AI Assignment 3

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1. Which of the following are correct?

a. False ⊨ True. TRUE

b. True ⊨ False. FALSE

c. $(A \land B) \models (A \Leftrightarrow B)$. TRUE

d. $A \Leftrightarrow B \models A \lor B$. FALSE

e. $A \Leftrightarrow B \models \neg A \lor B$. TRUE

f. $(A \land B) \Rightarrow C \models (A \Rightarrow C) \lor (B \Rightarrow C)$. TRUE

g. $(C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$. TRUE

h. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B)$. TRUE

i. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B) \land (\neg D \lor E)$. FALSE

2. Either prove or find a counterexample to each of the following assertions:

a. If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \land \beta) \models \gamma$. TRUE

$(\alpha \land \beta)$ is true when both are true. Thus, if $\alpha \models \gamma$ and $\beta \models \gamma$, $(\alpha \land \beta) \models \gamma$

b. If $\alpha \models (\beta \land \gamma)$ then $\alpha \models \beta$ and $\alpha \models \gamma$. TRUE

$(\beta \land \gamma)$ is true only if both are true. Thus, when α is true, both of them have to be true.

c. If $\alpha \models (\beta \lor \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both). FALSE

assume that $\beta = p$, $\gamma = -p$, and α = true. Now, $\alpha \models (\beta \lor \gamma)$, but neither $\alpha \models \beta$ or $\alpha \models \gamma$.

3. According to some political pundits, a person who is radical (R) is electable (E) if they are conservative (C), but otherwise is not electable. Which of the following are correct representations of this assertion? Explain why the representation is correct or why it is not.

a. $(R \land E) \Leftrightarrow C$ NO

We do not know pundits about not political people according to what is given. This representation, however, says that a person is conservative if and only if he is radical and electable. But there may be people who are conservative and not radical.

b. R ⇒ (E ⇔ C) YES

If a person is radical, he is either conservative and electable, or not conservative and not electable.

c. R ⇒ ((C ⇒ E) ∨¬E) NO

((C ⇒ E) ∨¬E) = ¬C ∨ E ∨¬E which is equivalent to 1. R ⇒ 1 is also equivalent to 1. Thus, this statement is always true, which contradicts with the problem given above.

4. For each of the following statements, either prove it is true or otherwise give a counterexample.

a. P (a|b ∧ a) = 1. TRUE

$$P(a \mid b \wedge a) = \frac{P(a \wedge b \wedge a)}{P(b \wedge a)} = \frac{P(b \wedge a)}{P(b \wedge a)} = 1 \checkmark$$

b. If P(a|b, c) = P(b|a, c), then P(a|c) = P(b|c). TRUE

Assume that $P(a \mid b,c) = P(b \mid a,c)$

$$P(a \mid b,c) = \frac{P(a \wedge b \wedge c)}{P(b \wedge c)}, \quad P(b \mid a,c) = \frac{P(a \wedge b \wedge c)}{P(a \wedge c)}$$

$$\Longrightarrow P(b \wedge c) = P(a \wedge c)$$

$$P(a \mid c) = \frac{P(a \wedge c)}{P(c)}, \quad P(b \mid c) = \frac{P(b \wedge c)}{P(c)}$$

Since $P(b \wedge c) = P(a \wedge c) \Longrightarrow P(a \mid c) = P(b \mid c)$ ✓

c. If P(a|b,c) = P(a), then P(b|c) = P(b). FALSE

First sentence says a is independent from b and c. There is no info about relation between b and c. If we say a = p, b = q, and c = -q; first sentence still holds but second sentence does not hold.

d. If P(a|b) = P(a), then P(a|b, c) = P(a|c). FALSE

First sentence says a and b are independent. If we say a = p, b = q, and c = a and b, first sentence still holds but second sentence does not hold.

a	b	$c = a \wedge b$
0	0	0
0	1	0
1	0	0
1	1	1

$P(a=0 \mid b=0, c=0) = 1/2$

$p(a=0 \mid c=0) = 2/3$

Thus, $P(a \mid b,c) \neq P(a \mid c)$

5. Consider the full joint distribution in Table 1, calculate the following:

	tootache		*¬toothache*	
	change	*¬change*	*change*	*¬change*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Table 1. Full joint distribution of three Boolean variables: Toothache, Cavity, ChangeOnXRay.

a. P(Cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2

b. P(Toothache | cavity) = (0.108 + 0.012) / 0.2 = 0.6

c. P(Cavity | toothache V change) = (0.108 + 0.012 + 0.072) / (1 – 0.584) ~ 0.4615

6. After Mr. Willoughby's yearly checkup, the doctor has some bad news: Mr. Willoughby tested positive for a rare disease and the test is 99% accurate (i.e., the probability of testing positive when one has the disease is 0.99, as is the probability of testing negative when one doesn't). The good news is that this disease affects only 1 in 10000 people of his age. Why is it good news that the disease is rare? What are the chances Mr. Willoughby actually has the disease? Calculate.

$P(T \mid D) = 0.99$, $P(\neg T \mid \neg D) = 0.99$, $P(D) = 0.0001$, $P(D \mid T) = ?$

$P(D \mid T) = P(T \mid D)*P(D) / P(T) = 0.99 * 0.0001 / P(T)$

$P(T) = P(T \mid D) * P(D) + (T \mid \neg D) * P(\neg D) = 0.99*0.0001 + 0.01*0.9999 = 0.010098$

$P(D \mid T) = 0.99 * 0.0001 / 0.010098 = 0{,}00980392156$

So he tested positive, but probability of having the disease given positive test result is 0,00980392156. Thus, this is good news.