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How to define the base units of the revised SI from seven constants with fixed numerical values

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Édité par le BIPM
Pavillon de Breteuil
F-92312 Sèvres Cedex
FRANCE

Foreword

As part of a revision to the SI expected to be approved later this year and to take effect in May 2019, the seven base units will be defined by giving fixed numerical values to seven “defining constants”.

The report shows how the definitions of all seven base units can be derived efficiently from the defining constants, with the result appearing as a table. The table’s form makes evident a number of connections between the defining constants and the base units.

Appendices show how the same methodology could have been used to define the same base units in the present SI, as well as the mathematics which underpins the methodology.

How to define the base units of the revised SI from seven constants with fixed numerical values Contents

1. Introduction

Preparations for the upcoming revision of the International System of Units (SI) began in earnest with Resolution 1 of the 24th meeting of the General Conference on Weights and Measures (CGPM) in 2011 [1]. The 26th CGPM in November 2018 is expected to give final approval to a revision of the present SI [2] based on the guidance laid down in Ref. [1]. The SI will then become a system of units based on exact numerical values of seven defining constants, $\Delta\nu_{Cs}$, c, h, e, k, N_A and K_{cd} exactly as specified in the following bullets:

- the unperturbed ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{Cs}$ is 9192 631 770 hertz,
- the speed of light in vacuum c is 299 792 458 metres per second,
- the Planck constant h is 6.626 070 15 $\times$ 10^{-34} joule second,
- the elementary charge e is 1.602 176 634 $\times$ 10^{-19} coulomb,
- the Boltzmann constant k is 1.380 649 $\times$ 10^{-23} joule per kelvin,
- the Avogadro constant N_A is 6.022 140 76 $\times$ 10^{23} reciprocal mole,
- the luminous efficacy of monochromatic radiation of frequency 540 $\times$ 1012Hz, K_{cd}, is 683 lumens per watt.

The hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm and W, respectively, are related to the seven base units: second, metre, kilogram, ampere, kelvin, mole and candela, with unit symbols s, m, kg, A, K, mol and cd, respectively, through the relations $\mathrm{Hz} = \mathrm{s}^{-1}$, $\mathrm{J} = \mathrm{kg\,m^2\,s^{-2}}$, $\mathrm{C} = \mathrm{A\cdot s}$, $lm = \mathrm{cd\cdot sr}^{(1)}$ and $\mathrm{W} = \mathrm{kg\,m^2\,s^{-3}}$ (see Ref. [2], see p. 20). Only so-called “coherent” units [2] are used here and in the following. This means that, just as in the bullets, we will not affix numerical prefixes (such as mega or nano) either to the base units or to combinations of base units that have special names (such as joule or watt).

(1) sr is the symbol for steradian, the unit of solid angle. Although $\mathrm{sr} = \mathrm{m^2\cdot m^{-2}} = 1$, sr is used when needed for clarity [2].

The numerical values of $\Delta\nu_{Cs}$, c and K_{cd} given in the bullets have been fixed (defined to be exact) since 1967, 1983 and 1979 respectively [2]. It was premature in 2011 to specify exact values for h, e, k and N_A because their experimentally-determined values were not yet known with sufficiently small uncertainty to assure a smooth transition to the proposed new definitions. That has changed, and the numerical values given above are those recommended in October 2017 by the CODATA Task Group on Fundamental Constants [3]. They have been accepted by the CIPM and are expected to be confirmed by the CGPM at its next meeting, in November 2018.

This note describes an efficient method to convert the information contained in the seven bullets to definitions of the SI base units, which are, not coincidentally, seven in number [2]. The reasons for this particular choice of defining constants are important but have been presented elsewhere [4].

Starting with the seven bullets, we derive in one step the seven combinations of defining constants whose unit is a base unit of the SI. The algorithm used results in an easy-to-read table. The exact numerical values given in the bullets are then introduced to complete the definitions of the base units. Appendix 1 applies the same method to the present SI, illustrating the method’s generality as well as providing a novel contrast to the upcoming revision. Each base unit will be defined independently of the others, although typical derivations take a different approach. Appendix 2 describes a more abstract method of defining the base units from the values of the defining constants and compares the abstract formalism with that presented in the next two sections.

If all seven base units of the SI can be defined in terms of the seven defining constants, an obvious but important corollary follows: ***All SI units can be defined in terms of the seven defining constants.*** The distinction between base and derived units remains useful, but not essential for many purposes.(2)

The following method is consistent with a more rigorous analysis provided by Mohr in 2008 [6], which the interested reader is encouraged to consult.

(2) Even early editions of the SI Brochure remarked that separate classes of base and derived units are "not essential to the physics of the subject" [5], but added that the classifications were useful, considering the goal of "a single, practical, worldwide system [of units] for international relations, for teaching and scientific work".

2. Defining constants written in terms of base units

We begin with Table 1, which presents much of the information given in the bullets of Chapter 1 in more usable form. Note that the defining constants are shown as labels in the first column. The four new defining constants and the base units which are redefined in consequence are shown in red. The units of the defining constants can be expressed as the product of powers of the base units, $\mathrm{s}^{\alpha}\mathrm{m}^{\beta}\mathrm{kg}^{\gamma}\mathrm{A}^{\delta}\mathrm{K}^{\varepsilon}\mathrm{mol}^{\zeta}\mathrm{cd}^{\eta}$ [2], as specified in the bullets of Chapter 1. The exponents that are required appear in the rows of Table 1 for each of the defining constants; for example, the coherent unit of the Planck constant h is $\mathrm{J \cdot s = kg\ m^2\ s^{-1}}$, so that for the row labelled h the exponents α through η are (-1, 2, 1, 0, 0, 0, 0). The columns show whether a unit appears in a particular bullet. We see, for instance, that the second appears in the unit of every constant except N_A, but with four different exponents.

Table 1

	s	m	kg	A	K	mol	cd
$\Delta\nu_{\mathrm{Cs}}$	-1	0	0	0	0	0	0
c	-1	1	0	0	0	0	0
h	-1	2	1	0	0	0	0
e	1	0	0	1	0	0	0
k	-2	2	1	0	-1	0	0
N_A	0	0	0	0	0	-1	0
K_{cd}	3	-2	-1	0	0	0	1

The sequence of the seven defining constants in the left column of labels follows the order in which they are presented in the CGPM Resolution and in Chapter 1. The sequence of base units in the top row of labels follows the order in which these units are defined in Ref. [4]. This results in a table where the exponents above the diagonal cells are all zero (as are most exponents below the diagonal). The diagonal cells (those with a violet background) associate each defining constant with a unique base unit. The cells with a yellow background show that "helping" units are also needed. For instance, the Planck constant h is key to redefining the kilogram (violet cell), but the unit of h also contains the inverse second and the metre squared (yellow cells).

3. Base units as defined by the “defining constants”

The seven SI base units can be defined in terms of the seven defining constants. To do this, we create a second table, Table 2, that shows the combination of defining constants required to define each base unit. Table 2 is the major contribution of this report.

The numbers in Table 2 are also exponents, this time used to show the combination of defining constants (labelled in the top row) that has the same unit as each base unit (labelled in the left column). Except for an exact scaling factor, which is easily derived as shown below, each base unit is defined by the product $\Delta\nu_{\mathrm{Cs}}^{\alpha'} c^{\beta'} h^{\gamma'} e^{\delta'} k^{\varepsilon'} N_{\mathrm{A}}^{\zeta'} K_{\mathrm{cd}}^{\eta'}$, where the required exponents for each row appear in the table. If an exponent is zero, it means that its constant is not needed, and its cell, though containing zero, has been left blank for visual clarity. Each column shows which defining constants are needed in the definition of the base units. We see that $\Delta\nu_{\mathrm{Cs}}$ is needed to define six of the seven base units (using three different exponents), c is only needed to define two base units, etc.

All exponents have been derived from Table 1 in one step using the following mathematical operation. Note that the cells containing numbers in Table 1 constitute a 7 × 7 matrix. Invert that matrix using, for example, the MINVERSE command in Excel. The inverse obtained is the 7 × 7 matrix of exponents shown in Table 2.[1] Remember that the blank cells actually contain zero.

[1] The 7 × 7 squares of numbers in Table 1 and Table 2 are called “lower triangular matrices” because all numbers above the diagonal are zero. The inverse of a triangular matrix is triangular as well, provided that the inverse exists. The inverse exists if and only if none of the numbers in the diagonal cells is zero.

Table 2. The revised SI [4]. Blank cells all contain zero (not displayed).

	$\Delta\nu_{\mathrm{Cs}}$	c	h	e	k	N_{A}	K_{cd}
s	-1						
m	-1	1					
kg	1	-2	1				
A	1			1			
K	1		1		-1		
mol						-1	
cd	2		1				1

From the “unitsml(kg)” row of Table 2 we may infer that the following combination of three defining constants has the kilogram as its unit [6]:

$$\Delta\nu_{\mathrm{Cs}}^{1} c^{-2} h^{1} e^{0} k^{0} N_{\mathrm{A}}^{0} K_{\mathrm{cd}}^{0} = \Delta\nu_{\mathrm{Cs}} c^{-2} h = \frac{\Delta\nu_{\mathrm{Cs}} h}{c^{2}}. \tag{1}$$

In the revised SI, all mass determinations must ultimately be traceable to this quantity because its numerical value in kilograms has been fixed. The exact values of $\Delta\nu_{\mathrm{Cs}}$, c and h given in the bullets of Chapter 1 provide the fixed value. Substituting the information in the first three bullets into the left side of the following equation, which is a combination of physical constants, gives us the right side, which is the value of the combination in the revised SI:

$$\frac{\Delta\nu_{Cs}h}{c^2} = \frac{(9192\ 631\ 770)\left(6.626\ 070\ 15 \times 10^{-34}\right)}{(299\ 792\ 458)^2}\mathrm{kg}.$$

The numbers in parentheses are obviously the exact numerical values of $\Delta\nu_{Cs}$, c and h specified in Chapter 1. The base units associated with these numerical values cancel out (unit symbols can be treated algebraically)—except for the kilogram! Then by simple arithmetic,

$$1\mathrm{kg} = \frac{(299\ 792\ 458)^2}{(9192\ 631\ 770)\left(6.626\ 070\ 15 \times 10^{-34}\right)}\frac{\Delta\nu_{Cs}h}{c^2} = 1.475\ 521\ 3997\ldots \times 10^{40}\frac{\Delta\nu_{Cs}h}{c^2}. \qquad (2)$$

Any given base unit can be defined similarly, without knowing the definitions of any other base units. Only the exact numerical values of the defining constants are required. This definition of the kilogram appears in section 2.3.1 of Ref. [4]. A more formal derivation of the seven definitions, of which Equation (2) is an example, is provided in Appendix 2 along with a comparison to the simplified approach adopted above.

It is irrelevant that the mass $\Delta\nu_{Cs}h/c^2$ is so miniscule that it must be scaled up by 40 orders of magnitude to equal one kilogram. It has always been true that "any method consistent with the laws of physics could be used to realize any SI unit" [2] and such methods already exist for the kilogram as it will be defined by Equation (2) [4]. Appendix 1 discusses in more detail the condition that assures continuity of the redefined kilogram with the present kilogram, and by extension the continuity conditions for the three other redefined units.

4. Summary and Discussion [corrected April 2018]

Several pictorial illustrations of the revised SI are already available [7], [8]. In one case, readers are cautioned that the illustration is not an explanation [7]. By contrast, Table 2 has been derived mathematically from the seven defining constants, knowing only their units. It is easily observed from Table 2 that:

- The violet cells on the diagonal connect a base unit in the left column with the constant which defines it, in the top row. This is loose terminology because in most instances one or two "helping constants" are required, and these are shown in the yellow cells of each row. All other cells contain zero, and these are left blank;
- There are only three helping constants, $\Delta\nu_{\mathrm{Cs}}$, c and h, and these also serve as the defining constants for the second, metre and kilogram, respectively. [It is perhaps noteworthy that the second, metre and kilogram are the mechanical units of the old metre-kilogram-second (MKS) system, from which the SI evolved];
- In each row, the product of powers of the constants in the violet cell and any yellow cells form a quantity (which is also a constant) whose unit is the base unit of the row. The exponents needed are shown;
- At most, two helping constants are required to define any base unit. [The appearance of helping constants can be viewed as a mathematical requirement which reconciles continuity of the historical base units with the most useful selection of defining constants. See Appendix 1, which shows that the present SI [2] is not very different in this respect];
- Helping constants are not needed to define either the second or the mole.
- The ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{\mathrm{Cs}}$ is needed in the definitions of all base units except the mole;
- The speed of light in vacuum c is needed *only* in the definitions of the metre and kilogram;
- The Planck constant h is needed *only* in the definitions of the kilogram, kelvin and candela;
- The elementary charge e, the Boltzmann constant k, the Avogadro constant N_{A} and the luminous efficacy of a specified wavelength K_{cd} are each needed to define a single base unit (ampere, kelvin, mole and candela respectively). They are not used as helping constants.

There is no general requirement that exponents in Table 1 and Table 2 must be displayed as lower triangular matrices (see Appendix 2, see p. 17), although this arrangement makes the tables easier to scan visually and therefore has merit. Because Table 1 is a lower triangular matrix, one can see that the units can also be defined in seven separate steps rather than as a group, as we have done. The step-by-step method, also used in the draft 9th edition of the SI Brochure [4], first defines the SI second from the upper left corner of Table 1. The metre can then be defined from the next row because the helping unit of c, the second, has already been defined. The kilogram can be defined from the third row because the two helping units of h have already been defined. All helping units have now been defined and so the remaining four SI units can be defined in any order one wishes, including of course the order found in [4].

The seven unique combinations of defining constants whose unit is a base unit (Equation (1) shows the combination for the kilogram) were derived together by matrix inversion. Since any order of units and defining constants used as labels in Table 1 leads to identical definitions of the base units, we have chosen an order that makes Table 2 visually simple. It is also the order found in the major reference for the revised SI [4].

5. Acknowledgements

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Appendix 1. The present SI

The present SI [2] could also have been formulated in terms of the six defining constants and one defining quantity that had been specified by the CGPM, either explicitly or implicitly, between 1889 and 1983:

- the unperturbed ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{\mathrm{Cs}}$ is 9192 631 770 hertz, (1967)
- the speed of light in vacuum c is 299 792 458 metres per second, (1983)
- the mass of the international prototype of the kilogram $m_{\mathcal{K}}$ is 1 kilogram, (1889)
- the permeability of vacuum μ_0 is $4\pi \times 10^{-7}$ newton per ampere squared, (1948, 1954)
- the thermodynamic temperature of the triple point of water T_{TPW} is 273.16 kelvin, (1954)
- the molar mass of carbon 12, $M^{12}\mathrm{C}$, is 0.012 kilogram per mole, (1971)
- the luminous efficacy of monochromatic radiation of frequency 540 × 1012Hz, K_{cd}, is 683 lumens per watt. (1979)

The newton (symbol: N) is expressed in terms of base units as $\mathrm{N} = \mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-2}$ [2]. Expressions for the hertz, lumen and watt in terms of base units are found in Chapter 1. The defining quantity and three defining constants that will be replaced in the revised SI are shown in blue.

Carrying out the same procedure as described in Chapter 2 and Chapter 3 for the revised SI, we start with Table 1.1, which contains an embedded 7 × 7 matrix. Again, this matrix is a table of exponents inferred from seven bullets, but now they are the bullets found in this Appendix. For ease of comparison with Table 1 and Table 2, the order of units in the top row of Table 1.1 is chosen to be identical to that of Table 1, and the order of quantities in the left column is chosen to produce a lower triangular matrix.

Table 1.1

	s	m	kg	A	K	mol	cd
$\Delta\nu_{\mathrm{Cs}}$	-1	0	0	0	0	0	0
c	-1	1	0	0	0	0	0
$m_{\mathcal{K}}$	0	0	1	0	0	0	0
μ_0	-2	1	1	-2	0	0	0
T_{TPW}	0	0	0	0	1	0	0
$M(^{12}\mathrm{C})$	0	0	1	0	0	-1	0

K_{cd}	3	-2	-1	0	0	0	1

Now transpose the labels of Table 1.1 and invert its embedded matrix to arrive at Table 1.2. As with Table 2, cells containing zero are left blank.

Table 1.2. The present SI [2]. The blank cells all contain zero (not displayed).

	$\Delta\nu_{\mathrm{Cs}}$	c	$m_{\mathcal{K}}$	μ_0	T_{TPW}	$M(^{12}\mathrm{C})$	K_{cd}
s	-1						
m	-1	1					
kg			1				
A	1/2	1/2	1/2	-1/2			
K					1		
mol			1			-1	
cd	1	2	1				1

This table can easily be compared with Table 2. Note that only the first two rows, those for the second and metre, are identical in the two tables.

There is no reason that the exponents must be integers, as this example illustrates. Thus, in the present SI [2], the ampere is realized by traceability to the quantity $\left(\frac{\Delta\nu_{\mathrm{Cs}} c m_{\mathcal{K}}}{\mu_0}\right)^{\frac{1}{2}}$ which, according to the information in the first four bullets, has an exact value of order 1.5×10^{12}A. Note that the SI unit of $(\Delta\nu_{\mathrm{Cs}} c m_{\mathcal{K}})$ is the newton and that of μ_0 is the newton per ampere squared.

The exponents appearing in Table 1.2 and the exact numerical values of the six constants and one physical quantity listed in the bullets of this Appendix would have been sufficient to define all base units of the present SI. For example, a definition equivalent to the present definition of the ampere [2] would be:

$$1\mathrm{A} = \left(\frac{(4\pi \times 10^{-7})}{(9192\ 631\ 770)\ (299\ 792\ 458)\ (1)}\right)^{\frac{1}{2}} \left(\frac{\Delta\nu_{\mathrm{Cs}} c m_{\mathcal{K}}}{\mu_0}\right)^{\frac{1}{2}} = 6.789\ 687\ldots \times 10^{-13} \left(\frac{\Delta\nu_{\mathrm{Cs}} c m_{\mathcal{K}}}{\mu_0}\right)^{\frac{1}{2}}$$

In the present SI, $m_{\mathcal{K}}$ is the sole defining quantity which is not some kind of constant. Rather, it is the mass of an artefact known as the international prototype of the kilogram, $\mathcal{K}$, which has been used since 1889 to define one kilogram [2]. This artefact definition of the kilogram is simple, understandable and independent of the six constants. Unfortunately, since the mass of $\mathcal{K}$ is not a physical constant, the stability over time of the unit it defines cannot be assured. The same lack of assurance affects, at least in principle, the three units for which $m_{\mathcal{K}}$ is a "helper", one of which is the ampere (see yellow cells in the column of Table 1.2 labelled "$m_{\mathcal{K}}$"). When the SI was first approved by the 11th CGPM in 1960, it was recognized that the artefact definition of the kilogram was a weakness of the International System of Units—to be remedied “sooner or later” [9].

The present definition of the kilogram [2] is contained entirely in the third bullet of this Appendix. In symbols,

$$1\text{kg} = m_{\mathcal{K}}.$$

The revised definition of the kilogram [4] is given by Equation (2), $1\text{kg} = 1.475\ 521\ 3997\ldots \times 10^{40}\frac{\Delta\nu_{\text{Cs}}h}{c^2}$. The value of the prefactor on the right-hand side ensures that there will be no perceptible discontinuity in the kilogram unit when it is redefined [3], [4]. The continuity condition requires that the weighted mean of the most accurate experimental values of h will have been fixed [3] so that, *just after* the redefinition comes into force,

$$x \cdot m_{\mathcal{K}} = 1.475\ 521\ 3997\ldots \times 10^{40}\frac{\Delta\nu_{\text{Cs}}h}{c^2},$$

where the experimental value of x is unity to within an uncertainty that is sufficiently small to make the redefinition imperceptible to the vast majority of users. (Subsequently, the experimental value of x might change simply because $m_{\mathcal{K}}$ is not a physical constant. Time will tell.) The impact of the revised SI on most users of the present SI has been assessed to be small by international experts [10].

Appendix 2. Derivation of definitions of the base units in the revised SI (see Ref. [4] section 2.3.1); comparison with the method presented in Chapter 2 and Chapter 3 of this report [added March 2018]

Preliminary consideration of the defining constants specified in the bullets of Chapter 1

Let C_i be the symbol for the i^{th} defining constant and let its fixed numerical value be N_i when expressed in the SI coherent unit U_i. Because C_i is one of seven defining constants, the index i runs from 1 to 7. The set of C_i is comprised of $\Delta\nu_{\text{Cs}}$, c, h, e, k, N_{A} and K_{cd}. The subscript i assigned to the defining constants is an arbitrary choice. We have chosen here the order in which the bullets are listed in Chapter 1.

The i^{th} bullet of Chapter 1 can be written in generic symbols as

$$C_i = N_i \cdot U_i. \tag{2.1}$$

The defining constants are quantities which appear in the equations of physics. The right side of Equation (2.1) is the exact value of each C_i in the revised SI because the numerical values N_i have been chosen to be exact. The bullets of Chapter 1 therefore define the coherent unit U_i in terms of the quantity C_i, which is a constant of some type [4]. In the bracket notation introduced in section 2.1 of Ref. [4], Equation (2.1) would be written

$$C_i = \{C_i\}[C_i].$$

Any coherent unit of the SI can be expressed as the product of powers of the seven base units [2], [4]. We refer below to the i^{th} base unit as B_i. The symbols for the seven base units are: s, m, kg, A, K, mol and cd. Because units are commutative, the index i assigned to each base unit is also an arbitrary choice and need not follow the order shown here, although this is the order adopted in Chapter 2 and Chapter 3 of this report and in Ref. [4]. The SI unit U_i of each defining constant is given in terms of the base units by

$$U_i = \prod_{j=1}^{7} B_j^{a_{ij}}. \tag{2.2}$$

The exponents a_{ij} are easily inferred from the seven bullets in Chapter 1 and the supplementary information written just below them. The exponents turn out to be integers ranging from -2 through +3. Note that Equation (2.2) is merely a consequence of each defining constant being a quantity which has an SI unit.

The definitions of the base units

Combining Equation (2.1) and Equation (2.2) to eliminate U_i, we obtain

$$\frac{C_i}{N_i} = \prod_{j=1}^{7} B_j^{a_{ij}}. \tag{2.3}$$

The task now is to express any given base unit B_i as the product of powers of the seven ratios C_j/N_j. We therefore seek the "inverse form" of Equation (2.3).

Temporarily treating the symbols of quantities and units as algebraic abstractions, we take the logarithm[(1)] of both sides of Equation (2.3):

[(1)] See Ref. [6] for a formal derivation that avoids any use of logarithms.

$$\ln\left(\frac{C_i}{N_i}\right)=\sum_{j=1}^{7}a_{ij}\ln\left(B_j\right). \tag{2.4}$$

Let

$$w_i=\ln\left(C_i/N_i\right)$$

and

$$z_i=\ln\left(B_i\right).$$

The set of seven equations represented by Equation (2.4) can now be written compactly in matrix form as

$$\mathbf{W}=\mathbf{AZ} \tag{2.5}$$

where **W** and **Z** are 1×7 arrays containing seven logarithmic elements of the form w_i and z_i respectively and **A** is the 7×7 matrix of exponents consisting of the elements a_{ij}.

To solve for **Z**, multiply Equation (2.5) from the left by $\mathbf{A}^{-1}$:

$$\mathbf{Z}=\mathbf{A}^{-1}\mathbf{W}. \tag{2.6}$$

The existence of $\mathbf{A}^{-1}$ is obviously a necessary condition.

Equation (2.6) represents seven individual equations. We now exponentiate each of these to eliminate the logarithms they contain. If the elements of $\mathbf{A}^{-1}$ are symbolized by d_{ij}, then the definition of the i^{th} base unit is found to be:

$$B_i=\prod_{j=1}^{7}\left(\frac{C_j}{N_j}\right)^{d_{ij}}, \tag{2.7}$$

which defines each base unit in terms of the defining constants and their fixed numerical values. The same procedure can be used to show that $B_i=\prod_j U_j^{d_{ij}}$ is the inverse form of Equation (2.2). The definitions of the base units given in Ref. [4] can be recognized as following from Equation (2.7) when it is written with separate factors for the terms containing the N_j and the C_j:

$$B_i=\left(\prod_{j=1}^{7}N_j^{-d_{ij}}\right)\left(\prod_{j=1}^{7}C_j^{d_{ij}}\right). \tag{2.8}$$

Equation (2.7) can also be used to define the same base units in terms of the present SI when account is taken of the different set of seven bullets, which are shown in Appendix 1.

Comparison with the approach taken in Chapter 2 and Chapter 3

Chapter 2 and Chapter 3 take advantage of the simplicity of Equation (2.2). The information required for each U_i seems practically self-evident. Nevertheless, the set of equations contains all exponents a_{ij} which are needed to create matrix **A**.

In Chapter 2, Table 1 is described as representing "much of the information" contained in the bullets of Chapter 1. The information contained is that that the unit U_i of the i^{th} defining constant C_i can be expressed in terms of the base units as $\mathrm{s}^{\alpha}\mathrm{m}^{\beta}\mathrm{kg}^{\gamma}\mathrm{A}^{\delta}\mathrm{K}^{\varepsilon}\mathrm{mol}^{\zeta}\mathrm{cd}^{\eta}$, where the required exponents are inferred from the i^{th} bullet of Chapter 1 (and, when needed, the supplementary information written below the bullets). Table 1 thus provides an example of how the information contained in Equation (2.2) can be displayed. The important point is that Table 1 contains **A**, and the cells of Table 1 contain the array of individual exponents a_{ij} for the chosen ordering of base units and defining constants. The ordering is arbitrary from a mathematical point of view and was therefore chosen in Chapter 2 for didactic reasons.

It is also true that the exact **value** of the i^{th} defining constant C_i will be expressed in the revised SI in terms of the base units multiplied by an exact number. The SI **value** of each C_i in the revised SI will therefore be $N_i\mathrm{s}^{\alpha}\mathrm{m}^{\beta}\mathrm{kg}^{\gamma}\mathrm{A}^{\delta}\mathrm{K}^{\varepsilon}\mathrm{mol}^{\zeta}\mathrm{cd}^{\eta}$, where the exact number N_i is stated in the i^{th} bullet of Chapter 1. Multiplication by a pure number does not affect the exponents.

Table 2 is designed to present a useful picture of the revised SI. With that in mind, Chapter 3 refers to combinations of the defining constants that have the same SI **unit** as the i^{th} base unit B_i. The i^{th} row of Table 2 is labelled with base unit B_i; the j^{th} column of Table 2 is labelled with the defining constant C_j. The elements d_{ij} of the table were obtained by inverting matrix **A**, which is contained in Table 1. Table 2 can therefore be used to find the combination of defining constants that has B_i as its unit. The **definition** of each B_i can be obtained by changing the column labels from C_j to C_j/N_j thereby making Table 2 equivalent to Equation (2.7). However, a more intuitive method is adopted for the example presented in Chapter 3.

References

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[10] Joint statement prepared in 2017 by the CIPM Consultative Committees on the proposed revision of the SI. https://www.bipm.org/utils/common/pdf/SI-statement.pdf (accessed 23 February 2018)

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Authors: Richard Davis (BIPM)

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