2)

Generate random data points from three bivariate Gaussian densities. Firstly, define class_means and class_covariances and class_sizes with given parameters.

```
In [2]: np.random.seed(421)
        # mean parameters
        class_means = np.array([[0.0,2.5],
                                [-2.5,-2.0],
                                [2.5,-2.0]])
        # standard covariance parameters
        class_covariances = np.array([[[3.2,0.0],
                                       [0.0,1.2]],

                                      [[1.2,0.8],
                                       [0.8,1.2]],

                                      [[1.2,-0.8],
                                       [-0.8,1.2]]])
        # sample sizes
        class_sizes = np.array([120, 80, 100])
```

Then, created random 3 classes based on a multivariate normal distribution. random samples created and its corresponding labels created with given class_sizes.

```
In [3]: # generate random samples
        points1 = np.random.multivariate_normal(class_means[0,:], class_covariances[0,:,:], class_sizes[0])
        points2 = np.random.multivariate_normal(class_means[1,:], class_covariances[1,:,:], class_sizes[1])
        points3 = np.random.multivariate_normal(class_means[2,:], class_covariances[2,:,:], class_sizes[2])
        X = np.vstack((points1, points2, points3))

        # generate corresponding labels
        y = np.concatenate((np.repeat(1, class_sizes[0]), np.repeat(2, class_sizes[1]), np.repeat(3, class_sizes[2])))
```

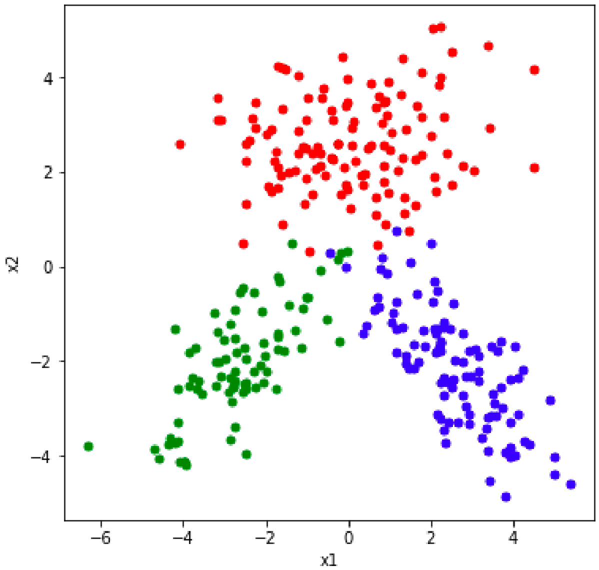
I saved these data point to the csv file to use later. This is not necessary in this Homework, I wanted to save them for later.

```
In [4]: # write data to a file
        np.savetxt("HW01_data_set.csv", np.hstack((X, y[:, None])), fmt = "%f,%f,%d")
```

Our data points should be checked to see how it looks like like. I used matplotlib.pyplot library top rint data points.

In [5]:
```
# plot data points generated
columns = ["x1","x2"]
plt.figure(figsize=(6, 6))
plt.plot(points1[:, 0], points1[:, 1], "r.", markersize=10)
plt.plot(points2[:, 0], points2[:, 1], "g.", markersize=10)
plt.plot(points3[:, 0], points3[:, 1], "b.", markersize=10)

plt.xlabel(columns[0])
plt.ylabel(columns[1])
plt.show()
```



3)

In the next part we will estimate parameters and prior probabilities.

First, I read data points that I saved to the csv file. I got number of classes, by getting max(y_truth) value which is equal 3. Then I got number of samples from data set (data set shape is (300,3)) which is equal 300.

In [6]:
```
# read data into memory
data_set = np.genfromtxt("HW01_data_set.csv", delimiter = ",")

# get X and y values
X = data_set[:,[0, 1]]
y_truth = data_set[:,2].astype(int)

# get number of classes and number of samples
K = np.max(y_truth)
N = data_set.shape[0]
```

I calculated sample means from given data with corresponding formula below.

$$\hat{\mu} = \frac{\sum_{i=1}^{N} x_i}{N}$$

In [7]:
```
# calculate sample means
sample_means = [np.mean(X[y_truth == (c + 1)], axis=0) for c in range(K)]

print("\033[4msample_means\033[0m \n")
for sample_mean in sample_means:
    print(sample_mean,"\n")
```

```
sample_means

[0.04453807 2.61225132]

[-2.65871584 -2.04611636]

[ 2.5605445  -2.12492713]
```

Next, I calculated sample covariances from data. By using following formula. However, in following formula x_i vectors is considered as column vectors. However, in the python code this is different. when writing code, each row in matrix X corresponds to a vector x_i.

$$\hat{\Sigma} = \frac{\sum_{i=1}^{N} (x_i - \widehat{\mu_c})(x_i - \widehat{\mu_c})^T}{N}$$

In [12]:
```
# calculate sample covariences
# in this code there is something that I couldn't handle it. in sample covarience formula we have (xi - mu)(xi-mu)T
# However in the code, whenever I do it that way, it creates meaningles matrix. So I tried other way.
sample_covariances = [
    (np.matmul(np.transpose(X[y == (c + 1)] - sample_means[c]),
               (X[y_truth == (c + 1)] - sample_means[c])) / class_sizes[c]) for c in range(K)]
print("\033[4msample_covariances\033[0m \n")
for sample_covariance in sample_covariances:
    print(sample_covariance,"\n")
```

```
sample_covariances

[[2.81619315 0.22436505]
 [0.22436505 1.00404695]]

[[1.42028643 1.01066174]
 [1.01066174 1.36103143]]

[[ 1.40686636 -1.07838139]
 [-1.07838139  1.5075388 ]]
```

I calculated prior probabilities: $\hat{P}(y=1), \hat{P}(y=2), \hat{P}(y=3)$

$$\widehat{P}(y_i = c) = \frac{\sum_{i=1}^{N} (y_i = c)}{N}$$

In [134]:
```
# calculate prior probabilities
class_priors = [np.mean(y_truth == (c + 1)) for c in range(K)]

print("\033[4mclass_priors\033[0m \n", class_priors)
```

```
class_priors
 [0.4, 0.26666666666666666, 0.3333333333333333]
```

4)

In this step we will need to calculate confusion matrix for the data points in your training set using the parametric classification rule you will develop using the estimated parameters from the previous step.

Before calculating score function, we will need to calculate W_c, w_c, w_{c0} matrices.

$$W_c = \frac{-1}{2} \hat{\Sigma}_c^{-1}$$

In [135]:
```
Wc = np.array([np.linalg.inv(sample_covariances[c]) / -2 for c in range(K)])
```

$$w_c = \hat{\Sigma}_c^{-1} \hat{\mu}_c$$

In [136]:
```
wc = np.array([np.matmul(np.linalg.inv(sample_covariances[c]), sample_means[c]) for c in range(K)])
```

$$w_{c0} = \frac{-1}{2} \hat{\mu}_c^T \hat{\Sigma}_c^{-1} \hat{\mu}_c - \frac{D}{2} \log(2\pi) - \frac{1}{2} \log|\hat{\Sigma}_c| + \log[\hat{P}(y=c)]$$

$$w_{c0} = \frac{-1}{2}\hat{\mu}_c^T \hat{\Sigma}_c^{-1} \hat{\mu}_c - \frac{D}{2}\log(2\pi) - \frac{1}{2}\log|\hat{\Sigma}_c| + \log[\hat{P}(y=c)]$$

-> $\frac{D}{2}\log(2\pi)$

this part will cancel each other out in the score function, since it is constant and has no effect for the results.

```
In [137]: wc0 = np.array([-(np.matmul(np.matmul(np.transpose(sample_means[c]),
                                      np.linalg.inv(sample_covariances[c])),sample_means[c])) / 2
                      - np.log(np.linalg.det(sample_covariances[c])) / 2
                      + np.log(class_priors[c]) for c in range(K)])

          print("\033[4mWc\033[0m \n", Wc)
          print("\033[4mwc\033[0m \n", wc)
          print("\033[4mwc0\033[0m \n", wc0)
```

```
Wc
 [[[-0.18076277  0.04039338]
  [ 0.04039338 -0.50701102]]

 [[-0.74649274  0.55432346]
  [ 0.55432346 -0.77899267]]

 [[-0.78681631 -0.56283   ]
  [-0.56283    -0.73427324]]]
wc
 [[-0.19493365  2.6452823 ]
 [-1.70100354 -0.24024219]
 [ 1.63741087 -0.23825172]]
wc0
 [-4.87775153 -3.78251262 -3.4266236 ]
```

I used python function to define score function formula.

$$g_c(x) = x^T W_c x + w_c^T x + w_{c0}$$

```
In [138]: def score_def(x):
              scores = np.array([0, 0, 0])
              for i in range(K):
                  score = np.matmul(np.matmul(np.transpose(x), Wc[i]), x) + np.matmul(np.transpose(wc[i]), x) + wc0[i]
                  scores[i] = score
              return scores
```

I used score_def function to calculate g scores with given X matrix data. Then predict y labels by using g scores.

```
In [36]: g_scores = [score_def(X[i]) for i in range(len(X))]

         # by using this I am checking maximum value(maximum g score). Then adds label max one.

         predict = [None] * X.shape[0]
         for i in range(len(g_scores)):
             max_g=np.max(g_scores[i])
             if g_scores[i][0]==max_g:
                 predict[i]=1
             elif g_scores[i][1]==max_g:
                 predict[i]=2
             else :
                 predict[i]=3

         predicted_label=np.array(predict)
         print(predicted_label)
```

```
[1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
 2 2 2 2 2 2 2 2 2 2 2 2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 2 3 3 3 3 3 3 3 3 3 3 3 3 3
 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
 1 3 3 3 1 3 3 3 3 3 3 3 3 3 3 3 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
 3 3 3 3]
```

Printed confusion matrix, I used pandas crosstab function.

```
In [141]: confusion_matrix = pd.crosstab(predicted_label, y_truth, rownames=['y_pred'], colnames=['y_truth'])
          print("\033[4mconfusion_matrix\033[0m \n", confusion_matrix)

confusion_matrix
 y_truth    1   2   3
y_pred
1        119   1   2
2          1  79   2
3          0   0  96
```

5) Visualization

First plot y_truth values each point. Then, mark misclassified data points.

```
In [35]: x1_interval = np.linspace(-6, +6, 121)
         x2_interval = np.linspace(-6, +6, 121)
         x1_grid, x2_grid = np.meshgrid(x1_interval, x2_interval)
         discriminant_values = np.zeros((len(x1_interval), len(x2_interval), K))

         # plot figure
         plt.figure(figsize=(10, 10))
         plt.plot(X[y_truth == 1, 0], X[y_truth == 1, 1], "r.", markersize=10)
         plt.plot(X[y_truth == 2, 0], X[y_truth == 2, 1], "g.", markersize=10)
         plt.plot(X[y_truth == 3, 0], X[y_truth == 3, 1], "b.", markersize=10)

         # pointing out misclassified points
         plt.plot(X[predicted_label != y_truth, 0], X[predicted_label != y_truth, 1], "ko", markersize=12, fillstyle="none")
```

$$g_c(x) = x^T W_c x + w_c^T x + w_{c0}$$

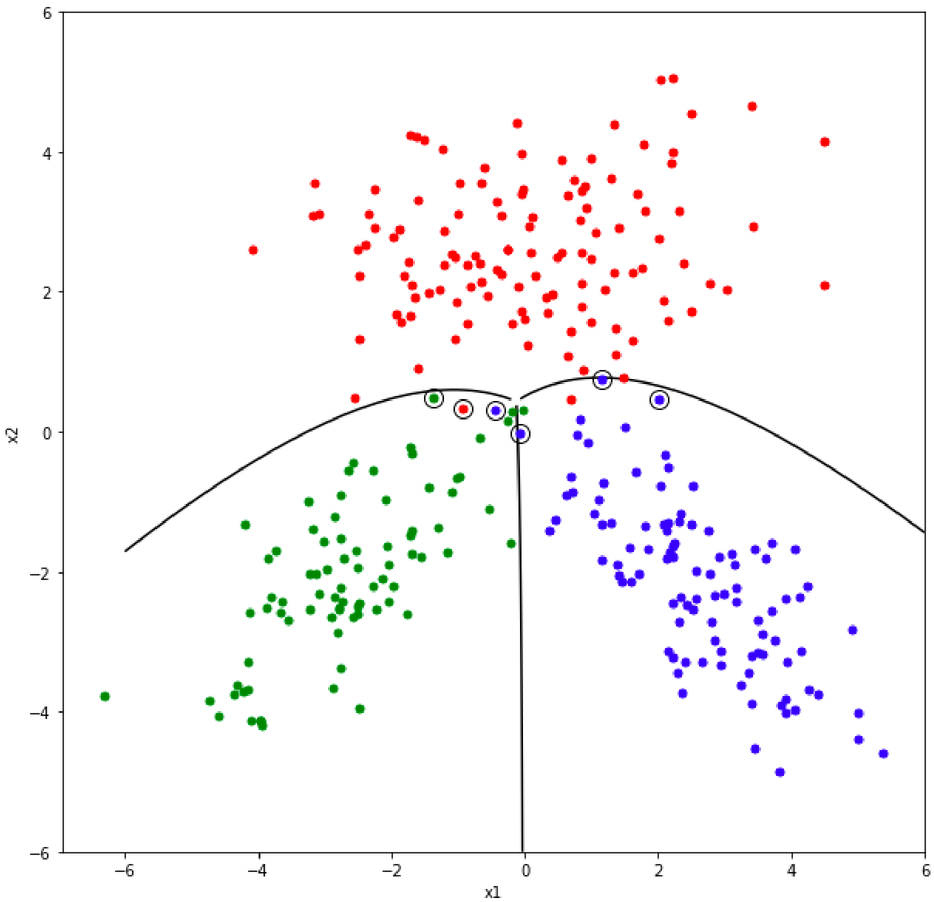
calculate discriminant values for 3 classes using the score function which is defined above. Then draw decision boundaries of each classes. When I don't assign nan values to out pf area, it gets unwanted lines. By using discriminant values, plotted contour graph lines

```
#for each label calculate discriminant values
for c in range(K):
    for i in range(x1_grid.shape[0]):
        for j in range(x2_grid.shape[1]):
            x = np.array([x1_grid[i][j],x2_grid[i][j]]).reshape(2,1)
            discriminant_values[i, j, c] = (np.matmul(np.matmul(np.transpose(x),Wc[c]),x)+
                                            np.matmul(np.transpose(wc[c]),x)+
                                            wc0[c])

A = discriminant_values[:, :, 0]
B = discriminant_values[:, :, 1]
C = discriminant_values[:, :, 2]
A[(A < B) & (A < C)] = np.nan
B[(B < A) & (B < C)] = np.nan
C[(C < A) & (C < B)] = np.nan
discriminant_values[:, :, 0] = A
discriminant_values[:, :, 1] = B
discriminant_values[:, :, 2] = C

plt.contour(x1_grid, x2_grid, discriminant_values[:, :, 0] - discriminant_values[:, :, 1], levels=0, colors="k")
plt.contour(x1_grid, x2_grid, discriminant_values[:, :, 0] - discriminant_values[:, :, 2], levels=0, colors="k")
plt.contour(x1_grid, x2_grid, discriminant_values[:, :, 1] - discriminant_values[:, :, 2], levels=0, colors="k")


plt.xlabel(columns[0])
plt.ylabel(columns[1])
plt.show()
```


6
4
2
0
−2
−4
−6
x2
−6
−4
−2
0
2
4
6
x1