Artificial Intelligence
Exercise Set 3

9p 1. Which of the following are correct?

a. $False \models True$.
b. $True \models False$.
c. $(A \land B) \models (A \Leftrightarrow B)$.
d. $A \Leftrightarrow B \models A \lor B$.
e. $A \Leftrightarrow B \models \neg A \lor B$.
f. $(A \land B) \Rightarrow C \models (A \Rightarrow C) \lor (B \Rightarrow C)$.
g. $(C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$.
h. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B)$.
i. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B) \land (\neg D \lor E)$.

3p 2. Either prove or find a counterexample to each of the following assertions:

a. If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \land \beta) \models \gamma$.
b. If $\alpha \models (\beta \land \gamma)$ then $\alpha \models \beta$ and $\alpha \models \gamma$.
c. If $\alpha \models (\beta \lor \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

3p 3. According to some political pundits, a person who is radical (R) is electable (E) if they are conservative (C), but otherwise is not electable. Which of the following are correct representations of this assertion? Explain why the representation is correct or why it is not.

a. $(R \land E) \Leftrightarrow C$
b. $R \Rightarrow (E \Leftrightarrow C)$
c. $R \Rightarrow ((C \Rightarrow E) \lor \neg E)$

5p 4. For each of the following statements, either prove it is true or otherwise give a counterexample.

a. $P(a|b \land a) = 1$. 2p
b. If $P(a|b, c) = P(b|a, c)$, then $P(a|c) = P(b|c)$. 1p
c. If $P(a|b,c) = P(a)$, then $P(b|c) = P(b)$. 1p
d. If $P(a|b) = P(a)$, then $P(a|b, c) = P(a|c)$. 1p

4p 5. Consider the full joint distribution in Table 1, calculate the following:

	$tootache$		$\neg toothache$	
	$change$	$\neg change$	$change$	$\neg change$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Table 1. Full joint distribution of three Boolean variables: $\mathrm{Toothache}$, Cavity, $\mathrm{ChangeOnXRay}$.

1p a. **P**(Cavity).
1p b. **P**(Toothache | *cavity*) .
2p c. **P**(Cavity | *toothache* ∨ *change*).

OBS!!! In all calculations above, you are required to calculate **bold P**.

6p 6. After Mr. Willoughby's yearly checkup, the doctor has some bad news: Mr. Willoughby tested positive for a rare disease and the test is 99% accurate (i.e., the probability of testing positive when one has the disease is 0.99, as is the probability of testing negative when one doesn't). The good news is that this disease affects only 1 in 10000 people of his age. Why is it good news that the disease is rare?

What are the chances Mr. Willoughby actually has the disease? Calculate.