Solution Manual

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36.6 Exercises

36.6.1

This is fairly direct.

$$J = \left| \det \begin{pmatrix} x'_u & x'_v & x'_w \\ y'_u & y'_v & y'_w \\ z'_u & z'_v & z'_w \end{pmatrix} \right| = \left| \det \begin{pmatrix} \frac{1}{v} & -\frac{u}{v^2} & 0 \\ 0 & \frac{1}{w} & -\frac{v}{w^2} \\ -\frac{w}{u^2} & 0 & \frac{1}{u} \end{pmatrix} \right| = 0$$

36.6.6

First change the variables to find that $u+v+w \leq a$, which is the region under a plane that intersects the u, v, and w axes at a (meaning the points are $(0,0,a)$, $(0,a,0)$, $(a,0,0)$). It is easiest to give the bounds in a vertically simple manner, meaning to start with $0 \leq w \leq a-u-v$. Then to find the bounds in v we may give the line where the plane intersects with the uv plane as $v = 5-u$, so evidently $0 \leq v \leq a-u$. And u will then vary from 0 to a.

Then we compute the Jacobian as follows for $dxdy = Jdudv$. We must rewrite the equations given into $x = u^2$, $y = v^2$, and $z = w^2$. Then:

$$J = \left| \det \begin{pmatrix} 2u & 0 & 0 \\ 0 & 2v & 0 \\ 0 & 0 & 2w \end{pmatrix} \right| = 8uvw$$

Then the triple integral becomes

$$\int_0^a \int_0^{a-u} \int_0^{a-u-v} (8uvw)dwdvdu$$

$$\to \int_0^a 4u \int_0^{a-u} (a^2v - 2auv + u^2v - 2av^2 + 2uv^2 + v^3)dvdu$$

$$\frac{1}{3}\int_0^a u(a-u)^4du = \frac{1}{90}a^6$$

It may be helpful to use auxiliary substitutions for the later integrals.

36.6.13

From the bounds give $u = \frac{x}{3}$, $v = \frac{y}{2}$, and $w = z$. The Jacobian is given by $\frac{1}{J}$ where J is given by

$$J = \left| \det \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \right| = \frac{1}{6}$$

The bound given by the paraboloid remains a paraboloid, but it becomes a circular paraboloid $w = u^2 + v^2$ that intersects with the same plane $z = w = 10$. Call this region E'.

The triple integral after substitution becomes

$$216 \iiint_{E'} (u^2 - v^2) dA'$$

However, notice that there is symmetry of both the integrand (skew symmetry) and the region of integration (geometric symmetry) across the line $u = v$, so we may apply the transformation $(u, v, w) \to (v, u, w)$ to find that the sign of the integral will flip and so the integral vanishes.