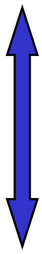
Slide 1

6.001 SICP
Data abstraction revisited

- Data structures: association list, vector, hash table
- Table abstract data type
- No implementation of an ADT is necessarily "best"
- Abstract data types are a technique for information hiding
  - in the types as well as in the code

CONCRETE



ABSTRACT CONCEPT

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Table: a set of bindings

- binding: a pairing of a key and a value
- Abstract interface to a table:
  - make
    create a new table
  - put! key value
    insert a new binding
    replaces any previous binding of that key
  - get key
    look up the key, return the corresponding value
- This definition IS the table abstract data type
  - Code shown later is an implementation of the ADT

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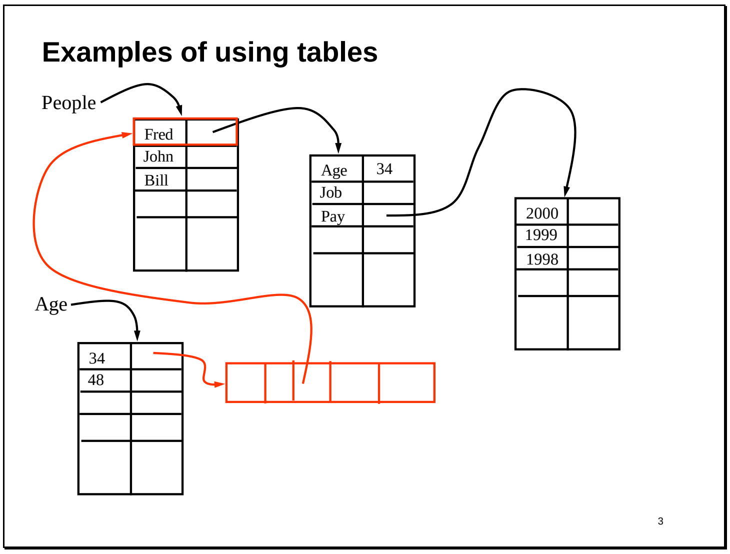

Examples of using tables
People
Fred
John
Bill
Age
34
Job
Pay
2000
1999
1998
Age
34
48
3

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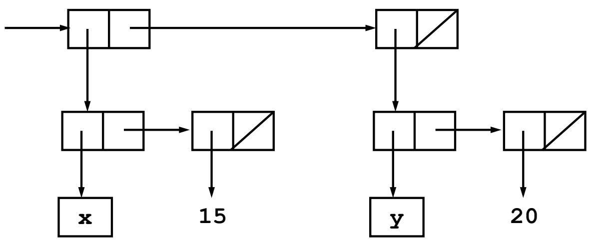
Traditional LISP structure: association list

- A list where each element is a list of the key and value.
- Represent the table

```
x: 15
y: 20
```

as the alist: `((x 15) (y 20))`



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Alist operation: find-assoc

```
(define (find-assoc key alist)
  (cond
    ((null? alist) #f)
     ((equal? key (caar alist)) (cadar alist))
     (else (find-assoc key (cdr alist)))))

(define a1 '((x 15) (y 20)))
(find-assoc 'y a1)  ==> 20
```

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Alist operation: add-assoc

```
(define (add-assoc key val alist)
   (cons (list key val) alist))

(define a2 (add-assoc 'y 10 a1))

a2                 ==> ((y 10) (x 15) (y 20))

(find-assoc 'y a2) ==> 10
```


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Alists are not an abstract data type

- Missing a constructor:
  - Use **quote** or **list** to construct

```
(define a1 '((x 15) (y 20)))
```

- There is no abstraction barrier:
  - Definition in scheme language manual:
    "An alist is a list of pairs, each of which is called an association. The car of an association is called the key."

- Therefore, the implementation is exposed. User may operate on alists using list operations.

```
(filter (lambda (a) (< (cadr a) 16)) a1))
          ==> ((x 15))
```

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Why do we care that Alists are not an ADT?

- Modularity is essential for software engineering
  - Build a program by sticking modules together
  - Can change one module without affecting the rest
- Alists have poor modularity
  - Programs may use list ops like `filter` and `map` on alists
  - These ops will fail if the implementation of alists change
  - Must change whole program if you want a different table
- To achieve modularity, hide information
  - Hide the fact that the table is implemented as a list
  - Do not allow rest of program to use list operations
  - ADT techniques exist in order to do this

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Table1: Table ADT implemented as an Alist

```
(define table1-tag 'table1)

(define (make-table1) (cons table1-tag nil))

(define (table1-get tbl key)
  (find-assoc key (cdr tbl)))

(define (table1-put! tbl key val)
  (set-cdr! tbl (add-assoc key val (cdr tbl))))
```

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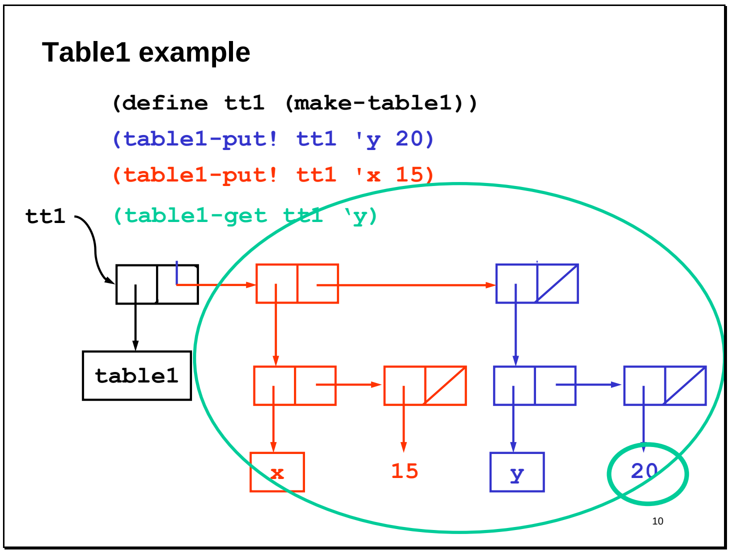
Table1 example
(define tt1 (make-table1))
(table1-put! tt1 'y 20)
(table1-put! tt1 'x 15)
(table1-get tt1 'y)
tt1
table1
x
15
y
20
10

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How do we know Table1 is an ADT implementation

- Potential reasons:
  - Because it has a type tag — No
  - Because it has a constructor — No
  - Because it has mutators and accessors — No
- Actual reason:
  - Because the rest of the program does not apply any functions to Table1 objects other than the functions specified in the Table ADT
  - For example, no **`car`**, **`cdr`**, **`map`**, **`filter`** done to tables
- The implementation (as an Alist) is hidden from the rest of the program, so it can be changed easily

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Information hiding in types: opaque names

- Opaque: type name that is defined but unspecified
- Given functions **m1** and **m2** and unspecified type MyType:

```
(define (m1 number) ...)   ; number → MyType
(define (m2 myt) ...)      ; MyType → undef
```

- Which of the following is OK? Which is a type mismatch?

```
(m2 (m1 10))      ; return type of m1 matches
                  ; argument type of m2
(car (m1 10))     ; return type of m1 fails to match
                  ; argument type of car
                  ; car: pair<A,B> → A
```

- Effect of an opaque name:
  no functions will match except the functions of the ADT

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Types for table1

- Here is everything the rest of the program knows

Table1<k,v>	opaque type
`make-table1`	void → Table1<anytype,anytype>
`table1-put!`	Table1<k,v>, k, v → undef
`table1-get`	Table1<k,v>, k → (v | null)

- Here is the hidden part, only the implementation knows it:

Table1<k,v> = symbol × Alist<k,v>
Alist<k,v> = list< k × v × null >

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Lessons so far

- Association list structure can represent the table ADT
- The data abstraction technique (constructors, accessors, etc) exists to support information hiding
- Information hiding is necessary for modularity
- Modularity is essential for software engineering
- Opaque type names denote information hiding

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Hash tables

- Suppose a program is written using Table1
- Suppose we measure that a lot of time is spent in **`table1-get`**
- Want to replace the implementation with a faster one
- Standard data structure for fast table lookup: hash table
- Idea:
  - keep N association lists instead of 1
  - choose which list to search using a hash function
    - given the key, hash function computes a number x where 0 <= x <= (N-1)

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Example hash function

- A table where the keys are points

point	graphic object
(5,5)	(circle 4)
(10,6)	(square 8)

```
(define (hash-a-point point N)
   (modulo (+ (x-coor point) (y-coor point))
           N))

; modulo x n = the remainder of x ÷ n
; 0 <= (modulo x n) <= n-1    for any x
```

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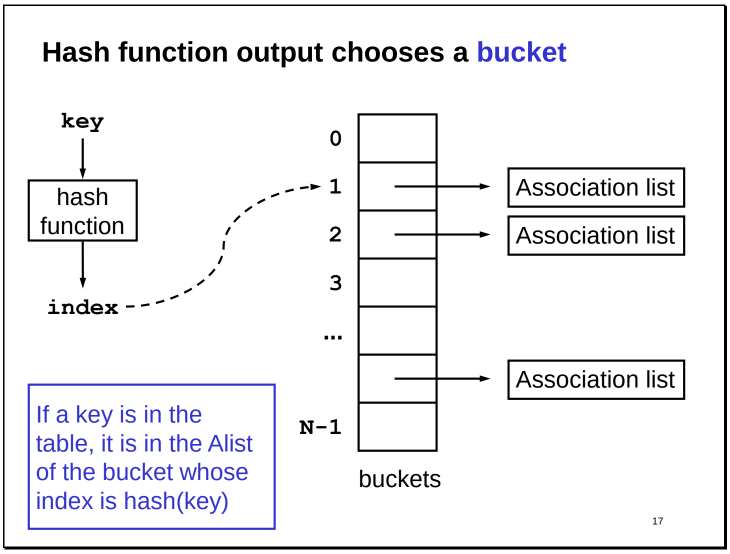

Hash function output chooses a bucket
key
hash function
index
0
1
2
3
...
N-1
buckets
Association list
Association list
Association list
If a key is in the table, it is in the Alist of the bucket whose index is hash(key)
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Store buckets using the vector ADT

- Vector: fixed size collection with indexed access

vector<A>	opaque type
`make-vector`	number, A → vector<A>
`vector-ref`	vector<A>, number → A
`vector-set!`	vector<A>,number, A → undef

(make-vector size value) ==> a vector with size locations;
each initially contains value

(vector-ref v index) ==> whatever is stored at that index of v
(error if index >= size of v)

(vector-set! v index val) stores val at that index of v
(error if index >= size of v)

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Table2: Table ADT implemented as hash table

```
(define t2-tag 'table2)
(define (make-table2 size hashfunc)
   (let ((buckets (make-vector size nil)))
     (list t2-tag size hashfunc buckets)))
(define (size-of tbl) (cadr tbl))
(define (hashfunc-of tbl) (caddr tbl))
(define (buckets-of tbl) (cadddr tbl))
```

- For each function defined on this slide, is it
  - a constructor of the data abstraction?
  - an accessor of the data abstraction?
  - an operation of the data abstraction?
  - none of the above?

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get in table2

```
(define (table2-get tbl key)
 (let ((index
        ((hashfunc-of tbl) key (size-of tbl))))
   (find-assoc key
         (vector-ref (buckets-of tbl) index))))
```

- Same type as table1-get

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put! in table2

```
(define (table2-put! tbl key val)
  (let ((index
          ((hashfunc-of tbl) key (size-of tbl)))
        (buckets (buckets-of tbl)))
    (vector-set! buckets index
                 (add-assoc key val
                   (vector-ref buckets index)))))
```

- Same type as table1-put!

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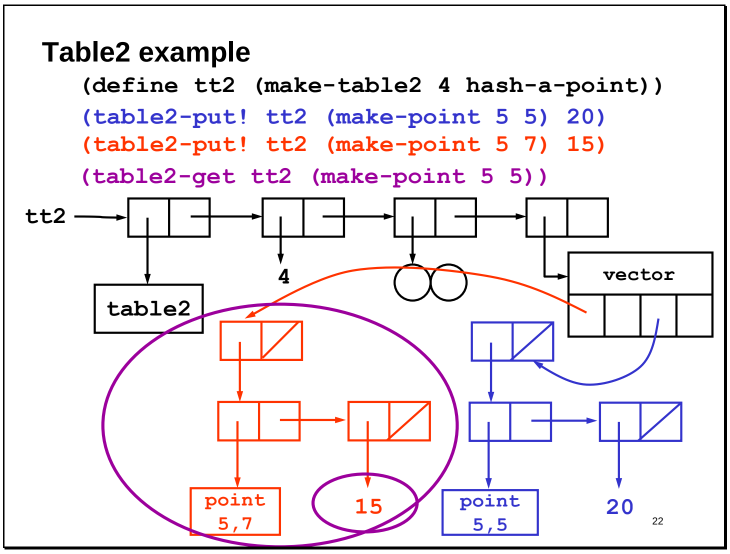

Table2 example
(define tt2 (make-table2 4 hash-a-point))
(table2-put! tt2 (make-point 5 5) 20)
(table2-put! tt2 (make-point 5 7) 15)
(table2-get tt2 (make-point 5 5))
tt2
table2
4
vector
point
5,7
15
point
5,5
20
22

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Is Table1 or Table2 better?

- Answer: it depends!
  - Table1:

make	extremely fast
put!	extremely fast
get	O(n) where n=# calls to put!

  - Table2:

make	space N where N=specified size
put!	must compute hash function
get	compute hash function plus O(n) where n=average length of a bucket

- Table1 better if almost no gets or if table is small
- Table2 challenges: predicting size, choosing a hash function that spreads keys evenly to the buckets

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End of lecture

- Introduced three useful data structures
  - association lists
  - vectors
  - hash tables
- Operations not listed in the ADT specification are internal
- The goal of the ADT methodology is to hide information
- Information hiding is denoted by opaque type names

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```
(define (add-assoc key val alist)
   (cons (list key val) alist))
(define (add-assoc key val alist)
   (cons (list key val) alist))

(define table1-tag 'table1)

(define (make-table1) (cons table1-tag nil))

(define (table1-get tbl key)
  (find-assoc key (cdr tbl)))

(define (table1-put! tbl key val)
  (set-cdr! tbl (add-assoc key val (cdr
 tbl))))
```

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```
(define (make-table2 size hashfunc)
   (let ((buckets (make-vector size nil)))
     (list t2-tag size hashfunc buckets)))
(define (table2-get tbl key)
 (let ((index
        ((hashfunc-of tbl) key (size-of tbl))))
   (find-assoc key
         (vector-ref (buckets-of tbl) index))))
(define (table2-put! tbl key val)
  (let ((index
          ((hashfunc-of tbl) key (size-of tbl)))
         (buckets (buckets-of tbl)))
    (vector-set! buckets index
                 (add-assoc key val
                   (vector-ref buckets index)))))
```