Define $\phi = x^2yz^3$ and $\vec{V} = \hat{\mathbf{i}}xz - \hat{\mathbf{j}}y^2 + \hat{\mathbf{k}}2x^2y$. Then find

(a) $\vec{\nabla}\phi$

$$\vec{\nabla}\phi = \hat{\mathbf{i}}\frac{\partial\phi}{\partial x} + \hat{\mathbf{j}}\frac{\partial\phi}{\partial y} + \hat{\mathbf{k}}\frac{\partial\phi}{\partial z} = \boxed{\hat{\mathbf{i}}2xyz^3 + \hat{\mathbf{j}}x^2z^3 + \hat{\mathbf{k}}3x^2yz^2}$$

(b) $\vec{\nabla}\cdot\vec{V}$

$$\vec{\nabla}\cdot\vec{V} = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z} = \boxed{z - 2y}$$

(c) $\vec{\nabla}\times\vec{V}$

$$\begin{aligned}\vec{\nabla}\times\vec{V} &= \hat{\mathbf{i}}\left(\frac{\partial V_3}{\partial y} - \frac{\partial V_2}{\partial z}\right) + \hat{\mathbf{j}}\left(\frac{\partial V_1}{\partial z} - \frac{\partial V_3}{\partial x}\right) + \hat{\mathbf{k}}\left(\frac{\partial V_2}{\partial x} - \frac{\partial V_1}{\partial y}\right)\\ &= \boxed{\hat{\mathbf{i}}2x^2 + \hat{\mathbf{j}}(x - 4xy) + \hat{\mathbf{k}}(0)}\end{aligned}$$

(d) $\vec{\nabla}\cdot\left(\vec{\nabla}\phi\right)$

$$\begin{aligned}\vec{\nabla}\cdot\left(\vec{\nabla}\phi\right) &= \frac{\partial}{\partial x}\left(2xyz^3\right) + \frac{\partial}{\partial y}\left(x^2z^3\right) + \frac{\partial}{\partial z}\left(3x^2yz^2\right)\\ &= \boxed{2yz^3 + 6x^2yz}\end{aligned}$$

(e) $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{V}\right)$

$$\begin{aligned}\vec{\nabla}\times\left(\vec{\nabla}\times\vec{V}\right) &= \hat{\mathbf{i}}\left(\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}(x - 4xy)\right) + \hat{\mathbf{j}}\left(\frac{\partial}{\partial z}\left(2x^2\right) - \frac{\partial}{\partial x}(0)\right) + \hat{\mathbf{k}}\left(\frac{\partial}{\partial x}(x - 4xy) - \frac{\partial}{\partial y}\left(2x^2\right)\right)\\ &= \boxed{\hat{\mathbf{i}}(0) + \hat{\mathbf{j}}(0) + \hat{\mathbf{k}}(1 - 4y)}\end{aligned}$$