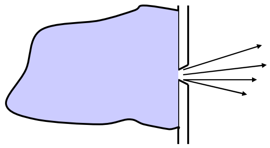
Gas Flow thru a Hole

P_o > Outside P



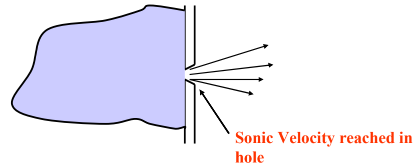
1. Pressure is driving force

2. Frictional losses

3. Gas expands as it escapes due to pressure drop

Isentropic process --> use Equation (4-48)

Choked Flow of Gas thru Hole



Flow rate a function only of supply or upstream pressure and is independent of downstream pressure.

Sonic Velocity

For ideal gases:

$$a = \sqrt{\gamma g_c R_g T / M}$$

For air at 20°C sonic velocity = 344 m/s = 1129 ft/s

This represents the maximum speed that information can be transmitted through the gas.

Choked Flow Equation - Equation (4-50)

$$(Q_m)_{choked} = C_o A P_o \sqrt{\frac{\gamma g_c M}{R_g T_o}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}}$$

Q_m = Mass Flow

C_o = Discharge coef. $\rightarrow$ 1.0 for choked gas flow

A = Area

P_o = Upstream pressure (absolute)

M = Molecular weight

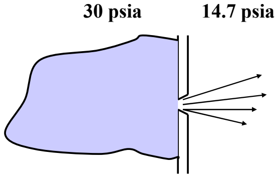
T_o = Temperature (absolute)

g_c = grav. constant

R_g = Ideal gas constant

Conditions for Choked Flow

30 psia 14.7 psia

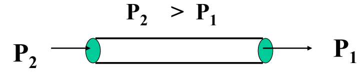


An absolute pressure ratio of greater than 1.67 to 2 will insure choked flow.

…. Choked flow is the usual case.

Gas Flow thru Pipes

P_2 > P_1



1. Pressure is driving force.

2. As P decreases, gas expands and velocity increases

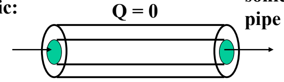
3.T can increase or decrease depending upon relative effect of gas expansion and friction.

Gas Flow thru Pipes - Sonic Conditions

Two Cases:

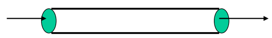
1. Adiabatic: **Q = 0**

Gas velocity is sonic at end of pipe



2. Isothermal: **(long pipelines approach this)**

T = const.



Gas velocity = Sonic Velocity / $\sqrt{\gamma}$

at end of pipe

Several Modeling Approaches (see text)

Adiabatic choked flow

---- Real Case here???

Isothermal choked flow

Adiabatic choked mass flow >

Isothermal choked mass flow

Adiabatic Choked Flow thru Pipe

Rigorous solution requires a trial and error solution of equation (4-67) coupled with equations (4-63) to (4-66).

$$\frac{T_{\text{choked}}}{T_1} = \frac{2Y_1}{\gamma + 1}, \qquad (4\text{-}63)$$

$$\frac{P_{\text{choked}}}{P_1} = \text{Ma}_1 \sqrt{\frac{2Y_1}{\gamma + 1}}, \qquad (4\text{-}64)$$

$$\frac{\rho_{\text{choked}}}{\rho_1} = \text{Ma}_1 \sqrt{\frac{\gamma + 1}{2Y_1}}, \qquad (4\text{-}65)$$

$$G_{\text{choked}} = \rho \bar{u} = \text{Ma}_1 P_1 \sqrt{\frac{\gamma g_c M}{R_g T_1}} = P_{\text{choked}} \sqrt{\frac{\gamma g_c M}{R_g T_{\text{choked}}}}, \qquad (4\text{-}66)$$

$$\frac{\gamma + 1}{2} \ln\left[\frac{2Y_1}{(\gamma + 1)\text{Ma}_1^2}\right] - \left(\frac{1}{\text{Ma}_1^2} - 1\right) + \gamma\left(\frac{4fL}{d}\right) = 0. \qquad (4\text{-}67)$$

Simplified Adiabatic Choked Flow thru Pipe

$$G = \frac{\dot{m}}{A} = Y_g \sqrt{\frac{2 g_c \rho_1 (P_1 - P_2)}{\sum K_f}}$$ **Equation (4-68)**

Y_g = expansion factor (Figure 4-14 or Table 4-4)

$P_1 - P_2$ = sonic pressure drop (Figure 4-13 or Table 4-4)

Direct solution possible with this approach.

See Example 4-5.

Adiabatic Choked Flow thru Pipe

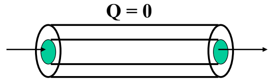
Given: Type, length and diameter of pipe

Pressure drop across pipe

Molecular weight, heat capacity ratio of gas

Temperature

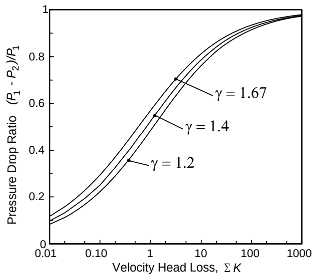
Q = 0



Simplified Approach: Adiabatic Choked Flow thru Pipe

1. **Determine friction factor, *f*. Usually assume fully developed turbulent flow. $f = f(d, \varepsilon)$**
2. **Determine ΣK_f from pipe length and fittings.**
3. **Determine sonic pressure drop from Figure 4-13 or Table 4-4.**
4. **Determine expansion factor, Y_g from Figure 4-14 or Table 4-4.**
5. **Substitute into Equation 4-68 to get mass flux, *G***
6. **Mass flow = *GA*.**

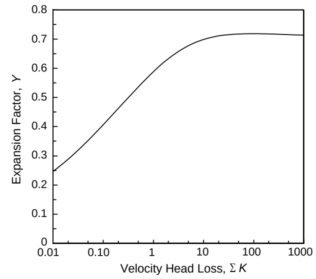
Adiabatic Sonic Pressure Drop



This is an improved version of Figure 4-13 since this is a log scale.

Equations provided in Table 4-4

Adiabatic Expansion Factor – Figure 4-14



This is an improved version of Figure 4-14 since this is a log scale.

Equations provided in Table 4-4

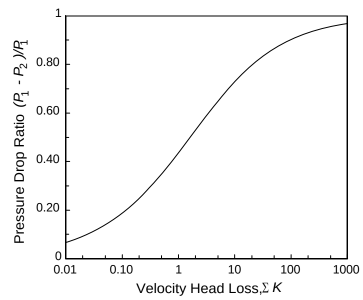
Simplified Isothermal Choked Flow thru Pipe

Correlations for the expansion factor Y and the sonic pressure drop ratio $(P_1-P_2)/P_1$ as a function of the pipe loss ΣK for isothermal flow conditions. The equation used to fit the functions is of the form ln Y = A (ln K)³ + B (ln K)² + C (ln K) + D.

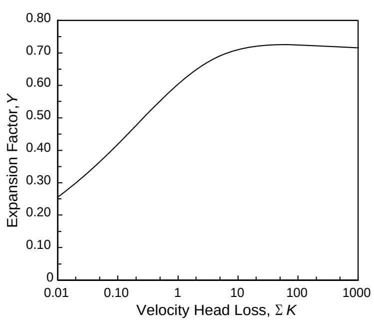
Function value	A	B	C	D
Expansion factor Y	0.0003	-0.0080	0.0611	-0.4588
Sonic pressure drop ratio γ = 1.2	0.0007	-0.0237	0.2409	-0.7678
Sonic pressure drop ratio γ = 1.4	0.0007	-0.0237	0.2408	-0.7677
Sonic pressure drop ratio γ = 1.67	0.0007	-0.0237	0.2407	-0.7677

This is not in the text!

Isothermal Pressure Drop Ratio



Isothermal Expansion Factor



Asymptotic Solution: Isothermal and Adiabatic

$$\dot{m} = A\sqrt{\frac{\rho_1 P_1 g_c}{\sum K}}$$

For a circular pipe, with friction due to pipe length:

$$\dot{m} = \frac{\pi}{8}\sqrt{\frac{\rho_1 P_1 D^5 g_c}{fL}}$$

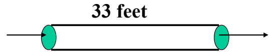
For an ideal gas,

$$\dot{m} = \frac{\pi}{8}\sqrt{\frac{P_1^2 M D^5 g_c}{R T_1 f L}}$$

Example 4-5 in Text:

Nitrogen 200 psig

33 feet



1-inch sch. 40

1 atm 0 psig

Example 4-5

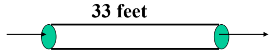
1. Friction factor, $f = 0.00564$ (assume fully developed turbulent flow).
2. $K_f = 4fL/d = 8.56$ due to pipe length only.
3. From Figure 4-13: $\frac{P_1 - P_2}{P_1} = 0.770 \Rightarrow P_2 = 49.4 \text{ psia}$

   Since actual downstream P is less than this, flow is sonic.
4. From Figure 4-14, $Y_g = 0.69$.
5. From Equation 4-68, $\dot{m} = 1.78 \text{ lb}_\text{m}/\text{sec}$

Example 4-5 in Text:

Nitrogen 200 psig

33 feet



1-inch sch. 40

1 atm 0 psig

Modeling Approaches:

Choked flow thru hole: 4.16 lb/sec

Adiabatic choked flow thru pipe: 1.81 lb/sec

Isothermal choked flow thru pipe: 1.76 lb/sec

<-Real Case??

Recommendation: Used adiabatic choked flow, or choked flow thru a hole

Asymptotic Solution

$$\dot{m} = A\sqrt{\frac{\rho_1 P_1 g_c}{\sum K}}$$

$$\dot{m} = (6.00\times10^{-3} \text{ ft}^2)\sqrt{\frac{\left(1.037 \frac{\text{lb}_\text{m}}{\text{ft}^3}\right)\left(214.7 \frac{\text{lb}_\text{f}}{\text{in}^2}\right)\left(144 \frac{\text{in}^2}{\text{ft}^2}\right)\left(32.17 \frac{\text{ft-lb}_\text{m}}{\text{lb}_\text{f}\text{-s}^2}\right)}{8.56}}$$

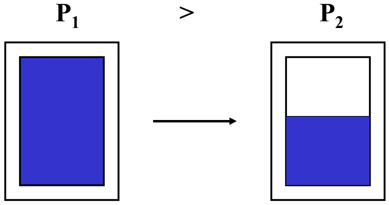
$\dot{m} = 2.08 \text{ lb}_\text{m}/\text{sec}$

Compared to a rigorous solution of $1.81 \text{ lb}_\text{m}/\text{sec}$

% error = 14.9%

Flashing Liquids

P_1 > P_2



Modeling Flashing Liquids

Energy for flashing comes from sensible energy in liquid

$$f_v = \frac{C_p\left(T_o - T_{BP}\right)}{\Delta H_{vap}} = \text{Mass Fraction Vap.}$$

T_o = Storage / Ambient Temperature

T_{BP} = Normal Boiling Point Temperature

Other Source Models (see textbook)

Flashing liquid flowing thru hole: assume liquid flashes outside of the hole.

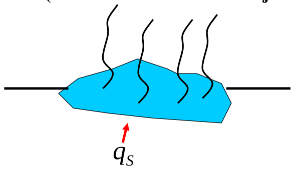
Flashing liquid flowing thru pipe:

- See equation 4-91 for liquids stored at P higher than saturation vapor pressure.
- See equation 4-104 for liquids stored at saturation vapor pressure.

Other Source Models (see textbook)

Boiling Pool: See Equations (4-105) and (4-106)

Initially, when liquid is first spilled on ground, boiling is limited by heat transfer from ground (Equation 4-105). After some time, heat transfer from air (conduction and convection) and radiant heat transfer (i.e. from the sun or an adjacent fire) contribute.



$$q_S = \frac{k_S\left(T_g - T\right)}{\sqrt{\pi \alpha_S t}} = \text{heat flux}$$

k_S = thermal cond. of soil

T_g = temp. of ground

α_S = thermal diffusivity of soil

Source Models do not need to be exact!

If uncertain about model, physical property, geometry, etc., select the one to obtain maximum discharge. See Table 4-5.

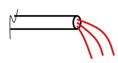
Maximum discharge ---> Maximum Consequence

Problem: can lead to a very large result.

We should always try to do best we can using good engineering judgement!

Realistic Release Incidents

Process Pipes: Rupture of largest diameter as follows:



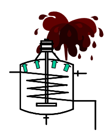
For d < 2 in., assume full bore rupture

For 2-4 in. assume rupture equal to 2-inch pipe

For d > 4 in, assume rupture area = 20% of pipe area

Vessels: Assume rupture based on largest diameter pipe and then use criteria above.

Relief Device: Use calculated total release rate at set pressure. Assume everything is airborne.



Worst Case Release Incidents – From RMP

Assume release of the largest quantity of substance handled on site in a single process vessel at any time. Assume entire quantity is released in 10-minutes.

Assume release on ground.

Assume F-stability, 1.5 m/s wind speed (Chapter 5)

Assume highest daily max. T and average humidity.

See Table 4-5