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# Aufgabe 4

## (i)

Es muss gelten

$$\int_{-\infty}^{\infty} f(x)dx = 1 \quad (1)$$

$$E[X] = 0 \quad (2)$$

$$\begin{aligned}\int_{-\infty}^{\infty} f(x)dx &= \int_{-2}^{0} f(x)dx + \int_{0}^{1} f(x)dx + 0 \\ &= \int_{-2}^{0} c_1 dx + \int_{0}^{1} c_2 x dx \\ &= -2c_1 + \frac{1}{2}c_2 (*)\end{aligned}$$

$$\begin{aligned}E[x] &= \int_{-\infty}^{\infty} x \cdot f(x)dx \\ &= \int_{-2}^{0} x \cdot f(x)dx + \int_{0}^{1} x^2 \cdot f(x)dx + 0 \\ &= \int_{-2}^{0} x \cdot c_1 dx + \int_{0}^{1} c_2 x^2 dx \\ &= c_1 + \frac{1}{3}c_2 (**)\end{aligned}$$

Mit (1) $\wedge$ $(*)$ und (2) $\wedge$ $(**)$ folgt:
$2 \cdot c_1 + 0.5c_2 = 1 \wedge c_1 + \frac{1}{3}c_2 = 0$
Daraus folgt dass $c_1 = \frac{2}{7} \wedge c_2 = \frac{6}{7}$

**(ii)**

$$\begin{aligned}
V[x] &= E[X^2] + E[X]^2 \\
&= E[X^2] + 0 \\
&= \int\limits_{-\infty}^{\infty} x \cdot x \cdot f(x) dx \\
&= \int\limits_{-2}^{0} x^2 \cdot f(x) dx + \int\limits_{0}^{1} x^3 \cdot f(x) dx + 0 \\
&= \int\limits_{-2}^{0} x^2 \cdot c_1 dx + \int\limits_{0}^{1} c_2 x^3 dx \\
&= \frac{2 \cdot 8}{3 \cdot 7} + \frac{6}{4 \cdot 7} \\
&= \frac{41}{42}
\end{aligned}$$