Homework 34.

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34.1

a) With $G(\vec{r},\vec{r}') = -\frac{1}{4\pi|\vec{r}-\vec{r}'|}$,

we have that

$$\Phi(\vec{r}) = \int_V \frac{\rho(\vec{r}')}{\varepsilon_0 4\pi|\vec{r}-\vec{r}'|} dV' .$$

b) With $r \gg r'$,

$$|\vec{r}-\vec{r}'| = \sqrt{|r^2 - 2(\vec{r}\cdot\vec{r}') + r'^2|}$$
$$= r\sqrt{\left|1 - 2\left(\frac{\hat{r}\cdot\vec{r}'}{r}\right) + \left(\frac{r'}{r}\right)^2\right|}$$
$$\approx r\sqrt{\left|1 - 2\left(\frac{\hat{r}\cdot\vec{r}'}{r}\right)\right|}$$

and by binomial expansion

$$\approx r\left(1 - \left(\frac{\hat{r}\cdot\vec{r}'}{r}\right) + O(1/r^2)\right)$$
$$\approx r - \frac{\vec{r}\cdot\vec{r}'}{r} + O(1/r) \quad \text{as desired.}$$

Instead we $\frac{1}{|\vec{r}-\vec{r}'|} \approx \frac{1}{r}\left(\left|1 - 2\left(\frac{\hat{r}\cdot\vec{r}'}{r}\right)\right|\right)^{-1/2} \approx \frac{1}{r}\left[1 + \left(\frac{\hat{r}\cdot\vec{r}'}{r}\right) + O(1/r^2)\right]$.

c)

$$\Phi(\vec{r}) = \int_V \frac{\rho(\vec{r}')}{\varepsilon_0 4\pi|\vec{r}-\vec{r}'|} dV' .$$
$$\approx \frac{1}{4\pi\varepsilon_0}\int_V \rho(\vec{r}')\left[1/r + \frac{\vec{r}\cdot\vec{r}'}{r^3} + O(1/r^3)\right] dV'$$
$$\approx \frac{1}{4\pi\varepsilon_0}\left[\frac{M}{r} + \frac{\vec{r}\cdot\vec{d}}{r^3} + O(1/r^3)\right],$$

and so $M = \frac{1}{4\pi\varepsilon_0}\int_V \rho(\vec{r}')\, dV'$

$d = \frac{1}{4\pi\varepsilon_0}\int_V \vec{r}'\, dV'$