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Conference Paper · May 2022

DOI: 10.1109/GlobConET53749.2022.9872465

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Implementation and Validation of NSGA-II Algorithm for Constrained and Unconstrained Multi-Objective Optimization Problem

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2022 IEEE IAS Global Conference on Emerging Technologies (GlobConET) | 978-1-6654-4357-9/22/$31.00 ©2022 IEEE | DOI: 10.1109/GlobConET53749.2022.9872465

***Abstract*—Multi-Objective optimization (MOO) algorithms are gaining more attention among designers or decision makers working on optimization problems in practice, compared to the single objective optimization algorithms that are limited to single objectives. Evolutionary Computation (EC) techniques are employed for solving these MOO problems. Techniques such as Non-dominated Sorting Genetic Algorithm (NSGA-I), its improved version called NSGA-II, Strength-Pareto Evolutionary Algorithm (SPEA), Multi-Objective Particle Swarm Optimization (MOPSO) are some widely employed EC techniques. The NSGA-II algorithm implementation and validation is carried out in this paper. The mathematical analysis of the NSGA-II algorithm and its implementation in the MATLAB environment are presented here. Developed algorithm is also tested using unconstrained and constrained standard test functions for MOO problems. The simulation results revealed that the developed algorithm is providing results similar to the standard test results. Hence, incorporating a suitable mathematical model of any practical MOO problem into the implemented algorithm will yield optimal solution for the multiple objectives under consideration.**

***Index Terms*—Constrained Optimization, Evolutionary Computation, Multi-Objective Optimization, NSGA-II, Pareto Optimal Solutions, Unconstrained Optimization.**

I. Introduction

The conventional gradient based numerical optimization techniques are unable to handle non-differentiable functions, discontinuous functions etc. and are inefficient in handling discrete variables. The inability of classical optimization methods in dealing with the multi-modal and multi-objective optimization problems were resolved by the introduction of evolutionary computation (EC) techniques [1]. They help to solve complex optimization problems by generating, evaluating and modifying a population of possible solutions. They are nature inspired algorithms and are based on natural evolution or genetics. The ability to handle mixed variables such as continuous, discrete, and permutation have boosted the research in the field of EC.

Any optimization problem has objectives, constraints and bounds. Objectives are functions that are to be either minimized or maximized through the optimization process. Single objective optimization refers to problems with only one objective function, whereas multi and many optimization problems refer to problems with more than two objectives and more than three objectives respectively.

While single objective optimization problems provide single optimal solution, MOO problems provide multiple solutions. Practically, most of the optimization problems have multiple objectives that yields multiple optimal solutions [2]. There are two approaches in dealing such MOO problems. First one is preference-based approach, wherein each objective is weighted and a single objective function is developed thus obtaining a single optimal solution.

The classical optimization methods like weighted sum approach [3], epsilon constraint approach [4], weighted metric method [5] and Benson's method [6] comes under preference based, where the multi-objective problem is formulated as sum of different single objective functions. The second approach, the Ideal MOO approach which considers convergence and diversity as the two prime goals, can handle conflicting objectives as in [2] and [7]. The collection of best solutions of the MOO problem are called non-dominated solutions or pareto optimal solutions and the path along which such solutions lie in the objective space is termed pareto optimal front. Better convergence means, the solutions are lying close to the expected pareto optimal front and better divergence means, the solutions are separated as far as possible in the objective space.

One of the popular and well cited ideal MOO method is NSGA-II [8], an elitist and developed version of NSGA-I [9]. Here, the convergence is ensured through the non-dominated sorting while the divergence is guaranteed through crowding distance sorting. The NSGA-II algorithm implementation provides pareto optimal solutions. The best solution have to be found out using Multi-Criteria Decision Making (MCDM) methods such as Technique for Order of Preference by Similiarity to Ideal Solution (TOPSIS), Analytic Hierarchy Process(AHP) etc.

This paper deals with the implementation of NSGA-II algorithm followed by its validation using a set of unconstrained standard test functions like Fonseca-Fleming function, Kursawe function, Schaffer function 1, Schaffer function 2, ZDT1 and ZDT3 and also using constrained standard test functions like Binh & Korn function and Chankong & Haimes function. The simulation results revealed that the designed algorithm produces findings that are comparable to those of standard test functions. As a result, combining a proper mathematical model of any practical MOO problem into the developed algorithm will result in the optimal solution with respect to the multiple objectives under study.

The rest of this paper is organized as follows: Section II and III introduces MOO and NSGA-II algorithm respectively. Section IV includes the simulation results and discussions. Section V concludes the paper with acknowledgement at the end.

II. Multi-Objective Optimization

As the term suggests, a multi-objective optimization problem (MOOP) deals with two or three objective functions. A MOOP in its general form is stated as shown below:

$$Minimize/Maximize \;\; f_m(Z), \quad m = 1, 2, ..., M;$$

subject to:-

$$g_j(Z) \leq 0, \quad j = 1, 2, ..., J;$$

$$h_k(Z) = 0, \quad k = 1, 2, ..., K;$$

$$Z_i^{(L)} \leq Z_i \leq Z_i^{(U)}, i = 1, 2, ..., n.$$

Here, Z is the vector of n decision variables represented as $Z = (z_1, z_2, ..., z_n)^T$. M is the total number of objectives considered in MOOP. Objectives could be minimized or maximized based on the problem statement. A common practise is to minimize all objectives by using duality property on maximizing objectives. The duality property states that by multiplying the objective function by -1, we can change a maximization problem into a minimization problem. The constraints can be of two types, inequality and equality constraints. $g_j(Z)$ represents j^{th} inequality constraint and $h_k(Z)$ represents k^{th} equality constraint. K and J gives the total number of equality and inequality constraints resp. The inequality constraints treated here as 'less-than-equal-to' type can also be replaced with 'greater-than-equal-to' type. Here less-than-equal-to type is used for the ease of constraint handling. n represents the total number of decision variables. Each decision variable z_i is limited within a lower bound of $z_i^{(L)}$ and an upper bound of $z_i^{(U)}$.

The decision variables can be mapped on n dimensional space called as decison variable space. The M objectives of MOOP introduces the additional space of M dimensions called objective space. The mapping takes places from the n-dimensional decison variable space to the M dimensional objective space. Fig. 1 illustrates the two spaces and the mapping between them. It is clear from this figure that not all solutions in the decision space are feasible. They are restricted by constraints and bounded limits. As a result, the objective value mapped by those decision variables will be

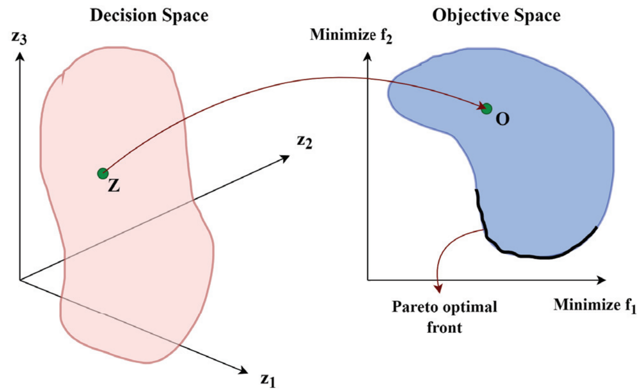

Fig. 1: Decision space and objective space representation

also constrained.

In the objective space, a solution $f_m(Z_a)$ is said to be dominating another solution $f_m(Z_b)$, if

1) $f_m(Z_a)$ is no worse than $f_m(Z_b)$ in all m objectives.
2) $f_m(Z_a)$ is strictly better than $f_m(Z_b)$ in at least one of the m objectives.

III. Non-dominated Sorting Genetic Algorithm (NSGA-II)

In NSGA-II, the feasible solutions found from the objective space is made to compete each other using the dominance concept and the best collection of solutions termed as non-dominating solutions or pareto optimal solutions are found out. The curve formed by joining these solutions are called pareto optimal front. The pareto optimal front of a two objective minimization problem is shown at the objective space of Fig.1.

The overall block diagram of the NSGA-II algorithm is shown in Fig. 2. Randomly generated initial population gives N possibilities of the decision variables. Similarly N solutions of the old generation or the last iteration is combined together to make a pool of $2N$ solutions. Out of these $2N$ solutions best N solutions are sorted out using non-dominated sorting as well as crowding distance sorting. The convergence of the algorithm is ensured through non-dominated sorting procedure, which rank the solutions based on dominance. Crowding distance sorting is done for the sake of maintaining diversity. This method helps to hunt the solution across the whole search space and ensures that the obtained optimal point is not a local optima. The survival selection collects the sorted best N solutions by removing worst solutions and these solutions are subjected to crossover and mutation variations to generate the next generation population of decision variables. After the fitness evaluation, this new population is again merged with the old population to continue the next iteration. The process continues updating the solutions at each iteration leaving behind the non-dominated solutions at the end of the iteration.

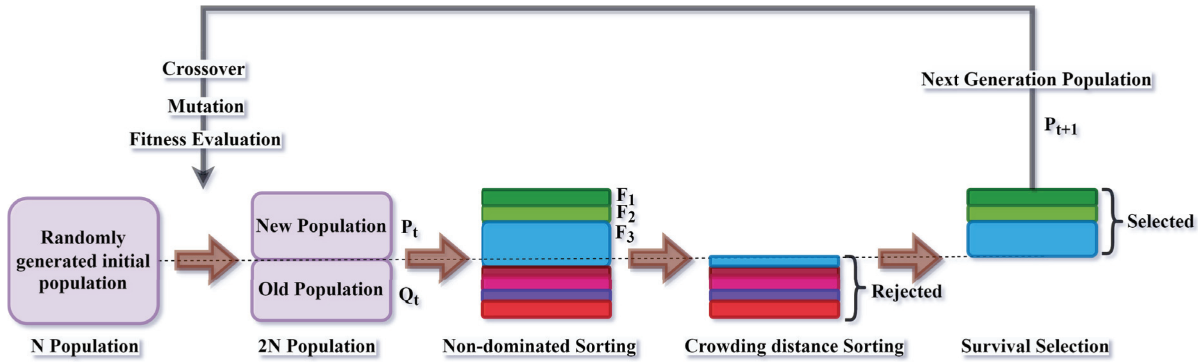

Fig. 2: NSGA-II Overall Block Diagram

IV. Simulation Results and Discussions

Even though the NSGA-II algorithm is widely discussed in EC techniques, its algorithm implementation is not yet standardized. Even if the pseudo code of the algorithm remains same, authors have used different types of crossover and mutation operators, selection procedures, constraint handling approaches etc. Fig.3, Fig. 4 and Fig. 5 shows the standard test functions [10] at left and the corresponding pareto optimal front results obtained at right for the purpose of comparison.

The results obtained using the implemented algorithm is observed to be similar to the standard test results, which validates the accuracy of the algorithm. Mathematical modelling of any multi objective optimization problem followed by replacing the problem parameters in the current algorithm is also expected to give the similar accuracy. The pareto optimal front thus obtained can be used at the decision making stages of the respective multi objective problem.

A. *Simulation Setup*

Inorder to get an unbiased comparison of CPU times, all the test function evaluations are performed for 100 iterations and population count of 50 using the same PC with the specifications as shown in Table 1.

TABLE I: Specifications of the PC used for simulations

Name	Specifications
Hardware	
Processor	AMD Ryzen 3 3200U
RAM	4.00 GB
Frequency	2.6 GHz
Software	
Operating System	Windows 10
Language	MATLAB R2020a

B. *Unconstrained Standard Test Functions*

The NSGA-II algorithm is tested for some unconstrained standard test functions to check its performance. The test functions considered here are Fonseca-Fleming function, Kursawe function, Schaffer function 1, Schaffer function 2, ZDT1 and ZDT3. Fig. 3 and 4 shows standard pareto optimal solutions of these unconstrained test functions with simulation results. Each of the test functions are defined as follows:

1. Fonseca-Fleming function [11]: This is an unconstrained n variable two objective test problem. The function is defined as follows:

$Minimize \quad F(f_1(z), f_2(z))$, where

$$f_1(z) = 1 - exp[-\sum_{i=1}^{n}(z_i - (1/\sqrt{n}))^2]$$
$$f_2(z) = 1 - exp[-\sum_{i=1}^{n}(z_i + (1/\sqrt{n}))^2]$$

Bounded Constraint: $-4 \leq z_i \leq 4$
$1 \leq i \leq n$

2. Kursawe function [12]: This is an unconstrained three variable two objective test problem. The function is defined as follows:

$Minimize \quad F(f_1(z), f_2(z))$, where

$$f_1(z) = \sum_{i=1}^{2}[-10exp(-0.2\sqrt{(z_i)^2 + (z_{i+1})^2})]$$
$$f_2(z) = \sum_{i=1}^{3}\left[|z_i|^{0.8} + 5sin(z_i^3)\right]$$

Bounded Constraint: $-5 \leq z_i \leq 5$
$1 \leq i \leq 3$

3. Schaffer function 1 [13]: This is an unconstrained single variable two objective test problem. The function definition is as shown below.

$Minimize \quad F(f_1(z), f_2(z))$, where

$$f_1(z) = z^2$$
$$f_2(z) = (z-2)^2$$

Bounded Constraint: $-A \leq z \leq A$
[Values of A from 10 to 10^5 are used successfully.]

4. Schaffer function 2: This is a modified test function of Schaffer function 1. The function definition is as shown below.

$Minimize \quad F(f_1(z), f_2(z))$, where

$$f_1(z) = \begin{cases} \text{-z} & \text{if } z \leq 1 \\ \text{z-2} & \text{if } 1 < z \leq 3 \\ \text{4-z} & \text{if } 3 < z \leq 4 \\ \text{z-4} & \text{if } z > 4 \end{cases}$$
$$f_2(z) = (z-5)^2$$

Bounded Constraint: $-5 \leq z \leq 10$

5. ZDT1 [14]: Zitzler–Deb–Thiele's function 1 is an unconstrained 30 variable two objective test problem. The function

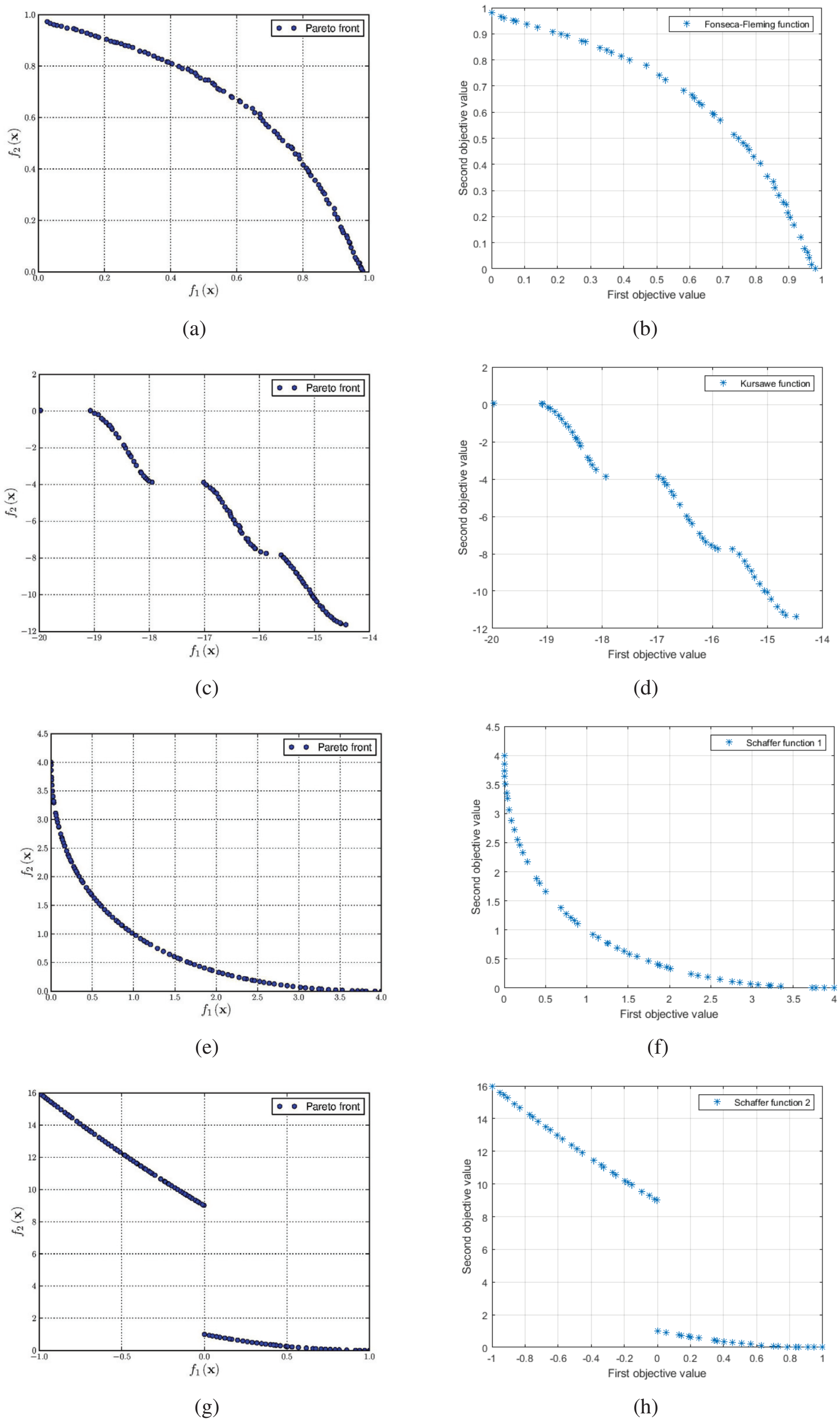

Fig. 3: Simulation results of standard test functions. (a) Expected pareto optimal front of Fonseca-Fleming function. (b) Obtained pareto optimal front of Fonseca-Fleming function. (c) Expected pareto optimal front of Kursawe function. (d) Obtained pareto optimal front of Kursawe function. (e) Expected pareto optimal front of Schaffer function 1. (f) Obtained pareto optimal front of Schaffer function 1. (g) Expected pareto optimal front of Schaffer function 2. (h) Obtained pareto optimal front of Schaffer function 2.

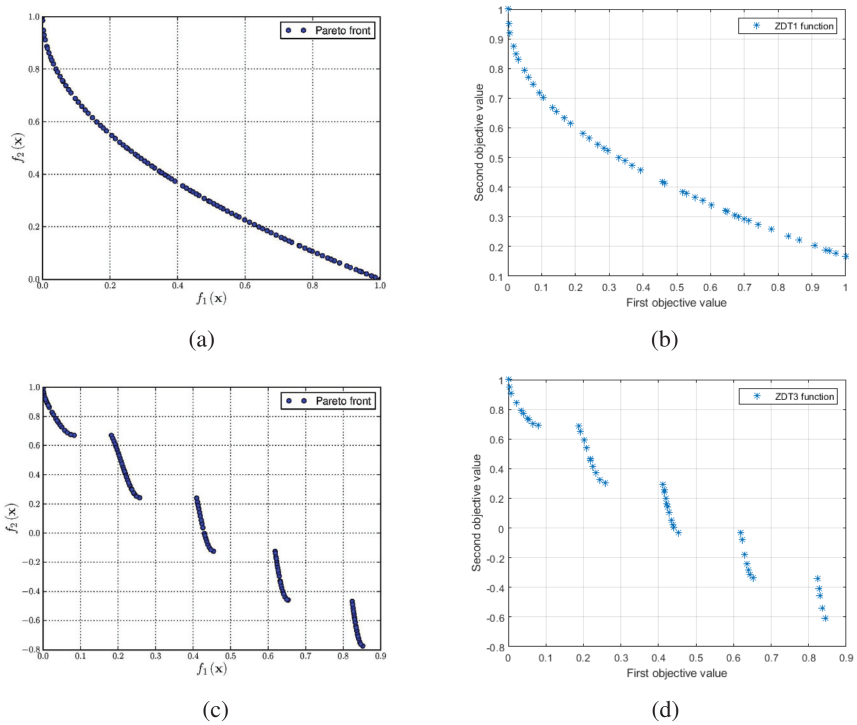

Fig. 4: Simulation results of standard test functions. (a) Expected pareto optimal front of ZDT1 function. (b) Obtained pareto optimal front of ZDT1 function. (c) Expected pareto optimal front of ZDT3 function. (d) Obtained pareto optimal front of ZDT3 function.

is defined as follows:

$Minimize \quad F(f_1(z), f_2(z))$, where

$$f_1(z) = z_1$$
$$f_2(z) = g(z)h(f_1(z), g(z))$$
$$g(z) = 1 + (9/29)\sum_{i=2}^{30} z_i$$
$$h(f_1(z), g(z)) = 1 - \sqrt{f_1(z)/g(z)}$$

Bounded Constraint: $0 \leq z_i \leq 1$
$1 \leq i \leq 30$

6. ZDT3: Zitzler–Deb–Thiele's function 3 is a modified test problem of ZDT1. The function is defined as follows:

$Minimize \quad F(f_1(z), f_2(z))$, where

$$f_1(z) = z_1$$
$$f_2(z) = g(z)h(f_1(z), g(z))$$
$$g(z) = 1 + (9/29)\sum_{i=2}^{30} z_i$$
$$h(f_1(z), g(z)) = 1 - \sqrt{f_1(z)/g(z)}$$
$$-(f_1(z)/g(z))sin(10\pi f_1(z))$$

Bounded Constraint: $0 \leq z_i \leq 1$
$1 \leq i \leq 30$

C. Constrained Standard Test Functions

The same NSGA-II algorithm tested using unconstrained functions is now tested for some constrained standard test functions to verify its performance. The test functions considered here are Binh & Korn function and Chankong & Haimes function. Fig. 5 shows these test functions along with obtained results. The details of these test functions are as follows:

1. Binh & Korn function [15]: This is a constrained two variable two objective test problem. The function is defined as follows:

$Minimize \quad F(f_1(z), f_2(z))$, where

$$f_1(z) = 4(z_1)^2 + 4(z_2)^2$$
$$f_2(z) = (z_1 - 5)^2 + (z_2 - 5)^2$$

Bounded Constraint: $0 \leq z_1 \leq 5$
$0 \leq z_2 \leq 3$

Inequality Constraint: $(z_1 - 5)^2 + z_2^2 \leq 25$
$(z_1 - 8)^2 + (z_2 + 3)^2 \geq 7.7$

2. Chankong and Haimes function [16]: This is a constrained two variable two objective test problem. The function is defined as follows:

$Minimize \quad F(f_1(z), f_2(z))$, where

$$f_1(z) = 2 + (z_1 - 2)^2 + (z_2 - 1)^2$$
$$f_2(z) = 9z_1 - (z_2 - 1)^2$$

Bounded Constraint: $-20 \leq z_1 \leq 20$
$-20 \leq z_2 \leq 20$

Inequality Constraint: $(z_1)^2 + z_2^2 \leq 225$
$z_1 - 3z_2 + 10 \leq 0$

V. Conclusion

In this paper a multi-objective optimization algorithm was implemented considering minimization of two objective functions using Non-dominated Sorting Genetic Algorithm (NSGA-II). The developed NSGA-II algorithm was validated using various unconstrained standard test functions like Fonseca-Fleming function, Kursawe function, Schaffer function 1, Schaffer function 2, ZDT1 and ZDT3 and also using

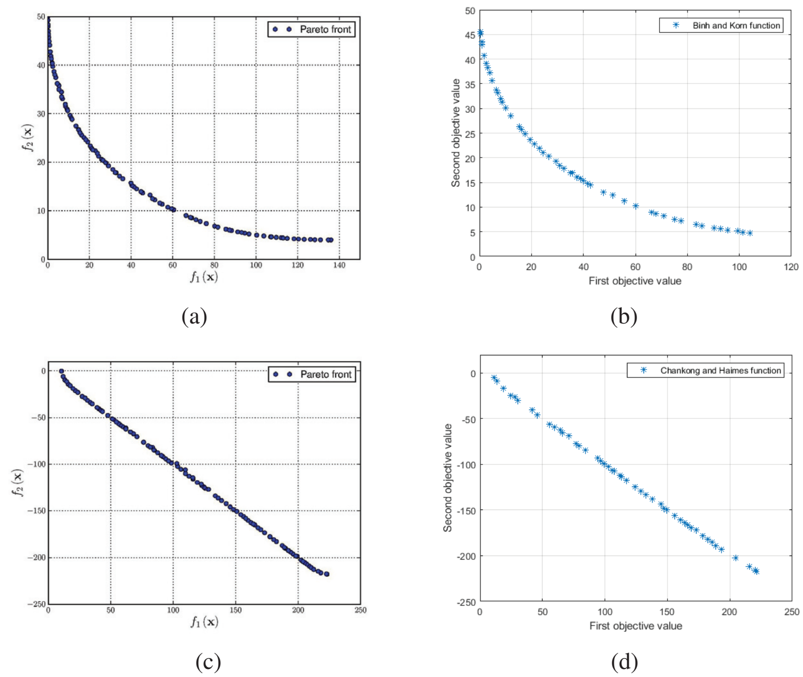

Fig. 5: Simulation results of constrained test functions. (a) Expected pareto optimal front of Binh & Korn function. (b) Obtained pareto optimal front of Binh & Korn function. (c) Expected pareto optimal front of Chankong and Haimes function. (d) Obtained pareto optimal front of Chankong and Haimes function.

constrained standard test functions like Binh & Korn function and Chankong & Haimes function. The obtained results are found to be matching with the standard test results thus proving the reliability of the algorithm. Hence, the developed algorithm can be incorporated for optimization of practical MOO problems with suitable mathematical modelling.

VI. Acknowledgment

The work is carried out under the project "Developing Smart Controller for Optimum Utilization of Energy and Trustworthy Management in a Micro Grid Environment-IMP/2019/000251" funded by IMPacting Research, INnovation and Technology (IMPRINT) 2C.1 from Science and Engineering Research Board, Government of India.

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