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Tesi di Laurea in

MACROECONOMIA

Skill Heterogeneity, Idiosyncratic Risk, and Labor Market Fluctuations

Relatore:	**Candidato:**
Ch.mo.	Antonio Cozzolino
Saverio Simonelli	P09000193

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Chapter 1

Introduction

Understanding the connection between skills and occupational preferences represents a challenge for economists. In a highly heterogeneous environment, it becomes a formidable task to discern and forecast the consequences of shifts in labor supply resulting from unobservable individual traits, thereby influencing the broader economy.

The primary objective of this thesis is to explore the connection between the observed heterogeneity in income and workers' skills, and their decision-making regarding occupations. Furthermore, it aims to evaluate the aggregate-level effects of occupational mobility resulting from non-pecuniary motivations. In order to do so, I formulate a Dynamic Discrete Choice Model in which such heterogeneity is captured along different dimensions.

In this economy, workers are heterogeneous in skills. They differ through a productivity parameter in each different occupation: in the style of Roy 1951, I establish a comparative advantage parameter to efficiently represent this dimension.

Moreover, workers possess diverse preferences for particular occupations, with a specific emphasis on non-pecuniary benefits that hold significance in their job choices. To incorporate this aspect, I introduce a taste shock component inspired by Faia, Kudlyak, and Shabalina 2021 and Grigsby 2022, allowing for a comprehensive exploration of the non-monetary factors valued by workers when selecting a job.

Finally, workers face the inter-temporal decision of consumption and saving in a scenario in which which markets are incomplete, drawing inspiration from Aiyagari 1994.

The thesis is structured as follows: Chapter 2 provides an extensive review of relevant literature pertaining to the three main topics explored in this work. In Chapter 3, a simple model that attempts to capture all the issues presented before is presented, accompanied by some analytical results. Chapter 4 examines the model from a partial equilibrium perspective, offering insights on the behaviors of agents within this framework at a micro level. Subsequently, Chapter 5 takes a macro-level approach, studying the broader implications and effects of these forces within a general equilibrium framework.

To give an intuition of the potential macro-level impact of non-pecuniary effects, consider an economy composed by two types of workers, each capable of supplying $\gamma_{i,j}$ efficient units of labor to different occupations. Here, $i = 1, 2$ denotes the type index and $j = 1, 2$ denotes the job[1] index. The effective labor supply for each occupation is therefore defined as:

$$L_1^S = M_{1,1}\gamma_{1,1} + M_{2,1}\gamma_{2,1}$$
$$L_2^S = M_{1,2}\gamma_{1,2} + M_{2,2}\gamma_{2,2}$$

Here, $M_{i,j}$ represents the mass of type i workers in occupation j. In this scenario, there are no frictions to occupational mobility, so workers are free to transition between occupations.

If we assume that type 1 workers are more productive in occupation 1, while type 2 workers excel in occupation 2, and further assume that workers do not value non-pecuniary benefits from jobs, the only viable equilibrium is one in which individuals self-select into occupations based on their comparative advantage. In this setting, the labor supply is fixed and determined only by the skills of the agents and the total mass of type 1 and type 2:

$$L_1^S = M_1\gamma_{1,1}$$
$$L_2^S = M_2\gamma_{2,2}$$

On the other hand, if workers attach non-pecuniary benefits to specific occupations, the outcome may be different. Under the free mobility assumption, consider a scenario where we fix $M_{2,1}$ and $M_{2,2}$ and allow some individuals to transition from $M_{1,1}$ to $M_{1,2}$. This movement occurs due to reasons unrelated to wages or pecuniary motives but rather associated with non-pecuniary aspects of job 2 that attract a portion of type 1 workers away from occupation 1 and towards occupation 2. As a result, this flow induces opposite shifts in the two labor supplies[2]:

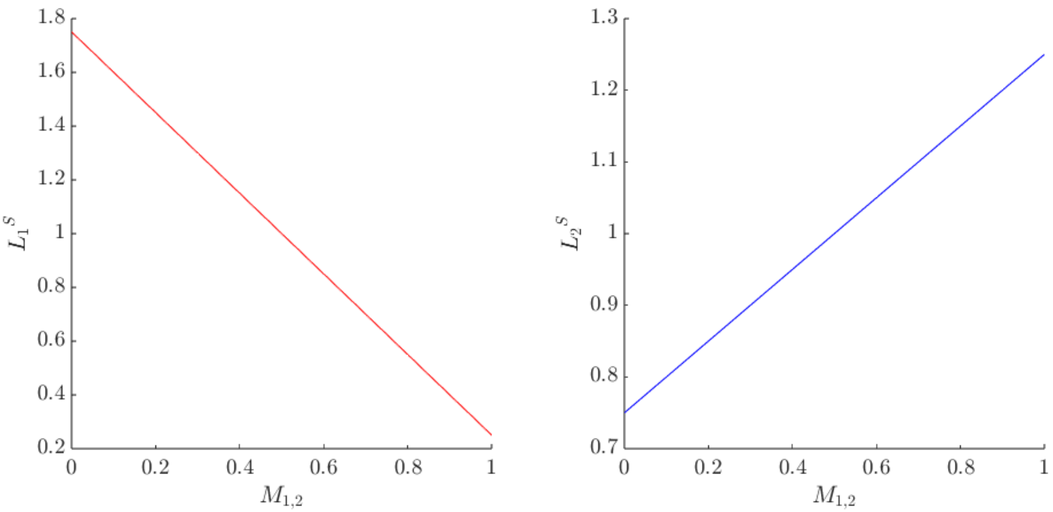


Figure 1.1: Response of the two supplies when Type 1's move from occupation 1 to occupation 2

[1]The concept of "occupation," "job," and "task" can be used interchangeably in this context, as I'm not focusing on these distinctions.

[2]For the purpose of this illustration, I am using the following parameter values: $M_{1,2} = 1 - M_{1,1}$, $M_{2,1} = M_{2,2} = 0.5$, $\gamma_{1,1} = \gamma_{2,2} = 1.5$, and $\gamma_{1,2} = \gamma_{2,1} = 0.5$

Figure 1.1 plots labor supplies on the vertical axis and the mass of individual of type 1 in occupation 2 on the horizontal axis. It is easy to check both graphically and analytically that, an increase in $M_{1,2}$ leads to a decrease of L_1^S and to an increase in L_2^S.

Moreover, though general equilibrium, such movements will have an impact on wages. At first glance, one might be inclined to deduce that, following a decrease in supply, wages would increase, all else being equal, while the opposite would occur for the first occupation. This holds true in the given setting, as long as we keep fixed mobility for type 2 workers. Once we relax this assumption, this may not necessarily be the case. Depending on the specific combination of flows into occupations, we can observe more complex dynamics on the supply side:

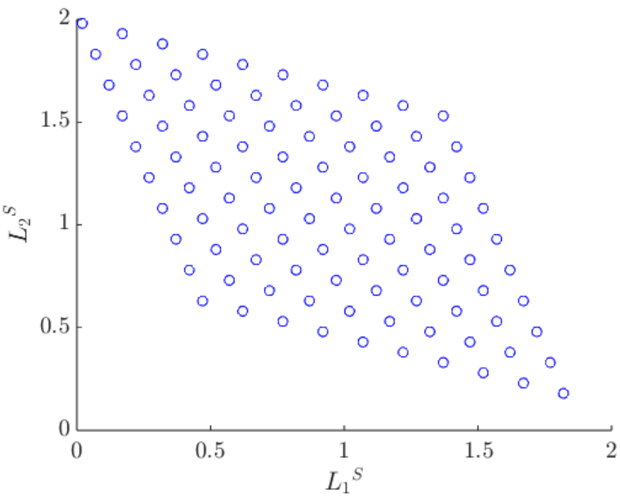


Figure 1.2: Different allocations

Figure 1.2 illustrates the relationship between the couple $\left(L_1{}^S, L_2{}^S\right)$ for various occupational allocations, represented by the quartet $(M_{1,1}, M_{1,2}, M_{2,1}, M_{2,2})$. Each dot represent a specific allocation and the axis measure the supply in each occupation for that specific allocation. Allocations shifts lead to the movements of $\left(L_1{}^S, L_2{}^S\right)$ within the boundaries of this parallelogram. In this setting, the impact of occupational mobility on labor supply is not uniquely determined. Depending on the specific allocation, the magnitude of $L_1{}^S$ and $L_2{}^S$ may vary. It is important to note that, at this stage, we have not yet specified an equilibrium definition, therefore, in principle in a dynamic setting, any allocation path could be feasible, resulting in shifts in the supplies in different directions and generating diverse effects at the aggregate level.

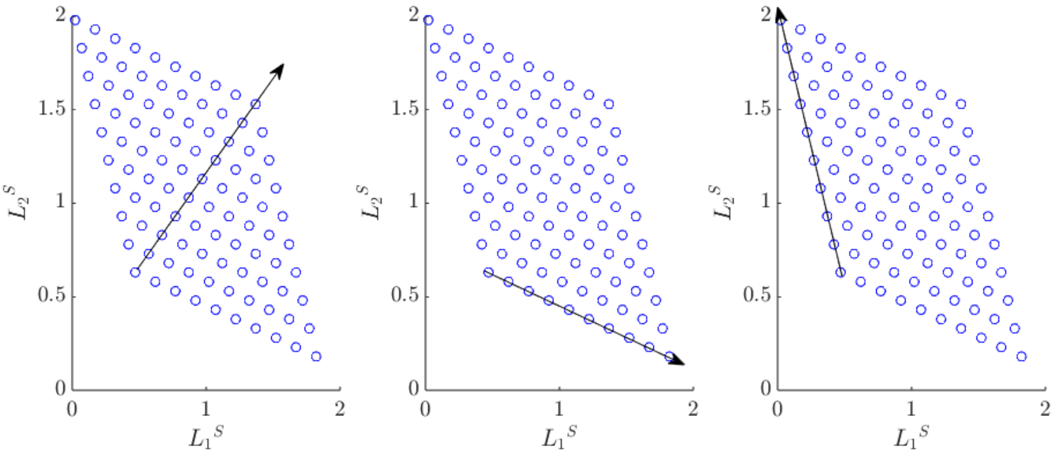


Figure 1.3: Different Directions for $L_1{}^S$ and $L_2{}^S$

For exposition purpose, if we take a look at a naive simulation of allocations overtime, we observe that the effect of the mobility on wages is not always well defined over a specific direction.

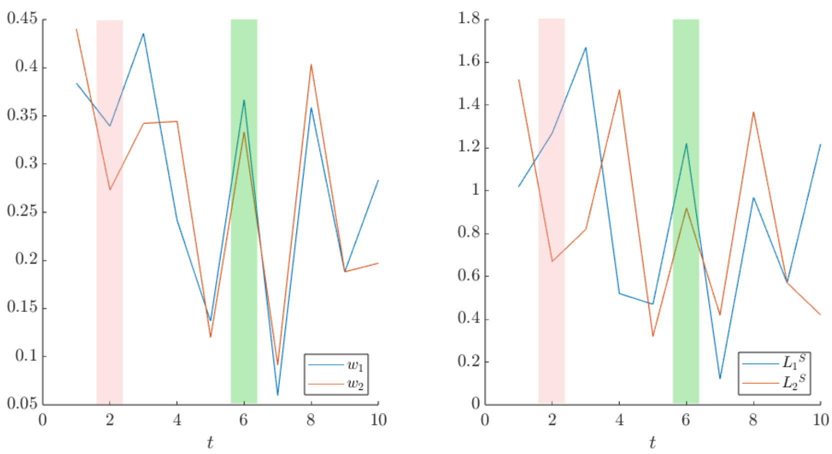


Figure 1.4: Dynamic simulation of labor supplies and wages

In Figure 1.4, in each period, a random allocation is drawn, this determines labor supply and therefore wages[3]. The red rectangle highlights opposite movements in labor supplies, resulting in the same movement in wages. On the other hand, the green rectangle depicts the same movements in labor supplies, which correspond to identical changes in wages.

Even with some simplification, this simple example motivates why non-pecuniary benefits may be relevant at macro level and what can be the potential effects. However, it lacks on some aspect: it does not investigate the motives of these shifts and/or does not take into account other aspects such as wealth. In order to make a more complete picture on this topic, it is therefore needed a more complete analysis, which is the purpose of this work.

[3]In this example I assume a Cobb-Douglas production function $f(L_1, L_2)$ with same labor share.

Chapter 2

Literature Review

This work is intricately connected with the extensive literature on skills and self-selection, with foundational insights drawn from Roy 1951. Roy challenges the prevalent notion that income distribution is arbitrary, contending that it is profoundly influenced by skill heterogeneity and productivity dynamics. In his model, individuals have the flexibility to choose between different occupations, such as hunting and fishing. The resulting distribution of earnings emerges from a selective process grounded in comparative advantages. Heckman and Honore 1990 subsequently undertakes the task of formalizing and estimating the Roy model. Their key finding is that the selection process endures when differences in skills follow a log-concave distribution.

Furthermore, this thesis establishes a crucial link with research that seeks to unravel the intricate relationship between earnings and skill heterogeneity, as extensively analyzed by Acemoglu and Autor 2011. Their insightful observation notes that, despite a growing number of college-educated workers, the return to skills, measured by relative wages, continues its upward trajectory for college graduates in comparison to high school graduates. This underscores the pivotal connection between skill demand and technology, where advancements create an increasing need for highly skilled workers. This phenomenon is evident in various sectors, notably in manufacturing and automation.

In addition to these themes, the work ventures into the rich literature surrounding labor market fluctuations. The historical debate between Keynes and Lucas on the relationship between employment and real wages serves as a backdrop. Models such as Christiano and Eichenbaum 1992 accentuate intertemporal labor-leisure substitution, while Real Business Cycle models focus on technology shocks, resulting in diverse predictions regarding the behavior of real wages over the business cycle. Empirical evidence, highlighted by seminal works like Bodkin 1969 and Sargent 1978, has yielded mixed results, underscoring the intricate and multifaceted nature of the relationship and emphasizing the critical importance of considering underlying assumptions when interpreting empirical findings.

Finally, the work is inspired by, Grigsby 2022, which investigates the relationship between skill heterogeneity and aggregate labor market dynamics. The paper focuses on the study of the aggregate labor market, considering a scenario where workers possess varying skills for different tasks and are subjected to non-uniform labor demand shocks. When workers exhibit diverse skill sets, changes in aggregate wages reflect not only shifts in overall labor demand but also the redistribution of workers across tasks and into employment. The author identifies three main effects that occur when workers differ in skills and labor demand shocks impact job availability. To illustrate this point, the author presents the following equation, demonstrating the general presence of these three effects:

Let $\bar{\omega}$ be the aggregate wage and $\bar{E}$ the aggregate employment. His economy is composed by J types of workers with mass m_j. Such workers can be employed into K different occupations. Moreover, let $d\ln\mathbf{z}$ be some arbitrary shock to labor demand, then the first order response of employment to this shock is:

$$\frac{d\ln \mathrm{E}}{d\ln \mathbf{z}} = \sum_j \underbrace{\left(\frac{m_j\tilde{E}_j}{\bar{E}}\right)}_{\text{Employment Share of j}} \sum_k \underbrace{\left(\frac{E_{jk}}{\tilde{E}_j}\right)}_{\substack{\text{Employment}\\ \text{Share of k}\\ \text{amongst type j}}} \underbrace{\frac{d\ln \mathrm{E}_{jk}}{d\ln \mathbf{z}}}_{\substack{\text{Elasticity of } E_{jk}\\ \text{to shock}}}$$

From this equation we understand that the response of aggregate employment to a shock is larger if the shock affects workers and jobs with more elastic employment and which constitute a larger share of employment. Instead, on aggregate wages we have:

$$\frac{d\ln \bar{\omega}}{d\ln \mathbf{z}} = \sum_j \left(\frac{m_j\tilde{E}_j}{\bar{E}}\right)\left(\frac{\tilde{\omega}_j}{\bar{\omega}}\right)\left(\underbrace{\frac{d\ln \tilde{E}_j}{d\ln \mathbf{z}} - \frac{d\ln \bar{E}}{d\ln \mathbf{z}}}_{\text{Composition Effect}} + \sum_k \underbrace{\left(\frac{E_{jk}\omega_{jk}}{\tilde{E}_j\tilde{\omega}_j}\right)}_{\substack{\text{Earnings}\\ \text{Share of k}\\ \text{amongst type j}}}\left(\underbrace{\frac{d\ln E_{jk}}{d\ln \mathbf{z}} - \frac{d\ln \tilde{E}_j}{d\ln \mathbf{z}}}_{\text{Reallocation Effect}} + \underbrace{\frac{d\ln \omega_{jk}}{d\ln \mathbf{z}}}_{\text{Direct Effect}}\right)\right) \tag{2.1}$$

Therefore, the direct effect implies that if the shock increases the earnings of workers within their job, wages will rise. The reallocation effect arises if workers move to different jobs in response to the shock. The composition effect is positive if workers who are elastic to the shock have average high wages relative to the aggregate wage. Composition and reallocation effects are present only if we have heterogeneity in workers and jobs: with a representative agent model only the direct effect operates. What is important to notice here is that, reallocation and composition effects could operate in a direction that is opposite to the direct effect. If these two effects are strong enough, aggregate wages could fall even if every worker types realizes wage gains within every job type. The implication is that, demand shocks might induce aggregate employment increases even while aggregate wages are stable or falling.

This paper holds significance for three key reasons. Firstly, the model presented has the ability to replicate certain statistics from the Great Recession within a frictionless economy, and this is important to keep in mind since all the models that try to replicate countercyclical wages often features frictions. Specifically, when provided with a sequence of industry-specific profitability shocks, it successfully reproduces the observed changes in employment and wages during that period. In the Great Recession, real wages increased by 2.7% while hours worked declined by 8.8%. The model generates wage increases of 2.5% and employment declines of 10%. Secondly, the paper captures the heterogeneity in workers' skills by calibrating a skill matrix by taking the idea of Roy 1951. Lastly, following the approach of Rosen 1986 and Sorkin 2018, the paper introduces idiosyncratic preference shocks for each occupation, which result in non-pecuniary benefits for workers, known as compensating differentials. However, this is a static picture. In the following chapters, my attempt is to extend this static model by deriving a simpler dynamic version that takes into account the intertemporal consumption-saving decision, together with occupational choice.

Chapter 3

Theoretical Framework

3.1 Workers

In this section I present a framework that can accommodate a variety of environment. Time is discrete[1] and the economy is populated by a continuum of agents in the interval $[0, 1]$.

Let $\mathcal{I} = \{1, ..., I\}$ be a finite set of types and $\mathcal{J} = \{1, ..., J\}$ a finite set of possible occupations. We can think about I as a classification of individuals in terms of various attributes such as skill level or education. For instance if we have two types we can think as Blue vs White Collars or Non-Graduated vs Graduated.

The heterogeneity in skill is captured by the fact that every type can supply a different efficient unit of labor in each possible occupation. Let $\Gamma_i = [\gamma_{i,1}, ..., \gamma_{i,J}]$ be a vector of possible efficient unit of labor for type $i \in \mathcal{I}$ in every possible occupation and let $w = [w_1, ..., w_J]$ be a vector of wage per efficient unit of labor in each occupation. For each individual in the economy we can define the current real income $\omega_{i,j}$ as:

$$\omega_{i,j} = w_o \gamma_{i,j} \tag{3.1}$$

Each type $i \in I$ has measure m_i and the following identity must hold:

$$\sum_{i=1}^{I} m_i = 1 \tag{3.2}$$

So that we can interpret m_i as the fraction of types i in the economy. At any point in time, the worker observes his current wealth a_i and the prevailing wages in the market that are in each occupation. When taking the occupational choice, workers are hitted by a preference shock $\phi_{i,o}$ which is introduced for two reasons: first, in this framework the occupational choice is not driven only by wage differentials across occupations, but it is also driven by other characteristics which could be job amenities, non-pecuniary motives, motivation and so on. Second, in real life, those variables are not always observables, so I want to treat them as an unobserved state variable that drives the occupational choice.

Denote with $\phi_i = [\phi_{i,0}, .., \phi_{i,J}]$ a vector of preference shocks, then every type $i \in I$ solves the following problem:

[1]Since the environment is stationary, I suppress the use of the time subscript and use a recursive notation

$$V_i(a_i,\varepsilon_i,\phi_i) = \max_{c_i\in C,\, a_i'\in A,\, j_i\in\mathcal{J}} \{u(c_i) + \phi_{i,j} + \beta\mathbb{E}\left[V_i(a_i',\varepsilon_i',\phi_i')|a_i,\varepsilon_i,\phi_i,j_i,c_i\right]\}$$

$$\begin{aligned} \text{s.t.}\quad & a_i' = (1+r)a_i + \omega_{i,j} - c_i \\ & \varepsilon_i' = \rho\varepsilon + \eta' \\ & a_i' \geq 0 \\ & c_i \geq 0 \\ & \phi_{i,j} \sim EV1(0,\sigma_\phi) \qquad\qquad \forall\, j \in J \\ & \eta' \sim \mathcal{N}(0,\sigma_\eta) \end{aligned} \tag{3.3}$$

The expectation of the continuation value is taken with respect to the transition density $f\left(a_i',\varepsilon_i',\phi_i'|a_i,\varepsilon_i,\phi_i,j_i,c_i\right)$. Following Rust 1987 I make a conditional independence assumption that allows to decompose the transition density in the following way:

$$f\left(a_i',\varepsilon_i',\phi_i'|a_i,\varepsilon_i,\phi_i,j_i,c_i\right) = g(\phi_i'|a_i',\varepsilon_i')h(a_i',\varepsilon_i'|a_i,\varepsilon_i,j_i,c_i) \tag{3.4}$$

By the budget constraint a_i' does not depend on ε_i' and we can rewrite:

$$f\left(a_i',\varepsilon_i',\phi_i'|a_i,\varepsilon_i,\phi_i,j_i,c_i\right) = g(\phi_i')h(\varepsilon'|\varepsilon)\mathbb{1}(a_i'|a_i,w,\varepsilon,j_i,c_i) \tag{3.5}$$

For each $o \in \mathcal{O}$ we can define the choice specific value function to be:

$$v_i(a_i,\varepsilon_i,j_i) \max_{c_{i,j}\in C,\, a_{i,j}'\in A} \{u(c_i) + \beta\mathbb{E}\left[V_i(a_i',\varepsilon_i',\phi_i')|a_i,\varepsilon_i,\phi_i,j_i,c_i\right]\} \tag{3.6}$$

By doing so we can rewrite the value function as:

$$V(a_i,\varepsilon_i,\phi_i) = \max_{j_i\in\mathcal{J}} \{v_i(a_i,\varepsilon_i,j_i) + \phi_{i,j}\} \tag{3.7}$$

Therefore the solution to this problem is a discrete occupational choice for $i \in I$ is:

$$j_i \in \arg\max_{j_i\in\mathcal{J}} \{v_i(a_i,\varepsilon_i,j_i) + \phi_{i,j}\} \tag{3.8}$$

together with choice specific continuous policy functions $\{c_i(a_i,\varepsilon_i,j_i),\ a'(a_i,\varepsilon_i,j_i)\}\ \forall\ j_i \in \mathcal{J}$.

3.1.1 Conditional Choice Probabilities

In order to make the problem tractable, I assume that ϕ_i is distributed as an Extreme Value Type 1 distribution with variance σ_ϕ. This assumption is usual in discrete choice models and have the advantage of delivering closed form solutions (McFadden et al. 1973) for the expectation of the value function with respect to ϕ_i'. If we take the total expectation of equation (3.7) using the transition density, we have:

$$\mathbb{E}[V(a_i,\varepsilon_i,\phi_i)] = \int_{\mathcal{E}}\int_{\Phi} \max_{j_i\in\mathcal{J}} \{v_i(a_i,\varepsilon_i,j_i) + \phi_{i,j}\}\, g(\phi_i)h(\varepsilon_i|\varepsilon_i^-)d\varepsilon_i d\phi_i \tag{3.9}$$

Where $\mathcal{E}$ and Φ are the supports of ε and ϕ. The distributional assumption on ϕ_i' allows us to define the following expectation function:

$$EV(a_i, \varepsilon_i) = \sigma_\phi \left(log \left(\sum_{j \in \mathcal{J}} exp \left(\frac{v_i(a_i, \varepsilon_i, j_i)}{\sigma_\phi} \right) \right) \right) \tag{3.10}$$

Which is the expectation of the value function with respect to the taste shock and has the advantage of reducing the state space. By using this function and taking the expectation with respect to the idiosyncratic shock, the expectation of the continuation value in equation (7) can be written as:

$$\mathbb{E}[V(a_i', \varepsilon_i', \phi_i')] = \int_{\mathcal{E}} EV(a_i', \varepsilon_i') h(\varepsilon_i' | \varepsilon_i) d\varepsilon_i \tag{3.11}$$

By Rust 1987 we know that the EV function defines an operator that is a contraction:

$$T(EV)(a_i, \varepsilon_i) = \sigma_\phi \left(log \left(\sum_{j \in \mathcal{J}} exp \left(\frac{\max_{c_{i,j} \in C, a_{i,j}' \in A} \left\{ u(c_i) + \beta \int_{\mathcal{E}} EV(a_i', \varepsilon_i') h(\varepsilon_i' | \varepsilon_i) d\varepsilon_i \right\}}{\sigma_\phi} \right) \right) \right)$$

Therefore EV^* is the solution to the functional equation $T(EV^*) = EV^*$ and can be found by solving the relative fixed point problem.

Working with the expected value function in this settings allows us to obtain conditional choice probabilities on occupations that, thanks to the Extreme Value assumption, have a closed logit form:

$$\theta_i(j_i | a_i, \varepsilon) = \frac{exp(v_i(a_i, \varepsilon_i, j_i))}{\sum_{j \in \mathcal{J}} exp(v_i(a_i, \varepsilon_i, j_i))} \quad \forall \; j_i \in \mathcal{J} \tag{3.12}$$

3.2 Firms

The production side of this economy is left as simple as possible for tractability: there is a representative firm which has constant returns to scale production function. There is no aggregate uncertainty and the firm produce accordingly to:

$$Y_t = {K_t}^\alpha {L_{1,t}}^\omega {L_{2,t}}^\eta \tag{3.13}$$

The produced good is a numeraire and the price is normalized to be 1. Firm chooses capital and labor in order to solve the following profit maximization problem:

$$\Pi \quad = \max_{\{K_t, L_{1,t}, L_{2,t}\}} Y_t - R_t K_t - w_{1,t} L_{1,t} - w_{2,t} L_{2,t} \tag{3.14}$$

with δ being the depreciation rate of the capital and $R_t = r_t + \delta$. Therefore competitive factor prices are determined by:

$$\begin{aligned} r_t &= \alpha {K_t}^{\alpha-1} {L_{1,t}}^\omega {L_{2,t}}^\eta - \delta \\ w_{1,t} &= \omega {K_t}^\alpha {L_{1,t}}^{\omega-1} {L_{2,t}}^\eta \\ w_{1,t} &= \eta {K_t}^\alpha {L_{1,t}}^\omega {L_{2,t}}^{\eta-1} \end{aligned} \tag{3.15}$$

3.3 Aggregate Measures

The aggregate law of motion of the economy is defined as:

$$C + K' = Y + K(1-\delta) \tag{3.16}$$

Define $\mu(a,\varepsilon)$ to be a measure of the households that are in state $(a_t = a, \varepsilon_t = \varepsilon)$ and $\mu(a,\varepsilon,j) = \mu^j(a,\varepsilon)$ the same conditional on occupation measure, i.e., in state $(a = \bar{a}, \varepsilon_t = \bar{\varepsilon}, j = \bar{j})$. The following identity holds:

$$\mu(a,\varepsilon) = \sum_{j=1}^{J} \mu(a,\varepsilon,j) \qquad \forall\ a_t = a, \varepsilon_t = \varepsilon \tag{3.17}$$

Given such measures and choice probabilities, one can define the following aggregate measures:

$$\begin{aligned} A' &= \sum_{i=1}^{I} m_i \sum_{j=1}^{J} \int a'(a,\varepsilon,j)\theta_i(j|a,\varepsilon)d\mu(a,\varepsilon) \\ C &= \sum_{i=1}^{I} m_i \sum_{j=1}^{J} \int c_j(a,\varepsilon,j)\theta_i(j|a,\varepsilon)d\mu(a,\varepsilon) \end{aligned} \tag{3.18}$$

Where A' is the aggregate level of savings, while C_t is the aggregate consumption. Notice that since the economy has a unit mass over types, then they coincide with per-capita measures.
Effective labor supplies are defined as:

$$L^S_{j,t} = \sum_{i=1}^{I} m_i \gamma_{i,1} \int \varepsilon_t \theta_i(j|a,\varepsilon)d\mu(a,\varepsilon) \tag{3.19}$$

A **Competitive Equilibrium** in this economy is:

1. A sequence of optimal action pair $\{c_t, a_{t+1}\}_{t=0}^{\infty}$ that solves the households problem given a_0, the distribution of ε_{t+1} and $\phi_{\mathbf{t+1}}$, the realization of ε_t and $\phi_{\mathbf{t}}$ and prices r_t, $w_{1,t}$, $w_{2,t}$
2. The sequence of $\{\theta_t(o = j \mid a_t, \varepsilon_t)\}_{t=0}^{\infty}$ solves the occupational choice problem for every households
3. Firms optimally choose $\{K_t, L_{1,t}, L_{2,t}\}_{t=0}^{\infty}$
4. Aggregate Demand equals Aggregate Supply and Market clears

3.3.1 Parametrization of the Numerical Exercise

The following table summarizes the baseline parametrization for the model:

Table 3.1: Baseline parametrization

Parameters	Description	Value
Skills and occupations		
I	Number of types in the economy	2
J	Number of occupation in the economy	2
M_i	Mass of households of type $i \in I$	$\frac{1}{I} = 0.5$
Firms		
α	Capital Share	0.34
ω	Occupation 1 Share	0.33
η	Occupation 2 Share	0.33
δ	Capital Depretiation	0.03
Households		
β	Time Discount Factor	0.9615
γ	Inverse Elasticity of Intertemporal Substitution	2
ρ	Persistency Parameter of the AR(1) Income process	0.9
σ_ε^2	Variance of the AR(1) Income process	0.2
B	Borrowing limit	0
μ	Location Parameter for Taste Shock	0
σ	Scale Parameter for Taste Shock	1

The skill matrix is:

$$\Gamma = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 1.5 \end{bmatrix}$$

I discretize the income process into two states $\left(\varepsilon^L, \varepsilon^H\right) = (0.5709, 1.4291)$ using Tauchen 1986's method with transition matrix:

$$\Pi = \begin{bmatrix} 0.9371 & 0.0629 \\ 0.0629 & 0.9371 \end{bmatrix}$$

This parametrization will be the maintained thoughout the work. Notice that, in order to simplify the exposition, I impose symmetry in the problem with the skill matrix and occupational shares, so that consideration for type 1 are the opposite for type 2. Unfortunately, at the current stage of the work, the parametrization does not match some particular empirical fact, but is used in order to have a qualitative exposion of the model.

Chapter 4

Decision Rules Analysis

Recall that the value function in the problem is:

$$V(a_i, \phi_i, \varepsilon_i) = \max_{o_i \in O} \{v_i(a_i, \varepsilon_i, o_i) + \phi_{i,o}\} \tag{4.1}$$

and the associated occupational decision rule is:

$$o_i \in \arg\max_{o_i \in O} \{v_i(a_i, \varepsilon_i, o_i) + \phi_{i,o}\} \tag{4.2}$$

Figure 4.1 depicts the household problem solution. We observe that the household may face constraints depending on their occupation. Specifically, when the household is engaged in the occupation where they possess lower skills and, receive lower labor income, both the asset and consumption policies exhibit a kink. In the region before the kink, it is important to note that the Euler's equation holds with inequality, implying that it is optimal for the household to refrain from saving assets and instead consume everything. On the other hand, when the occupational choice aligns with the occupation in which the household is more productive and earns an higher labor income, constraints are no longer present. Consequently, the household can continue accumulating assets and consume while satisfying the Euler's equation with equality.

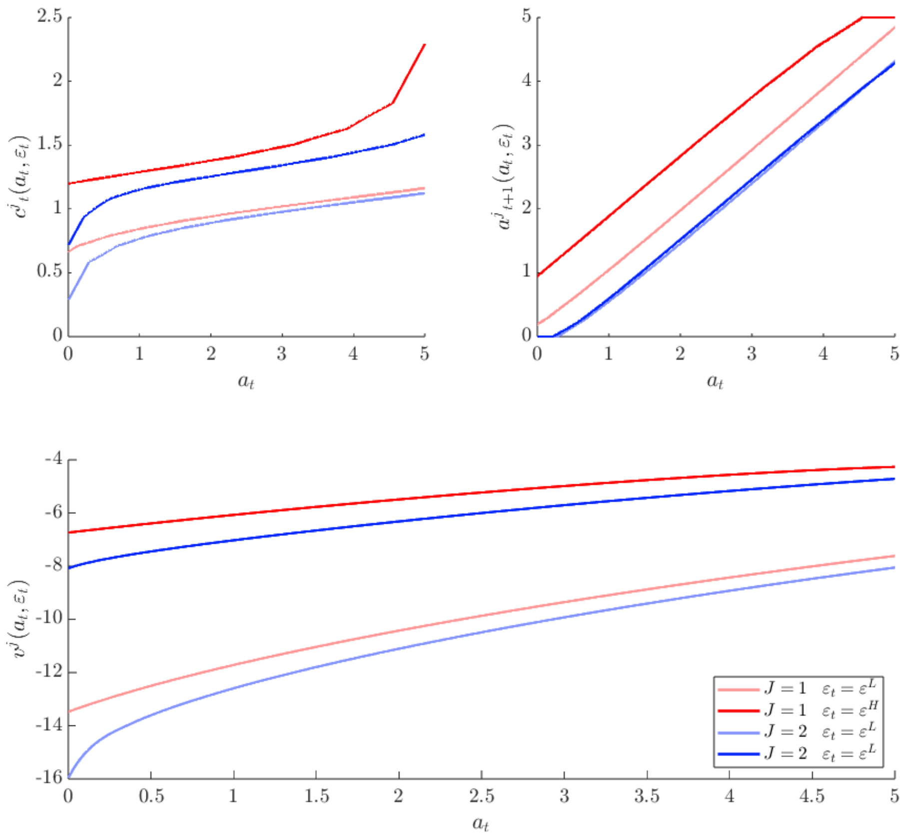
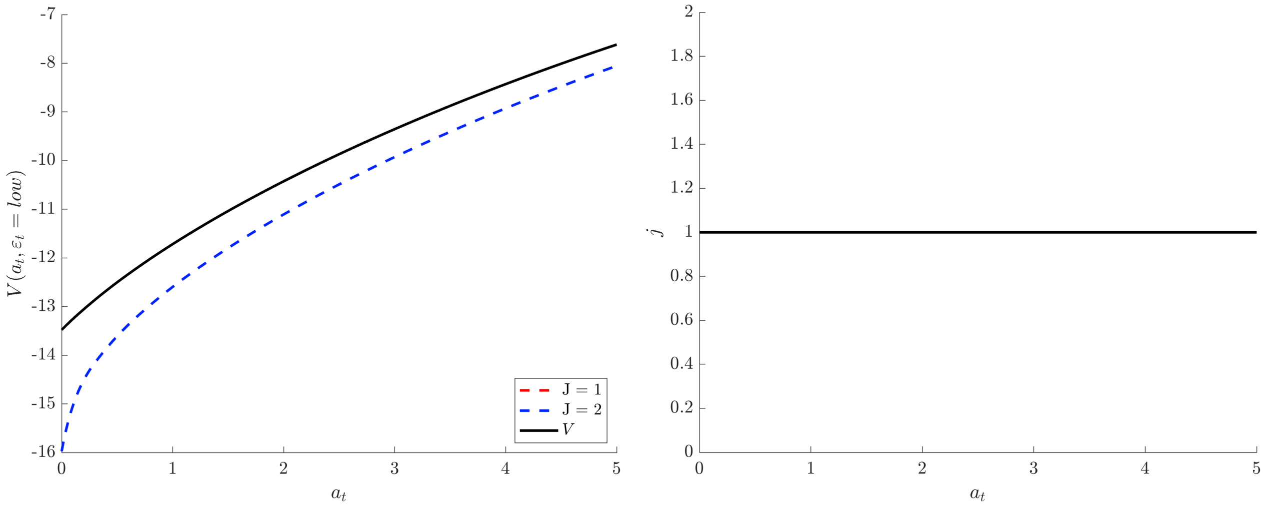


Figure 4.1: Value, Consumption and Asset functions conditional on occupation and on idiosyncratic shock

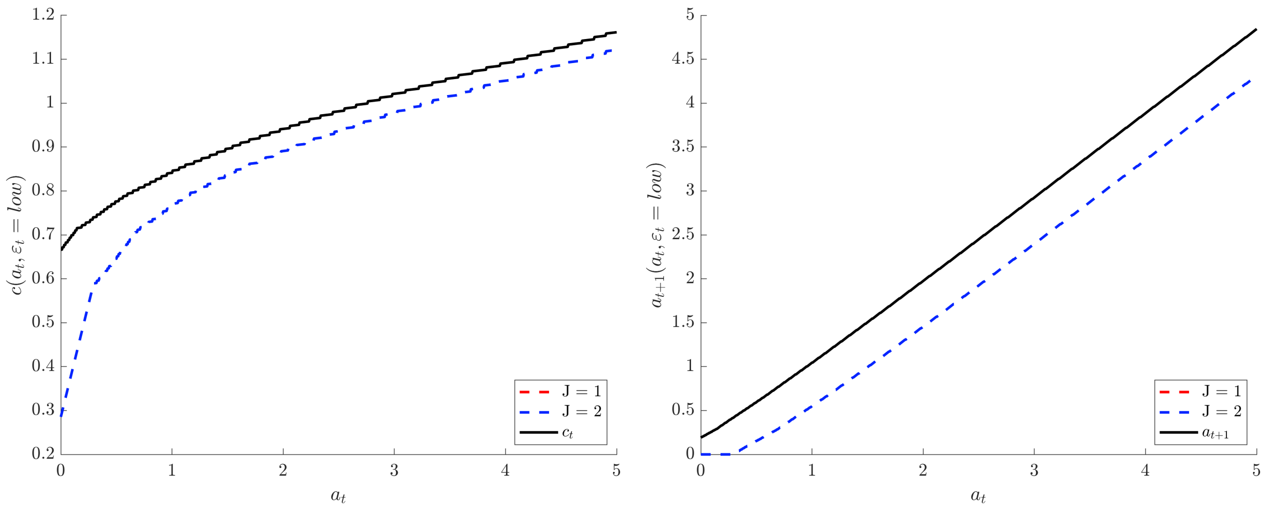
This figure however does not take into account any taste shock. Indeed, conditional on non-pecuniary motives, the Value Function, and the decision rules, may exhibit discontinuities. To provide a qualitative description of agents behavior in this economy, I present three distinct cases.

Case 1 ($\phi_1 = 0$, $\phi_2 = 0$): in Figure 4.2, there is complete separation between the two Value Functions, resulting in a continuous decision rule favoring occupation 1. This outcome is expected: given that type 1 individuals are more productive in $J = 1$ an therefore have higher wage, it is optimal for them to persist in occupation 1, regardless of their level of wealth and in absence of non-pecuniary motives. The same conclusion holds when considering $\varepsilon_t = \varepsilon^H$.



(a) $\phi^1 = 0$ and $\phi^2 = 0$ (b) Decision Rule

Figure 4.2: Value Function and Decision Rule for type 1



(a) Consumption Policy (b) Asset Policy

Figure 4.3: Policy Functions

Case 2 ($\phi_1 = 0$, $\phi_2 = 0.8$): instead, in Figures 4.4 and 4.5, a positive taste shock is associated to occupation 2. This taste shock creates a discontinuity both in the value and decision functions. Depending on the level of wealth, it may not be optimal to stay in occupation 1. If wealth does not influence occupational choices, we would expect the individual to select occupation 2 purely based on taste motives, regardless of their level of wealth (considering that occupation 2 has lower productivity and therefore lower wages). However, in practice, we see that when type 1 individuals are "poor," i.e., when $a_t \leq 1.30$, it is optimal for them to remain in the occupation that provides a higher wage. This suggests that the non-monetary characteristics captured by the taste shock in occupation 2 are not sufficient to offset the loss in consumption resulting from their lower wealth. However, when the individual becomes "rich" enough, i.e., when $a_t > 1.30$, the non-monetary characteristics of occupation 2 start to matter more. At this point, even though it leads to a lower consumption level, it becomes optimal to switch to occupation 2, since there is a compensation at utility level. This theoretical explanation could potentially shed light on the systematic observation of people transitioning to lower-paying jobs in Sorkin 2018.

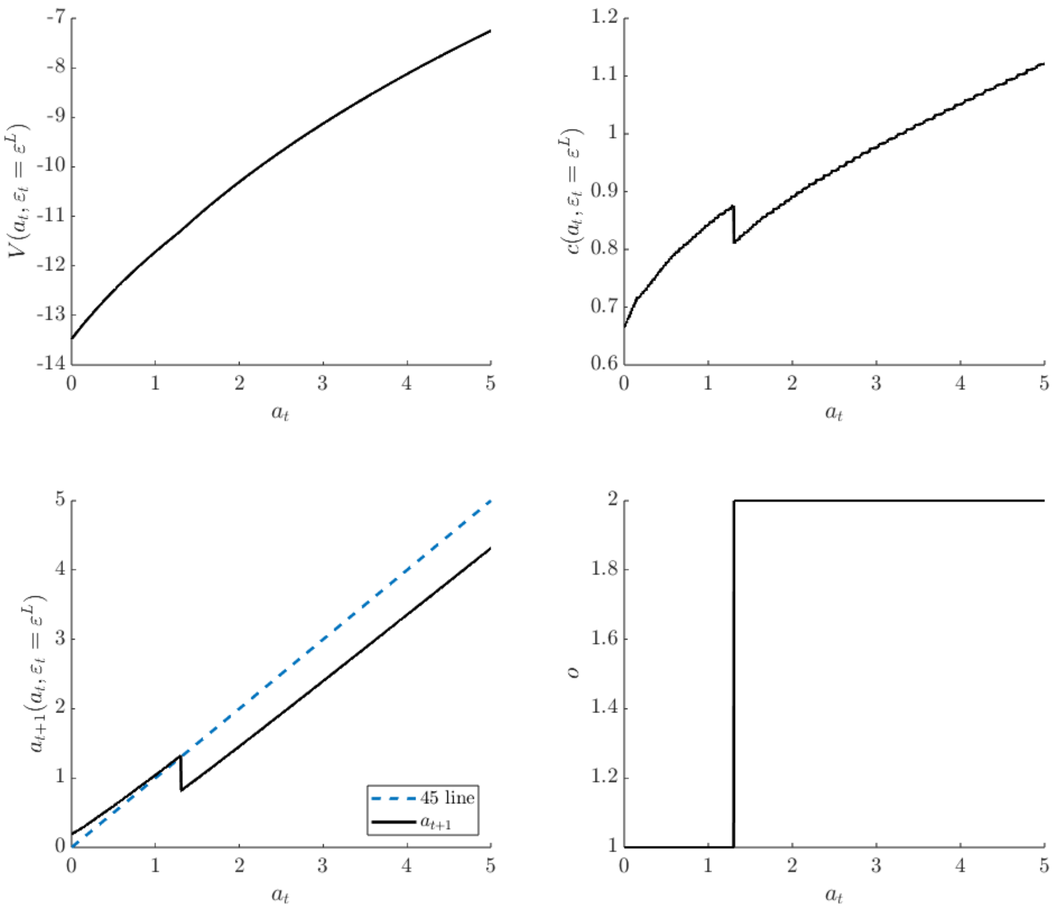


Figure 4.4: Value, Consumption and Asset functions in the low state of the idiosyncratic risk

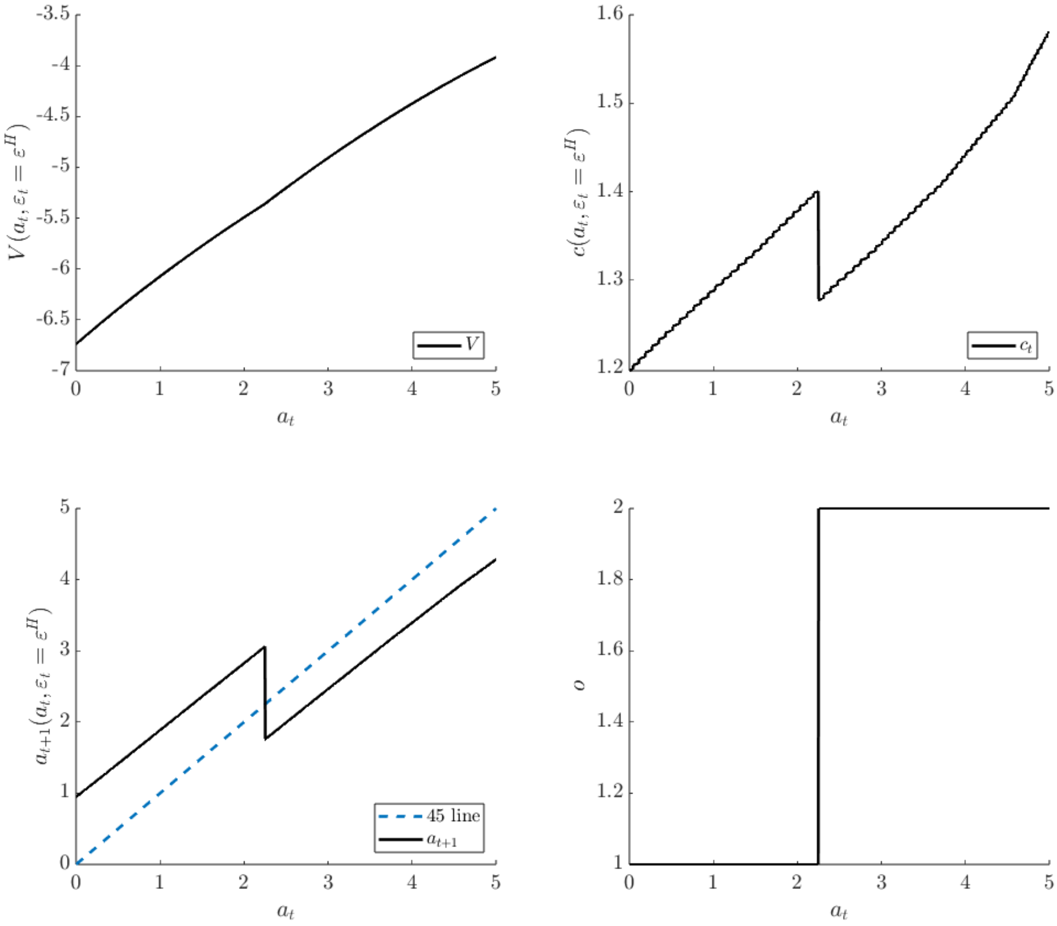


Figure 4.5: Value, Consumption and Asset functions in the high state of the idiosyncratic risk

This picture, only gives us an idea of the decision rule at fixed point in time and conditional on a specific state of the economy. It is also interesting examine the dynamics of asset accumulation since, the discontinuity in the decision functions could lead to non-trivial dynamics:

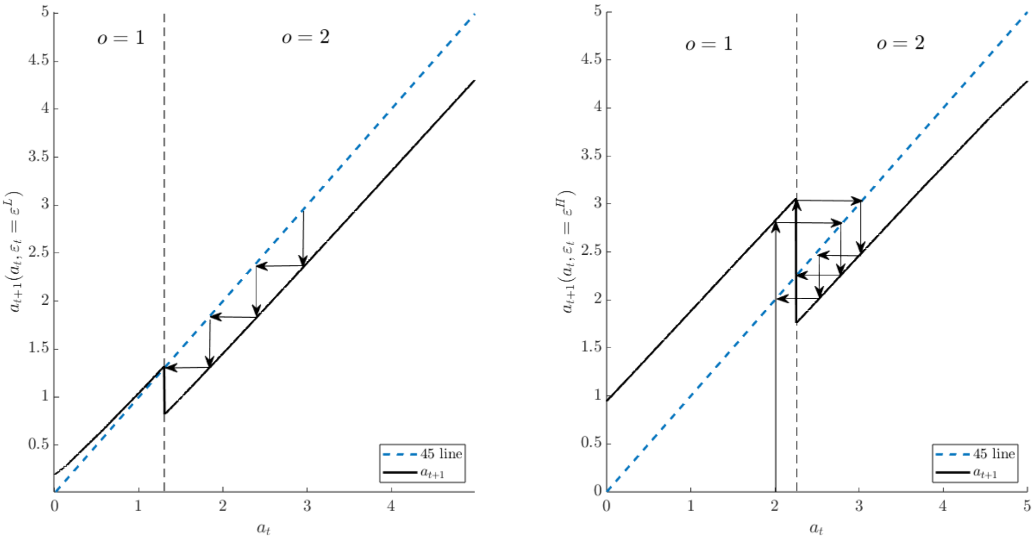


Figure 4.6: Wealth accumulation and occupational decision

In Figure 4.6, I present two wealth paths for a given initial condition while keeping the taste and idiosyncratic shocks fixed. In the left plot, the household begins with an initial wealth of $a_t = 3$ and faces a low idiosyncratic state. In this scenario, it is optimal for the household to choose occupation 2 and gradually consume their wealth until they become "poor" and switch to occupation 1. Furthermore, in this specific case, the occupational switch occurs at the fixed point of the asset policy function, resulting in convergence towards occupation 1. Generally, it is visually apparent that, regardless of the initial condition, and with all other factors held constant, the household will always converge to a stationary state where they remain in occupation 1.

On the other hand, the right plot maintains the same taste shock but assumes a high idiosyncratic state. In this situation, the wealth dynamics is considerably more complex. As seen in the figure, the discontinuity in the asset function implies that no fixed point exists for this policy. However, even if a fixed point is absent, the function is first above and then below the 45° line. Given an initial wealth condition, the agent wealth path will continuously cycle, frequently switching between occupation 1 and occupation 2. In general, when introducing idiosyncratic and preference shocks, one cannot guarantee the existence of a fixed point for the asset function and cannot rule out explosive wealth dynamics, posing in this setting a challenge for the stationarity and the property of the subsequent wealth distribution.

4.1 Occupational Probabilities

The previous section gave an intuition of the static decision rule and what can happen for an individual along the dynamics of wealth accumulation. The whole picture can be enriched by analyzing some properties of occupational probabilities. As a remainder, the occupational probabilities are given by the following formula:

$$\theta_i(o_i|a_i, \varepsilon) = \frac{exp(v_i(a_i, \varepsilon_i, o_i))}{\sum_{o \in \mathcal{O}} exp(v_i(a_i, \varepsilon_i, o_i))} \quad \forall\ o_i \in O \tag{4.3}$$

In Figure 4.7 we have the occupational probabilities for type 1 in both the occupation and idiosyncratic risk state.

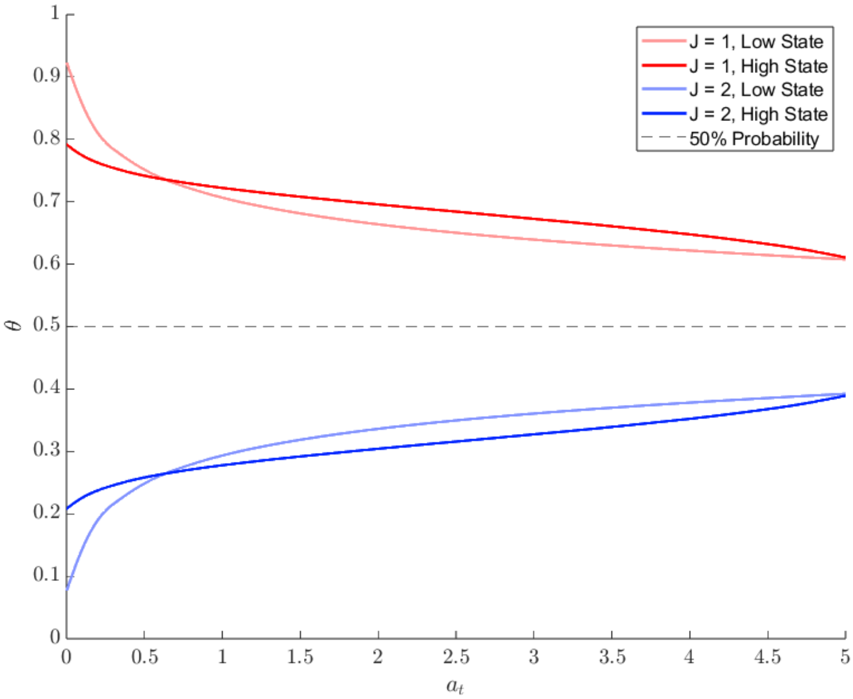


Figure 4.7: Type 1's Occupational Probabilities

By a law of large number argument, for a given level of wealth, we can interpret this probability both as the probability of switching occupation or as the fraction of people that will be in that occupation. In general, the likelihood of switching in the occupation which pays the higher wage is always higher. Qualitatively this is coherent with the empirical finding that job switchs occurs the most when the destination wage is higher (Sorkin 2018).

However, in the mobility pattern we also observe a fraction of people that switch to less paying job. In this theoretical framework, it can be explained by noting the strong link between skill heterogeneity and non-pecuniary motives attached to the occupation. When type 1 is "poor", the likelihood of switching to the occupation that pays more is higher relative to $J = 2$. Idiosyncratic risks make this likelihood even higher. Again, this is not suprising, if type 1 is really poor, in order to maximize his utility he must go in the occupation that pays more to sustain his optimal level of consumption. However, as he gets richer, non-pecuniary motives start to matter more, and we observe more frequently switches to the occupation that pays a lower wage. To illustrate better why this is the case and to provide some more insights, consider again the occupational probability, by fixing for simplicity ε_t so that we can drop the expectation[1]:

$$\theta_t(o = 1 \mid a_t, \varepsilon_t) = \frac{exp\left(v^1(a_t, \varepsilon_t)\right)}{exp\left(v^1(a_t, \varepsilon_t)\right) + exp\left(v^2(a_t, \varepsilon_t)\right)} \tag{4.4}$$

I'm interested in analyzing the effect of a change of wealth, therefore, after some computation, we obtain this derivative:

[1] However, the result is still valid if the expectation remains

$$\frac{\partial \theta_t(o=1 \mid a_t, \varepsilon_t)}{\partial a_t} = \underbrace{\theta_1(1-\theta_1)}_{\text{Variance of job switching}} \underbrace{\left(\frac{\partial v^1}{\partial a_t} - \frac{\partial v^2}{\partial a_t}\right)}_{\text{Marginal loss from job switching}} \tag{4.5}$$

From this equation we recognize two things. First, the direction of the probability after an increment in the wealth level depends directly on the variance of job switching[2]. Such variance depends on the structural parameters of the Gumbel distribution, in particular to the scale parameter which is related to the variance of non-pecuniary motives. As the variance of occupational switching goes to 0, an increment in the wealth has no effect on the occupational probability.

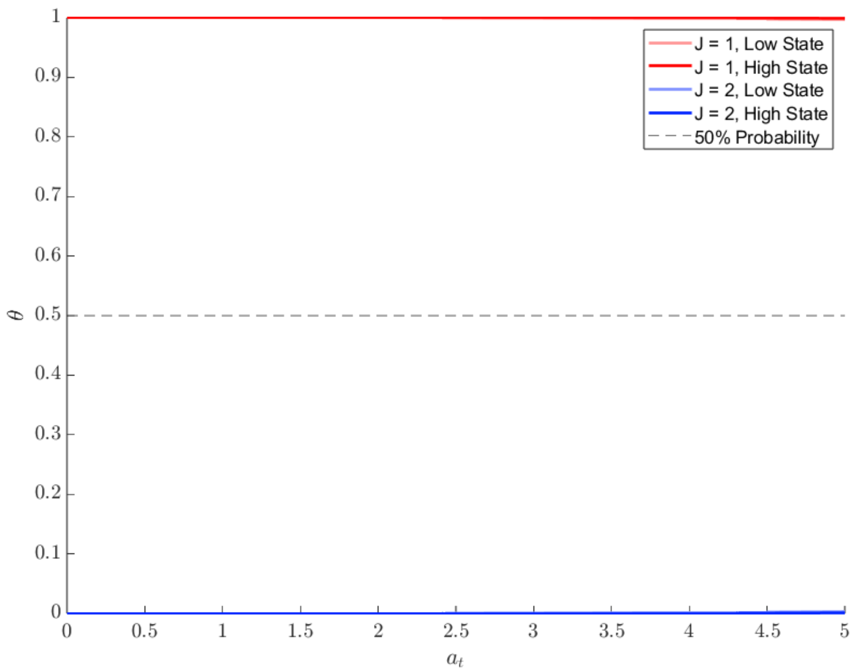


Figure 4.8: Type 1's Occupational Probabilities with $\sigma = 0.05$

On the other hand, as long as the variance increase, the effect of an increase in wealth to the occupational probability increase as well.

Second, such variation in probability depends also on the marginal loss in utility due to consumption that occurs when switching jobs. In Figure 4.7 we see that the probability of going in occupation 1 (occupation 2) is a decreasing (increasing) function of the asset level, which is the case if we have:

$$\frac{\partial v^1}{\partial a_t} \leq \frac{\partial v^2}{\partial a_t} \tag{4.6}$$

From the envelope condition we have:

$$\frac{\partial u(c_t^1)}{\partial c_t^1} \frac{\partial c_t^1}{\partial a_t} \leq \frac{\partial u(c_t^2)}{\partial c_t^2} \frac{\partial c_t^2}{\partial a_t} \tag{4.7}$$

$$\partial c^1 u'(c^1) \leq \partial c^2 u'(c^2) \tag{4.8}$$

[2]Since by the parametrization we have two occupation, we can interpret the random variable occupation as a Bernoulli.

In other words, as wealth increases, the likelihood of selecting occupation 1 decreases, provided that the change in consumption associated with occupation 1, weighted by marginal utility, is smaller than the change in consumption associated with occupation 2. Conversely, if an increase in wealth leads to a greater increase in consumption in occupation 2 compared to occupation 1, it results in an increased probability of choosing occupation 2. Therefore wealth plays a fundamental role, joint with non pecuniary benefits, on occupational choices. Figure 4.9 shows graphically the right and left hand side of Equation 4.8.

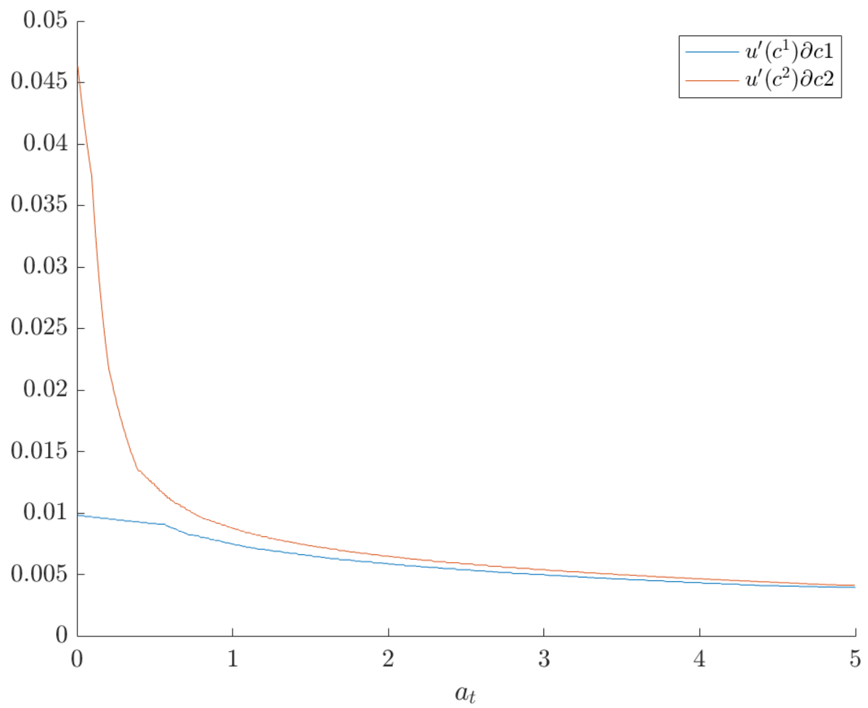


Figure 4.9: Consumption Weighted Marginal Utility in Both Occupations

This approximation shows clearly that, for low level of asset such difference is big, and so it is the curvature of occupational probabilities. Instead, with higher wealth level, this difference approaches to zero, that is the region for which the choice probabilities become more flat.

One should keep in mind that this is a partial equilibrium analysis, therefore prices effects are totally ignored. The picture becomes more rich if we consider prices effect. In particular, let $\mu_t(a_t, \varepsilon_t)$ a measure on the state space and let $w_{1,t}(\mu_t)$, $w_{2,t}(\mu_t)$ and $r_t(\mu_t)$ be the prices. Recall that, for a type i, conditional on occupation, the budget constraint is:

$$c_t^j = (1 + r_t(\mu_t))a_t + w_{j,t}(\mu_t)\gamma_j\varepsilon_t - a_{t+1}^j$$

Therefore we have:

$$\frac{\partial c_t^j}{\partial a_t} = (1 + r) + \frac{\partial \mu_t}{\partial a_t}\left(\frac{\partial r_t}{\partial \mu_t} + \gamma_j\varepsilon_t\frac{\partial w_{j,t}}{\partial \mu_t}\right) - \frac{\partial a_{t+1}^j}{\partial a_t}$$

If we define the savings as $s_t^j = a_{t+1}^j(a_t, \varepsilon_t) - a_t$, the marginal propensity to save is $\frac{\partial s_t^j}{\partial a_t} = MPS^j = \frac{\partial a_{t+1}^j}{\partial a_t} - 1$, therefore if we substitute we obtain:

$$\frac{\partial c_t^j}{\partial a_t} = MPC^j = r(\mu_t) + \frac{\partial \mu_t}{\partial a_t}\left(\frac{\partial r_t}{\partial \mu_t} + \gamma_j \varepsilon_t \frac{\partial w_{j,t}}{\partial \mu_t}\right) - MPS^j$$

So, when we take prices effects into account, the picture becomes more complete.

$$\frac{u'(c^1)}{u'(c^2)} \leq \frac{r(\mu_t) + \frac{\partial \mu_t}{\partial a_t}\left(\frac{\partial r_t}{\partial \mu_t} + \gamma_2 \varepsilon_t \frac{\partial w_{2,t}}{\partial \mu_t}\right) - MPS^2}{r(\mu_t) + \frac{\partial \mu_t}{\partial a_t}\left(\frac{\partial r_t}{\partial \mu_t} + \gamma_1 \varepsilon_t \frac{\partial w_{1,t}}{\partial \mu_t}\right) - MPS^1}$$

Therefore we recognize that the degree of substitution between occupation depends first from the idiosyncratic shocks, which plays a role through the variance, and second from wage movements and marginal propensity to save in the two occupation. To makes things even clear, suppose that for a given level of asset and idiosyncratic risk, marginal utility and marginal propensity to save are equal in both occupations, then we have:

$$\gamma_1 \frac{\partial w_{1,t}}{\partial \mu_t} u'(c^1) \leq \gamma_2 \frac{\partial w_{2,t}}{\partial \mu_t} u'(c^2)$$

Then, even in this case, the heterogeneity in skill level matters, since there can be movement in wages that can offset the fact that alone $\gamma_1 - \gamma_2 > 0$, letting the strict inequality hold true.

Chapter 5

General Equilibrium Analysis

In order to assess the effect of the heterogeneity in preferences on the overall economy, I simulate a panel 100 Type 1 and 100 Type 2 for 10000 time periods that are subject both to idiosyncratic risks and taste shocks. On the simulated time series, for each variable of interest, I apply an Hodrick-Prescott Filter in order to separate the cyclical component from the trend, and then I compute a correlation matrix between all the relevant variables' cycle.

Table 5.1: Correlation Matrix at Cycle level

$\rho(X,Y)$	C	K	$M_{1,1}$	$M_{1,2}$	$M_{2,1}$	$M_{2,2}$	L_1	L_2	r	w_1	w_2	Y	I
C	1	0.35	0.1	-0.1	-0.26	0.26	0.01	0.21	0.39	0.2	0.32	0.33	-0.14
K	0.35	1	-0.12	0.12	0.08	-0.08	-0.08	-0.04	0.28	0.01	0.03	0.1	-0.06
$M_{1,1}$	0.1	-0.12	1	-1	0.02	-0.02	0.95	-0.33	0.61	0.95	0.14	0.63	0.61
$M_{1,2}$	-0.1	0.12	-1	1	-0.02	0.02	-0.95	0.33	-0.61	-0.95	-0.14	-0.63	-0.61
$M_{2,1}$	-0.26	0.08	0.02	-0.02	1	-1	0.33	-0.95	-0.63	-0.16	-0.96	-0.65	-0.55
$M_{2,2}$	0.26	-0.08	-0.02	0.02	-1	1	-0.33	0.95	0.63	0.16	0.96	0.65	0.55
L_1	0.01	-0.08	0.95	-0.95	0.33	-0.33	1	-0.61	0.37	0.85	-0.17	0.39	0.4
L_2	0.21	-0.04	-0.33	0.33	-0.95	0.95	-0.61	1	0.41	-0.15	0.86	0.42	0.33
r	0.39	0.28	0.61	-0.61	-0.63	0.63	0.37	0.41	1	0.79	0.8	0.97	0.83
w_1	0.2	0.01	0.95	-0.95	-0.16	0.16	0.85	-0.15	0.79	1	0.36	0.82	0.76
w_2	0.32	0.03	0.14	-0.14	-0.96	0.96	-0.17	0.86	0.8	0.36	1	0.82	0.71
Y	0.33	0.1	0.63	-0.63	-0.65	0.65	0.39	0.42	0.97	0.82	0.82	1	0.89
I	-0.14	-0.06	0.61	-0.61	-0.55	0.55	0.4	0.33	0.83	0.76	0.71	0.89	1

Since this analysis is for qualitative purpose, I focus more on the sign of the correlations and not much on the magnitude. Recall the skill matrix:

$$\Gamma = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 1.5 \end{bmatrix}$$

As anticipated in the introduction, if preference shock are not present, then there exists a unique equilibrium, instead, in this simulation, if we compute the time average of the masses we have:

Table 5.2: Steady State Comparison

	No Taste Shocks	Taste Shocks	$\Delta\%$
M_i	0.5	0.5	-
$M_{1,1}$	0.5	0.33	-0.34
$M_{1,2}$	0	0.17	-
$M_{2,1}$	0	0.17	-
$M_{2,2}$	0.5	0.33	-0.34
$\bar{\gamma}$	1.5	1.16	-0.23
L_i	0.75	0.58	-0.23
K	4.14	3.27	-0.21
Y	1.33	1.05	-0.21
C	0.9	0.75	-0.2

On average, people sort in the occupation for which they are more productive, coherently both with the partial equilibrium analysis and with the empirical literature. However, we have a 34% of individuals that sort in the occupations for which they are less productive. Now I show what are the effects of this phenomenon at aggregate level. If we focus on this submatrix:

Table 5.3: Sub-Matrix of Correlations

$\rho(X,Y)$	C	K	$M_{1,1}$	$M_{1,2}$	$M_{2,1}$	$M_{2,2}$	L_1	L_2	r	w_1	w_2	Y	I
$M_{1,1}$	0.1	-0.12	1	-1	0.02	-0.02	0.95	-0.33	0.61	0.95	0.14	0.63	0.61
$M_{1,2}$	-0.1	0.12	-1	1	-0.02	0.02	-0.95	0.33	-0.61	-0.95	-0.14	-0.63	-0.61
$M_{2,1}$	-0.26	0.08	0.02	-0.02	1	-1	0.33	-0.95	-0.63	-0.16	-0.96	-0.65	-0.55
$M_{2,2}$	0.26	-0.08	-0.02	0.02	-1	1	-0.33	0.95	0.63	0.16	0.96	0.65	0.55

Recall that type 1 and 2 are more productive respectively on occupation 1 and occupation 2. We see that "mis-matched" masses negatively correlates with all aggregate measures and prices. The interpretation is the following: when we have a flow of people from the occupation for which they are more skilled, to the occupation for which they are less skilled, for non-pecuniary motives, we have a contraction in the whole economy. Since in this economy the skill is a measure of productivity of workers, the effect of this mobility pattern on aggregate measures is a contraction of the economy. Moreover, coherently again with the partial equilibrium analysis, we see a positive correlation between K, $M_{1,2}$ and $M_{2,1}$. Again, this fact is not surprising, an increment in K is overall, an increment of wealth in the economy, and as seen in Chapter 4, an increase in the wealth determine an increase in the probability of switching to the occupation for which people are less productive, and therefore since K is the sum of all asset level, it must correlate positively with those 2 masses.

5.1 Response to a one-period preference shock

It is also interesting to analyze the impact that non-pecuniary benefits could have on the economy. I report three different scenarios: the first one is depicted in Figure 5.1 and Figure 5.2 in which a one time preference shock towards occupation 2 is given to the 10% of type 1 agents and everyone is in the low idiosyncratic state.

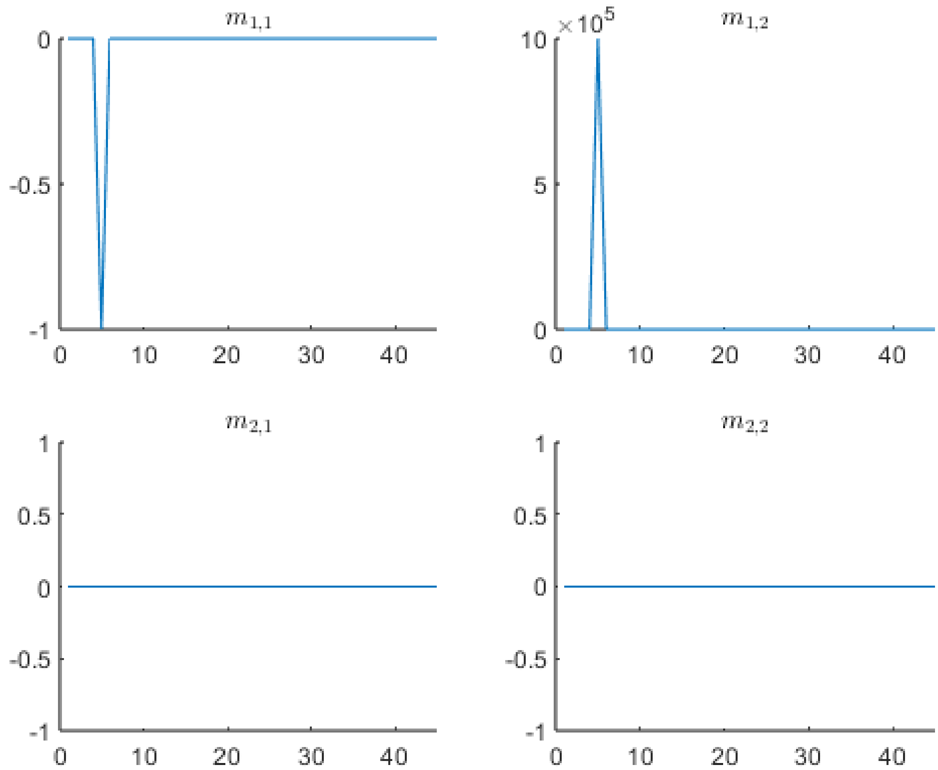


Figure 5.1: Mass Change after a transitory preference shock

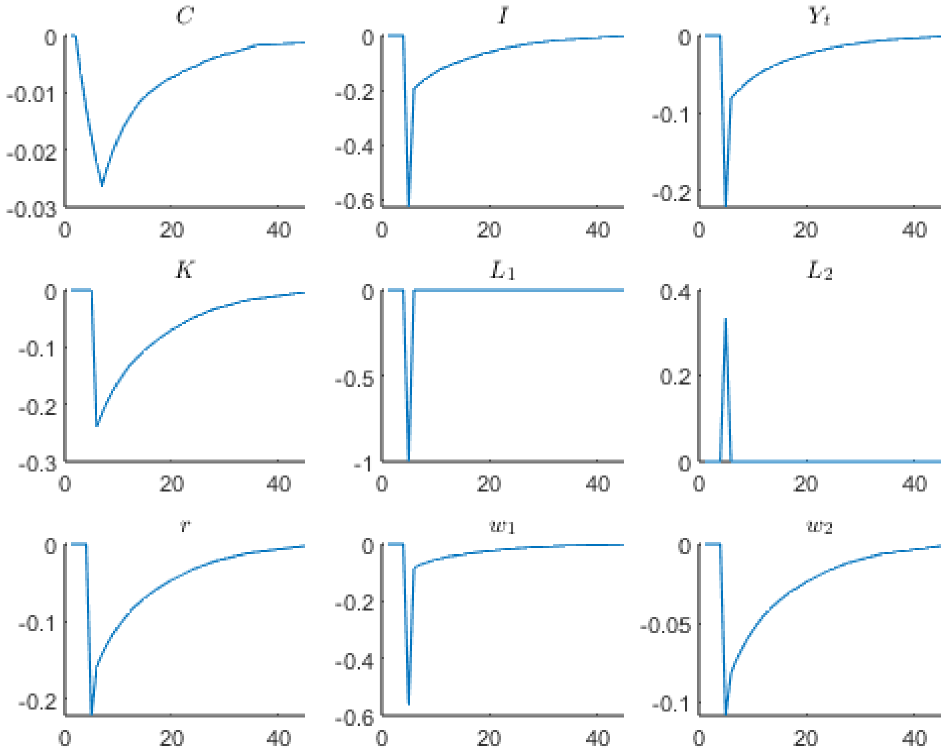


Figure 5.2: Impulse Response Functions

As seen from the correlation matrix, an increase in $m_{1,2}$ is negatively correlated to all aggregate measure, and this is seen also in the impulse repsonse functions. First notice that, such shock have, at least qualitative, the same effects of a negative TFP shocks, but in this case is not driven by a shock in demand, instead by a shock in supply due to non-pecuniary benefits. To highlight this complex effect, one must start from the policy functions:

Since we are starting from the steady state, which is the situation in which a type 1 is in occupation 1, a preference shock towards occupation 2 let optimal for an individual to decumulate asset on impact, selecting as decision function a^2_{t+1}. On the other hand, he switches also consumption function, by lowering it, and these two effects on aggregate traduce in lower consumption and capital level. After the shock period, the preference shifts again towards occupation 1, therefore since the new asset level is below the individual steady state level, they will start to accumulate again, and this can be seen on aggregate by the recover of consumption and capital to the previous steady state. This change in the aggregate capital should increase the interest rate. However we should take into account the interplay with the change in labor supply, that is given by:

$$
\begin{aligned}
\Delta {L_1}^S &= -\underbrace{1.5}_{\gamma_{1,1}}\underbrace{0.01}_{\text{Shock}} m_{1,1} \\
\Delta {L_2}^S &= \underbrace{0.5}_{\gamma_{1,2}}\underbrace{0.01}_{\text{Shock}} m_{1,1}
\end{aligned} \tag{5.1}
$$

First, $\Delta {L_1}^S < 0$, $\Delta {L_2}^S > 0$ and in particular $\mid \Delta {L_1}^S \mid > \mid \Delta {L_2}^S \mid$ as shown in Figure 5.2. Since this is a one-time shock, we can notice that the effect on labor supply will last only one period and immediately revert

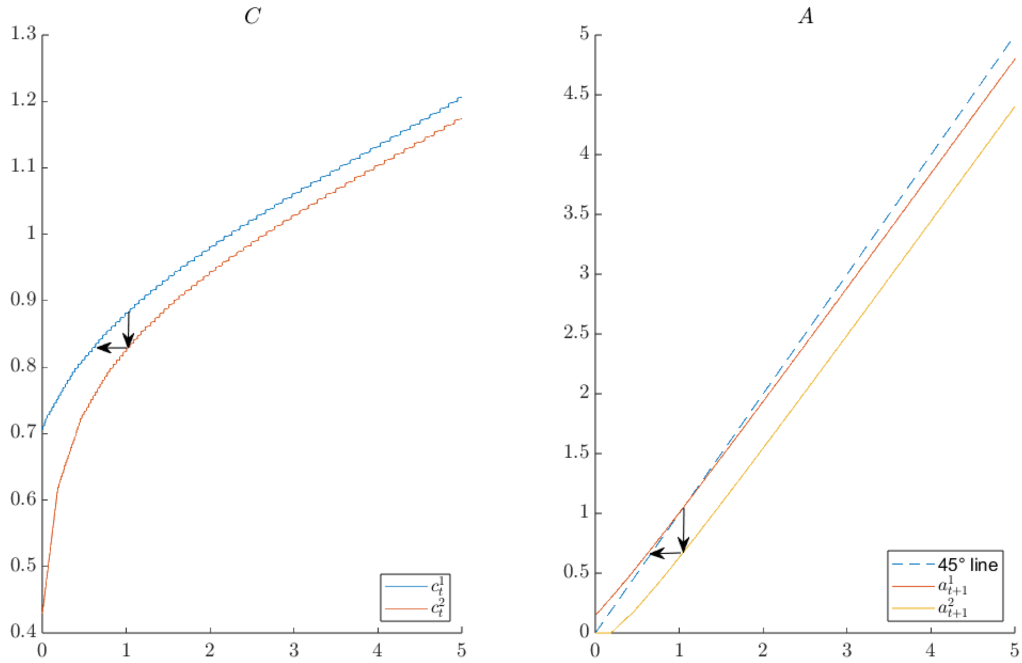


Figure 5.3: Policy Function and Transition Dynamics

back, together with the masses. However, the effect on prices do not follow the same pattern because visually we recognize first an overall negative impact on prices and second a certain degree of persistency. To explain why this is the case, we can look at percentage change in factor prices. Let $\Delta\%{L_j}^S$ be the percentage change for labor supply. In equilibrium supply is equal to demand, therefore we can use $\Delta\%L_j$. Factor prices change accordingly to:

$$\begin{aligned} \Delta\%r &= -(1-\alpha)\Delta\%K + \omega\Delta\%L_1 + \eta\Delta\%L_2 \\ \Delta\%w_1 &= -(1-\omega)\Delta\%L_1 + \alpha\Delta\%K + \eta\Delta\%L_2 \\ \Delta\%w_2 &= -(1-\eta)\Delta\%L_2 + \alpha\Delta\%K + \omega\Delta\%L_1 \end{aligned} \tag{5.2}$$

In order to reconcile the negative percentage drop that we see in the simulation, recall that $\Delta\%K < 0$, $\Delta\%L_1 < 0$ while $\Delta\%L_2 > 0$ and $\mid \Delta {L_1}^S \mid > \mid \Delta {L_2}^S \mid$. If we have an overall drop in prices, it means that there exists some off-setting forces in production factor. In this particular example I'm assuming $\eta = \omega = s$, so equally shared labor factor. Therefore we can rewrite the equations above as:

$$\begin{aligned} \Delta\%r &= -(1-\alpha)\Delta\%K + s(\Delta\%L_1 + \Delta\%L_2) \\ \Delta\%w_1 &= -\Delta\%L_1 + \alpha\Delta\%K + s(\Delta\%L_1 + \Delta\%L_2) \\ \Delta\%w_2 &= -\Delta\%L_2 + \alpha\Delta\%K + s(\Delta\%L_1 + \Delta\%L_2) \end{aligned} \tag{5.3}$$

and we have the second wage just decreases because we are increasing the supply and this effect is amplified by the negative variation in capital stock and the difference in percentage variation of the two labor supplies which is always negative. While for interest rate and the first wage we have two opposite forces, the first one is an increase that is due to lower capital stock and labor supply.

However, this increase is offset by a negative difference in the percentage change of labor supplies for the interest rate. The first wage suffer from this effect plus a decrease of the capital stock.

Chapter 6

Conclusions

To summarize, the examination of skill heterogeneity and its impact on earnings inequality offers valuable insights into the interplay of skills, wages, occupational choices, compensating differentials, and labor market fluctuations. Various perspectives, including recent studies by Acemoglu and Autor, contribute to our understanding of this relationship.

Acemoglu and Autor's work in 2011 emphasizes the link between skill heterogeneity, technology, and earnings. They argue that technological advancements increase the demand for highly skilled workers, resulting in an escalating return to skills. This leads to wage polarization, where high and low-wage jobs grow at the expense of middle-wage jobs. Their framework, distinguishing between skills and tasks, provides a comprehensive understanding of how skill allocation and technological changes influence earnings distribution.

On a theoretical basis, Roy's classic work in 1951 challenged the idea of arbitrary earnings distribution by demonstrating the role of skill heterogeneity. Heckman and Honore later formalized Roy's model and extended it to empirical analysis, finding support for the log-normal distribution of skills and wages under certain conditions. Acemoglu and Autor further build on this, developing a model that captures polarization.

Compensating differentials offer an alternative perspective on earnings inequality, suggesting that wage differentials can be explained by job characteristics and the need to compensate individuals for unpleasant or demanding jobs. The ongoing debate surrounding the impact of compensating differentials on the economy involves various contributions attempting to identify, quantify, and assess their effects.

Examining the relationship between employment cyclicality and wages during labor market fluctuations, Abraham and Haltiwanger's survey provides comprehensive insights. Recent research by Grigsby bridges key themes, incorporating skill heterogeneity within a Roy framework and accounting for idiosyncratic shocks in occupational preferences. Grigsby's findings suggest both pro and counter-cyclicality of employment and wages within a frictionless environment.

This work aims to build upon existing research by investigating the model in a dynamic setting. Introducing idiosyncratic risks and a consumption-saving problem adds another layer of heterogeneity, generating complex dynamics. Unlike the Real Business Cycle literature, this study highlights the possibility of generating cyclical movement on the supply side, without relying on total factor productivity shocks. The presence of non-pecuniary motives to occupational mobility creates an effect similar to a negative total factor productivity shock, with a precise interpretation of the forces driving this dynamics.

Despite these contributions, there is room for improvement. For example, the inelastic supply of labor could be addressed by endogenizing the number of hours worked. Additionally, a multisector model could be considered to explain job reallocation across sectors in a dynamic version of Grigsby's work.

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