A Proof of Proposition 1

For all x and x' in R^d we have

$$
\begin{aligned}
& f(\lambda x + (1-\lambda)x') \\
=& f(x') + \lambda \int_0^1 \langle \nabla f(\lambda t x + (1-\lambda t)x'), x - x' \rangle \, dt \\
=& f(x') + \lambda [f(x) - f(x')] + \lambda \left[\int_0^1 \langle \nabla f(\lambda t x + (1-\lambda t)x'), x - x' \rangle \, dt - (f(x) - f(x')) \right]
\end{aligned}
$$

Therefore

$$
\begin{aligned}
& |f(\lambda x + (1-\lambda)x') - (\lambda f(x) + (1-\lambda) f(x')| \\
=& \lambda \left| \int_0^1 \langle \nabla f(\lambda t x + (1-\lambda t)x') - \nabla f(tx + (1-t)x'), x - x' \rangle \, dt \right| \\
\leq& \lambda \int_0^1 |\langle \nabla f(\lambda t x + (1-\lambda t)x') - \nabla f(tx + (1-t)x'), x - x' \rangle| \, dt \\
\leq& \lambda \int_0^1 \|\nabla f(\lambda t x + (1-\lambda t)x') - \nabla f(tx + (1-t)x')\| \|x - x'\| dt \\
\leq& \lambda \int_0^1 (1-\lambda) t L \|x - x'\|^2 dt \\
=& \frac{\lambda(1-\lambda)L}{2} \|x - x'\|^2,
\end{aligned}
$$

where the second inequality follows Cauchy-Schwarz inequality and the third inequality is the property (**??**).