Solution Manual

N. Kapsos, M. Schrank, S. Sivakumar

38.3 Exercises

38.3.1

First we need to find out the parameterization for C. So give $x = 2\cos(t)$ and $y = 2\sin(t)$, but in order to ensure that x remains nonnegative and we only trace out a half circle, we give the parameter the range $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

The arclength element ds for this problem is $\sqrt{(-2\sin(t))^2 + (2\cos(t))^2}dt = 2dt$ (this parameterization is a natural one). So then we may substitute in the line integral like so:

$$16\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2(t)\cos(t)dt = \frac{32}{3}$$

38.3.2

Recall that a line segment with initial point $\vec{r}_0$ and terminal point $\vec{r}_1$ are most easily parameterized as $\vec{r}(t) = (1-t)\vec{r}_0 + t\vec{r}_1$, for $0 \leq t \leq 1$. Apply this definition to the line segment given in the problem to find that $\vec{r}(t) = \langle bt, (1-t)a \rangle$, for t in that same interval as before. Then find ds:

$$ds = \sqrt{(-a)^2 + (b)^2} = \sqrt{a^2 + b^2}$$

Then the integral becomes

$$\int_C x\sin(y)ds \to \sqrt{a^2 + b^2}\int_0^1 bt\sin((1-t)a)dt$$

Give $u = 1 - t$ so that $t = 1 - u$ and $du = -dt$. Change the bounds as well (they actually remain unchanged):

$$\sqrt{a^2 + b^2}\int_0^1 (1-u)b\sin(au)du \to \sqrt{a^2 + b^2}\int_0^1 (b\sin(au) - bu\sin(au))\,du$$

$$= \frac{b}{a}\sqrt{a^2 + b^2}\left[1 - \frac{\sin(a)}{a}\right]$$

38.3.4

We are given the parameterization for the curve C from the get go, and it is easy to deduce that the range of t is from 0 to 1. Then the arclength element ds is given by $\sqrt{(1)^2 + (2t)^2 + (3t^2)^2} dt = \sqrt{9t^4 + 4t^2 + 1} dt$.

Substitute into the line integral and resolve:

$$\int_0^1 (2t + 9t^3)(\sqrt{9t^4 + 4t^2 + 1})dt \to \frac{1}{4}\int_1^{14} \sqrt{u} du = \frac{1}{6}\left(14^{\frac{3}{2}} - 1\right)$$

38.3.18

We seek to take a line integral over the path given by an arc of the parabola in the problem where the integrand is the linear mass density.

Give $t = y$ and $x = \frac{t^2}{2a}$. Since $0 \leq \frac{t^2}{2a} \leq \frac{a}{2}$ (from substitution) it is apparent that $-a \leq t \leq a$. Find the arclength element ds as $\sqrt{(1)^2 + \left(\frac{t}{a}\right)^2} dt = \frac{1}{a}\sqrt{a^2 + t^2}$. Then we may substitute to find the following:

$$\frac{1}{a}\int_{-a}^{a} |t|\sqrt{a^2 + t^2} dt \to \frac{1}{a}\int_{a^2}^{2a^2} u^{\frac{1}{2}} du = \frac{2}{3}a^2(2\sqrt{2} - 1)$$

The evaluation of the integral may be done in the piecewise manner or by symmetry (and substitution) as above.