# HOMEWORK 1

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Show, if $s \in \mathbb{Q}$, that $s^3 \neq 5$.

*Proof.* Let $s \in \mathbb{Q}$, and assume via contradiction that $s^3 = 5$. We cannot have that $s \leq 0$, since non-positive quantities cubed could not produce 5, as 5 is positive. Thus $s$ must be a strictly positive rational number.

Since $s$ is a positive rational number, there exists $n, m \in \mathbb{N}$ such that we may write $s = n/m$, where without loss of generality we may also choose $n$ and $m$ so that $\gcd(n, m) = 1$.

Then since $s^3 = n^3/m^3 = 5$, $n^3 = 5m^3$, and from the definition of divisibility $5 \mid n^3$. Since 5 is a prime number, we can use the fact that integers have unique prime factorizations to find that $5 \mid n$, so that there exists $k \in \mathbb{Z}$ such that $5k = n$. So then $n^3 = 125k^3 = 5m^3$ implies that $25k^3 = m^3$, from which we find that $25 \mid m^3$. But $5 \mid 25$, and because divisibility is transitive, $5 \mid m^3$ and like before we find that $5 \mid m$. We have arrived at a contradiction, since $\gcd(n, m) \geq 5$. $\square$