**1)**

To apply Discrimination by Regression we are need to some updates on Linear discrimination.

a)

Instead of the softmax function, 𝐾 sigmoid functions to generate values.

b)

Instead of the negative log-likelihood, use the sum squared errors as the error function to minimize

**2)**

Generate random data points from three bivariate Gaussian densities. Firstly, define class\_means and class\_covariances and class\_sizes with given parameters.

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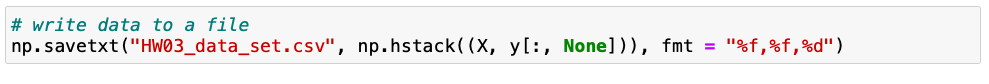
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Then, created random 3 classes based on a multivariate normal distribution. random samples created and its corresponding labels created with given class\_sizes.

Text

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I saved these data point to the csv file to use later. This is not necessary in this Homework, I wanted to save them for later.



Our data points should be checked to see how it looks like like. I used matplotlib.pyplot library top rint data points.

Chart, scatter chart

Description automatically generated with medium confidence

Chart, scatter chart

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**3)**

First defined learning parameters

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Sigmoid function is defined to use in the parameter estimation

Graphical user interface, text, application, chat or text message

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Derivative of Error is derived with respect to w and w0. By multiplying with (-)step size we are going to get gradients(which is steps).

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Initialize parameter with small w,w0 (to get rid of memory issues.)

W has shape (N, K)

w0 has shape (1,K)

Text

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Now iterate over parameters to converge objective value. If improvement is less than epsilon, we stop. That means we found good enough parameter estimation. Here we could use iteration limit, however epsilon does same thing.

for y\_predicted I used sigmoid function instead of softmax for discrimination by regression.

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When we look at the parameter estimates, we finally get such a result. Text

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**4)**

Here objective function values throughout iterations. As we can see after some iterations improvement is very limited. We could limit our iterations, if particular error is enough for us.

Graphical user interface

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**5)**

Here confusion matrix for the data points which is developed using parameters from step 3

Graphical user interface, application

Description automatically generated

Draw decision boundaries which is calculated from estimated parameters. Then marked misclassified data points. When I don’t assign nan values to out pf area, it gets unwanted lines. By using discriminant values, plotted contour graph lines

Chart, scatter chart

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