TheGI4 HA4

David Konopek(349333),
Paul Walger(349968),
Lukas Klammt(332263)

14. Juli 2014

Aufgabe 1

Aufgabe 1 a)

Wir bestimmen zunächst $[\![X]\!]$:
Sei $F_X = \langle a \rangle X \wedge [c] \, f\!f$

$$
\begin{aligned}
\mathcal{O}_{F_X}(\mathsf{Proc}) &= \langle \cdot a \cdot \rangle \, \mathsf{Proc} \cap [\cdot c \cdot] \, f\!f \\
&= \{p_1, p_2, p_5\} \cap \{p_2, p_3, p_5, p_6\} \\
&= \{p_2, p_5\} \\
(\mathcal{O}_{F_X})^2(\mathsf{Proc}) &= \langle \cdot a \cdot \rangle \{p_2, p_5\} \cap [\cdot c \cdot] \, f\!f \\
&= \{p_2\} \cap \{p_2, p_3, p_5, p_6\} \\
&= \{p_2\} \\
(\mathcal{O}_{F_X})^3(\mathsf{Proc}) &= \langle \cdot a \cdot \rangle \{p_2\} \cap [\cdot c \cdot] \, f\!f \\
&= \{p_2\} \cap \{p_2, p_3, p_5, p_6\} \\
&= \{p_2\} \Rightarrow \mathit{Fixpunkt}!
\end{aligned}
$$

Nun können wir $Safe(\langle b \rangle X)$ berechnen.
Wir wissen: $Safe(F) : Y \overset{\max}{=} F \wedge ([\mathit{Act}] \, f\!f \vee \langle \mathit{Act} \rangle Y)$
Sei nun $F_{Safe(G)} = G \wedge ([\mathit{Act}] \, f\!f \vee \langle \mathit{Act} \rangle Y)$

$$
\begin{aligned}
\mathcal{O}_{F_{Safe(\langle b \rangle X)}}(\mathsf{Proc}) &\overset{\text{Def. } \mathcal{O}_{F_{Safe(G)}}, X}{=} \langle \cdot b \cdot \rangle \{p_2\} \cap ([\cdot a \cdot] \, f\!f \cup [\cdot b \cdot] \, f\!f \cup [\cdot c \cdot] \, f\!f \cup \langle \cdot a \cdot \rangle \, \mathsf{Proc} \cup \langle \cdot b \cdot \rangle \, \mathsf{Proc} \cup \langle \cdot c \cdot \rangle \, \mathsf{Proc}) \\
&\overset{\text{Def. } \langle \cdot \cdot \rangle, [\cdot \cdot]}{=} \{p_1\} \cap (\{p_3, p_4, p_6\} \cup \{p_2, p_5, p_6\} \cup \{p_2, p_3, p_5, p_6\} \\
&\qquad \cup \{p_1, p_2, p_5\} \cup \{p_1, p_3, p_4\} \cup \{p_1, p_4\}) \\
&\overset{\text{Def. } \cap, \cup}{=} \{p_1\} \\
(\mathcal{O}_{F_{Safe(\langle b \rangle X)}})^2(\mathsf{Proc}) &\overset{\text{Def. } \mathcal{O}_{F_{Safe(G)}}, X}{=} \langle \cdot b \cdot \rangle \{p_2\} \cap ([\cdot a \cdot] \, f\!f \cup [\cdot b \cdot] \, f\!f \cup [\cdot c \cdot] \, f\!f \cup \langle \cdot a \cdot \rangle \{p_1\} \cup \langle \cdot b \cdot \rangle \{p_1\} \cup \langle \cdot c \cdot \rangle \{p_1\}) \\
&\overset{\text{Def. } \langle \cdot \cdot \rangle, [\cdot \cdot]}{=} \{p_1\} \cap (\{p_3, p_4, p_6\} \cup \{p_2, p_5, p_6\} \cup \{p_2, p_3, p_5, p_6\} \cup \emptyset \cup \emptyset \cup \emptyset) \\
&\overset{\text{Def. } \cap, \cup}{=} \emptyset \\
(\mathcal{O}_{F_{Safe(\langle b \rangle X)}})^3(\mathsf{Proc}) &\overset{\text{Def. } \mathcal{O}_{F_{Safe(G)}}, X}{=} \langle \cdot b \cdot \rangle \{p_2\} \cap ([\cdot a \cdot] \, f\!f \cup [\cdot b \cdot] \, f\!f \cup [\cdot c \cdot] \, f\!f \cup \langle \cdot a \cdot \rangle \emptyset \cup \langle \cdot b \cdot \rangle \emptyset \cup \langle \cdot c \cdot \rangle \emptyset) \\
&\overset{\text{Def. } \langle \cdot \cdot \rangle, [\cdot \cdot]}{=} \{p_1\} \cap (\{p_3, p_4, p_6\} \cup \{p_2, p_5, p_6\} \cup \{p_2, p_3, p_5, p_6\} \cup \emptyset \cup \emptyset \cup \emptyset) \\
&\overset{\text{Def. } \cap, \cup}{=} \emptyset \Rightarrow \mathit{Fixpunkt}!
\end{aligned}
$$

$\Rightarrow Safe(\langle b \rangle X) = \emptyset$

Aufgabe 1 b)

Wir bestimmen zunächst $[\![X]\!]$:
Sei $F_X = \langle a \rangle X \vee \langle b \rangle X$

$$
\begin{aligned}
\mathcal{O}_{F_X}(\mathsf{Proc}) &\overset{\text{Def. } \mathcal{O}_{F_X}}{=} && \langle\cdot a\cdot\rangle\, \mathsf{Proc} \cup \langle\cdot b\cdot\rangle\, \mathsf{Proc} \\
&\overset{\text{Def. } \langle\cdot\cdot\rangle, [\cdot\cdot]}{=} && \{p_1, p_2, p_5\} \cup \{p_1, p_3, p_4\} \\
&\overset{\text{Def. } \cup}{=} && \{p_1, p_2, p_3, p_4, p_5\} \\
(\mathcal{O}_{F_X})^2(\mathsf{Proc}) &\overset{\text{Def. } \mathcal{O}_{F_X}}{=} && \langle\cdot a\cdot\rangle\{p_1, p_2, p_3, p_4, p_5\} \cup \langle\cdot b\cdot\rangle\{p_1, p_2, p_3, p_4, p_5\} \\
&\overset{\text{Def. } \langle\cdot\cdot\rangle, [\cdot\cdot]}{=} && \{p_1, p_2, p_5\} \cup \{p_1, p_3\} \\
&\overset{\text{Def. } \cup}{=} && \{p_1, p_2, p_3, p_5\} \\
(\mathcal{O}_{F_X})^3(\mathsf{Proc}) &\overset{\text{Def. } \mathcal{O}_{F_X}}{=} && \langle\cdot a\cdot\rangle\{p_1, p_2, p_3, p_5\} \cup \langle\cdot b\cdot\rangle\{p_1, p_2, p_3, p_5\} \\
&\overset{\text{Def. } \langle\cdot\cdot\rangle, [\cdot\cdot]}{=} && \{p_1, p_2, p_5\} \cup \{p_1\} \\
&\overset{\text{Def. } \cup}{=} && \{p_1, p_2, p_5\} \\
(\mathcal{O}_{F_X})^4(\mathsf{Proc}) &\overset{\text{Def. } \mathcal{O}_{F_X}}{=} && \langle\cdot a\cdot\rangle\{p_1, p_2, p_5\} \cup \langle\cdot b\cdot\rangle\{p_1, p_2, p_5\} \\
&\overset{\text{Def. } \langle\cdot\cdot\rangle, [\cdot\cdot]}{=} && \{p_2\} \cup \{p_1\} \\
&\overset{\text{Def. } \cup}{=} && \{p_1, p_2\}
\end{aligned}
$$

$$
\begin{aligned}
(\mathcal{O}_{F_X})^5(\mathsf{Proc}) &\overset{\text{Def. } \mathcal{O}_{F_X}}{=} && \langle\cdot a\cdot\rangle\{p_1, p_2\} \cup \langle\cdot b\cdot\rangle\{p_1, p_2\} \\
&\overset{\text{Def. } \langle\cdot\cdot\rangle, [\cdot\cdot]}{=} && \{p_2\} \cup \{p_1\} \\
&\overset{\text{Def. } \cup}{=} && \{p_1, p_2\} \Rightarrow \mathit{Fixpunkt}!
\end{aligned}
$$

Nun können wir $[\![Y]\!]$ bestimmen:
Sei $F_Y = [\, b\,]\, X \vee [\, b\,]\, Y$

$$
\begin{aligned}
\mathcal{O}_{F_Y}(\emptyset) &\overset{\text{Def. } \mathcal{O}_{F_Y}, \text{ X}}{=} && [\cdot b\cdot]\{p_1, p_2\} \cup [\cdot b\cdot]\emptyset \\
&\overset{\text{Def. } \langle\cdot\cdot\rangle, [\cdot\cdot]}{=} && \{p_1, p_2, p_5, p_6\} \cup \{p_2, p_5, p_6\} \\
&\overset{\text{Def. } \cup}{=} && \{p_1, p_2, p_5, p_6\} \\
(\mathcal{O}_{F_Y})^2(\emptyset) &\overset{\text{Def. } \mathcal{O}_{F_Y}, \text{ X}}{=} && [\cdot b\cdot]\{p_1, p_2\} \cup [\cdot b\cdot]\{p_1, p_2, p_5, p_6\}
\end{aligned}
$$

$$
\begin{aligned}
& \overset{\text{Def. } \langle\cdot\cdot\rangle, [\cdot\cdot]}{=} && \{p_1, p_2, p_5, p_6\} \cup \{p_1, p_2, p_4, p_5, p_6\} \\
& \overset{\text{Def. } \cup}{=} && \{p_1, p_2, p_4, p_5, p_6\} \\
(\mathcal{O}_{F_Y})^3(\emptyset) & \overset{\text{Def. } \mathcal{O}_{F_Y}, \text{ X}}{=} && [\cdot b\cdot]\{p_1, p_2\} \cup [\cdot b\cdot]\{p_1, p_2, p_4, p_5, p_6\} \\
& \overset{\text{Def. } \langle\cdot\cdot\rangle, [\cdot\cdot]}{=} && \{p_1, p_2, p_5, p_6\} \cup \{p_1, p_2, p_3, p_4, p_5, p_6\} \\
& \overset{\text{Def. } \cup}{=} && \{p_1, p_2, p_3, p_4, p_5, p_6\} \\
& = && \mathsf{Proc}
\end{aligned}
$$

$\Rightarrow$ F gilt in allen Zuständen des LTS!

Aufgabe 2

Aufgabe 3

3 a)

$X_{I1} \overset{\text{max}}{=} \langle a \rangle X_R \wedge \langle b \rangle X_G \wedge \langle a \rangle X_W \wedge [\,a\,]\ (X_R \vee X_W) \wedge [\,b\,]\, X_G \wedge [\,c\,]\ \mathit{ff}$

$X_R \overset{\text{max}}{=} \langle b \rangle X_W \wedge \langle c \rangle X_T \wedge \langle c \rangle X_{I1} \wedge [\,c\,]\ (X_T \vee X_{I1}) \wedge [\,b\,]\, X_W \wedge [\,a\,]\ \mathit{ff}$

$X_W \overset{\text{max}}{=} [\,a\,]\ \mathit{ff} \wedge [\,b\,]\ \mathit{ff} \wedge [\,c\,]\ \mathit{ff}$

$X_S \overset{\text{max}}{=} \langle c \rangle X_{I1} \wedge [\,c\,]\, X_{I1} \wedge [\,a\,]\ \mathit{ff} \wedge [\,b\,]\ \mathit{ff}$

$X_D \overset{\text{max}}{=} \langle c \rangle X_{I1} \wedge [\,c\,]\, X_{I1} \wedge [\,a\,]\ \mathit{ff} \wedge [\,b\,]\ \mathit{ff}$

$X_{I2} \overset{\text{max}}{=} \langle a \rangle X_T \wedge \langle b \rangle X_S \wedge [\,a\,]\, X_T \wedge [\,b\,]\, X_S \wedge [\,c\,]\ \mathit{ff}$

$X_N \overset{\text{max}}{=} \langle a \rangle X_U \wedge \langle a \rangle X_G \wedge \langle b \rangle X_D \wedge [\,a\,]\ (X_U \vee X_G) \wedge [\,b\,]\, X_D \wedge [\,c\,]\ \mathit{ff}$

$X_T \overset{\text{max}}{=} \langle b \rangle X_U \wedge [\,b\,]\, X_U \wedge [\,a\,]\ \mathit{ff} \wedge [\,c\,]\ \mathit{ff}$

$X_G \overset{\text{max}}{=} \langle b \rangle X_U \wedge [\,b\,]\, X_U \wedge [\,a\,]\ \mathit{ff} \wedge [\,c\,]\ \mathit{ff}$

$X_U \overset{\text{max}}{=} \langle b \rangle X_U \wedge [\,b\,]\, X_U \wedge [\,a\,]\ \mathit{ff} \wedge [\,c\,]\ \mathit{ff}$

3 b)

$\{\ W\ \}$

$\{\ I1\ \}$

$\{\ R\ \}$

$\{\ D,\ S\ \}$

$\{\ I2,\ N\ \}$

$\{\ G,\ U,\ T\ \}$

3 c)

Wir sind gut

Aufgabe 4

Aufgabe 4 a)

$F = Inv(\langle\, oeffnen \,\rangle\, tt)$

also $F = X$ mit:

$X \overset{\max}{=} \langle\, oeffnen \,\rangle\, tt \wedge [\ \mathsf{Act}\ ]\, X$

Aufgabe 4 b)

$F = Even(\langle\, beenden \,\rangle\, tt)$

also $F = X$ mit:

$X \overset{\min}{=} \langle\, beenden \,\rangle\, tt \vee (\langle\ \mathsf{Act}\ \rangle\, tt \wedge [\ \mathsf{Act}\ ]\, X)$

Aufgabe 4 c)

$F = Pos(\langle\, fahren \,\rangle\, tt) U^S\ [\, oeffnen \,]\, ff$

also $F = X$ mit:

$X \overset{\min}{=} [\, oeffnen \,]\, ff \vee (Y \wedge \langle\ \mathsf{Act}\ \rangle\, tt \wedge [\ \mathsf{Act}\ ]\, X)$

$Y \overset{\min}{=} \langle\, fahren \,\rangle\, tt \vee \langle\ \mathsf{Act}\ \rangle Y$