Homework 32.

Sai Sivakumar

32.1 The electrostatic potential $V(r,\theta)$ on the surface of a hollow, empty sphere of radius R is maintained to be $V(R,\theta) = \cos(\theta)$. The potential satisfies Laplace's equation with azimuthal symmetry.

Since V satisfies Laplace's equation (with azimuthal symmetry), we have that the general solution for V is given by:

$$V(r,\theta) = \sum_{\ell=0}^{\infty} \left[A_\ell r^\ell + B_\ell r^{-(\ell+1)}\right] P_\ell(\cos\theta).$$

a) Find $V(r,\theta)$ <u>outside</u> the sphere.

The voltage is not allowed to go to infinity, and since $R>0$, outside the sphere, the voltage may not go to infinity so the coefficients A_ℓ for $\ell>0$ vanish. For $\ell=0$, A_0 must be zero also since $V\to 0$ as $r\to 0$.

Furthermore, P_ℓ for $\ell>1$ has degree >1 so it is impossible for these terms to appear (since $V(R,\theta)=\cos(\theta)$). Similarly, the $P_0=1$ term must not appear ($B_0=0$).

Hence

$$V(r,\theta) = B_1 r^{-2}\cos(\theta), \text{ and with } V(R,\theta)=\cos\theta,$$

$B_1 R^{-2} = 1$ so that $\underline{B_1 = R^2}$. Thus $\boxed{V(r,\theta) = R^2 r^{-2}\cos(\theta)}$

b) Find $V(r,\theta)$ <u>inside</u> the sphere.

Since the interior of the sphere contains the origin, <u>none</u> of the B_ℓ terms may appear, otherwise $V\to\infty$ as $r\to 0$. Also A_0, P_ℓ for $\ell\neq 1$, due to the boundary condition,

Hence

$$V(r,\theta) = A_1 r\cos\theta, \text{ and with } V(R,\theta)=\cos(\theta), \text{ we have}$$

$A_1 R = 1$ so that R^{-1}. Thus $\boxed{V(r,\theta) = R^{-1} r\cos(\theta)}$