1. Show that equation (1) is equivalent to $\partial_\mu F^{\mu\nu} = 0$. We have

$$\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$= \left(\left(\tfrac{\partial}{\partial x}E_x + \tfrac{\partial}{\partial y}E_y + \tfrac{\partial}{\partial z}E_z\right), \left(-\tfrac{\partial}{\partial t}E_x + \tfrac{\partial}{\partial y}B_z - \tfrac{\partial}{\partial z}B_y\right), \left(-\tfrac{\partial}{\partial t}E_y - \tfrac{\partial}{\partial x}B_z + \tfrac{\partial}{\partial z}B_x\right), \left(-\tfrac{\partial}{\partial t}E_z + \tfrac{\partial}{\partial x}B_y - \tfrac{\partial}{\partial y}B_x\right)\right)$$

$$= \left(\left(\vec{\nabla}\cdot\vec{E}\right), \left((\vec{\nabla}\times\vec{B})_x - \left(\tfrac{\partial \vec{E}}{\partial t}\right)_x\right), \left((\vec{\nabla}\times\vec{B})_y - \left(\tfrac{\partial \vec{E}}{\partial t}\right)_y\right), \left((\vec{\nabla}\times\vec{B})_z - \left(\tfrac{\partial \vec{E}}{\partial t}\right)_z\right)\right) = \vec{0},$$

which is equivalent to (two of) Maxwell's equations, since the divergence of the electric field in the vacuum is zero and each of the remaining components being equal to zero means that the curl of the magnetic field is equal to the time derivative of the electric field.

2. Prove that $\vec{\nabla}\times(\vec{\nabla}\times\vec{v}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{v}) - \nabla^2\vec{v}$.

Proof. We compute the i-th component of both sides.

$$\left(\vec{\nabla}\times(\vec{\nabla}\times\vec{v})\right)_i = \epsilon_{ijk}\partial_j(\vec{\nabla}\times\vec{v})_k = \epsilon_{ijk}\epsilon_{kij}\partial_j\partial_k v_i = \delta_{ik}\delta_{ji}\partial_j\partial_k v_i - \delta_{ii}\delta_{jk}\partial_j\partial_k v_i = \partial_i(\partial_k v_k) - \partial_k\partial_k v_i$$

$$\left(\vec{\nabla}(\vec{\nabla}\cdot\vec{v}) - \nabla^2\vec{v}\right)_i = \partial_i(\partial_j v_j) - \partial_j\partial_j v_i,$$

and since repeated indices are summed over these two are equivalent. □