Homework 29.

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29.1 (a) Find the Fourier inverse transformation $f(x)$ of

$$\hat{f}(\xi) = \begin{cases} 1/\sqrt{\varepsilon} & \text{for } -\varepsilon/2 \le \xi \le \varepsilon/2 \\ 0 & \text{for } |\xi| > \varepsilon/2 \end{cases}.$$

We have by the Fourier inversion theorem that

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \hat{f}(\xi)\, e^{ix\xi}\, d\xi$$

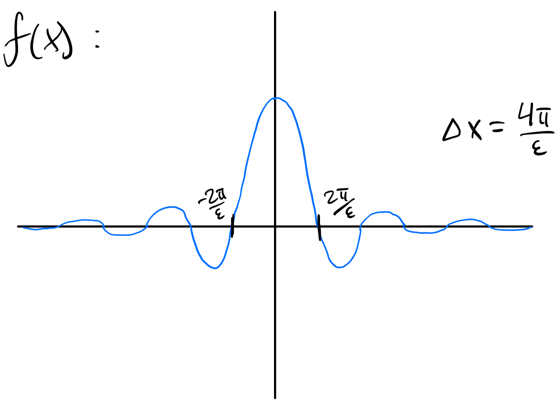
$$= \frac{1}{\sqrt{2\pi\varepsilon}} \int_{-\varepsilon/2}^{\varepsilon/2} e^{ix\xi}\, d\xi$$

$$= \frac{1}{\sqrt{2\pi\varepsilon}} \left(\frac{e^{ix\xi}}{ix} \Bigg|_{-\varepsilon/2}^{\varepsilon/2} \right)$$

$$= \frac{2}{\sqrt{2\pi\varepsilon}\, x} \left(\frac{e^{ix\varepsilon/2} - e^{-ix\varepsilon/2}}{2i} \right)$$

$$\boxed{= \sqrt{\frac{\varepsilon}{2\pi}} \left[\frac{\sin(x\varepsilon/2)}{x\varepsilon/2} \right]}$$

(b) $f(x)$:



$\Delta x = \frac{4\pi}{\varepsilon}$

and from definition of $\hat{f}(\xi)$, $\Delta \xi = \varepsilon$.

$$\boxed{\Delta x \Delta \xi = 4\pi > 1.}$$