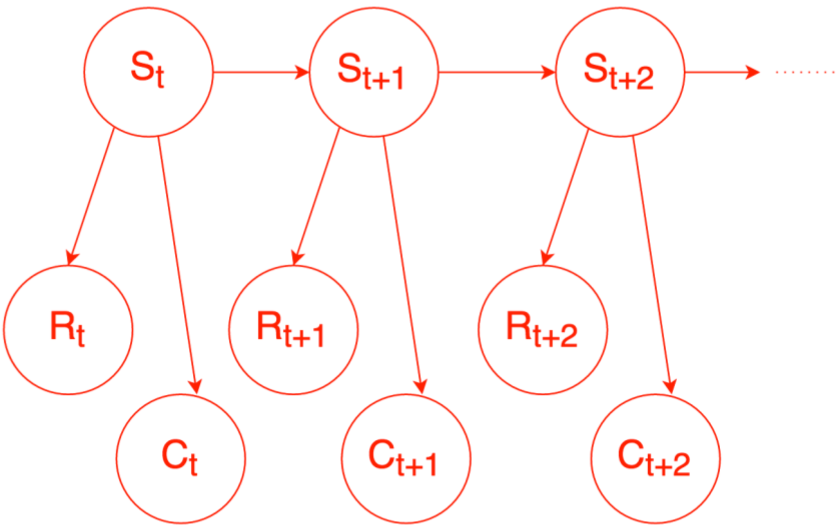
Artificial Intelligence
Exercise Set 5 - Solutions

20p 1. A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class, and whether they have red eyes. Let S_{t} be the random variable of the student having enough sleep, R_t be the random variable for the student having red eyes, and C_t be the random variable of the student sleeping in class on day t. The professor has the following domain theory:

- The prior probability of getting enough sleep at time t, with no observations, is 0.6.
- The probability of getting enough sleep on night t is 0.9 given that the student got enough sleep the previous night, and 0.2 if not.
- The probability of having red eyes is 0.1 if the student got enough sleep, and 0.7 if not.
- The probability of sleeping in class is 0.2 if the student got enough sleep, and 0.4 if not.



7p a. Give the complete probability tables for the model. Assume S_0 $as\ a\ prior$.

S_0	P(S_0)
s	0.6
$\neg s$	0.4

R_t	S_t	$P(R_t\|S_t)$
r	s	0.1
$\neg r$	s	0.9
r	$\neg s$	0.7
$\neg r$	$\neg s$	0.3

S_{t+1}	S_t	$P(S_{t+1}\|S_t)$
s_{t+1}	s_t	0.9
$\neg s_{t+1}$	s_t	0.1
s_{t+1}	$\neg s_t$	0.2
$\neg s_{t+1}$	$\neg s_t$	0.8

C_t	S_t	$P(C_t\|S_t)$
c	s	0.2
$\neg c$	s	0.8
c	$\neg s$	0.4
$\neg c$	$\neg s$	0.6

13p b. Given evidence:

- $e_1 = \neg r_1, \neg c_1 =$ not red eyes, not sleeping in class
- $e_2 = r_2, \neg c_2 =$ red eyes, not sleeping in class
- $e_3 = r_3, c_3$ = red eyes, sleeping in class,

perform the following computations:

(i) State estimation: compute P($S_t|e_{1:t}$) for each of t = 1; 2; 3. 7p

$P(S_1|e_1)$

$S_0 - prior$

Predict:

$P(s_1) = \sum_{s_0} P(s_1|s_0)P(s_0)$= $0.6 \times 0.9 + 0.4 \times 0.2 = 0.62.$

$P(\neg s_1) = 1 - P(s_1) = 0.38.$

Update:

$P(s_1|e_1) = \alpha P(e_1|s_1)\, P(s_1) = \alpha \times 0.9 \times 0.8 \times 0.62 = \alpha \times 0.4464.$

$P(\neg s_1|e_1) = \ \alpha P(e_1|\neg s_1)\, P(\neg s_1) = \alpha \times 0.3 \times 0.6 \times 0.38 =$

$= \ \alpha \times 0.0684.$

$$\alpha = \frac{1}{0.4464 + 0.0684} = 0.5148.$$

$P(s_1|e_1) = 0.867.$

$P(\neg s_1|e_1) = 0.133.$

$P(S_2|e_{1:2})$

Predict:

$$P(s_2|e_1) = \sum_{s_1} P(s_2|s_1)P(s_1|e_1) = 0.9 \times 0.867 + 0.2 \times 0.133 =$$
$$= 0.807.$$
$$P(\neg s_2|e_1) = 1 - P(s_2|e_1) = 0.193.$$

Update:

$$P(s_2|e_{1:2}) = \alpha P(e_2|s_2)\, P(s_2|e_1) = \alpha \times (0.1 \times 0.8 \ \times 0.807) =$$
$$= \alpha \times 0.06456.$$
$$P(\neg s_2|e_{1:2}) = \alpha P(e_2|\neg s_2)\, P(\neg s_2|e_1) = \alpha \times (0.7 \times 0.6 \times 0.193) =$$
$$= \alpha \times 0.08156.$$
$$\alpha = \frac{1}{0.14612} = 6.8436.$$
$$P(s_2|e_{1:2}) = 0.442.$$
$$P(\neg s_2|e_{1:2}) = 0.558.$$

$P(S_3|e_{1:3})$

Predict:

$$P(s_3|e_{1:2}) = \sum_{s_2} P(s_3|s_2)P(s_2|e_{1:2}) = 0.9 \times 0.442 + 0.2 \times 0.558 =$$
$$= 0.5094.$$

$$P(\neg s_3|e_{1:2}) = 1 - P(s_3|e_{1:2}) = 0.4906.$$

Update:

$$P(s_3|e_{1:3}) = \alpha P(e_3|s_3)\, P(s_3|e_{1:2}) = \alpha \times (0.1 \times 0.2 \times 0.5094) =$$

$$= \alpha \times 0.0102.$$
$$P(\neg s_3|e_{1:3}) = \ \alpha P(e_3|\neg s_3)\, P(\neg s_3|e_{1:2}) = \alpha \times (0.7 \times 0.4 \times 0.4906)$$
$$= \ \alpha \times 0.1374.$$
$$P(s_3|e_{1:3}) = 0.069.$$
$$P(\neg s_3|e_{1:3}) = 0.931\ .$$

(ii) Smoothing: compute $P(S_t|e_{1:3})$ for each t = 1; 2; 3. 6p

$P(S_3|e_{1:3})$: same as before.

$$P(s_3|e_{1:3}) = 0.069.$$
$$P(\neg s_3|e_{1:3}) = 0.931\ .$$

$P(S_2|e_{1:3})$

$$P(s_2|e_{1:3}) = \alpha P(s_2|e_{1:2})\, P(e_3|s_2)$$
$$P(e_3|s_2) = \sum_{s_3} P(e_3|s_3)P(s_3|s_2) = 0.02 \times 0.9 + 0.28 \times 0.1 = 0.046.$$
$$P(e_3|\neg s_2) = 0.228.$$
$$P(s_2|e_{1:3}) = \alpha P(s_2|e_{1:2})\, P(e_3|s_2) = \alpha \times 0.442 \times 0.046 =$$
$$= \alpha \times 0.0203.$$
$$P(\neg s_2|e_{1:3}) = \alpha \times 0.558 \times 0.228 = \alpha \times 0.127.$$
$$\alpha = 6.7888.$$
$$P(s_2|e_{1:3}) = 0.138.$$
$$P(\neg s_2|e_{1:3}) = 0.862$$

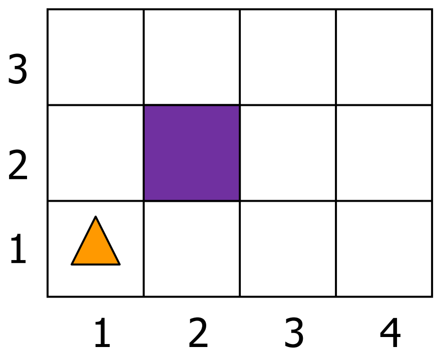
$P(S_1|e_{1:3})$

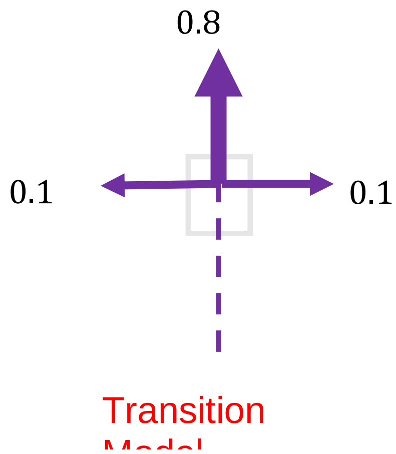
$P(s_1|e_{1:3}) = \alpha P(s_1|e_1)\, P(e_{2:3}|s_1)$

$P(s_1|e_1) = 0.867$. from (i)

$$P(e_{2:3}|s_1) = \sum_{s_2} P(e_2|s_2)P(e_3|s_2)\, P(s_2|s_1) = \cdots.$$

10p 2. For the 4×3 world shown in the Figure above, calculate which squares can be reached from (1,1) by the action sequence [Up, Up] and with what probabilities, by filling in the Table on the next page. Write down the calculations, not only the values.





	Up	Up
(1,1)	0.1	0.02
(1,2)	0.8	0.24
(1,3)		0.64
(2,1)	0.1	0.09
(2,3)		
(3,1)		0.01
(3,2)		
(3,3)		
(4,1)		
(4,2)		
(4,3)		

Example calculations:

- **P**([1,1] | (U,U).[1,1]) =
  P([1,1] | U.[1,2]) x **P**([1,2] | U.[1,1])
  \+ **P**([1,1] | U.[2,1]) x **P**([2,1] | U.[1,1])
  \+ **P**([1,1] | U.[1,1]) x **P**([1,1] | U.[1,1]) = 0 + 0.1*0.1+0.1*0.1=0.02.

- **P**([1,2] | (U,U).[1,1]) =
  P([1,2] | U.[1,1]) x **P**([1,1] | U.[1,1])
  \+ **P**([1,2] | U.[1,2]) x **P**([1,2] | U.[1,1])
  \+ **P**([1,2] | U.[1,3]) x **P**([1,3] | U.[1,1]) = 0.8*0.1 + 0.2*0.8 + 0 = 0.24.

- **P**([1,3] | (U,U).[1,1]) =
  P([1,3] | U.[2,3]) x **P**([2,3] | U.[1,1])
  \+ **P**([1,3] | U.[1,2]) x **P**([1,2] | U.[1,1])
  \+ **P**([1,3] | U.[1,3]) x **P**([1,3] | U.[1,1]) = 0 + 0.8*0.8 + 0 = 0.64.

The rest of the values are computed in a similar manner.