Contradictory Procedures

# Chapter 0: Preamble and Introduction

## 0.1 Background and Motivation

### 0.1.1 What are constructive procedures?

### 0.1.2 Why are constructive procedures important?

### 0.1.3 Historical context and key figures

## 0.2 Overview of the Book

### 0.2.1 Structure of the book

### 0.2.2 Main topics covered in each chapter

### 0.2.3 Target audience and prerequisites

# Part I: Consistent Paracompleteness

## 1. Chomsky Hierarchy of Formal Grammars

1.1 Recursive Languages

Regular Languages

Context-Free Languages

Context-Sensitive Languages

1.2 Recursively Enumerable Languages

1.3 Non-Recursive Languages

## 2. Tarskian Hierarchy of Formal Languages

2.1 Introduction to Metalanguages and Tarskian Languages

2.2 Tarski's Undefinability Theorem

2.2.1 Metalanguages and Tarskian Languages: Levels of Undefinability

2.3 Levels of the Tarskian Hierarchy:

2.3.1 Level 0: Languages without Truth Definitions

2.3.2 Level 1: Languages with Truth Definitions for Level 0 Languages

2.3.3 Level n: General Representation for Finite Hierarchies

2.3.4 Level ω: General Representation for Transfinite Hierarchies

## 3. Kripkean Hierarchies of Formal Languages

3.1 Introduction to Kripkean Semantics:

3.1.1 Limitations of Tarskian Semantics

3.1.2 Modal Operators and the Kripke Hierarchy

3.1.3 Fixed-Point Operators and Constructive Truth

3.1.4 Kripkean Theories of Truth: Strong Kleene, Partial Truth, and Fixed-Point Logics

3.2 Modal Operators and Possible Worlds:

3.2.1 Kripke Frames and Accessibility Relations

3.2.2 Truth in Different Possible Worlds

3.2.3 Propositional Modal Logic: □ (Necessity) and ◇ (Possibility)

3.2.4 Quantified Modal Logic: Bringing in Quantifiers

3.3 Fixed-Point Operators and Constructive Truth:

3.3.1 Introduction to Fixed-Point Operators

3.3.2 Least and Greatest Fixed-Points: Finding the "Right" Truth Value

3.3.3 Recursion and Fixed-Point Operators

3.3.4 Fixed-Point Constructions in Formal Languages

3.4 Kleene Evaluation and Strong Kleene Fixed-Points:

3.4.1 Partial Truth Values: Beyond True and False

3.4.2 Kleene Evaluation: Assigning Truth Values in Steps

3.4.3 Strong Kleene Fixed-Points: Reaching Stable Truth Values

3.5 Partial Truth and the Hierarchy of Languages:

3.5.1 Languages with Partial Truth: Expanding Expressive Power

3.5.2 The Kripke Hierarchy of Fixed-Point Languages

3.5.3 Jumping Between Levels and Truth Definitions

## Chapter 2: Consistency and Paracompleteness in Formal Systems

**2.3 Many-Valued and Paracomplete Logics**

While bivalent logic is powerful, it sometimes struggles to represent real-world situations with varying degrees of truth or uncertainty. This section introduces many-valued and paracomplete logics, which offer alternative approaches.

**2.3.1 Many-Valued Logics:**

**2.3.2 Paracomplete Logics:**

* These logics relax the principle of bivalence by allowing for "indeterminate" or "gap" truth values. This allows for reasoning with incomplete information or situations where truth is not fully determined.
* Kleene logic and Łukasiewicz logic are examples of paracomplete logics.

**2.3.3 The Power of the Monoid:**

Many-valued and paracomplete logics often utilize the concept of a monoid, a mathematical structure that allows for combining truth values in a meaningful way. This enables reasoning and inference even with incomplete or uncertain information.

**2.4 Introduction to Many-Valued Logics (Specific Examples):**

This section could delve into specific examples of many-valued logics, such as:

* **Three-Valued Logic:** Explain how truth values are assigned and how propositions are evaluated. Discuss applications and limitations.
* **Fuzzy Logic:** Introduce the concept of fuzzy sets and fuzzy truth values. Explore practical applications in areas like control systems and decision-making.
* **Łukasiewicz Logic:** Explain its distinctive features and how it handles propositions with intermediate truth values. Discuss its connection to probability theory.

### Bivalent and Classical Consistencies: Definitions and Properties

#### Introduction to Bivalence Logics

**Bivalence:** Every proposition in the language is assigned one of two truth values: true (T) or false (F). There are no "unknown" or "indeterminate" values.

**Classical Consistency:** A formal system is classically consistent if it does not admit a contradiction.

##### 2.1.2 Properties of Bivalent and Classical Consistencies:

**2-Valued T-Schema is a metatheorem for the object language.**

**Law of Excluded Middle:** For any proposition P, either P is true or not-P is true. There is no third option.

**Law of Contradiction:** It is not possible for P and not-P to both be true at the same time.

**Law of Double Negation:** Not (not-P) is logically equivalent to P.

**Validity of Classical Negation**

**Classical Propositional Logic:** The basic logic of propositions and connectives like "and," "or," and "not."

**Classical First Order Predicate Logic:** Introduces quantifiers ("all" and "some") to express propositions about individuals and properties.

**Classical Modal Logic:** Deals with concepts like possibility and necessity, adding modal operators like "necessarily" and "possibly."

### Many-Valued and Paracomplete Consistencies: many-values but one and only one designated value.

#### Introduction to Many-Valued Logics

These logics relax the principle of bivalence by allowing for "indeterminate" or "gap" truth values. This allows for reasoning with incomplete information or situations where truth is not fully determined.

##### Finitely Many-Valued Languages with Singular Designation.

##### Infinitely Many-Valued Languages with Singular Designation.

## Chapter 3: Constructive Proof and Intuitionistic Logic

### **3.1 Introduction: Beyond Classical Logic - The Rise of Constructive Proofs**

### Motivation for constructive approaches: Limitations of classical logic in representing proofs and reasoning about incomplete knowledge.

### Historical context: Brouwer, Heyting, Kolmogorov - Pioneers of constructive mathematics and intuitionistic logic.

### **3.2 Brouwer-Heyting-Kolmogorov (BHK) Interpretation: Formalizing Constructive Proof**

### Intuitionistic negation: Not-P as the absence of a constructive proof for P.

### Intuitionistic implication: P implies Q only if there is a procedure for constructing Q from a proof of P.

### Logical connectives: Formalization of BHK interpretations for conjunction, disjunction, existential quantification, etc.

### **3.3 Constructive Dilemma and Intuitionistic Logic: Formalization and Consequences**

### Law of excluded middle: Not-P or P is not valid in intuitionistic logic, reflecting the possibility of gaps in knowledge.

### Law of double negation: Not-not-P is not equivalent to P, highlighting the distinction between proving a proposition and proving the non-existence of a proof.

### Derivations and proofs in intuitionistic logic: Formalization of sequent calculus and natural deduction systems for intuitionistic reasoning.

### Gödel's incompleteness theorems: Applicability and limitations within the intuitionistic framework.

### **3.4 Superclassical and Subclassical Calculi: Expanding the Logical Landscape**

### Superclassical calculi: Extensions of classical logic incorporating features from intuitionism, like constructive implication.

### Subclassical calculi: Weakenings of classical logic, capturing different degrees of constructive content.

### Examples: Minimal logic, relevant logic, fuzzy logic - exploring alternative truth values and reasoning patterns.

### **3.5 Superintuitionistic and Subintuitionistic Calculi: Pushing the Boundaries of Constructivism**

### Superintuitionistic calculi: Extensions of intuitionistic logic with additional axioms or rules, enabling reasoning about potential counterexamples and hypothetical scenarios.

### Subintuitionistic calculi: Weakenings of intuitionistic logic, allowing for reasoning with incomplete information or non-constructive proofs in specific contexts.

### Examples: Brouwer logic, Glivenko logic - exploring different ways of handling gaps and uncertainties within a constructive framework.

### **3.6 Consistent Paracomplete Calculi as Generalizations: Bridging the Gap**

### Paracomplete logics: Handling gaps in knowledge through additional truth values or designated "unknown" states.

### Consistent paracomplete calculi as generalizations of subintuitionistic and subclassical calculi: Unifying diverse approaches to reasoning with incomplete information.

### Examples: Kleene logic, Łukasiewicz logic - demonstrating the power of paracompleteness for representing and reasoning about real-world situations.

### Brouwer-Heyting-Kolmogorov Interpretation: Formalization of Constructive Proof

### Constructive Dilemma and Intuitionistic Logic: Formalization and Consequences

# Part II: Radical Paraconsistency

## Chapter 4: Many-Signified Logics and Languages

### Transcending the dichotomy of true xor false. Contradictions as multi-signifiers and the impossibility of a paraconsistent discrete many-signified bivalent language.

## Chapter 5: Constructive and Destructive Interference in Formal Reasoning

### Constructive Interference: Inference Patterns for Proof Generation from measurement.

### Destructive Interference: Inference Patterns for Refutation and Contradictions from measurement.

## Chapter 6: Contradictory Languages and Procedures

### **6.1 Introduction: Beyond the Binary Wall of Truth**

### 6.1.1 The allure of contradiction: Challenging classical logic's limitations

### 6.1.2 A spectrum of truth values: Beyond the binary dichotomy of true and false

### **6.2 Defining Contradictory Languages: Embracing Uncertainty and Paradox**

### 6.2.1 Extending the truth spectrum: Multi-valued logics and paraconsistency

### 6.2.2 Formalizing contradictions: Symbolizing and reasoning with contradictory statements

### 6.2.3 Examples of contradictory languages: From fuzzy logic to dialectical logic

### **6.3 Navigating Compatibility: When Contradictions Collide**

### 6.3.1 Consistent compatibility: Finding a common ground without explosions

### 6.3.2 Paraconsistent compatibility: Embracing contradictions within a unified framework

### 6.3.3 The incompatibility of contradictions: Why a consistent language can't coexist with paradoxes

### **6.4 Demystifying Inessential Paraconsistency: When the Scaffolding Falls Away**

### 6.4.1 Unmasking the inessential: Paraconsistency without contradictions

### 6.4.2 Reduction to bivalence: Simplifying the language without losing meaning

### 6.4.3 The Tarskian comfort zone: Maintaining a classical foundation for the metalanguage

### **6.5 Unveiling Essential Paraconsistency: Living with Contradictions as Neighbors**

### 6.5.1 The price of contradictions: Explosions and the need for paraconsistency

### 6.5.2 Splitting the paraconsistent world: Reducing a language to pairs of consistent systems

### 6.5.3 The multi-valued metalanguage: Beyond bivalence to accommodate paradoxes

### 6.5.4 Non-Tarskian options: Exploring alternative foundations for the metalanguage

### **6.6 Applications and Implications: Where Contradictions Matter**

### 6.6.1 Quantum mechanics: Embracing superposition and uncertainty with paraconsistency

### 6.6.2 Artificial intelligence: Handling ambiguity and vagueness in real-world data

### 6.6.3 Philosophical discourse: New perspectives on truth, paradox, and the limits of language

### **6.7 Conclusion: A Glimpse into a World of Contradictions**

### 6.7.1 The future of contradictory languages: Challenges, opportunities, and ongoing research

### 6.7.2 Contradictions as a lens: Rethinking logic, language, and the nature of reality

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# Part III: Supercalculi, Subcalculi, Isomorphic Calculi, and Dual Calculi

## Chapter 7: Superclassical and Subclassical Logics: Expanding and Restricting Logical Power

### Superclassical Logics: Extending Classical Reasoning

### Subclassical Logics: Weakening Classical Axioms and Completeness

### Generalized Functional Incompleteness or Constructive Completeness in Subclassical Logics

## Chapter 8: Paracomplete and Paraconsistent Supercalculi and Subcalculi

### Paracompleteness: Supercalculi and Subcalculi

#### Superintuitionistic Logics: Strengthening Intuitionistic Axioms

#### Subintuitionistic Logics: Weakening Intuitionistic Axioms and Constructive Completeness

### Paraconsistent Supercalculi and subcalculi

#### Super-counter-intuitive logics

#### Sub-counter-intuitive logics

# Part IV: Correspondence Theories Between Logic, Proof, Computation, and Experiment

## Chapter 9: Tarski's T-Schema and its Generalizations

### Models and Interpretations: Formalizing Classical Semantic Truth

### Generalizations of Tarski's T-Schema: the Liar’s sentence as a Metatheorem for Self-Reference.

## Chapter 10: Curry-Howard Correspondence: Proof as Program

### Logical Formulas as Types: The Curry-Howard Isomorphism

### Extracting Algorithms from Proofs: Operational Semantics and Program Extraction

## Chapter 11: Church-Turing-Deutsch Thesis: Proof, Program, and Measurement

### Formalizing Algorithmic Power: The Church-Turing Thesis

### Extending the Thesis: The Church-Turing-Deutsch Thesis and Quantum Computation

# Part V: Metamathematics, Metalanguages, and the Limits of Formalization

## Chapter 12: Hierarchical Languages and Metamathematics

### Uncomputable Languages: These languages cannot be enumerated by a Turing machine, and are therefore considered uncomputable.

### Languages of Doubt, Denial, and Relativity: Expanding the Formal Landscape

### Quantum Languages and Quantum Logic: Formalizing Quantum Phenomena

## Chapter 13: Self-Metalanguages and Metalogics

### The Unique Power of Paraconsistent Languages.

### Formal Systems Reflecting on Themselves: Self-Reference, Gödel's Theorems, and Tarski’s Theorems.

### Metalanguages and Metalogics: Formalizing Formalization

V3

Part I: Consistency: The Bedrock of Logic

1.1 Classical Consistency: The Foundation of Reasoning

1.2 Limits of Consistency: The Gödel Incompleteness Theorems

1.3 Beyond Bivalence: Fuzzy Logic and Many-Valued Systems

Part II: Lattice of Consistency: A Spectrum of Logics

2.1 Paraconsistent Logics: Embracing Contradictions

2.2 Dialetheism: Accepting True and False Simultaneously

2.3 Weak Kleene Logic: Accommodating Inconsistent Information

2.4 Paracompleteness: Leaving Truth Values Undetermined

Part III: Correspondences: Bridging Logic and Other Disciplines

3.1 Measurement and Paraconsistency: Reconciling Quantum Indeterminacy

3.2 Instrumentation and Inconsistency: Can We Measure the Unmeasurable?

3.3 Formal Languages and Paraconsistent Logic: Grammars of the Unknowable

Part IV: Metalinguistics, Metamathematics, Metalogic, and Metaphysics: Ascending the Ladder of Abstraction

4.1 The Scientific Method: A Paraconsistent Framework?

4.2 Hypothetico-Deductive Model: Reasoning with Uncertainty

4.3 Deductive Reasoning and Paraconsistency: Can We Reason from Contradictions?

4.4 Chomsky Hierarchy and Paraconsistent Logics: Formalizing the Unformalizable

Part V: Paraconsistency and the Edges of Knowledge

5.1 Semantics and Paraconsistency: Meaning in the Face of Contradictions

5.2 Metalanguage and Paraconsistency: Talking about the Untalkable

5.3 Metatheory and Paraconsistency: Justifying the Unjustifiable

5.4 Possibility Theory and Paraconsistency: Reasoning about the Unknowable

5.5 Possible Worlds and Paraconsistency: Navigating the Multiverse of Contradictions

V4

**I. Foundations of Certainty: The Lattice of Logic**

1.1 Classical Logic: Cornerstone of Deduction 1.2 Beyond Bivalence: Many Valued and Fuzzy Systems 1.3 The Lattice of Logics: A Spectrum of Formal Reasoning

**II. Measuring the Unmeasurable: Quantum Entanglements and Paraconsistency**

2.1 Quantum Mechanics: Where Measurement Defies Certainty 2.2 Paraconsistency: Embracing Contradictions within Formal Systems 2.3 Dialetheism: Two Sides of the Coin - True and False

**III. Formalizing the Unknowable: Languages of Possibility and Impossibility**

3.1 Epistemic and Subjunctive Modalities: Reasoning about the Potentially True 3.2 Possible Worlds: Exploring Alternative Realities 3.3 Van Wijngaarden Grammar: Formalizing Uncertainty and Openness

**IV. The Scientific Scaffolding: Theory, Method, and Observation**

4.1 The Scientific Method: Navigating the Labyrinth of Uncertainty 4.2 Hypothetico-Deductive Reasoning: Building Bridges between Theory and Observation 4.4 Deductive Reasoning: Chains of Implication and the Quest for Certainty

**V. The Meta-level: Language about Language, Knowledge about Knowledge**

5.1 Metalanguages: Speaking about the Language of Logic itself 5.2 Metatheory: Justifying the Foundations of Formal Systems 5.3 Semantic Invariants: Meaning Amidst Shifting Sands 5.4 Systems and Automorphisms: Structure and Transformations

**VI. Beyond the Horizon: Open Questions and Future Directions**

6.1 Limits of Computation: Landauer's Principle and the Bekenstein Bound 6.2 Holographic Principle and the Information Paradox 6.3 Quantum Speed Limit and the Church-Turing-Deutsch Principle

V5

## Part I: Consistency

## Part II: Lattice of Consistency

## Part III: Correspondences

## Part IV: Languages of Possibility

## Proof of Possibility

## Mathematical possibility

## Physical possibility

## Quantum possibility

## Part V: Languages of Impossibility

## Proof of Impossibility

## Mathematical Impossibility

## Physical Impossibility

## Quantum Impossibility

## Part VI: Metalinguistics, Metamathematics, and Metaphysics

V6

I. Beyond Bivalence: Rethinking the Foundations of Logic

1.1. Classical Logic: The Cornerstone of Reason 1.2. Gödel's Incompleteness: Unveiling the Limits of Certainty 1.3. Fuzzy Logic and Many-Valued Systems: Expanding the Spectrum of Truth

II. Navigating the Lattice of Logics: A Landscape of Possibilities

2.1. Paraconsistent Logics: Reasoning with Contradictions 2.2. Dialetheism: Embracing Opposing Truths 2.3. Weak Kleene Logic: Integrating Open Information 2.4. Paracompleteness: Embracing Undetermined Truths

III. Bridging the Gap: From Logic to the Real World

3.1. Measurement and the Unmeasurable: Quantum Mechanics and Paraconsistency 3.2. Instrumentation and the Indeterminate: Can We Measure the Unknown? 3.3. Formal Languages and Paraconsistency: Grammars for the Unconventional

IV. Languages of Possibility: Exploring the Uncharted

4.1. The Scientific Method: A Paraconsistent Framework for Discovery? 4.2. Hypothetico-Deductive Reasoning: Building Theories Beyond Certainty 4.3. Deductive Reasoning and Open Premises: Can We Derive Knowledge from Uncertainty? 4.4. The Chomsky Hierarchy and Paraconsistency: Formalizing the Unformalizable

V. Languages of Impossibility: Confronting the Unknowable

5.1. Semantics and the Unspeakable: Meaning-Making in the Face of Contradictions 5.2. Metalanguage and the Unutterable: Speaking about the Unthinkable 5.3. Metatheory and the Unjustifiable: Foundations Beyond Proof 5.4. Possibility Theory and the Unforeseen: Reasoning about the Unknowable

VI. Metalinguistics, Metamathematics, Metalogic, and Metaphysics: Ascending the Abstraction Ladder

6.1. Invariance and Paraconsistency: Fixed Points Amidst Shifting Sands 6.2. Systems and Paraconsistency: Coherence Within Contradiction 6.3. Automorphisms and Paraconsistency: Transformations Preserving the Unconventional 6.4. Relations and Paraconsistency: Connecting the Unconnectable

# Chapter: Contradictory Languages and Contradictory Procedures

**6.1 Introduction: Beyond the Binary Barrier**

6.1.1 Motivation: Challenging the limitations of classical logic and its binary framework.

6.1.2 Expanding the Truth Spectrum: Exploring alternative truth values beyond true and false.

**6.2 Navigating the Realm of Contradictory Languages:**

6.2.1 Representation of Contradictions:

6.2.1.1 Gluts and Gaps: Formalizing states beyond the binary truth spectrum.

6.2.1.2 Modal Operators: Embracing uncertainty and possibility through modalities.

6.2.1.3 Fuzzy Logic: Quantifying degrees of truth and falsity.

6.2.2 Inessential Paraconsistency: Languages with latent paraconsistent features.

6.2.2.1 Identification and Reduction: Recognizing and simplifying inessentially paraconsistent systems.

6.2.2.2 Role of the Metalanguage: Maintaining bivalence or venturing beyond?

6.2.3 Essential Paraconsistency: Languages embracing inherent contradictions.

6.2.3.1 Formalization of Contradictions: Integrating paradoxes within the language structure.

6.2.3.2 Decomposition into Consistent Systems: Splitting the paraconsistent world into compatible parts.

6.2.3.3 Multi-Valued Metalanguages: Accommodating paradoxes beyond classical bivalence.

6.2.4 Compatibility and Consistency: Finding common ground amidst contradictions.

6.2.4.1 Consistent Compatibility: Merging languages without triggering logical explosions.

6.2.4.2 Paraconsistent Compatibility: Embracing contradictions within a unified framework.

6.2.4.3 Limits of Compatibility: Why some paradoxes cannot coexist peacefully.

**6.3 Contradictory Procedures and Algorithms**

6.3.1 Processing Contradictory Inputs: Algorithms handling ambiguous data.

6.3.1.1 Uncertainty and Ambiguity: Fuzzy reasoning and probabilistic approaches.

6.3.1.2 Conflicting Evidence: Aggregation and filtering of contradictory information.

6.3.2 Generating Contradictory Outputs: Algorithms producing paradoxical results.

6.3.2.1 Quantum Algorithms: Exploring superposition and entanglement as sources of contradiction.

6.3.2.2 Non-deterministic Algorithms: Embracing multiple possibilities and contradictory outcomes.

Two languages L0 and L1 are said to be consistently compatible if they have a common consistent extension; this is equivalent to saying the union of L0 and L1 is consistent.

Two languages L0 and L1 are said to be paraconsistently compatible if they have a common paraconsistent consistent extension; this is equivalent to saying the union of L0 and L1 is paraconsistent.

If either L0 or L1 is paraconsistent and has a contradiction then the other can not be consistent if they are consistently compatible.

if L0 and L1 are consistently compatible then L1 and L0 can be reduced to a consistent languages or are a consistent languages.

A paraconsistent language is inessentially paraconsistent if and only if it has no contradictions; it is reducible to a consistent language; inessentially paraconsistent languages are reducible to bivalent languages.

Every inessentially paraconsistent language has a 2-value T-schema that holds in its metalanguage, and its metalanguage is Tarskian.

If a paraconsistent language L is essentially paraconsistent then L has contradictions; it is reducible to a pair of consistent language for each contradiction in the paraconsistent language. Every essentially paraconsistent language L has a (3 or more)-valued T-schema that holds in L's metalanguage or L's metalanguage has no T-Schema; the metalanguage can not be bivalent.

If the metalanguage of an essentially paraconsistent language was bivalent then any contradiction in the essentially paraconsistent language would be logically explosive.

Every essentially paraconsistent language is at least partially independent of some consistent language.

Incompleteness of consistent extensions: While essentially paraconsistent languages might have consistent extensions, these extensions often do not capture the full richness and nuances of the original language. This suggests that some aspects of the paraconsistent language remain independent of the consistent extension.

# References

L.E.J. Brouwer,

Arend Heyting,

Alfred Tarski,

Kurt Gödel,

Paola Zizzi

Noam Chomsky