ANSWERS

MATHEMATICS

1. (c)	2. (c)	3. (b)	4. (d)	5. (c)	6. (c)	7. (d)	8. (b)
9. (b)	10. (d)	11. (b)	12. (c)	13. (c)	14. (a)	15. (c)	16. (a)
17. (c)	18. (a)	19. (d)	20. (c)	21. (a)	22. (d)	23. (d)	24. (a)
25. (b)	26. (b)	27. (a)	28. (a)	29. (d)	30. (c)	31. (a)	32. (c)
33. (b)	34. (b)	35. (a)	36. (b)	37. (d)	38. (c)	39. (a)	40. (a)
41. (c)	42. (c)	43. (d)	44. (a)	45. (c)			

PHYSICS

46. (b)	47. (d)	48. (a)	49. (c)	50. (a)	51. (b)	52. (c)	53. (d)
54. (b)	55. (c)	56. (c)	57. (c)	58. (c)	59. (b)	60. (c)	61. (b)
62. (c)	63. (b)	64. (c)	65. (b)	66. (b)	67. (b)	68. (c)	69. (b)
70. (b)	71. (c)	72. (b)	73. (a)	74. (b)	75. (b)	76. (b)	77. (c)
78. (d)	79. (d)	80. (d)	81. (d)	82. (d)	83. (d)	84. (b)	85. (a)

CHEMISTRY

86. (b)	87. (d)	88. (c)	89. (b)	90. (b)	91. (a)	92. (c)	93. (a)
94. (b)	95. (a)	96. (a)	97. (b)	98. (b)	99. (a)	100. (b)	101. (b)
102. (d)	103. (b)	104. (c)	105. (b)	106. (b)	107. (c)	108. (a)	109. (d)
110. (d)	111. (c)	112. (b)	113. (b)	114. (d)	115. (b)	116. (c)	117. (d)
118. (b)	119. (a)	120. (a)	121. (b)	122. (c)	123. (b)	124. (b)	125. (b)

ENGLISH

126. (d)	127. (c)	128. (c)	129. (b)	130. (b)	131. (c)	132. (b)	133. (d)
134. (a)	135. (d)	136. (a)	137. (d)	138. (d)	139. (a)	140. (c)	

REASONING

141. (c)	142. (b)	143. (d)	144. (d)	145. (d)	146. (b)	147. (a)	148. (c)
149. (d)	150. (a)						

HINTS & SOLUTIONS

Mathematics

1. Given equation of line is

$$x + y = 0 \quad \ldots(i)$$

and equation of circle is

$$x^2 + y^2 + 4y = 0 \quad \ldots(ii)$$

On solving Eqs. (i) and (ii),

$$x^2 + (-x)^2 + 4(-x) = 0$$

$\Rightarrow \quad 2x^2 - 4x = 0$

$\Rightarrow \quad 2x(x-2) = 0$

$\Rightarrow \quad x = 0, 2$ and $y = 0, -2$

Now taking option (c)

i.e., $\quad y^2 = 2x$

at poing $(0, 0) \Rightarrow 0 = 0$

and at point $(2, -2)$

$\Rightarrow \quad (-2)^2 = 2(2) \Rightarrow 4 = 4$

$\therefore$ option (c) is the correct answer.

2. Given equation of ellipse is $4x^2 + 5y^2 = 1$

or $\quad S \equiv 4x^2 + 5y^2 - 1 = 0 \quad \ldots(i)$

At point $(4, -3)$

$$S \equiv 4(4)^2 + 5(-3)^2 - 1$$

$$\equiv 108 > 0$$

Therefore the given point lies outside the ellipse.

3. Given that

$\vec{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\vec{\mathbf{b}} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

Now, $\quad \vec{\mathbf{a}} + \vec{\mathbf{b}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}} + 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$= 4\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

and $\vec{\mathbf{a}} - \vec{\mathbf{b}} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) - (3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= -2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

Let θ be the angle between $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ and $\vec{\mathbf{a}} - \vec{\mathbf{b}}$

$$\therefore \quad \cos\theta = \frac{(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{a}} - \vec{\mathbf{b}})}{|\vec{\mathbf{a}} + \vec{\mathbf{b}}|\;|\vec{\mathbf{a}} - \vec{\mathbf{b}}|}$$

$$= \frac{(4\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) \cdot (-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}})}{|4\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}|\;|-2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}|}$$

$$= \frac{-8 + 3 + 5}{\sqrt{16 + 1 + 1}\,\sqrt{4 + 9 + 25}} = 0$$

$\Rightarrow \quad \theta = 90°$

4. Let A and B be two subsets of S. There are following cases to make a subset of S, under the given condition *i.e.* $A \cup B = S$ and $A \cap B = \phi$

Case I : If set A has no element. The number of ways of selection of 0 element from set S is ${}^nC_0$.

Case II : If set A has one element. The number of ways of selection of one element from set S is ${}^nC_1$.

Case III : If set A has two elements. The number of ways of selection of two element from set S is ${}^nC_2$.

....

....

....

....

Case (*n*) : If set A has *n* elements. The number of ways of selection of *n* elements from set *S* is ${}^nC_n$.

$\therefore$ Total set of $A = {}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n$

$$= 2^n$$

Total set of A and $B = 2^n \times 2^n = 2^{2n}$

$\therefore$ Required probability $= \dfrac{2^n}{2^{2n}} = \dfrac{1}{2^n}$

5. Let $A \equiv \begin{vmatrix} 1+\sin^2\theta & \sin^2\theta & \sin^2\theta \\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta \\ 4\sin 4\theta & 4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix} = 0$

Applying $R_1 \to R_1 + R_2$

$$\Rightarrow \begin{vmatrix} 2 & 2 & 1 \\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta \\ 4\sin 4\theta & 4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - 2C_3, C_2 \to C_2 - 2C_3$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ -\cos^2\theta & 1-\cos^2\theta & \cos^2\theta \\ -2-4\sin 4\theta & -2-4\sin 4\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

$\Rightarrow$

$$[\cos^2\theta(2+4\sin 4\theta) + (1-\cos^2\theta)(2+4\sin 4\theta)] = 0$$

$$\Rightarrow [2\cos^2\theta + 4\cos^2\theta\sin 4\theta + 2 + 4\sin 4\theta - 2\cos^2\theta - 4\cos^2\theta\sin 4\theta] = 0$$

$$\Rightarrow \quad 2 + 4\sin 4\theta = 0$$

$$\Rightarrow \quad \sin 4\theta = -\frac{1}{2}$$

6. Given that,

$$\lim_{x\to\infty}\left(\frac{x^2+1}{x+1} - \alpha x - \beta\right) = 0$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{x^2+1-\alpha(x^2+x)-\beta(x+1)}{x+1}\right) = 0$$

Using L-Hospital's rule, we get

$$\lim_{x\to\infty}\left(\frac{2x-\alpha(2x+1)-\beta(1)}{1}\right) = 0$$

If this limit is zero, then the function

$$2x - \alpha(2x+1) - \beta = 0$$

or $\quad x(2-2\alpha) - (\alpha+\beta) = 0$

Equating the coefficient of x and constant terms, we get

$$2 - 2\alpha = 0 \text{ and } \alpha + \beta = 0$$

$$\Rightarrow \quad \alpha = 1, \beta = -1$$

7. Given equations are

$px + y + z = 0,\ x + qy + z = 0,\ x + y + rz = 0$

Since the system have a non-zero solution, then

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$

and $\quad C_3 \to C_3 - C_2$

$$\Rightarrow \begin{vmatrix} p & 1-p & 0 \\ 1 & q-1 & 1-q \\ 1 & 0 & r-1 \end{vmatrix} = 0$$

$$\Rightarrow (1-p)(1-q)(1-r)\begin{vmatrix} \frac{p}{1-p} & 1 & 0 \\ \frac{1}{1-q} & -1 & 1 \\ \frac{1}{1-r} & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \quad (1-p)(1-q)(1-r)\left[\frac{p}{1-p}(1) - 1\left(-\frac{1}{1-q} - \frac{1}{1-r}\right)\right] = 0$$

Since, $p, q, r \neq 1$

$$\therefore \quad \frac{p}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

$$\Rightarrow \quad \frac{1}{1-p} - 1 + \frac{1}{1-q} + \frac{1}{1-r} = 0$$

$$\Rightarrow \quad \frac{1}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 1$$

8. Given that (α, β) lies on the circle $x^2 + y^2 = 1$.

$$\therefore \quad \alpha^2 + \beta^2 = 1$$

or it can be rewritten as

$$\frac{1}{9}(9\alpha^2 + 4 + 12\alpha) + \beta^2 = 1 + \frac{1}{9}(4 + 12\alpha)$$

$$\Rightarrow \frac{1}{9}(3\alpha^2 + 2)^2 + \beta^2 = 1 + \frac{4}{9}(1 + 3\alpha + 1) - \frac{4}{9}$$

$$\Rightarrow \frac{1}{9}(3\alpha + 2)^2 + \beta^2 = \frac{5}{9} + \frac{4}{9}(3\alpha + 2)$$

The locus of $(3\alpha + 2, \beta)$ is

$$\frac{1}{9}x^2 + y^2 = \frac{5}{9} + \frac{4}{9}x$$

or $\quad x^2 - 4x + 9y^2 - 5 = 0$

On comparing this equation with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$\Rightarrow \; a = 1, b = 9, h = 0, g = -2, f = 0, c = -5$

Now,

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= 1 \times 9 \times (-5) + 2(0) - 1(0)^2 - 9(-2)^2 - 0$

$= -45 - 36 = -81 \neq 0$

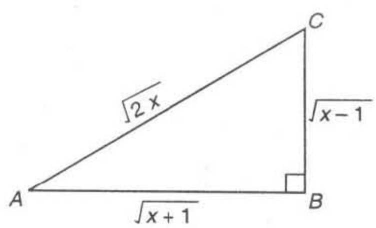
Now, $h^2 - ab = 0 - 9(1) = -9 < 0$

$\therefore \quad \Delta \neq 0$ and $h^2 < ab$,

Hence, it is an ellipse.

9. Given that

$$\sin\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$$



In $\Delta\, ABC$

$$\tan\frac{\theta}{2} = \sqrt{\frac{x-1}{x+1}}$$

$$\therefore \quad \tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

$$= \frac{2\sqrt{\frac{x-1}{x+1}}}{1-\frac{x-1}{x+1}} = \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{2}{x+1}}$$

$$= \sqrt{x^2-1}$$

10. Let

$$f(\theta) = \left[\int_0^{\sin^2\theta} \sin^{-1}\sqrt{\phi}\, d\phi + \int_0^{\cos^2\theta} \cos^{-1}\sqrt{\phi}\, d\phi\right]$$

$$f'(\theta) = \frac{d}{d\theta}\sin^2\theta\left[\sin^{-1}\sqrt{\sin^2\theta}\right] + \frac{d}{d\theta}\cos^2\theta\left[\cos^{-1}\sqrt{\cos^2\theta}\right]$$

$= (2\sin\theta\cos\theta)\theta - (2\sin\theta\cos\theta)\theta$

$= 0$

$\therefore \quad f(\theta) = \text{constant} = a$ (say)

$\therefore \quad f\left(\frac{\pi}{4}\right) = a$

$$\Rightarrow \quad \int_0^{1/2} \sin^{-1}\sqrt{\phi}\, d\phi + \int_0^{1/2} \cos^{-1}\sqrt{\phi}\, d\phi = a$$

$$\Rightarrow \quad \int_0^{1/2} (\sin^{-1}\sqrt{\phi} + \cos^{-1}\sqrt{\phi})\, d\phi = a$$

$$\Rightarrow \quad \frac{\pi}{2}[\phi]_0^{1/2} = a$$

$$\Rightarrow \quad \frac{\pi}{4} = a$$

11. Let $I = \int_0^{2n\pi} \left\{|\sin x| - \left|\frac{1}{2}\sin x\right|\right\} dx$

$$= \int_0^{2n\pi} \left\{|\sin x| - \frac{1}{2}|\sin x|\right\} dx$$

$$= \int_0^{2n\pi} \frac{1}{2}|\sin x|\, dx$$

$$= \frac{1}{2}\left[\int_0^{2\pi} |\sin x|\, dx + \int_{2\pi}^{4\pi} |\sin x|\, dx + \ldots\ldots + \int_{2(n-1)\pi}^{2n\pi} |\sin x|\, dx\right]$$

Now, $I_1 = \int_0^{2\pi} |\sin x|\, dx$

$$I_1 = \int_0^{\pi} \sin x\, dx - \int_{\pi}^{2\pi} \sin x\, dx$$

$= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi} = -[-1-1] + [+1+1]$

$= 2 + 2$

$= 4$

$\therefore \quad I = \frac{1}{2}[4 + 4 + 4 + \ldots n \text{ times}]$

$= \frac{1}{2}(4n) = 2n$

12. Given that $f(x) = \frac{x^2}{x^2+1}$

Since, it is an even function therefore its values is always greater than equal to 0 and we know

$$x^2 < x^2 + 1 \quad \text{or} \quad \frac{x^2}{x^2+1} < 1$$

$\therefore$ Required range is [0, 1).

13. Given $\sin^{-1}(1-x) + 2\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}(x)$

$\Rightarrow \quad (1-x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$

$\Rightarrow \quad (1-x) = \cos(2\sin^{-1}x)$

$\Rightarrow \quad (1-x) = \cos[\cos^{-1}(1-2x^2)]$

$\Rightarrow \quad (1-x) = 1-2x^2$

$\Rightarrow \quad 2x^2 - x = 0$

$\Rightarrow \quad x = 0, \frac{1}{2}$

But $x = \frac{1}{2}$ does not satisfy the given equation,

So, $x = \{0\}$ is the answer.

14. Since $\sin A$, $\sin B$ and $\cos A$ are in GP

$\therefore \quad \sin^2 B = \sin A \cos A \quad$ ...(i)

$x^2 + 2x\cot B + 1 = 0 \quad$ (given)

Now, $\quad b^2 - 4ac = 4\cot^2 B - 4$

$$= \frac{4\cos^2 B - 4\sin^2 B}{\sin^2 B} = \frac{4(1-\sin^2 B) - 4\sin^2 B}{\sin^2 B}$$

$$= \frac{4[1-2\sin^2 B]}{\sin^2 B}$$

$$= \frac{4[1-2\sin A\cos A]}{\sin^2 B} \quad \text{[from (i)]}$$

$$= 4\left(\frac{\sin A - \cos A}{\sin B}\right)^2 > 0$$

$\therefore$ Roots are always real.

15. Let $I = \int_{\log 2}^{x} \frac{du}{(e^u - 1)^{1/2}}$

or $I = \int_{\log 2}^{x} \frac{e^u}{e^u(e^u-1)^{1/2}} du$

Let $e^u - 1 = t^2 \Rightarrow e^u\, du = 2t\, dt$

$$= \int_1^{\sqrt{e^x-1}} \frac{2t}{(t^2+1)t} dt = 2\int_1^{\sqrt{e^x-1}} \frac{dt}{(1+t^2)}$$

$$= [\tan^{-1} t]_1^{\sqrt{e^x-1}} = 2\tan^{-1}\sqrt{e^x-1} - \tan^{-1} 1]$$

$\Rightarrow \quad 2\left[\tan^{-1}\sqrt{e^x-1} - \frac{\pi}{4}\right] = \frac{\pi}{6} \quad$ (given)

$\Rightarrow \quad \tan^{-1}\sqrt{e^x-1} = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3}$

$\Rightarrow \quad \sqrt{e^x-1} = \tan\left(\frac{\pi}{3}\right)$

$\sqrt{e^x-1} = \sqrt{3}$

$\Rightarrow \quad e^x = 3+1 = 4$

16. Since

$$(1+x)^{2n+1} = C_0 + C_1x + \ldots + C_n x^n + C_{n+1}x^{n+1} + \ldots + x^{2n+1}$$

$$= 2(C_0 + C_1 + \ldots + C_n x^n)$$

Put $x = 1$

$(1+1)^{2n+1} = 2(C_0 + C_1 + \ldots + C_n)$

$\Rightarrow \quad 2^{2n} = (C_0 + C_1 + \ldots + C_n)$

$\Rightarrow \quad 2^{2n} - 1 = C_1 + C_2 + \ldots + C_n$

$\Rightarrow \quad 2^{2n} - 1 = 63$

$\Rightarrow \quad 2^{2n} = 64 \Rightarrow 2^{2n} = 2^6$

$\Rightarrow \quad 2n = 6 \Rightarrow n = 3$

17. Given that

$x^2 = xy$

Let $\quad x, y \in R$

$xRy = x^2 = xy$

and $\quad yRz = y^2 = yz$

Now, $\quad x^2y^2 = xy^2z$

$\Rightarrow \quad x^2 = xz$

$\Rightarrow \quad xRz$

$\therefore$ It is transitive.

18. Given that $\det(A) = 6 \quad$ ...(i)

Now, $\quad B = 5A^2$

$\Rightarrow \quad \det(B) = \det(5A^2)$

$= 5\det(A^2) = 5\det(A)^2$

$= 5(6)^2 \quad$ (from (i))

$\Rightarrow \quad \det(B) = 180$

19. Given that

$$f(x) = \begin{cases} 1 & \forall \quad x < 0 \\ 1 + \sin x & \forall \quad 0 \le x \le \pi/2 \end{cases}$$

At $\quad x = 0$

$$\text{LHD} = \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h\to 0} \frac{1-1}{-h} = 0$$

$$\text{RHD} = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{1 + \sin(0+h) - 1}{h}$$

$$= \lim_{h\to 0} \frac{\sin h}{h} = 1$$

$\Rightarrow \quad \text{LHD} \ne \text{RHD}$

$\therefore \quad f'(x)$ does not exist at $x = 0$.

20. Given curve is $\quad y^2 = 4x \quad$ ...(i)

Let the equation of line by $y = mx + c$

Since $\frac{dy}{dx} = m = 1$ and this line is passing through the point (0, 1).

$\therefore \quad 1 = 1(0) + c \Rightarrow c = 1$

$\therefore \quad y = x + 1 \quad$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$(x+1)^2 = 4x$$

$\Rightarrow$ $(x-1)^2 = 0$

$\Rightarrow$ $x = 1$ and $y = 2$

This shows that line touch the curve at one point. So length of intercept is zero.

21. Given curves are

$$y = x^2 \quad \text{...(i)}$$

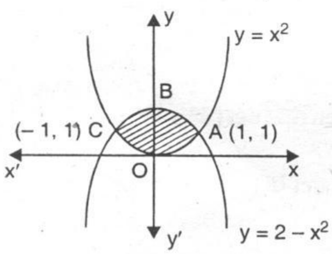
and $y = 2 - x^2$

or $x^2 = -(y-2)$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$x = -1, 1$$

and $y = 1, 1$



$\therefore$ Required area = Area of curve $OABCO$

$= 2$ Area of curve $OABO$

$$= 2\int_0^1 y\,dx$$

$$= 2\int_0^1 [(2-x^2)-(x^2)]\,dx$$

$$= 2\int_0^1 (2-2x^2)\,dx$$

$$= 4\left[x - \frac{x^3}{3}\right]_0^1$$

$$= 4\left[1 - \frac{1}{3}\right]$$

$$= \frac{8}{3} \text{ sq units.}$$

22. $\lim_{\theta\to 0} \dfrac{4\theta(\tan\theta - 2\theta\tan\theta)}{1-\cos 2\theta}$

$$= \lim_{\theta\to 0} \frac{4(\theta\tan\theta - 2\theta^2\tan\theta)}{1-\cos 2\theta}$$

Using L' Hospital's rule

$$= \lim_{\theta\to 0} \frac{4(\theta\sec^2\theta + \tan\theta - 4\theta\tan\theta - 2\theta^2\sec^2\theta)}{2\sin 2\theta}$$

Again using L' Hospital's rule

$4(\sec^2\theta + 2\theta\sec^2\theta\tan\theta + \sec^2\theta - 4\tan\theta)$

$$= \lim_{\theta\to 0} \frac{-4\theta\sec^2\theta - 4\theta\sec^2\theta - 4\theta^2\sec^2\theta\tan\theta}{4\cos 2\theta}$$

$$= \frac{4(1+0+1)}{4} = 2$$

23. Let $f(x) = 2x^3 - 3x^2 - 12x + 5$

$\therefore$ $f'(x) = 6x^2 - 6x - 12$

Put $f'(x) = 0$, for maxima or minima.

$\therefore$ $6x^2 - 6x - 12 = 0$

$\Rightarrow$ $x^2 - x - 2 = 0$

$\Rightarrow$ $x^2 - 2x + x - 2 = 0$

$\Rightarrow$ $(x-2)(x+1) = 0$

$\Rightarrow$ $x = -1, 2$

Now, $f''(x) = 12x - 6$

$f''(-1) = -12 - 6 = -18 < 0$

$\therefore$ $f(x)$ is maximum at $x = -1$.

But $x = 4$

$f(x) = 37.$

$\therefore$ The largest value of $f(x)$ is at $x = 4$

24. Let $z = x + iy$

$\therefore$ $|z-1| = |z-2| = |z-i|$

$\Rightarrow$ $|(x-1)+iy| = |(x-2)+iy|$

$= |(x + i(y-1)|$

$\Rightarrow x^2 - 2x + 1 + y^2 = x^2 + 4 - 4x + y^2$

$= x^2 + y^2 + 1 - 2y$

Taking Ist and IInd term

$\Rightarrow$ $-2x + 1 = 4 - 4x$

$\Rightarrow$ $2x = 3$ ...(i)

Taking IInd and IIIrd term

$\Rightarrow$ $4 - 4x = 1 - 2y$

$\Rightarrow$ $4x - 2y = 3$ ...(ii)

Taking Ist and IIIrd term

$\Rightarrow$ $-2x + 1 = 1 - 2y$

$\Rightarrow$ $2x - 2y = 0$

$\Rightarrow$ $x = y$ ...(iii)

From (i) $x = \dfrac{3}{2}$

On putting value of x in Eq. (iii), we get

$$y = \frac{3}{2}$$

On putting the value of x and y in Eq. (ii), we get $4\left(\dfrac{3}{2}\right) - 2\left(\dfrac{3}{2}\right) = 3$

$\Rightarrow$ $3 = 3$

$\therefore$ One solution exist.

25. Given that, $P(A')=0.3, P(B)=0.4$

and $P(A\cap B')=0.5$

$P(B')=1-P(B)=1-0.4=0.6$

$P(A)=1-P(A')=1-0.3=0.7$

$P(A\cup B')=P(A)+P(B')-P(A\cap B')$

$=0.7+0.6-0.5=0.8$

26. $(10101101)_2$

$=1\times 2^7+0\times 2^6+1\times 2^5+0\times 2^4+1\times 2^3+1\times 2^2+0\times 2^1+1\times 2^0$

$=128+0+32+0+8+4+0+1$

$=173$

27. Given that $f(x)=\dfrac{e^{2x}-1}{e^{2x}+1}$

On differentiating w.r.t. x, we get

$$f'(x)=\frac{2(e^{2x}+1)(e^{2x})-2(e^{2x}-1)(e^{2x})}{(e^{2x}+1)^2}$$

$$=\frac{2(e^{2x}+e^{2x})}{(e^{2x}+1)^2}=\frac{4e^{2x}}{(e^{2x}+1)^2}>0$$

$\therefore$ Function is an increasing

28. Given equation is

$$x^2+y^2-2xy\frac{dy}{dx}=0$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{x^2+y^2}{2xy} \quad \text{...(i)}$$

This is a homogeneous equation

$\therefore$ we put $y=vx$ and $\dfrac{dy}{dx}=v+x\dfrac{dv}{dx}$

The Eq. (i) is reduces to

$$v+x\frac{dv}{dx}=\frac{x^2(1+v^2)}{2x^2v}$$

$$\Rightarrow \quad x\frac{dv}{dx}=\frac{1+v^2}{2v}-v=\frac{1-v^2}{2v}$$

$$\Rightarrow \quad -\frac{2v}{1-v^2}dv=-\frac{dx}{x}$$

On integrating both sides, we get

$\log(1-v^2)=-\log x+\log c$

$\Rightarrow \log(x^2-y^2)-2\log x=-\log x+\log c$

$\Rightarrow \quad \log(x^2-y^2)=\log xc$

$\Rightarrow \quad x^2-y^2=xc$

29. $\because f(x)=ax^2+bx+c$

and $g(x)=px^2+qx$

Since, $g(1)=f(1)$

$\Rightarrow \quad p+q=a+b+c \quad \text{...(i)}$

and $g(2)-f(2)=1$

$\Rightarrow \quad 4p+2q-4a-2b-c=1 \quad \text{...(ii)}$

also $g(3)-f(3)=4$

$\Rightarrow \quad 9p+3q-9a-3b-c=4 \quad \text{...(iii)}$

From Eqs. (i) and (ii)

$2p=2a-c+1$

Now, $g(4)-f(4)$

$=16p+4q-16a-4b-c$

$=12p+4(p+q)-16a-4b-c=6-3c$

30. Since the given vectors $\alpha\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}, \hat{\mathbf{i}}+\beta\hat{\mathbf{j}}+\hat{\mathbf{k}}$ and $\hat{\mathbf{i}}+\hat{\mathbf{j}}+\gamma\hat{\mathbf{k}}$ are coplanar, then

$$\begin{vmatrix}\alpha & 1 & 1\\ 1 & \beta & 1\\ 1 & 1 & \gamma\end{vmatrix}=0$$

Applying $C_2\to C_2-C_1, C_3\to C_3-C_2$

$$\Rightarrow \begin{vmatrix}\alpha & 1-\alpha & 0\\ 1 & \beta-1 & 1-\beta\\ 1 & 0 & \gamma-1\end{vmatrix}=0$$

$$\Rightarrow (1-\alpha)(1-\beta)(1-\gamma)\begin{vmatrix}\dfrac{\alpha}{1-\alpha} & 1 & 0\\ \dfrac{1}{1-\beta} & -1 & 1\\ \dfrac{1}{1-\gamma} & 0 & -1\end{vmatrix}=0$$

$$\Rightarrow (1-\alpha)(1-\beta)(1-\gamma)\left[\frac{\alpha}{1-\alpha}(1)-1\left(-\frac{1}{1-\beta}-\frac{1}{1-\gamma}\right)\right]=0$$

But $\alpha\ne 1, \beta\ne 1$ and $\gamma\ne 1$

$$\therefore \quad \frac{1}{(1-\alpha)}-1+\frac{1}{1-\beta}+\frac{1}{1-\gamma}=0$$

$$\Rightarrow \quad \frac{1}{1-\alpha}+\frac{1}{1-\beta}+\frac{1}{1-\gamma}=1$$

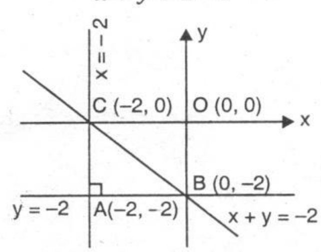
31. Given equation of lines are

$xy+2x+2y+4=0$

or $(x+2)(y+2)=0$

or $x+2=0, y+2=0 \quad \text{...(i)}$

and $x+y+2=0 \quad \text{...(iii)}$



These three lines makes an right triangle CAB right angled at A.

The circumcentre of a triangle is the mid point of BC *i.e.* $(-1,-1)$.

32. The centre and radius of the first circle $x^2 + y^2 + 2x + 8y - 23 = 0$ are $C_1(-1, -4)$ and $r_1 = \sqrt{40}$

Similarly, the centre and radius of second circle $x^2 + y^2 - 4x - 10y + 9 = 0$ are $C_2(2, 5)$ and $r_2 = \sqrt{20}$

Now, $C_1C_2 = \sqrt{(2+1)^2 + (5+4)^2}$

$= \sqrt{9 + 81} = \sqrt{90}$

and $r_1 + r_2 = \sqrt{40} + \sqrt{20}$

also $r_1 - r_2 = \sqrt{40} - \sqrt{20}$

Here, $r_1 - r_2 < C_1C_2 < r_1 + r_2$

$\therefore$ Two common tangents can be drawn.

33. Since the line $\frac{x}{\alpha} + \frac{y}{\beta} = 1$ touches the circle $x^2 + y^2 = a^2$.

$\therefore$ The perpendicular distance from centre (0, 0) to the tangent = radius of the circle

$$\Rightarrow \quad \frac{|-1|}{\sqrt{\frac{1}{\alpha^2} + \frac{1}{\beta^2}}} = a$$

$$\Rightarrow \quad \frac{1}{a^2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

The locus of $\left(\frac{1}{\alpha}, \frac{1}{\beta}\right)$ is

$$\frac{1}{a^2} = \frac{1}{x^2} + \frac{1}{y^2}$$

$\therefore$ It represents a circle.

34. Given equation of line is

$$\frac{x-1}{3} = \frac{y+2}{4} = \frac{z-3}{-2} = k \quad \text{(say)}$$

Any point on the line is

$(3k + 1, 4k - 2, -2k + 3)$.

If the given line intersect the plane $2x - y + 3z - 1 = 0$, then any point on the line lies in the plane.

$\therefore 2(3k + 1) - (4k - 2) + 3(-2k + 3) - 1 = 0$

$\Rightarrow \quad -4k + 12 = 0 \Rightarrow k = 3$

$\therefore$ Point is $(9 + 1, 12 - 2, -6 + 3)$

i.e., $(10, 10, -3)$.

35. Equation of director circle of given hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ is $x^2 + y^2 = 25 - 16$

$\Rightarrow \quad x^2 + y^2 = 9 \quad$ ...(i)

This circle passes through $(2\sqrt{2}, 1)$ and we know that director circle is the locus of point of intersection of perpendicular tangents drawn to a hyperbola.

Thus the angle between the tangents is $\pi/2$

36. Given that, it is given

$\alpha\beta\gamma\delta = 1 \quad$ ...(i)

As, we know A.M. $\geq$ G.M.

$\Rightarrow \quad \frac{1+\alpha}{2} \geq \sqrt{\alpha}$

$\Rightarrow \quad 1 + \alpha \geq 2\sqrt{\alpha} \quad$ ...(ii)

Similarly, $1 + \beta \geq 2\sqrt{\beta} \quad$ ...(iii)

$1 + \gamma \geq 2\sqrt{\gamma} \quad$ ...(iv)

and $1 + \delta \geq 2\sqrt{\delta} \quad$ ...(v)

Multiplying Eqs. (ii), (iii), (iv) and (v), we get

$(1+\alpha)(1+\beta)(1+\gamma)(1+\delta) \geq 16\sqrt{\alpha\beta\gamma\delta}$

$\Rightarrow \quad (1+\alpha)(1+\beta)(1+\gamma)(1+\delta) = 16$

37. Given that

$$\sum_{k=1}^{6}\left(\sin\left(\frac{2k\pi}{7}\right) - i\cos\left(\frac{2k\pi}{7}\right)\right)$$

$$= -i\sum_{k=1}^{6}\cos\left(\frac{2k\pi}{7}\right) + i\sin\left(\frac{2k\pi}{7}\right)$$

$$= -i\sum_{k=1}^{6}\left(e^{\frac{2\pi i}{7}}\right)^k$$

$$= -i\sum_{k=1}^{6} r^k \qquad \left(\text{let } r = e^{\frac{2\pi i}{7}}\right)$$

$$= -i\,(r^1 + r^2 + \ldots + r^6)$$

$$= -i\,r\frac{(1 - r^6)}{1 - r} = \frac{-i(r - r^7)}{1 - r}$$

$$= \frac{-i\,(r-1)}{1-r} = i \qquad [\because r^7 = e^{2\pi i} = 1]$$

38. Given that

$$y(x) = 1 + \frac{dy}{dx} = \frac{1}{1\cdot 2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{1\cdot 2\cdot 3}\left(\frac{dy}{dx}\right)^3 + \ldots$$

or $$y(x) = 1 + \frac{1}{1!}\left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots$$

$y(x) = e^{dy/dx}$

Taking log on both sides, we get

$$\log y(x) = \frac{dy}{dx}$$

$\therefore$ The degree of this equation is 1.

39. Given x_1, x_2 are the roots of the equation

$x^2 + 2x - 3 = 0$

$\Rightarrow \quad x^2 + 3x - x - 3 = 0$

$\Rightarrow \quad x(x + 3) - 1(x + 3) = 0$

$\Rightarrow \quad (x - 1)(x + 3) = 0$

$\Rightarrow \quad x_1 = -3, x_2 = 1$

and y_1, y_2 are the roots of the equation

$y^2 + 4y - 12 = 0$

$\Rightarrow \quad y^2 + 6y - 2y - 12 = 0$

$\Rightarrow \quad y(y+6) - 2(y+6) = 0$

$\Rightarrow \quad (y-2)(y+6) = 0$

$\Rightarrow \quad y_1 = -6, y_2 = 2$

$\therefore$ Points are $P(-3, -6)$ and $Q(1, 2)$.

Since, P and Q are the end points of a diameter.

$\therefore$ Centre = mid point of PQ

$$= \left(\frac{-3+1}{2}, \frac{-6+2}{2}\right)$$

$$= (-1, -2)$$

40. The equation of any plane through $(2, -1, 3)$ is

$a(x-2) + b(y+1) + c(z-3) = 0$ ...(i)

where a, b and c are direction ratios, Since Eq. (i) is parallel to $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$

$\therefore \quad 3a + 0b - c = 0$ ...(ii)

and $\quad -3a + 2b - 2c = 0$ ...(iii)

Solving Eqs. (ii) and (iii), we get

$$\frac{a}{2} = -\frac{b}{6-3} = \frac{c}{6} = k \quad \text{(say)}$$

$\Rightarrow \quad a = 2k, b = -3k, c = 6k$

Putting the values of a, b and c in Eq. (i), we get

$2k(x-2) - 3k(y+1) + 6k(z-3) = 0$

$\Rightarrow \quad 2x - 3y + 6z - 25 = 0$

which is a required equation of a plane.

41. Equation of parabola is

$$y^2 = -4x$$

$\therefore$ focus is $(-1, 0)$.

The equation of line passing through $(-1, 0)$ is

$y - 0 = m(x+1)$ ...(i)

Since, the line makes an angle $\theta = 120°$

$\therefore \quad m = \tan\theta = \tan 120°$

$\Rightarrow \quad m = -\sqrt{3}$

On putting the value of m in Eq. (i), we get

$$y = -\sqrt{3}(x+1)$$

42. Given that

$x = \alpha = \beta, y = \alpha\omega + \beta\omega^2, z = \alpha\omega^2 + \beta\omega$

Now, $xyz = (\alpha+\beta)(\alpha\omega + \beta\omega^2)(\alpha\omega^2 + \beta\omega)$

$= (\alpha+\beta)(\alpha^2\omega^3 + \alpha\beta\omega^2 + \alpha\beta\omega^4 + \beta^2\omega^3)$

$= (\alpha+\beta)(\alpha^2 + \alpha\beta(\omega^2+\omega) + \beta^2)$

$$\left[\begin{array}{r}\therefore 1 + \omega + \omega^2 = 0 \\ \text{and } \omega^3 = 1\end{array}\right]$$

$= (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= \alpha^3 + \beta^3$

43. Given that

$$r = \left[2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)\right]^{1/2}$$

On differentiating w.r.t ϕ, we get

$$\frac{dr}{d\phi} = \frac{\left[2 - 2\cos\left(2\phi + \frac{\pi}{4}\right)\sin\left(2\phi + \frac{\pi}{4}\right)\cdot 2\right]}{2\sqrt{2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)}}$$

$$= \frac{\left[1 - \sin\left(4\phi + \frac{\pi}{2}\right)\right]}{\sqrt{2\phi + \cos^2\left(2\phi + \frac{\pi}{4}\right)}}$$

$$\Rightarrow \left(\frac{dr}{d\phi}\right)_{\phi = \pi/4} = \frac{\left[1 - \sin\left(\pi + \frac{\pi}{2}\right)\right]}{\sqrt{2\cdot\frac{\pi}{4} + \cos^2\left(\frac{\pi}{2} + \frac{\pi}{4}\right)}}$$

$$= \frac{1+1}{\sqrt{\frac{\pi}{2} + \frac{1}{2}}} = 2\sqrt{\frac{2}{1+\pi}}$$

44. Since, $\vec{\alpha}$ lie in the plane of $\vec{\beta}$ and $\vec{\gamma}$.

It means that all three vectors are coplanar.

$\therefore \quad [\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = 0$

45. Given that

$$\vec{\alpha} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}, \vec{\beta} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

and $\quad \vec{\gamma} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Now,

$$\vec{\alpha}\times\vec{\beta} = \begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -1 \\ -1 & 2 & -4\end{vmatrix}$$

$= \hat{\mathbf{i}}(-12+2) - \hat{\mathbf{j}}(-8-1) + \hat{\mathbf{k}}(4+3)$

$= -10\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

and $$(\vec{\alpha}\times\vec{\gamma}) = \begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -1 \\ 1 & 1 & 1\end{vmatrix}$$

$= \hat{\mathbf{i}}(3+1) - \hat{\mathbf{j}}(2+1) + \hat{\mathbf{k}}(2-3)$

$= -4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$

Now, $(\vec{\alpha}\times\vec{\beta})\cdot(\vec{\alpha}\times\vec{\gamma})$

$= (-10\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 7\hat{\mathbf{k}})\cdot(4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$

$= -40 - 27 - 7$

$= -74$

46. Gravitational acceleration is given by

$$g = \frac{GM}{R^2}$$

where G =gravitational constant

$$\therefore \quad \frac{g}{G} = \frac{M}{R^2}$$

47. In this case the internal force is applied on the system, so he will not succeed. According to Newton's law the state of a body can only be changed if some external force is applied on it.

48. $y = \frac{\text{stress}}{\text{strain}} = \text{N/m}^2$ or pascal (in SI system)

and $y = \frac{\text{dyne}}{\text{cm}^2}$ (in CGS System)

Thus, Nm^{-1} is not the unit of Young's modulus.

49. According to Stefan's law the energy emitted by a body per second is directly proportional to the fourth power of the temperature of the body. Here, the temperature of blue glass is more than that of red glass, so it will look brighter.

50. Chemical energy reduced

$$= VIt$$
$$= 6 \times 5 \times 6 \times 60$$
$$= 10800$$
$$= 1.08 \times 10^4 \text{ V}$$

51. Let the original resistance is $R\,\Omega$.

$$\therefore \quad V = IR$$
$$V = 5 \times R = 5R \quad \text{...(i)}$$

When 2Ω resistance is inserted, then total resistance $= (R + 2)\,\Omega$

$$\therefore \quad V = I'(R + 2) = 4(R + 2) \quad \text{...(ii)}$$

From Eqs. (i) and(II), we get

$$5R = 4(R + 2)$$
$$\therefore \quad R = 8\Omega$$
$$= 100 + \frac{75}{4} = \frac{475}{4}\Omega$$
$$= 118.75\Omega$$

52. Let S be the large and R be the smaller resistance.

From formula for metre bridge

$$S = \left(\frac{100 - l}{l}\right) R$$
$$= \frac{100 - 20}{20} R = 4R$$

Again,

$$S = \left(\frac{100 - l}{100}\right)(R + 15)$$
$$= \frac{100 - 40}{40}(R + 15)$$
$$= \frac{3}{2}(R + 15)$$
$$\therefore \quad 4R = \frac{3}{2}(R + 15)$$
$$\frac{8R}{3} - R = 15 \Rightarrow \frac{5R}{3} = 15$$
$$R = 9\Omega$$

53. If we take $R_1 = 4\Omega\, R_2 = 12\Omega$,

then in series resistance

$$R = R_1 + R_2$$
$$= 4 + 12$$
$$= 16\Omega$$

In parallel, resistance $R = \frac{4 \times 12}{4 + 12} = 3\Omega$

So, $R_1 = 4\Omega$ and $R = 12\Omega$

54. Let the resistance of voltmeter is $G\,\Omega$.

$\therefore$ Total resistance of the circuit

$$R = \left(\frac{G \times 100}{G + 100} + 50\right)\Omega$$

Total current $i = \frac{V}{R}$

$$= \frac{10}{\left(\frac{G \times 100}{G + 100} + 50\right)}$$

Voltage across $100\,\Omega$ resistance

$$= i\left(\frac{G \times 100}{G + 100}\right) = \frac{10}{\left(\frac{G \times 100}{G + 100} + 50\right)} \times \left(\frac{G \times 100}{G + 100}\right)$$

Reading of voltmeter = 5 V

$\therefore$ Voltage across $100\,\Omega = 5$ V

$$\therefore \quad 5 = \frac{10}{\left(\frac{G \times 100}{G + 100} + 50\right)} \times \left(\frac{G \times 100}{G + 100}\right)$$

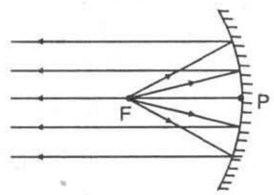
On solving $G = 100\,\Omega$.

55. According to Wien's law

$$\lambda \propto \frac{1}{T}$$

i.e., it depends on the temperature of the surface.

56. It lamp is placed at the focus of concave mirror, then we get parallel beam of light.



58. Here, $E = 1500 \text{V/m}$, $B = 0.4 \text{ Wb/m}^2$

Minimum speed of electron along the straight line $v = \frac{E}{B}$

$$= \frac{1500}{0.4}$$
$$= 3750$$
$$= 3.75 \times 10^3 \text{ m/s}$$

59. Shunt resistance

$$S = \frac{I_g G}{I - I_g}$$
$$= \frac{0.1\,G}{1 - 0.1}$$
$$= \frac{G}{9}$$

60. Diamagnetic materials have negative susceptibility. Thus, (c) is wrongly stated.

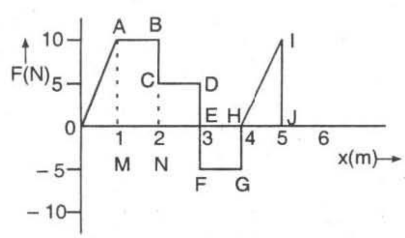
61. The induction coil works on the principle of mutual induction.

62. We know $f = \frac{1}{2\pi\sqrt{LC}}$

or $\sqrt{LC} = \frac{1}{2\pi f} = \text{time}$.

Thus, $\sqrt{LC}$ has the dimension of time.

63. Work done = area enclosed by F-x graph.



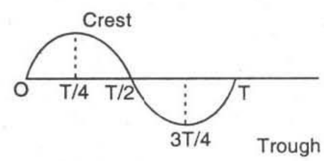
= area of $ABNM$ + area of $CDEN$ – area of $EFGH$ + area of HIJ

$$= 1 \times 10 + 1 \times 5 - 1 \times 5 + \frac{1}{2} \times 1 \times 10$$

$$= 10 + 5 - 5 + 5 = 15\text{J}$$

64. Both the stones will have the same speed when they hit the ground.

66. The time taken by the particle to come to mean position from the trough $= \frac{T}{4}$



67. Speed = 360 rev/min

$$= \frac{360}{60} \text{ rev/s}$$
$$= 6$$

$\therefore$ Frequency $= 6 \times 60$

$$= 360$$

68. Velocity of sourd $v = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{v_H}{v_O} = \sqrt{\frac{M_O}{M_H}}$$
$$= \sqrt{\frac{16}{1}}$$
$$= 4 : 1$$

69. For sonometer $n \propto \frac{1}{l}$

$$\therefore \quad \frac{n_1}{n_2} = \frac{l_2}{l_1} \Rightarrow \frac{256}{n_2} = \frac{16}{25}$$
$$n_2 = \frac{256 \times 25}{16}$$
$$= 400 \text{ Hz}$$

70. Wave theory of light was first proposed by Christian Huygens.

71. For the liquids, which do not wet the glass, the liquid meniscus is convex upward, so angle of contact is obtuse.

72. Radius of path of electron

$$r = \frac{mv}{Bq}$$

m and q remain unchanged.

So, $\frac{r_1}{r_2} = \frac{v_1}{v_2} \cdot \frac{B_2}{B_1}$

$$= \frac{v}{2v} \cdot \frac{B/2}{B} = \frac{1}{4} \Rightarrow r_2 = 4r$$

73. As $I \propto a^2$ or $a \propto \sqrt{I}$

$$\therefore \quad \frac{a_1}{a_2} = \sqrt{\frac{I}{4I}}$$

$\Rightarrow \quad \frac{a_1}{a_2} = \frac{1}{2}$

$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

$$= \left(\frac{1+2}{1-2}\right)^2 = \frac{9}{1}$$

$\therefore \quad I_{max} = 9I, I_{min} = I$

74. In a hydrogen atom the time period is given by

$$T \propto n^3$$

$$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^2 \Rightarrow \frac{8}{1} = \left(\frac{n_1}{n_2}\right)^3$$

$\therefore \quad \frac{n_1}{n_2} = \frac{2}{1}$

Thus, $\quad n_1 = 4$ and $n_1 = 2$

75. On increasing the forward bias voltage, the barrier energy decreases. This results in the flow of majority charge carriers. Hence, width of depletion region decreases.

76. Nuclear forces are charge independent so,

$$F_1 = F_2 = F_3.$$

79. Potential $V = \frac{Q}{C} \Rightarrow V = \frac{Q}{\frac{A\varepsilon_0}{d}}$

Hence, potential depends on the amount of charge, area or geometry and size of the conductor.

78. The potential at each point on the circular path will be equal.

So, work done $= q \times$ potential difference

$= q \times 0$

$= 0$

79. Capacitance with air

$$C = \frac{A\varepsilon_0}{d}$$

When interspace between the plates is filled with wax, then

$$C' = \frac{KA\varepsilon_0}{2d}$$

or $\quad C' = \left(\frac{A\varepsilon_0}{d}\right)\frac{K}{2}$

or $\quad C' = C\frac{K}{2}$

$\therefore \quad 6 = 2 \cdot \frac{K}{2} \Rightarrow K = 6$

80. de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$\therefore \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}} \Rightarrow \frac{1 \times 10^{-9}}{0.5 \times 10^{-9}} = \sqrt{\frac{E_2}{E_1}}$

$\Rightarrow \quad 2 = \sqrt{\frac{E_2}{E_1}} \Rightarrow \frac{E_2}{E_1} = 4$

$\therefore \quad E_2 = 4E_1$

$\therefore$ Energy to be added $= E_2 - E_1$

$= 4E_1 - E_1 = 3E_1$

81. Half-life $T/2 = \frac{T}{1.44} = \frac{100}{1.44}$ s

$= 69.44$ s

$= \frac{69.44}{60} \approx 1.155$ min

82. Radioactive decay does not depend upon the time of creation.

84. Coulomb's law is appliable for charged particles, it is not responsible to bind the protons and neutrons in the nucleus of an atom.

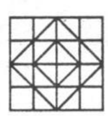
85. If unpolarised light is incident at polarising angle, then reflected light is completely, *i.e*, 100% polarised.

Reasoning

141. (c) Second is the result of the first.

142. (b) In all other groups, the first second and third letters are respectively moved one, five and one step forward to obtain second, third and fourth letters respectively.

143. Clearly, fig. (d) when placed in the blank space of fig (x) will complete the pattern, as shown below.



Hence, the answer is (d).

145. In fig. X, the right half of the rectangular paper sheet is folded over the left half. In fig. Y, two semicircles are punched into the folded paper. When the paper is unfolded, the semicircles in the two halves will join to form circles. Thus, two circles will appear in the unfolded position of fig. Y.

Hence, fig. (d) is the correct answer.

$$\Rightarrow \qquad \frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

$$= \left(\frac{1+2}{1-2}\right)^2 = \frac{9}{1}$$

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or $$C' = \left(\frac{A\varepsilon_0}{d}\right)\frac{K}{2}$$

or $$C' = C\frac{K}{2}$$

$$\therefore \qquad 6 = 2 \cdot \frac{K}{2} \Rightarrow K = 6$$

80. de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}} \Rightarrow \frac{1 \times 10^{-9}}{0.5 \times 10^{-9}} = \sqrt{\frac{E_2}{E_1}}$$

$$\Rightarrow \qquad 2 = \sqrt{\frac{E_2}{E_1}} \Rightarrow \frac{E_2}{E_1} = 4$$

$$\therefore \qquad E_2 = 4E_1$$

$\therefore$ Energy to be added $= E_2 - E_1$

$$= 4E_1 - E_1 = 3E_1$$

81. Half-life $T/2 = \frac{T}{1.44} = \frac{100}{1.44}$ s

$$= 69.44 \text{ s}$$

$$= \frac{69.44}{60} \approx 1.155 \text{ min}$$

82. Radioactive decay does not depend upon the time of creation.

84. Coulomb's law is appliable for charged particles, it is not responsible to bind the protons and neutrons in the nucleus of an atom.

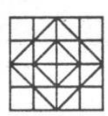
85. If unpolarised light is incident at polarising angle, then reflected light is completely, *i.e,* 100% polarised.

Reasoning

141. (c) Second is the result of the first.

142. (b) In all other groups, the first second and third letters are respectively moved one, five and one step forward to obtain second, third and fourth letters respectively.

143. Clearly, fig. (d) when placed in the blank space of fig (x) will complete the pattern, as shown below.



Hence, the answer is (d).

145. In fig. X, the right half of the rectangular paper sheet is folded over the left half. In fig. Y, two semicircles are punched into the folded paper. When the paper is unfolded, the semicircles in the two halves will join to form circles. Thus, two circles will appear in the unfolded position of fig. Y.

Hence, fig. (d) is the correct answer.