.158)	(0.162)
			[0.133]	[0.044]			[0.701]	[0.905]
Δ Search space $\times$ Early				−1.136				
				(0.280)				
				[0.000]				
Search space $(t-5)\times$ Early				−0.291				
				(0.143)				
				[0.042]				
Early				0.412				
				(0.292)				
				[0.158]				
Age					−0.000502			−0.00965
					(0.000551)			(0.00392)
					[0.362]			[0.014]
Constant	−3.940	−4.102	−4.544	−4.784	−0.685	−2.506	−2.572	−1.842
	(0.0456)	(0.0911)	(0.128)	(0.184)	(0.863)	(0.542)	(0.565)	(0.637)
	[0.000]	[0.000]	[0.000]	[0.000]	[0.427]	[0.000]	[0.000]	[0.004]]
Observations	25,843	9506	7687	7687	25,843	126	124	124
Year FE	No	No	No	No	Yes	No	No	No

Panels 1–5 are based on firm-year observations. Panels 5–6 are averages by the cumulative number of innovations. Standard errors in brackets. *P*-s in square brackets.

Models 2–4 also generally corroborate the notion that the size of the search space impacts innovation rates as in equation (4), but the highest coefficient is within 5 years, indicating the presence of knowledge depreciation. For the average firm, the estimate of search length λ is slightly higher than one (1.142), which indicates a slower rate of innovation than predicted by the general model in equation (4) and equation (6) if innovation is recombinant. Models 4–5, however, show that this is mostly due to an impact of relatively slower innovation rates of older incumbent firms. When differentiating firms starting from 1910 and onwards and older firms, the baseline estimate of average search length λ is 1.803.

Table 4 estimates key parameters related to the distribution of innovations across organizations (models 1 and 2) and Heaps' law (model 3), corresponding to tests of H1c and H2. Models 1 and 2 estimate the slope of the distribution of innovations across firms and the cumulative distribution of innovations respectively. The results agree with the expectation (hypothesis H1c) of a power-law exponent of 2 for the distribution across firms and 1 for the corresponding cumulative distribution, shown in Figure 4b. Model 3 suggests a log-log dependence of product diversity on cumulative innovations with an estimated exponent $\nu/\rho = 0.587$, in line with hypothesis H2.

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Table 4. Baseline estimates of key parameters

VARIABLES	(1) $P(k)$ (log)	(2) CDF (log)	(3) Product diversity (log)
Cumulative inno. (log) (k)	−2.213	−1.111	0.587
	(0.122)	(0.0323)	(0.00172)
	[0.000]	[0.000]	[0.000]
Constant	6.453	5.953	0.00564
	(0.275)	(0.0960)	(0.00170)
	[0.000]	[0.000]	[0.001]
Observations	20	32	25,677
R-squared	0.948	0.975	0.820

Model 1 estimates the slope of the distribution of innovations. Model 2 estimates a linear baseline regression for the relative frequency of new product introductions across organizations. Model 3 estimates Heaps' law equation (11) with product diversity (log) and cumulative innovations (log). The coefficient is an estimate of the ratio ν/ρ. Standard errors in brackets. P-values in square brackets.

Table 5. Estimates of α, goodness of fit (Kolmogorov–Smirnov statistic KS) for discrete power law distributions

	Power law			Log-normal	
Period	α	KS	p	R	p
1970	2.55	0.01	1.00	−2.86	0.00
1990	2.01	0.03	1.00	−0.88	0.38
2010	2.07	0.04	0.99	−0.65	0.52

The P-value tests the null hypothesis that the empirical distribution stems from a power law distribution. The log-likelihood ratios R and P-values test the null hypothesis that both a log-normal and power-law are equally far from the true distribution. Significant negative values imply that the log-normal has a better fit.

As is well-known, however, these results do not necessarily confirm a power-law shape of the distribution, especially since econometric tests do not respect the requirement that the probability distribution must sum up to one. Following Clauset *et al.* (2009), Table 5 tests the goodness of fit of the power-law distribution and compares it with the alternative log-normal distribution. The results, for three benchmark distributions in 1970, 1990, and 2010 do not reject the null that the distribution of innovations across organizations stems from a power-law distribution. Follow-up tests using a log-normal cannot, however, exclude that the log-normal is an equally good or better fit. Overall, these results are in line with expectations from our theoretical framework, which proposes either a power-law with an exponent of ≈ 2 or a log-normal distribution (see Appendix C).

As discussed before, the data on products is based only on the engineering industry and the generic engineering trade journals. For this reason, as a robustness check, Appendix D also presents the main results of this study using the full data for all 15 trade journals for the sub-period 1970–2016. Figure D1 and Table D1 show that the results are very similar when taking into account product types not captured by the main data.

In sum, the results show a broad agreement between the theoretical framework and the empirical picture. As summarized in Table 6 none of the hypotheses derived from the theoretical framework can be rejected, although the results for testing hypothesis H1b suggest that there may be important variations in recombination length between firms, and important exceptions from the assumption of innovation as recombination, especially for older incumbent firms. This exception may also be due to data limitations, e.g. because of shifts in the types of innovations developed by older incumbent firms toward process innovation or because of internationalization.

4.2 Product space

The previous results have shown that the general theoretical model proposed in this work agrees with the empirical picture as regards the dependence of innovation rates on cumulative

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Table 6. Summary of results

Test and parameter	Tested equation	Result
H1a: Linearity σ	$\frac{dk}{dt} = k^{\sigma}$	Linearity cannot be rejected
H1b: Recombination length λ	$\frac{dk}{dt} \propto k^{1-\gamma} D^{*\lambda}$	1.14 on average, 1.8 for firms entering later than 1910
H1c: Heaps' law ν/ρ	$D \sim (\rho - \nu)^{\nu/\rho} k^{\nu/\rho}$	0.587
H2: Power law distribution of innovations across firms	$P_k \sim k^{-2}$	Power law with exponent ≈ 2, or log-normal

Table 7. Network statistics for sub-networks and the total period network 1908–2016

		1930	1970	2010	1908–2016
1	N	36.00	66.00	76.00	78.00
2	E	99.00	245.00	259.00	588.00
3	Density (%)	7.86	5.71	4.54	9.79
4	Av. degree	2.75	3.71	3.41	7.54
5	Std. degree	2.16	4.11	4.11	6.88
6	Transitivity (%)	37.14	22.66	17.02	33.31
7	Modularity	0.38	0.37	0.36	0.23
8	Av. path length	3.17	3.65	3.44	2.49
9	Connectivity	2.00	1.00	1.00	1.00
10	Biggest component	34.00	66.00	76.00	78.00
11	Small worldness	2.19	1.70	1.65	1.36

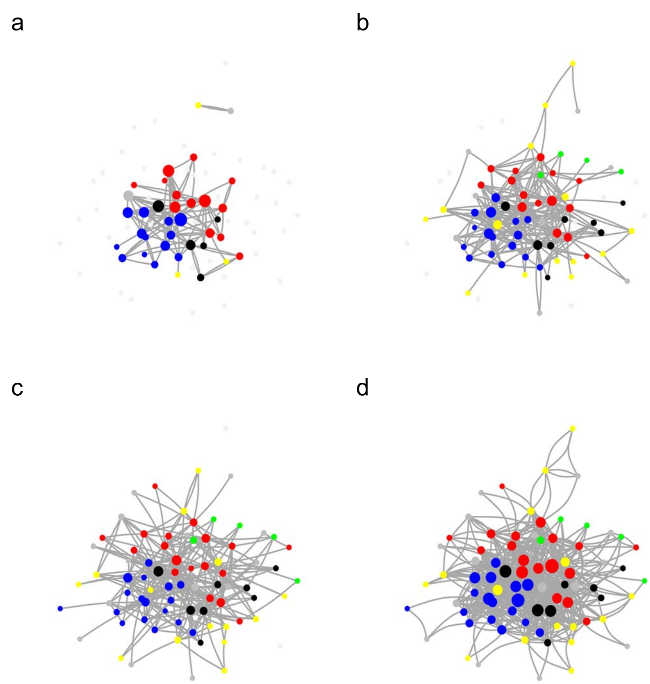

Figure 5. Product networks for (a) 1930s, (b) 1970s, (c) 2010s, and (d) 1908–2016. Blue, red, black, yellow and green nodes are, respectively, ICTs, metals and machinery, transportation equipment, non-metallic mineral products, and forest and wood products.

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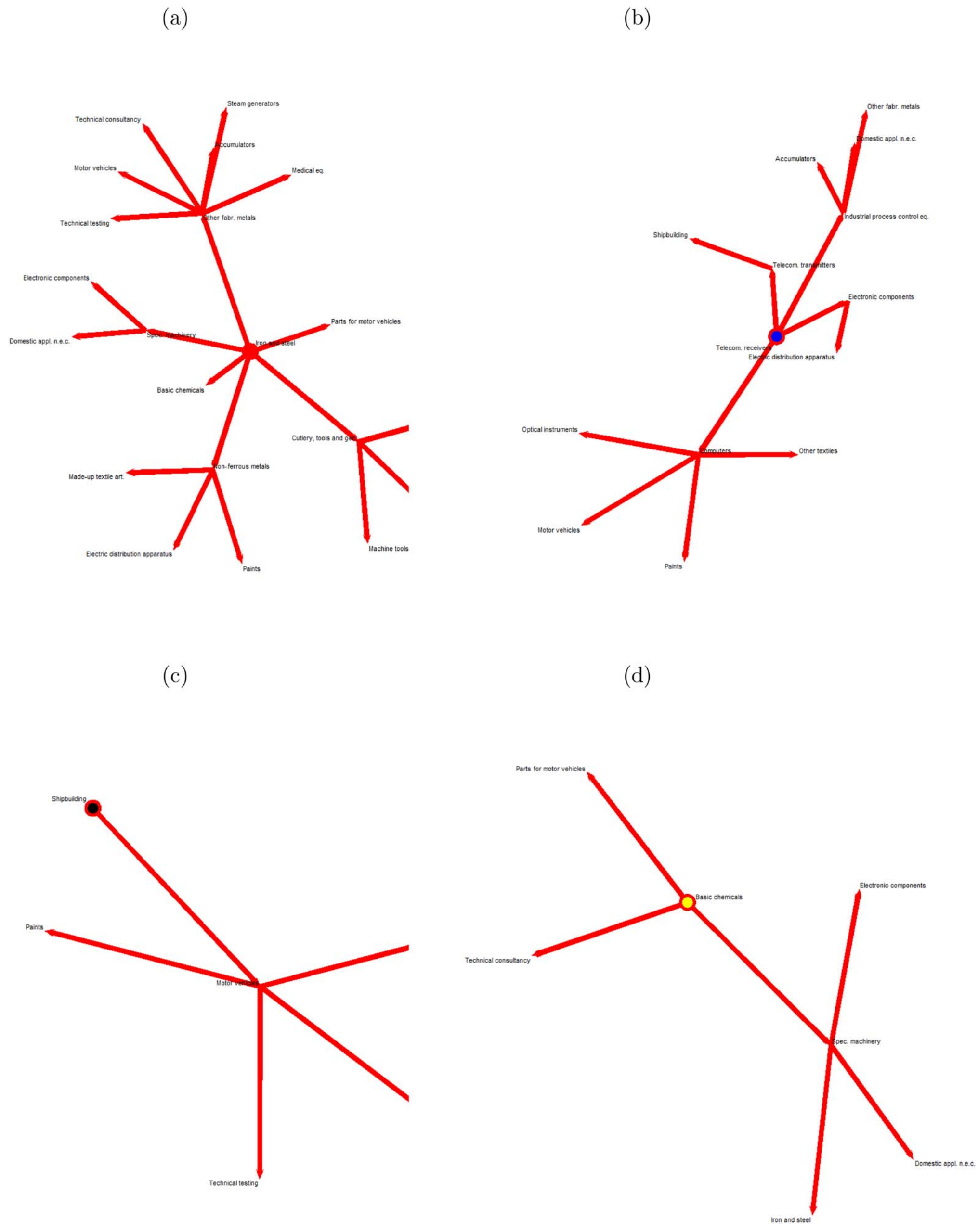

Figure 6. (a) Available pathways of product diversification for firms initially active in 1970 in (a) iron and steel (b) radio (telecommunication receivers), (c) shipbuilding, and (d) basic chemicals.

innovations and the size of the search space, the distribution of innovations across firms, and the dependence of product diversity on cumulative innovations. However, as discussed in Section 2.5, it remains an open question whether there are constraints on what knowledge types can be combined, or whether the types of products in firms' product portfolios restrict the adjacent possible. To further this discussion, this section tests hypotheses H3a and H3b that concern whether recombination is locally constrained to subgroups within the product space and whether the product network predicts the adjacent possible in terms of the future product groups of firms.

The main results are presented in Figures 5–6 and Tables 7–8. A first result is that the network can be characterized as a relatively sparse small-world network with a moderate-to-weak

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Table 8. Comparison of prediction performance of random graph benchmark, co-occurrence-based proximity measures, and machine-learning (XGBoost).

	Random	Co-occurrence (binary)	Co-occurrence (weighted)	XGBoost
Sensitivity	0.92	0.76	0.77	0.81
Specificity	0.10	0.67	0.67	0.83
G-mean	0.30	0.71	0.72	0.82
Balanced accuracy	0.51	0.71	0.72	0.82

community structure (see Table 7). The small worldness statistic (Humphries and Gurney, 2008) compares the transitivity and average path lengths to a random network, considering a network to have "small worldness" if the ratio is higher than one. The results consequently suggest higher "small worldness" than a random network. This means that, although direct linkages are sparse, organizations in certain industries can relatively easily reach other product types by diversifying their product portfolio over time. Small-worldness and high reachability should be conducive to novelty generation (Björneborn, 2020). The modularity statistics from a fast greedy algorithm (Clauset *et al*., 2004) suggest the presence of community structure for the sub-networks of the 1930s, 1970s, and 2010s, but only a weak community structure for the total network 1908–2016.[6] These results align with the notion that technological search has constraints, but is unconstrained over longer time spans (Fleming, 2001).

There is substantial variation in the ability of certain industries to diversify, once link direction is taken into account. Figure 5 shows the product space for benchmark decades and the full period. The product space consists of a few central nodes, in the machinery (red) and ICT industries (blue), whereas other industries are more peripheral with less diversification paths. To illustrate this point further, Figure 6 shows the available pathways of diversification for producers in four industries that were severely affected by the oil crisis of the 1970s. Producers in the basic chemicals and shipbuilding industries had less opportunities to diversify into new fields, than did producers of e.g. radio, and iron and steel products.

The second main result is that the product density ω_{jl}, the proximity of firms to a certain product is associated with the probability of a firm making an innovation in that field in the next period. To gauge the prediction performance, the density ω_{jl} in period $t-1$ is compared to whether a firm l has at least one innovation of product type j in period t. Table 8 provides key statistics. Sensitivity is the fraction of true positives out of all firm-year products with at least one innovation. Specificity is the fraction of true negatives out of all firm-year products with no innovations. Since the dataset is strongly imbalanced, the model performance is assessed through the geometric and arithmetic means of sensitivity and specificity. The results show that both density variables predict positive outcomes. Machine learning (XGBoost) predicts both positive and negative outcomes to a higher degree. Overall, these results provide support for hypothesis H3b, that the product space has information about patterns of discovery of the adjacent possible.

5. Discussion

The present study makes two main contributions to the framework of the adjacent possible in the context of long-run innovation dynamics. The results of this study suggest that the view of innovation as a process of recombination to discover the adjacent possible can be reconciled with empirical patterns. Specifically, the theoretical model predicts, with reasonable accuracy, the empirical results as regards (i) the dependence of innovation rates on cumulative innovations and the size of the search space, (ii) the distribution of innovations across organizations, and (iii) the rate of introduction of new types of products (Heaps' law). While it is possible that other models could produce the same results, the model here devised is, to the best of my knowledge, the first to achieve an integration of the notion of innovation as the discovery of the

[6] A modularity of above 0.3 can be taken as an indicator of significant community structure in large networks (Clauset *et al*., 2004)

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adjacent possible through recombination with all three of these empirical features of innovation activity.

Second, the results also support earlier literature in showing that the structure of the product space can provide important insights into how organizations diversify their product portfolios over time and that it is possible to model the adjacent possible as a discovery process on a network (compare Iacopini *et al.*, 2018), using empirical information within the product space.

While this work shows at once the broad appeal and usefulness of the notion of innovation as a search for recombinations to find adjacent possible novelties, there are also important limitations and nuances. First, while recombination drives innovation, it is fundamentally hampered by (resource) constraints. The arguably most surprising result of this study is that, on average, innovation may have a natural tendency to gravitate toward a linear dependence on cumulative innovations, and exponential growth curves, due to resource constraints. Here one must remind oneself that the analytical unit of this work is the single organization, often a private firm. Hence, these results do not exclude that super-linear patterns may appear in the macroeconomy if, for example the population of inventors or firms increases to make possible the discovery of untapped adjacent possible innovations beyond the capacity of individual incumbent firms. Ultimately, however, one may conjecture that checks to growth make such scenarios intermittent rather than permanent, which again would engender exponential rather than super-exponential rates of innovation in the long run.

A key question is also whether adjacent possible products can be predicted from product portfolios. A key result is that proximities in the product space displayed a general correlation to the adjacent possible innovations. Similarly, the product space also reveals a hierarchical pattern where complex product types, mainly machinery, transportation equipment, and ICTs have high (out)degrees, and other products, like pulp and paper or pharmaceuticals, appear to imply high specialization and relatively low likelihood of diversification to other product groups (compare Hidalgo *et al.*, 2007). This suggests that the product space has important constraints, enabling predictability. However, the results also show that the network is a small-world network and that substructures in the network vanish in the long run, supporting a view similar to Fleming (2001), where virtually any elements can be recombined. Hence, these results suggest that the product space and the adjacent possible can be viewed as characterized by strong constraints, enabling high degrees of short-run predictability, that however, appear to vanish in the long run. If these results are generalizable, long-run dynamics in innovation outcomes cannot be straightforwardly accounted for by the structure of the product space.

From a broader perspective, this study has explored the interaction between search behavior, industrial dynamics, and the dynamics of organization' discovery of the adjacent possible and has suggested ways to theorize these connections. The framework is possible to extend to other situations where multiple agents explore the space of the adjacent possible through recombination, under behavioral or resource constraints. The results of this study suggest that this is a process that takes place in a kind of balance between the highly dynamic expansion of the adjacent possibilities and the checks and constraints that are placed on organizations. These tensions are plausibly responsible for long-run outcomes across the life cycles of industries, where, early on, some organizations may cut ahead, building upon cumulated advances, but at the same time search, long-run resource constraints prevent the emergence of winner-take-all situations, instead leaving enough space for new innovative entries. The dynamics of novelty creation and innovation appears thus to be fated to oscillate between cumulativity and renewal, continuity and disruption, and the rise and fall of major innovators.

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Conflicts of interest. None declared.

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A. Derivation of Heaps' law

We start with the equation for the rate of introduction of new product types:

$$\frac{dD}{dk} = \frac{\nu D}{\nu D + \rho k} \tag{A.1}$$

To solve this differential equation, one may use substitution and plug in $z = D/k$, giving $\frac{dzk}{dk} = kdz/dk + z = \nu zk/(\nu zk + \rho k)$ and

$$\int \frac{\nu z + \rho}{(\nu - \rho)z - \nu z^2} dz = \int \frac{1}{k} dk \tag{A.2}$$

This gives $\frac{\rho}{\nu-\rho}\log z - \frac{\nu}{\nu-\rho}\log(\nu z - \nu + \rho) = \log k$ which can be rearranged to

$$\frac{\rho}{\nu}\log z - \log(\nu z - \nu + \rho) = \frac{\nu - \rho}{\nu}\log k \tag{A.3}$$

Solving out D,

$$\frac{\rho}{\nu}\log D - \log\left(\nu\frac{D}{k} - \nu + \rho\right) = \log k \tag{A.4}$$

and rearranging

$$\frac{\rho}{\nu}\log D = \log\left(\nu D - \nu k + \rho k\right) \tag{A.5}$$

which after taking the exponential and rearranging becomes

$$D^{\rho/\nu} - \nu D = (\rho - \nu)k \tag{A.6}$$

For large D different results are obtained depending on the parameters. For deepening search regimes, $\nu < \rho$, and one can ignore the second term on the left of equation (A.6) and derive

$$D \sim (\rho - \nu)^{\nu/\rho} k^{\nu/\rho} \tag{A.7}$$

This scaling is sub-linear in k and is known as Heaps' law. For widening search regimes, $\nu > \rho$ and one can ignore the first term on the left hand side of equation (A.6) and derive

$$D \sim \frac{\nu - \rho}{\nu} k \tag{A.8}$$

In other words, for deepening search regimes, the relative rate of new product types decreases over time. For widening search regimes, the relative rate of new product types approaches a fixed share of total new product introductions.

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B. Derivation of the Bianconi-Barabási equation

The rate of innovation is given by the size of the effective adjacent possible, according to

$$\frac{dk}{dt} = \left(\nu + \rho\frac{k}{D}\right)|\mathcal{R}| \tag{B.1}$$

with $|\mathcal{R}| = \frac{D^*!}{\lambda!(D-\lambda)!} \sim \frac{1}{\lambda!}D^{*\lambda}$

The search space D^* can be rewritten in terms of a limiting growth rate η

$$D^* = \eta^t \tag{B.2}$$

We are interested in long-run dynamics and large k and hence the equation can be approximated as

$$\frac{dk}{dt} = \frac{\rho}{\lambda!}\frac{k}{D}\eta^{\lambda t} \tag{B.3}$$

Using Heaps' law equation (A.6) and equation (A.7) one can rewrite D in general as a sub-linear function of k. In turn, this means that one can also write $\frac{k}{D} = \Lambda k^{1-\gamma}$ with Λ some constant and $0 < \gamma \leq 1$.

Using these two observations, one can rewrite equation (B.3) as

$$\frac{dk}{dt} = \frac{\rho\Lambda}{\lambda!}k^{1-\gamma}\exp\left(\lambda \ln \eta t\right) \tag{B.4}$$

If search space approaches a stable level, $\ln \eta$ is zero and the rate of innovation will be sub-linear in cumulative innovations. If $\ln \eta$ is negative, implying that the organization decreases its search space in the limit (stops searching), the rate of innovation will follow a sub-linear power law with an aging function, producing a curvilinear pattern. If $\ln \eta$ is positive, it is useful to note that equation (B.4) is equivalent to the Bianconi-Barabási model. From equation (B.4) it is straightforward to use integration to get an expression for k, being equal to

$$k = \left(\frac{\rho\Lambda}{\lambda!}\right)^{\frac{1}{\gamma}}\left(\lambda\frac{\ln \eta}{\gamma}\right)^{-\frac{1}{\gamma}}\exp\left(\frac{\lambda \ln \eta}{\gamma}t\right) \tag{B.5}$$

Plugging this back into equation (B.4), or taking the derivative of the expression for k, gives the result:

$$\frac{dk}{dt} = \frac{\lambda}{\gamma}\ln \eta k \tag{B.6}$$

whose solution is an exponential function of time:

$$k = \exp\left(C + \frac{\lambda}{\gamma}\ln \eta t\right) \tag{B.7}$$

where C is a constant of integration.

C. Derivation of distribution of new product introductions

There are two approaches to derive the distribution of new product introductions across organizations from the linear Bianconi-Barabási model discussed in Appendix B. One is to simplify and ignore the fitness distributions, or assume that the deviations from the mean are small. In this case, the framework leads to a power-law distribution with exponent −2 (Section C.2). If, however, the effect of varying "fitness" is non-negligible, it can be shown that the cumulative number of innovations across fields will follow a log-normal distribution, as has been suggested in the discussion about Gibrat's law and firm growth (Section C.1).

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Lognormal

The lognormal follows if one considers cross-sectional variation in η and η is i.i.d. across firms and over time with a given mean and standard deviation. In this case, the limiting distribution follows trivially from equation (B.6). Simplifying slightly, one may write

$$\frac{dk}{dt}\frac{1}{k} = \eta \tag{C.1}$$

One may note that $\frac{d(\log k)}{dt} = \frac{dk}{dt}\frac{1}{k}$, which gives the equation

$$\log k = \int \eta dt \tag{C.2}$$

The Central Limit Theorem of probability theory states that the sum of i.i.d variables approaches a normal distribution. k then clearly will approach a lognormal distribution.

Power law

Say we have N independent organizations at time t. New entrants occur at a rate m at each point in time. In the simplest model, with a linear or sub-linear attachment kernel, the probability of making a new innovation is

$$\Pi_k = \frac{k^\sigma}{\sum_k k^\sigma N_k} \tag{C.3}$$

The distribution is given by first constructing the master equation for the number of firms with innovations k:

$$N_k(t+1) = N_k + \Pi_{k-1}N_{k-1} - \Pi_k N_k \tag{C.4}$$

We define the number of firms as $N = mt + N_0$. The share of firms with k innnovations is $P_k = N_k/N$. This means that

$$\frac{\partial N_k}{\partial t} = \frac{\partial N_k}{\partial N}\frac{\partial N}{\partial t} = mP_k \tag{C.5}$$

which combined with the master equation gives

$$mP_k = N\Pi_{k-1}P_{k-1} - N\Pi_k P_k \tag{C.6}$$

which can be rearranged to

$$P_k = \frac{N\Pi_{k-1}}{m + N\Pi_k}P_{k-1} \tag{C.7}$$

In our case, we plug in the attachment kernel (equation C.3) and use $\mu = \sum_k \left(k^\sigma N_k\right)/N$ to simplify to:

$$P_k = \mu^{-1}\frac{\left(k-1\right)^\sigma}{m + \mu^{-1}k^\sigma}P_{k-1} \tag{C.8}$$

or

$$P_k \frac{1}{k^\sigma}\prod_j \frac{j^\sigma}{\mu m + j^\sigma}P_0 \tag{C.9}$$

We now have two cases. If $\lambda = 1$

$$P_k = \frac{1}{k}\frac{k!}{\left(\mu m + k\right)!}\mu m!\, P_0 \tag{C.10}$$

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Using the fact that the continuous approximations of $x!/(a+x)!$ is similar to x^{-a}, one may see that this is similar to a power law

$$P_k \sim k^{-1-\mu m} \tag{C.11}$$

With $\sigma = 1$ and $\mu = \sum_k (k\sigma N_k)/N = \frac{t}{mt} = \frac{1}{m}$ and hence

$$P_k \sim k^{-2} \tag{C.12}$$

For sub-linear attachment kernels, i.e. $\sigma < 1$, it is possible to write the distribution as

$$P_k = \frac{1}{k^\sigma} P_0 \exp\left(\sum_j \ln\left(\frac{j^\sigma}{\mu m + j^\sigma}\right)\right) = \frac{1}{k^\sigma} P_0 \exp \sum_j \left(-\ln(\mu m j^{-\sigma} + 1)\right) \tag{C.13}$$

If one takes the series representation

$$\ln\left(\mu m j^{-\sigma} + 1\right) = -\sum_n \frac{(-1)^n (\mu m j^{-\sigma})^n}{n} \tag{C.14}$$

and noting terms in the sum vanishes for large n and that the sum can be approximated by the first-order term, one can take the integral and obtain a stretched exponential:

$$P_k = \frac{1}{k^\sigma} P_0 \exp \int -\left(\mu m j^{-\sigma}\right) dj = \frac{1}{k^\sigma} P_0 \exp\left(-\frac{\mu m}{1-\sigma} k^{1-\sigma}\right) \tag{C.15}$$

D. Results with extended data 1970–2016

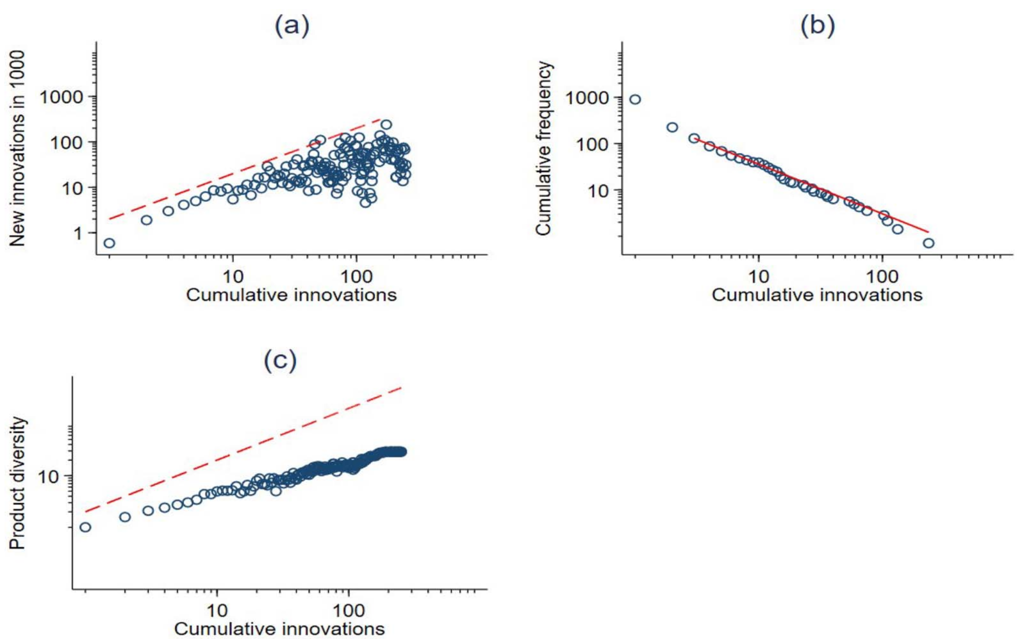

Figure D1. (a) Relative rate of new innovations by total cumulative innovations (k). Dashed lines demarcate linearity. The inset graph shows an early superlinear exponent of 1.5, (b) Distribution of the cumulative number of innovations $P(\kappa \geq k)$. The straight line shows the fitted power-law based on minimization of Kolmogorov-Statistics, (c) Product diversity (D) vs total cumulative innovation (k). The dashed line demarcates linearity.

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Table D1. Negative binomial regressions

VARIABLES	(1) New inno	(2) New inno	(3) New inno	(4) New inno	(5) New inno	(6) New inno	(7) New inno	(8) New inno
Cumulative innovations (log)	0.970	0.960	0.946	0.985	0.985	0.697	1.044	1.040
	(0.0179)	(0.0249)	(0.0317)	(0.0396)	(0.0191)	(0.0874)	(0.132)	(0.130)
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Av. growth rate η (log)		1.610						
		(0.710)						
		[0.023]						
Δ Search space			0.993	1.598			−0.275	−0.309
			(0.126)	(0.191)			(0.269)	(0.270)
			[0.000]	[0.000]			[0.307]	[0.252]
Search space $(t-5)$			0.104	0.224			−0.600	−0.592
			(0.0668)	(0.105)			(0.160)	(0.162)
			[0.120]	[0.032]			[0.000]	[0.000]
Δ Search space × Early				−0.951				
				(0.241)				
				[0.000]				
Search space × Early				−0.240				
				(0.118)				
				[0.042]				
Early				0.294				
				(0.231)				
				[0.204]				
Age					−0.000744			−0.00351
					(0.000487)			(0.00275)
					[0.127]			[0.201]
Constant	−3.725	−3.842	−4.284	−4.516	−0.681	−2.492	−2.760	−2.374
	(0.0404)	(0.0793)	(0.104)	(0.154)	(0.848)	(0.411)	(0.441)	(0.526)
	[0.000]	[0.000]	[0.000]	[0.000]	[0.422]	[0.000]	[0.000]	[0.000]
Observations	26,359	9637	7799	7799	26,359	152	151	151
Year FE	No	No	No	No	Yes	No	No	No

Panels 1–4 are based on firm-year observations. Panels 4–6 are averages by the number of cumulative number of innovations. Standard errors in brackets. P-values in square brackets.

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