Artificial Intelligence Exercise Set 4

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5p. 1. Consider two medical tests, A and B, for a virus. Test A is 95% effective at recognizing the virus when it is present, but has a 10% false positive rate (indicating that the virus is present, when it is not). Test B is 90% effective at recognizing the virus, but has a 5% false positive rate. The two tests use independent methods of identifying the virus. The virus is carried by 1% of all people. Say that a person is tested for the virus using only one of the tests, and that test comes back positive for carrying the virus. Which test returning positive is more indicative of someone really carrying the virus? **Tip**: Which posterior is larger:

$$P(CarryVirus|TestA) \text{ or } P(CarryVirus|TestB)?$$

$P(A|V) = 0.95$ $\qquad$ $P(B|V) = 0.90$

$P(A|V') = 0.10$ $\qquad$ $P(B|V') = 0.05$

$P(V) = 0.01$, $P(V') = 0.99$

$P(V|A) = ?$ $\qquad$ $P(V|B) = ?$

$$P(V|A) = \frac{P(A|V)\cdot P(V)}{P(A)} = \frac{P(A|V)\cdot P(V)}{P(A|V)\cdot P(V) + P(A|V')\cdot P(V')}$$

$$P(V|A) = \frac{0.95 \times 0.01}{0.95\times 0.01 + 0.1 \times 0.99} = \underline{0.0875576}$$

$$P(V|B) = \frac{P(B|V)\cdot P(V)}{P(B)} = \frac{P(B|V)\cdot P(V)}{P(B|V)\cdot P(V) + P(B|V')\cdot P(V')}$$

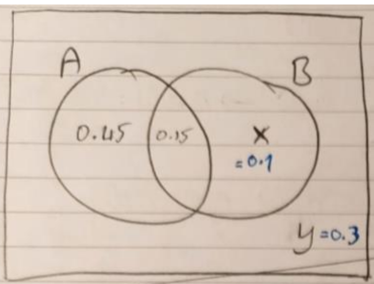
$$P(V|B) = \frac{0.9\times 0.01}{0.9\times 0.01 + 0.05\times 0.99} = \underline{0.153846}$$

$$P(V|B) > P(V|A)$$

Since P(V|B) > P(V|A), test B returning positive is more indicative.

6p. 2. Given two **independent** Boolean variables A and B, determine x and y in the joint distribution of $\mathbf{P}(A; B)$ shown below:

$$P(A = T; B = T) = 0{,}15$$
$$P(A = T; B = F) = 0{,}45$$
$$P(A = F; B = T) = x$$
$$P(A = F; B = F) = y$$



$P(A) \times P(B) = P(A \cap B)$

$P(A) = 0.45 + 0.15 = 0.6$

$P(B) = x + 0.15$

$P(A \cap B) = 0.15$

$0.6 \times (x + 0.15) = 0.15$

$x + 0.15 = \dfrac{0.15}{0.6} = \dfrac{1}{4} = 0.25$

$x = 0.25 - 0.15 = 0.1$

$y = 1 - (0.45 + 0.15 + 0.1) = 1 - 0.7 = 0.3$

$x = P(A = F; B = T) = 0.1$

$y = P(A = F; B = F) = 0.3$

3p 3. Prove the conditionalized version of the chain rule, e = fixed background evidence:

$$P(A, B|\, e) \;=\; P(A|B, e)\, P(B|e).$$

$$P(A,B|e) = \frac{P(A,B,e)}{P(e)}, \quad P(A|B,e) = \frac{P(A,B,e)}{P(B,e)}$$

$$P(B|e) = \frac{P(B,e)}{P(e)}$$

$$\frac{P(A,B,e)}{P(e)} \overset{?}{=} \frac{P(A,B,e)}{\cancel{P(B,e)}} \times \frac{\cancel{P(B,e)}}{P(e)}$$

$$\Rightarrow \frac{P(A,B,e)}{P(e)} = \frac{P(A,B,e)}{P(e)} \quad ✓$$

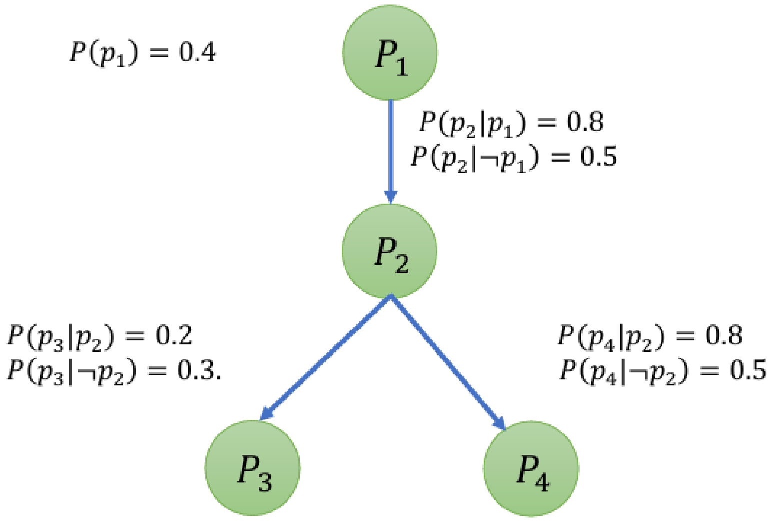
3p 4. Give a proof or a counterexample for the assertion: the independence of A and B implies the independence of $\neg A$ and B, **i.e.**

$$P(A|B) \;=\; P(A) \;\Rightarrow P(\neg A|B) \;=\; P(\neg A).$$

Assume A and B are independent → P(A|B) = P(A)

P($\neg A|B$) = 1− P(A|B) = 1− P(A)= P(¬A).

13p 5. Given the network in the figure below, calculate the following probabilities using *inference by enumeration*. **Tip**: similar to the example in class.

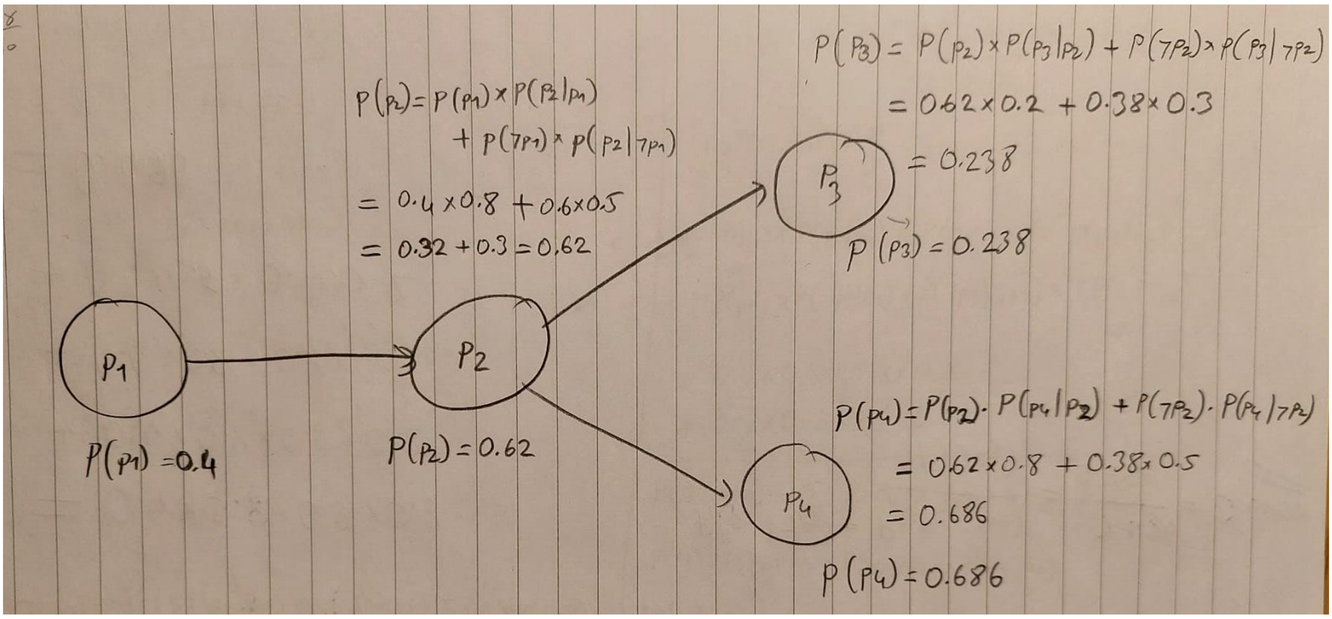


2p a. $P(\neg p_3)$

3p b. $P(p_2|\neg p_3)$

4p c. $P(p_1|p_2, \neg p_3)$

4p d. $P(p_1|\neg p_{3,} p_4)$



a. $P(\neg p3)$ = 1 - 0.238 = 0.762

b. $P(p2|\neg p3)$ **= P(p2,** $\neg p3$**)/** $P(\neg p3)$ **= 0.496/0.762 = 0.6509**

$$P(p_2, \neg p_3) = \sum_{p_1, p_4} P(p_1, p_2, \neg p_3, p_4)$$

$$= P(p_1) \times P(p_2|p_1) \times P(\neg p_3|p_2) \times P(p_4|p_2)$$
$$+ P(p_1) \times P(p_2|p_1) \times P(\neg p_3|p_2) \times P(\neg p_4|p_2)$$
$$+ P(\neg p_1) \times P(p_2|\neg p_1) \times P(\neg p_3|p_2) \times P(p_4|p_2)$$
$$+ P(\neg p_1) \times P(p_2|\neg p_1) \times P(\neg p_3|p_2) \times P(\neg p_4|p_2)$$

$$= 0.4 \times 0.8 \times 0.8 \times 0.8$$
$$+ 0.4 \times 0.8 \times 0.8 \times 0.2$$
$$+ 0.6 \times 0.5 \times 0.8 \times 0.8$$
$$+ 0.6 \times 0.5 \times 0.8 \times 0.2$$
$$= 0.496$$

$$P(p_2|\neg p_3) = \frac{P(p_2, \neg p_3)}{P(\neg p_3)} = \frac{0.496}{0.762} = 0.6509$$

c. $P(p1|p2, \neg p3)$ **= P(**$p1, p2, \neg p3$**) / P(**$p2, \neg p3$**) = 0.256 / 0.496 = 0.5161**

$$P(p_1|p_2, \neg p_3) = \frac{P(p_1, p_2, \neg p_3)}{P(p_2, \neg p_3)}$$

$$P(p_1, p_2, \neg p_3) = \sum_{p_4} P(p_1, p_2, \neg p_3, p_4)$$

$$= P(p_1) \times P(p_2|p_1) \times P(\neg p_3|p_2) \times P(p_4|p_2)$$
$$+ P(p_1) \times P(p_2|p_1) \times P(\neg p_3|p_2) \times P(\neg p_4|p_2)$$

$$= 0.4 \times 0.8 \times 0.8 \times 0.8$$
$$+ 0.4 \times 0.8 \times 0.8 \times 0.2$$
$$= 0.4 \times 0.8 \times 0.8 = 0.256$$

$$P(p_1|p_2, \neg p_3) = \frac{P(p_1, p_2, \neg p_3)}{P(p_2, \neg p_3)} = \frac{0.256}{0.496} = 0.5161$$

d. $P(p1|\neg p3\ ,p4)$ **=** $P(p1,\neg p3\ ,p4)$ **/** $P(\neg p3\ ,p4)$ **= 0.2328 / 0.5298 = 0.439**

$$P(p_1|\neg p_3,p_4) = \frac{P(p_1,\neg p_3,p_4)}{P(\neg p_3,p_4)}$$

$$P(p_1,\neg p_3,p_4) = \sum_{P_2} P(p_1,P_2,\neg p_3,p_4)$$

$$= P(p_1) \times P(p_2|p_1) \times P(\neg p_3|p_2) \times P(p_4|p_2)$$

$$+ P(p_1) \times P(\neg p_2|p_1) \times P(\neg p_3|\neg p_2) \times P(p_4|\neg p_2)$$

$$= 0.4 \times 0.8 \times 0.8 \times 0.8$$

$$+ 0.4 \times 0.2 \times 0.7 \times 0.5 = \underline{0.2328}$$

$$P(\neg p_3,p_4) = \sum_{P_1,P_2} P(P_1,P_2,\neg p_3,p_4)$$

$$= P(p_1) \times P(p_2|p_1) \times P(\neg p_3|p_2) \times P(p_4|p_2)$$

$$+ P(p_1) \times P(\neg p_2|p_1) \times P(\neg p_3|\neg p_2) \times P(p_4|\neg p_2)$$

$$+ P(\neg p_1) \times P(p_2|\neg p_1) \times P(\neg p_3|p_2) \times P(p_4|p_2)$$

$$+ P(\neg p_1) \times P(\neg p_2|\neg p_1) \times P(\neg p_3|\neg p_2) \times P(p_4|\neg p_2)$$

$$= 0.4 \times 0.8 \times 0.8 \times 0.8$$

$$+ 0.4 \times 0.2 \times 0.7 \times 0.5$$

$$+ 0.6 \times 0.5 \times 0.8 \times 0.8 = 0.5298$$

$$+ 0.6 \times 0.5 \times 0.7 \times 0.5$$

$$\rightarrow P(p_1|\neg p_3,p_4) = \frac{0.2328}{0.5298} = 0.439$$