COM2536 – Ders 2

Crisp Sets, Fuzzy Sets, Membership Functions, Operations, Hedges

Dr. Metehan GÜZEL
Uzman

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Crisp Sets

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Klasik Setler (Crisp Sets)

Evrensel Set (Universal Set)

Klasik Set (Classical Set/Crisp Set)

Bulanık Set (Fuzzy Set)

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Klasik Setler (Crisp Sets)

Temel Notasyon (Notation)

* $\varnothing$: Boş Set (Empty Set)
* $x \in A$: element x belongs to set A
* $x \notin A$: element x does not belong to set A
* $A \subseteq B$: set A is a subset of set B
* $A \supseteq B$: set A is a superset of set B
* $A \subset B$: set A is a proper subset of set B
* $A \supset B$: set A is a proper superset of set B
* $A = B$: set A and set B are equal
* $A \neq B$: set A and set B are not equal
* $|A|$: number of elements in set A
* $p(A)$: number of subsets of set A

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Klasik Setler (Crisp Sets)

Operasyonlar (Operations) on Crisp Sets

Sets: A, B

* A - B : Fark (Difference)
* ¬A : Tamlayan (Complement)
* A∩B : Kesişim (Intersection)
* A∪B : Birleşim (Union)

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Klasik Setler (Crisp Sets)

Properties of Crisp Sets

Sets : A, B, C
Universal Set: X

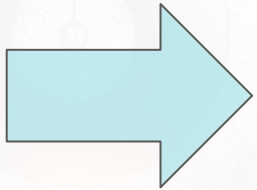
* $\neg\neg A = A$
* $A \cup B = B \cup A$
  * $A \cap B = B \cap A$
* $A \cup (B \cup C) = (A \cup B) \cup C$
  * $A \cap (B \cap C) = (A \cap B) \cap C$
* $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  * $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
* $A \cup A = A$
  * $A \cap A = A$
* $A \cap (A \cup B) = A$
  * $A \cup (A \cap B) = A$
* $A \cup X = X$
  * $A \cap X = A$
* $A \cup \varnothing = A$
  * $A \cap \varnothing = \varnothing$
* $\neg(A \cup B) = \neg A \cap \neg B$,
  * $\neg(A \cap B) = \neg A \cup \neg B$
* $A \cup \neg A = X$
* $A \cap \neg A = \varnothing$

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Klasik Setler (Crisp Sets)

Remove precision from boundaries

Crisp Sets



Fuzzy Sets

Better handling of imprecision and uncertainty

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Fuzzy Sets

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Fuzzy Sets

Crisp Sets,

* $A = \{a1, a2, a3, \ldots, an\}$

* $A = \{x \mid P(x)\}$

Fuzzy Sets,

* $A = \{(x, \mu_A(x), x \in X)$

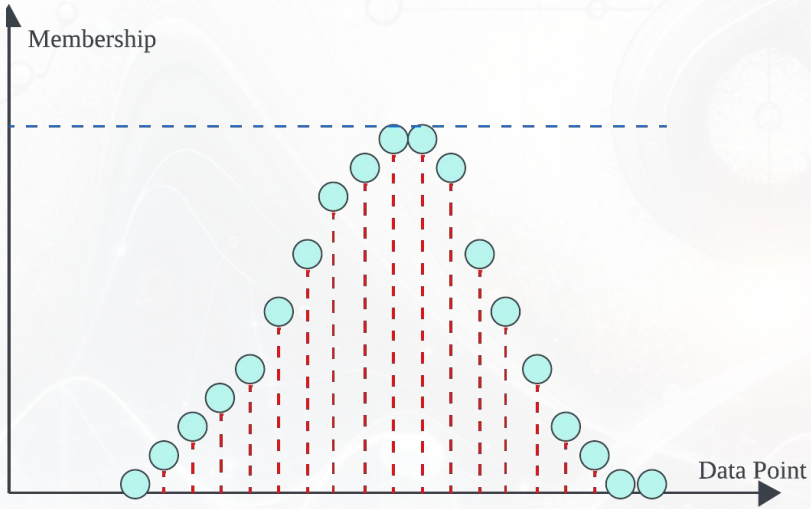
Probability vs Membership 😕

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Fuzzy Sets

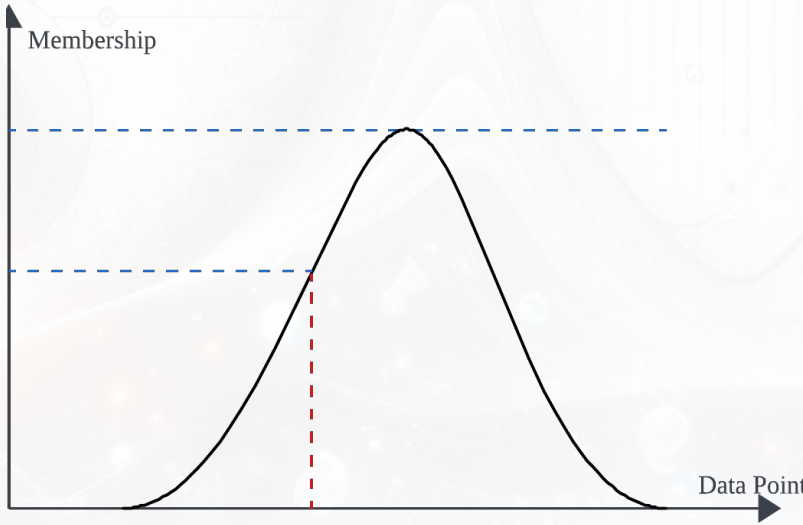
Discrete Fuzzy Set

(Ayrık Bulanık Set)



Continuous Fuzzy Set

(Devamlı Bulanık Set)



Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

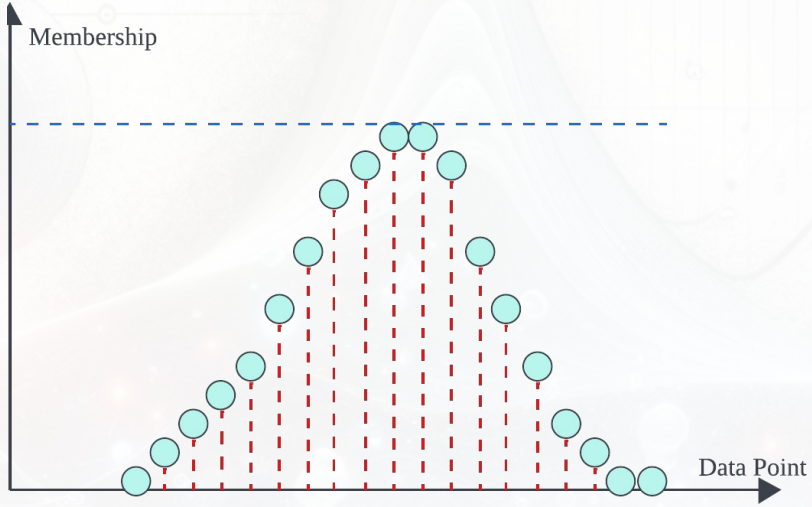
Fuzzy Sets

Discrete Fuzzy Set

(Ayrık Bulanık Set)

$$A(x) = \sum \mu_A(x_i) / x_i \ (i=1 \rightarrow n)$$

n: number of elements of set A



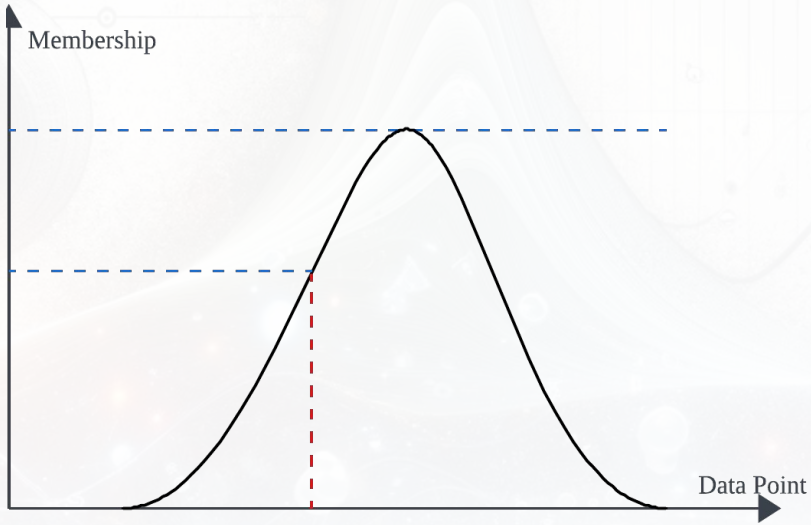
Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Fuzzy Sets

Continuous Fuzzy Set

(Devamlı Bulanık Set)

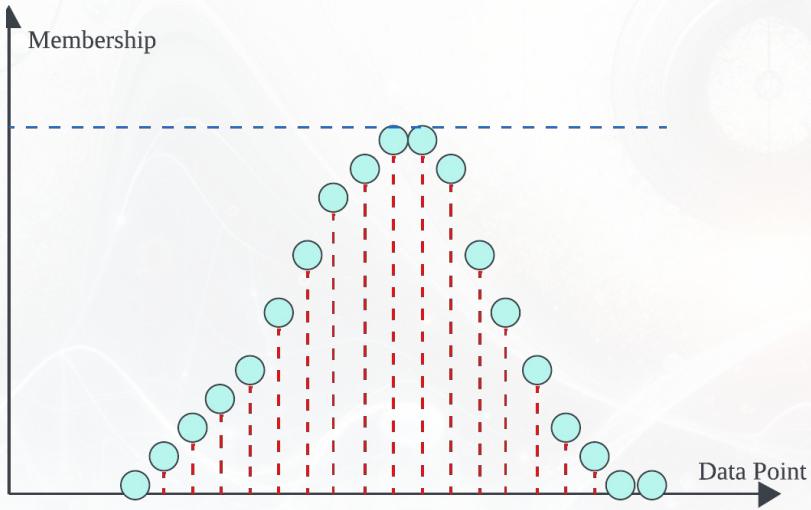
$$A(x) = \int \mu_A(x)/x$$

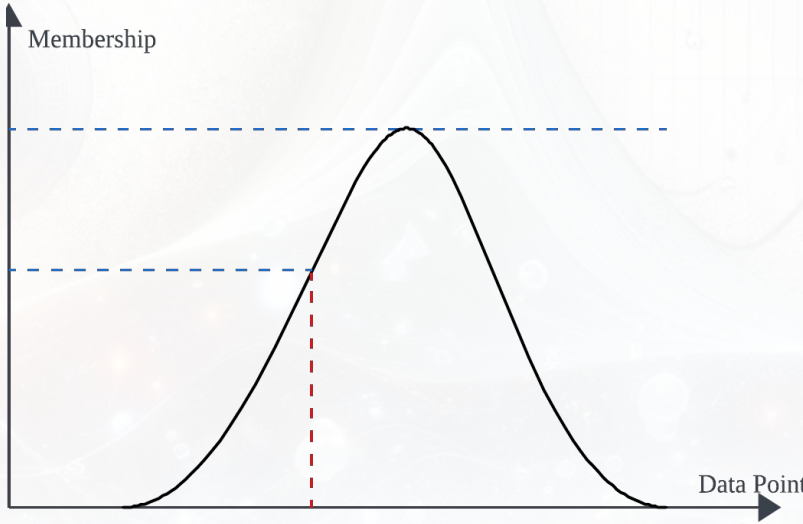


Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Fuzzy Sets

Dikkatinizi çeken bir şey oldu mu?



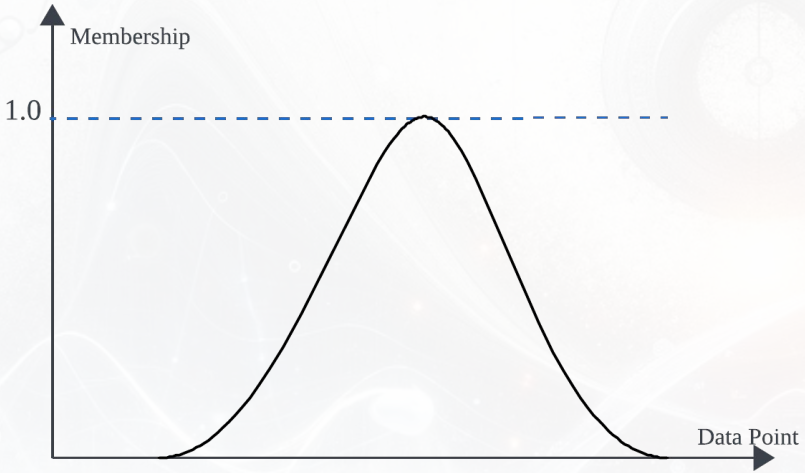


Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Fuzzy Sets

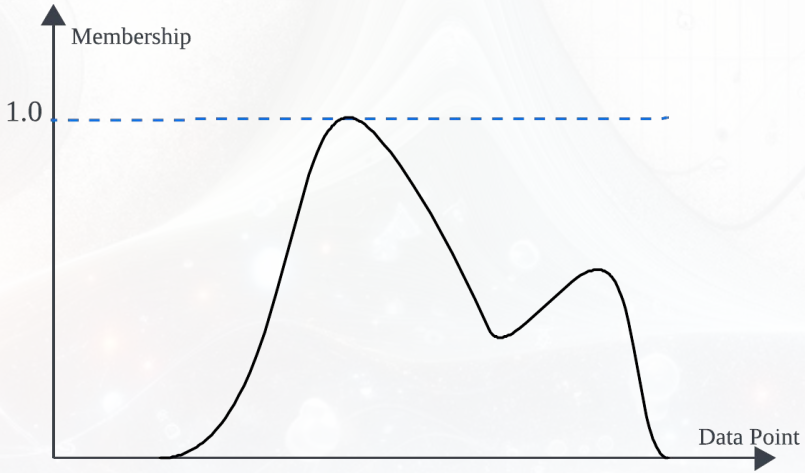
Convex Membership Function

(Konveks Üyelik Fonksiyonu)



Non-Convex Membership Function

(Konveks Olmayan Üyelik Fonksiyonu)



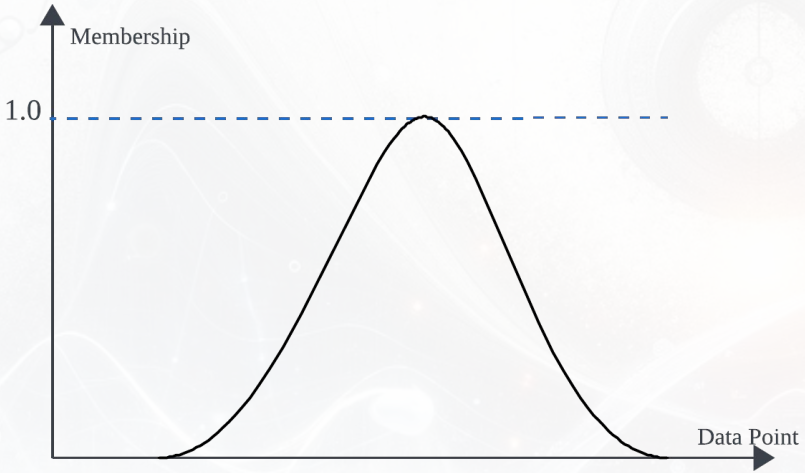
Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

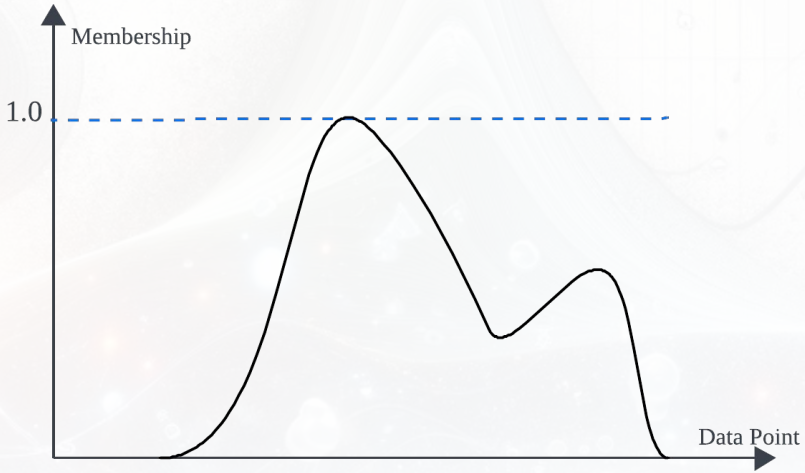
Fuzzy Sets

Set A is convex if

$$\mu_A\{\lambda x_1 + (1 - \lambda)x_2\} \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$$

for $0.0 < \lambda < 1.0$





Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

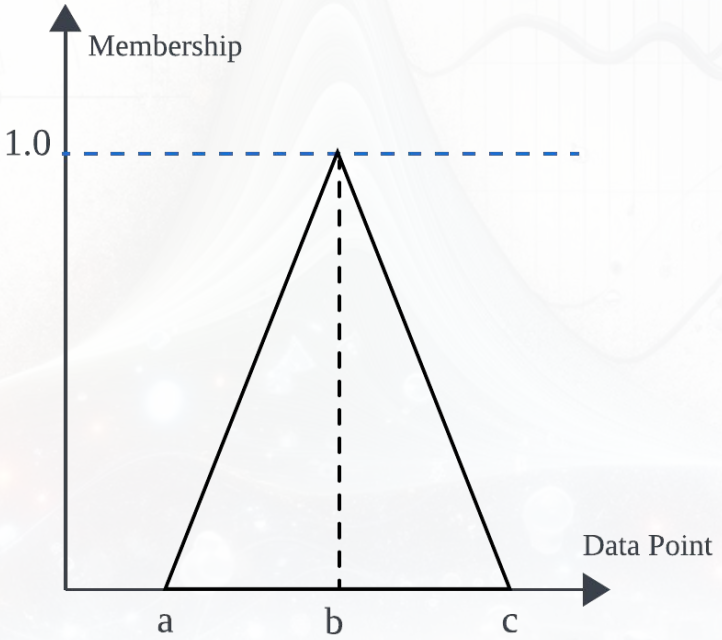
Üyelik Fonksiyonları

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Membership Functions - Triangular

$$\mu = \max(\min((x-a)/(b-a), (c-x)/(c-b)), 0)$$

$$\mu = \begin{cases} 0 & \text{if } x \le a \\ (x-a)/(b-a) & \text{if } a \le x \le b \\ (c-x)/(c-b) & \text{if } b \le x \le c \\ 0 & \text{if } x \ge c \end{cases}$$

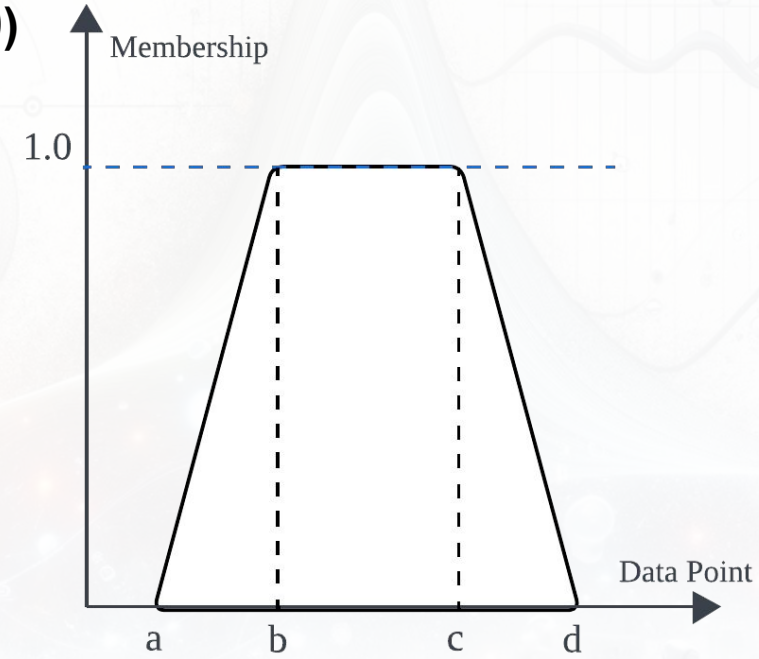


Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Membership Functions - Trapezodial

$$\mu = \max(\min((x-a)/(b-a), 1, (d-x)/(d-c)), 0)$$

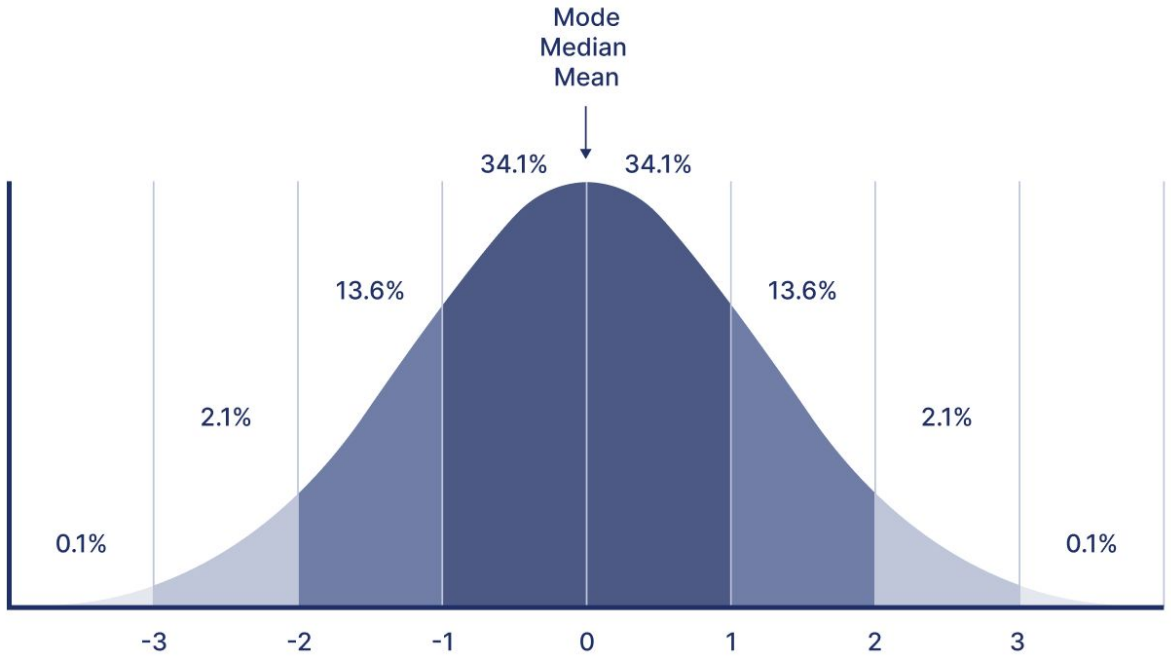
$$\mu = \begin{cases} 0 & \text{if } x \le a \\ (x-a)/(b-a) & \text{if } a \le x \le b \\ 1 & \text{if } b \le x \le c \\ (d-x)/(d-c) & \text{if } c \le x \le d \\ 0 & \text{if } x \ge d \end{cases}$$



Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Membership Functions - Gaussian

Gaussian Distribution

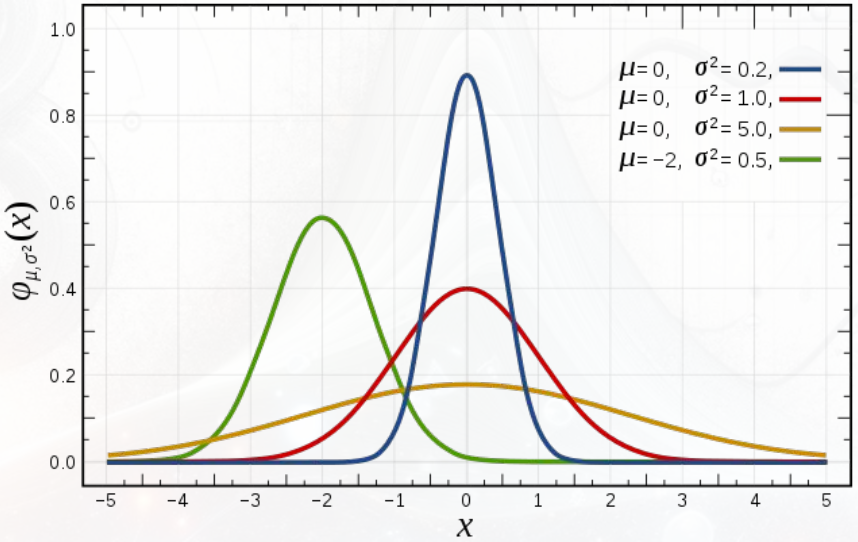


Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Membership Functions - Gaussian

Parameters:

* Mean (Ortalama)
* Standart Deviation
  (Standart Sapma)



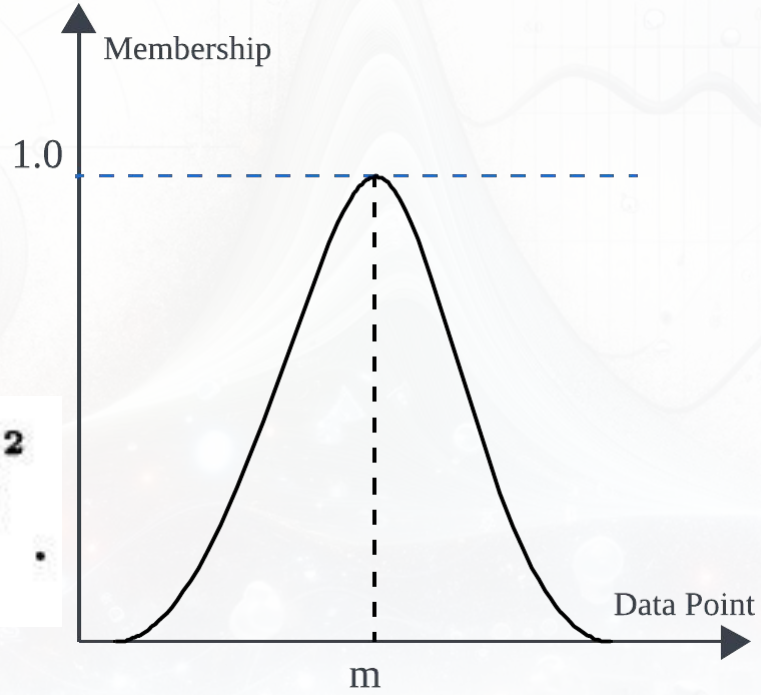
Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Membership Functions - Gaussian

mean (merkez): c
sd (standart sapma): σ

gaussian membership value can be calculated as:

$$\text{gaussian}(x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}.$$



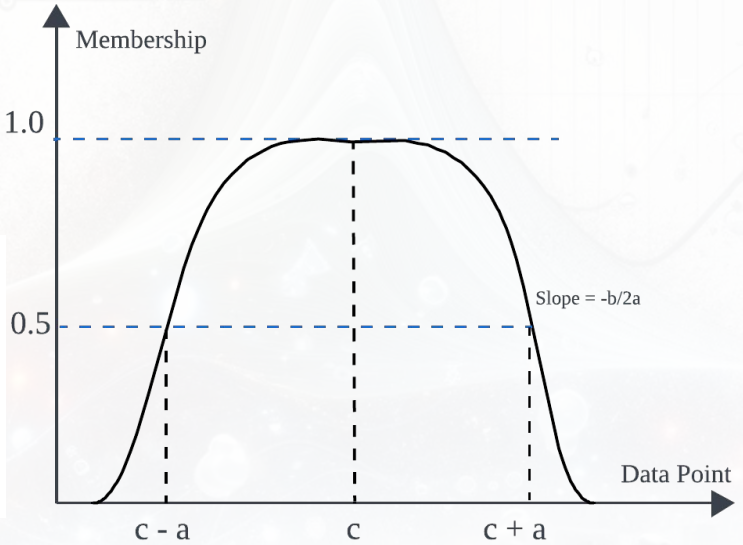
Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Membership Functions - Bell Shaped

a: width (genişlik)
b: slope, gradient (eğim)
c: center (merkez)

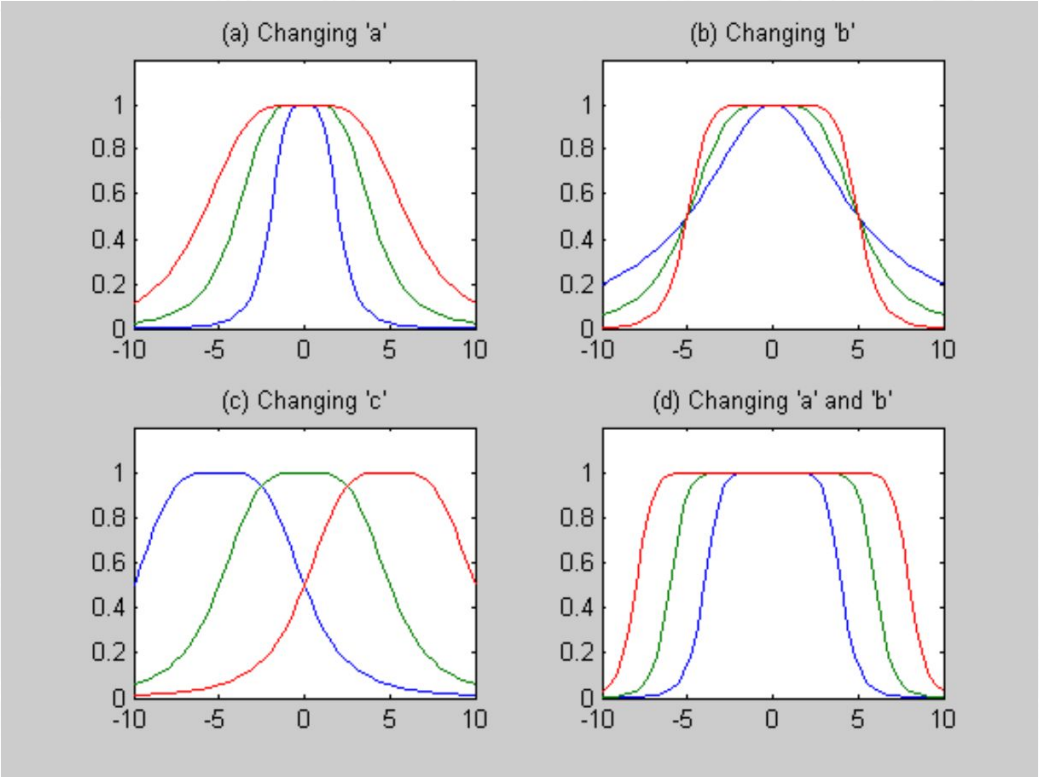
generalized bell membership value can be calculated as:

$$gbell(x, a, b, c) = \frac{1}{1 + \left|\frac{x - c}{a}\right|^{2b}}$$



Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Membership Functions - Bell Shaped



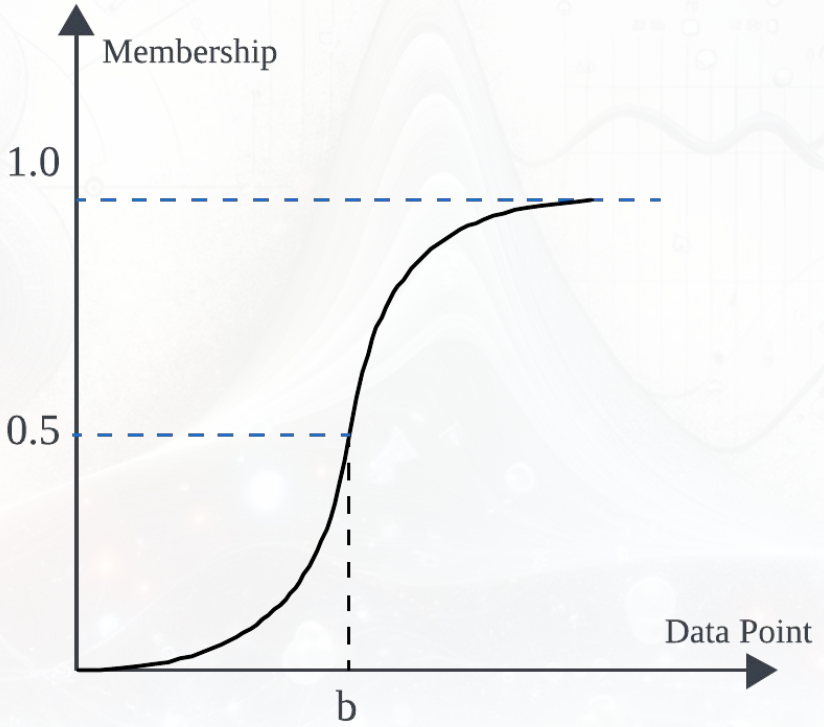
Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Membership Functions - Sigmoid

mean (merkez): c
slope (eğim): a

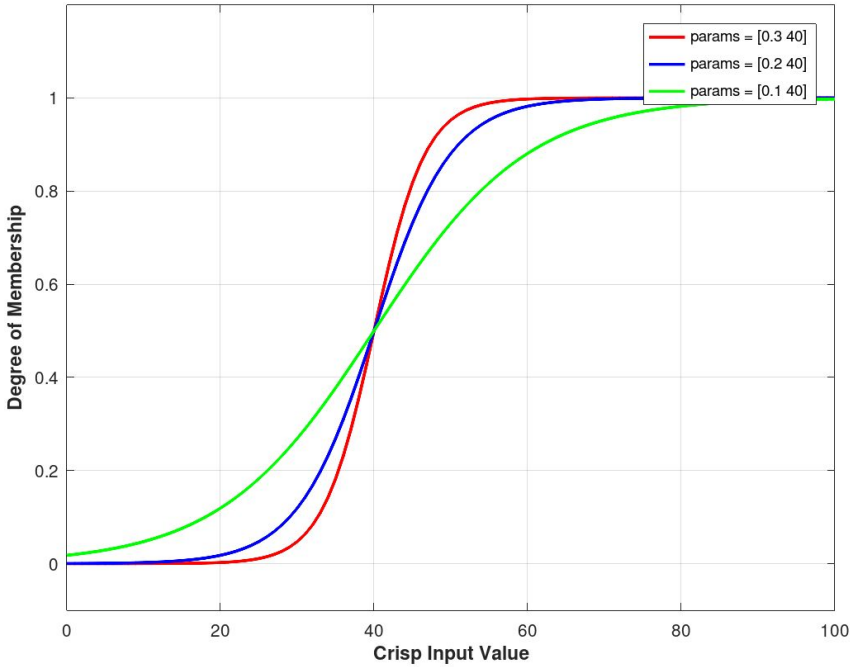
sigmoid membership value can be calculated as:

$$sigmf\,(x;a,b,c) = \frac{1}{1+e^{-a(x-c)}}$$



Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Membership Functions - Sigmoid



Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Operations on Fuzzy Sets

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Operations on Fuzzy Sets

Lets remember operations on crips sets,

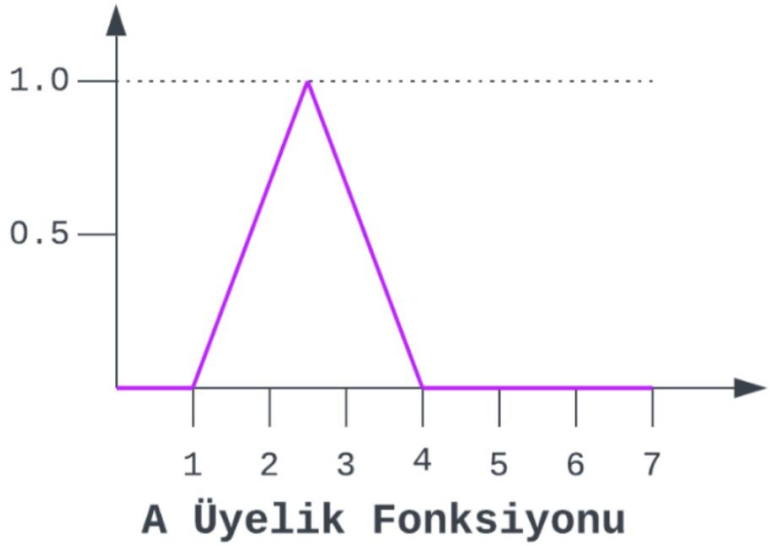
* A - B : Fark (Difference)
* ¬A : Tamlayan (Complement)
* A∩B : Kesişim (Intersection)
* A∪B : Birleşim (Union)

How can ve perform given operations on fuzzy sets 😕

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Operations on Fuzzy Sets

Let’s give it a try 💪



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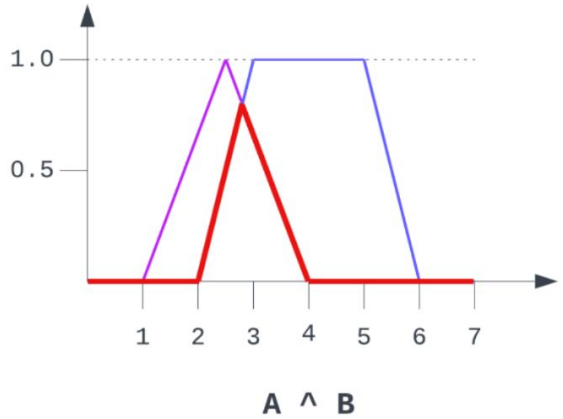
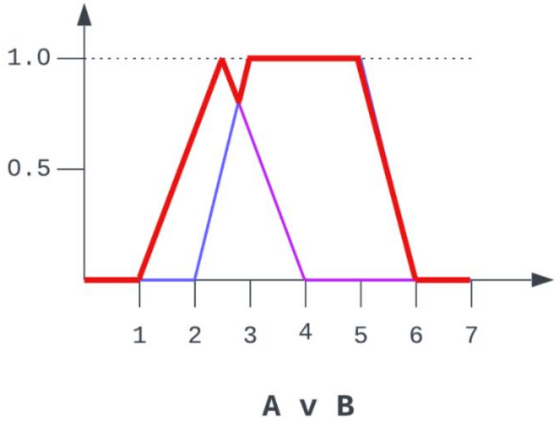
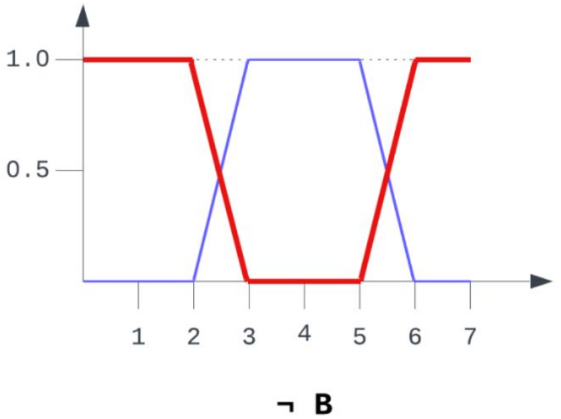
Operations on Fuzzy Sets



A ^ B



A v B



¬ B

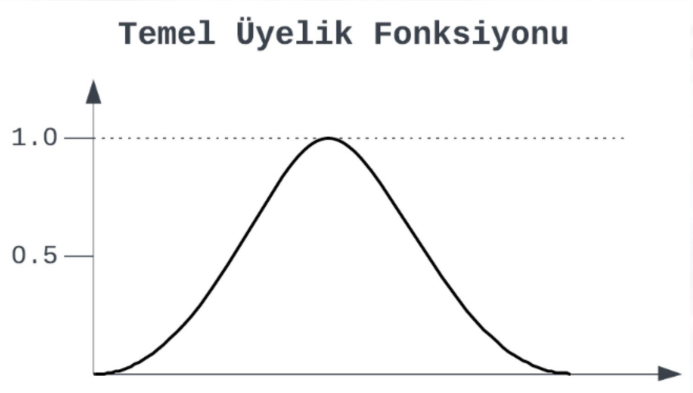
Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Hedges

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Hedges

* Linguistic Term (Dilsel Terim)
* Hedge (Türkçesini bilmiyorum :P)
    * Modifies distributions of membership functions



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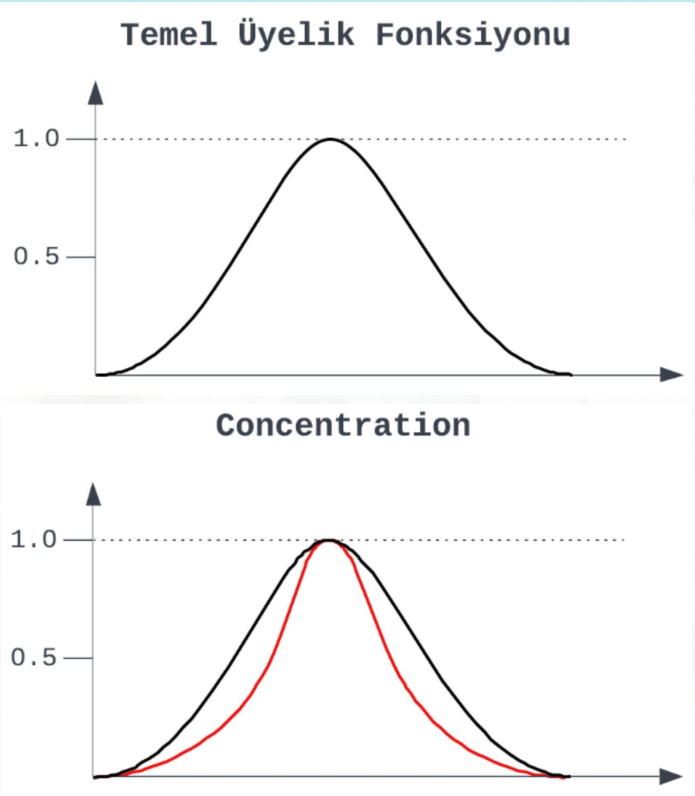
Hedges

Concentration



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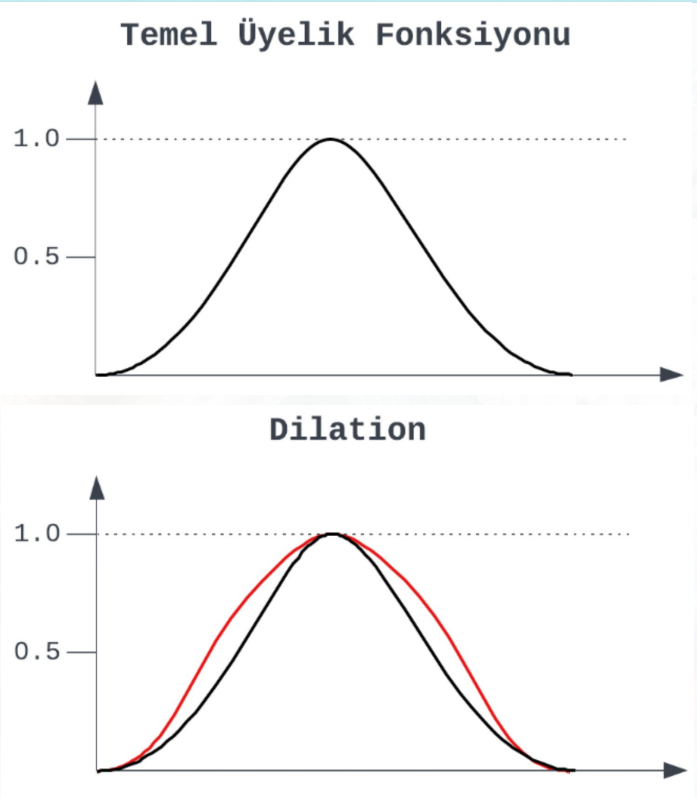
Hedges

Dilation



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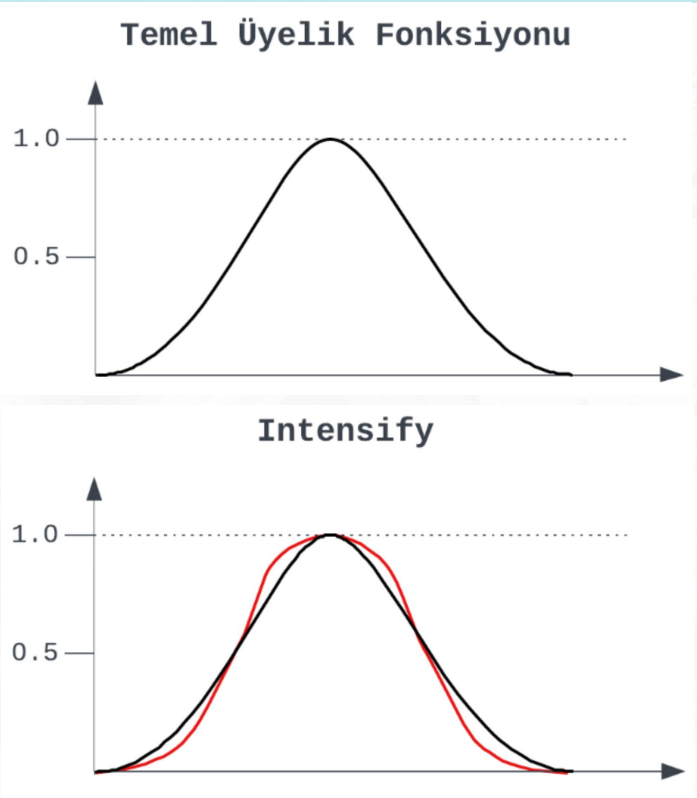
Hedges

Intensify



Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Hedges

Hedge	Operatör	Yön
Biraz (A little)	μ_A^ 1.3	Concentration
Kısmen (Slightly)	μ_A^ 1.7	Concentration
Very (Çok)	μ_A^ 2	Concentration
Aşırı (Extremely)	μ_A^ 3	Concentration
Çok Çok (Very Very)	μ_A^ 4	Concentration
Sayılır (Somewhat)	μ_A^ 0.5	Dilation
Aslında (Indeed)	2(μ_A^ 2) if (0 <= μ_A <= 0.5) 1 - 2(1 - μ_A)^2 if (0.5 < μ_A <= 1.0)	Intensify

Final Remarks

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Some Updates About the Course

* I expect you to submit your research assignments at midterm.

    * Don’t worry, I will make a presentation about how to prepare the assignments.

* I expect you to prepare your final projects (both presentation and report) at the last week of the course.

    * Don’t worry about the final exam, focus on the project

Next Week

* Some theoretical stuff about fuzzy logic
* More practice on this week's stuff
* Maybe some coding too :P
* You have a **QUIZ** next week :P
    * Prepare well on this week’s stuff

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io

Beni Dinlediğiniz İçin Teşekkür Ederim

Sorularınız???

Metehan GÜZEL, Ph.D. , e-mail: metehanguzel@kafkas.edu.tr, web: https://metehanguzel.github.io