Artificial Intelligence Assignment 5

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1.

a. Give the complete probability tables for the model. Assume $S\ as\ a\ prior$.

S_0	$P(S_0)$
s	0.6
$\neg s$	0.4

R_t	S_t	$P(R_t \mid S_t)$
r	s	0.1
$\neg r$	s	0.9
r	$\neg s$	0.7
$\neg r$	$\neg s$	0.3

S_{t+1}	S_t	$P(S_{t+1} \mid S_t)$
s_{t+1}	s_t	0.9
$\neg s_{t+1}$	s_t	0.1
s_{t+1}	$\neg s_t$	0.2
$\neg s_{t+1}$	$\neg s_t$	0.8

C_t	S_t	$P(C_t \mid S_t)$
c	s	0.2
$\neg c$	s	0.8
c	$\neg s$	0.4
$\neg c$	$\neg s$	0.6

b. Given evidence:

- e =$\neg r1$,¬c1 = not red eyes, not sleeping in class
- e =$r2$,¬c2 = red eyes, not sleeping in class
- e =$r3$,c3 = red eyes, sleeping in class, perform the following computations:

(i) State estimation: compute P($St|e$ 1:t) for each of t = 1; 2; 3.

t = 1 → P(S_1 | $e_{1:1}$) = <0.867, 0.132>

i) $t=1 \rightarrow P(S_1 \mid e_1) = P(S_1 \mid \neg e_1, \neg e_1)$

$P(s_1) = P(s_0) \times P(s_1 \mid s_0) + P(\neg s_0) \cdot P(s_1 \mid \neg s_0)$

$P(s_1) = 0.6 \times 0.9 + 0.4 \times 0.2 = 0.62$

$P(\neg s_1) = 1 - 0.62 = 0.38$

$P(s_1 \mid e_1) = \alpha \cdot P(e_1 \mid s_1) \times P(s_1) = \alpha \cdot 0.9 \times 0.8 \times 0.62 = \alpha \cdot 0.4464$

$P(\neg s_1 \mid e_1) = \alpha \cdot P(e_1 \mid \neg s_1) \times P(\neg s_1) = \alpha \cdot 0.6 \times 0.3 \times 0.38 = \alpha \cdot 0.0684$

$P(s_1 \mid e_1) + P(\neg s_1 \mid e_1) = 1$

$\alpha \cdot (0.4464 + 0.0684) = 1$

$\alpha = \dfrac{1}{0.4464 + 0.0684} = \dfrac{1}{0.5148}$

$P(s_1 \mid e_1) = 0.4464 / 0.5148 = 0.867$

$P(\neg s_1 \mid e_1) = 0.0684 / 0.5148 = 0.132$

t = 2 → P(S_2 | $e_{1:2}$) = <0.4432, 0.5568>

$t=2 \rightarrow P(S_2 \mid e_{1:2})$

$P(S_2 \mid e_1) = \sum_{s_1} P(s_2 \mid s_1) \times P(s_1 \mid e_1) = 0.9 \times 0.867 + 0.2 \times 0.133 = 0.8069$

$P(\neg s_2 \mid e_1) = 0.1931$

$P(s_2 \mid e_{1:2}) = \alpha \cdot P(e_2 \mid s_2) \cdot P(s_2 \mid e_1) = \alpha \times 0.1 \times 0.8 \times 0.8069 = \alpha \cdot 0.06456$

$P(\neg s_2 \mid e_{1:2}) = \alpha \cdot P(e_2 \mid \neg s_2) \cdot P(\neg s_2 \mid e_1) = \alpha \times 0.7 \times 0.6 \times 0.1931 = \alpha \cdot 0.081102$

$\alpha \cdot (0.06456 + 0.081102) = 1 \rightarrow \alpha = 1 / 0.145662$

$P(s_2 \mid e_{1:2}) = \dfrac{0.06456}{0.145662} = 0.4432 \quad P(\neg s_2 \mid e_{1:2}) = \dfrac{0.081102}{0.145662} = 0.5568$

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t = 3 → $P(S_3 \mid e_{1:3})$ = <0.0692, 0.9308>

$t=3 \rightarrow P(S_3 \mid e_{1:3})$

$$P(S_3 \mid e_{1:2}) = \sum_{s_2} P(s_3 \mid s_2) \cdot P(s_2 \mid e_{1:2}) = 0.9 \times 0.4432 + 0.2 \times 0.5568$$
$$= 0.51024$$

$$P(\neg S_3 \mid e_{1:2}) = \sum_{S_2} P(\neg S_3 \mid s_2) \cdot P(S_2 \mid e_{1:2}) = 0.1 \times 0.4432 + 0.8 \times 0.5568$$
$$= 0.48976$$

$$P(S_3 \mid e_{1:3}) = \alpha \cdot P(e_3 \mid s_3) \cdot P(S_3 \mid e_{1:2}) = \alpha \times 0.1 \times 0.2 \times 0.51024 = \alpha \times 0.0102$$

$$P(\neg S_3 \mid e_{1:3}) = \alpha \cdot P(e_3 \mid \neg S_3) \cdot P(\neg S_3 \mid e_{1:2}) = \alpha \times 0.7 \times 0.4 \times 0.48976 = \alpha \times 0.1371$$

$$\alpha \cdot (0.0102 + 0.1371) = 1 \rightarrow \alpha = \frac{1}{0.1473}$$

$$P(S_3 \mid e_{1:3}) = \frac{0.0102}{0.1473} = 0.0692$$

$$P(\neg S_3 \mid e_{1:3}) = \frac{0.1371}{0.1473} = 0.9308$$

(ii) Smoothing: compute P(*S*t|*e* 1:3), for each t = 1; 2; 3.

t = 3 → $P(S_3 \mid e_{1:3})$ = <0.0692, 0.9308>

t = 2 → $P(S_2 \mid e_{1:3})$ = <0.138, 0.862>

$t=2 \rightarrow P(S_2 \mid e_{1:3}) = \alpha \times P(s_2 \mid e_{1:2}) \times P(e_3 \mid s_2)$

$$P(e_3 \mid s_2) = \sum_{S_3} P(e_3 \mid S_3) \times P(S_3 \mid s_2) = 0.02 \times 0.9 + 0.28 \times 0.1 = 0.046$$

$$P(e_3 \mid \neg S_2) = \sum_{S_3} P(e_3 \mid S_3) \times P(S_3 \mid \neg S_2) = 0.02 \times 0.2 + 0.28 \times 0.8 = 0.228$$

$$P(s_2 \mid e_{1:3}) = \alpha \times P(s_2 \mid e_{1:2}) \times P(e_3 \mid s_2) = \alpha \times 0.4432 \times 0.046 = \alpha \times 6.0204$$

$$P(\neg s_2 \mid e_{1:3}) = \alpha \times P(\neg s_2 \mid e_{1:2}) \times P(e_3 \mid \neg s_2) = \alpha \times 0.5568 \times 0.228 = 0.1269$$

$$\alpha \cdot (0.0204 + 0.1269) = 1 \longrightarrow \alpha = 1/0.1473$$

$$P(s_2 \mid e_{1:3}) = \frac{0.0204}{0.1473} = 0.138 \quad , \quad P(\neg s_2 \mid e_{1:3}) = \frac{0.1269}{0.1473} = 0.862$$

$t = 1 \rightarrow P(S_1 \mid e_{1:3}) = <0.512, 0.488>$

$t=1 \rightarrow P(S_1 \mid e_{1:3})$

$P(s_1 \mid e_{1:3}) = \alpha \times P(s_1 \mid e_1) \times P(e_{2:3} \mid s_1)$

$P(s_1 \mid e_1) = 0.867 \quad , P(\neg s_1 \mid e_1) = 0.133$

$P(e_{2:3} \mid s_1) = \sum_{s_2} P(e_2 \mid s_2) \times P(e_3 \mid s_2) \times P(s_2 \mid s_1)$

$P(e_{2:3} \mid s_1) = 0.1 \times 0.8 \times 0.046 \times 0.9 + 0.7 \times 0.6 \times 0.228 \times 0.1 = 0.012888$

$P(e_{2:3} \mid \neg s_1) = 0.1 \times 0.8 \times 0.046 \times 0.2 + 0.7 \times 0.6 \times 0.228 \times 0.8 = 0.079994$

$P(s_1 \mid e_{1:3}) = \alpha \times 0.867 \times 0.012888, \quad P(\neg s_1 \mid e_{1:3}) = \alpha \times 0.133 \times 0.079994$

$\alpha \cdot (0.867 \times 0.012888 + 0.133 \times 0.079994) = 1 \rightarrow \alpha = 1/0.0218$

$P(s_1 \mid e_{1:3}) = 0.512564 \quad , \quad P(\neg s_1 \mid e_{1:3}) = 0.488037$

2. For the 4×3 world shown in the figure below, calculate which squares can be reached from (1,1) by the action sequence [Up, Up] and with what probabilities, by filling in the table on the next page.

	Up	Up
(1,1)	0.1	0.02
(1,2)	0.8	0.24
(1,3)	0	0.64
(2,1)	0.1	0.09
(2,3)	0	0
(3,1)	0	0.01
(3,2)	0	0
(3,3)	0	0
(4,1)	0	0
(4,2)	0	0
(4,3)	0	0

	1st Up	2nd Up
(1,1)	0.1	$0.1 \times 0.1 + 0.1 \times 0.1 = 0.02$
(1,2)	0.8	$0.1 \times 0.8 + 0.8 \times 0.2 = 0.24$
(1,3)	0	$0.8 \times 0.8 = 0.64$
(2,1)	0.1	$0.1 \times 0.1 + 0.1 \times 0.8 = 0.09$
(2,3)	0	0
(3,1)	0	$0.1 \times 0.1 = 0.01$
(3,2)	0	0
(3,3)	0	0
(4,1)	0	0
(4,2)	0	0
(4,3)	0	0