#### **HOMEWORK 1**

#### 1. The Chess Game

#### Question 1.1

Since Gates plays *bold*, he will either *win* or *lose* each game. Therefore, at the end of first 3 games, one of the players will win strictly more games than the other player. A player wins the match if he wins at least 2 games.

P(G wins the match) = P(G wins 2 games out of 3) + P(G wins 3 games out of 3)

$$= \binom{3}{2} * P(G \text{ wins a game})^2 * P(G \text{ loses a game})^1 + \binom{3}{3} * P(G \text{ wins a game})^3$$
$$= 3 * 0.6^2 * 0.4^1 + 1 * 0.6^3 = 0.648$$

Note: G stands for Gates, S stands for Steve.

### Question 1.2

Since Gates plays *timid*, he will either lose or draw each game. Therefore, during the first 3 games, he should NOT lose any game and return *bold* to win the 4<sup>th</sup> game, and the match.

$$P(G \text{ wins the match}) = P(G \text{ draws first 3 games}) * P(G \text{ wins the 4}^{th} \text{ game})$$
  
=  $P(G \text{ draws a game} \mid \text{timid})^3 * P(G \text{ wins a game} \mid \text{bold}) = 0.65^3 * 0.6 = 0.164775$ 

### Question 1.3

Since G chooses styles randomly, P(style = bold) = P(style = timid) = 1/2.

$$P(style = bold \mid game = lose) = \frac{P(style = bold \& game = lose)}{P(game = lose)}$$

$$P(style = bold \& game = lose)$$

 $= \frac{1}{P(game = lose \mid style = bold) * P(style = bold) + P(game = lose \mid style = timid) * P(style = timid)}$ 

$$= \frac{\frac{1}{2} * 0.4}{\frac{1}{2} * 0.4 + \frac{1}{2} * 0.35} = \frac{8}{15} = 0.5\overline{3}$$

## Question 1.4

Steve may guess Gates's strategy correctly or not. Therefore, both cases should be considered.

$$P(S \text{ wins a game}) = P(S = \text{win \& prediction} = +) + P(S = \text{win \& prediction} = -)$$

$$= P(S = \text{win} \mid p = +) * P(p = +) + (1 - P(p = +)) * (P(\text{style} = \text{timid \& S} = \text{win}) + P(\text{style} = \text{bold \& S} = \text{win}))$$

$$= 0.85 * 0.75 + 0.25 * (\frac{1}{2} * 0.35 + \frac{1}{2} * 0.4) = 0.73125$$

### 2. Medical Diagnosis

### Question 2.1

$$P(S = disease) = \frac{5}{1000} = \frac{1}{200} \text{ (already given)}$$

$$P(S = healthy) = 1 - P(S = disease) = 1 - \frac{1}{200} = \frac{199}{200}$$

$$P(T = + | S = disease) = \frac{97}{100} \text{ (already given)}$$

$$P(T = - | S = disease) = 1 - P(T = + | S = disease) = 1 - \frac{97}{100} = \frac{3}{100}$$

$$P(T = + | S = healthy) = 1 - P(T = - | S = healthy) = 1 - \frac{98}{100} = \frac{2}{100}$$

$$P(T = - | S = healthy) = \frac{98}{100} \text{ (already given)}$$

# Question 2.2

$$P(S = disease \mid T = +) = \frac{P(S = disease \& T = +)}{P(T = +)}$$

$$= \frac{P(S = disease) * P(T = + \mid S = disease)}{P(S = disease) * P(T = + \mid S = disease)}$$

$$= \frac{1}{200} * \frac{97}{100} / (\frac{1}{200} * \frac{97}{100} + \frac{199}{200} * \frac{2}{100}) = \frac{97}{97 + 398} = \frac{97}{495} = 0.1\overline{95} \approx 0.2$$

Given that the prediction is positive, the probability of the disease is close to 20% is smaller than 50%. Therefore, patient should be considered as healthy.

#### 3. MLE and MAP

#### Question 3.1

Given  $\lambda$ , the probability of  $X = \{x_1, x_2, x_3, ..., x_n\}$  is given below.

$$P(X_1 = x_1, ..., X_n = x_n \mid \lambda) = \prod_{i=1}^n \frac{\lambda^{x_i} * e^{-\lambda}}{x_i!}$$

Taking logarithm of it, we get the summation below.

$$\sum_{i=1}^{n} -\lambda + x_i * \log(\lambda) - \log(x_i!)$$

Taking derivative of it with respect to  $\lambda$ , we get the MLE of  $\lambda$ .

$$-n + \sum_{i=1}^{n} \frac{x_i}{\lambda} = 0$$

$$\hat{\lambda} = \frac{\sum_{i=1}^{n} x_i}{n}$$

This means that  $\hat{\lambda} = \arg \max P(X|\lambda) = Mean \ of \ x_i s$ .

#### Question 3.2

$$\hat{\lambda} = \arg \max \frac{P(X \mid \lambda) * P(\lambda)}{P(X)}$$

Taking logarithm of it, we get the expression below.

$$(\alpha * \log(\beta) + (\alpha - 1) * \log(\lambda) - \beta * \lambda - \log \Gamma(\alpha)) - \log(P(X)) \sum_{i=1}^{n} -\lambda + x_i * \log(\lambda) - \log(x_i!)$$

Taking derivative of it with respect to  $\lambda$ , we get the MAP of  $\lambda$ .

$$\frac{\alpha - 1}{\lambda} - \beta - n + \frac{\sum_{i=1}^{n} x_i}{\lambda} = 0$$

$$\hat{\lambda} = \frac{(\alpha - 1) + \sum_{i=1}^{n} x_i}{\beta + n}$$

#### Question 3.3

$$\hat{\lambda}_{MAP} = \arg\max \frac{P(X \mid \lambda) * P(\lambda)}{P(X)} = \arg\max P(X \mid \lambda) * P(\lambda) = \arg\max P(X \mid \lambda) = \hat{\lambda}_{MLE}$$

The reason behind the first reduction is that X is given. The reason behind the second reduction is that  $P(\lambda)$  is constant since  $\lambda \sim U(a,b)$ . Therefore, MLE estimate of  $\lambda$  and MAP estimate of  $\lambda$  with the uniform prior is the same for any (a, b).

#### 4. Sentiment Analysis on Tweets

### Question 4.1

For each tweet, we aim to find a class that maximizes the given formula. For each class, the denominator stays the same. In other words, the denominator has the same effect on each candidate class. Hence, we can ignore it and just look for the numerator.

### Question 4.2

In the training dataset,  $N_{neutral}=2617$ ,  $N_{positive}=2004$  and  $N_{negative}=7091$  out of N=11712 tweets.  $\pi_{neutral}\approx 0.223$ ,  $\pi_{positive}\approx 0.171$  and  $\pi_{negative}\approx 0.605$ . As it is seen, the training set is skewed towards the negative class.

Considering Eq. (4.6), density of negative class does NOT affect summation part since denominator of theta values deals with this. Also,  $t_{wj,i}$  is related to length of a tweet, not a class. However, left part of the formula changes according to the  $\pi$  value which denotes density of a class. Since the negative class dominates the training data, and therefore the formula, it will be harder to detect positive and negative classes. Also, suppose that another class dominates the test data. In that case, *false positive* (*FP*) for negative class would be high.

Many solutions can be proposed to this problem. However, guessing which one is the best is a little bit difficult. Effect of  $\pi$  can be removed, a new algorithm and formula can be generated, new training data can be added, training data can be separated to different classes and same number of tweets can be chosen for each class

#### Question 4.3

Simply, we need  $n_{class} = 3$  for  $\pi$  and  $n_{class} * n_{word} = 3 * 5722 = 17166$  for  $\theta$ . However, if I assume that I know  $n_{tweet} = 11712$ , then I can predict P(negative) since  $\pi_i$  values sum up to 1.

$$P(negative) = 1 - (P(positive) + P(neutral))$$

Therefore, 17166 + 3 - 1 = 17168 parameters are needed.

### Question 4.4

Out of 2928 tests, #  $correct\ prediction = 1839\ and\ #\ wrong\ prediction = 1089$ .

Also,  $accuracy \approx 0.628$ . For each pair of predicted and actual class, number of tests can be seen below.

Prediction / Actual	A = neutral	A = positive	A = negative
P = neutral	207	107	500
P = positive	18	80	35
P = negative	257	172	1552

Table 1. Multinomial Naïve Bayes classifier with MLE estimation

Considering Eq. (4.6), in the case of  $\alpha^*log(0)$ , value in the equation becomes -inf. If a tweet encounters this issue for all classes, then it will be harder to compare and take maximum among them. As a solution, I made slight changes in the method and counted -inf for each candidate class to predict the class of a given specific tweet. I prioritized the number of -inf and take the minimum among candidate classes. Then, I observed # correct prediction' = 2138 and accuracy'  $\approx 0.73$ .

It seems that MLE estimation has a terrible accuracy. If the label *negative* was chosen, we would have #  $correct\ prediction'' = 2087$  and  $accuracy'' \approx 0.713 > 0.628$ . However, this is only one of the metrics used to understand how good the estimation is. For instance, for positive and neutral tweets, precision and recall would be zero since there will be no *true positive* (*TP*) for positive and neutral tweets.

# Question 4.5

Out of 2928 tests, # correct prediction = 2205 and # wrong prediction = 723.

Also,  $accuracy \approx 0.753$ . For each pair of predicted and actual class, number of tests can be seen below.

Prediction / Actual	A = neutral	A = positive	A = negative
P = neutral	169	46	131
P = positive	20	123	43
P = negative	293	190	1913

Table 2. Multinomial Naïve Bayes classifier with MAP estimation

Since a value  $\alpha > 1$  is added to the maximization problem, there will be no issue originating from *-inf*. It will be impossible for a  $\theta$  value to be 0. Therefore, the predictions will be better. As a sign of it,

increase in accuracy by 13% can be given. Compared to Multinomial Naïve Bayes classifier with MLE estimation, *true positive (TP)* of positive and negative class increases where it decreases for neutral class. In my opinion, the reason behind this is that in case of ties, the predicted class was neutral for MLE estimation and we don't face with this condition when using MAP estimation. Therefore, this leads to less amount of correctly predicted neutral tweets.

#### Question 4.6

Out of 2928 tests, #  $correct\ prediction = 1878\ and\ #\ wrong\ prediction = 1050$ .

Also,  $accuracy \approx 0.641$ . For each pair of predicted and actual class, number of tests can be seen below.

Prediction / Actual	A = neutral	A = positive	A = negative
P = neutral	267	160	515
P = positive	25	85	46
P = negative	190	114	1526

Table 3. Bernoulli Naïve Bayes classifier with MLE estimation

A similar situation to the Q4.4 can be seen here:  $accuracy' \approx 0.713 > 0.641$ . Also, it can be claimed that Bernoulli Naïve Bayes is better than Multinomial Naïve Bayes used before. It seems that whatever approach we use to learn, as long as MLE estimation is used, there is no a massive change in the accuracy. I made the similar change what I did at Multinomial Naïve Bayes with MLE estimation, counted number of -inf. Then, I observed #  $correct\ prediction'' = 2175$  and  $accuracy'' \approx 0.743$ .

Compared to Multinomial Naïve Bayes with MLE estimation, the number of (correctly) predicted neutral and (correctly) predicted positive tweets increase whereas (correctly) predicted negative tweets decrease. The accuracy increases by 1.2% which means instead of counting the same word multiple times, checking the existence of it is more useful.

### Question 4.7

For each class, words exist in the vocabulary list are listed and sorted by frequency in non-increasing order.

**Neutral:** ['@jetblue', '@united', '@southwestair', 'flight', '@usairways', '@virginamerica', 'flights', 'help', 'fleek', "fleet's", 'dm', 'tomorrow', 'time', 'flying', 'cancelled', 'fly', 'change', 'travel', 'today', 'check']

**Positive:** ['@southwestair', '@jetblue', '@united', 'flight', '@usairways', 'great', '@virginamerica', 'service', 'love', 'best', 'guys', 'customer', 'time', 'awesome', 'airline', 'help', 'amazing', 'today', 'fly', 'flying']

**Negative:** ['@united', 'flight', '@usairways', '@southwestair', '@jetblue', 'cancelled', 'service', 'hours', 'hold', 'time', 'customer', 'help', 'delayed', 'plane', 'hour', 'flights', 'bag', 'gate', 'late', 'flightled']

Given lists have many words in common; including twitter usernames with @s, some simple words related to the airlines in U.S like *flight*, and words with suffixes like *flying*. From my perspective, the words that don't exist in the actual English vocabulary, common technical words related to the domain and different words with the same origin make harder to understand what label is related with what words.

Among the most commonly used 20 words in positive tweets, only 5 of them (*great, love, best, awesome* and *amazing*) are related to positivity. Among the most commonly used 20 words in negative tweets, only 3 of them (*cancelled, delayed* and *late*) are related to negativity. Having a few keywords is a sign of how bad this approach and vocabulary are. Among the most commonly used 20 words in neutral tweets, 2 of them (*fleek* and *cancelled*) are related to other classes and it seems good actually.

As a solution, the words existing in the English vocabulary can be considered, and among them, the ones with low IDF (Inverse Document Frequency) can be removed. In that way, the words occurring too much in any case will NOT occupy a good candidate word. However, this may affect prediction of neutral tweets in a bad way because the words like *flight* would be eliminated. Therefore, it would be better not to use this method for prediction of neutral tweets.

### 5. Code

### **5.1 main.py** (Main source code to run)

```
from helper import get train theta t map
from helper import get train s
from helper import get train theta s
from tester import multinomial naive bayes
train theta t = get train theta t(train t, train t total)
```

```
train theta t map = get train theta t map(train t, train t total)
train_n_positive, train_n_negative)
print('train_theta_s')
test naive bayes = multinomial naive bayes(train pis, test features matrix,
find accuracy(test naive bayes, test labels)
test naive bayes map = multinomial naive bayes(train pis,
print(test naive bayes map)
find accuracy (test naive bayes map, test labels)
test bernoulli naive bayes = bernoulli naive bayes(train pis,
find accuracy(test bernoulli naive bayes, test labels)
vocab = read vocab file()
print('vocab')
all freq words = get frequent word ids(train t, vocab)
print('all freq word ids')
print('Neutral:', all_freq_words[0])
print('Positive:', all_freq_words[1])
print('Negative:', all freq words[2])
```

**5.2** helper.py (Deals with calculation of parameters related to training)

```
import os
import pickle

def get_label_counter(train_labels):
    pickle_file_name = 'train_n_label.pkl'
    if not os.path.isfile(pickle_file_name):
        train_n_neutral = 0
        train_n_positive = 0
        train_n_negative = 0
        for i in range(len(train_labels)):
            if train_labels[i] == 'neutral':
                train_n_neutral += 1
        elif train_labels[i] == 'positive':
                train_n_positive += 1
        elif train_labels[i] == 'negative':
                train_n_negative += 1
        pickle.dump([train_n_neutral, train_n_positive, train_n_negative],
```

```
open(pickle file name, "wb"))
pickle.load(pickle file)
pickle.load(pickle file)
def get_train_theta_t(train_t, train_t_total):
```

```
pickle.dump(train theta t, open(pickle file name, "wb"))
       with open (pickle file name, 'rb') as pickle file:
def get_train_theta_t_map(train_t, train_t_total, alpha=1):
           train theta t map.append([train theta t map neutral,
   pickle file name = 'train s.pkl'
   if not os.path.isfile(pickle file name):
           train s.append([0, 0, 0])
def get train theta s(train s, train n neutral, train n positive,
```

```
pickle_file_name = 'train_theta_s.pkl'
if not os.path.isfile(pickle_file_name):
    train_theta_s = []
    n_feature = len(train_s)
    for feature in range(n_feature):
        train_theta_s_neutral = float(train_s[feature][0]) /
train_n_neutral
        train_theta_s_positive = float(train_s[feature][1]) /
train_n_positive
        train_theta_s_negative = float(train_s[feature][2]) /
train_n_negative
        train_theta_s.append([train_theta_s_neutral,
train_theta_s_positive, train_theta_s_negative])
        pickle.dump(train_theta_s, open(pickle_file_name, "wb"))
else:
    with open(pickle_file_name, 'rb') as pickle_file:
        train_theta_s = pickle.load(pickle_file)
return train_theta_s
```

**5.3 tester.py** (Tests the model, calculates accuracy, the functions counting *-inf* are commented out)

```
def multinomial naive bayes (train pis, test features matrix,
            test_result.append('neutral')
            test result.append('positive')
            test result.append('negative')
```

```
naive_bayes(train_pis, test_features_matrix, train_theta_t):
n_tweet = len(test_features_matrix)
n_feature = len(test_features_matrix[0])
test_result = []
for tweet in range(n_tweet):
    score = [0, 0, 0]
    minf = [0, 0, 0]
    for k in range(3):
def bernoulli naive bayes(train pis, test features matrix, train theta s):
```

```
if best label == -1 or score[k] > best score:
if math.fabs(score[1] - score[2]) < 0.000001:</pre>
   test result.append('neutral')
    test_result.append('negative')
```

```
return 1
elif label == 'negative':
    return 2
return -1

def find_accuracy(test_result, test_labels):
    correct = 0
    failure = 0
    counter = [[0, 0, 0], [0, 0, 0], [0, 0, 0]]
    n_tweet = len(test_result)
    for tweet in range(n_tweet):
        prediction = get_class_id(test_result[tweet])
        actual = get_class_id(test_labels[tweet])
        counter[prediction][actual] += 1
        if test_result[tweet] == test_labels[tweet]:
            correct += 1
        else:
            failure += 1
        print('correct: ', correct)
        print('failure: ', failure)
        print('accuracy: ', float(correct) / (correct + failure))
        print('counter: ', counter)
```

## **5.4 read\_data.py** (Reads 4 csv files and gets the input data)

```
return [train_labels, train_features_matrix, test_labels,
test_features_matrix]
```

# **5.5 observe\_common.py** (Separated to Question 4.7)